

The Risks and Rewards of Selling Volatility

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BUYING AND SELLING CERTAIN KINDS OF VOLATILITY-SENSITIVE OPTIONS PORTFOLIOS IS A POPULAR PRACTICE EVEN THOUGH THIS ACTIVITY IS ASSOCIATED WITH SUBSTANTIAL RISK. WHEN SUCH PORTFOLIOS ARE FORMED, TWO FACTORS SIGNIFICANTLY INFLUENCE THE VALUE OF THE OPTIONS: THE PRICE OF THE UNDERLYING ASSET AND THE FUTURE VOLATILITY EXPECTED TO

prevail until the options expire. It is possible to form a portfolio of call and put options so that the portfolio's payoff is very sensitive to the volatility of the underlying asset but only minimally sensitive to changes in the level of the underlying asset. Traders and investors who frequently buy or sell such portfolios do so with a view of the volatility of the underlying asset that does not correspond to the expected future volatility embedded in the option price, or the implied volatility.¹ For example, the former hedge fund Long Term Capital Management had created portfolios of options on certain stock indexes based on its view that expected future volatility was different from the prevailing implied volatilities (Dunbar 1999).

There is often substantial risk, however, in options portfolios set up to exploit a perceived mispricing in the expected volatility of the underlying asset without initial sensitivity to the level of the asset. This risk stems from possible subsequent abrupt changes in the level of the asset price. Changes in volatilities are highly negatively correlated with changes in levels in many asset markets. If a short position in implied volatility on a market is created and a subsequent sharp market decline

results in much higher implied volatility, the value of the implicit short position in volatility could dramatically decrease. For example, the former Barings PLC sustained huge losses from short positions in volatility (initiated by a trader, Nick Leeson) on Nikkei futures as the Nikkei plunged in early 1995 (Jorion 2000).

The objective of this article is to delineate the risks and rewards associated with the popular practice of selling volatility through selling a particular portfolio of options called straddles. Toward this end, the article first examines the statistical properties of the returns generated by selling straddles on the Standard and Poor's (S&P) 500 index. Although it is theoretically possible to construct other options portfolios to make volatility bets (Carr and Madan 1998), straddles are by far the most popular type of portfolio. The article also demonstrates that the usual practice of selling volatility by comparing the observed implied volatility (from option prices) with the volatility expected to prevail (given the history of asset prices) could be flawed. This flaw could arise if the underlying asset has a positive risk premium—that is, if its expected return over a given horizon exceeds the risk-free rate over the

same horizon—and the returns of the underlying asset are negatively correlated with changes in volatility. Thus, basing the decision to sell a straddle on a comparison of seemingly irrational high implied volatilities with much lower expected volatility could itself be an irrational choice.

Straddles

Straddles are very popular but quite volatility-sensitive options portfolios. A straddle consists of a call and put option of the same strike price and maturity so that the strike price is equal to (or very close to) the current price of the underlying asset. The higher the volatility until the option

expires, the higher the expected profits from buying the straddle, and vice versa. For example, suppose an investor buys a European straddle that matures in fifty days, the current price of the underlying stock is \$100, and the strike prices of both the European call and put options are \$100. Under the (BSM) Black-Scholes-Merton model (out-

lined in Black and Scholes 1973 and Merton 1973) with an annualized implied volatility of 20 percent and a risk-free rate of 5 percent, the price of the call option is \$3.29, and the put option price is \$2.61. The cost of the straddle is thus \$5.90 (\$3.29 + \$2.61). If the stock price at the expiration of the option is at least \$105.90 (payoff from the call option is \$5.90, and payoff from the put option is zero) or \$94.10 (payoff from the put option is \$5.90, and payoff from the call option is zero), the buyer breaks even. The greater the stock price is than \$105.90 or the lower it is then \$94.10, the greater the buyer's profits are. Conversely, if the stock price is greater than \$94.10 but less than \$105.90, the seller of the straddle turns in a profit because the revenue generated by the sales exceeds the losses when the straddle expires. Of course, the narrower the dispersion of the possible stock prices at maturity, the higher the seller's profits. The chart shows the payoff from buying the straddle when the options expire.

When the straddle is created, the change in its value is not very sensitive to the change in the value of the underlying asset. The previous example and

the chart clearly show, however, that the higher the volatility of the underlying asset between the present time and option expiration, the higher the potential payoff from a long position in the straddle. A straddle might thus seem to be an ideal vehicle through which to express a view on the future volatility of the underlying asset without necessarily having an initial view on the future direction of the asset price—that is, a trader might buy a straddle if volatility is expected to be high or sell one if volatility is expected to be low.

To see how a straddle is a bet on volatility, assume that the implied volatility (using the BSM model) from a straddle on an equity option that expires in one month is 40 percent (annualized). On the other hand, assume that the maximum historical volatility of the underlying asset that has been observed over any one-month horizon is 30 percent (annualized). Based on this observation, would one sell the straddle—in other words, sell volatility—because it appears to be high? If volatility is constant or evolves deterministically, that is, if the assumptions of the BSM model hold true, it may be tempting to sell the straddle. It has been well documented, however, that in most asset markets volatility evolves randomly through time (Bollerslev, Chou, and Kroner 1992). Even if volatility were constant or predictable, the price of the underlying asset could periodically undergo an abrupt level shift—sometimes referred to as a jump in the price process.² Because of these factors, a risk premium related to the randomness of volatility or unpredictable jumps in asset prices could get incorporated into the price of an option (Bates 1996). The implied volatility determined from the price of an option using the BSM model would thus be contaminated with the relevant risk premium, making any comparison with historical volatility problematic.³

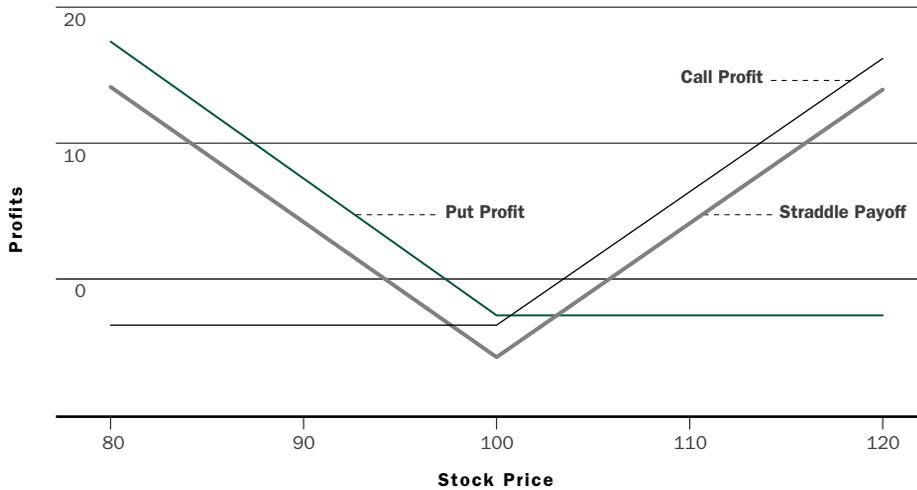
Box 1 gives an example in which the volatility of the underlying asset evolves randomly and the changes in volatility and returns are negatively correlated. The box shows that as long as the underlying asset has a positive risk premium, the implied volatility from the options could be higher than a historical measure of volatility. Because the BSM model may be an inadequate description of the observed option prices, caution is warranted in using it to compare implied volatility to historical volatility.

The Implied and Historical Volatility of the S&P 500

In trading straddles on a market index such as the S&P 500, it is important to have a good understanding of the dynamics of volatility through time. This section offers a computation of the implied

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A Hypothetical Payoff Scenario



Note: Chart shows profits at option expiration from buying the straddle as well as the profits at expiration from buying a call and put option with strike prices of 100.

volatilities from near-the-money options (options in which the strike price is as close as possible to the price of the underlying asset) on the S&P 500 and a simple but frequently used measure of historical volatility for each trading day over a six-year horizon, 1990 through 1995.⁴ On each day, historical volatility is computed as the standard deviation from a sample of the last thirty days of daily S&P 500 returns (close-to-close) and then multiplied by the square root of 252 (as there are about 252 trading days in a year with daily returns assumed to be independently and identically distributed).

The means and standard deviations of the annualized implied volatilities and the annualized historical volatilities are shown in Table 1. This table demonstrates that the implied volatility from near-the-money S&P 500 index options tends to be above (on average, 3 percent) the measure of historical volatility frequently used in practice.⁵ In addition, the minimum and maximum historical volatilities observed over this period tend to be lower than min-

imum and maximum implied volatilities. Someone comparing the implied volatility with the simple measure of historical volatility might often be tempted to sell straddles. Box 1 shows, however, that implied volatilities can be above historical volatilities without any trading opportunities.

The Impact of Correlation and Implied Volatility Skew

Although it is possible to sell straddles based on an incorrect comparison of volatilities, the larger risk from selling straddles often arises from the correlation between returns and volatility as well as large changes in asset price over a short period. In many equity markets, returns and volatility tend to be negatively correlated—volatility goes up if returns go down, and vice versa. From 1990 to 1995, the average correlation between daily returns on the S&P 500 and the daily changes in implied volatilities was -0.525 . For example, on August 23, 1990, as the S&P 500 closed at 307.06, down from its previous

1. Because a very high implied volatility (currently observed from a near-the-money option) is likely to revert to a much lower level over a period of time—a phenomenon known as mean reversion in volatility—selling option portfolios to take advantage of this mean reversion may seem appealing.
2. The stock market crash of 1987 could be viewed as a jump in the price process of an asset.
3. Even if the payoff from the option can be perfectly replicated by trading in the underlying asset and the risk-free asset, as in the BSM model (that is, the option is a redundant security), a risk premium that is related to the riskiness of the underlying asset would still show up in the option values and therefore in the implied volatilities as long as volatility is path-dependent (Heston and Nandi 2000).
4. Options on the S&P 500 are traded at the Chicago Board Options Exchange. The description of the data from which these implied volatilities are computed appears in Box 2.
5. However, if a measure of historical volatility were computed using a model that permits the volatility to be time-varying and unpredictable, it would not necessarily lie below the implied volatility.

Comparing Historical Implied Volatility

If volatility is random or path-dependent, and asset returns and volatilities are negatively correlated, then implied volatilities from option prices could be higher than the observed historical volatility from the underlying asset. Assume that the natural logarithm of the asset price conforms to the following GARCH-type process (Heston and Nandi 2000) over time steps of length Δ :

$$\log[S(t)] = \log[S(t - \Delta)] + r + \lambda h(t) + \sqrt{h(t)} z(t),$$

$$h(t) = \omega + \beta h(t - \Delta) + \alpha [z(t - \Delta) - \gamma \sqrt{h(t - \Delta)}]^2,$$

where r is the continuously compounded risk-free rate for the time interval Δ , $z(t)$ is a standard normal disturbance, and $h(t)$ is the conditional variance of the log return between $t - \Delta$ and t and is known from the information set at time $t - \Delta$. In this model, if γ is positive, there is negative correlation between asset returns and

volatility. In particular, the variance, $h(t)$, depends on the entire path of asset prices until time $t - \Delta$.

Options are typically valued not under the distribution that generates the observed asset prices but under an adjusted distribution called the risk-neutral distribution (Cox and Ross 1976; Harrison and Kreps 1979). Heston and Nandi (2000) show that under negative correlation between returns and volatility ($\gamma > 0$), as long as the expected return of the risky asset exceeds the risk-free rate, the conditional mean of the variance, $h(t)$, under the risk-neutral distribution exceeds the conditional mean of $h(t)$ under the data-generating distribution. The drift of the variance under the risk-neutral distribution tends to be higher than the drift of the variance under the data-generating distribution. Since implied volatility (to a large extent) reflects the drift or the expected value of the variance under the risk-neutral distribution, it is quite possible for the implied volatility to be higher than the expected volatility.¹

1. This assertion ignores the small bias that can arise from Jensen's inequality as option values are nonlinear functions of volatility. However, the degree of nonlinearity tends to be small for at-the-money options.

overnight close of 316.55, the implied volatility from short-maturity straddles (seven to forty days in expiration) went up to 0.322 from the previous day's 0.265.⁶ Similarly, on August 27, 1990, as the S&P 500 closed at 321.44, up from its overnight close of 311.31, the implied volatility from the straddles went down to 0.239 from previous day's 0.307.

If a trader sells volatility through straddles and the market goes down, the value of the short position in the straddles drops considerably, more than the loss incurred on a long position in the market. For example, if a trader had sold a straddle on the S&P 500 index at the beginning of the day on November 11, 1991, and held it through the day as the market went down around 3.6 percent, the action would have incurred a loss of around 9 percent on the straddle position over the day. At the same time, sharp upswings in the asset price could also adversely impact the value of a straddle. As the S&P 500 increased from 343.95 to 356.95 from January 31, 1991, to February 7, 1991, the return from selling straddles on January 31 and closing out on February 7, was -12.2 percent.

Another feature of implied volatilities that could adversely impact the seller of the straddle if the price of the underlying asset goes up is the so-called skew or smirk in implied volatilities. It is well known that the S&P 500 index options market consistently exhibits a skew in implied volatilities—the lower the strike price of an option, the higher the implied volatility, and vice versa (Rubinstein 1994; Nandi and Waggoner 2000). If the S&P 500 rises sharply, the near-the-money call option of the straddle becomes an in-the-money call (the strike is less than the price of the underlying asset). The loss in the value of the short-call position thus would stem not only from the change in the level of the S&P 500 but also perhaps from the skew in implied volatilities because an in-the-money call has a higher implied volatility than a near-the-money call.

Selling Straddles on the S&P 500 Index

An empirical exercise helps gauge the actual risks and rewards associated with selling straddles in a liquid options market over a long enough period. In this empirical exercise, a

TABLE 1
Statistics Computed from Near-the-Money S&P 500 Index Options, 1990–95

	Mean	Standard Deviation	Min	Max
Implied Volatility	0.136	0.039	0.074	0.325
Historical Volatility	0.106	0.036	0.050	0.226

TABLE 2 Returns from Unconditionally Selling Straddles

Days	Annual Mean	Annual Standard Deviation	Skewness	Max	Min
Short-Maturity Straddles					
10	0.231	0.228	-1.79	0.117	-0.292
15	0.385	0.239	-1.67	0.162	-0.295
20	0.334	0.289	-2.12	0.136	-0.424
Medium-Maturity Straddles					
10	0.106	0.187	-2.25	0.116	-0.282
15	0.225	0.221	-2.49	0.145	-0.373
20	0.279	0.239	-2.23	0.172	-0.416

trader sells straddles of two different maturities in the market for S&P 500 index options and then liquidates the positions (buys back the straddles) after a certain number of days.⁷ The market for S&P 500 options is one of the most active options markets in the world. (The description of options data used for the empirical exercise for the 1990–95 period appears in Box 2.) This exercise of selling straddles adheres to all the margin requirements (initial margin and maintenance margin) of the Chicago Board Options Exchange (where S&P 500 options are traded). The round-trip transactions cost in the form of bid-ask spreads is also explicitly recognized because these spreads are not insignificant.

As a first exercise, the trader sells the straddles every day (1990–95) without comparing their implied and historical volatilities. Table 2 shows some statistics of holding period returns generated by selling short- and medium-maturity straddles at the bid prices and then liquidating the positions (buying back the options at the ask prices) after ten, fifteen, and twenty trading days.⁸ In computing these returns, the initial margin the trader must put up according to

exchange rules is treated as the initial investment. The mean and standard deviations of the holding period returns shown in these tables are annualized. The average annual returns from short- and medium-maturity straddles that are liquidated after twenty days are 33.4 percent and 27.9 percent with annualized standard deviations of 28.9 percent and 23.9 percent. In contrast, the corresponding average annual return from holding the S&P 500 index for twenty trading days over this period is 12.9 percent with an annualized standard deviation of 10.9 percent.

This exercise shows that although selling the straddle may appear to achieve risk-adjusted returns comparable to holding the market if standard deviation is used as the measure of risk, the returns from selling the straddles are much more negatively skewed than returns from holding the market: the coefficients of skewness from selling the short- and medium-maturity straddles are -2.12 and -2.23 while that from holding the market is 0.075. The probability of negative returns exceeds the probability of positive returns of equal magnitude. Further, the probability of large negative returns far exceeds

6. The strike price of these straddles is set so that the options are at-the-money-forward, that is, $K = S(t)\exp[r(\tau)\tau]$, where K is the strike price, $S(t)$ is the current asset price, τ is the time to maturity of the option, and $r(\tau)$ is the risk-free interest rate applicable for the maturity.

7. The straddle sold resembles as closely as possible an at-the-money-forward straddle.

8. Short-maturity straddles have maturities of twenty to forty calendar days, and medium-maturity straddles have maturities of forty to seventy-five calendar days.

S&P 500 Data Used in Creating Straddles

The market for S&P 500 index options is one of the most active index options market in the United States; it is also the largest in terms of open interest in options. The data set used for creating the straddles is a subset of the tick-by-tick data on the S&P 500 options that includes both the bid-ask quotes and the transaction prices; the raw data set was obtained directly from the exchange.

As many of the stocks in the S&P 500 index pay dividends, a time series of dividends for the index is required for computing straddles. This case uses the daily cash dividends for the S&P 500 index collected from the S&P 500 information bulletin. The present value of the dividends is subtracted from the current index level. For the risk-free rate, the continuously compounded Treasury bill rate (from the average of the bid and ask discounts reported in the *Wall Street Journal*), interpolated to match the maturity of the option, is used. Because the S&P 500 level that appears with the options records may be stale, this article's computations use index levels implied from the S&P 500 futures traded at the Chicago Mercantile Exchange. The nearest matches for maturity of S&P 500 futures prices (in terms of the difference in time stamps between the options record and the futures record) are used to get the implied S&P 500 index levels. These futures prices are created from tick-by-tick S&P 500 futures data sets obtained from the Futures Industry Institute. Given a discrete dividend series, the following equation (Hull 1997) is used to compute the implied spot price (S&P 500 index level): $F(t) = [S(t) - PVDIV]e^{r(t)(T-t)}$, where $F(t)$ denotes the futures price, $PVDIV$ denotes the present value of divi-

dends to be paid from time t until the maturity of the futures contract at time T , and $r(t)$ is the continuously compounded Treasury bill rate (from the average of the bid and ask discounts reported in the *Wall Street Journal*), interpolated to match the maturity of the futures contract. In terms of maturity, options with less than six days or more than one hundred days to maturity are excluded.¹

An option of a particular moneyness and maturity is represented only once in the sample on any particular day. Although the same option may be quoted again in this time window (with same or different index levels) on a given day, only the first record of that option is included in this sample for that day.

A transaction must satisfy the no-arbitrage relationship (Merton 1973) in that the call price must be greater than or equal to the spot price minus the present value of the remaining dividends and the discounted strike price. Similarly, the put price has to be greater than or equal to the present value of the remaining dividends plus the discounted strike price minus the spot price.

The call and put options that the straddle comprises are chosen each day from the bid-ask quotes after 9:00 A.M. central standard time (CST) so that they are closest to being at-the-money-forward; that is, $K = S(t)\exp[r(\tau)\tau]$, where K is the strike price, $S(t)$ is the current asset price, τ is the time to maturity of the option, and $r(\tau)$ is the risk-free interest rate applicable for the maturity. Only one straddle is formed each day, and on the day of the liquidation (when the straddle is bought back), the option positions are closed out from the bid-ask quotes after 9:00 A.M. CST.

1. See Dumas, Fleming, and Whaley (1998) for a justification of the exclusionary criteria about moneyness and maturity.

the probability of large positive returns. For example, the highest twenty-day return (not annualized) from selling short maturity straddles is 13.6 percent whereas the lowest twenty-day return is -42.4 percent. How important is the negative correlation between returns and volatility in determining some of the extreme negative returns? When the S&P 500 lost around 8 percent over a fifteen-day trading period starting in July 20, 1990, the

implied volatility spiked from 14 percent to around 25.5 percent (a change of around 82 percent), and the return from selling a straddle was -28 percent.

The returns from short positions in the straddles could result from a change in the level of the market or changes in the implied volatilities of the options constituting the straddles. The high returns from selling straddles are related to the correlations between returns from short positions in straddles and the

returns on the S&P 500, as well as the correlations between straddle returns and changes in the implied volatilities over the holding period of the options. In Table 3, $\rho(R_s, R_m)$ denotes the correlation between returns from selling straddles and returns from buying the S&P 500 over the holding period, and $\rho(R_s, IV)$ denotes the correlation between returns from selling the straddles and the changes in the implied volatilities of the options. Table 3 also shows the correlation between returns from selling medium-maturity straddles and buying the S&P 500 as well as the correlation between returns from selling medium-maturity straddles and changes in implied volatilities.

Short positions in straddles stand to gain mainly from the decrease in implied volatilities over the holding period. The correlations between the returns from selling the straddles and market returns tend to be negative, but for some holding periods they are only slightly negative.⁹ In contrast, the correlations between straddle returns and changes in implied volatilities are substantially negative across all holding periods. This finding suggests that the positive returns from the short straddle positions mainly result from decreases in implied volatilities of the options in the straddle. Similarly, the negative returns from selling the straddles primarily result from the increases in implied volatilities of the options in the straddle.

Often, straddles are sold only if the implied volatility from the options exceeds the historical volatility that has been observed over a certain period of time. Thus, instead of selling straddles unconditionally, it is less risky to sell the straddle only if both the put and call implied volatility exceed the thirty-day historical volatility by at least 1 percent.¹⁰ Table 4 shows some of the returns generated by selling short-maturity straddles using this decision rule, based on comparisons between implied and historical volatilities.

The mean returns are actually lower using the trading rule than they would be if the straddles were sold unconditionally. For example, if we liquidate the straddles after fifteen trading days, the mean (annualized) return under the trading rule is 26.1 percent compared to 38.5 percent without the trading rule. At the same time, however, these data suggest that using the mechanical trading rule to sell straddles is

TABLE 3
Correlation between Returns

Days	$\rho(R_s, R_m)$	$\rho(R_s, IV)$
Short-Maturity Straddles		
10	-0.05	-0.61
15	-0.28	-0.71
20	-0.38	-0.89
Medium-Maturity Straddles		
10	-0.09	-0.74
15	-0.08	-0.73
20	-0.20	-0.72

no better than selling the straddles unconditionally: the standard deviations of the returns are a little higher whereas the coefficient of skewness (of the returns) is a little lower using the trading rule. Further, substantial downside risk still remains—the minimum return from selling implied volatility on the S&P 500 over a fifteen-day holding period is -29.5 percent, much lower than could be achieved by being either long or short on the S&P 500 over the same holding period in our sample.

Does Rebalancing Help?

In the preceding trading exercises, as the level of the S&P 500 varied from the strike prices of the options constituting the straddle, the trader did not rebalance the straddle to make it delta-neutral. Delta measures the change in the value of the option relative to the change in the value of the underlying asset (see Box 3 for the delta of a straddle).¹¹ The straddle is only approximately delta-neutral at the beginning, and the value of the straddle becomes sensitive to changes in the level of the S&P 500 as the index starts moving away from its initial level. It is possible that some of the excessive skewness and some of the extreme negative returns from selling the straddles can be reduced or eliminated by rebalancing the straddle to be delta-neutral each day. An option valuation model (such as the BSM model) should be used to arrive at the number of units of the underlying asset that need to be bought or sold

9. One should not necessarily conclude that the correlations between returns from buying the straddles and buying the S&P 500 are positive because initial margins and maintenance margins are taken into account in computing the returns from selling straddles. Buying a straddle does not require any initial or maintenance margins.
10. Historical volatility was also computed from the last forty-five days and sixty days of returns, but the results were not significantly different. Similarly, more sophisticated volatility models, such as the asymmetric GARCH models used in Engle and Ng (1993) and Heston and Nandi (2000), have been used to construct various measures of volatility without any substantial differences in results.
11. Although a straddle may have a low delta, it has a high gamma (that is, the rate of change in delta is high), especially at short maturities.

TABLE 4
Returns from Conditionally Selling Short-Maturity Straddles

Days	Annual Mean	Annual Standard Deviation	Skewness	Max	Min
10	0.181	0.222	-1.24	0.103	-0.181
15	0.261	0.271	-1.33	0.161	-0.295
20	0.199	0.319	-2.06	0.129	-0.413

TABLE 5
Returns from Selling Short-Maturity Straddles with Rebalancing

Days	Annual Mean	Annual Standard Deviation	Skewness	Max	Min
10	0.184	0.364	-1.45	0.265	-0.415
15	0.352	0.542	-0.93	0.638	-0.971
20	0.379	0.611	-0.45	0.854	-0.732

to rebalance the straddle. If the theoretical model used for rebalancing is different from the model generating the observed option prices, however, a delta-neutral position will not really result from rebalancing. Thus, the value of the portfolio may be adversely affected both by a change in the level of the market and volatility. In short, selling straddles and rebalancing to maintain delta-neutrality could expose the seller to model risk.

In the final exercise, in addition to selling a straddle, a certain number of units of the S&P 500 are bought or sold every day according to the BSM model so that the straddle stays delta-neutral.¹² Table 5 shows the various statistics of the three different holding-period returns of the short-maturity straddles from this exercise. (Note that the initial and final values of the options, the initial margin requirement, and cash flows that would accrue from buying/selling the S&P 500 are taken into account in computing the returns.) The table shows that rebalancing reduces the negative skewness of the returns from selling the straddles but makes the standard deviation of the returns go up. Thus, after taking into account both the standard deviation and skewness of returns, rebalancing the portfolio to be delta-neutral does not substantially alter the risk profile from selling straddles.

Since the number of units of the underlying asset that need to be bought or sold for rebalancing is derived from a theoretical option valuation model, the process of rebalancing is subject to model risk. Choosing a valuation model that does not capture the dynamics of option prices very well can lead to results that are no better or worse than without rebalancing.

In this case, the BSM model is used. However, existing research (Bakshi, Cao, and Chen 1997; Nandi 1998; Dumas, Fleming, and Whaley 1998) suggests that even more sophisticated rebalancing schemes such as those using stochastic volatility models may not lead to substantially better results than the simple BSM model in the S&P 500 index options market.

Conclusion

Selling market volatility through selling straddles exposes traders and investors to substantial risk, especially in equity markets. Selling straddles has resulted in substantial losses at banks and hedge funds such as the former Barings PLC and Long Term Capital Management. Although the returns from selling straddles can sometimes be very lucrative, especially if the volatility at which the options are sold quickly reverts to a much lower level, the probability of large negative returns far exceeds the probability of large positive returns. Moreover, the negative correlation between equity market returns and implied volatilities could make the straddle values highly sensitive to the direction of the market. In other words, if a trader sells the volatility implicit in option prices and the market subsequently goes down, the mark-to-market value of the portfolio could significantly decrease as the implied volatility increases. While rebalancing the straddle to maintain minimal exposure to the direction of the market is theoretically feasible, the rebalancing process exposes a trader to model risk and may not always help. In short, selling volatility through selling straddles can be lucrative but is quite risky.

12. See Nandi and Waggoner (2000) for an example that shows how to make a portfolio of options delta-neutral.

The Delta of a Straddle

A short-maturity, at-the-money-forward straddle has a low delta under the BSM model. If Δ_c is the delta of the call option and Δ_p is the delta of the put option, then the delta of the straddle, Δ_s , is

$$\Delta_s = \Delta_c + \Delta_p = 2N(d1) - 1,$$

where $N(d1)$ is the standard normal distribution function and $d1 = 0.5\sigma\sqrt{\tau}$, with σ being the volatility and τ being the time to expiration of the option. Expanding $N(d1)$ using a first-order Taylor expansion around zero yields

$$\begin{aligned} N(d1) &= N(0) + (0.5\sigma\sqrt{\tau})/\sqrt{2\pi} \\ &= 0.5[1 + (\sigma\sqrt{\tau})/\sqrt{2\pi}]. \end{aligned}$$

Hence, $\Delta_s \approx (\sigma\sqrt{\tau})/\sqrt{2\pi}$.

As an example, the table shows the delta of a straddle with thirty days ($\tau = 30/365$) to maturity for a few values of annualized σ .

Delta of a Straddle with Thirty Days to Maturity

Value of Annualized Volatility (σ)	Delta (Δ_s)
0.10	0.0114
0.15	0.0171
0.20	0.0228

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