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Abstract: This paper relates predictable gains from positions in fed funds futures contracts to violations of the expectations hypothesis of the term structure of interest rates. Although evidence for predictable gains from positions in short-horizon contracts is mixed, we find that gains in longer-horizon contracts can be well described using Markov switching models, with predictability associated with particular episodes in which economic activity was weak and variability in the returns to these contracts was quite high.

JEL classification: E40, E50, G13

Key words: federal funds, futures, monetary policy, Markov switching

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1 Introduction

The strong formulation of the expectations hypothesis of the term structure of interest rates conjectures that the expected holding yields from long-term and short-term bonds should be the same, while the weak form of the expectations hypothesis allows the holding yields to differ by a constant term premium. These are approximately equivalent¹ to the claims that the long rate should equal the average of expected future short rates under the strong form, or an average plus a constant term premium under the weak form. Although these are a priori appealing hypotheses, even the weak forms have been consistently rejected by empirical researchers; see Campbell and Shiller (1991), Evans and Lewis (1994), Bekaert, Hodrick, and Marshall (1997), and Cochrane and Piazzesi (2005), among many others. Notwithstanding, Rudebusch (1995) and Longstaff (2000) suggested that the expectations hypothesis may not be a bad approximation at the shortest end of the term structure.

Another well-established empirical finding is that short-term interest rates can be usefully described using Markov regime-switching models such as proposed by Hamilton (1989); see for example Hamilton (1988), Garcia and Perron (1996), Gray (1996), Ang and Bekaert (2002), Sims and Zha (2006), and Inoue and Okimoto (2008). A number of researchers have reported considerable improvements in models of the term structure of interest rates by incorporating these regime changes, including Bansal and Zhou (2002), Evans (2003), Dai, Singleton, and Yang (2007), and Ang, Bekaert, and Wei (2008).

A separate literature has examined whether expected gains from long positions in fed funds futures contracts are zero, or equivalently, whether these contracts incorporate an optimal forecast of future fed funds rates. Previous studies include Krueger and Kuttner (1996), Rudebusch (1998), Kuttner (2001), Sack (2004), Gürkaynak, Sack, and Swanson (2007), Piazzesi and Swanson (2008), and Hamilton (2009a). While these studies all support the view that fed funds futures provide an excellent forecast of fed funds rates, they have often also found some evidence of predictable gains, particularly for fed funds futures contracts at longer maturities.

The purpose of the present paper is to investigate the extent to which changes in regime can account for these predictable gains from positions in fed funds futures contracts. A

¹See Campbell, Lo, and MacKinlay (1997, Section 10.2).

companion paper by Bundick (2007) has also fit regime-switching models to fed funds futures. Bundick’s analysis begins in 1995, whereas some of the most interesting features we highlight occurred in the early 1990s. Bundick studied the n -month gains from holding these contracts to maturity, whereas we use 1-month holding gains throughout, which simplifies the implied dynamic structure under both null and alternative hypotheses. And whereas Bundick’s primary focus was on constructing an optimal forecast of the future fed funds rate, here our emphasis is on characterizing the nature of the implicit risk premium associated with these contracts.

Like Piazzesi and Swanson (2008) and Hamilton (2009a), we find the most persuasive evidence of predictable gains for the longer horizon futures contracts. Our results suggest that predictable gains are primarily associated with one particular regime that is characterized by unusual volatility in the holding gains and tends to coincide with episodes of weak economic growth or recession. These results also complement the findings by Cochrane and Piazzesi (2005) that predicted excess holding yields from long-term bonds are countercyclical, and by Piazzesi and Swanson (2008) that expected holding gains from long positions in fed funds futures contracts are countercyclical. We suggest that at least in the case of fed funds futures contracts, this phenomena is better characterized as a feature confined to particular historical episodes rather than present throughout the sample.

The plan of the paper is as follows. In Section 2 we briefly explore the theoretical connection between the term structure of interest rates and predictable gains from fed funds futures contracts. Empirical results are presented in Section 3, and implications for the most recent behavior of fed funds futures are noted in Section 4. Section 5 concludes.

2 Risk premia in interest rates and fed funds futures.

Let P_{nt} denote the price paid at time t for a pure-discount bond that will be worth \$1 with certainty at $t+n$ and let M_t denote the stochastic discount factor. Standard finance theory (e.g., Campbell, Lo, and MacKinlay, 1997, p.428) asserts that

$$P_{nt} = E_t [P_{n-1,t+1} M_{t+1}] \tag{1}$$

implying

$$P_{nt} = E_t[M_{t,t+n}]$$

for $M_{t,t+n} = \prod_{i=1}^n M_{t+i}$. Consider a forward contract agreed to at date t to buy a one-period bond at $t+n$, whose value will be \$1 at $t+n+1$. Let S_{nt} denote the price agreed to under the contract and $P_{1,t+n}$ what the actual price of the bond turns out to be. Absence of arbitrage requires the forward price to satisfy

$$E_t [M_{t,t+n}(P_{1,t+n} - S_{nt})] = 0$$

or

$$S_{nt} = \frac{E_t [M_{t,t+n}P_{1,t+n}]}{E_t [M_{t,t+n}]} \tag{2}$$

Evaluating (1) for a 1-period bond at date $t+n$,

$$P_{1,t+n} = E_{t+n}[M_{t+n+1}] \tag{3}$$

Substituting (3) into (2) and using the Law of Iterated Expectations,

$$S_{nt} = \frac{E_t [M_{t,t+n+1}]}{E_t [M_{t,t+n}]} = \frac{P_{n+1,t}}{P_{nt}} \tag{4}$$

This forward price S_{nt} will be recognized as the reciprocal of the gross forward interest rate; see Campbell, Lo and MacKinlay (1997, equation 10.1.7).

When the term structure of interest rates slopes up (bonds of higher maturities n have higher yields), the value of S_{nt} for given t would typically be a decreasing function of the horizon n . Thus in a stationary environment, a term premium that induces an average positive slope to the term structure would result in $S_{n,t-1} - S_{n-1,t} < 0$ on average.

Fed funds futures contracts are settled based on the average value of the overnight fed funds rate during the expiry month. Insofar as these behave like forward contracts on the future price of one-period bonds, fed funds futures contracts should inherit the same pricing premia as the term structure of interest rates. Let f_{nt} denote the interest rate implied as of the last day of month t by a fed funds futures contract expiring n months subsequently, and r_{t+n} the actual average fed funds rate for month $t+n$. In the absence of arbitrage, the futures contracts must be priced such that

$$E_t(M_{t,t+n}f_{n,t}) = E_t(M_{t,t+n}r_{t+n}).$$

Note that if the fed funds rate at time $t + n$ were given by one minus the risk-free one-period bond price at that date, that is, if

$$r_{t+n} = 1 - P_{1,t+n}, \quad (5)$$

then the fed funds futures price f_{nt} would be exactly equal to $1 - S_{nt}$ under the assumption of no arbitrage. In practice, however, we should not expect this condition to hold exactly. Fed funds contracts are based on a monthly average of daily values for overnight fed funds, which differs from the end-of-month concept P_{1t} . Moreover, lending overnight fed funds is not completely risk free, the yield is the nonlinear function $(1/P_{1t}) - 1$ rather than $1 - P_{1t}$, and margin requirements are marked to market daily unlike a pure forward contract. For these reasons, $f_{nt} \simeq 1 - S_{nt}$ is only an approximation. Nonetheless, it would be surprising if fed funds futures contracts behave grossly out of line with risk premia in the term structure.

Following Piazzesi and Swanson (2008), we define the one-month holding gain u_{nt} to be the profit (or loss if negative value) from taking the long position in an n -month contract on the last day of month $t - 1$ and closing the position on the last day of month t :

$$u_{nt} = f_{n,t-1} - f_{n-1,t}. \quad (6)$$

For $n = 1$, the value of u_{1t} is the difference between the 1-month futures rate at the end of month $t - 1$ and the actual fed funds rate experienced in month t :

$$u_{1t} = f_{1,t-1} - r_t.$$

The strong form of the expectations hypothesis would require $E(u_{nt} = 0)$, and the weak form $E_{t-1}(u_{nt}) = c_n$. A violation of the strong form of the expectations hypothesis in the form of a positively sloped yield curve would cause $S_{n,t-1} - S_{n-1,t} < 0$ on average, under which we would expect u_{nt} to be positive on average. If the long rate is above the average of expected future short rates, then the forward rate should be above the expected future spot rate, and the interest rate f_{nt} associated with a given contract should on average decline as the contract gets closer to expiry, i.e., as n decreases and t increases, implying a positive average value for u_{nt} . The positive average slope to the yield curve is of course well-documented, and Piazzesi and Swanson (2008) found evidence for positive values of

u_{nt} for all n . However, just as the evidence against the expectations hypothesis of the term structure is weakest for securities with maturity n less than 2 months (Rudebusch, 1995; Longstaff, 2000), evidence for predictable variation in the fed funds futures revisions u_{nt} is weakest for small n (Piazzesi and Swanson, 2008; Hamilton, 2009a).

3 Empirical Results

3.1 Autoregressive specifications

The basic data used in this study are the monthly changes in the prices of contracts with one to six months of settlement and purchased from the Chicago Board of Trade. We calculate f_{nt} (in basis points) from the end-of-month closing contract price Q_{nt} as $f_{nt} = 100 \times (100 - Q_{nt})$ and u_{nt} from (6). The sample period is from November 1990 to June, 2006.² The main purpose of the paper is to examine possible regime switches in the one-month holding gains associated with long positions in contracts of various maturities.

As a starting point, we estimated a simple Gaussian AR(1) model,

$$u_{nt} = \alpha_n + \phi_n u_{n,t-1} + \sigma_n \varepsilon_{nt}, \quad \varepsilon_{nt} \sim N(0, 1), \quad (7)$$

for horizons $n = 1, \dots, 6$ months using maximum likelihood estimation (MLE), in this case equivalent to ordinary least squares. The estimation results are summarized in Table 1. As can be seen, the constant terms are estimated to be positive and statistically significant for all horizons, consistent with an upward-sloping yield curve or positive risk premia on longer term bonds. The AR(1) terms are significantly positive for all horizons longer than one month, suggesting variation over time in these risk premia. The implied average holding gain given by $\mu_n = \alpha_n / (1 - \phi_n)$ is increasing in the horizon n , ranging from 2.7 to 6.1 basis points per month (32-73 bp per year), which is fairly consistent with the results of Piazzesi and Swanson (2008). Regarding the explanatory power, the coefficient of determination, R^2 , suggests that the AR(1) model can explain about 13% of the variation in one-month holding gains on federal funds futures if the horizon is longer than two months.

²Federal funds futures price data are available from October 1988. However, we chose November 1990 as the beginning of our sample, since the longer horizon contracts were not traded frequently in the first two years of the data. Our sample ends prior to the most recent turbulence in fed funds and other financial markets.

We then compared this specification with a two-state Markov switching AR(1) (MSAR(1)) model:

$$u_{nt} = \alpha_{n,s_t} + \phi_{n,s_t} u_{n,t-1} + \sigma_{n,s_t} \varepsilon_t, \quad \varepsilon_{nt} \sim N(0, 1), \quad (8)$$

where s_t is an unobserved latent variable presumed to follow a two-state first-order Markov chain. The maximum likelihood estimates reported in Table 2 identify two distinct regimes in fed funds futures prices. The first regime is characterized by a high risk premium ($\alpha_{n1} > 0$) and high volatility ($\sigma_{n1} \gg \sigma_{n2}$), while the second regime is associated with little risk premium and low volatility. Note that we fail to reject the null hypothesis of zero average risk premium in regime 2 ($\alpha_{n2} = 0$) for every maturity n .

Table 3 reports tests of the hypothesis that parameters are constant across the two regimes.³ The differences in the constant term and volatility are highly significant for all horizons. On the other hand, the differences in the AR coefficient are not significant at the 10% significance level for any horizon longer than one month.

To assess the in-sample fit of the Markov switching models we use the Markov switching criterion (MSC) recently proposed by Smith, Naik, and Tsai (2006). The MSC asymptotically is a generalization of the Akaike Information Criterion. The MSC is based on the Kullback-Leibler divergence between the true and candidate models and is calculated as

$$MSC = -2 \log(f(Y, \hat{\theta})) + \sum_{i=1}^N \frac{\hat{T}_i(\hat{T}_i + \lambda_i K)}{\delta_i \hat{T}_i - \lambda_i K - 2},$$

where $\log(f(Y, \hat{\theta}))$ is the maximized log-likelihood value, N is the number of regimes, $\hat{T}_i = \sum_{t=1}^T \hat{\xi}_{ti}$, $\hat{\xi}_{ti}$ is the smoothed probability of the regime i at time t , K is the number of regressors, and δ_i and λ_i are tuning parameters. Following the suggestion of Smith, Naik, and Tsai (2006), we set $\delta_i = 1$ and $\lambda_i = N$. In our case, $N = 2$ for the Markov-switching specifications and $N = 1$ with no Markov-switching. The smaller the value for MSC, the better the model is judged to be. The values for MSC given in Panel A of Table 4 suggest that the MSAR(1) model would be selected over the AR(1) model for every horizon n .

We also compared models based on the out-of-sample root-mean-squared forecast errors (RMSE) on data since 2000 as follows. First, we estimated both AR(1) and MSAR(1)

³Standard tests of the null hypothesis of no Markov switching are plagued by the problem that nuisance parameters are unidentified under the null. This problem is avoided here, insofar as the null hypothesis of constant coefficients still allows the regime transition probabilities to be identified from differences in variances, as in Engel and Hamilton (1990).

models using data from November 1990 to December 1999 and evaluated the terminal 1-month-ahead forecast error based on the estimation results. The data were then updated by 1-month, and the terminal 1-month-ahead forecast error was re-calculated from the updated sample (specifically, from November 1990 to January 2000). This procedure was repeated until one month before the end of the sample period, namely May 2006. Panel B of Table 4 reports the values of RMSE for the AR(1) and MSAR(1) models. As can be seen, the RMSE of the MSAR(1) model is uniformly smaller than that of the AR(1) model, meaning the MSAR(1) model provided better post-sample forecasts than the AR(1) model.

Figure 1 plots the smoothed probabilities of regime 1 (high risk premium with high volatility regime). As was also reported by Bundick (2007), for the 1-month contracts the moves in and out of regime 1 occur quite frequently, whereas regime 1 for the longer horizon contracts is associated with a few episodes of longer duration. Two of these occur prior to and following the recessions of 1990-91 and 2001, and a third is associated with 1994-95.

Figure 1 also plots gray regions for episodes in which the Fed was lowering its target for the fed funds rate. These dates also correlate with occurrences of regime 1. The fact that $\alpha_{n1} > 0$ means that in regime 1 one could predict that r_{t+n} will be lower on average than f_{nt} , or the fed funds market seems to underestimate the likelihood or magnitude of a funds rate cut at these times. On the other hand, although our model implies that the Fed will likely cut rates in these episodes, it also maintains that it is most difficult to predict interest rates at these times ($\sigma_{n1} \gg \sigma_{n2}$). Interpreted as a risk premium, the claim is that at these times, holders of long-term bonds receive extra compensation for their risk, and this pricing of risk is mirrored in that those who take the long position in fed funds futures are rewarded on average for those positions.

Following Hamilton (2009a, 2009b), we also investigated whether the improved performance of the Markov-switching specifications results from its description of the conditional variance rather than the conditional mean. We generalized the baseline autoregression (7) to allow for outliers and heteroskedastacity, using an EGARCH(1,1) specification with

a time trend⁴ in the variance,

$$u_{nt} = \alpha_n + \phi_n u_{n,t-1} + e_{nt}, \quad (9)$$

with e_{nt} drawn from a Student t distribution with ν_n degrees of freedom and scale parameter h_{nt} evolving according to

$$\log h_{nt} - \gamma_n - \xi_n t = \delta[\log h_{n,t-1} - \gamma_n - \xi_n(t-1)] + \eta_n(|e_{n,t-1}/h_{n,t-1}| - k_{\nu_n}). \quad (10)$$

Estimates for the parameters are reported in Table 5. Consistent with the results in Hamilton (2009a), when heteroskedasticity and fat tails are allowed for in the estimation, the intercept α_n for $n = 1$ to 3 changes signs and is no longer statistically significant. The autoregressive parameter is statistically significant for $n \geq 2$ but not for $n = 1$. As seen in row 3 of Table 4A, the EGARCH-AR(1) specification (9)-(10) has a better in-sample fit in terms of the Markov Switching Criterion for $n = 1$ to 4, while the Markov-switching specification (8) does better at horizons 5 and 6. The success of the Markov-switching specification for short horizons relative to the homoskedastic Gaussian AR(1) thus results in part from its description of a time-varying variance which might be more parsimoniously captured with an EGARCH specification. For the longer horizons, however, the broad episodes of predictability described by the Markov-switching model appear to be a genuine feature of the data.

In row 3 of Table 4B we compare the EGARCH-AR(1) specification with the others in terms of the post-sample forecasts. The EGARCH specification does as well as Markov-switching for $n = 1$, while the MS models do uniformly better than EGARCH at longer horizons.

We also estimated more restricted versions of the EGARCH specification that imposed a constant risk premium ($\phi_n = 0$) or zero risk premium ($\alpha_n = \phi_n = 0$). Both in sample and out of sample these tend to be dominated by both the unrestricted EGARCH-AR(1) and the Markov-switching AR(1).

3.2 Predictive regression models

We now explore the role of a set of other variables besides lagged gains that might help predict holding gains on fed funds contracts. We use the set of variables proposed by

⁴As in Hamilton (2009a), we scaled the time trend by dividing by 1000 for better numerical stability.

Piazzesi and Swanson (2004, 2008) and also used by Hamilton (2009a). These explanatory variables include the employment growth rate, the credit spread, and assorted treasury yield spreads. For employment growth we looked at two measures. The first is the 12-month change in the logarithm of seasonally adjusted nonfarm payrolls for the period ended in month $t - 2$. Although a value for this number would have been known to market participants as of the end of month $t - 1$, the currently revised series is not the same as the number actually available at the time. We therefore also conducted these tests using the 12-month growth in seasonally unadjusted nonfarm payrolls as reported at the time from the real-time data set of the Federal Reserve Bank of Philadelphia.

Our measure for the credit spread is the difference between Baa-corporate and 10-year treasury yields (in basis points). We also used four different treasury spreads based on differences of the 6-month, 1-, 2-, 5-, and 10-year treasury yields. All data other than the real-time employment series were obtained from the FRED database of the Federal Reserve Bank of St. Louis.

We explored the role of the above list of explanatory variables, taken one at a time, in predictive regressions for three sets of specifications. The first is a homoskedastic Gaussian regression:

$$u_{nt} = \alpha_n + \phi_n u_{n,t-1} + \beta_n x_{t-1} + \sigma_n \varepsilon_{nt}, \quad \varepsilon_{nt} \sim N(0, 1). \quad (11)$$

We compared this with a Markov-switching predictive regression (MSPR),

$$u_{nt} = \alpha_{n,s_t} + \phi_{n,s_t} u_{n,t-1} + \beta_{n,s_t} x_{t-1} + \sigma_{n,s_t} \varepsilon_{nt}, \quad \varepsilon_{nt} \sim N(0, 1) \quad (12)$$

and a predictive regression with EGARCH Student t errors,

$$u_{nt} = \alpha_n + \phi_n u_{n,t-1} + \beta_n x_{t-1} + e_{nt} \quad (13)$$

where e_{nt} is drawn from a Student t distribution with ν_n degrees of freedom and scale parameter h_{nt} evolving according to (10).

Figure 2 illustrates the difference between the predictions of the Markov-switching formulation with no explanatory variables (8) and the simple predictive regression (11) for the case when employment growth is the explanatory variable and $n = 6$. The actual 1-month gain from taking the long position in a 6-month fed funds futures contract at the

end of month $t - 1$ and closing the position at the end of month t is plotted as the dotted line. The predicted gain as of time $t - 1$ according to the Markov-switching specification is given by the gray line, while that from the homoskedastic predictive regression is shown in bold. The two formulations are both capturing the tendency for there to be bigger gains in the early 1990s and early 2000s, the predictive regression because these were episodes of falling employment, and the Markov-switching model because they were identified as associated with regime 1. The latter does a better job of recognizing the shift in 2003 to a regime in which there were no longer expected gains associated with these contracts.

Comparisons among the different predictive regression specifications based on the in-sample MSC are reported in Table 6. For the $n = 6$ example just discussed, the Markov-switching model with no explanatory variables was seen in Table 4A to achieve an MSC of 1850.75, better than that for any of the three specifications using employment reported in Table 6. Indeed, for the $n = 6$ case, the Markov-switching autoregression does better than any predictive regression using any explanatory variable, with the single exception of the Markov-switching regression based on the 1 year minus 6 month spread.

The Markov-switching autoregression also performs well among this group at predicting the 5-month contract gains, but for shorter horizons it tends to be dominated by the EGARCH predictive regressions. More generally, we can make the following observations from the results in Table 6. First, for every explanatory variable and every horizon, the Gaussian predictive regression (11) is dominated by both the Markov-switching and EGARCH predictive regressions. Second, for most explanatory variables and horizons, the EGARCH predictive regressions do better than the Markov-switching predictive regressions.

In comparing across explanatory variables and specifications, for horizons 5 and 6 the Markov-switching predictive regression based on the 1-year minus 6-month treasury spread has the best fit to the data, while for horizons 2 and 3, an EGARCH specification based on this same variable does best. For $n = 1$, an EGARCH formulation based on the 2-year minus the 1-year spread does best, while for $n = 4$, EGARCH using 2-year minus 1-year does about the same as Markov-switching using 1-year minus 6-month.

Table 7 reports parameter estimates for the Markov-switching predictive regressions based on the 1-year minus 6-month spread. Interestingly, the spread enters with a

negative coefficient in regime 1 and a positive coefficient in regime 2. Using the $n = 5$ results for illustration, the net contribution of the intercept and spread is positive on average in regime 1,

$$\alpha_{51} + \beta_{51}\bar{s}_{1y,6m} = 15.8 - (0.186)(17.6) = 12.5,$$

for $\bar{s}_{1y,6m} = 17.6$ the average 1-year minus 6-month spread. If the spread is above its average value, the expected value of u_{5t} would be less 12.5— a more steeply sloped yield curve lessens the expected profitability of a long position in 5-month fed funds futures in this regime.

By contrast, the typical expected drift in regime 2 is essentially zero, just as it was in the simple MSAR formulation:

$$\alpha_{52} + \beta_{52}\bar{s} = -6.4 + (0.33)(17.6) = -0.6.$$

In this case, if the yield curve becomes more steeply sloped than average, it suggests there is a small profit opportunity from taking a long position in fed funds futures. Figure 3 displays the difference between the predicted values from MSAR (8) and MSPR (12) for the case $n = 5$.

Table 8 reports the post-sample root-mean-square errors for the various predictive regressions estimated. Predictive EGARCH regressions based on employment growth do best for the 1- and 2-month contracts, while predictive Markov-switching regressions using the 1y-6m or 10y-5y spreads do best for the 3- to 6-month contracts among the predictive regressions. The MSAR specification turns out to dominate all other models for $n = 5$ in terms of post-sample performance.

4 Application: Interpreting fed funds futures in July 2009.

We illustrate some implications and potential uses of these estimates by taking a look at fed funds futures prices in the summer of 2009. During the first half of 2009, the Fed had announced a target range for the fed funds rate of between 0 and 25 basis points. The actual average monthly fed funds rate was between 15 and 22 basis points over these 6 months, averaging 18 bp.

The interest rates implied by fed funds futures contracts as of June 30 are plotted as the solid line in Figure 4. These rise steadily to a value of 35 basis points for the December contract. Some analysts had interpreted that modest slope to a belief by traders that there was some possibility that the Fed could increase its target before the end of the year, despite statements from Fed officials that seemed to suggest such a change was unlikely.

To what extent is it plausible to attribute this slope instead to the pricing of risk implicit in these contracts? One quick way to answer this question is to look at the average historical discrepancy between the fed funds futures contract and what the actual interest rate turned out to be. The average value of u_{1t} in our sample is 2.9 basis points, from which, as in Piazzesi and Swanson (2008), one might calculate a “risk-adjusted” market forecast that is 2.9 basis points below the 1-month fed funds futures rate f_{1t} . The average value of u_{2t} is 6.9, implying that f_{2t} exceeded r_{t+2} on average by $2.9 + 6.9 = 9.8$ basis points, for a risk-adjusted implied forecast of $f_{2t} - 9.8$.

The dashed line in Figure 4 uses these calculations to calculate what we’d expect the fed funds futures rate to be as of June 30, 2009 if investors in fact believed that the actual rate each month would be the same 18 basis points observed in the first half of the year, but these contracts incorporated risk premia consistent with their average ex post gains on fed funds contracts of different maturities. Such a calculation suggests that the modest upward slope in fed funds futures in the summer of 2009 is fully consistent with a belief by market participants that the actual fed funds rate was unlikely to change.

Our Markov-switching framework suggests that average pricing of risk over the entire sample in fact arises from a few episodes associated with weak economic activity and highly volatile interest rates in which the risk premium was quite large. Conditions in 2009 are clearly outside of the range included in the sample, both in the severity of the economic downturn and in the fact that the fed funds rate had effectively bumped against the zero lower bound, which is why we did not attempt to use these data in the estimation. However, it is interesting to take a look at what the model would imply if we assumed that the economy was in regime 1 in June of 2009 and that the 1-year minus 6-month spread at that time (21 basis points) was expected to persist through the end of the year.

Let

$$\begin{aligned} & \hat{u}_{n,t+j|t}(i_k, i_{k-1}, \dots, i_1) \\ = & E(u_{n,t+j}|u_{nt}, s_{n,t+k} = i_k, s_{n,t+k-1} = i_{k-1}, \dots, s_{n,t+1} = i_1, s_t = 1) \end{aligned}$$

denote the forecast conditional on knowing the future regimes, which can be generated recursively from

$$\hat{u}_{n,t+j|t}(i_k, i_{k-1}, \dots, i_1) = \phi_{n,i_j} \hat{u}_{n,t+j-1|t}(i_k, i_{k-1}, \dots, i_1) + c_{n,i_j}$$

for $c_{nj} = \alpha_{nj} + 21\beta_{nj}$. The unconditional forecast is then

$$E(u_{n,t+k}|u_{nt}, s_t = 1) = \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_k=1}^2 \hat{u}_{n,t+k|t}(i_k, i_{k-1}, \dots, i_1) p_{n,1,i_1} p_{n,i_1,i_2} \cdots p_{n,i_{k-1},i_k}. \quad (14)$$

Note the identity

$$\begin{aligned} f_{kt} &= r_{t+k} + (f_{1,t+k-1} - r_{t+k}) + (f_{2,t+k-2} - f_{1,t+k-1}) + \cdots + (f_{k,t} - f_{k-1,t+1}) \\ &= r_{t+k} + u_{1,t+k} + u_{2,t+k-1} + \cdots + u_{k,t+1}. \end{aligned} \quad (15)$$

Taking conditional expectations of (15),

$$f_{kt} = E(r_{t+k}|u_{nt}, s_t = 1) + \sum_{j=1}^k E(u_{j,t+j}|u_{nt}, s_t = 1). \quad (16)$$

Thus if we supposed that the market expectation of fed funds rates for the remainder of 2009 was a constant 18 basis points,

$$E(r_{t+k}|u_{nt}, s_t = 1) = 18 \quad k = 1, 2, \dots, 6,$$

equations (16) and (14) imply a value for f_{kt} that we would expect to see. These values are plotted as the dotted line in Figure 4. This prediction is uniformly above the average risk premia correction, because with a 1-year minus 6-month spread of 21 basis points, the MS predictive regressions in Table 7 imply bigger risk premia when the economy is in regime 1 than observed on average.

The results of this exercise suggest that the modest upward slope of the futures curve in the summer of 2009 could easily be entirely accounted for by risk premia in these contracts, and need not be interpreted as a belief by market participants that an increase in the target fed funds rate before the end of 2009 was a material possibility.

5 Conclusions.

In this paper we have confirmed the findings of previous researchers that while it is difficult to make a better prediction of the fed funds rate than is incorporated in near-term fed funds futures contracts, for longer horizons there is a systematic gain for those taking the long position in these contracts, that is, the actual fed funds rate is likely to end up below the forward rate implied by the contract. We related this to the tendency of treasury securities of longer maturities to provide a higher yield on average than those of shorter maturities, and studied time-variation in the predictable contract gain over the period 1990-2006. We found that the predictable gain for these contracts is primarily coming from particular episodes of weak economic activity in the early 1990s, 1995, and 2001-2002. Although a speculator could expect a positive average gain from a long position in fed funds futures at such times, they are also associated with much more variance of the gain.

References

- [1] Ang, Andrew, and Geert Bekaert (2002). “Regime Switches in Interest Rates.” *Journal of Business and Economic Statistics*, 20, 163-82.
- [2] Ang, Andrew, Geert Bekaert, and Min Wei (2008). “The Term Structure of Real Rates and Expected Inflation.” *Journal of Finance*, 63, 797-849.
- [3] Bansal, Ravi, and Hao Zhou (2002). “Term Structure of Interest Rates with Regime Shifts.” *Journal of Finance*, 57, 1997-2043.
- [4] Bekaert, Geert, Robert J. Hodrick, and David A. Marshall (1997). “On Biases in Tests of the Expectations Hypothesis of the Term Structure of Interest Rates.” *Journal of Financial Economics*, 44, 309-48.
- [5] Bundick, Brent (2007). “Do Federal Funds Futures Need Adjustment for Excess Returns? A State-Dependent Approach.” Working Paper 07-08, Federal Reserve Bank of Kansas City.
- [6] Campbell, John Y., Andrew W. Lo, and A. Craig MacKinlay (1997). *The Econometrics of Financial Markets*, Princeton: Princeton University Press.
- [7] Campbell, John Y., and Robert J. Shiller (1991). “Yield Spreads and Interest Rate Movements: A Bird’s Eye View.” *Review of Economic Studies*, 58, 495-514.
- [8] Cochrane, John H., and Monika Piazzesi (2005). “Bond Risk Premia.” *American Economic Review*, 95, 138-160.
- [9] Dai, Qiang, Kenneth J. Singleton, and Wei Yang (2007). “Regime Shifts in a Dynamic Term Structure Model of U.S. Treasury Bond Yields.” *Review of Financial Studies*, 20, 1669-1706.
- [10] Engel, Charles, and James D. Hamilton (1990). “Long Swings in the Dollar: Are They in the Data and Do Markets Know It?” *American Economic Review*, 80, 689-713
- [11] Evans, Martin D.D. (2003). “Real Risk, Inflation Risk, and the Term Structure.” *Economic Journal*, 113, 345-389.

- [12] Evans, Martin D.D., and Karen K. Lewis (1994). “Do Stationary Risk Premia Explain it All? Evidence from the Term Structure.” *Journal of Monetary Economics*, 33, 285-318.
- [13] Garcia, René, and Pierre Perron (1996). “An Analysis of the Real Interest Rate under Regime Shifts.” *Review of Economics and Statistics* 78, 111-125.
- [14] Gray, Stephan F. (1996). “Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process.” *Journal of Financial Economics*, 42, 27-62.
- [15] Gürkaynak, Refet S., Brian Sack, and Eric Swanson (2007). “Market-Based Measures of Monetary Policy Expectations,” *Journal of Business and Economic Statistics*, 25, 201-212.
- [16] Hamilton, James D. (1988). “Rational-Expectations Econometric Analysis of Changes in Regime: An Investigation of the Term Structure of Interest Rates.” *Journal of Economic Dynamics and Control*, 12, 385-423.
- [17] Hamilton, James D. (1989). “A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle.” *Econometrica*, 57, 357-84.
- [18] Hamilton, James D. (2008). “Daily Monetary Policy Shocks and New Home Sales.” *Journal of Monetary Economics*, 55, 1171-1190.
- [19] Hamilton, James D. (2009a). “Daily Changes in Fed Funds Futures Prices.” *Journal of Money, Credit, and Banking*, 41, 567-582.
- [20] Hamilton, James D. (2009b). “Macroeconomics and ARCH.” In *Festschrift in Honor of Robert F. Engle*, edited by Tim Bollerslev, Jeffrey R. Russell and Mark Watson. Oxford University Press.
- [21] Inoue, Tomoo, and Tatsuyoshi Okimoto (2008). “Were There Structural Breaks in the Effect of Japanese Monetary Policy?: Re-evaluating the Policy Effects of the Lost Decade.” *Journal of the Japanese and International Economies*, 22, 320-42.

- [22] Kuttner, Kenneth N. (2001). "Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Market." *Journal of Monetary Economics*, 47, 523-544.
- [23] Krueger, Joel T., and Kenneth N. Kuttner (1996). "The Fed Funds Futures Rate as a Predictor of Federal Reserve Policy." *Journal of Futures Markets*, 16, 865-879.
- [24] Longstaff, Francis A. (2000). "The Term Structure of Very Short-Term Rates: New Evidence for the Expectations Hypothesis." *Journal of Financial Economics*, 58, 397-415.
- [25] Piazzesi, Monika, and Eric Swanson (2004). "Futures Prices as Risk-Adjusted Forecasts of Monetary Policy." Working Paper 10547, National Bureau of Economic Research.
- [26] Piazzesi, Monika, and Eric Swanson (2008). "Futures Prices as Risk-Adjusted Forecasts of Monetary Policy." *Journal of Monetary Economics*, 55, 677-91.
- [27] Rudebusch, Glenn D. (1995). "Federal Reserve Interest Rate Targeting, Rational Expectations, and the Term Structure." *Journal of Monetary Economics*, 35, 245-74.
- [28] Rudebusch, Glenn D. (1998). "Do Measures of Monetary Policy in a VAR Make Sense?" *International Economic Review*, 39, 907-31.
- [29] Sack, Brian (2004). "Extracting the Expected Path of Monetary Policy from Futures Rates." *Journal of Futures Markets*, 24, 733-754.
- [30] Sims, Christopher, and Tao Zha (2006). "Were There Regime Switches in U.S. Monetary Policy?" *American Economic Review*, 96, 54-81.
- [31] Smith, Aaron, Prasad A. Naik, and Chih-Ling Tsai (2006). "Markov-switching model selection using Kullback-Leibler divergence." *Journal of Econometrics*, 134, 553-77.

Table 1: Estimation results for the AR(1) model

n	1	2	3	4	5	6
α	2.716	3.106	2.511	3.071	3.536	4.076
std error	0.744	1.000	1.015	1.284	1.466	1.667
φ	-0.011	0.147	0.366	0.372	0.357	0.330
std error	0.076	0.074	0.074	0.069	0.071	0.071
σ	9.70	13.05	14.50	16.77	19.24	21.91
std error	0.54	0.68	0.76	0.87	0.95	1.06
R^2	0.000	0.024	0.139	0.143	0.129	0.110
Log-likelihood	-686	-742	-761	-788	-814	-838

Table 2: Estimation results for the MSAR(1) model

n	1	2	3	4	5	6
p_{11}	0.788	0.792	0.920	0.937	0.935	0.937
std error	0.074	0.103	0.057	0.052	0.047	0.040
p_{22}	0.840	0.910	0.970	0.966	0.971	0.973
std error	0.047	0.042	0.019	0.027	0.019	0.017
α_1	7.15	11.9	9.83	10.3	21.8	24.1
std error	1.76	3.37	3.81	3.61	5.68	6.20
α_2	-0.393	0.040	0.140	-0.009	-1.217	-1.275
std error	0.291	0.512	0.697	1.040	1.221	1.552
φ_1	-0.146	0.024	0.248	0.274	0.019	-0.001
std error	0.120	0.132	0.157	0.126	0.158	0.154
φ_2	0.188	0.079	0.390	0.341	0.308	0.284
std error	0.060	0.063	0.067	0.094	0.082	0.088
σ_1	13.4	20.8	23.5	24.2	24.0	26.5
std error	1.19	1.97	2.65	2.54	2.46	2.96
σ_2	2.33	4.82	7.48	9.45	13.29	15.91
std error	0.25	0.44	0.52	0.83	0.95	1.08
Log-likelihood	-613	-675	-716	-758	-793	-822

Table 3: Hypothesis testing results of the equality of each parameter across regimes for the MSAR(1) model

n		1	2	3	4	5	6
α	Wald-stat	17.9	12.1	6.21	7.85	16.0	15.9
	p-value	0.000	0.001	0.013	0.005	0.000	0.000
φ	Wald-stat	5.65	0.129	0.717	0.175	2.52	2.28
	p-value	0.017	0.720	0.397	0.675	0.112	0.131
σ	Wald-stat	91.11	69.45	37.00	33.53	15.88	10.96
	p-value	0.000	0.000	0.000	0.000	0.000	0.001

Table 4: Comparison of autoregressive models

Panel A: In-sample Markov Switching Criterion (MSC)

n	1	2	3	4	5	6
AR(1)	1565.05	1675.66	1714.84	1768.94	1819.94	1868.38
MSAR(1)	1433.96	1557.92	1638.92	1722.84	1794.34	1850.75
EGARCH-AR(1)	1421.57	1531.28	1633.79	1719.53	1795.68	1854.06
EGARCH-AR(1) ($\varphi_n=0$)	1435.74	1551.53	1672.97	1760.72	1831.03	1886.88
EGARCH-AR(1) ($\alpha_n=\varphi_n=0$)	1433.78	1550.41	1671.03	1759.03	1830.02	1887.29

Panel B: Out-of-sample root mean squared error (RMSE)

n	1	2	3	4	5	6
AR(1)	8.33	12.94	13.88	16.16	18.54	21.42
MSAR(1)	7.95	12.78	13.40	15.64	17.40	20.57
EGARCH-AR(1)	7.95	12.84	13.82	16.09	18.41	21.32
EGARCH-AR(1) ($\varphi_n=0$)	8.06	12.83	15.02	17.66	19.96	22.34
EGARCH-AR(1) ($\alpha_n=\varphi_n=0$)	8.15	13.28	15.55	18.26	20.76	23.27

Table 5: Estimation results for the AR(1)-EGARCH model

n	1	2	3	4	5	6
α	-0.118	-0.333	-0.089	0.674	1.066	1.749
std error	0.243	0.332	0.612	0.815	1.208	1.317
φ	0.096	0.162	0.354	0.373	0.382	0.366
std error	0.066	0.054	0.056	0.057	0.057	0.059
γ	4.85	5.52	5.60	5.61	5.89	6.09
std error	0.45	0.43	0.42	0.36	0.38	0.32
δ	0.600	0.828	0.880	0.879	0.893	0.895
std error	0.181	0.171	0.078	0.096	0.099	0.134
η	0.188	0.109	0.100	0.076	0.049	0.034
std error	0.072	0.064	0.068	0.065	0.053	0.065
ξ	-23.2	-23.9	-15.3	-9.78	-7.18	-5.97
std error	3.93	4.11	3.82	3.11	3.02	2.77
ν	2.05	2.08	3.07	3.42	4.07	4.12
std error	0.43	0.44	0.76	0.83	1.25	1.19
Log-likelihood	-610	-665	-717	-759	-798	-827

Table 6: Comparison of predictive regressions based on in-sample MSC

Panel A: Homoskedastic

n	1	2	3	4	5	6
12-month job growth	1566.26	1673.13	1711.58	1764.43	1815.35	1863.12
Real-time 12-month job growth	1566.54	1674.70	1713.64	1766.93	1817.81	1865.49
Baa minus 10-year treasury spread	1566.63	1676.79	1716.21	1770.27	1821.27	1869.73
10-year minus 5-year treasury spread	1567.01	1677.64	1716.68	1770.82	1821.83	1870.39
5-year minus 2-year treasury spread	1567.02	1677.68	1716.82	1770.66	1821.50	1869.40
2-year minus 1-year treasury spread	1566.29	1677.71	1716.76	1770.22	1820.44	1867.56
1-year minus 6-month treasury spread	1567.10	1677.03	1716.81	1770.98	1821.78	1869.90

Panel B: Markov-switching

n	1	2	3	4	5	6
12-month job growth	1439.27	1558.97	1643.92	1728.14	1801.18	1858.37
Real-time 12-month job growth	1441.06	1561.91	1646.36	1730.29	1802.20	1859.26
Baa minus 10-year treasury spread	1439.60	1560.77	1645.47	1730.43	1803.21	1859.45
10-year minus 5-year treasury spread	1442.78	1566.31	1647.51	1730.70	1801.81	1857.21
5-year minus 2-year treasury spread	1443.24	1566.75	1646.64	1729.55	1800.77	1860.16
2-year minus 1-year treasury spread	1439.64	1563.01	1642.21	1722.93	1793.31	1852.81
1-year minus 6-month treasury spread	1438.48	1559.17	1632.84	1715.87	1789.70	1849.41

Table 6 (continued)

Panel C: EGARCH

n	1	2	3	4	5	6
12-month job growth	1423.75	1533.39	1633.55	1718.55	1794.34	1852.41
Real-time 12-month job growth	1423.73	1533.43	1634.93	1719.92	1795.81	1853.85
Baa minus 10-year treasury spread	1423.63	1532.44	1635.71	1721.58	1797.41	1855.77
10-year minus 5-year treasury spread	1423.23	1533.47	1635.41	1721.59	1797.85	1856.24
5-year minus 2-year treasury spread	1423.75	1532.84	1633.70	1719.69	1796.38	1854.72
2-year minus 1-year treasury spread	1420.76	1525.05	1629.78	1715.84	1793.01	1851.49
1-year minus 6-month treasury spread	1421.97	1524.99	1629.40	1716.97	1795.75	1854.79

Table 7: Estimation results for the MSPR model (1y-6m treasury yield spread)

n	1	2	3	4	5	6
p_{11}	0.796	0.800	0.933	0.938	0.947	0.945
std error	0.071	0.096	0.038	0.035	0.032	0.031
p_{22}	0.841	0.918	0.966	0.962	0.962	0.958
std error	0.048	0.038	0.019	0.019	0.019	0.021
α_1	8.72	19.9	14.26	15.1	15.8	15.7
std error	2.38	5.59	4.30	4.62	4.91	4.96
α_2	-0.969	-0.739	-3.68	-4.81	-6.39	-8.18
std error	0.498	0.650	1.13	1.42	1.78	2.62
φ_1	-0.131	-0.071	0.164	0.196	0.197	0.210
std error	0.112	0.158	0.143	0.128	0.127	0.116
φ_2	0.096	0.094	0.311	0.314	0.260	0.210
std error	0.145	0.061	0.078	0.069	0.092	0.102
β_1	-0.076	-0.313	-0.206	-0.200	-0.186	-0.148
std error	0.076	0.160	0.120	0.123	0.131	0.150
β_2	0.035	0.054	0.196	0.267	0.331	0.412
std error	0.024	0.032	0.050	0.064	0.086	0.110
σ_1	13.3	20.0	21.6	23.2	24.8	27.5
std error	1.16	2.10	2.19	2.11	2.08	2.39
σ_2	2.26	4.86	6.70	8.47	10.7	12.7
std error	0.24	0.42	0.48	0.64	0.80	1.01
Log-likelihood	-611	-671	-708	-749	-786	-816

Table 8: Comparison of predictive regressions based on out-of-sample RMSE

Panel A: Homoskedastic

n	1	2	3	4	5	6
12-month job growth	8.62	13.52	14.34	16.45	18.75	21.40
Real-time 12-month job growth	8.56	13.40	14.24	16.41	18.75	21.46
Baa minus 10-year treasury spread	8.64	13.19	14.29	16.67	19.08	22.14
10-year minus 5-year treasury spread	8.40	13.02	13.92	16.20	18.59	21.50
5-year minus 2-year treasury spread	8.43	13.07	14.04	16.32	18.71	21.52
2-year minus 1-year treasury spread	8.54	13.16	14.22	16.50	18.84	21.54
1-year minus 6-month treasury spread	8.45	13.04	14.13	16.51	18.89	21.80

Panel B: Markov-switching

n	1	2	3	4	5	6
12-month job growth	8.20	13.27	13.26	15.78	17.79	20.64
Real-time 12-month job growth	8.09	13.07	13.46	15.89	18.15	21.27
Baa minus 10-year treasury spread	8.30	13.51	13.45	16.18	18.13	21.06
10-year minus 5-year treasury spread	7.97	12.81	13.44	15.73	17.55	20.33
5-year minus 2-year treasury spread	8.10	12.91	13.53	15.73	17.77	20.86
2-year minus 1-year treasury spread	8.21	13.03	13.47	15.68	17.71	20.65
1-year minus 6-month treasury spread	8.15	13.27	13.09	15.02	17.61	20.48

Table 8 (continued)

Panel C: EGARCH

n	1	2	3	4	5	6
12-month job growth	7.78	12.49	13.46	15.60	18.00	20.81
Real-time 12-month job growth	7.83	12.67	13.64	15.79	18.16	21.00
Baa minus 10-year treasury spread	8.15	13.50	14.63	17.05	18.81	21.72
10-year minus 5-year treasury spread	7.83	12.82	13.90	16.20	18.55	21.41
5-year minus 2-year treasury spread	7.87	12.80	13.86	16.15	18.47	21.31
2-year minus 1-year treasury spread	8.18	12.95	13.95	16.27	18.50	21.31
1-year minus 6-month treasury spread	8.14	13.03	14.38	16.46	18.58	21.53

Figure 1: Smoothed probabilities for the high risk premium and volatility regime based on the MSAR(1) model

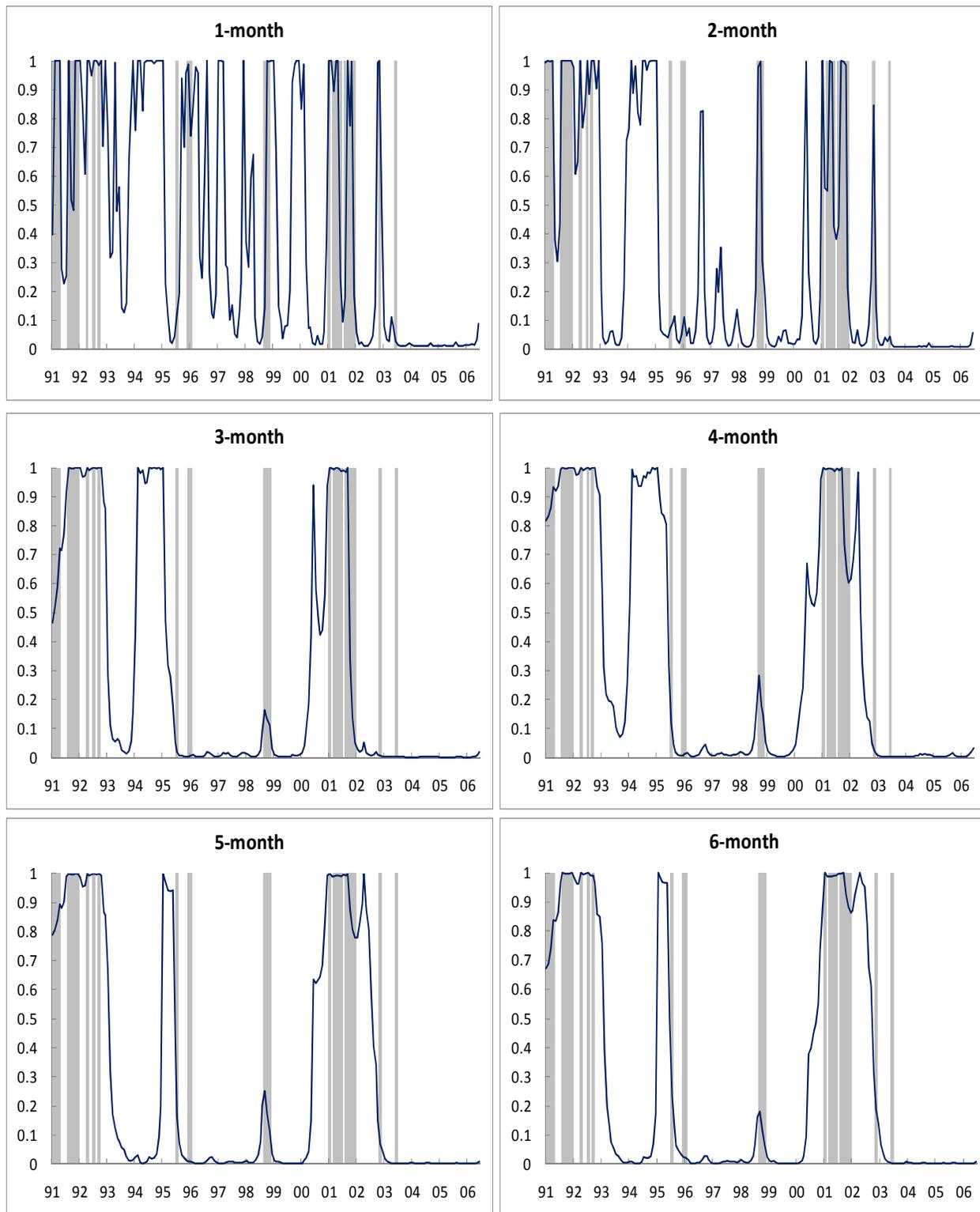
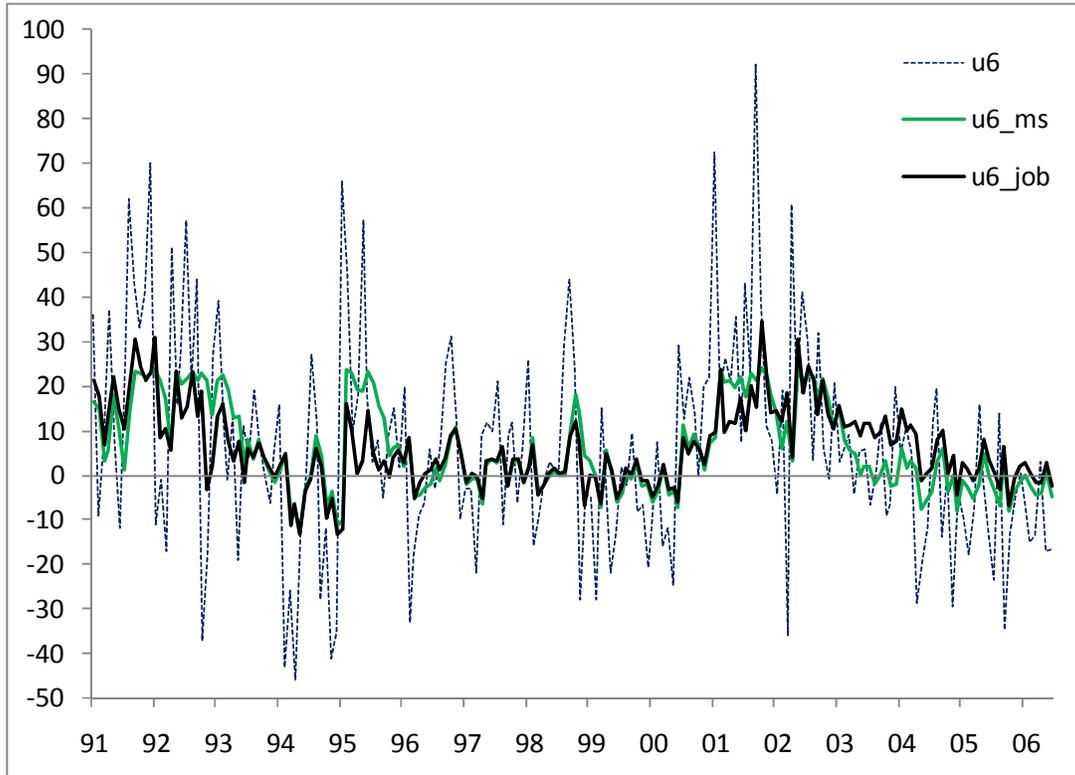
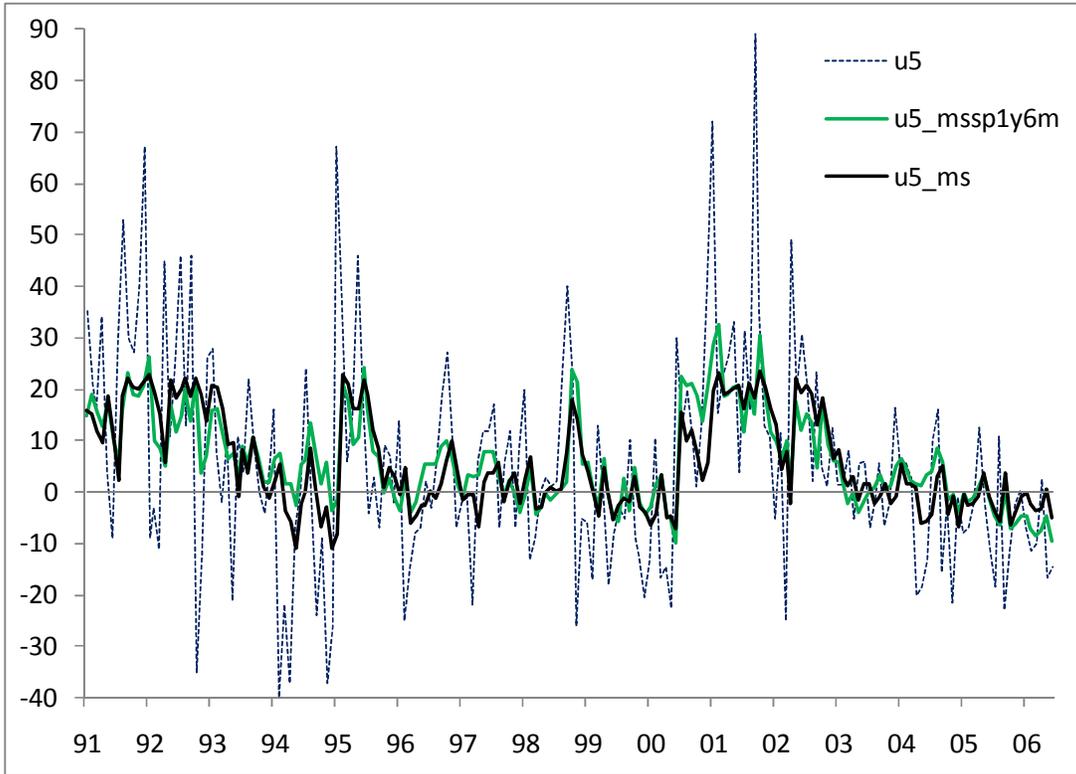


Figure 2: Comparison of the predicted values from the MSAR model and predictive regression for 6-month futures contracts



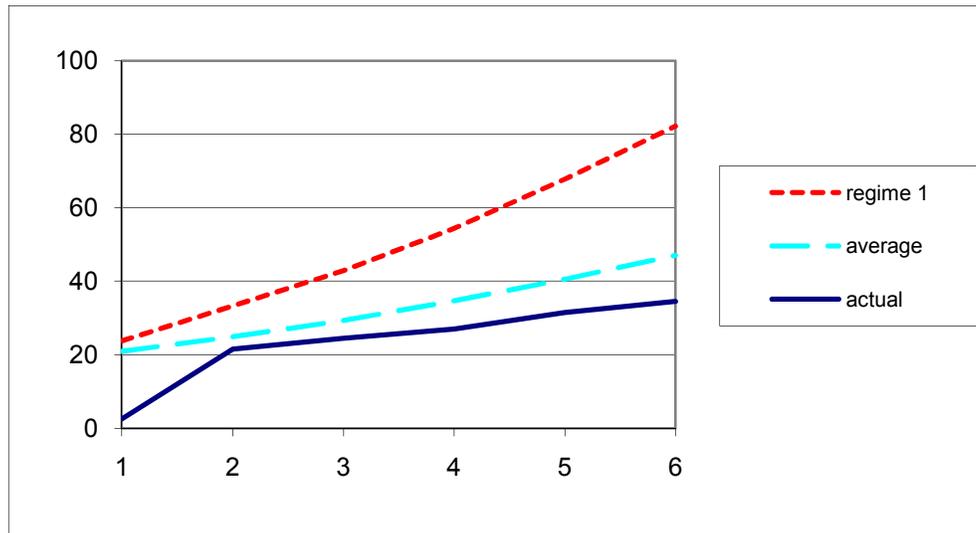
Notes to Figure 2. *Dotted line*: observed value of u_{6t} for each month t . *Bold line*: predicted value of u_{6t} based on information available as of month $t - 1$ using the predictive regression based on growth in nonfarm payrolls. *Gray line*: predicted value of u_{6t} based on information available as of month $t - 1$ using Markov-switching autoregression and no other explanatory variables.

Figure 3: Comparison of the predicted values from the MSAR and MSPR models for 5-month futures contracts



Notes to Figure 3. *Dotted line*: observed value of u_{5t} for each month t . *Bold line*: predicted value of u_{5t} based on information available as of month $t - 1$ using Markov-switching autoregression and no other explanatory variables. *Gray line*: predicted value of u_{5t} based on information available as of month $t - 1$ using Markov-switching predictive regression autoregression based on 1-year minus 6-month spread.

Figure 4: Predicted interest rates on fed funds futures contracts as of June 30, 2009 under alternative scenarios, in basis points



Notes to Figure 4. *Average*: implied fed funds futures rate if market expects fed funds rate to remain at 18 basis points through the end of 2009 but futures contracts embody historical average risk premia. *Regime 1*: market expects fed funds rate to remain at 18 basis points through the end of 2009 but futures contracts embody risk premia appropriate if June 2009 is known to have been governed by regime 1. *Actual*: actual fed funds futures rates on June 30, 2009.