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Lance A. Fisher, Hyeon-seung Huh, Ellis W. Tallman

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Lance A. Fisher, University of New South Wales
Hyeon-seung Huh, Hallym University
Ellis W. Tallman, Federal Reserve Bank of Atlanta

Abstract: The authors use the permanent income hypothesis as the framework to analyze a number of results from recent empirical macroeconomic research. First, they demonstrate that the "productivity" shock isolated in both the three- and six-variable models of King, Plosser, Stock, and Watson (1991) depends mainly on the time-series behavior of the reduced form consumption residual. A permanent income hypothesis interpretation of this finding is that consumption fully reflects the implications of long-lived shocks for the common stochastic trend in consumption, investment, and output. Further, for the three variable model, shocks to consumption have permanent effects on the levels of the series while shocks to investment and output have only transitory effects, given that consumption is ordered first in a causal ordering. The permanent income interpretation is that shocks to productivity are changes in permanent income, and, hence, are fully reflected through consumption decisions. From this perspective, consumption shocks are proxies for the changes in permanent income generated by the "productivity" shocks. The results complement the findings in Hall (1978) and Campbell (1987) and generalize Cochrane's (1994) analysis to a model that includes investment.

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Key words: permanent income, transitory shocks, structural VEC, productivity

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Please address questions regarding content to Lance A. Fisher, School of Economics, University of New South Wales Sydney, NSW, 2052, Australia, 61-2-9385-3323, 61-2-9313-6337 (fax), L.Fisher@unsw.edu.au; Hyeon-seung Huh, School of Economics, Hallym University, 1 Okchon-dong, Chunchon, Kangwon-do 200-702, South Korea, 82-33-240-1309, 82-33-255-9109 (fax), hshuh@sun.hallym.ac.kr; or Ellis W. Tallman, Research Department, Federal Reserve Bank of Atlanta, 1000 Peachtree Street, N.E., Atlanta, Georgia 30309-4470, 404-498-8915, 404-498-8956 (fax), ellis.tallman@atl.frb.org.

Permanent Income and Transitory Variation in Investment and Output

1. Introduction

The permanent income hypothesis of Friedman (1957) implies that consumption is proportional to a measure of permanent income. Two notable implications are: 1) that consumption follows a random walk process (Hall, 1978), and, 2) that consumption and income are cointegrated (Campbell, 1987) so that the ratio of consumption to income is a stationary process. Cochrane (1994) exploited these implications to characterize the permanent and transitory components of GDP in a bivariate framework. He found that consumption was a good proxy for the permanent component of GDP because shocks to consumption had permanent effects on GDP while shocks to GDP that left consumption unchanged had only transitory effects.

King, Plosser, Stock and Watson (KPSW, 1991) show implications for the time series properties of consumption, investment and output in a three-variable model. In this model, a shock to total factor productivity follows a random walk process that is the common component of the driving forces in consumption, investment and output. Also, in the steady state, the “great ratios” of consumption and investment to output are stationary. From an econometric perspective, the model implies that there is one structural shock with permanent effects on the levels of the series and two with only transitory effects or, equivalently, that there is a single common stochastic trend and two cointegrating relationships, which correspond to the “great ratios”. KPSW identify econometrically the permanent structural shock, which they call the productivity shock, from a reduced-form vector-error correction model of consumption, investment and output.

In this paper, we employ the permanent income hypothesis as an organizing framework to interpret empirical evidence from two oft-cited papers. First, we show that the time-series behavior of the productivity shock in King, Plosser, Stock, and Watson (1991) reflects the movements of the residuals from the reduced form consumption equation. We further show that the productivity shock isolated in this model depends only on the reduced form consumption residuals under weak exogeneity of consumption, a condition supported by the data. Then, the common stochastic trend in their three-variable model depends only on the accumulated reduced form consumption residuals. Using the permanent income hypothesis as a lens, we interpret observed consumption changes as reflecting fully shocks to total factor productivity, which are not directly observable in the data. Hence, consumption is a good proxy for (a sufficient indicator of) permanent income which, in turn, is ultimately determined by total factor productivity.[1]

We extend Cochrane's analysis by imposing weak exogeneity of consumption in a three variable model framework. We demonstrate that if consumption is ordered first in a recursive causal ordering, shocks to consumption have permanent effects on the levels of consumption, investment, and output, while shocks to investment and output have only transitory effects. The consumption shock here is analogous to the permanent shock in KPSW's analysis while shocks to investment and output are the analogs to the two transitory shocks in their analysis, which they did not identify. The two features of our results that allow Cochrane's findings to generalize to a real business cycle framework are: 1) consumption is weakly exogenous and 2) the two cointegrating vectors have predictive power for long run changes in investment and output.

[1] Our results bear out Fama's assertion that "consumption is a random walk that immediately captures the implications of shocks (demand shocks, supply shocks, whatever) for the long term stochastic trend in consumption, investment and GNP" (Fama, 1992, p.469). We are able to provide evidence for Fama's assertion because our analysis is conducted in the context of a complete VEC system whereas his analysis was not.

In the last section of the paper, the model is augmented to include nominal variables following KPSW. In this model of six variables, there are three permanent structural shocks. They are identified under the same set of long run identifying restrictions as KPSW use. As before, we find that the productivity shock is mainly driven by the reduced form consumption residual. Of the variables in the extended model, observed consumption reflects the implications of shocks to total factor productivity, which is not directly observable.

2. Econometric Overview

In order to facilitate interpretation of the empirical results that follow, first consider a general specification of a *p*-dimensional dynamic linear simultaneous equation model:

$$AX_t = \mu + A_1X_{t-1} + \ldots + A_kX_{t-k} + v_t \qquad (1)$$

where $X_t = (x_{1,t}, x_{2,t}, \ldots, x_{p,t})'$, μ is a vector of constant terms, the A's are *pxp* coefficient matrices and $v_t \sim N(0, I_p)$. For the structural model (1), the reduced form model is:

$$X_t = \mu^* + A_1^*X_{t-1} + \ldots + A_k^*X_{t-k} + \varepsilon_t \qquad (2)$$

where $A_i^* = A^{-1}A_i$, $\mu^* = A^{-1}\mu$, $\varepsilon_t = A^{-1}v_t$ and the covariance matrix of the reduced form errors is given by Ω, where $\Omega = A^{-1}A^{-1'}$. Equation (2) is a standard vector auto-regressive (VAR) model in levels that can be re-parameterized as:

$$\Delta X_t = \mu^* + \Pi X_{t-1} + \Pi_1\Delta X_{t-1} + \ldots + \Pi_{k-1}\Delta X_{t-k+1} + \varepsilon_t \qquad (3)$$

where Δ is the difference operator, $\Pi = (A_1^* + \ldots + A_k^* - I)$ and $\Pi_i = -(A_{i+1}^* + \ldots + A_k^*)$. The matrix Π provides information about the long-run relationships among the series. If the series are I(1) and co-integrated, then $0 < \text{rank}(\Pi) < p$ and (3) is a vector-error correction (VEC) model. In this case, there are *r* long-run or cointegrating relationships and $m = p - r$ common trends among the series. In particular, $\Pi = \alpha\beta'$ where α and β are *pxr* matrices, the columns of

β are the coefficients in the cointegrating vectors and the rows of α are the loadings on the error correction terms $\beta' X_{t-1}$ in each equation, respectively. If the loadings on the error correction terms in a particular equation are all zero, then the dependent variable of that equation is said to be weakly exogenous with respect to the matrix of cointegrating vectors β. More formally, if $x_{i,t}$ is weakly exogenous, then the *r* error correction terms do not enter the equation for $x_{i,t}$ so that $\alpha_{i1} = \alpha_{i2} = \ldots = \alpha_{ir} = 0$. Note that since α has rank *r* by construction, there can be at most *m* weakly exogenous variables.

As shown in Johansen (1991), the VEC model (3) can be inverted to obtain the moving average representation for ΔX_t:

$$\Delta X_t = \rho + C(L)\varepsilon_t \tag{4}$$

where $\rho = C(1)\mu^*, C(l) = I + \sum_{i=1}^{\infty} C_i L^i$, and

$$C(1) = \beta_{\perp}\gamma\alpha'_{\perp} \tag{5}$$

where the subscript $(\perp)$ denotes a matrix orthogonal to the original matrix, $\gamma = (\alpha'_{\perp}\Psi\beta_{\perp})^{-1}$ and $\Psi = I - \sum_{i=1}^{k-1}\Pi_i$ from (3). Subject to identification, a structural moving average representation, corresponding to (4) is:

$$\Delta X_t = \rho + \Gamma(L)v_t \tag{6}$$

where $\Gamma(L) = \Gamma_0 + \Gamma_1 L + \Gamma_2 L^2 + \ldots$, and $\Gamma(1) = \sum_{i=0}^{\infty}\Gamma_i$. The relationships between the reduced form and structural parameters are:

$$\varepsilon_t = \Gamma_0 v_t = A^{-1} v_t \tag{7}$$

and

$$\Gamma(1) = C(1)A^{-1} \qquad (8).$$

KPSW (1991) provide a procedure to identify the *m* structural shocks with permanent effects on the levels of the series or, equivalently, the *m* common trends in the system. Since there are *m* structural shocks with permanent effects and *r=p-m* with transitory effects, KPSW assume that

$$\Gamma(1) = [K \mid 0] \qquad (9)$$

where K is a (*pxm*) matrix whose columns are orthogonal to the cointegrating vectors.
In addition to the cointegrating relations, a further *m(m-1)/2* restrictions are required for exact identification of the *m* structural shocks with permanent effects when *m*>1. KPSW impose *m(m-1)/2* zero restrictions on the elements of K. Once this is done, they show how the remaining elements of K can be determined. Once K is known, the first *m* rows of A, denoted A_m can be found and the *m* structural shocks with permanent effects, $v_{m,t}$, are given as $A_m\varepsilon_t$. For the discussion that follows, it is important to note that under the KPSW procedure, $\alpha_\perp$ spans the column vectors of $A_m^{/}$ so that

$$A_m^{/} = \alpha_\perp P \qquad (10)$$

where P is any non-singular (*mxm*) matrix. This means that the structural shocks with permanent effects are proportional to $\alpha_\perp$. When there is only one common trend (i.e. *m*=1), P is a normalizing constant chosen so that the permanent shock has unit variance (i.e. $P = (\alpha_\perp^{/}\Omega\alpha_\perp)^{-1/2}$). In that case (10) becomes

$$A_m = (\alpha_\perp^{/}\Omega\alpha_\perp)^{-1/2}\alpha_\perp^{/} \qquad (11)$$

which provides exactly the same result for A_m as that obtained from the KPSW procedure when *m*=1.

3. Long-run Relationships

In this section, we consider a model of the log levels of real per-capita U.S. consumption, investment and private output. Consumption is defined as real personal consumption expenditure on goods and services. The investment measure is real private gross domestic investment. Following KPSW, the measure of output is a private measure that excludes the government sector. Specifically, private output is defined as real GDP less total real government expenditure. The series are quarterly and seasonally adjusted, and divided by population so that each series is expressed in real per-capita dollar values.[2]

In the analysis, the series enter the reduced form model in log levels and the ordering of the variables in X_t is maintained as $X_t = (c_t, i_t, y_t)'$ where c_t is log consumption, i_t is log investment and y_t is log private output. The full sample period is 1948:1-2000:3. As a check on the robustness of our results, the analysis is also performed for the sample period used by KPSW, 1948:1-1988:4. Prior to testing for long run relationships among the series, the reduced form model (2) was estimated, over both sample periods, with five lags as the Sims' likelihood ratio test and the AIC criterion both chose this lag length.

Table 1 shows the results of tests for long run relationships among the series over the full sample using Johansen's (1991) maximum likelihood procedure. Both the trace and $\lambda-$max statistics reject the null hypotheses of $r = 0$ and $r \leq 1$ but do not reject the null hypothesis of $r \leq 2$, at the 5% level. Thus, we conclude that there are two cointegrating vectors.

[2] The series were obtained from the FREDS database of the Federal Reserve Bank of St Louis. The series are personal consumption expenditures (PCECC96), gross private domestic investment (GDPIC1) and private output defined as GDP (GDPC1) less government consumption and gross investment (GCEC1). The FRED designators for the series are shown in brackets. The series are in billions of chained 1996 dollars. They are quarterly and given as seasonally adjusted annual rates. The series were divided by the civilian non-institutional population, aged 16 years and over, also obtained from the FREDS database (CNP16OV). This series is not seasonally adjusted and is expressed as thousands of persons. Since the series is monthly, the value for the second month of the quarter is taken as the quarterly value.

Their normalized coefficients are reported in the table. The coefficient on private output in first cointegrating vector is close to -1.0 so that there is almost a long-run one-for-one proportional relationship between c_t and y_t, as implied by the permanent income hypothesis. The coefficient on private output in the second cointegrating vector is also close to -1.0. However, since both coefficients are statistically different from -1.0, at the 5% level, balanced growth is not a feature of the data.

In this model, there is only one structural shock that has permanent effects on the levels of the series, as there are two cointegrating vectors. From Section 2, it is apparent that permanent structural shock, denoted $v_{1,t}$, is equal to $A_m\varepsilon_t$ where A_m is $\alpha_\perp^{/}$ normalized so that $v_{1,t}$ has unit variance (see the discussion surrounding (10) and (11)). The table reports normalized $\alpha_\perp^{/}$ so that the relationship between the permanent structural shock and the reduced form errors is given by

$$v_{1,t} = 153.87\varepsilon_{1,t} + 4.16\varepsilon_{2,t} - 31.08\varepsilon_{3,t} \qquad (12)$$

The permanent structural shock, which KPSW refer to as a productivity shock in this set-up, has its largest coefficient associated with the residual from the reduced form equation for consumption. Even allowing for the greater variance of the residual series for investment and output, the variance of the consumption residuals comprise over 75 percent of the variance of the permanent structural shock series. The consumption residual tracks the permanent structural shock series extremely closely (the correlation is 0.99) whereas the investment residual bears little resemblance to it (the correlation is only 0.17). As a result, the correlation of the output residual with the permanent structural shock series is considerably less (0.56) than that of the consumption residual. Finding that the permanent structural shock depends mostly on the consumption residual is consistent with the permanent income hypothesis. Consumption proxies for permanent income. Permanent income itself depends on productivity growth in the economy, so permanent productivity shocks are essential determinants of permanent income. Thus, a consumption time-series that embodies the permanent income hypothesis would likely reflect the permanent shocks that affect productivity and hence estimated permanent income.

When consumption is assumed weakly exogenous, the permanent structural shock depends only on the residual from the reduced form equation for consumption since $\alpha_{\perp}^{\prime} = (\alpha_{1\perp} \;\; 0 \;\; 0)$ in this case.[3] Table 1 reports Johansen's likelihood ratio test statistic of the null that consumption is weakly exogenous. This null is formulated as a test of a H_4 hypothesis in the terminology of Johansen and Juselius (1990). The likelihood ratio test statistic is distributed as a chi-squared with two degrees of freedom as there are two restrictions being tested here. The likelihood ratio test statistic does not reject the null that consumption is weakly exogenous at standard significance levels since the p-value of the test is 0.82. This is consistent with the earlier result that most of the weight falls on the consumption residual in the determination of the permanent structural shock. Under weakly exogenous consumption, the permanent structural shock is

$$v_{1,t} = 129.60\varepsilon_{1,t} \tag{13}$$

where, as before, $v_{1,t}$ has unit variance. When consumption is weakly exogenous, the permanent structural shock is simply the residuals from the reduced form equation for consumption. As we will see in Section 5, this implies that the accumulated residuals from the reduced form consumption equation constitute the common stochastic trend.

[3] Under weak exogeneity of consumption, $\alpha = \begin{pmatrix} 0 & 0 \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{pmatrix}$. Since α has rank 2, it follows that the sub-matrix $\begin{pmatrix} \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{pmatrix}$ has full rank. It then follows from $\alpha^{\prime}\alpha_{\perp} = 0$ that $\alpha_{\perp} = (\alpha_{1\perp} \quad 0 \quad 0)^{\prime}$.

Table 1 also reports that Johansen's test statistic emphatically rejects the null that investment and private output are, respectively, weakly exogenous. Taken together, the exogeneity test results show that the two cointegrating vectors have jointly predictive power for changes in investment and private output but not for consumption. Note that because some of the coefficients on the ΔX_{t-i}'s in the consumption equation in (3) were statistically significant (not shown), consumption, although weakly exogenous, is not strictly a random walk process. Lastly, the normalized cointegrating vectors, estimated under the restriction that consumption is weakly exogenous, are reported in Table 1. These are virtually identical to the unrestricted vectors, providing further support for the weak exogeneity of consumption.

Table 2 reports the results of the cointegration analysis over the sample considered by KPSW. These results generate the same statistical inferences as those reported for the full sample, although the degrees of freedom differ. In this sub-sample, 88-percent of the weight is given to the consumption residual in the determination of the permanent structural shock. Again, the null that consumption is weakly exogenous is not rejected at standard significance levels.[4]

4. Dynamic Interactions

In this section, we consider the dynamic interactions between consumption, investment and private output under the restriction that consumption is weakly exogenous.[5] Under that restriction $\alpha_{\perp}^{\prime} = (\alpha_{1\perp} \;\; 0 \;\; 0)$ and it immediately follows from (5) that the long-run impact matrix of the reduced form shocks is

$$C(1) = \begin{pmatrix} \beta_{1\perp}\gamma\alpha_{1\perp} & 0 & 0 \\ \beta_{2\perp}\gamma\alpha_{1\perp} & 0 & 0 \\ \beta_{3\perp}\gamma\alpha_{1\perp} & 0 & 0 \end{pmatrix} \qquad (14)$$

This says that the last two reduced form errors in ε_t have zero long-run impacts on investment and private output.

Identification of all the structural shocks in the model is achieved once the matrix A (or A^{-1}) in (7) is determined. The KPSW procedure determines only the first *m* rows of A leaving the transitory structural shocks unidentified. To exactly identify all the structural shocks in the model, we follow Sims (1980) and assume a recursive structure on the *contemporaneous* interactions among the series by specifying A as a lower triangular matrix. In that case, A^{-1} is also lower triangular and is obtained uniquely from the Choleski decomposition of Ω since $\Omega = A^{-1}A^{-1/}$.

Under a Sims recursive structure, weak exogeneity of consumption has an important implication for the form of the long-run impact matrix of the structural shocks $\Gamma(1)$. Since A^{-1} is lower triangular, it follows immediately from (8) and (14) that the last two columns of $\Gamma(1)$ are column vectors of zeros. But this is precisely the form of the long run impact matrix for the structural shocks *assumed* by KPSW (see (9) above). Here we have shown that their form of the long run impact matrix of structural shocks is an implication of weak exogeneity of consumption and a Sims contemporaneous recursive causal ordering when consumption is ordered first.[6] Thus structural shocks to consumption have permanent effects on the levels of the series whereas structural shocks to investment and private output have only transitory effects.

[4] If expenditure on consumer durable goods is included in investment rather than consumption as in Cochrane (1994) and Fama (1992), the results of the analysis over both samples are qualitatively unchanged. In particular, weak exogeneity of consumption is not rejected in either sample.

[5] We note that the dynamics are not materially different when weak exogeneity is not imposed.

[6] In the context of a bivariate model of consumption and GDP, Cochrane (1994) showed that the last column of $\Gamma(1)$ is a column vector of zeros when consumption is a random walk and is ordered first in a Sims causal ordering. It is not necessary to assume that consumption is a random walk to arrive at this result, only that it is weakly exogenous with respect to the single cointegrating vector between consumption and GDP (Ribba (1997)).

Figure 1 shows the responses of each series to one standard error shocks from the corresponding structural equations in (1) together with one standard error confidence bounds.[7] These responses are based on the VEC model (3) estimated over the full sample under the restriction that consumption is weakly exogenous. In that case, the cointegrating vectors are the restricted ones reported in table 1. The estimated VEC model is then re-parameterized to the reduced form model in levels, given by (2), prior to the impulse response analysis.

The first column of Figure 1 shows that consumption, investment and private output increase permanently in the long run in response to the consumption shock. The long-run responses of c_t, i_t and y_t to a one-standard error consumption shock are 0.92, 1.21 and 0.98 percent, respectively.[8] These long-run responses satisfy the condition that $\beta^{/}\Gamma(1)_1 = 0$ where $\Gamma(1)_1$ is the first column of $\Gamma(1)$. Consumption is almost a random walk since its response to its own shock is almost flat. However, there is a significant transitory component in private output, and particularly investment. In response to the consumption shock, investment increases by a much larger percentage in the short run (4.5 percent after five quarters) than in the long run. Notice also the hump-shaped response of private output to the consumption shock. At five quarters, private output has increased by 1.52 percent, around one-half of a percentage point higher than its long run response.

The responses in the first column of figure 1 are similar to those reported by KPSW. The responses of investment and private output to the permanent structural shock are hump-shaped whereas the response of consumption is immediate and flat in their analysis. The similarity of the responses is not surprising since we have shown that the permanent structural shock in their

[7] One-standard errors for the impulse responses were approximated using 500 bootstrap replications following the procedure described in Runkle (1987).

analysis is very heavily weighted towards the residuals from the consumption equation (see the discussion of (12)).[9]

The second and third columns of figure 1 show that shocks to investment and private output have short run but no long run effects on the levels of the series. KPSW did not report responses like these, since in their analysis, the two structural shocks with transitory effects are not identified by their procedure. Transitory variation in the series also shows up in the short run responses to investment and private output shocks. In response to a one-standard error investment shock, investment increases initially by 4 percent and private output by about 1 percent. In response to a one-standard error private output shock, private output at first rises and, as it returns to its original level, investment and to a lesser extent consumption fall in the short run as may happen when producers and consumers run down inventory.

Figure 1 shows that Cochrane's results hold in a model that includes investment. Shocks to investment and private output have only transitory effects on the levels of the series when they do not also have a contemporaneous impact on consumption, that is, when consumption is placed first in a Sims recursive causal ordering. Here, a shock to private output with no contemporaneous consumption change must mean that consumers view the output shock as transitory and, thus, do not change their consumption levels in response. In contrast, shocks to consumption have permanent effects on the levels of the series. Consumption shocks can be thought of as shocks to permanent income. In the permanent income hypothesis, consumption responds only to permanent income changes. Consumers permanently change their consumption

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[8] The coefficient greater than 1.0 for investment indicates its sensitivity to productivity shocks – when productivity is high, one wants to invest more to produce greater amounts during that highly productive period.

[9] KPSW did not report the weights on the reduced form residuals that comprise the permanent shock nor did they test for the weak exogeneity of consumption.

levels in response to these shocks, so these shocks proxy shocks to total factor productivity, key determinants of permanent income.

5. Permanent and Transitory Decompositions

In this section, we show that under weak exogeneity of consumption, the permanent component of each series depends only on the accumulated consumption shocks, thus providing support for Fama's (1992) assertion that consumption is a good proxy for the common stochastic trend.

By the Granger Representation Theorem, the VEC model (3) can be inverted and summed to yield the common trends representation:

$$X_t = X_0 + C(1)\sum_{i=1}^{t}(\varepsilon_i + \mu^*) + X_t^s \qquad (15)$$

where X_0 is a vector of initial permanent values, and X_t^s is the vector of transitory components that are stationary by construction.[10] The permanent components are given as $X_t^p = X_t - X_t^s$. From (7) and (8), the common trends structural representation is:

$$X_t = X_0 + \Gamma(1)\sum_{i=1}^{t}(v_i + \mu) + X_t^s \qquad (16)\,.$$

Under the KPSW procedure, the last two columns of $\Gamma(1)$ are column vectors of zeros (see (9)). Then from (9) the permanent components of (16) become:

$$X_t^p = X_0 + K\sum_{i=1}^{t}(v_{1,i} + \mu_1) \qquad (17)$$

where K is the first column of $\Gamma(1)$. Under weak exogeneity of consumption, the permanent structural shock in (17) depends only on the reduced form errors from the consumption equation;

[10] We estimate each initial permanent component by forecasting 206 periods ahead beginning in 1949:2 and subtracting off the deterministic portion of the forecast.

specifically, it is given by (13). Thus (17) says that the permanent component of each series depends only on the accumulated values of the consumption shock. Note that this result also follows from the analysis of the previous section since it was shown there that the last two columns of $\Gamma(1)$ are column vectors of zeros.[11]

In the stochastic growth model of KPSW, the permanent structural shock arises from the unit root in total factor productivity. However, total factor productivity is not identifiable in an empirical model that uses consumption, investment and private output data. Our results show that, of these three series, it is consumption that captures the implications of unobservable total factor productivity since the permanent structural shock is very heavily weighted towards the residual from the consumption equation. Indeed, it is entirely so weighted under weak exogeneity of consumption which we could not reject. These results are consistent with the permanent income hypothesis in which consumption is a forward-looking variable.

To investigate this further, figure 2 shows the growth rate of the permanent component of private output (y_t^p) and of total factor productivity (TFP), both measured as annual percentage rates at quarterly frequency. TFP is recovered from a constant returns to scale Cobb-Douglas production function setting the share of capital equal to 0.3.[12] Also shown in the figure are the business cycle recessions (indicated by the shaded regions) as dated by the National Bureau of

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[11] Formally, Fisher and Huh (1999) prove that under weak exogeneity of consumption, the permanent shock in KPSW's analysis (given here as (13)) is identical to the consumption shock obtained from a Sims recursive causal ordering with consumption ordered first.

[12] In calculating TFP, data for output, capital and labor, exclusive of the government sector, were used. Except where stated, the FREDS designators are shown. The series for private output is GDPC1 less GCEC1, as before. The capital stock series is the current-cost net stock of fixed assets (excluding government fixed assets) obtained from the Survey of Current Business, September 2000. The series is the annual year-end estimate of the private capital stock from 1948 to 1999 measured in billions of 1996 dollars. Quarterly series are obtained by interpolating the annual series using the procedure INTERPOL in RATS and then dividing by the implicit price deflator for private output to obtain real estimates of the capital stock. The implicit price deflator is the ratio of nominal to real private output. Nominal output is GDP less GCE. The labor force is the number of civilians, 16 years and over, who are in employment (CE16OV) less the number of government employees. The latter is obtained from the U.S. Department of Labor, Bureau of Labor Statistics, and is designated as series EE90000001. Both are monthly, seasonally adjusted

Economic Research. The model of consumption, investment and private output in this paper cannot directly recover a productivity shock series. Equation (17) in conjunction with (13) shows that the growth rate of y_t^p depends only on the current consumption shock, which we have interpreted as a proxy for productivity shocks. The consumption shock moves similarly to the rate of change in TFP observed in the recessions of 1974/75, 1982 and 1990/91. Overall, the consumption shock is less variable but moves in the same direction as the change in TFP. The sample correlation coefficient between the two series over the full sample is 0.63. These results are robust to other empirically plausible values of capital's share in output. Figure 2 and the sample correlation coefficient are practically unchanged for capital's share in output ranging from 0.3 to 0.4. These observations indicate that the consumption shock reflects a pattern of productivity shocks that are comparable to those productivity shocks derived from a standard, yet different paradigm.

In figure 3, the transitory components of the series are shown, together with the growth cycle recessions (shaded regions), documented by the Foundation for International Business and Economic Research.[13] A notable feature is that the peaks and troughs in the transitory components coincide well to the peaks and troughs in the growth cycle. The transitory component in consumption is small consistent with our earlier observation that consumption is approximately a random walk while the transitory component of investment is large and highly variable.

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and measured in thousands of persons. The value for the second month of the quarter was taken as the quarterly value.

[13] The growth cycle refers to deviations of macroeconomic time series from trend whereas the classical cycle refers to their deviations from zero growth. The National Bureau of Economic Research has documented the dates for the classical cycle. However, because the transitory components are deviations from the common stochastic trend, it is more appropriate to refer to the growth cycle dates here.

6. Inclusion of Nominal Variables

We have seen that in the three variable model, the permanent structural shock, which KPSW refer to as the productivity shock, is heavily weighted towards the residuals from the reduced form equation for consumption. Under the permanent income hypothesis, this result may be expected since consumption proxies for permanent income which, in turn, ultimately depends on total factor productivity. In this section we investigate the robustness of this result when the model is augmented to include nominal variables. Specifically, the log of real per-capita M1 ($m-p$), the nominal three month treasury bill rate (R) and the rate of inflation (Δp) are added to the three variable model.[14] Following KPSW we expect that, in this augmented model, there is a third cointegrating vector that corresponds to the long run demand for money.

Crowder, Hoffman and Rasche (1999) report that there is a long run relationship between M1 velocity and the nominal interest rate for the sample that ends in 1994:4. They also note (p. 120) that after 1994:4 the stability of this relationship deteriorates. Part (a) of table 3 shows the results of Johansen's tests for cointegration between M1 velocity and the nominal interest rate from 1959:1-1994:4. On the basis of these tests, we conclude that there is a long run relationship between M1 velocity and the nominal interest rate. The estimated long run relationship is reported in the form of a long run demand for money function in the table. The estimated interest rate semi-elasticity of -0.109 is statistically different from zero and is very close to that reported by Crowder, Hoffman and Rasche. This is the only coefficient that is estimated since the velocity formulation imposes a unitary income elasticity on the demand for real money balances. Consistent with Crowder, Hoffman and Rasche, we find no evidence for a

[14] The data are obtained from the FREDS database. M1 (M1SL) is seasonally adjusted and starts in 1959:1. It is expressed in per-capita terms by dividing by CNP16OV. The three month treasury bill rate (TB3MS) is from the secondary market and is measured as an annual percentage. The inflation rate is calculated from the implicit price deflator for private output and is also measured as an annual percentage.

long run stable relationship between M1 velocity and the nominal interest rate over the full sample 1959:1-2000:3. For that reason, all the results reported in this section are for the sample 1959:1-1994:4.

Part (b) of table 3 shows, on the basis of Johansen's test statistics, that there is evidence for two cointegrating relationships between consumption, investment, private output and the real interest rate ($R - \Delta p$), at the 10-percent significance level. The point estimate of the coefficient on private output in each vector is close to unity, although the null of balanced growth is rejected at standard significance levels. For each vector, the estimated coefficient on the real interest rate is close to zero. Formally, Johansen's likelihood ratio test shows that neither coefficient is statistically different from zero at standard significance levels. The null of long run exclusion of the real interest rate from both cointegrating vectors is not rejected by Johansen's likelihood ratio test, since the p-value of the test statistic is 0.8. The cointegrating vectors, estimated under this null, are reported in the table. The estimated coefficients on private output are essentially unchanged, consistent with the restriction.[15]

On the basis of these results, we estimate two reduced form VEC models. In the first, we do not impose long run exclusion of the real interest rate while, in the second, we impose this restriction. For each model, the three error correction terms are given by the corresponding cointegrating vectors reported in table 3. Also for each model, the lag length is six in levels of the series as that was the larger of the two lag lengths used in the cointegration analysis, reported in the table.

[15] It is interesting to note that KPSW find a similar result for their sample that ends in 1988:4. In the cointegrating relationships corresponding to the "great ratios", they report coefficients on the real interest rate that are considerably smaller (in absolute value) than twice the coefficient's standard error (table 2, p.828). Moreover, on the basis of a Wald test, they cannot reject at the 5-percent level, that the vectors $(c-y)$, $(i-y)$ and $(m-p-1.152y+0.009R)$ span the cointegrating space (table 3, part B, p.829).

Because there are three cointegrating vectors in the six variable models, there are three structural shocks with permanent effects on the levels of the series. To identify the permanent shocks, KPSW impose three zero restrictions on the long run impact matrix of the structural shocks in such a way that first permanent shock ($v_{1,t}$) can be interpreted as a productivity shock, the second as an inflation rate shock ($v_{2,t}$) and the third as a real interest rate shock ($v_{3,t}$). Formally, in equation (9), KPSW assume that $K = \tilde{K}\pi$ where the columns of $\tilde{K}$ are specified a priori and are orthogonal to the cointegrating vectors. The (*3x3*) matrix π is a lower triangular matrix to be estimated. The productivity shock is identified under the assumption that it is the only shock that can have a long run effect on output. This gives two zero long run restrictions. The third is from the identifying assumption that the real interest rate shock does not have a long run effect on the inflation rate. KPSW order the variables in X_t as $(y_t, c_t, i_t, m_t - p_t, R_t \ \Delta p_t)'$. For this ordering, these restrictions are imposed by setting the elements (1,2), (1,3) and (6,3) of $\tilde{K}$ to zero. Because π is lower triangular, these same three elements of K are also zero. KPSW show how the elements of π can be determined. Once this is done, the non-zero elements of K are determined and from this, the (*6x3*) matrix A_m, which gives the permanent structural shocks.[16]

From the estimated VEC models, we recover the permanent structural shocks in exactly the same way as KPSW. For the variables (and equations) ordered as in KPSW, the first permanent structural shock, interpreted as the productivity shock, in the unrestricted model, is given as

$$v_{1,t} = 36.80\varepsilon_{1,t} + 121.65\varepsilon_{2,t} + 1.75\varepsilon_{3,t} - 10.45\varepsilon_{4,t} - 0.59\varepsilon_{5,t} - 0.14\varepsilon_{6,t} \qquad (18)$$

[16] Fisher, Fackler and Orden (1995) provide a technical description of the KPSW procedure.

where $\varepsilon_{i,t}$ is the residual from the *i*th reduced form equation and the weights are the first row of A_m. The productivity shock here as well is driven mainly by the time-series behavior of the consumption residual. Even in this extended model, it appears that the consumption residual displays the implication of the permanent income hypothesis, embodying the effects of productivity shocks on permanent income. The productivity shocks are not directly observed by the econometrician nor directly recoverable from the six variable model.

For the restricted model (i.e. estimated under long run exclusion of the real interest rate), the first permanent structural shock is given as

$$v_{1,t} = 1.14\varepsilon_{1,t} + 151.01\varepsilon_{2,t} + 5.97\varepsilon_{3,t} - 3.34\varepsilon_{4,t} - 0.33\varepsilon_{5,t} - 0.14\varepsilon_{6,t} \qquad (19)$$

Here even more weight is placed on the residuals from the reduced form equation for consumption. Specifically, 93-percent of the weight is placed on the consumption residual. In fact, the weight on the consumption residual is twenty times more than the combined weight on both the output and investment residuals. The weights on the nominal residuals are negligible. This only strengthens our earlier conclusion that, in the data, it is the consumption residual that captures the implications of shocks to unobserved total factor productivity.

Figure 5 shows the impulse responses for consumption, investment, and output that come from the unrestricted six variable model together with one standard error confidence bounds. The dynamic patterns in the impulses are similar to those generated from the three-variable model, indicating further the robustness of the finding that consumption shocks have permanent impacts on real variables, but the shocks to investment and output have only transitory effects.

7. Conclusions

In this paper, we use the permanent income hypothesis to interpret several empirical results. First, we show that the time-series behavior of the productivity shock in both the three and six variable model of KPSW is driven by the reduced form consumption residual. In particular, in the three variable model, the series interpreted as the productivity shock depends only on the consumption residual and the common stochastic trend only on the accumulated consumption residuals, under weak exogeneity of consumption, which was not rejected by the data. The interpretation we give to these findings is that, in the data, consumption reflects the implications of the permanent income hypothesis. The consumption reduced form residuals embody essentially the permanent shocks to the common stochastic trend in consumption, investment and output. Theoretically, the common stochastic trend arises from the random walk in total factor productivity but, in the data, total factor productivity is not directly verifiable. It is consumption that reflects the implications of shocks to total factor productivity in the data, irrespective of whether the data under consideration include nominal variables.

Furthermore, we show that Cochrane’s analysis applies to a model that includes investment. Shocks to consumption have permanent effects on the levels of the series while shocks to investment and output have only transitory effects, given that consumption is weakly exogenous and is ordered first in the causal ordering. The consumption shock has the same effects as the permanent shock in KPSW’s model while the investment and output shocks can be thought of as the analogs to the their transitory shocks, which they did not identify.

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TABLE 1
Results of Cointegration Analysis: 1948:1-2000:3

Hypothesis	$r = 0$	$r \leq 1$	$r \leq 2$
Trace statistic			
Observed value	44.83	20.63	0.26
95% critical value	29.68	15.41	3.76
$\lambda -$ max statistic			
Observed value	24.20	20.38	0.26
95% critical value	20.97	14.07	3.76
Variable	c	i	y
Normalized Cointegrating Vectors	1.0	0.0	-0.931(0.012)
	0.0	1.0	-1.231 (0.042)
Normalized $\alpha'_{\perp}$	153.87	4.16	-31.08
Weak Exogeneity			
Johansen's test statistic	0.40	19.78	13.86
p-value	0.82	0.00	0.00
Restricted Cointegrating Vectors	1.0	0.0	-0.930 (0.012)
	0.0	1.0	-1.233 (0.042)

Notes: Results are based on a reduced form model of the log of per-capita consumption, investment and private output estimated with five lags. The 95% critical values are from Osterwald-Lenum (1992, Table 1). The numbers in parentheses are one-standard errors. $\alpha'_{\perp}$ is normalized so that the variance of the structural shock is unity and is A_m in the notation of the text. The null of weak exogeneity is formulated as a test of a H_4 hypothesis, in the terminology of Johansen and Juselius (1990). Johansen's likelihood ratio test statistic is distributed as a chi-squared with degrees of freedom being equal to the number of restrictions being tested. Here, the test statistic for weak exogeneity is distributed as $\chi^2(2)$ and the 5-percent critical value is 5.99.

TABLE 2
Results of Cointegration Analysis: 1948:1-1988:4

Hypothesis	$r = 0$	$r \leq 1$	$r \leq 2$
Trace statistic			
Observed value	47.89	18.37	0.88
95% critical value	29.68	15.41	3.76
$\lambda - \max$ statistic			
Observed value	29.52	17.49	0.88
95% critical value	20.97	14.07	3.76
Variable	c	i	y
Normalized Cointegrating	1.0	0.0	-0.933 (0.019)
Vectors	0.0	1.0	-1.164 (0.034)
Normalized $\alpha'_{\perp}$	128.89	-6.14	-10.18
Weak Exogeneity			
Johansen's test statistic	3.20	25.24	19.18
p-value	0.20	0.00	0.00
Restricted Cointegating Vectors	1.0	0.0	-0.929 (0.019)
	0.0	1.0	-1.175 (0.036)

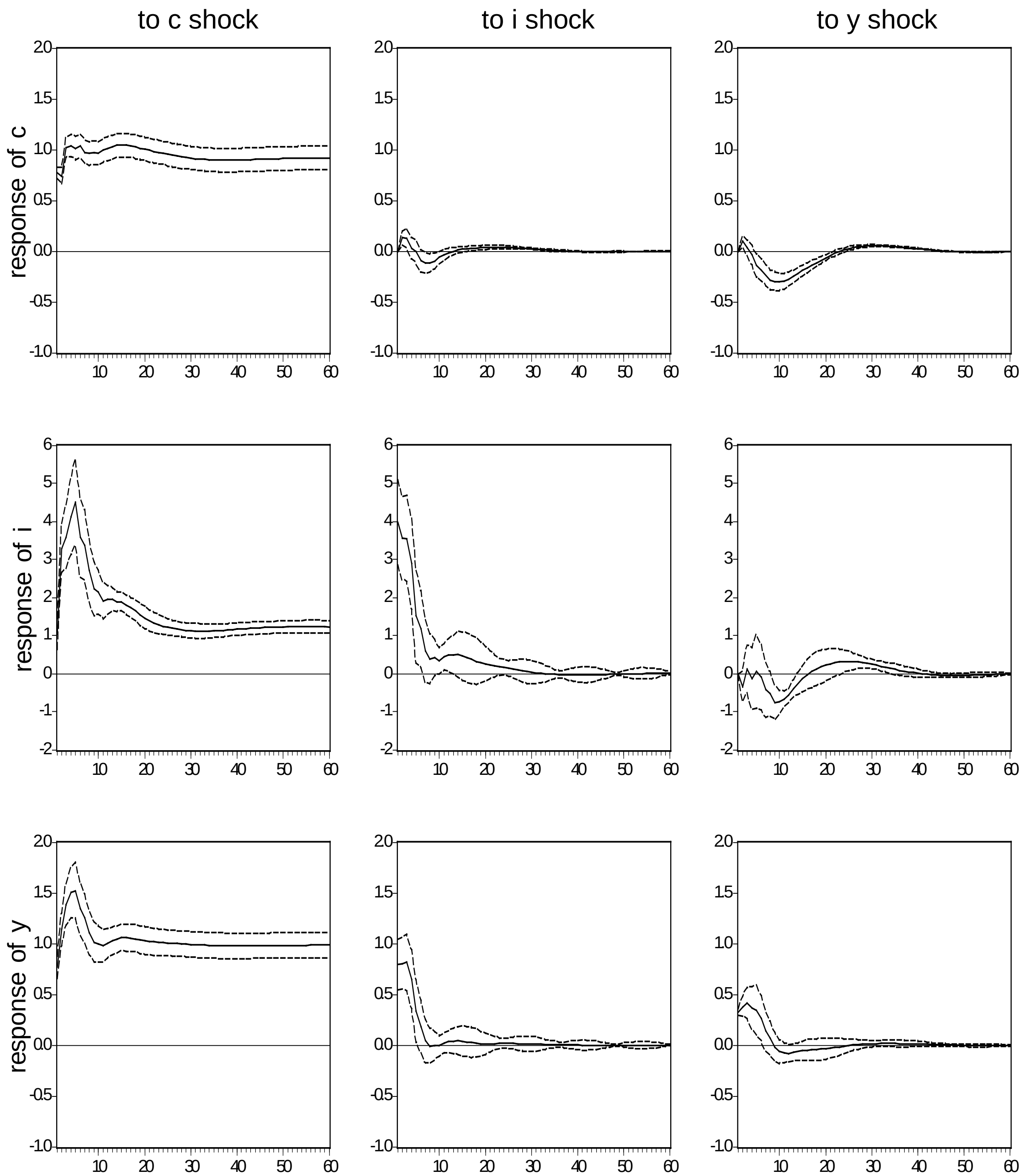
Notes: Results are based on a reduced form model of the log of per-capita consumption, investment and private output estimated with five lags. The 95% critical values are from Osterwald-Lenum (1992, Table 1). The numbers in parentheses are one-standard errors. $\alpha'_{\perp}$ is normalized so that the variance of the structural shock is unity and is A_m in the notation of the text. The null of weak exogeneity is formulated as a test of a H_4 hypothesis, in the terminology of Johansen and Juselius (1990). Johansen's likelihood ratio test statistic is distributed as a chi-squared with degrees of freedom being equal to the number of restrictions being tested. Here, the test statistic for weak exogeneity is distributed as $\chi^2(2)$ and the 5-percent critical value is 5.99.

TABLE 3
Results of Cointegration Analysis in the Extended Model: 1959:1-1994:4

(a) Money Demand Variables: Velocity $(p + y - m)$, R	
AIC Criterion: Lags = 6 Number of cointegrating vectors: (r = 0):Trace = 15.96** λ-max = 12.39** $(r \leq 1)$: Trace = 3.57 λ-max = 3.57	Estimated cointegrating vector: $m - p - y + 0.109R = 0$ (0.020)
(b) Real Variables: c, i, y, $R - \Delta p$	
AIC Criterion: Lags = 5 Number of cointegrating vectors: $(r = 0)$: Trace = 55.63** λ-max = 28.51** $(r \leq 1)$: Trace = 27.13* λ-max = 13.48 $(r \leq 2)$: Trace = 13.65 λ-max = 11.56 $(r \leq 3)$: Trace = 2.09 λ-max = 2.09 LR test of long-run exclusion of $(R - \Delta p)$ $\chi^2(2) = 0.44$, p-value = 0.80	Estimated cointegrating vectors: $c - 0.917y - 0.004(R - \Delta p) = 0$ (0.021) (0.002) $i - 1.136y + 0.003(R - \Delta p) = 0$ (0.051) (0.005) Estimated restricted cointegrating vectors: $c - 0.939y = 0$ (0.018) $i - 1.114y = 0$ (0.048)

Notes: The AIC criterion is used to determine the number of lags in levels in the VAR, prior to the cointegration analysis. The number of cointegrating vectors is denoted by r and * and ** represent rejection of the null hypothesis at the 10-percent and 5-percent levels, respectively. Critical values for Johansen's trace and $\lambda - \max$ statistics were obtained from Osterwald-Lenum (1992, table 1). The null of long run exclusion of the real interest rate from both cointegrating vectors is formulated as a test of a H_3 hypothesis, in the terminology of Johansen and Juselius (1990). Johansen's likelihood ratio test statistic is distributed as a chi-squared with degrees of freedom equal to the number of restrictions being tested. The critical value at the 5-percent significance level for $\chi^2(2)$ is 5.99. Standard errors of the estimated coefficients in the cointegrating vectors are shown in parentheses.

Figure 1. Impulse Responses (*percent*)



Quarters

Figure 2. Permanent Output and Total Factor Productivity

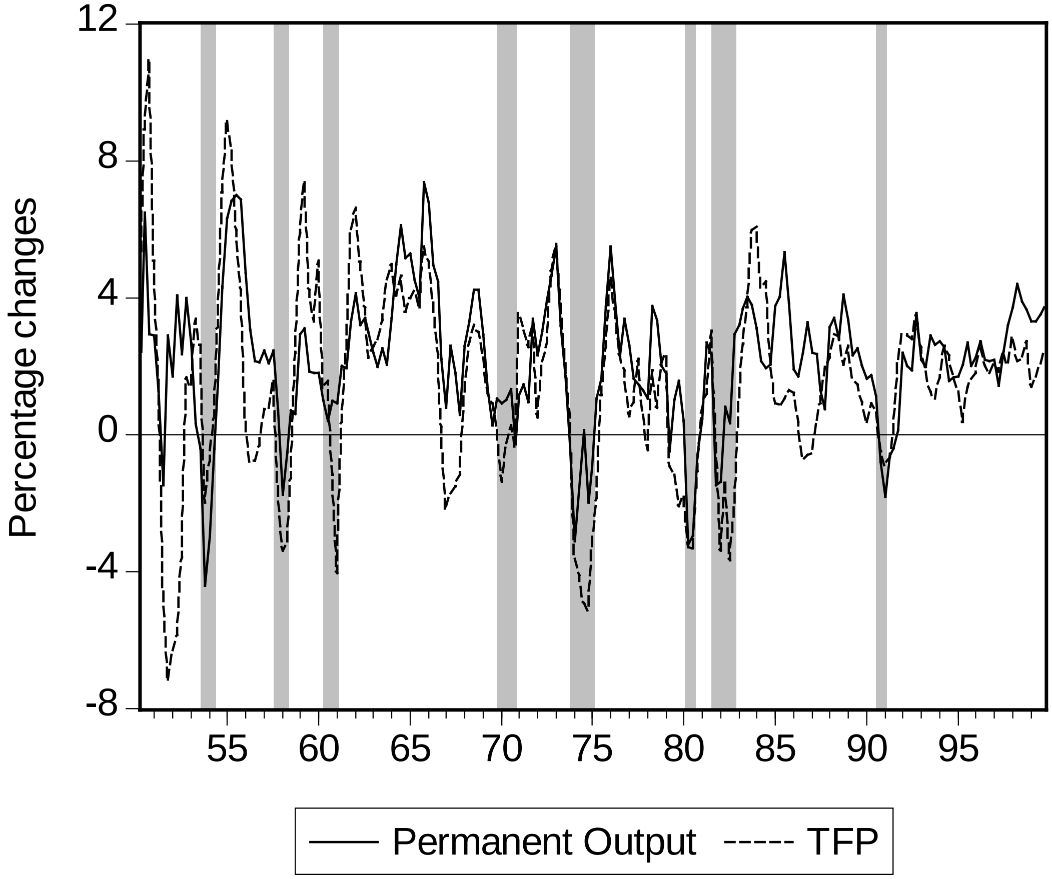


Figure 3. Transitory Components of Series (*percent*)

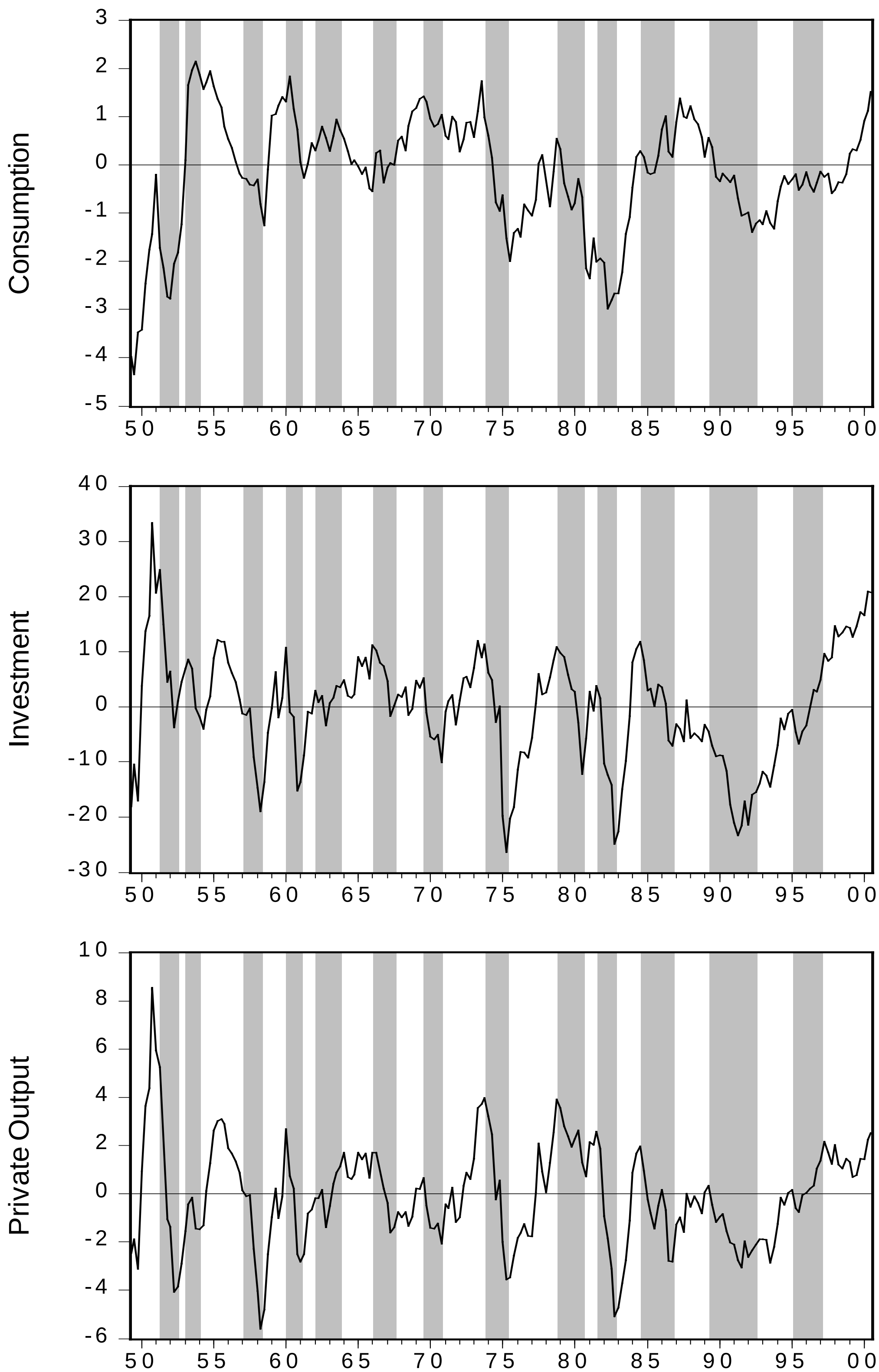
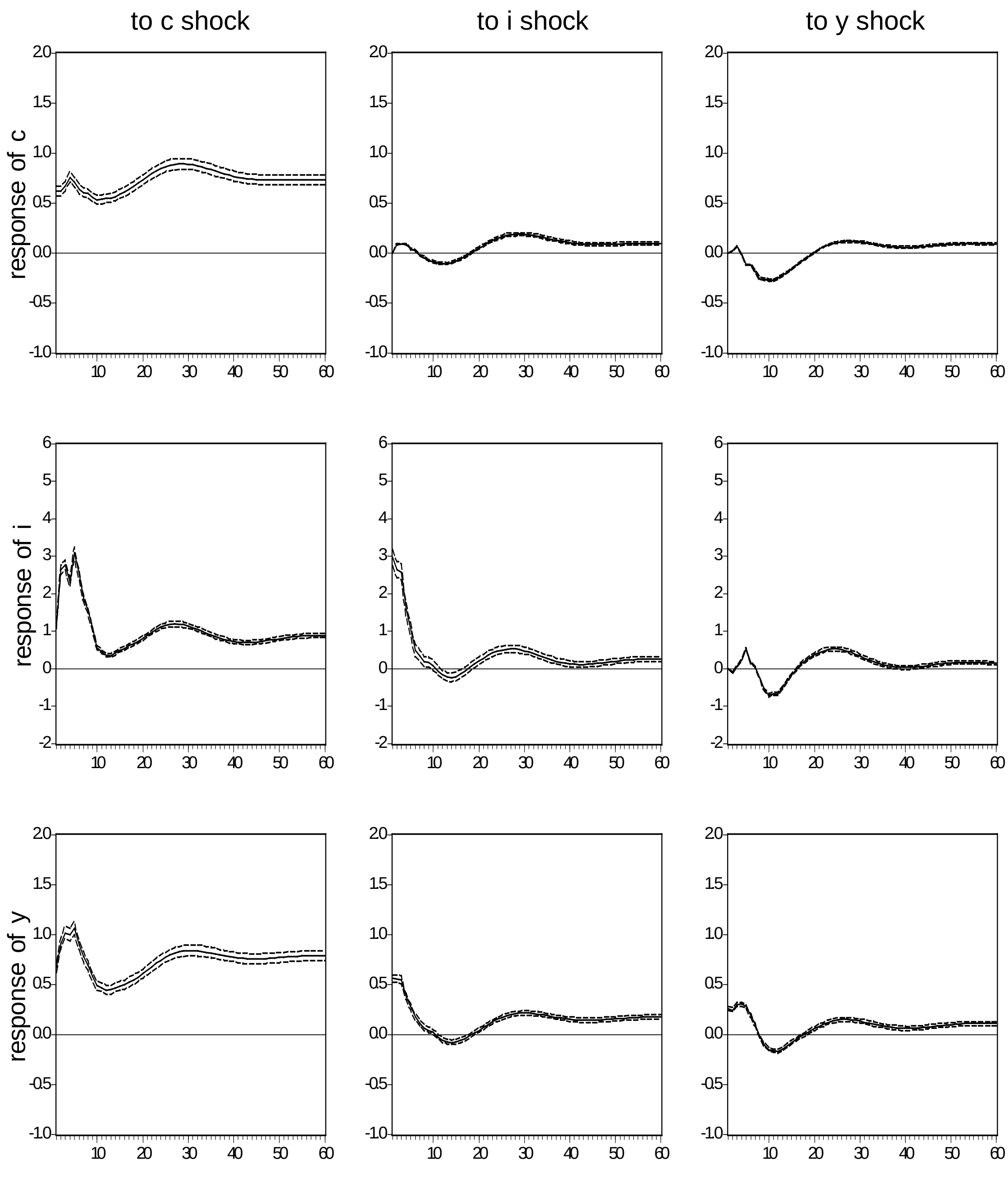


Figure 4. Impulse Responses in Six Variable Model (*percent*)



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