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When, Why, How Big, and for Whom?**

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Abstract: Until recently, the trend in world capital markets has been toward increasing "globalization." Recent events in Latin America and Asia have forced a rethinking of the desirability of unrestricted world capital flows. In this paper we ask whether simple restrictions on capital mobility can succeed in reducing the volatility of funds flows, whether such restrictions are consistent with the long-term development of the countries that might impose them, whether such restrictions are beneficial for poorer countries while harming wealthier countries, and whether barriers to capital movements should be reduced in magnitude as the development process proceeds.

We find first that appropriately selected barriers to capital movements can be used by a poorer country to eliminate the short-term volatility of capital flows and other economic volatility as well. Second, we find that these barriers are consistent with *increased* rather than reduced levels of economic development in both the short and long run. Third, we show that it is empirically plausible that such barriers will be reduced over time as economies develop. Fourth, we show that, in the long run, all countries can benefit from the presence of barriers to capital mobility. And, fifth, we show that barriers to capital mobility can *increase* the magnitude of net capital flows in a steady state.

JEL classification: F21, F34, G18

Key words: barriers, international capital flows, volatility

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Barriers to International Capital Flows: When, Why, How Big, and for Whom?

1. Introduction

Until recently, the trend in world capital markets has been towards increasing "globalization." During the 1990's even economies with long and successful histories of barriers to capital flows[1] have moved in the direction of opening their financial systems to largely unrestricted inflows and outflows of funds.

Recent events in Latin America and Asia have caused many in academic and policy-making circles to question whether this trend should be wholly, or at least partially, reversed. Indeed, it is now commonplace to see IMF and World Bank officials advocating the desirability of various barriers to free capital mobility. And, it is commonly argued that -at a minimum - countries should be given the discretion to erect such barriers, at least in certain circumstances.

Recent events, then, have forced a rethinking of the desirability of unrestricted world capital flows. And, the events that have been central to this rethinking are the frequently observed, massive reversals in capital flows to the developing world. But, if by some criteria, restrictions on international capital movements are desirable, this raises several questions. Can simple restrictions on capital mobility succeed in reducing the volatility of funds flows? Are such restrictions consistent with the long-term development of the countries that might impose them? And, if such restrictions are beneficial for poorer countries, do they necessarily harm wealthier countries? Finally, if the volatility of short-run capital movements is primarily a problem for economies in the relatively earlier stages of development, does this imply that barriers to capital movements should naturally be reduced in magnitude as the development process proceeds?

At this point, there is relatively little available in the way of analytical frameworks for evaluating the answers to these questions.[2] What features should such a framework have? First, the enormous volatility in international capital flows has

[1] For instance Korea and Taiwan.

[2] See Eichengreen, Tobin and Wyplosz (1995) for an exception applied to foreign exchange markets.

proven difficult to explain based on fundamentals alone.[3] Thus a model that is useful for evaluating the consequences of barriers to capital flows should contain a mechanism whereby the unrestricted operation of world capital markets can – perhaps dramatically– amplify economic volatility. Second, while sharp reversals in capital flows have received the most attention, there are other features of international capital markets that are of concern to developing countries. For instance, about two-thirds of the world's poorest economies receive no extension of private long-term, non guaranteed credit.[4] And, with the exception of short episodes, such as 1990-96, for extended periods of time the net average flow of capital has been from poorer to wealthier countries.[5] Thus not only do capital flows tend to be very volatile in the short-run, but the unrestricted operation of international capital markets appears to act to the detriment of long-term development in poor economies. In our view, a model that is useful for analyzing the consequences of barriers to capital mobility should also be able to explain these observations.

In this paper we put forward a framework capable of explaining why capital might, on net, flow from poorer to richer economies. This framework also has the feature that the unrestricted operation of the international financial system can easily lead to endogenously generated volatility in capital flows, and in all other endogenous variables. We then use the model to analyze the consequences of barriers to capital mobility. The barriers we analyze are meant to crudely capture several features of the widely praised "Chilean model." And, we are able to obtain the following results.

First, appropriately selected barriers to capital movements can be used by a poorer country to eliminate the short-term volatility of capital flows and other economic volatility as well. Second, these barriers are consistent with increased rather than reduced levels of economic development, in both the short and long-run. Thus developing economies need not face a trade-off whereby short-term volatility can be reduced only at the expense of long-term development. Long-run real performance can be improved at the same time that short-run fluctuations are reduced. Third, there is no presumption that the barriers to capital mobility will or should be reduced as the development process proceeds. However, we show that it is empirically plausible that such barriers will indeed be reduced over time

[3] See, for example, Calvo et.al. (1993), Chuhan, Claessens and Mamingi (1993), and Fernandez-Arias (1994).

[4] See, for example, the World Development Report 1989, table 21. And, this has been true for the last 30 years.

[5] See the Human Development Report 1992, table 4-3, and the World Economic Outlook, May 1999.

as economies develop. Fourth, we show that, in the long-run, all countries can benefit from the presence of barriers to capital mobility. And, fifth, we show that barriers to capital mobility can actually <u>increase</u> the magnitude of net capital flows, in a steady state.

Finally, we consider an issue that, so far as we know, has remained entirely unaddressed. If it is desirable to erect barriers to capital flows, who should erect them? The general presumption appears to be that the "victims" of highly volatile capital flows should be allowed to limit or restrict inflows and outflows of funds. But, outflows of funds from smaller and less developed economies often represent inflows of funds to larger and more developed economies. This raises the issue of whether there would be benefits associated with larger and wealthier economies taking actions to limit capital mobility? We show that, in fact, it is a matter of indifference who erects the barriers to capital flows. The consequences of any barriers erected by a developing country can be replicated by having a developed country implement some analogous restrictions on capital mobility.

Our vehicle for analyzing these issues is a two country model where both countries are identical except, possibly, for their policies with respect to capital flows and their initial capital stocks. In particular, each country produces a single final good using the same constant returns to scale technology with capital and labor as inputs. In each country capital investment requires some credit finance. This credit extension is complicated, in both domestic and world markets, by the presence of a standard informational friction. In order to bring the consequences of this friction to the forefront, we abstract from other features of real world economies that might lead to capital flight from certain countries, or to an international diversification motive for investors.

In this context, Boyd and Smith (1997) demonstrated that there would, under weak conditions, exist locally stable steady state equilibria where one country is permanently wealthy and one is permanently poor. Indeed, this is true even though all conditions implying the equality of long-run income levels are satisfied if the economies are closed, or if the informational friction is absent. Moreover, Boyd and Smith show that, in such steady states, the world is poorer than it would be if all economies were closed. And, the pattern of international funds flows is necessarily perverse: that is, poor countries are net investors in rich countries. Finally, Boyd and Smith illustrate that the unrestricted operation of world capital markets can easily lead to endogenously generated volatility along equilibrium paths that approach a steady state. Large fluctuations in incomes, capital flows, and rates of return can be observed along such paths. Such a finding, of course, suggests why we might observe so much volatility in world capital markets that seems hard to explain based on fundamentals alone. And the Boyd-Smith results suggest that endogenously arising volatility may predominantly affect the poorer country.

These results raise the possibility that barriers to capital flows are, indeed, "desirable," at least for some countries. Here we explore the consequences of some such barriers, which are crudely based on those employed by Chile and Malaysia. We then derive the results described above. And, finally, we establish sufficient conditions under which increased barriers to capital mobility increase the ex ante expected steady state utility of <u>all</u> residents in both the rich and poor countries. Obviously, the satisfaction of such conditions implies that the erection of such barriers may be beneficial –in the long-run– not just to the residents of the country erecting the barriers. Indeed, such barriers can –in the long-run– have positive consequences for other economies.

The remainder of the paper proceeds as follows. Section 2 describes the economic environment that we consider, and section 3 discusses trade in world credit and factor markets. Section 4 examines the model in the absence of any barriers to capital flows. Section 5 introduces such barriers, loosely motivated by Chile's and Malaysia's widely discussed effective taxation of certain investments by foreigners. It then analyzes the shorter-run consequences of restrictions on capital flows, while Section 6 considers some implications of these restrictions for steady state equilibria including an analysis of their implications for steady state welfare. Section 7 shows that, in order to eliminate "excess volatility," barriers to capital flows must evolve over time. Section 8 briefly considers the consequences of who it is that erects barriers to capital mobility, and some concluding remarks are offered in section 9.

2. The Model

2.1. Environment

Consider a world economy consisting of two countries, each populated by an infinite sequence of two-period lived, overlapping generations. In each country, every generation is identical in size and composition, and contains a continuum of agents with unit mass. Within each generation, agents are divided into two types: "potential borrowers" and "lenders". A fraction $\alpha \in (0, 1)$ of the population is potential borrowers; this fraction is the same in each country.

Let t = 0,1,... index time. At date t a single final good is produced in each country, using a constant returns to scale technology with capital and labor of that country as inputs. We assume the final good to be mobile across borders, while factors of production are immobile. Let K_t^i $(i = 1, 2)$ denote the time t capital input, and L_t^i denote the time t labor input of a representative firm of country i. Then final output is $Y_t^i = F(K_t^i, L_t^i)$. The production function F satisfies the following conditions: it is increasing in each argument, strictly concave, and $F(0, L) = F(K, 0) = 0$ holds, for all K, L. In addition, if $k^i \equiv \frac{K^i}{L^i}$ is the capital-labor ratio of country i, and if $f(k^i) \equiv F(k^i, 1)$ denotes the intensive production function, then $f' > 0 > f''$ holds $\forall k^i$, and f satisfies the standard Inada conditions. Finally, we assume that the inherited capital stock at date t is used in production, and that thereafter it depreciates completely. Notice that the final goods production technology is the same in each country.

With respect to endowments, all young agents are endowed with one unit of labor, which is supplied inelastically, while agents are retired when old. Individuals other than the old of period zero have no endowment of capital or final goods, while the initial old agents of country i have an aggregate capital endowment of $K_0^i > 0$.

Agents of all types are assumed to care only about old age consumption and, in addition, all agents are risk neutral. Thus all young period income is saved.[6]

Potential borrowers and lenders are differentiated by the fact that each potential borrower has access to a stochastic linear technology for converting date t final goods into date $t + 1$ capital. Lenders have no access to this technology.

The capital investment technology has the following properties. First, it is indivisible and nontradable: each potential borrower has one investment project which can only be operated at the scale q. In particular, $q > 0$ units of the final good invested in one project at t yield zq units of capital at $t+1$, where z is an iid (across borrowers and periods) random variable, which is realized at $t+1$. We let G denote the probability distribution of z, and assume that G has a differentiable density function g with support $[0, \bar{z}]$. We let $\hat{z}$ denote the expected value of z, i.e., $\hat{z} \equiv \int_0^{\bar{z}} zg(z)dz$. We also assume that q and the probability distribution of investment returns are the same in each economy.

The amount of capital produced by any investment project can be observed costlessly by the project owner. Any agent other than the project owner can

[6] The fact that savings is supplied inelastically implies that the taxation of capital income will have no effect on the steady state capital stock in a closed economy version of our model. As we will see, this will not typically be the case in open economies.

observe the project return only by bearing a fixed cost of $\gamma > 0$ units of capital.[7]

3. Trade

3.1. Factor Markets

We assume that capital and labor are traded in competitive domestic markets at each date. Thus, if w_t^i denotes the time t real wage rate and ρ_t^i is the time t capital rental rate in country i, the standard factor pricing relationships obtain:

$$\rho_t^i = f'(k_t^i) \tag{3.1}$$
$$w_t^i = f(k_t^i) - k_t^i f'(k_t^i) \equiv w(k_t^i). \tag{3.2}$$

Notice that $w'(k) > 0$ holds and, in addition, we will assume the following.

Assumption 1. $w''(k) < 0; \forall k \geq 0.$

Assumption 1 is satisfied if, for example, f is any CES production function with elasticity of substitution no less than one. Assumption 1 guarantees the uniqueness of a non-trivial steady state equilibrium in a closed economy version of the model.

3.2. Credit Markets

All young agents in country i at t supply one unit of labor inelastically, earning the real wage rate w_t^i. For lenders this income is supplied inelastically in world credit markets. We can think of all credit extension as being intermediated in the manner described by Williamson (1986).

Potential borrowers also have young period income w_t^i, and we assume they must obtain external financing to operate their investment projects.

Assumption 2. *$q > w(k_t^i)$ for all "relevant" values of k_t^i.*

Let b_t^i denote the amount borrowed by the operator of a funded project (in real terms) in country i at t; clearly

$$b_t^i = q - w(k_t^i). \tag{3.3}$$

[7] That is, in verifying the project return, γ units of capital are used up. The assumption that capital is consumed in the verification process follows Bernanke and Gertler (1989). This cost is again identical across countries.

To obtain external funding borrowers announce loan contract terms. These terms are either accepted or rejected by intermediaries: borrowers whose terms are accepted then receive funding and operate their projects. Following Williamson (1986, 1987), a loan contract consists of the following objects. One is a set of project return realizations A_t^i for which verification of the return occurs in country i at t. Verification does not occur if $z \in B_t^i \equiv [0, \bar{z}] - A_t^i$.[8] Second, if $z \in A_t^i$, there is a promised payment (per unit borrowed) $R_t^i(z)$. If $z \in B_t^i$, then the loan contract offers an uncontingent payment of x_t^i (per unit borrowed). All payments specified by any loan contract are in real terms.

Loan contracts offered by borrowers are either accepted or rejected by intermediaries who can be thought of as making all loans. In addition intermediaries take deposits and conduct monitoring of project returns as called for by loan contracts. Any lender can establish an intermediary. In equilibrium, intermediaries will be perfectly diversified, earn zero profits, and have a nonstochastic return on their portfolios. Thus they need not be monitored by their depositors.

Intermediaries behave competitively in deposit markets and therefore take the gross rate of interest on deposits at t, r_{t+1}, as given.[9] It follows that intermediaries are willing to accept loan contract offers yielding an expected return of at least r_{t+1}. Loan contract offers must therefore satisfy the expected return constraint

$$\int\limits_{A_t^i} [R_t^i(z) b_t^i - \rho_{t+1}^i \gamma] g(z) dz + x_t^i b_t^i \int\limits_{B_t^i} g(z) dz \geq r_{t+1} b_t^i. \tag{3.4}$$

Notice that expected repayments must at least cover the intermediary's cost of funds - $r_{t+1} b_t^i$ - plus the real expected monitoring cost

$$\rho_{t+1}^i \gamma \int\limits_{A_t^i} g(z) dz.$$

The latter term depends on ρ_{t+1}^i because γ units of *capital* are expended when project returns are verified. Finally, project owners must have the proper incentives to correctly reveal when a monitoring state has occurred. The appropriate incentive constraint is

$$R_t^i(z) \leq x_t^i; \quad z \in A_t^i. \tag{3.5}$$

[8] We thus abstract from stochastic state verification. Boyd and Smith (1994) show that the welfare gains from stochastic monitoring are trivial when realistic parameter values are assumed.

[9] Since funds are mobile internationally, this rate of return must be the same in each country.

In addition, the repayments specified by any contract must be feasible for the borrower, so that

$$R_t^i(z) \leq \frac{\rho_{t+1}^i zq}{b_t^i}; \quad z \in A_t^i \tag{3.6}$$

$$x_t^i \leq \inf_{z \in B_t^i} [\frac{\rho_{t+1}^i zq}{b_t^i}]. \tag{3.7}$$

Equations (3.6) and (3.7) require that repayments never exceed the real value of the capital yielded by an investment project, which in state z is $zq\rho_{t+1}^i$ at $t+1$.

Borrowers will maximize their own expected utility by choice of contract terms, subject to the constraints just described. Therefore, announced loan contracts at date t will be selected to maximize

$$\rho_{t+1}^i \hat{z} q - b_t^i \int_{A_t^i} R_t^i(z) g(z) dz - x_t^i b_t^i \int_{B_t^i} g(z) dz$$

subject to (3.4)-(3.7).

The solution to the borrower's problem is to offer a standard debt contract. In particular, the borrower either repays x_t^i (principal plus interest) or else defaults. In the event of default the lender monitors the project, and retains its proceeds net of monitoring costs. Formally, we have[10]

Proposition 3.1. *Suppose $q > b_t^i$. Then the optimal contractual loan terms satisfy*

$$R_t^i(z) = \frac{\rho_{t+1}^i zq}{b_t^i}; \quad z \in A_t^i \tag{3.8}$$

$$A_t^i = [0, \frac{x_t^i b_t^i}{q\rho_{t+1}^i}) \tag{3.9}$$

$$r_{t+1} = \int_{A_t^i} \left[R_t^i(z) - \frac{\rho_{t+1}^i \gamma}{b_t^i} \right] g(z) dz + x_t^i \int_{B_t^i} g(z) dz. \tag{3.10}$$

[10] The proof of proposition 3.1 is standard: See Gale and Hellwig (1985) or Williamson (1986).

For future reference, the expected return received by a lender under the optimal contract, per unit of funding, is given by

$$\begin{aligned}
&\int\limits_{A_t^i} \left[R_t^i(z) - \frac{\rho_{t+1}^i \gamma}{b_t^i}\right] g(z)dz + x_t^i \int\limits_{B_t^i} g(z)dz \\
= \quad & x_t^i - \frac{\rho_{t+1}^i \gamma}{b_t^i} G(\frac{x_t^i b_t^i}{\rho_{t+1}^i q}) - \frac{\rho_{t+1}^i q}{b_t^i} \int\limits_0^{\frac{x_t^i b_t^i}{\rho_{t+1}^i q}} G(z)dz \equiv \pi\left[x_t^i; \frac{b_t^i}{\rho_{t+1}^i}\right]. \qquad (3.11)
\end{aligned}$$

The function π gives the expected return to the lender as a function of the gross loan rate, x_t^i, the amount of external finance required, b_t^i, and the future relative price of capital, ρ_{t+1}^i.

It will be useful to make some assumptions regarding the function π. Following Williamson (1986, 1987) we assume the following:

Assumption 3. $g(z) + (\frac{\gamma}{q})g'(z) \geq 0$; *for all* $z \in [0, \bar{z}]$.

Assumption 4. $\pi_1[0, (\frac{b_t^i}{\rho_{t+1}^i})] > 0$.

Assumption 3 implies that $\pi_{11} < 0$. Assumptions 3 and 4 imply that π has the configuration depicted in Figure 1. Evidently, depending on the value of b_t^i/ρ_{t+1}^i, there is a unique value of x_t^i which maximizes the expected return that can be offered to any lender. We will denote this value by $\hat{x}(b_t^i/\rho_{t+1}^i)$. The function $\hat{x}$ satisfies

$$\pi_1\left[\hat{x}(\frac{b_t^i}{\rho_{t+1}^i}); \frac{b_t^i}{\rho_{t+1}^i}\right] \equiv 1 - (\frac{\gamma}{q})g\left[\hat{x}(\frac{b_t^i}{\rho_{t+1}^i})\frac{b_t^i}{\rho_{t+1}^i q}\right] - G\left[\hat{x}(\frac{b_t^i}{\rho_{t+1}^i})\frac{b_t^i}{\rho_{t+1}^i q}\right] \equiv 0. \quad (3.12)$$

Equation (3.12) and Assumption 3 imply that

$$\hat{x}\left(\frac{b_t^i}{\rho_{t+1}^i}\right)\frac{b_t^i}{\rho_{t+1}^i q} \equiv \eta \qquad (3.13)$$

where $\eta > 0$ is a constant satisfying $1 - (\frac{\gamma}{q})g(\eta) - G(\eta) \equiv 0$. When all potential borrowers are offering the interest rate $\hat{x}\left(\frac{b_t^i}{\rho_{t+1}^i}\right)$, η is the critical project return for which a borrower's project income exactly covers loan principal plus interest. In other words, a default occurs if $z \in [0, \eta)$.

3.3. Credit Rationing

As noted by Gale and Hellwig (1985) and Williamson (1986, 1987), in this environment it is possible for credit to be rationed in either or both countries. In particular, if all borrowers want to operate their projects at date t, the total (per capita) demand for funds in each country is αq. The total per capita supply of saving is $w(k_t^i)$ in country i at t. World credit demand exceeds world credit supply, and hence credit must be rationed in the world economy, if the following assumption holds for all $t \geq 0$.

Assumption 5. $2\alpha q > w(k_t^1) + w(k_t^2)$.

When credit is rationed in country i, it also must be the case that

$$x_t^i = \hat{x}(\frac{b_t^i}{\rho_{t+1}^i}) \equiv \hat{x}\left[\frac{q - w(k_t^i)}{\rho_{t+1}^i}\right]. \tag{3.14}$$

In particular, all potential borrowers must offer the interest rate that maximizes a prospective lender's expected rate of return. Rationed (unfunded) potential borrowers cannot then obtain credit by changing loan contract terms, since doing so simply reduces the expected return perceived by (all) lenders. Thus if assumption 5 and equation (3.14) hold at date t, credit rationing is an equilibrium outcome for the world economy. We henceforth focus on the case where credit rationing occurs at all dates in both countries.[11]

3.3.1. Payoffs Under Credit Rationing

We now describe the expected payoffs of lenders and (funded) borrowers at t when credit is rationed. Equations (3.11) and (3.14) imply that the expected return on bank loans is given by

$$\pi\left[\hat{x}(\frac{b_t^i}{\rho_{t+1}^i}); \frac{b_t^i}{\rho_{t+1}^i}\right] \equiv \frac{\rho_{t+1}^i q}{b_t^i}\left\{\hat{x}(\frac{b_t^i}{\rho_{t+1}^i})\frac{b_t^i}{\rho_{t+1}^i q} - (\frac{\gamma}{q})G\left[\hat{x}(\frac{b_t^i}{\rho_{t+1}^i})\frac{b_t^i}{\rho_{t+1}^i q}\right] - \int_0^{\frac{\hat{x}b_t}{\rho_{t+1}q}} G(z)dz\right\}$$

[11]The assumption that credit is rationed results in a substantial technical simplification. However, credit rationing is common in developing countries (McKinnon, 1973), and there is evidence of significant rationing of credit even in the United States (Japelli, 1990). Therefore this does not seem to be an empirically unreasonable assumption. We comment below on how the analysis would need to be modified if credit rationing were observed in only one (the poorer) country.

$$\equiv \quad \frac{\rho^i_{t+1}q}{b^i_t}\left[\eta - (\frac{\gamma}{q})G(\eta) - \int_0^{\eta} G(z)dz\right] \equiv \psi\frac{\rho^i_{t+1}q}{b^i_t}. \tag{3.15}$$

Note that the return to a lender between t and $t+1$ is proportional to the ratio ρ^i_{t+1}/b^i_t when credit rationing obtains. The expected utility of a funded borrower under credit rationing is given by

$$\rho^i_{t+1}\hat{z}q - r_{t+1}b^i_t - \rho^i_{t+1}\gamma G\left[\hat{x}(\frac{b^i_t}{\rho^i_{t+1}})\frac{b^i_t}{\rho^i_{t+1}q}\right] \equiv \rho^i_{t+1}q\left[\hat{z} - (\frac{\gamma}{q})G(\eta)\right] - r_{t+1}b^i_t.$$

Since any potential borrower could always forego operating his project and deposit his income in a bank, all potential borrowers can guarantee themselves the utility level $r_{t+1}w(k^i_t)$. Potential borrowers then prefer borrowing to lending iff

$$\rho^i_{t+1}q\left[\hat{z} - (\frac{\gamma}{q})G(\eta)\right] - r_{t+1}b^i_t \geq r_{t+1}w(k^i_t). \tag{3.16}$$

We now define

$$\phi \equiv \hat{z} - (\frac{\gamma}{q})G(\eta). \tag{3.17}$$

The variable ϕ represents the expected project yield per unit invested, under credit rationing, net of capital consumed by monitoring.[12] Then equation (3.16) can be reduced to

$$\phi\rho^i_{t+1} \geq r_{t+1}. \tag{3.18}$$

If (3.18) fails, then borrowers will prefer not to borrow.

4. The Open Economy

As a point of reference we begin by considering equilibria when there are no barriers to international capital flows. For reasons of technical simplicity, we also restrict attention to the case where credit is rationed in both countries.[13]

With unrestricted international borrowing and lending, clearly one equilibrium condition is that the expected returns offered by borrowers be the same in each

[12] Of course, we assume that $\phi > 0$.

[13] See Huybens and Smith (1998) for some discussion of the relaxation of this assumption in the case of a small open economy.

country for $t \geq 0$. Given the form of the expected return function, this condition obtains iff

$$\frac{f'(k^1_{t+1})}{q - w(k^1_t)} = \frac{f'(k^2_{t+1})}{q - w(k^2_t)}. \tag{4.1}$$

The second condition of equilibrium is that world saving equal world investment. Let μ^i_t be the fraction of potential borrowers who obtain credit in country i at t. Then an equality between sources and uses of funds requires[14]

$$\alpha q(\mu^1_t + \mu^2_t) = w(k^1_t) + w(k^2_t). \tag{4.2}$$

Since there is no aggregate uncertainty, the time $t+1$ capital stock available for use in worldwide production, not inclusive of capital expended on state verification, is $\widehat{z}\alpha q(\mu^1_t + \mu^2_t) = \widehat{z}[w(k^1_t) + w(k^2_t)]$. Total capital expenditures on state verification are $\alpha\gamma G(\eta)(\mu^1_t + \mu^2_t) = (\gamma/q)G(\eta)[w(k^1_t) + w(k^2_t)]$. In equilibrium, therefore,[15]

$$k^1_{t+1} + k^2_{t+1} = \phi[w(k^1_t) + w(k^2_t)]. \tag{4.3}$$

Finally, to guarantee that credit is rationed in each country, we require

$$\max(k^1_t, k^2_t) < \phi\alpha q$$

for all $t \geq 1$.

[14] Notice that unfunded borrowers do not wish to consume their income when young. Hence each unfunded borrower saves $w(k^i_t)$ to consume when old.

[15] Suppose that credit is not rationed (at any date) in country 1, and define $\theta_t \equiv x^1_t b^1_t / q\rho^1_{t+1}$. Then the equilibrium conditions (4.1) and (4.2) must be replaced with

$$f'(k^1_{t+1})\left[\theta_t - (\frac{\gamma}{q})G(\theta_t) - \int_0^{\theta_t} G(z)dz\right] = \frac{[q - w(k^1_t)]\, f'(k^2_{t+1})}{[q - w(k^2_t)]}\left[\eta - (\frac{\gamma}{q})G(\eta) - \int_0^{\eta} G(z)dz\right],$$

$$k^1_{t+1} = \alpha q\left[\hat{z} - (\frac{\gamma}{q})G(\theta_t)\right], \text{ and } k^2_{t+1} = \phi\left\{w(k^1_t) + w(k^2_t) - \alpha q\right\}$$

Solution sequences to this system would have to satisfy $\theta_t \leq \eta$, $k^1_t \geq \phi\alpha q$, and $k^2_t < \phi\alpha q$, $\forall t$. Because this system of equations is not symmetric in k^1_t and k^2_t, it is much harder to analyze than the system consisting of (4.1) and (4.3).

4.1. Steady State Equilibria

We begin by examining steady states. Dynamical equilibria are discussed in the next section. To facilitate our analysis, we define the function $H(k)$ by

$$H(k) = \frac{f'(k)}{[q - w(k)]}. \tag{4.4}$$

In order to characterize steady states, it will be useful to know more about the function $H(k)$. Some of its properties are stated in the following lemma.[16]

Lemma 4.1. *The function H satisfies*
(a) $\lim_{k\to 0} H(k) = \infty,$
(b) $\lim_{k\to \hat{k}} H(k) = \infty,$ *where* $\hat{k} \equiv w^{-1}(q),$
(c)$H'(k) \le 0; k \le f^{-1}(q),$ *and*
(d)$H'(k) \ge 0; k \ge f^{-1}(q).$

Lemma 4.1 implies that the function H has the configuration depicted in Figure 2.

In a steady state, (4.1) and (4.3) reduce to

$$k^1 + k^2 = \phi[w(k^1) + w(k^2)]. \tag{4.5}$$

$$H(k^1) = H(k^2). \tag{4.6}$$

Boyd and Smith (1997) show that equations (4.5) and (4.6) defined loci as depicted in Figure 3. As is apparent from the figure, there is the distinct possibility that two kinds of steady state equilibria exist. (i) One is a "symmetric" steady state in which $k^1 = k^2 = k^*$, where $k^* \equiv \phi w(k^*) > 0$ is the steady-state equilibrium level of the capital stock for a closed economy. The capital stocks in each country coincide with the nontrivial steady state capital stocks that would be observed in steady state if each country were closed.[17] (ii) There may also be "asymmetric" steady states in which $k^1 \neq k^2$. Boyd and Smith (1997) establish the following properties of asymmetric steady states.
(a) If $k^1 > k^2$, $k^1 > k^* > k^2$ holds.

[16] See Boyd and Smith (1997) for a proof.

[17] This is to be expected, since the economies are identical in all respects except for their initial capital stocks. Assumption 1 implies that there is a unique nontrivial symmetric steady state.

(b) In a steady state with $k^1 > k^2$, there are net capital flows *from* country 2 *to* country 1. That is, the poor country is a net investor in the rich country.[18]
(c) In an asymmetric steady state, $k^1 + k^2 < 2k^*$ holds. Thus the world as a whole is poorer than it would be if both countries were closed economies.[19]
(d) An asymmetric steady state exists[20] if $f(k^*) > q$ holds.

Property (a) asserts that wealthy economies are wealthier –and poor countries are poorer – as open economies than they would be as closed economies. This gives a clear sense in which unrestricted international capital markets operate to the long-run detriment of less developed countries. And it suggests that such countries might want to restrict the operation of such markets for reasons that have nothing to do with –or for reasons in addition to – the short-term volatility of capital flows. Property (b) indicates that this impoverishment of poorer countries occurs because of perverse long-run flows of capital from poor to rich economies. Evidence that this pattern of long-run financial flows is actually observed– at least for sufficiently poor countries– was cited in the introduction. Property (c) indicates that international capital flows not only make poor countries poorer, but that they make the world as a whole poorer. This property at least suggests the possibility that –from a steady state perspective– all countries can benefit from the erection of some barriers to capital mobility. Finally, property (d) indicates that asymmetric steady states will exist if, in closed economies, the share of capital investment financed internally exceeds labor's share of income.[21] In the U.S., the U.K., and Germany such a situation does indeed obtain, and it approximately obtains in France and Japan.[22] Thus the existence of locally stable asymmetric steady states is, in fact, an empirically plausible situation.

Together these conditions imply that the income level of the initially poorer economy need not converge to that of the initially wealthier economy. Such a result is consistent with the findings of Parente and Prescott (1993) that poorer

[18] Formally, $k^1 - \phi w(k^1) > 0 > k^2 - \phi w(k^2)$ holds.

[19] This result depends on the assumed concavity of $w(k)$. Under this assumption, world savings (and hence the world capital stock) is maximized by allocating a fixed stock of capital equally across countries. Parenthetically, $w''(k) < 0$ holds if $f(k)$ is any CES production function with elasticity of substitution no less than one.

[20] In fact, at least two asymmetric steady states exist under these conditions, since asymmetric steady states come in pairs.

[21] To see this, observe that the share of investment financed internally in a closed economy is $w(k^*)/q$, in a steady state. And, $f(k^*) > q$ holds iff $w(k^*)/q > w(k^*)/f(k^*)$.

[22] See Hamid and Singh (1992) for figures on the fraction of investment finaced internally in each country.

economies tend to retain their income level relative to the U.S. over time. Together with the results on local dynamics described below, these findings indicate why some countries might be tempted to erect complete or partial barriers to international capital flows.

4.2. Local Dynamics

In the case where $f(k) = Dk^{\delta}$ (Cobb-Douglas production), Boyd and Smith (1997) show that (a) the symmetric steady state is a saddle. Its stable manifold is the locus $k_t^1 = k_t^2$. It is not possible to converge to the steady state if $k_0^1 \neq k_0^2$. (b) They provide sufficient conditions under which an asymmetric steady state will be a sink. Together these results imply that an initially poor economy can be permanently impoverished by the operation of unrestricted world capital markets. Intuitively, borrowers in the rich country have relatively high incomes, and hence they have a superior ability to provide internal finance for their own projects. In the presence of the CSV problem, as noted by Bernanke and Gertler (1989), the ability to provide internal finance acts to mitigate financial market frictions. The Boyd-Smith results specify conditions under which this factor is so important that it can never be overcome by an initially poor economy, at least if international capital flows are unrestricted.[23]

As we have noted, there is considerable evidence that the observed volatility of international capital flows cannot easily be understood by reference to the behavior of economic "fundamentals." The economy considered here, however, is perfectly capable of giving rise to endogenous, or "market-generated" volatility. The model is therefore consistent with the high observed volatility of capital flows, as we now illustrate.

To analyze local dynamics at any steady state, we replace (4.1) and (4.5) with the linear approximation

$$(k_t^1 - k^1, k_t^2 - k^2)' = J(k_{t-1}^1 - k^1, k_{t-1}^2 - k^2)',$$

where J is the Jacobian matrix

$$J = \begin{bmatrix} \frac{\partial k_t^1}{\partial k_{t-1}^1} & \frac{\partial k_t^1}{\partial k_{t-1}^2} \\ \frac{\partial k_t^2}{\partial k_{t-1}^1} & \frac{\partial k_t^2}{\partial k_{t-1}^2} \end{bmatrix}$$

[23] As indicated by Gertler and Rogoff (1990), the same forces are likely to be at work in an economy where investment is subject to a moral hazard problem.

with all partial derivatives evaluated at the appropriate steady state. The properties of local dynamics are then governed by the eigenvalues of J.

Suppose that the common production function has the Cobb-Douglas form $f(k) = Dk^{\delta}$, with $\delta \in (0,1)$. Then Boyd and Smith (1997) establish that the determinant and trace of J satisfy

$$Det(J) = \delta\left(\frac{1+y^{\delta-1}}{1+y^{\delta}}\right)\left[\frac{\delta f(k^2)}{q-(1-\delta)f(k^2)}\right]$$

$$Trace(J) = \delta + \left[\frac{2}{\delta(1+y)}\right]\left(\frac{1+y^{\delta}}{1+y^{\delta-1}}\right)Det(J)$$

where $y \equiv k^2/k^1$ is the ratio of the two capital stocks. In addition, the eigenvalues of J, λ_1 and λ_2, are given by the relations $\lambda_1\lambda_2 = Det(J)$, and $\lambda_1+\lambda_2 = Trace(J)$.

It is quite difficult to derive general results about the eigenvalues of J, and hence about local dynamics. Therefore we content ourselves here with making statements about "empirically plausible" local dynamics with unrestricted international capital flows.

4.3. A Calibration Exercise

We calibrate the following economy using available statistics on internal financing of investments, since the role of internal finance is the central feature of our analysis. In the U.S., for instance, about 80% of capital investment is financed internally (in Germany about 70% is financed internally), while in Mexico, Thailand, Turkey, or Korea, only about 20% of investment comes from internal funds.[24] Under the assumption that $f(k^i) = D(k^i)^{\delta}$, internal financing of investment in country i is $w(k^i) = (1-\delta)f(k^i)$. Hence we choose parameters so that the steady state equilibrium values k^2 and k^1 satisfy $w(k^1) = .8q$ and $w(k^2) = .2q$. This requires setting $\delta = .5$,[25] and $\phi = (68/25)(q/D^2)$. Credit is rationed in each country, for these parameter values, if $\alpha > 16/17$. Finally, condition (3.18) is satisfied if $\phi > \frac{5[\bar{z}-(\gamma/q)]^2}{2\bar{z}}$.

For these parameters, the eigenvalues of J are $0.4853 \pm 0.529i$. Thus the asymmetric steady state will be a sink, and dynamical equilibrium paths approaching it will display endogenously arising volatility for "plausible" parameter values.

[24] See Hamid and Singh (1992). These figures are for relatively large corporations.

[25] This is a relatively large value for capital's share. However, given the small number of free parameters in the model, we do not regard this as "bad" value for δ.

Remark

For every example we calculated, the asymmetric steady state is a sink. And, the eigenvalues of J are sometimes real and sometimes complex. However, even if local dynamics are monotone in this economy, global dynamics need not be. Indeed, large changes in net flows of capital are quite possible, as the next example illustrates.

Example 1. Let D =0.7, $\delta = 0.25, q = 0.5, \alpha = 0.925, \gamma = 0.8162$, and $g(z) = \overline{z}^{-1}$ with $\overline{z} = 2$. Then there is a symmetric steady state, and there are two asymmetric steady states: one with $(k^1, k^2) = (0.31, .214)$, and one with $(k^1, k^2) = (0.214, .31)$. The eigenvalues of J, evaluated at the second steady state, are $\lambda_1 = 0.9772$ and $\lambda_2 = 0.2588$. Nevertheless, depending on the values of initial conditions, large changes in the net flow of capital are possible, as Figure 4 illustrates.

As the discussion above indicates, this economy is capable of explaining why there is much more volatility in capital flows than movements in "fundamentals" would suggest. The model also suggests why the operation of international capital markets might work against the long-term development of initially poor economies. Both observations indicate that –at least in relatively poor economies– it may be desirable to impose barriers to capital flows. We now turn our attention to the erection of such barriers.

5. Barriers to Capital Flows: Eliminating Endogenous Volatility

As the previous section indicates, the unfetted functioning of international capital markets can itself easily give rise to economic volatility. In this section we show how various barriers to capital mobility –if selected appropriately- can be used to eliminate this volatility.

As we have noted, unless initial capital stocks are identical (i.e., $k_0^1 = k_0^2$), the economies we consider cannot converge to a symmetric steady state. Moreover, we have illustrated that there will be asymmetric steady states that are sinks for "plausible" parameterizations of the model. Near such steady states, funds will flow (on net) from the poor to the rich country. We henceforth restrict attention to this situation. And, without loss of generality, let country 1(2) be the rich (poor) country, so that the asymmetric steady state we consider has $k^1 > k^2$.

Suppose that country 2 wishes to stabilize its production level (or that country

1 wishes to stabilize the production level of country 2), and possibly to influence the magnitude and volatility of net capital flows. Then we imagine that country 2 chooses a “target” value for its capital stock (output level), so that $k_t^2 = k^2$ $\forall t \geq 1$, where k^2 is some constant satisfying $k^2 \in (0, k^*)$.[26] In addition, a tax is imposed on the interest income earned by residents of country 2 in country 1 at date t. This income is taxed at the proportional rate τ_t at date t. There are several interpretations that can be given to such a tax. These include: (a) country 2 taxes interest (or all) income earned abroad (that is, country 2 taxes capital outflows); (b) country 2 taxes all income earned by foreign investment, so that it taxes interest income earned by its residents abroad and interest income earned by residents of country 1 in country 2; (c) country 2 taxes all interest income; (d) country 1 taxes capital inflows. In any case, we assume that the country levying the tax rebates the proceeds to old lenders as a lump-sum at each date. Under our assumptions, such rebates do not affect the equilibrium conditions of the model.

In the presence of this barrier to capital mobility, residents of country 2 must be indifferent between lending internationally and lending domestically. The gross return on funds lent domestically will be

$$q\left[\eta - \left(\frac{\gamma}{q}\right)G(\eta) - \int_0^\eta G(z)dz\right] f'(k^2)/[q - w(k^2)].$$

And, the after-tax gross real return on funds invested abroad will be

$$(1-\tau_{t+1})q\left[\eta - \left(\frac{\gamma}{q}\right)G(\eta) - \int_0^\eta G(z)dz\right] f'(k_{t+1}^1)/[q - w(k_t^1)]. \tag{5.1}$$

Hence, when country 2 attempts to hold its production level (capital stock) constant,

$$H(k^2) = (1-\tau_{t+1})\frac{f'(k_{t+1}^1)}{[q - w(k_t^1)]},$$

$\forall t \geq 1$ must hold in order for after-tax real returns to be equated.[27]

[26] Recall that $k^* = \phi w(k^*) > 0$; that is, k^* is the unique non-trivial steady state capital stock in a closed economy.

Under our assumptions about the taxation of interest income, country 1 residents will invest in country 1 only. In addition, an equality of world sources and uses of funds continues to require that (4.5) obtain. Thus[28]

$$\begin{aligned} k^1_{t+1} &= \phi w(k^1_t) + [\phi w(k^2) - k^2] \\ &\equiv \Phi(k^1_t, k^2); t \geq 1 \end{aligned} \tag{5.2}$$

must obtain[29]

Equation (5.2) defines a law of motion for $\{k^1_t\}_{t=1}^{\infty}$ that is depicted in Figure 5. Since $k^2 < k^*$, $\phi w(k^2) > k^2$ holds. Hence, at all dates (except possibly the first), the country 1 capital stock converges monotonically to a steady state capital stock $k^1 > k^*$. No economic fluctuations (again, except possibly at the initial date) can be observed.

In order to validate the sequence of capital stocks $\{k^1_t\}_{t=1}^{\infty}$ given by (5.2) and $k^2_t = k^2, t \geq 1$, as equilibria, the "barrier" τ_{t+1} must be selected appropriately at each date. Equations (5.1) and (5.2) imply that τ_{t+1}must be set so that

$$\begin{aligned} 1 - \tau_{t+1} &= H(k^2)[q - w(k^1_t)]/f'(k^1_{t+1}) \\ &= H(k^2)[q - w(k^1_t)]/f'[\Phi(k^1_t, k^2)]; t \geq 1 \end{aligned} \tag{5.3}$$

[27] At t $= 0$, $\frac{f'(k^2)}{q - w(k^2_0)} = (1-\tau_1)\frac{f'(k^1_1)}{[q-w(k^1_o)]}$ must be satisfied in order for after-tax real returns to be equated.

[28] At t=0, $k^1_1 = \phi[w(k^1_0) + w(k^2_0)] - k^2$ holds. Note that $k^2 < \phi[w(k^1_0) + w(k^2_0)]$ is required for feasibility. Thus $k^2 < \max\{k^*, \phi[w(k^1_0) + w(k^2_0)]\}$ must hold.

[29] If credit is rationed in country 2, but not in country 1, then the system of equilibrium conditions is

$$k^1_{t+1} = \alpha q\left[\hat{z} - (\frac{\gamma}{q})G(\theta_t)\right];\ k^2_{t+1} = \phi\left\{w(k^1_t) + w(k^2_t) - \alpha q\right\} \text{ and}$$

$$\frac{(1-\tau_{t+1})f'(k^1_{t+1})}{[q - w(k^1_t)]}\left[\theta_t - (\frac{\gamma}{q})G(\theta_t) - \int_0^{\theta_t} G(z)dz\right] = \frac{f'(k^2_{t+1})}{[q - w(k^2_t)]}\left[\eta - (\frac{\gamma}{q})G(\eta) - \int_0^{\eta} G(z)dz\right],$$

with $\theta_t \leq \eta$. Clearly any barrier to capital mobility that stabilized the capital stock in country 2 would require both k^1_t and k^2_t to be at their steady state values forever. Since this is generically infeasible, if credit is not rationed in country 1, it may be impossible for country 2 to prevent variations over time in its capital stock and production level.

Conditions under which $\tau_{t+1} \in (0,1), \forall\, t \geq 1$, are stated below.

Evidently, (5.3) gives τ_{t+1} as a function of k_t^1 and k^2; say $\tau_{t+1} = T(k_t^1,\, k^2)$. It is straightforward to establish the following result.

Lemma 5.1. *If $k^2 \leq \max\{f^{-1}(q), (w')^{-1}(1/\phi)\}$, then $T_2(k_t^1,\, k^2) \leq 0$ holds. (b) If $(w')^{-1}(1/\phi) \leq k^2 \leq f^{-1}(q)$ holds, then $T_2(k_t^1,\, k^2) \geq 0$ holds. (c) Otherwise, $T_2(k_t^1,\, k^2)$ is of ambiguous sign.*

Lemma 5.1 establishes that, given k_t^1,the magnitude of the appropriate barrier to capital mobility at t+1 may be either increasing or decreasing in country 2's income level.

An interesting question concerns how barriers to capital flows will vary over time with the level of the country 1 capital stock. Appendix A establishes the following result.

Lemma 5.2. *Suppose $f(k) = Dk^\delta$. Then*

$$\left(\frac{k_t^1}{1-\tau_{t+1}}\right)\frac{\partial(1-\tau_{t+1})}{\partial k_t^1} = \delta\left\{\left[\frac{w(k_t^1)}{q-w(k_t^1)}\right] - (1-\delta) + (1-\delta)\left[\frac{\phi w(k^2) - k^2}{k_{t+1}^1}\right]\right\} \tag{5.4}$$

Since $k^2 < k^*$, $\phi w(k^2) > k^2$ necessarily holds. Thus $T_1(k_t^1,\, k^2) < 0$ ($(1-\tau_{t+1})$ is increasing in k_t^1) if $w(k_t^1)/q \geq (1-\delta)/(2-\delta)$ is satisfied. Since $w(k_t^1)/q$ is the fraction of capital investment financed internally in the rich country, this condition must be satisfied for any plausible parametrization of the model. Indeed, for $\delta \geq 0$, $T_1(k_t^1,\, k^2) < 0$ will hold if $w(k_t^1)/q \geq .5$ is satisfied. For all the developed countries examined, for instance, by Hamid and Singh (1992), the fraction of investment financed internally is no less than 50 percent. Hence we conclude that $T_1(k_t^1,\, k^2) < 0$ is the empirically plausible situation. Thus barriers to capital mobility will decline over time if $\{k_t^1\}$ is an increasing sequence. Or, in other words, barriers to capital mobility will decline as the world economy develops over time.

Of course (5.3) implies that $\lim_{t\to\infty}(1-\tau_{t+1}) = H(k^2)/H(k^1) = 1-\tau$. Hence $\tau \in [0,1)$ will hold if k^2 is not too small relative to k^1. Indeed $0 < \tau < 1$ will necessarily be satisfied if the "target" value k^2 exceeds the steady state country 2 capital stock that would obtain in an asymmetric steady state in the economy of section 4. Thus, if the long-run situation of country 2 is to be "improved" (from the perspective of production), some barriers to capital mobility will have to be retained even asymptotically.[30]

We make a final remark. When barriers to capital mobility are erected, (a) economic volatility can be reduced, and (b) both the short and long-run production levels of a country can be increased relative to the situation with unrestricted capital mobility. But, if all variation in country 2 income is eliminated (and if $k_1^1 < k^1$), one result will be that country 2 production levels will be "stagnant" while country 1 production levels will rise over time. Hence inequality in world production will increase over time along with world economic development.

Of course thus far the analysis has been silent regarding the long-run (steady state) consequences of erecting barriers to capital mobility. We now consider this issue.

6. Long-Run Consequences of Barriers to Capital Mobility

Let $(\widehat{k}^1, \widehat{k}^2)$ denote a steady state equilibrium of the model with no barriers to capital flows in place. Thus $(\widehat{k}^1, \widehat{k}^2)$ satisfies (4.7) and (4.8). Moreover, to fix ideas, suppose that $(\widehat{k}^1, \widehat{k}^2)$ is an asymmetric steady state with $\widehat{k}^1 > \widehat{k}^2$, so that country 1(2) is relatively wealthy (poor). Then, as we have seen, country 2 can select any value $k^2 \in [\widehat{k}^2, k^*)$, and can select an appropriate sequence of barriers to capital flows , $\{\tau_t\}_{t=1}^{\infty}$, so that $k_t^2 = k^2 \; \forall t \geq 1$, and so that $\lim_{t\to\infty} \tau_t = \tau \in [0, 1)$. Moreover, with $k^2 \in [\widehat{k}^2, k^*)$, $\lim_{t\to\infty} k_t^1 > k^*$ will hold. Thus, with appropriately selected –and appropriately evolving– impediments to capital mobility in place, country 2 can both eliminate any endogenous fluctuations in capital flows and production levels, <u>and</u> increase its long-run output and capital stock.

We now wish to consider the consequences of any barriers country 2 might impose for these purposes from the perspective of country 1. In addition, we consider the consequences of country 2's restrictions on capital flows for <u>world</u> production levels, and for the welfare of different agents in both countries. Throughout, we focus our attention on the long-run, or steady state consequences of any barriers imposed. We now own turn our attention to this task.

[30] If $T_1(k_t^1, k^2) < 0$ holds, it follows that $\tau_{t+1} \in (0, 1)$ will then hold for all t so long as k_t^1 is never "too far" from its steady state value.

6.1. Long-Run Production Levels

In order to describe the effects of barriers to capital flows on long-run production levels, it will be useful to define two benchmark levels of country 2's capital stock. First, let $\widetilde{k}$ be defined by $\phi w'(\widetilde{k}) = 1$. Under our assumptions on technology, $\widetilde{k}$ is the capital stock that, in steady state, maximizes the difference between country 2 saving and country 2 investment. Or, in other words, when $k^2 = \widetilde{k}$, net steady state capital flows from country 2 to country 1 are maximized. In addition, define $k^+ > k^*$ by the relation

$$k^+ - \phi w(k^+) = \phi w(\widetilde{k}) - \widetilde{k}.$$

Then, if $k^2 = \widetilde{k}$, $k^1 = k^+$. Or, in other words, k^+ is country 1's steady state capital stock if it receives the maximum possible level of capital inflows. Our assumptions imply that $k^1 \leq k^+$ must hold in any steady state.

In order to fully discuss the consequences of country 2's taxation of income earned abroad, it is useful to distinguish between two cases.

Case 1: $\widehat{k}^2 < \widetilde{k}$ and $k^2 \in [\widehat{k}^2, \widetilde{k}]$. This case transpires if, in the steady state of section 4, country 2 is fairly poor. In this situation, the following possibility arises. With $k^2 \in (\widehat{k}^2, \widetilde{k}]$, the imposition of taxes by country 2 on income earned abroad increases the quantity $\phi w(k^2) - k^2$, relative to its "original" steady state level. It is then immediate that the steady state capital stock of country 1 must rise; that is, $k^1 > \widehat{k}^1$ holds.

Thus, if country 2 is initially poor enough, its erection of barriers to capital flows actually raises long-run income levels in each country. That is, production in country 1 is not adversely affected by the attempts of country 2 to discourage capital outflows.

Indeed, in a steady state, these attempts actually increase the net outflow of capital from country 2. Intuitively, the barriers to capital mobility erected by country 2 do encourage country 2 residents to shift investment from country 1 to country 2, other things equal. But, this increase in country 2 investment raises the level of savings in country 2 as well. With $k^2 \leq \widetilde{k}$, domestic saving increases by more than domestic investment in the poor country, and the net effect is that country 2 has a greater net outflow of capital in the presence of barriers to capital outflows than it does in their absence. Notice that this result illustrates an important possibility: barriers to capital mobility need not lessen the net magnitude of capital flows.

Case 2: $\widehat{k}^2 \geq \widetilde{k}$. In this case, setting $k^2 > \widehat{k}^2$ must reduce the value $\phi w(k^2) - k^2$ (relative to the situation where foreign interest income is untaxed), and it must reduce the steady state value k^1. In other words, $k^1 < \widehat{k}^1$ must obtain. Intuitively, when $\widehat{k}^2 \geq \widetilde{k}$, barriers to capital mobility must reduce net capital flows to country 1. As a result, its steady state capital stock must fall. Having $\widehat{k}^2 < \widetilde{k}$ hold is a real possibility, as the following examples illustrate.

Example 2. Let $f(k) = 0.85k^{.66}, q = 0.045, \gamma = 0.1064711$, and $g(z) = \overline{z}^{-1}$ (uniform distribution), with $\overline{z} = 3.73205$. Then there is an asymmetric steady state, if there are no barriers to capital mobility, with $\widehat{k}^1 = 0.032879$ and $\widehat{k}^2 = 0.00161283$. Here $\widetilde{k} = 0.0076486$ so that $\widehat{k}^2 < \widetilde{k}$ holds. Parenthetically, the eigenvalues of J, at this steady state, are $0.517917 \pm 0.406507i$, so that endogenous volatility will be observed in the absence of barriers to capital mobility.

In our calibration exercise above, it is straightforward to demonstrate that $\phi w'(\widehat{k}^2) = 1.7 > 1$, so that $\widehat{k}^2 < \widetilde{k}$ holds. Thus, we regard it as a strong empirical possibility that some degree of barriers to capital mobility, along with an appropriate choice of k^2, can (a) increase long-run production levels in each country, and (b) increase the magnitude of net capital flows.

Discussion

So long as $\widehat{k}^2 < \widetilde{k}$ holds, and so long as country 2 sets $k^2 \in [\widehat{k}^2, \widetilde{k}]$, country 1 and country 2 experience the same production consequences from the creation of barriers to capital mobility. Hence, at least with respect to production, no conflicts of interest between the countries –at least in the long-run – are created if country 2 erects such barriers.

Once country 2 contemplates setting k^2 sufficiently high, however, it may cause country 1's long-run production level to decline as a result of the impediments it creates to capital mobility. In particular, with no barriers to capital mobility, country 1 experiences a net (steady state) inflow of funds equal to $\phi w(\widehat{k}^2) - \widehat{k}^2$. So long as $\phi w(k^2) - k^2 \geq \phi w(\widehat{k}^2) - \widehat{k}^2$ is satisfied[31], country 2's actions will not reduce net capital inflows to country 1, or country 1's production, relative to the steady state of section 4. Attempts by country 2 to set k^2 so that $\phi w(k^2) - k^2 < \phi w(\widehat{k}^2) - \widehat{k}^2$ will adversely affect country 1's production levels, however, and may invite retaliation. Thus the ability of country 2 to erect barriers to capital mobility –without any reaction by country 1– will be greatest when $\widehat{k}^2 < \widetilde{k}$ holds.

[31] If $\widehat{k}^2 < \widetilde{k}$, this condition can be satisfied if $k^2 > \widetilde{k}$ holds, so long as k^2 is not too large. If $\widehat{k}^2 \geq \widetilde{k}$, this condition fails, for all values $k^2 \geq \widehat{k}^2$.

6.1.1. World Production Levels

Consider the problem of maximizing the world steady state production level, $f(k^1) + f(k^2)$, subject to the constraint that world investment does not exceed world savings ($k^1 + k^2 \leq \phi[w(k^1) + w(k^2)]$). The following lemma states a result concerning the solution to this problem.

Lemma 6.1. Suppose that $f(k)$ has the CES form $f(k) = [ak^\rho + b]^{1/\rho}$, with $\rho \in [0, 1)$.[32] Then world output is maximized by setting $k^1 = k^2 = k^*$.

The proof of Lemma 6.1 appears in Appendix B. Intuitively, the concavity of $w(k)$ implies that world savings is maximized by setting $k^1 = k^2$. Similarly, the concavity of $f(k)$ implies that world production is maximized by setting $k^1 = k^2$. Thus $k^1 = k^2 = k^* = \phi w(k^*)$ must hold.

The lemma implies that world production will rise as k^2 is increased up to the point where $k^2 = k^*$. Moreover, if $k^2 = k^*$, $\lim_{t\to\infty} \tau_t = 0$, so that no barriers to capital mobility will be required, in such a steady state. However, as we have noted, setting k^2 at such a high level may invite retaliation by country 1, and hence may not be feasible for country 2.

6.1.2. Some Calibrated Barriers

Consider our calibration exercise of section 4. What would be the magnitude of the tax that country 2 would have to levy if they set $k^2 = \widetilde{k}$ (that is, if they maximized the steady state capital stock of country 1)? It is easy to calculate that $\widetilde{k} = [(17/25)(q/D)]^2$, and that $k^+ = (1+\sqrt{2})^2\widetilde{k}$. It is also easy to verify that $H(\widetilde{k})/H(k^+) = 0.65539$, so that in a steady state, country 2 world would have to set $\tau = 0.34461$.

Alternatively, suppose that country 2 set k^2 so that $k^1 = \widehat{k}^1$, and so that k^2 is the largest solution to $\widehat{k}^1 - \phi w(\widehat{k}^1) = \phi w(k^2) - k^2$. (This is the largest that the country 2 capital stock can be without lowering the country 1 capital stock relative to $\widehat{k}^1$.) One readily calculates that $k^2 = [(3.84/4)(q/D)]^2$, and that $H(k^2)/H(k^1) = 0.641$. Thus country 2 needs to tax interest income on foreign investment at the rate $\tau = 1 - 0.641 = 0.359$, in a steady state.

$f(\widetilde{k})/f(\widehat{k}^2) = 1.7$ is readily shown to hold. $f(k^+)/f(\widehat{k}^1) = 1.026$. Thus, by setting an appropriate tax sequence $\{\tau_t\}_{t=1}^{\infty}$, country 2 can have $k^2 = \widetilde{k} > \widehat{k}^2$

[32] Setting $\rho < 0$ would imply the violation of assumption 1.

$\forall t \geq 1$, thereby eliminating excess volatility of production and net capital flows, and it can increase its steady state output level by 70%, relative to the situation with $\tau = 0$. Country 1's long-run output level will also rise as a result, but only by 2.6%. Alternatively, if country 2 maximizes k^2 subject to $k^1 \geq \widehat{k}^1$, and sets a tax sequence $\{\tau_t\}_{t=1}^{\infty}$ so as to have $k_t^2 = k^2$ $\forall t \geq 1$, it is easy to check that $f(k^2)/f(\widehat{k}^2) = 2.4$. Thus country 2 can increase its steady state production level by 140% without impairing steady state production in country 1.

6.1.3. A Formal Result

We now address the following, relatively technical question. When will an asymmetric steady state with $\widehat{k}^2 < \widetilde{k}$ exist? The following result states a formal condition.

Proposition 6.1. *Suppose that $f(k^*) > q$ and $H(\widetilde{k}) < H(k^+)$ hold. Then there exists an asymmetric steady state with $\widehat{k}^2 < \widetilde{k}$.*

Proposition 6.1 is proved in appendix C.

6.2. Steady State Welfare

As we have shown, appropriately erected barriers to capital mobility can eliminate endogenously arising fluctuations in output, and can actually increase the steady state production levels of <u>all</u> economies (relative to a situation where no barriers to capital flows exist). Indeed, we have even argued that it is empirically plausible that such a situation might obtain. We now turn to the following question: what impact do these barriers have on the steady state welfare of different agents in different countries?

To answer this question, we assume that there exists an asymmetric steady state, $(\widehat{k}^1, \widehat{k}^2)$, satisfying $\widehat{k}^2 < \widetilde{k}$, when no barriers to capital mobility are in place. Under these circumstances, we know that it is possible –through the construction of appropriate taxes on capital flows– to increase k^2 without reducing k^1. And, indeed, it is possible to raise both k^1 and k^2.

Moreover, to simplify calculations, we assume throughout the remainder of this section that the production function has the Cobb-Douglas form $f(k) = Dk^{\delta}$, with $\delta \in (0, 1)$.

Since all lenders have the option of investing in their home country, where investment income is untaxed, lenders in country i earn the real gross return

$\psi H(k^i)$ on their savings. Moreover, under our assumptions, all young period income is saved. Thus, the utility of lenders in country 1 is $\psi H(k^1)w(k^1)$. The utility of lenders in country 2 is $\psi H(k^2)w(k^2)$, plus the value of any transfer received from the proceeds of taxing investment income earned from abroad. We will state conditions under which the term $\psi H(k)w(k)$ is increasing in k. These conditions are clearly sufficient to imply that lenders in both countries benefit from barriers to capital flows that raise both k^1 and k^2.

With respect to borrowers, we know from the discussion in section 3 that their utility, <u>conditional</u> on receiving funding, is given by the expression

$$\begin{aligned} \phi q f'(k^i) - rb^i &= \phi q f'(k^i) - \psi H(k^i)[q - w(k^i)] \\ &= (\phi q - \psi) f'(k^i) \end{aligned}$$

In addition, the probability of receiving funding for a potential borrower is $\mu^i = k^i/\phi\alpha q$. It follows that the ex ante (prior to the revelation of whether funding is received) expected utility of a potential borrower is

$$V(k^i) \equiv \left(\frac{\phi q - \psi}{\phi\alpha q}\right) k^i f'(k^i) + \left(\frac{\phi\alpha q - k^i}{\phi\alpha q}\right) \psi w(k^i) H(k^i); \; i = 1, 2 \qquad (6.1)$$

We now state our first welfare result. The result asserts conditions under which the utility of lenders is increasing in k^i.

Proposition 6.2. *(a) Suppose that $k^i \geq f^{-1}(q)$. Then $w(k^i)H(k^i)$ is strictly increasing in k^i. (b) Suppose that $\delta \geq 0.5$. Then $w(k^i)H(k^i)$ is strictly increasing in $k^i, \forall k^i$.*

The proofs of propositions 6.2-6.4 appear in appendices $D - F$, respectively. Proposition 6.2 implies that lenders in country 1 always benefit from actions that raise k^1. And, if $\delta \geq 0.5$, lenders in country 2 necessarily benefit from actions that raise k^2.

We now need to determine when potential borrowers benefit, in an ex ante sense, from increases in the capital stock. Differentiating (6.1) we obtain

$$\begin{aligned} \phi\alpha q V'(k^i) &= (\phi q - \psi) f'(k^i) \left\{1 + [k^i f''(k^i)/f'(k^i)]\right\} \\ &\quad + \left(\frac{\phi\alpha q - k^i}{k^i}\right) \psi w(k^i) H(k^i) \left\{\frac{k^i w'(k^i)}{w(k^i)} + \frac{k^i H'(k^i)}{H(k^i)}\right\} \\ &\quad - \psi w(k^i) H(k^i). \end{aligned} \qquad (6.2)$$

We now state conditions under which $V'(k^i)$ >0 holds for the relatively wealthy country.

Proposition 6.3. *Suppose that $\phi q > \psi$, and that $w(k^+) \leq q - (\psi/\phi)$. Then $V'(k^i) > 0$ holds for all $k^i \in [f^{-1}(q), k^+]$ if the following condition is satisfied:*

$$\delta[(1-\delta)D]^{(1/\delta)} \geq \frac{[q-(\psi/\phi)]^{(1/\delta)}}{\phi\alpha q}. \tag{6.3}$$

Since $k^1 \in [f^{-1}(q), k^+]$, it follows that $V'(k^1) > 0$ holds if the conditions of the proposition are satisfied. Thus, when these conditions obtain, both lenders and potential borrowers benefit, in an ex ante sense, from the creation of barriers to capital mobility that raise k^1.

Intuitively, the conditions of proposition 6.3 guarantee that the probability of a potential borrower receiving a loan is not "too great." It follows that the ex ante expected utility of potential borrowers will not differ too greatly from the welfare of savers. As a result, policy actions that increase the steady state capital stock will also increase the ex ante expected utility of potential borrowers.

Evidently, if the conditions of proposition 6.2 are satisfied, potential borrowers and lenders (at least from the perspective of the steady state) in country 1 will unanimously approve of the barriers to capital flows, if they do not reduce k^1. And, if $\delta \geq 0.5$, these barriers will also be beneficial to lenders residing in country 2. Obviously, it remains to describe when potential borrowers in country 2 also benefit from these barriers, in a steady state. Proposition 6.4 states a sufficient condition for this to occur.

Proposition 6.4. *Suppose that $\delta \geq 0.5$ and $\phi\alpha q \geq \psi(1-\delta+\delta^2)/\delta^2$. Then $V'(k^i) > 0$ holds for all $k^i \leq f^{-1}(q)$.*

It is now clear that if the conditions of proposition 6.3 and 6.4 hold simultaneously, all agents in all countries benefit –in an ex-ante sense– from appropriately selected barriers to capital mobility, at least from the perspective of a steady state. Moreover, the conditions of the propositions are merely sufficient, and by no means necessary conditions for all groups to benefit from these barriers. And, finally, it deserves emphasis that parameters can be selected to satisfy all of the conditions of proposition 6.2–6.4 within the context of our calibrated example. We would , therefore, argue that these conditions are not empirically implausible.

7. Static Barriers to Capital Mobility

Thus far we have focused on barriers to capital mobility that evolve over time, so as to prevent production levels in country 2 from fluctuating. Obviously, from the perspective of a steady state, anything that can be achieved with a sequence $\{\tau_t\}_{t=1}^{\infty}$ satisfying $\lim_{t\to\infty}\tau_t = \tau$ can also be achieved simply by setting $\tau_t = \tau \forall t$. However, erecting a static barrier to capital flows will generally not prevent endogenous volatility in production and capital flows, as we now demonstrate.

If $\tau_t = \tau$ holds for all t, where τ has the same interpretation as before (a tax imposed on interest income earned abroad that is paid by country 2 residents), then the sequence $\{k_t^1, k_t^2\}$ must evolve according to (4.5) and

$$\frac{(1-\tau)f'(k_{t+1}^1)}{q-w(k_t^1)} = \frac{f'(k_{t+1}^2)}{q-w(k_t^2)} \tag{7.1}$$

Again, we consider the local dynamics of this system, so we replace (4.5) and (7.1) with the linear approximation

$$(k_{t+1}^1 - k^1, k_{t+1}^2 - k^2)' = J(k_t^1 - k^1, k_t^2 - k^2)'$$

where, as before, J is the Jacobian matrix

$$J = \begin{pmatrix} \frac{\partial k_{t+1}^1}{\partial k_t^1} & \frac{\partial k_{t+1}^1}{\partial k_t^2} \\ \frac{\partial k_{t+1}^2}{\partial k_t^1} & \frac{\partial k_{t+1}^2}{\partial k_t^2} \end{pmatrix}$$

We now use an example to illustrate the fact that static barriers to capital flows cannot generally prevent endogenous volatility from appearing in capital flows and production levels.

Example 3. The example is the same as example 2. For these parameter values $\phi = 1$ and $\eta = 1.36603$ hold.

If $\tau = 0$ is satisfied, so that there is no interference with capital flows, then there are three nontrivial (k^1, k^2) pairs: $(k^1, k^2)^C = (0.00161283, 0.032879)$, $(k^1, k^2)^B = (0.025966,\ 0.025966)$ and $(k^1, k^2)^A = (0.032879,\ 0.00161283)$.[33]

As noted previously, the symmetric steady state is a saddle (here the relevant eigenvalues are .66 and 2.65). The eigenvalues at each asymmetric steady state are $0.517917 \pm 0.406507i$. It follows that the asymmetric steady states are sinks, and

[33] It is easy to verify that credit is rationed in each country in each of the steady states, and that (3.18) is satisfied. This is true for all of the tax rates considered here.

that equilibrium paths approaching them display endogenously arising volatility that dampens asymptotically.

If τ is set equal to 0.1, then there continue to be three steady states. One steady state has $k^1 = 0.033739$ and $k^2 = 0.002164$, so that country 2 continues to be quite poor relative to country 1. Nonetheless, an increase in the rate of capital income taxation by country 2 has led to an increase in the capital stock of each country (relative to the asymmetric steady state that obtains with $\tau = 0$). And the proportional effect on the capital stock of country 2 is much larger than that for country 1: the steady value of k^2 rises by more than 34% while the steady state value of k^1 rises by only 2.6%. Finally, we note that this asymmetric steady state continues to be a sink, and that paths approaching it continue to display endogenous volatility. The relevant eigenvalues of J are $\lambda = .05484 \pm 0.3949i$.

When $\tau_1 = .1$, there exists another steady state with $k^1 = 0.028339$ and $k^2 = 0.0234395$. This steady state is a saddle: the eigenvalues of J are 0.70183 and 1.77418.

For this example, if $\tau = 0.361124$ then there exists a steady state with $k^1 = k^+ = 0.03637976$, and $k^2 = \widetilde{k} = 0.0076486$. Notice that the capital stock of country 2, while still quite low relative to that of country 1, is more than 3.5 times greater than was the case with $\tau_1 = 0.1$. However, the asymmetric steady state continues to be a sink, and the eigenvalues of J are $0.78592 \pm 0.25543i$. Thus, in this example, static barriers to capital flows cannot eliminate endogenous volatility.

8. Barriers to Capital Flows: Who Should Erect Them?

So far we have discussed barriers to capital mobility put in place by the poor country. If we think of this as a "North-South" model, these barriers will take the form of effective taxation of capital outflows, as on net capital must –in the long-run– flow from the "South to the North." And, these barriers can be thought of as – and are, in fact, measures that prevent fluctuations in the magnitude of net capital flows.

It is important to emphasize, though, that (at least with respect to equilibrium capital stocks and production levels) identical outcomes can be achieved by having the rich country erect barriers against capital inflows. These barriers can take the form of taxation of interest income earned by foreign residents in the domestic country. Thus, since an equilibrium simply requires that

$$\frac{(1-\tau_{t+1})f'(k^1_{t+1})}{q - w(k^1_t)} = \frac{f'(k^2_{t+1})}{q - w(k^2_t)},$$

it is virtually irrelevant whether τ_{t+1} is a tax on capital outflows imposed by country 2, or a tax on capital inflows imposed by country 1. All that is affected by who imposes the tax is who collects the revenue it generates. This observation has an important corollary, however. If developed countries are concerned about the consequences of volatility in net capital flows between poor and rich countries, they can impose barriers to capital mobility that will rectify this situation as well poor countries can.

9. Conclusions

Capital flows potentially pose problems for relatively poor economies in both the short and the long-run. In the short-run they can generate economic volatility far in excess of fundamentals. And, in the long run, the net direction of capital flows is not generally conducive to the sustained development of initially poor countries.

As we have shown, barriers to capital mobility can be used to eliminate the excess volatility generated by international capital movements. And, these same barriers can be used to promote capital accumulation, and to increase steady state production levels in the poor economy. Finally, if the poor country would be sufficiently poor in the absence of any barriers to capital flows, these benefits can be had without reducing long-run production levels in wealthier economies. Indeed, at least with respect to steady states, such barriers can benefit all agents in all countries.

We have also looked at parameterizations of our model that generate equilibria displaying international differences in the internal financing of investment that are consistent with observation. We think that this important to do exactly because differences in the internal financing of investment are the central feature of our analysis. And, in such parameterizations, endogenous volatility in production and capital flows does arise. Barriers to capital mobility meant to eliminate these fluctuations will have the feature that the magnitude of these barriers will decline over time. And, these barriers can raise long-run production levels in both countries. However, at least if the poor country does not erect barriers that reduce long-run production levels in the wealthier country, these barriers will remain quite high, even asymptotically.

10. Appendices

A: Proof of Lemma 5.2. Differentiation of (5.3) yields

$$\begin{aligned}\left(\frac{k_t^1}{1-\tau_{t+1}}\right)\frac{\partial(1-\tau_{t+1})}{\partial k_t^1} &= -\left[\frac{k_t^1 w'(k_t^1)}{q-w(k_t^1)}\right]-\frac{k_t^1 f''(k_{t+1}^1)}{f'(k_{t+1}^1)}\frac{dk_{t+1}^1}{dk_t^1} \qquad (A.1)\\ &= -\left[\frac{k_t^1 w'(k_t^1)}{w(k_t^1)}\right]\left[\frac{w(k_t^1)}{q-w(k_t^1)}\right]-\left[\frac{k_t^1 f''(k_{t+1}^1)}{f'(k_{t+1}^1)}\right]\frac{k_t^1}{k_{t+1}^1}\frac{dk_{t+1}^1}{dk_t^1}\end{aligned}$$

Moreover, given the form of the production function, $\frac{kw'(k)}{w(k)}=\delta$ and $\frac{-kf''(k)}{f'(k)}=1-\delta$. And, $\frac{dk_{t+1}^1}{dk_t^1}=\phi w'(k_t^1)$. Using these facts in (A.1) yields

$$\begin{aligned}\left(\frac{k_t^1}{1-\tau_{t+1}}\right)\frac{\partial(1-\tau_{t+1})}{\partial k_t^1} &= -\delta\left[\frac{w(k_t^1)}{q-w(k_t^1)}\right]+(1-\delta)\frac{k_t^1}{k_{t+1}^1}\phi w'(k_t^1) \qquad (A.2)\\ &= -\delta\left[\frac{w(k_t^1)}{q-w(k_t^1)}\right]+(1-\delta)\left[\frac{k_t^1 w'(k_t^1)}{w(k_t^1)}\right]\left[\frac{\phi w(k_t^1)}{k_{t+1}^1}\right]\\ &= -\delta\left\{\left[\frac{w(k_t^1)}{q-w(k_t^1)}\right]-(1-\delta)\left[\frac{k_{t+1}^1+k^2-\phi w(k^2)}{k_{t+1}^1}\right]\right\}\end{aligned}$$

where the last inequality follows from (5.2). Finally, (A.2) can be rearranged to yield

$$\left(\frac{k_t^1}{1-\tau_{t+1}}\right)\frac{\partial(1-\tau_{t+1})}{\partial k_t^1}=\delta\left\{\left[\frac{w(k_t^1)}{q-w(k_t^1)}\right]-(1-\delta)+(1-\delta)\left[\frac{\phi w(k^2)-k^2}{k_{t+1}^1}\right]\right\},$$

as claimed.□

B: Proof of Lemma 6.1. Consider the problem $max\ f(k^1)+f(k^2)$ subject to

$$k^1+k^2\leq\phi[w(k^1)+w(k^2)]. \qquad (B.1)$$

Letting λ be the Lagrange multiplier associated with the constraint (B.1), the first order conditions for this problem set

$$f'(k^i)\left\{1+\lambda\phi\left[\frac{k^i w'(k^i)}{w(k^i)}\right]\left[\frac{w(k^i)}{k^i f'(k^i)}\right]\right\}=\lambda;i=1,2. \qquad (B.2)$$

Moreover, our assumptions on $f(k)$ imply that $\left[\frac{kw'(k)}{w(k)}\right]\left[\frac{w(k)}{kf'(k)}\right] = \frac{b(1-\rho)}{ak^{\rho}+b}$. It follows that the left-hand side of (B.2) is strictly decreasing in k^i. Hence an optimum has $k^1 = k^2$. Satisfaction of (B.1) then implies that $k^1 = k^2 = k^*$.

C: Proof of Proposition 6.1

The condition $f(k^*) > q$ guarantees the existence of an asymmetric steady state [Boyd-Smith (1997)]. And, there exists an asymmetric steady state with $k^2 < \widetilde{k}$ if $w(k^*) < q$, and if the downward sloping branch of $H(k^1) = H(k^2)$ passes through a point $(k^1, \widetilde{k})$, with $k^1 < k^+$ (see Figure 6). But this is guaranteed by $H(k^+) > H(\widetilde{k})$.□

D: Proof of proposition 6.2.

(a) $w'(k^i) > 0$ holds $\forall k^i$. And, for $k^i \in [f^{-1}(q), w^{-1}(q))$, $H'(k^i) \geq 0$ holds. Thus $w(k^i)H(k^i)$is strictly increasing in k^i.□

(b) Note that $w(k^i)H(k^i) = w(k^i)f'(k^i)/[q-w(k^i)]$. If $\delta \geq 0.5$, then $w(k^i)f'(k^i)$ is nondecreasing in k^i. The claim follows immediately.□

E: Proof of proposition 6.3.

Given our specification of the production technology,

$kf''(k)/f'(k) = \delta - 1$, $kw'(k)/w(k) = \delta$ and $kH'(k)/H(k) = \delta - 1 + \{\delta w(k)/[q - w(k)]\}$ hold. Substituting these relations into (6.2) and rearranging terms yields

$$\begin{aligned} \phi\alpha q V'(k^i) \quad = \quad & \delta(\phi q - \psi)f'(k^i) \\ & + \psi w(k^i)H(k^i)\left\{\left(\frac{\phi\alpha q - k^i}{k^i}\right)\left[(2\delta - 1) + \left(\frac{\delta w(k^i)}{q - w(k^i)}\right)\right] - 1\right\} \end{aligned} \tag{E.1}$$

It follows from (E.1) and the definition of H that $V'(k^i) > 0$ holds iff

$$\delta(\phi q - \psi) \geq \left[\psi\frac{w(k^i)}{q - w(k^i)}\right]\left\{1 - \left(\frac{\phi\alpha q - k^i}{k^i}\right)\left[\frac{\delta q}{q - w(k^i)} - (1-\delta)\right]\right\} \tag{E.2}$$

Since $w(k^+) \leq q - (\psi/\phi)$, it follows that $\psi/[q - w(k^i)] \leq \phi$ for all $k^i \leq k^+$. Therefore, $\psi w(k^i)/[q - w(k^i)] \leq \phi w(k^i) \leq \phi q - \psi$ holds for all $k^i \leq k^+$. Using this fact in (E.2), it is evident that $V'(k^i) > 0$ holds for all $k^i \in [f^{-1}(q), w^{-1}[q - (\psi/\phi)]]$

if

$$\left(\frac{\phi\alpha q - k^i}{k^i}\right)\left\{\frac{\delta q}{q - w(k^i)} - (1-\delta)\right\} \geq (1-\delta) \tag{E.3}$$

holds. Notice that for all $k^i \in [f^{-1}(q), w^{-1}[q-(\psi/\phi)]]$, we have $\left\{\frac{\delta q}{q-w(k^i)} - (1-\delta)\right\} \geq \delta$. Thus, (E.3) is satisfied if

$$\delta(\phi\alpha q - k^i) \geq (1-\delta)k^i. \tag{E.4}$$

Rearranging terms in (E.4) yields the equivalent condition

$$\delta\phi\alpha q \geq k^i \tag{E.5}$$

But, since $k^i \leq w^{-1}[q-(\psi/\phi)] = \{[q-(\psi/\phi)]/(1-\delta)D\}^{1/\delta}$, (E.5) must hold $\forall k^i \in [f^{-1}(q), k^+]$ if

$$\delta\phi\alpha q \geq \left[\frac{q-(\psi/\phi)}{(1-\delta)D}\right]^{1/\delta}. \tag{E.6}$$

This establishes the proposition.□

F. Proof of proposition 6.4

If $\delta \geq 0.5$, then

$$\psi\left[\frac{w(k^i)}{q-w(k^i)}\right]\left\{1-\left(\frac{\phi\alpha q-k^i}{k^i}\right)\left[\frac{\delta q}{q-w(k^i)}-(1-\delta)\right]\right\} < \psi\left[\frac{w(k^i)}{q-w(k^i)}\right]$$

holds. It follows that $V'(k^i) > 0$ if

$$\delta(\phi\alpha q - \psi) \geq \psi\left[\frac{w(k^i)}{q-w(k^i)}\right] \tag{F.1}$$

is satisfied. But, for $k^i \leq f^{-1}(q)$, $\frac{w(k^i)}{q-w(k^i)} \leq (1-\delta)/\delta$. Thus (E.2) (and $V'(k^i) > 0$) holds for all $k^i \leq f^{-1}(q)$ if

$$\phi\alpha q \geq \psi(1-\delta+\delta^2)/\delta^2.\square \tag{F.2}$$

References

[1] Bernanke, Ben S., and Mark Gertler, 1989, Agency Costs, Net Worth, and Business Fluctuations, *American Economic Review* 79, 14-31.

[2] Boyd, John H., and Bruce D. Smith, 1994, How Good Are Standard Debt Contracts?: Stochastic versus Nonstochastic Monitoring in a Costly State Verification Environment, *Journal of Business 67, 539-561.*

[3] Boyd, John H., and Bruce D. Smith, 1997, Capital Market Imperfections, International Credit Markets, and Nonconvergence, *Journal of Economic Theory* 73, 335-364.

[4] Calvo, Guillermo, L. Leiderman, and Carmen Reinhart, "Capital Flows and the Real Exchange Rate Appreciation in Latin America: the Role of External Factors," *IMF Staff Papers* 40 (1993), 108-151.

[5] Chuhan, P., S. Claessens, and N. Mamingi, "Equity and Bonds Flows to Latin America and Asia: the Role of Global and Country Factors," manuscript, The World Bank, 1993.

[6] Eichengreen, Barry, James Tobin and Charles Wyplosz, 1995, Two Cases for Sand in the Wheels of International Finance, *The Economic Journal* 162-172.

[7] Fernandez-Arias, E. "The New Wave of Private Capital Flows: Push or Pull?" manuscript, The World Bank 1994.

[8] Gale, Douglas and Martin Hellwig, 1985, Incentive Compatible Debt Contracts: the One Period Problem, *Review of Economic Studies* 52, 647-663.

[9] Gertler, Mark and Kenneth Rogoff, 1990, North-South Lending and Endogenous Domestic Capital Market Inefficiencies, *Journal of Monetary Economics,* 26, 245-266.

[10] International Monetary Fund, 1999, "World Economic Outlook," May.

[11] Hamid, Javed, and Ajit Singh, 1992, Corporate Financial Structures in Developing Countries, International Finance Corporation, Technical Paper No. 1, The World Bank Washington D.C.

[12] Huybens, Elisabeth and Bruce Smith, 1998, Financial Market Frictions, Monetary Policy and Capital Accumulation in a Small Open Economy. *Journal of Economic Theory 81,596-623.*

[13] Japelli, Tullio, 1990, Who is Credit Constrained in the U.S. Economy?, *Quarterly Journal of Economics* 105, 219-34.

[14] McKinnon, Ronald I., 1973, Money and Capital in Economic Development, Brookings Institute, Washington, D.C.

[15] Parente, Stephen, and Edward C. Prescott 1993, Changes in the Wealth of Nations, Federal Reserve Bank of Minneapolis, *Quarterly Review* 17, No. 2, 1-16.

[16] United Nations, 1992, Human Development Report 1992, Oxford University Press, New York.

[17] Williamson, Stephen, 1986, Costly Monitoring, Financial Intermediation, and Equilibrium Credit Rationing. *Journal of Monetary Economics* 18, 159-179.

[18] Williamson, Stephen, 1987, Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing. *Quarterly Journal of Economics* 102, 135-145.

[19] World Bank, "World Development Report, 1989," Oxford University Press, New York.

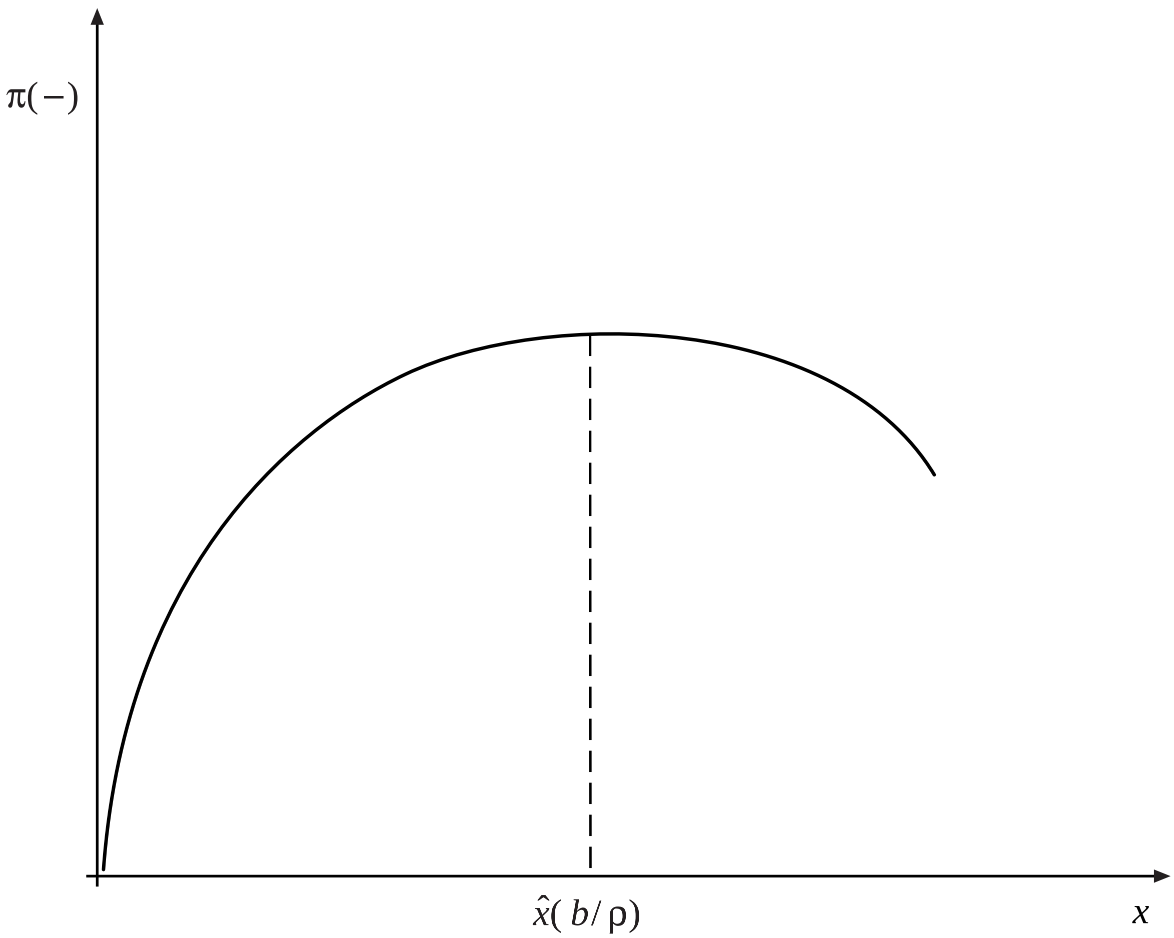

Figure 1
The Expected Return Function

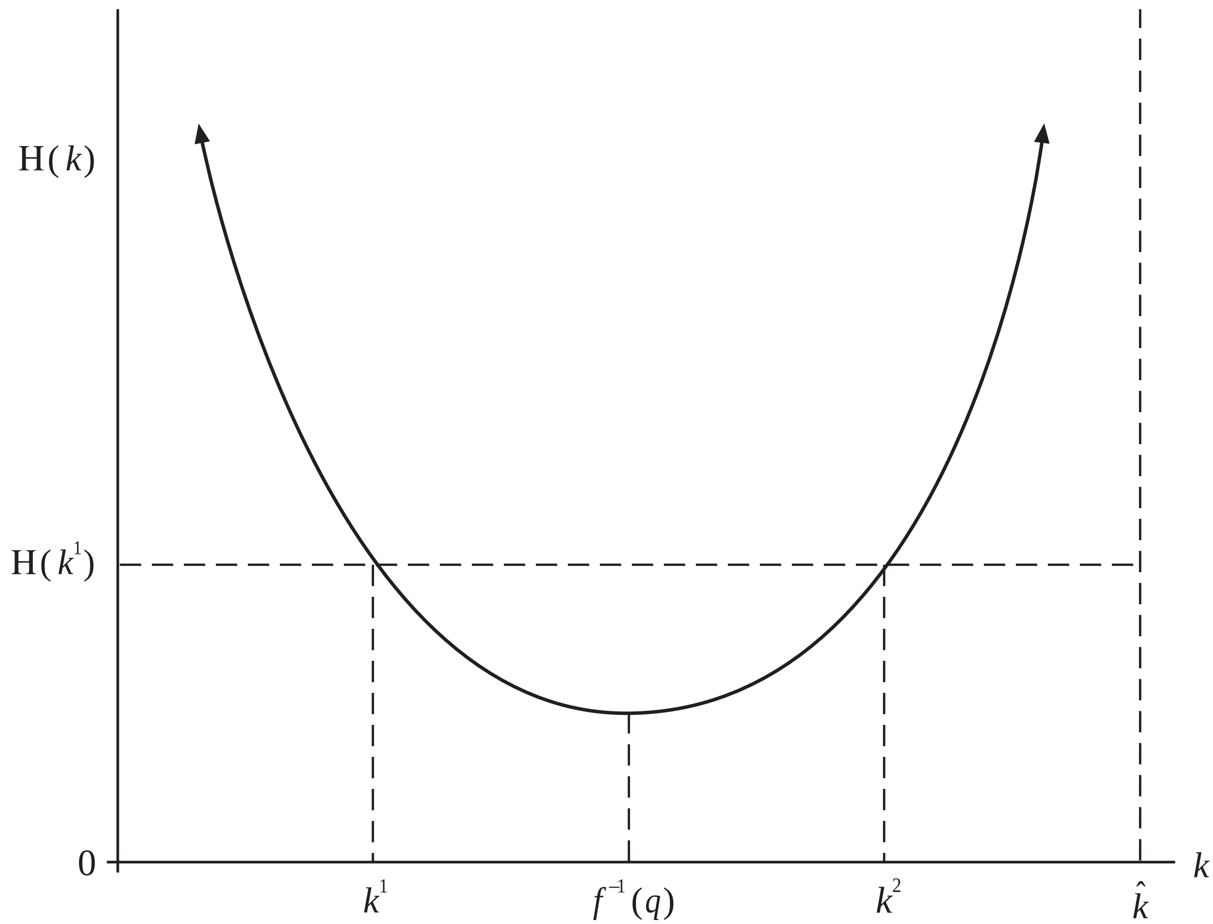

Figure 2

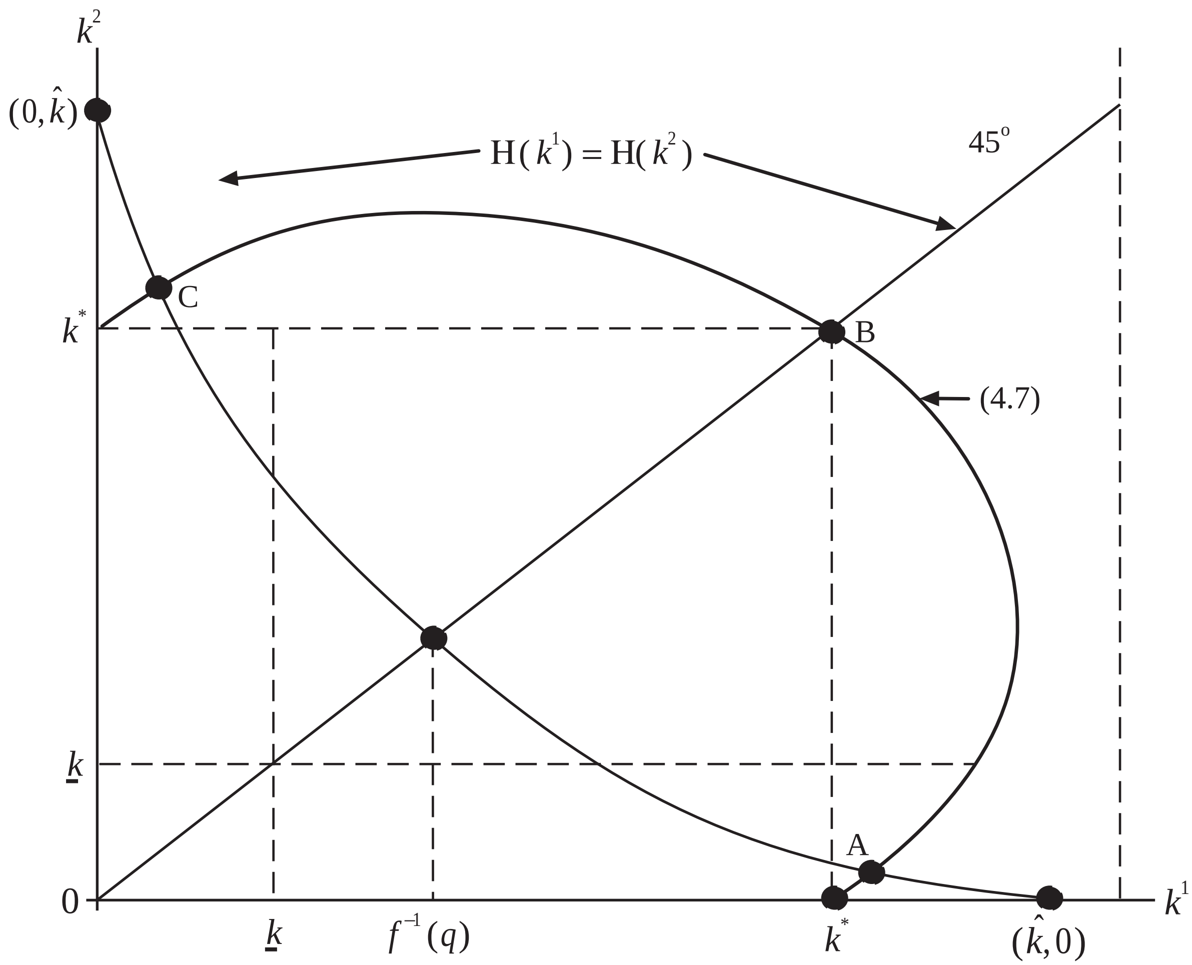

Figure 3
Steady State Equilibria with Unrestricted Capital Flows

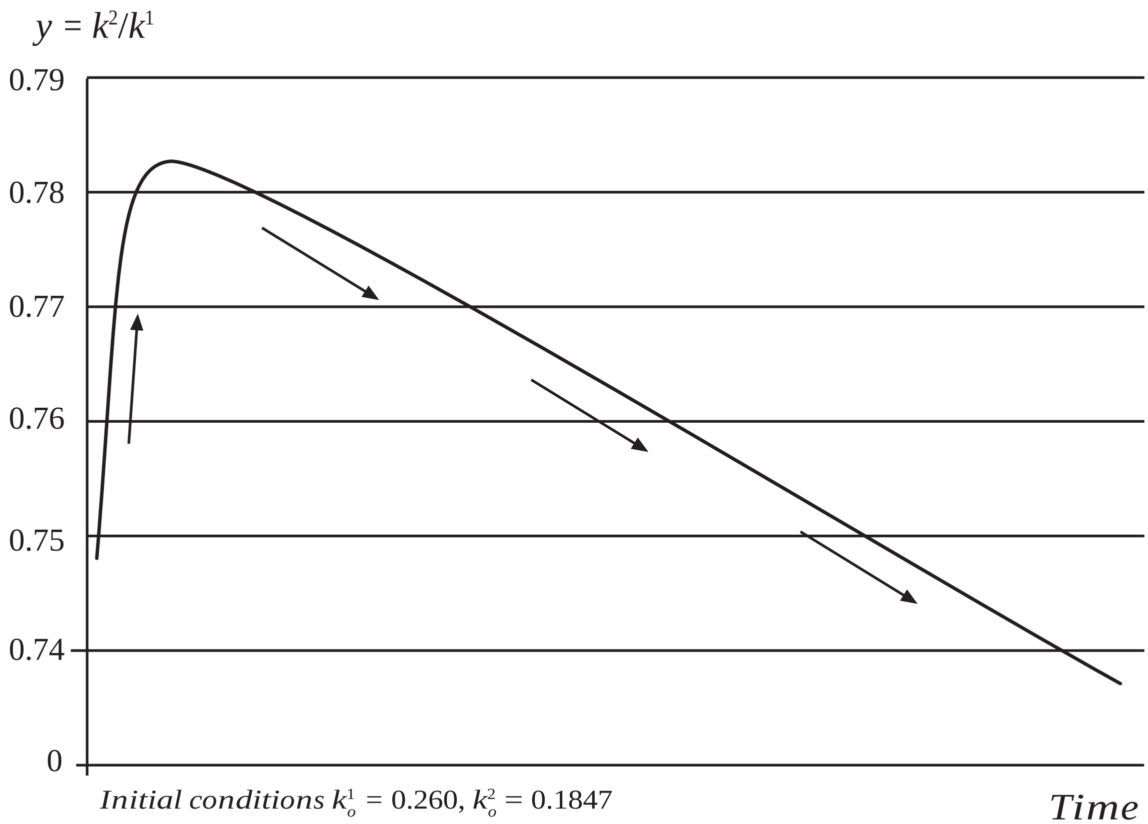

Figure 4
Example 1

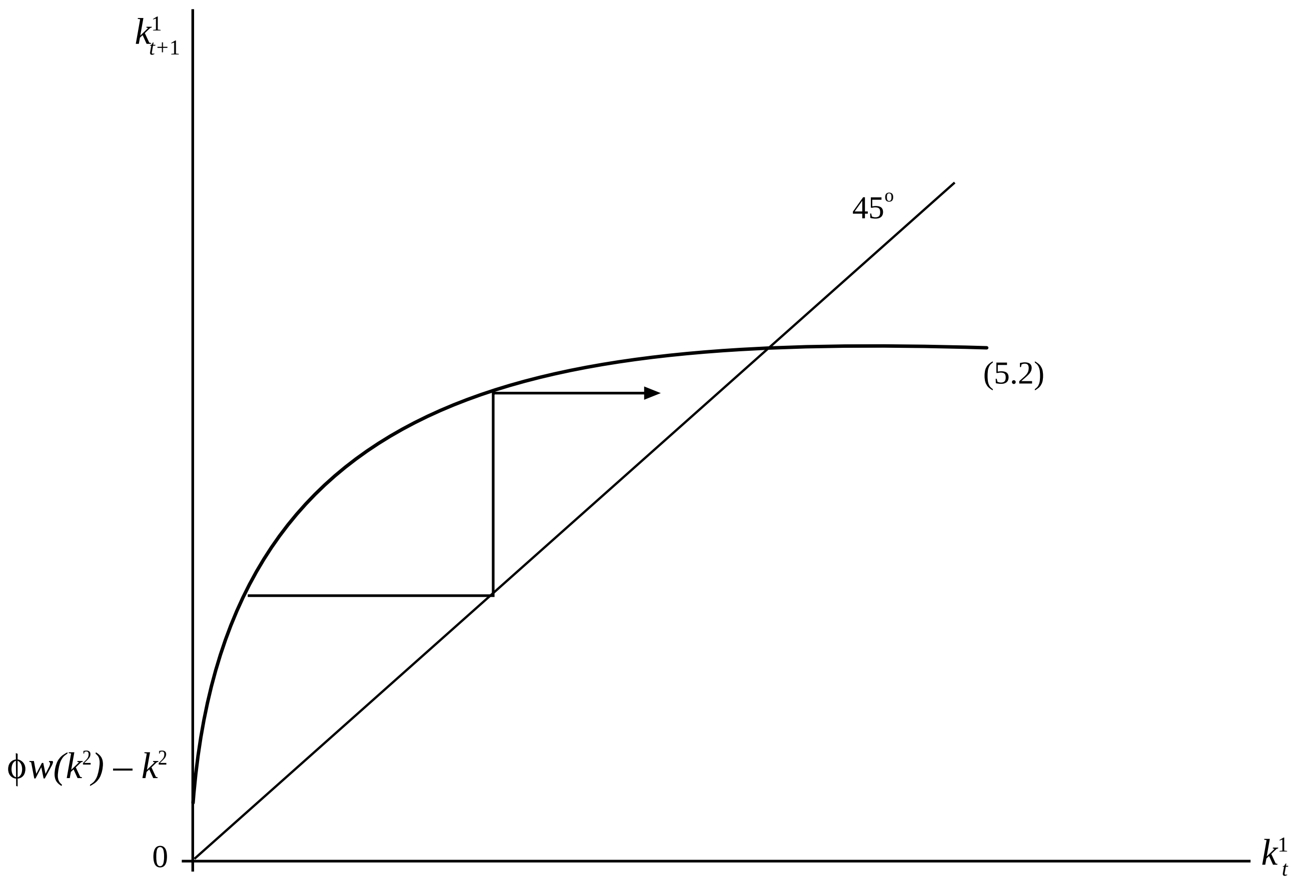

Figure 5
Law of Motion for $\{k^1_t\}$ when Variations in Country 2 Capital Stock are Eliminated

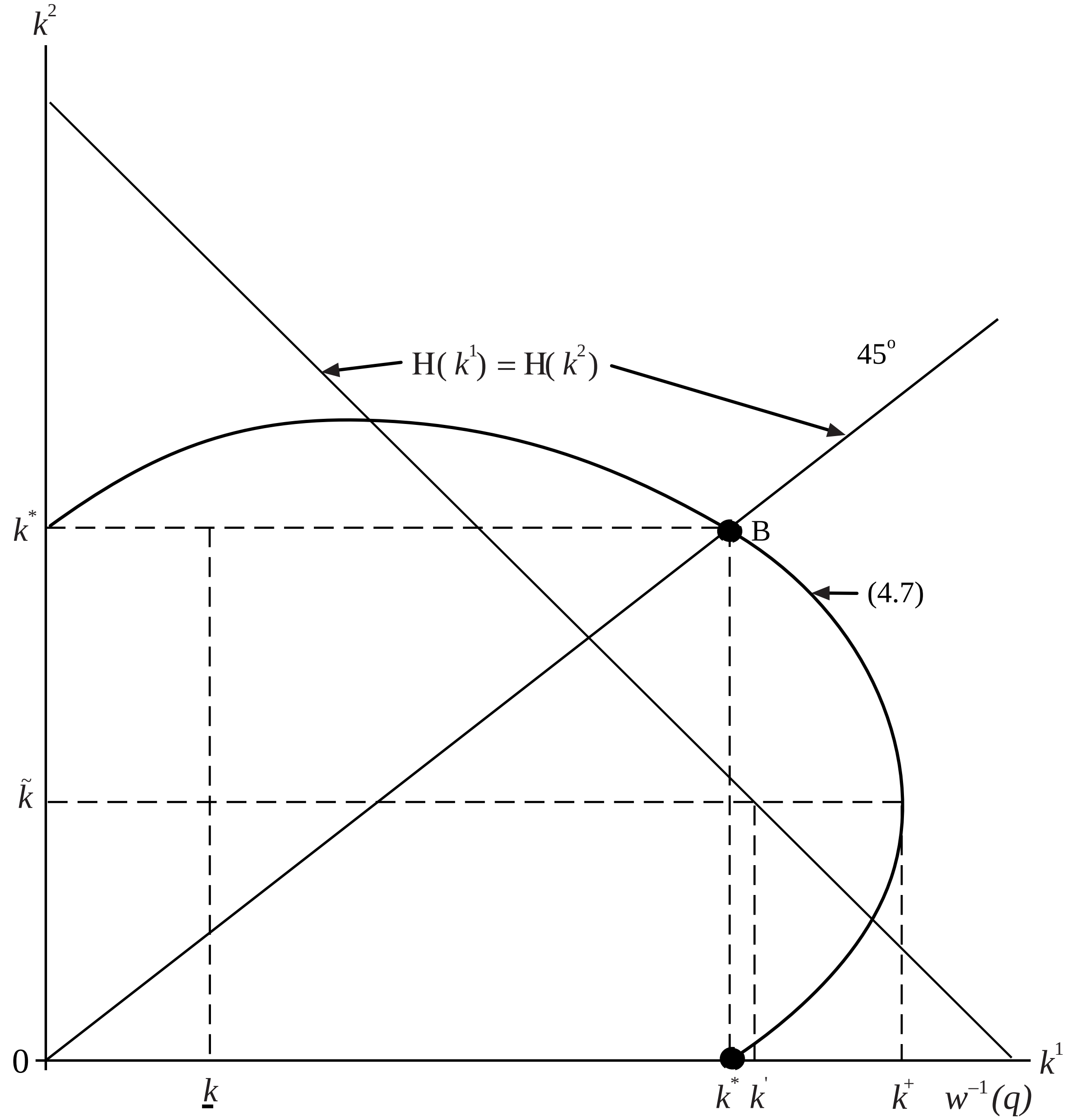

Figure 6
Existence of an Asymmetric Steady State with $\hat{k}_2 < \tilde{k}$