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The Price of Inflation and Foreign Exchange Risk in International Equity Markets

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Abstract: In this paper the author formulates and tests an international intertemporal capital asset pricing model in the presence of deviations from purchasing power parity (II-CAPM [PPP]). He finds evidence in favor of at least mild segmentation of international equity markets in which only global market risk appears to be priced. When using the Hansen & Jagannathan (1991, 1997) variance bounds and distance measures as testing devices, the author finds that, while all international asset pricing models are formally rejected by the data, their pricing implications are substantially different. The superior performance of the II-CAPM (PPP) is mainly attributable to significant hedging against inflation risk.

JEL classification: G12, G15

Key words: international intertemporal capital asset pricing model, purchasing power parity, hedging demands

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Introduction

In recent years, several papers have tested the international version of the Capital Asset Pricing Model (I–CAPM). Some of these papers have found that in addition to compensation for global market risk, investors require compensation for inflation and foreign exchange risk. However, the evidence on the ability of different specifications of the I–CAPM to correctly price international asset returns is mixed. Dumas and Solnik (1995) and De Santis and Gérard (1998) cannot reject a specification of the I–CAPM that includes global market risk and foreign exchange risk. Their analysis is consistent with the hypothesis of integration of international equity and foreign exchange markets. On the contrary, Vassalou (2000) rejects the adequacy of several nested versions of the I–CAPM with foreign exchange and inflation risks to explain most of the cross–sectional variation in stock excess returns. Her work seems to support the hypothesis of at least mild segmentation in international stock markets.

Moreover, the evidence regarding the size and significance of the economic risk premia is less than conclusive. While there is some evidence of time variation in the premia, the patterns of time variation are somewhat unclear. For example, De Santis and Gérard (1997,1998) find that exposure to global market risk commands a positive and significant average risk premium only when a conditional, fully parametric specification of the I–CAPM with time–varying price of risk is assumed. Similarly, Ferson and Harvey (1993) find a positive and significant risk premium for exposure to global market risk in a time–varying multi–beta international asset pricing model. Dumas and Solnik (1995) provide evidence of time variation in the global market risk premium, but do not report size and statistical significance of the average conditional market premium. Vassalou (2000) does not comment on the sign and the significance of the unconditional global market premium. The evidence on the premia associated with foreign exchange risk and inflation risk is also unclear. For example, De Santis and Gérard (1998) find support for a specification of the conditional I–CAPM that includes both global market risk and foreign exchange risk under the assumption of time variation in all prices of risk. Their results are globally consistent with the findings of Dumas and Solnik (1995). Even if both studies assume time–varying prices of risk, the main difference between the two approaches is that Dumas and Solnik (1995) test an unconditional version of the conditional I–CAPM using the generalized method of moments, while De Santis and Gérard (1998) explicitly model time variation in the first two conditional moments using VGARCH specifications. On the other hand, Vassalou (2000) finds strong support for versions of the I–CAPM that take inflation risk into account, mixed evidence for versions of the I–CAPM with foreign exchange risk, and no support for international asset pricing models where both foreign exchange premia and inflation premia are jointly estimated. In addition, the magnitudes of the estimated risk premia change substantially from one study to the other.

Two main issues emerge from the previous discussion. First, it is hard to reconcile home bias in international equity and foreign exchange markets with the findings that stock markets are perfectly integrated and that the I–CAPM holds. One of the reasons why the I–CAPM holds in some studies and not in others might be that the testing procedure used crucially depends on the level of aggregation of the equity portfolios used in the analysis. For example, Vassalou (2000) considers not only cross–country variation but also within–country variation in equity returns in order to control for home bias. On the other hand, Dumas and Solnik (1995) and De Santis and Gérard (1998) use aggregated stock return data from Morgan Stanley Capital International (MSCI). In the effort to control for home bias, the authors include in the analysis idiosyncratic risks and country–specific effects. Nonetheless, De Santis and Gérard (1997,1998) reach a quite puzzling result. They find that when the restriction of non–negativity of the conditional global market risk premium is imposed, there is some residual predictability in stock excess returns that can be explained by idiosyncratic risk and country–specific effects. On the other hand, when the restriction of non–negativity is relaxed, this residual predictability disappears and they cannot reject the hypothesis of integration of international stock markets. Second, the patterns of time variation in the first two conditional moments of excess asset returns and in the prices of risk are not clear. The I–CAPM in its unconditional version does not seem to hold.[1] It also does not seem to hold in its conditional version unless all prices of risk are assumed to be time–varying.[2] I try to

[1]See, for example, Solnik (1974), and Stehle (1977).

[2]See, for example, Hodrick (1981), De Santis and Gérard (1998), Dumas and Solnik (1995), and Bekaert and Harvey (1995).

clarify the above-mentioned two issues by separating the problem of economic risk premia estimation from the problem of testing intertemporal international asset pricing models and by implementing a convenient new approach to the estimation of economic risk premia. I also implement a comprehensive set of tests of asset pricing models with foreign exchange and inflation risks.

This paper contributes to the existing literature in three different ways. First, I use a new methodology to estimate the inflation and foreign exchange risk premia which is based on the minimum-variance stochastic discount factors of Hansen and Jagannathan (1991) (HJ). This pricing kernel prices, by construction, the asset returns under consideration, and has the minimum variance among all kernels consistent with asset returns. The economic risk premia that I estimate are those assigned by the minimum–variance kernel. This approach has several advantages on the traditional methods:

- I do not need to specify all relevant sources of risk; i.e., the estimate of a risk premium does not change depending on which other sources of uncertainty are simultaneously considered.

- The estimates are robust to the form of the pricing kernel. Any admissible pricing kernel has the same price implications and, hence, assigns the same risk premia as the HJ minimum–variance kernel.

- The estimation procedure is quite simple. I estimate the parameters of the minimum–variance kernel and the covariance of the kernel with the different sources of uncertainty. By comparison, studies based on multi–beta models require the estimation of linear factor models for each of the securities, and then the estimation of the risk premia.

- I show that when the non–negativity restriction on the pricing kernel is not imposed, the proposed economic risk premia reduce to the expected cash flows on the portfolios that best hedge the risk factors, financed at the riskless rate. This result makes it clear that the risk premia estimates crucially depend on the hedging portfolios composition, and hence on the assets available for investment.

Second, I model the time variation of economic risk premia by specifying a pricing kernel linear in the set of instruments used in the analysis. Finally, I use a new methodology to test the international CAPM, which is based on the construction of portfolios mimicking the behavior of global market risk, foreign exchange, and inflation risks. Specifically, I test the International Static CAPM (IS-CAPM), the International CAPM with demands hedging against foreign exchange and inflation risks (I-CAPM), and the International Intertemporal CAPM (II-CAPM) by using a variety of diagnostics: the standard J–test of overidentifying restrictions of Hansen and Singleton (1982); the Hansen–Jagannathan (1991) variance bound; and the Hansen–Jagannathan (1997) distance measure. I am not aware of any formal test of the I–CAPM with demands hedging against variations in the investment opportunity set of an international investor in presence of deviations from Purchasing Power Parity (PPP) and currency risk.[3]

The main results of the paper can be summarized as follows: First, I show that only global market risk commands a highly positive and significant unconditional risk premium as indicated by its relative Sharpe ratio. Inflation risk does not seem to be priced both unconditionally and conditionally, while global market and exchange risks exhibit interesting patterns of time variation. Second, none of the international asset pricing models are rejected by the data when, following Hansen and Singleton (1982), I perform tests of the overidentifying restrictions that they impose. On the contrary, testing the same specifications with the HJ variance bounds and distance measures leads to a strong rejection of all models.[4] Hence, even if all international asset pricing models deliver different pricing implications, none of them seems to hold when the testing procedure adopted is stringent enough. Large and statistically significant inflation-hedging demands explain the better pricing implications delivered by the II-CAPM in presence of deviations from PPP. Interestingly, this analysis is able to rationalize recent findings in the international asset pricing literature and, at the same time, shed some light on the controversial home bias puzzle. Even if I cannot explain

[3] Hodrick, Ng, and Sengmüller (1999) test an international version of Campbell's (1996) intertemporal CAPM, but do not consider deviations from PPP.

[4] See Zhang (2001) for an evaluation of domestic and international asset pricing models using the HJ (1997) distance measure.

home bias in international equity markets, I can argue that my results are consistent with the hypothesis of at least mild segmentation of international stock markets.

The remainder of this paper is organized as follows: Section I discusses the economic risk premia assigned by the minimum–variance admissible kernel and explicit international asset pricing models;[5] Section II illustrates the methods used for estimation and testing; Section III describes the data; Section IV identifies the investment opportunity set of an investor; Section V presents the results of the risk premia estimation and relative Sharpe ratios; Section VI presents the results of the tests of international asset pricing models and empirically identifies the size of the hedging demands; and Section VII concludes.

I. Methodology: Economic Risk Premia and International Asset Pricing Models

This section derives a general pricing result for nominal asset returns denominated in terms of a reference currency. Namely, I show that the expected nominal returns in excess of the nominal risk-free rate of the reference country are directly related to the covariance of a nominal pricing kernel, scaled by the risk-free rate, with nominal asset returns. I then consider economic variables that are believed to be relevant for international asset prices: the rate of return on the world market portfolio; the rate of appreciation of the reference currency relative to the other currencies; and the rates of inflation in the different countries. If I can construct portfolios that exactly replicate the conditional variability of these variables, then the cross moments of the scaled (or normalized) pricing kernel translate into risk premia. Specifically, these cross moments reduce to the expected nominal cash flows on zero-net-investment positions long in the mimicking portfolio and short in the riskless asset. If the nominal (normalized) pricing kernel is linear in the economic variables described above, as is often assumed in international asset pricing models, then the risk premia also enter familiar “beta” pricing results. In addition, I show how to test several specifications of the

[5]The analysis of risk premia mainly follows Balduzzi & Kallal (1997) and Balduzzi & Robotti (2001).

international capital asset pricing model using a variety of diagnostics.

A. A General International Asset Pricing Result

Assume there are $L+1$ countries and a set of $N = n+1$ assets, other than the measurement-currency nominally risk-free asset. These include n risky assets, or portfolios of risky assets, and the world portfolio of risky assets.

Consider now the $N \times 1$ vector of (gross) nominal returns on the N assets, r. From now on, I shall assume all returns to be denominated in terms of the reference currency. By the law of one price, I have

$$\mathrm{E}_t(m_{t+1}\mathbf{r}_{t+1}) = \mathbf{1} \tag{1}$$

for some admissible nominal pricing kernel m,[6] where $\mathbf{1}$ is an $N \times 1$ vector of ones. Note that the restriction stated in equation (1) prevents arbitrage in the security markets, but not in the commodity markets where Purchasing Power Parity (PPP) may not hold.

Let r_{ft} denote the nominally risk-free rate of the reference country. I then have

$$\mathrm{E}_t(m_{t+1}r_{ft}) = 1 \ . \tag{2}$$

It is convenient to perform the analysis that follows in terms of the pricing kernel scaled by the risk-free rate. That is, I use the normalized pricing kernel, $m_{t+1}r_{ft} \equiv q_{t+1}$, where $\mathrm{E}_t(q_{t+1}) = 1$. As a consequence, I circumvent the problem of explicitly modeling the conditional mean of the pricing kernel m.

Using equations (1) and (2), I obtain the familiar orthogonality conditions

$$\mathrm{E}_t[q_{t+1}(\mathbf{r}_{t+1} - \mathbf{1}\mathsf{r}_{\mathsf{ft}})] = \mathbf{0} \ . \tag{3}$$

Rearranging, I obtain

$$\mathrm{E}_t(q_{t+1})[\mathrm{E}_t(\mathbf{r}_{t+1}) - \mathbf{1}\mathsf{r}_{\mathsf{ft}}] \;=\; \mathrm{E}_t(\mathbf{r}_{t+1}) - \mathbf{1}\mathsf{r}_{\mathsf{ft}} = -\mathrm{Cov}_{\mathsf{t}}(\mathsf{q}_{\mathsf{t}+1}, \mathbf{r}_{\mathsf{t}+1}) \ . \tag{4}$$

[6] See Harrison and Kreps (1979). The set of pricing kernels can be interpreted as the set of nominal intertemporal marginal rates of substitution compatible with the distribution of returns.

The conditional risk premium on any asset equals the opposite of the conditional covariance between the normalized pricing kernel and the asset return.

B. Economic Variables and Exact Mimicking Portfolios

Consider now a $K \times 1$ vector of economic variables $\mathbf{y}$. Without loss of generality, I assume these variables to have constant zero mean and unit variance: i.e., $\mathrm{E}_t(y_{k,t+1}) = 0$ and $\mathrm{E}_t(y_{k,t+1}^2) = 1$, for $k = 1, \ldots, K$.

The reason for assuming a constant zero mean is that security markets only price unanticipated variability; the relevant economic variables are, in fact, innovations. Finally, the reason for scaling the risk factors to have constant unit variance is that I can compare the "prices" attached to the economic variables without having to worry about differences in variability.

Assume now that there exist portfolios that exactly mimic the behavior of the risk factors. Let $\mathbf{r}_{y,t+1} \equiv \mathbf{y}_{t+1}$, denote the $K \times 1$ vector of mimicking portfolio returns. According to (4), I have

$$\mathrm{E}_t(\mathbf{r}_{y,t+1}) - r_{ft}\mathbf{1} = -\mathrm{E}_t[\mathbf{y}_{t+1}(q_{t+1} - 1)] \equiv \boldsymbol{\lambda}_{yt} \ , \tag{5}$$

which is the vector of conditional risk premia on the corresponding economic variables. Hence, the conditional cross moments between the normalized pricing kernel q and the economic variable y_k equals, with the opposite sign, the conditional risk premium on the variable. Assuming stationarity and using the law of iterated expectations, I have

$$\boldsymbol{\lambda}_y \equiv \mathrm{E}(\boldsymbol{\lambda}_{yt}) \equiv -\mathrm{E}[(q_{t+1} - 1)\mathbf{y}_{t+1}] \ \ ,$$

which is the vector of unconditional or mean risk premia on $\mathbf{y}$. The unconditional cross moment between the normalized pricing kernel q and the economic variable y_k equals, with the opposite sign, the mean risk premium on the variable.

C. The Minimum-Variance Kernel

Using the definition of the normalized pricing kernel q, I can rewrite equations (1) and (2) as

$$\mathrm{E}_t(q_{t+1}\mathbf{r}_{t+1}) = r_{ft}\mathbf{1} \tag{6}$$

and

$$\mathrm{E}_t(q_{t+1}) = 1 \ . \tag{7}$$

Following HJ, I can construct an admissible (normalized) pricing kernel, i.e., a random variable that satisfies equations (6) and (7), that is linear in $\mathbf{r}_{t+1}$: $q^{\star}_{t+1} \equiv \alpha_{0t} + \mathbf{r}_{t+1}^{\top}\boldsymbol{\alpha}_t$, where α_{0t} is a scalar and $\boldsymbol{\alpha}_t$ is an $N \times 1$ coefficient vector. I then have

$$\mathrm{E}_t(q_{t+1})\mathrm{E}_t(\mathbf{r}_{t+1}) + \mathrm{Cov}_t(q_{t+1}, \mathbf{r}_{t+1}) = r_{ft}\mathbf{1} \ . \tag{8}$$

Using (6) and $q^{\star}_{t+1} \equiv \alpha_{0t} + \mathbf{r}_{t+1}^{\top}\boldsymbol{\alpha}_t$, I have

$$-\boldsymbol{\Sigma}_{rrt}\boldsymbol{\alpha}_t = \mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1} \ , \tag{9}$$

where $\boldsymbol{\Sigma}_{rrt}$ is the conditional covariance matrix of risky asset returns (which I assume to be invertible). I obtain

$$\boldsymbol{\alpha}_t = -\boldsymbol{\Sigma}_{rrt}^{-1}[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] \tag{10}$$

while

$$\alpha_{0t} = 1 - \mathrm{E}_t(\mathbf{r}_{t+1})^{\top}\boldsymbol{\alpha}_t \ . \tag{11}$$

Using the results in equations (10) and (11), I have

$$q^{\star}_{t+1} = 1 - [\mathbf{r}_{t+1} - \mathrm{E}_t(\mathbf{r}_{t+1})]^{\top}\boldsymbol{\Sigma}_{rrt}^{-1}[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] \ . \tag{12}$$

This minimum-variance normalized pricing kernel, $q^{\star}_{t+1}$, has several properties worth noting. First, the vector $\boldsymbol{\alpha}_t$ is proportional to the vector of portfolio weights of the tangency portfolio obtained from the risky security returns $\mathbf{r}_{t+1}$.[7] Hence, $q^\star_{t+1}$ is perfectly negatively correlated with the rate of return on the tangency portfolio, $r_{\tau,t+1}$. Another important property is that the conditional variance of $q^\star_{t+1}$ equals the squared conditional Sharpe ratio of the tangency portfolio, $\mathrm{S}_{\tau t}$. I have

$$[\mathrm{Cov}_t(q^\star_{t+1}, r_{\tau,t+1})]^2 = \mathrm{Var}_t(r_{\tau,t+1})\mathrm{Var}_t(q^\star_{t+1}) \ . \tag{13}$$

Since $q^\star_{t+1}$ correctly prices all the securities under consideration, it also correctly prices the tangency portfolio, and I have $-\mathrm{Cov}_t(q^\star_{t+1}, r_{\tau,t+1}) = \mathrm{E}_t(r_{\tau,t+1}) - r_{ft}$. Using this result and rearranging equation (13) above, I obtain

$$\left[\frac{\mathrm{E}_t(r_{\tau,t+1}) - r_{ft}}{\sqrt{\mathrm{Var}_t(r_{\tau,t+1})}}\right]^2 \equiv \mathrm{S}^2_{\tau t} = \mathrm{Var}_t(q^\star_{t+1}) \ . \tag{14}$$

Finally, since $\mathrm{Var}_t(q^\star_{t+1}) = \mathrm{E}_t[(q^\star_{t+1} - 1)^2]$, then $\mathrm{Var}(q^\star_{t+1}) = \mathrm{E}[(q^\star_{t+1} - 1)^2]$, and

$$\mathrm{Var}(q^\star_{t+1}) = \mathrm{E}(\mathrm{S}^2_{\tau t}) \ . \tag{15}$$

Hence, the unconditional variance of $q^\star_{t+1}$ equals the average squared Sharpe ratio of the tangency portfolio.

D. Hedging Portfolios and Risk Premia

Now consider the conditional projection of the risk variable $y_{k,t+1}$ on a constant and $\mathbf{r}_{t+1}$ ($y^\star_{k,t+1} \equiv \alpha_{0y_kt} + \mathbf{r}^\top_{t+1}\boldsymbol{\alpha}_{y_kt}$, where α_{0y_kt} is a scalar and $\boldsymbol{\alpha}_{y_kt}$ is an $N \times 1$ coefficient vector). Let $\lambda^\star_{kt} \equiv -\mathrm{Cov}_t(q_{t+1}y^\star_{k,t+1}) = -\mathrm{Cov}_t(q^\star_{t+1}y^\star_{k,t+1})$. Since $q^\star_{t+1}$ satisfies the conditional moment restriction (6) by construction, I have

$$\begin{aligned}\lambda^\star_{kt} &= -\mathrm{Cov}_t(q^\star_{t+1}, y^\star_{k,t+1}) = -\mathrm{Cov}_t(q^\star_{t+1}, \boldsymbol{\alpha}^\top_{y_kt}\mathbf{r}_{t+1}) \\ &= -\boldsymbol{\alpha}^\top_{y_kt}\mathrm{Cov}_t(q^\star_{t+1}, \mathbf{r}_{t+1}) = \boldsymbol{\alpha}^\top_{y_kt}[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] \ ,\end{aligned} \tag{16}$$

which is the mean cash flow generated by the portfolio hedging the risk variable y_k financed at the riskless rate. In other words, $\lambda^\star_{kt}$ is the risk premium on the hedging portfolio for the

[7] For example, see Ingersoll (1987), p. 89.

risk variable $y_{k,t+1}$. This implies that $\lambda^{\star}_{kt}$ does not depend on the choice of the (normalized) pricing kernel, but only on the asset returns under scrutiny. If the conditional volatility of $y_{k,t+1}$ equals one and the factor is exactly replicated by asset returns, then $\lambda^{\star}_{kt}$ is also the conditional Sharpe ratio on the exact hedging portfolio.

The hedging portfolios defined here are analogous to the "economic tracking portfolios" of Lamont (2001). The main difference in his approach is that the portfolios are constructed to track changes in expectations of future realizations of the economic variables. Instead, the proposed hedging portfolios are designed to track the contemporaneous realizations of economic variables.

The composition of the hedging portfolios is worth further discussion. First, the hedging portfolios contain allocations to the riskless asset in the amount α_{0y_kt}/r_{ft}. (The rate of return on the hedging portfolio contains a constant component, α_{0y_kt}, resulting from the investment in the riskless asset.) Second, the hedging portfolio quantities do not sum up to one. In order to have a "true" hedging portfolio I have to scale the coefficients α_{0y_kt} and $\boldsymbol{\alpha}_{y_kt}$ by $\alpha_{0y_kt}/r_{ft} + \mathbf{1}^{\top}\boldsymbol{\alpha}_{y_kt}$. Finally, the composition of the hedging portfolios corresponds to the coefficients of a regression of the risk factors on the asset returns

$$[\alpha_{0y_kt}, \boldsymbol{\alpha}^{\top}_{y_kt}]^{\top} = [\mathrm{E}_t(\mathbf{r}_{t+1}\mathbf{r}^{\top}_{t+1})]^{-1}\mathrm{E}_t(\mathbf{r}_{t+1}y_{k,t+1}) \ . \tag{17}$$

Hence, the hedging portfolios are the discrete-time counterparts of the portfolios held by a dynamic portfolio optimizer to hedge against changes in the investment-opportunity set.[8]

Two properties relating to the estimation of the risk premia are also worth noting. First, while the conditional expected excess cash flow on a mimicking portfolio equals the conditional covariance between the factor and the minimum-variance kernel, the realized excess cash flow on the mimicking portfolio, $\boldsymbol{\alpha}^{\top}_{y_kt}(\mathbf{r}_{t+1} - r_{ft}\mathbf{1})$, in general differs from $-(q^{\star}_{t+1}-1)y_{k,t+1}$. In fact,

$$\mathrm{E}_t(y_{k,t+1}\mathbf{r}^{\top}_{t+1})\mathrm{E}_t(\mathbf{r}_{t+1}\mathbf{r}^{\top}_{t+1})^{-1}(\mathbf{r}_{t+1} - r_f\mathbf{1}) \neq [\mathbf{r}_{t+1} - \mathrm{E}_t(\mathbf{r}_{t+1})]^{\top}\boldsymbol{\Sigma}^{-1}_{rrt}[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}]y_{k,t+1} \ . \tag{18}$$

This observation is important because the estimates of the risk premia may differ in small

[8] See Merton (1973).

samples, depending on which approach is used, and the precision of the estimates may also differ. Second, the conditional Sharpe ratios of the hedging portfolios, $\mathrm{S}_{y_k^\star t}$, depend on how closely a risk variable is replicated. Since the replication is not perfect, in general $\mathrm{Var}_t(y_{k,t+1}^\star) < \mathrm{Var}_t(y_{k,t+1})$. This means that

$$\mathrm{S}^2_{y_k^\star t} = \frac{(\lambda_{kt}^\star)^2}{\mathrm{Var}_t(y_{k,t+1}^\star)} > \frac{(\lambda_{kt}^\star)^2}{\mathrm{Var}_t(y_{k,t+1})} \ .$$

This observation is important because a hedging portfolio might receive a "small" risk premium and yet command a high Sharpe ratio. This is because it only captures a small fraction of the variability of a risk variable.

Appendix A relates my approach to the estimation of economic risk premia to two alternative approaches widely considered in the literature: 1) Principal components approach; and 2) Multi–beta models approach. The first approach assumes that a number of unobservable factors drive the variation in asset returns. The realizations of these factors, while not directly observable, can be inferred from the statistical properties of asset returns. The second approach explicitly identifies the factors with observable macro-economic variables, and assumes the pricing kernel and/or asset returns to be linear in the factors.

E. Hedging Portfolios and Linear Kernels

In this section, I describe the link between hedging portfolios and linear kernels. This link is then used in Section F to formulate and test competitive international asset pricing models. Without loss of generality, I assume $\mathrm{E}_t(\mathbf{y}_{t+1}) = \mathbf{0}$ and $\mathrm{E}_t(\mathbf{y}_{t+1}\mathbf{y}_{t+1}^\top) \equiv \mathbf{\Sigma}_{yyt} = \mathbf{I}$. Consider now an admissible kernel $q_{y,t+1}$ which is linear in the risk variables

$$q_{y,t+1} = 1 - \mathbf{y}_{t+1}^\top \boldsymbol{\lambda}_t \ . \tag{19}$$

Equation (4) above becomes

$$\mathrm{E}_t(\mathbf{r}_{t+1}) - \mathbf{1} r_{ft} = \boldsymbol{\beta}_t \boldsymbol{\lambda}_t \ , \tag{20}$$

where $\boldsymbol{\beta}_t = \mathrm{E}_t(\mathbf{r}_{t+1}\mathbf{y}_{t+1}^\top) \equiv \mathbf{\Sigma}_{ryt}$. This implies that returns satisfy the linear factor model

$$\mathbf{r}_{t+1} = \mathbf{1} r_{ft} + \boldsymbol{\beta}_t \boldsymbol{\lambda}_t + \boldsymbol{\beta}_t \mathbf{y}_{t+1} + \boldsymbol{\epsilon}_{t+1} \ , \tag{21}$$

where $\boldsymbol{\lambda}_t$ is a $K \times 1$ vector of conditional risk premia and $\boldsymbol{\epsilon}_{t+1}$ is an $N \times 1$ vector of disturbances orthogonal to $\mathbf{y}_{t+1}$.

Now consider the projection of the risk variables onto the span of asset returns augmented of a unit-constant. I have $\mathbf{y}^{\star}_{t+1} \equiv \boldsymbol{\alpha}_{0yt} + \boldsymbol{\alpha}_{yt}\mathbf{r}^{\top}_{t+1}$, where $\boldsymbol{\alpha}_{0yt}$ is a $K \times 1$ vector and $\boldsymbol{\alpha}_{yt}$ is a $K \times N$ coefficient matrix. I have

$$\begin{aligned} \boldsymbol{\alpha}_{yt} &= \boldsymbol{\Sigma}_{yrt}\boldsymbol{\Sigma}^{-1}_{rrt} , \qquad (22) \\ \boldsymbol{\alpha}_{0yt} &= -\boldsymbol{\alpha}_{yt}\mathrm{E}_t(\mathbf{r}_{t+1}) , \qquad (23) \end{aligned}$$

and

$$\mathbf{y}^{\star}_{t+1} \equiv \boldsymbol{\alpha}_{yt}[\mathbf{r}_{t+1} - \mathrm{E}_t(\mathbf{r}_{t+1})] . \qquad (24)$$

If, in the admissible linear pricing kernel q_{t+1}, I replace $\mathbf{y}_{t+1}$ with $\mathbf{y}^{\star}_{t+1}$, the pricing result (20) does not change. In other words, the projection of $q_{y,t+1}$ onto the augmented span of asset returns, $q_{y^{\star},t+1} \equiv (\mathbf{y}^{\star}_{t+1})^{\top}\boldsymbol{\lambda}_t$, is also an admissible pricing kernel.

Consider now the minimum-variance kernel constructed based on the cash flows of the hedging portfolios $q^{\star}_{y^{\star},t+1}$. This is the kernel with minimum variance that correctly prices the hedging portfolios. I have

$$q^{\star}_{y^{\star},t+1} = 1 - (\mathbf{y}^{\star}_{t+1})^{\top}\boldsymbol{\Sigma}^{-1}_{y^{\star}y^{\star}t}\boldsymbol{\alpha}_{yt}[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] , \qquad (25)$$

where $\boldsymbol{\Sigma}_{y^{\star}y^{\star}t}$ is the covariance matrix of the hedging-portfolio cash-flows. It is straightforward to verify that

$$\boldsymbol{\Sigma}_{y^{\star}y^{\star}t} = \boldsymbol{\Sigma}_{yrt}\boldsymbol{\Sigma}^{-1}_{rrt}\boldsymbol{\Sigma}_{ryt} \qquad (26)$$

holds.[9]

Hence,

$$q^{\star}_{y^{\star},t+1} = 1 - (\mathbf{y}^{\star}_{t+1})^{\top}(\boldsymbol{\Sigma}_{yrt}\boldsymbol{\Sigma}^{-1}_{rrt}\boldsymbol{\Sigma}_{ryt})^{-1}\boldsymbol{\Sigma}_{yrt}\boldsymbol{\Sigma}^{-1}_{rrt}[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] . \qquad (27)$$

[9] I have

$$\boldsymbol{\Sigma}_{\mathrm{y?y?t}} = \mathrm{E}_{\mathrm{t}}[\boldsymbol{\alpha}_{\mathrm{yt}}(\mathbf{r}_{\mathrm{t+1}} - \mathrm{E}_{\mathrm{t}}(\mathbf{r}_{\mathrm{t+1}}))(\mathbf{r}_{\mathrm{t+1}} - \mathrm{E}_{\mathrm{t}}(\mathbf{r}_{\mathrm{t+1}}))^{\top}\boldsymbol{\alpha}^{\top}_{\mathrm{yt}}] = \boldsymbol{\Sigma}_{\mathrm{yrt}}\boldsymbol{\Sigma}^{-1}_{\mathrm{rrt}}\boldsymbol{\Sigma}_{\mathrm{rrt}}\boldsymbol{\Sigma}^{-1}_{\mathrm{rrt}}\boldsymbol{\Sigma}_{\mathrm{ryt}} = \boldsymbol{\Sigma}_{\mathrm{yrt}}\boldsymbol{\Sigma}^{-1}_{\mathrm{rrt}}\boldsymbol{\Sigma}_{\mathrm{ryt}} .$$

Under the null of the multi-beta model, $\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1} = \boldsymbol{\Sigma}_{ryt}\boldsymbol{\lambda}_t$, and the expression above simplifies to

$$q^{\star}_{y^{\star},t+1} = 1 - (\mathbf{y}^{\star}_{t+1})^{\top}\boldsymbol{\lambda}_t \ . \tag{28}$$

In other words, under the null, the projection of $q_{y,t+1}$ onto the augmented span of asset returns equals the minimum-variance kernel constructed using the hedging portfolio cash-flows.

F. Hedging Portfolios and Tests of Asset-Pricing Models

One appealing property of the kernel $q^{\star}_{y^{\star},t+1}$ is that, under the null, it is the minimum-variance admissible kernel. In other words, any other admissible kernel is at least as volatile as $q^{\star}_{y^{\star},t+1}$. This minimum-volatility property is obviously appealing. When it comes to tests of overidentifying restrictions (the standard J test of Hansen, 1982), low volatility of the kernel implies low volatility of the pricing errors, $q^{\star}_{y^{\star},t+1}\mathbf{r}_{t+1} - r_{ft}\mathbf{1}$. This, in turn, reduces the likelihood that a poor model is not rejected simply because the volatility of the candidate pricing kernel is high.

One additional feature that makes the kernel $q^{\star}_{y^{\star},t+1}$ appealing is that the statistics associated with tests of the Hansen-Jagannathan variance bounds (HJV) and the Hansen-Jagannathan distance (HJD) are the same, and can be interpreted in terms of comparisons of expected squared conditional Sharpe ratios of the unrestricted tangency portfolio and a restricted tangency portfolio.

The unconditional variance of $q^{\star}_{y^{\star},t+1}$ equals the expectation of the squared conditional Sharpe ratio of the tangency portfolio obtained using the hedging portfolios, instead of all the assets available. Hence, when I test whether $q^{\star}_{y^{\star},t+1}$ satisfies the HJ variance bound, I test whether $\mathrm{Var}(q^{\star}_{t+1}) - \mathrm{Var}(q^{\star}_{y^{\star},t+1}) > 0$. This is equivalent to a test that $\mathrm{E}(\mathrm{S}^2_{\tau t}) - \mathrm{E}(\mathrm{S}^2_{y^{\star}\tau t}) > 0$ where $S_{y^{\star}\tau t}$ is the Sharpe ratio of the restricted tangency portfolio.

The HJD test is a test that the projection of a candidate kernel onto the augmented span of asset returns and the minimum-variance kernel are sufficiently "close": under the null, the second moment of the difference between the minimum-variance kernel and the projection of the candidate kernel onto the augmented span of asset returns should be "small". In particular, using $q_{y^\star,t+1}$ as the candidate kernel (note that the projection of $q^\star_{y^\star,t+1}$ onto a constant and $\mathbf{r}_{t+1}$ is $q^\star_{y^\star,t+1}$ itself) the HJD statistic is the sample counterpart of

$$\mathrm{E}[(q^\star_{t+1} - q^\star_{y^\star,t+1})^2] = \mathrm{E}\{\mathrm{E}_t(q^\star_{t+1} - q^\star_{y^\star,t+1})^2\} = \mathrm{E}[\mathrm{Var}_t(q^\star_{t+1} - q^\star_{y^\star,t+1})] \; . \tag{29}$$

Now note that

$$\begin{aligned}
\mathrm{Cov}_t(q^\star_{t+1} - q^\star_{y^\star,t+1}) &= [\mathrm{E}_t(\mathbf{r}_{t+1} - r_{ft}\mathbf{1})]^\top \boldsymbol{\Sigma}^{-1}_{rrt}\boldsymbol{\Sigma}_{rr}\boldsymbol{\alpha}_y^\top \boldsymbol{\Sigma}^{-1}_{rr}\boldsymbol{\alpha}_y[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] \\
&= [\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}]^\top \boldsymbol{\alpha}_y^\top \boldsymbol{\Sigma}^{-1}_{rr}\boldsymbol{\alpha}_y[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] \\
&= \mathrm{Var}_t(q^\star_{y^\star,t+1}) \; .
\end{aligned} \tag{30}$$

Hence

$$\begin{aligned}
\mathrm{E}[\mathrm{Var}_t(q^\star_{t+1} - q^\star_{y^\star,t+1})] &= \mathrm{E}[\mathrm{Var}_t(q^\star_{t+1})] - \mathrm{E}[\mathrm{Var}_t(q^\star_{y^\star,t+1})] \\
&= \mathrm{Var}(q^\star_{t+1}) - \mathrm{Var}(q^\star_{y^\star,t+1}) \; .
\end{aligned} \tag{31}$$

Up to this point, I have assumed the existence of a pricing kernel q_{t+1} which prices exactly the securities under consideration. I now investigate theories which postulate the form of q_{t+1}. For concreteness, I consider here four models: the International Static CAPM (IS–CAPM); the International CAPM in presence of deviations from PPP (I-CAPM (PPP)); the International CAPM in presence of currency risk (I-CAPM (SPOT)); and the International Intertemporal CAPM (II–CAPM). The IS–CAPM was first derived by Solnik (1974) and postulates a linear relationship between the cross-section of international expected excess equity returns and the excess return on a world market portfolio. The I-CAPM, as introduced by Adler and Dumas (1983), links nominal excess returns on international equity denominated in a reference currency to a world market portfolio and portfolios hedging against deviations from PPP. The II-CAPM is a combination of Merton's (1973) intertemporal CAPM and Adler and Dumas's (1983) international CAPM. Namely, the cross-section of nominal excess returns denominated in a reference currency is explained by three hedging funds: a nationless (or logarithmic) world market portfolio; portfolios hedging against deviations from PPP; and portfolios hedging against variations in the investment opportunity set of an international investor. Consider the II-CAPM in its discrete-time approximate version. The vector $\mathbf{w}_t^l$ of the l-country ($l = 1, ..., L+1$) optimal weights on the risky assets is given by

$$\mathbf{w}_t^l = \alpha^l \mathbf{\Sigma}_{rrt}^{-1}[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] + (1-\alpha^l)\mathbf{\Sigma}_{rrt}^{-1}\mathbf{s}_{r\pi t}^l + \beta^l \mathbf{\Sigma}_{rrt}^{-1}\mathbf{s}_{ry_k t}^l \ , \tag{32}$$

where $\mathbf{s}_{ry_k t}^l$ and $\mathbf{s}_{r\pi t}^l$ are the vectors of time-varying covariances between each country's security returns and the k-th state variable and level of inflation, respectively. Moreover, I define $\alpha^l \equiv -\frac{J_{Wt}^l}{J_{WWt}^l W_t^l}$ and $\beta^l \equiv (\alpha^l)^2 \times \frac{J_{Wy_k t}^l}{W_t^l}$.[10] Let $\mathbf{w}_{\tau t}$ denote the $N \times 1$ vector of unscaled weights of the tangency portfolio

$$\mathbf{w}_{\tau t} \quad = \quad \mathbf{\Sigma}_{rrt}^{-1}[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] \ . \tag{33}$$

Let $\mathbf{w}_{yt}^l$ and $\mathbf{w}_{\pi t}^l$ denote the $N \times 1$ vectors of unscaled weights hedging against variations in the investment opportunity set and against deviations from PPP, respectively, so that

$$\mathbf{w}_{yt}^l \quad = \quad \mathbf{\Sigma}_{rrt}^{-1}\mathbf{s}_{ry_k t}^l \quad \text{and} \tag{34}$$

$$\mathbf{w}_{\pi t}^l \quad = \quad \mathbf{\Sigma}_{rrt}^{-1}\mathbf{s}_{ryt}^l \ . \tag{35}$$

Then I can rewrite (32) as

$$\mathbf{w}_t^l = \alpha^l \mathbf{w}_{\tau t} + (1-\alpha^l)\mathbf{w}_{\pi t}^l + \beta^l \mathbf{w}_{yt}^l \ . \tag{36}$$

Aggregating over countries and defining $\alpha^m = \frac{\sum_{l=1}^{L+1} \alpha^l W^l}{\sum_{l=1}^{L+1} W^l}$, $\alpha_\pi^l = \frac{(1-\alpha^l)W^l}{\sum_{l=1}^{L+1} W^l}$, $\alpha_y^l = \frac{\beta^l W^l}{\sum_{l=1}^{L+1} W^l}$, I obtain

$$\mathbf{w}_{mt} = \alpha^m \mathbf{w}_{\tau t} + \sum_{l=1}^{L+1} \alpha_\pi^l \mathbf{w}_{\pi t}^l + \sum_{l=1}^{L+1} \alpha_y^l \mathbf{w}_{yt}^l \ . \tag{37}$$

Hence the tangency portfolio, which prices all security returns, is a combination of the global market portfolio and the portfolios hedging against deviations from PPP and movements in the investment opportunity set of an international investor. Notice that the II-CAPM collapses to the IS-CAPM when the investment opportunity set is constant and there are no deviations from PPP.[11] The II-CAPM collapses to the I-CAPM when the investment

[10] See Appendix B for a derivation of (32).

[11] Note that in this case, using either nominal or real stock returns to test the CAPM returns the same results.

opportunity set is constant or, equivalently, when the weights associated with the hedging demands are set equal to zero. Moreover, when the inflation rate of country l, expressed in its home currency, is zero or non stochastic, the $L+1$ inflation hedging funds of Adler and Dumas model collapse to L exchange rate hedging funds:[12] the I-CAPM (PPP) collapses to the I-CAPM (SPOT). The previous setup provides a rationale for using nominal returns in the analysis. The absence of money illusion also makes it possible to express nominal returns in a reference currency and, without loss of generality, in excess of a measurement currency risk-free rate.[13] The problem of testing these different versions of the international CAPM simply reduces to the problem of testing that the tangency portfolio is mean–variance efficient. The international asset pricing models discussed above can be conveniently stated in terms of their assumptions on the nominal (normalized) minimum–variance kernel $q^{\star}$.[14] Hence, I define the vector $\mathbf{y}_{t+1}$ as

$$\mathbf{y}_{t+1} \equiv \left[y_{m,t+1}, \mathbf{y}_{f,t+1}, \mathbf{y}_{\pi,t+1}, \mathbf{y}_{h,t+1}\right]^{\top} ,$$

where $y_{m,t+1}$ is the rate of return on the world market portfolio, $\mathbf{y}_{f,t+1}$ is the $L \times 1$ vector of logarithmic changes of the rates of appreciation of the measurement currency, $\mathbf{y}_{\pi,t+1}$ is the $(L+1) \times 1$ vector of innovations in the inflation rates, and $\mathbf{y}_{h,t+1}$ is the $K \times 1$ vector of demands hedging against variations in the investment opportunity set, where K represents the number of economic factors used in the analysis. The corresponding mimicking-portfolio returns, $\mathbf{y}^{\star}_{t+1}$, are formed using the methodology described in Section I.D.

Consider, for example, the II-CAPM. The corresponding normalized minimum–variance

[12]See, for example, Solnik (1974), Sercu (1980), and Grauer, Litzenberger and Stehle (1976).

[13]In the IS-CAPM, translating returns into a new currency and measuring excess returns relative to the new currency risk-free rate would leave the intercept term equal to zero. In the I-CAPM and II-CAPM, the new currency foreign exchange premium would be replaced by the old currency exchange risk premium. Nonetheless, the introduction of conditioning information and the expansion of the set of primitive securities to include managed portfolios might be affected by the choice of the measurement currency. Hence, the pricing implications delivered by alternative asset pricing models might differ according to the reference currency considered. Dumas and Solnik (1995) found that the choice of the measurement currency did not affect their conclusions in terms of rejection and acceptance of the international CAPM.

[14]See Dumas and Solnik (1995) for a similar interpretation of international asset pricing models.

pricing kernel, $q^{\star}_{t+1}$, can be written as

$$q^{\star}_{t+1} = 1 - y^{\star}_{m,t+1}\lambda_{mt} - \mathbf{y}^{\star\top}_{\pi,t+1}\boldsymbol{\lambda}_{\pi t} - \mathbf{y}^{\star\top}_{h,t+1}\boldsymbol{\lambda}_{ht} \ , \tag{38}$$

or, when inflation rates in each country are non-random, as

$$q^{\star}_{t+1} = 1 - y^{\star}_{m,t+1}\lambda_{mt} - \mathbf{y}^{\star\top}_{f,t+1}\boldsymbol{\lambda}_{ft} - \mathbf{y}^{\star\top}_{h,t+1}\boldsymbol{\lambda}_{ht} \ , \tag{39}$$

where λ_{mt}, $\boldsymbol{\lambda}_{ft}$, $\boldsymbol{\lambda}_{\pi t}$ and $\boldsymbol{\lambda}_{ht}$ are commensurable coefficient vectors.

Using (5), I have

$$\mathrm{E}_t(\mathbf{r}_{t+1}) - \mathbf{1}\mathsf{r}_{\mathsf{ft}} \ = \ \mathrm{E}_t(\mathbf{r}_{t+1}y^{\star}_{m,t+1})\lambda_{mt} + \mathrm{E}_t(\mathbf{r}_{t+1}\mathbf{y}^{\star\top}_{\pi,t+1})\boldsymbol{\lambda}_{\pi t} + \mathrm{E}_t(\mathbf{r}_{t+1}\mathbf{y}^{\star\top}_{h,t+1})\boldsymbol{\lambda}_{ht} \tag{40}$$

and

$$\mathrm{E}_t(\mathbf{r}_{t+1}) - \mathbf{1}\mathsf{r}_{\mathsf{ft}} \ = \ \mathrm{E}_t(\mathbf{r}_{t+1}y^{\star}_{m,t+1})\lambda_{mt} + \mathrm{E}_t(\mathbf{r}_{t+1}\mathbf{y}^{\star\top}_{f,t+1})\boldsymbol{\lambda}_{ft} + \mathrm{E}_t(\mathbf{r}_{t+1}\mathbf{y}^{\star\top}_{h,t+1})\boldsymbol{\lambda}_{ht} \ . \tag{41}$$

Let $\boldsymbol{\beta}_{mt} \equiv \mathrm{E}_t(\mathbf{r}_{t+1}y^{\star}_{m,t+1})$, $\boldsymbol{\beta}_{ft} \equiv \mathrm{E}_t(\mathbf{r}_{t+1}\mathbf{y}^{\star\top}_{f,t+1})$, $\boldsymbol{\beta}_{\pi t} \equiv \mathrm{E}_t(\mathbf{r}_{t+1}\mathbf{y}^{\star\top}_{\pi,t+1})$, and $\boldsymbol{\beta}_{ht} \equiv \mathrm{E}_t(\mathbf{r}_{t+1}\mathbf{y}^{\star\top}_{h,t+1})$ denote the (arrays of) the conditional "betas" associated with the economic variables $\mathbf{y}$. I can rewrite (40) and (41) to obtain the international intertemporal CAPM linear pricing results

$$\mathrm{E}_t(\mathbf{r}_{t+1}) - \mathbf{1}\mathsf{r}_{\mathsf{ft}} = \boldsymbol{\beta}_{\mathsf{mt}}\lambda_{\mathsf{mt}} + \boldsymbol{\beta}_{\pi\mathsf{t}}\boldsymbol{\lambda}_{\pi\mathsf{t}} + \boldsymbol{\beta}_{\mathsf{ht}}\boldsymbol{\lambda}_{\mathsf{ht}} \tag{42}$$

and

$$\mathrm{E}_t(\mathbf{r}_{t+1}) - \mathbf{1}\mathsf{r}_{\mathsf{ft}} = \boldsymbol{\beta}_{\mathsf{mt}}\lambda_{\mathsf{mt}} + \boldsymbol{\beta}_{\mathsf{ft}}\boldsymbol{\lambda}_{\mathsf{ft}} + \boldsymbol{\beta}_{\mathsf{ht}}\boldsymbol{\lambda}_{\mathsf{ht}} \ . \tag{43}$$

Equations (42) and (43) state that the conditional risk premium on any asset is a linear combination of the conditional risk premia on the different sources of economic and financial risks. The previous equations collapse to the IS-CAPM when PPP holds and hedging demands are equal to zero while they reduce to the Adler and Dumas I-CAPM when hedging demands are equal to zero.[15]

[15]Specifically, in testing the II-CAPM, I overparameterize the model and use demands hedging against inflation and foreign exchange risk simultaneously.

II. Estimation

This section describes the methodology I use in the empirical analysis. First, I illustrate the approach taken to document time variation in the first and second moments of international asset returns. Second, I illustrate the estimation of the coefficients of the minimum-variance kernel. Third, I illustrate the estimation of the economic risk premia using two approaches based on the minimum-variance kernel and mimicking portfolios. Fourth, I illustrate how the explicit asset pricing models discussed in the previous section are tested.

A. Selecting Instruments

Let $\mathbf{z}_t$ denote a $J \times 1$ vector of instruments whose realizations belong to the information set at the beginning of each investment period. Without loss of generality, I assume the first element of $\mathbf{z}_t$ to be unity, $z_{1t} = 1$. I model the conditional mean and conditional volatility of asset returns as linear functions of the instruments. Namely, I assume

$$\begin{aligned} \mathrm{E}_t(\mathbf{r}_{t+1}) &= \mu_{r1} + \mu_{r2} z_{2t} + \ldots + \mu_{J1} z_{Jt} \ , \text{ and} \\ \mathrm{E}_t[|\mathbf{r}_{t+1} - \mathrm{E}_t(\mathbf{r}_{t+1})|] &= v_{r1} + v_{r2} z_{2t} + \ldots + v_{J1} z_{Jt} \ . \end{aligned} \tag{44}$$

Hence, the mean and volatility coefficients can be estimated by exactly-identified GMM.

B. The Minimum-Variance Kernel

I postulate that the coefficients α_{0t} and $\boldsymbol{\alpha}_t$ of the minimum-variance pricing kernel are linear functions of the instruments $\mathbf{z}_t$;

$$\alpha_{0t} = 1 - \mathrm{E}_t(\mathbf{r}_{t+1})^\top \boldsymbol{\alpha}_t = \alpha_0(\mathbf{z}_t) \ , \tag{45}$$

and

$$\boldsymbol{\alpha}_t = -\boldsymbol{\Sigma}_{rrt}^{-1}[\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] = \boldsymbol{\alpha}(\mathbf{z}_t) \ , \tag{46}$$

where $\boldsymbol{\alpha}_j$, $j = 1, \ldots, J$, are $N \times 1$ coefficient vectors.

Note that this approach can be interpreted as the scaling of asset returns with instruments (or the expansion of the set of securities to include managed portfolios) which is common in the asset pricing literature.[16] In fact, I can write

$$\mathrm{E}_t(q^\star_{t+1})\mathbf{z}_t = \mathbf{z}_t \ , \text{ and} \tag{47}$$

$$\mathrm{E}_t(q^\star_{t+1}\mathbf{r}_{t+1}) \otimes \mathbf{z}_t = r_{ft}\mathbf{1} \otimes \mathbf{z}_t \ . \tag{48}$$

Assuming stationarity, and applying the law of iterated expectations, I have

$$\mathrm{E}(q^\star_{t+1}\mathbf{z}_t) = \mathrm{E}(\mathbf{z}_t) \ , \text{ and} \tag{49}$$

$$\mathrm{E}(q^\star_{t+1}\mathbf{r}_{t+1} \otimes \mathbf{z}_t) = \mathrm{E}(r_{ft}\mathbf{1} \otimes \mathbf{z}_t) \ . \tag{50}$$

The two conditions above ensure that, conditioning on $\mathbf{z}_t$, $q^\star_{t+1}$ has mean one and correctly prices the securities under consideration.

It is convenient at this point to define

$$\boldsymbol{\iota}_t \equiv \begin{bmatrix} 1 \\ r_{ft}\mathbf{1} \end{bmatrix} \tag{51}$$

and let $\boldsymbol{\iota}^z_t \equiv \boldsymbol{\iota}_t \otimes \mathbf{z}_t$. Also, I define

$$\mathbf{r}_{a,t+1} \equiv \begin{bmatrix} 1 \\ \mathbf{r}_{t+1} \end{bmatrix} \tag{52}$$

and $\mathbf{r}^z_{a,t+1} \equiv \mathbf{r}_{a,t+1} \otimes \mathbf{z}_t$. Hence, I can rewrite (49) and (50) as

$$\mathrm{E}(q^\star_{t+1}\mathbf{r}^z_{a,t+1}) = \mathrm{E}(\boldsymbol{\iota}^z_t) \ . \tag{53}$$

The minimum-variance kernel satisfying (53) has the form $q^\star_{t+1} \equiv \mathbf{r}^{z\top}_{a,t+1}\boldsymbol{\alpha}^z_a$, where $\boldsymbol{\alpha}^z_a$ is an $(N+1)J$ coefficient vector. I have

$$\boldsymbol{\alpha}^z_a = [\mathrm{E}(\mathbf{r}^z_{a,t+1}\mathbf{r}^{z\top}_{a,t+1})]^{-1}\mathrm{E}(\boldsymbol{\iota}^z_t) \ . \tag{54}$$

[16]This scaling procedure has an intuitive interpretation. The scaled returns are the returns on managed portfolios in which the manager invests more or less according to the signals $\mathbf{z}_t$.

Hence, the minimum-variance kernel satisfying (53) has the form

$$\begin{aligned} q^{\star}_{t+1} &= (\alpha^{z}_{a1} + \alpha^{z}_{a2} z_{2t} + \ldots + \alpha^{z}_{aJ} z_{Jt}) \\ &\quad + (\alpha^{z}_{a,J+1} + \alpha^{z}_{a,J+2} z_{2t} + \ldots \alpha^{z}_{a,2J} z_{Jt}) r_{1,t+1} \\ &\quad + \ldots \\ &\quad + (\alpha^{z}_{a,NJ+1} + \alpha^{z}_{a,NJ+2} z_{2t} + \ldots \alpha^{z}_{a,J(N+1)} z_{Jt}) r_{N,t+1} \ , \end{aligned} \tag{55}$$

which is equivalent to imposing the assumptions (45) and (46). The analysis above still applies when the positivity constraint is imposed.[17] In this case the minimum-variance kernel has the form $\tilde{q}_{t+1} \equiv (\mathbf{r}^{z\top}_{a,t+1} \tilde{\boldsymbol{\alpha}}^{z}_{a})^{+}$.

Note that when the set of instruments used in the analysis only contains a constant, this approach can be interpreted as projecting the minimum–variance kernel on the set of primitive securities.

C. Economic Risk Premia

In order to estimate the conditional risk premia associated with the variables y_{kt}, I use two approaches. First, I consider the conditional covariance between the minimum-variance kernel and y_k. Second, I construct mimicking portfolios and I estimate their conditional risk premia.

In implementing the first approach, I assume

$$\lambda^{\star}_{kt} \equiv -\mathrm{Cov}_t(q^{\star}_{t+1}, y_{k,t+1}) = -\mathrm{E}_t[(q^{\star}_{t+1} - 1) y_{k,t+1}] = \lambda_{k1} + \lambda_{k2} z_{2t} + \ldots + \lambda_{kJ} z_{Jt} \ . \tag{56}$$

Without loss of generality, I also assume $\mathrm{Var}(z_{jt}) = 1$. Hence the coefficients λ_{kj} can be interpreted as the change in the conditional risk premium for a one-standard-deviation change in the instrument. The assumption that the conditional risk premia are determined by the set of instruments $\mathbf{z}_t$ is quite natural: the conditional risk premia assigned by the minimum-variance kernel are the mean cash flows generated by the hedging portfolio financed at the

[17] Appendix C shows how to estimate economic risk premia under the no arbitrage condition of non-negativity of the normalized pricing kernel.

riskless rate. Hence, if the variables in $\mathbf{z}_t$ are predictors of asset returns, they should also predict excess returns on the hedging portfolios. In fact, modeling the time-variation of risk premia in this fashion is common to other studies (Ferson and Harvey (1991), for example). Note that, without loss of generality, I can assume $\mathrm{E}(z_{jt}) = 0$, $i = 2, \ldots, J$. This means that

$$\mathrm{E}(\lambda^{\star}_{kt}) = \lambda_{k1} \ . \tag{57}$$

The distinction between conditional and unconditional risk premia is important because, even if the unconditional premium is close to zero, the conditional premia may take values over time which are significantly different from zero.

When the positivity restriction is imposed, I have

$$\tilde{\lambda}_{kt} \equiv -\mathrm{Cov}_t(\tilde{q}_{t+1}, y_{k,t+1}) = -\mathrm{E}_t[(\tilde{q}_{t+1} - 1)y_{k,t+1}] = \lambda_{k1} + \lambda_{k2} z_{2t} + \ldots + \lambda_{kJ} z_{Jt} \ . \tag{58}$$

The second approach to the estimation of the economic risk premia is based on the construction of hedging portfolios. I postulate that the coefficients $\alpha_{0y_k t}$ and $\boldsymbol{\alpha}_{y_k t}$ of the hedging portfolio are linear functions of the instruments $\mathbf{z}_t$. Hence, I have $y^{\star}_{k,t+1} = \boldsymbol{\alpha}^{z}_{ay_k} \mathbf{r}^{z}_{a,t+1}$, where

$$\boldsymbol{\alpha}^{z}_{ay_k} = [\mathrm{E}(\mathbf{r}^{z}_{a,t+1} \mathbf{r}^{z\top}_{a,t+1})]^{-1} \mathrm{E}(\mathbf{r}^{z}_{a,t+1} y_{k,t+1}) \ . \tag{59}$$

As with the coefficients of the minimum-variance kernel, one can show that this approach is equivalent to projecting the risk variables on both the returns on the primitive securities and the cash flows of dynamic strategies. In particular, when $\mathbf{z}_t = z_{1t} = 1$ this approach is equivalent to projecting the risk variables only on the set of primitive securities.

Finally, I estimate the Sharpe ratios of the hedging portfolios, since they might differ substantially from the conditional risk premia (see discussion above). Hence, I take the ratio between the risk premium and the volatility of the hedging portfolios cash flows.

D. Tests of International Asset-Pricing Models

Each of the international asset-pricing models I considered has specific pricing implications, as previously discussed. For convenience, I discuss my tests with respect to the implications of the static international capital asset pricing model (IS-CAPM). The approach to test the other models is analogous.

Recall that the IS-CAPM implies that the portfolio hedging global market risk prices all securities. Using a standard GMM test I test whether a pricing kernel linear in the cash flows of this portfolio, $q^{\star}_{m,t+1}$, prices all securities. I have $q^{\star}_{m,t+1} = \mathbf{y}^{\star}_{ma,t+1}\boldsymbol{\alpha}^{z}_{ma}$, where

$$\boldsymbol{\alpha}^{z}_{ma} = \mathrm{E}(\mathbf{y}^{\star}_{ma,t+1}\mathbf{y}^{\star\top}_{ma,t+1})^{-1}\mathrm{E}(\boldsymbol{\iota}_{mt}) \tag{60}$$

and

$$\mathbf{y}^{\star}_{ma,t+1} \equiv \begin{bmatrix} \mathbf{z}_t \\ \mathbf{r}^{z\top}_{a,t+1}\boldsymbol{\alpha}^{z}_{ay_m} \end{bmatrix} \tag{61}$$

$$\boldsymbol{\iota}_{mt} \equiv \begin{bmatrix} \mathbf{z}_t \\ \boldsymbol{\iota}^{z\top}_{t}\boldsymbol{\alpha}^{z}_{ay_m} \end{bmatrix} . \tag{62}$$

I test the moment conditions

$$\mathrm{E}(q^{\star}_{m,t+1}\mathbf{r}^{z}_{a,t+1}) = \mathrm{E}(\boldsymbol{\iota}^{z}_{t+1}) . \tag{63}$$

This corresponds to the test of overidentifying restrictions pioneered by Hansen and Singleton (1982). I have $J+K$ coefficients in the vector $\boldsymbol{\alpha}^{z}_{ma}$ and $J+NJ$ moment conditions, for a total of $NJ-K$ overidentifying restrictions. If the test failed, this would mean that the pricing errors generated by the IS-CAPM are statistically significant, and the model is rejected.

Second, I compare the standard deviation of $q^{\star}_{m,t+1}$ to the standard deviation of the minimum-variance normalized kernel, $q^{\star}_{t+1}$. Namely, I estimate the difference

$$\sqrt{\mathrm{Var}(q^{\star}_{m,t+1})} - \sqrt{\mathrm{Var}(q^{\star}_{t+1})} . \tag{64}$$

This corresponds to the HJV test. Since the variances of the two kernels correspond to the average squared Sharpe ratios of the global market-hedging portfolio and the tangency portfolio, this test is equivalent to a test of the mean-variance efficiency of the world market-hedging portfolio.

Third, I calculate the standard deviation of the difference between $q^\star_{m,t+1}$ and $q^\star_{t+1}$,

$$\sqrt{\mathrm{Var}(q^\star_{t+1} - q^\star_{m,t+1})}\ . \tag{65}$$

This corresponds to the HJD test.[18] If the test failed, this would mean that the difference between the pricing kernel generated by the IS-CAPM and any admissible kernel is statistically significant, and the model is rejected.

III. Data

This section illustrates the data used in the empirical analysis. The period considered is April 1970 through October 1998 for stock returns and economic variables and March 1970 through September 1998 for instrumental variables. Data are monthly. The starting and ending dates for the sample are dictated by macroeconomic and financial data availability.

A. Asset Returns

I use the Morgan Stanley Capital International (MSCI) national equity indices. The nominal returns are denominated in U.S. dollars and are calculated with dividends. All indices have a common basis of 100 in December 1969. The indices are constructed using the Laspeyres method, which approximates value weighting.[19] U.S. dollar returns are calculated by using the closing European interbank currency rates from MSCI. I choose the four countries with the largest market capitalization: United States; United Kingdom; Japan; and Germany.[20] Table I shows summary statistics for monthly returns on stock indices from MSCI.

[18] Since I allow a constant in $q^?_{m,t+1}$, $E(q^?_{m,t+1}) = E(q^?_{t+1}) = 1$. Hence, $\sqrt{E(q^?_{t+1} - q^?_{m,t+1})^2}$, which is the Hansen-Jagannathan distance, equals $\sqrt{\mathrm{Var}(q^?_{t+1} - q^?_{m,t+1})}$.

[19] See MSCI Methodology & Index Policy for a detailed description of MSCI's indices and properties.

[20] As of 1996, the market capitalization weight for these countries is 76.2% of the market capitalization world.

B. Economic Variables and Instruments

I use inflation rates, spot exchange rates, and the rate of return on the world market portfolio as the relevant sources of risk in this study. Consumer price indices are from International Financial Statistics (IFS) and are denominated in local currency. Spot exchange rates are from MSCI. The world equity market index is a value–weighted combination of the country returns tracked by MSCI. I proxy foreign exchange risk with logarithmic changes in the spot exchange rates (SPOT); inflation risk with an ARIMA(0,1,1) for inflation (INFL); and global market risk with the level of world equity returns (WLDMK). Note that the variable INFL represents unexpected inflation and that the variable SPOT represents innovations in the exchange rate if I assume that spot rates follow a random walk. Table II contains summary statistics for the relevant risk variables. In choosing the set of instruments, I concentrate on a set of variables which have been previously used in tests of multiple-beta models and/or in studies of stock-return predictability.[21] These variables are statistically significant in multivariate predictive regressions of means and volatilities and/or they have special economic significance. The instruments include a constant, a January dummy, and the following five variables:

DINFLUS is the lagged difference in the U.S. monthly rate of inflation (IFS).

EURO represents the one–month Eurodollar deposit rate (DRI) and performs as the conditionally nominal risk–free asset in the analysis.

USDIVYLD denotes the U.S. monthly dividend yield (MSCI) in excess of the 1-month Eurodollar deposit rate. Specifically, the monthly dividend yield is equal to 1/12 of the ratio between the previous year dividend and the index at the end of each month.

WLDMK denotes the lagged value of the world stock market monthly returns (MSCI).

DEFPREM denotes the U.S. default premium as given by the return difference between Moody's Baa-rated and Aaa-rated bonds (SBBI Yearbook).

[21]See, for example, Ferson and Harvey (1993), Dumas and Solnik (1995), and De Santis and Gérard (1998).

I select as instruments the previous variables as a proxy for the information investors use to set prices in the market.

Table III presents summary statistics of the instrumental variables.

IV. Selecting Instruments

In this section I select a set of instruments by performing an analysis of predictability. Specifically, I look at the ability of some variables used in previous studies of international asset pricing to predict variation in the first and second conditional moments. This preliminary analysis is relevant for three reasons. First, it identifies the information set of an international investor. Second, it makes it possible to study the patterns of time variation in the conditional risk premia. Third, it provides a rationale for including a third hedging portfolio in alternative international CAPM specifications besides portfolios hedging against global market and inflation (or currency) risks.

Table IV presents my results. I report the coefficients of the mean and variance equations, as well as three statistics: $\bar{\mu}_r$ is the average slope coefficient in the mean equations; $\bar{v}_r$ is the average slope coefficient in the variance equations; and $\bar{\mu}_r - \bar{v}_r$ is the difference between the two average slope coefficients. These statistics provide an indication of the net effect of the instruments on the investment opportunity set.

The following patterns emerge from this analysis (see especially Panel C):

> The lagged change in the U.S. inflation rate (DINFLUS) has a negative and significant average impact on returns and a negative, but insignificant, average impact on return volatility. The net effect on the investment-opportunity set is strongly negative.
>
> The Eurodollar deposit rate (EURO) has a positive and significant average impact on returns and a negative and significant average impact on return volatility. The net effect is strongly positive.
>
> The U.S. dividend yield in excess of the EURO (USDIVYLD) has a positive and

significant average effect on returns and a negative and significant overall effect on volatility. The net effect is positive.

The lagged world market equity return (WLDMK) has an overall positive, but insignificant, impact on returns. The impact on volatility is negative and partially significant. The net effect is positive but not large.

The U.S. default premium (DEFPREM) positively affects returns and is partially significant. The overall effect on volatility is negative and partially significant. The net effect is positive.

In summary, I can rank the net effects of the different variables on the investment-opportunity set as follows (from largest to smallest): USDIVYLD, EURO, DEFPREM, WLDMK, DINFLUS.[22]

V. Risk Premia and Sharpe Ratios

In this section I report estimates of the risk premia associated with the economic variables and look at their patterns of time variation.[23] I use the instrumental variables selected in the previous section to document the patterns of time variation of the conditional premia.

Table V reports estimates of the coefficients of the economic risk premia estimated using the minimum-variance kernel $q^{\star}_{t+1}$. Since the instruments are demeaned, the intercept term can be interpreted as the unconditional risk premium on $y_{k,t+1}$. I do not report coefficient estimates of the economic risk premia estimated using the non-negative minimum-variance kernel $\tilde{q}_{t+1}$ because there is no substantial difference with respect to the estimates reported in Table VI. Table VII reports coefficient estimates of the economic risk premia estimated using the hedging portfolios.

[22]Note that Germany, among all countries in the study, exhibits the weakest patterns of predictability.

[23]Note that the instruments do not coincide with the past values of the economic variables included in the analysis, with the exception of the world market portfolio.

As noted in section I.D. above, the expected excess cash flows on the hedging portfolios coincide with risk premia assigned by $q^{\star}_{t+1}$. Yet, the realized excess cash flows on the hedging portfolios in general differ from $(q^{\star}_{t+1} - 1)y_{k,t+1}$. Hence, the estimates of the unconditional risk premia using the "$q^{\star}$" and the "hedging portfolio" approaches will coincide, although their standard errors may differ. In addition, the impact of the conditioning variables on the conditional risk premia will also differ. Note that when $y^{\star}$ and $q^{\star}$ are mapped onto the space spanned by the primitive securities and are estimated inside of the algorithm, the standard errors of the unconditional risk premia coincide (see Panel A of Tables V and VI).

The tables report two sets of t-ratios. The first t-ratio is obtained using a two-step procedure: I first estimate the coefficients of $q^{\star}_{t+1}$ and $y^{\star}_{k,t+1}$; I then estimate the risk premia by exactly-identified GMM algorithm. The second t-ratio is obtained estimating all parameters inside the GMM algorithm. The reason for the two separate approaches is that I am concerned with the large number of estimated parameters when the coefficients of the minimum-variance kernels and the hedging portfolios are estimated by GMM. As it turns out, the t-ratios change only marginally across the two procedures. In all tests, standard errors are adjusted for heteroskedasticity and serial correlation.

The results are also very similar across estimation methods. The main difference is that the estimation based on the $q^{\star}$ tends, in most of the cases, to yield tighter standard errors.

The following patterns emerge from the tables:

The unconditional inflation risk premia are negative (except for Germany) but not significant. When managed portfolios are ruled out from the analysis, only U.S. inflation seems to be priced and to be significant across estimation techniques. No significant pattern of time variation emerges from the analysis of the foreign inflation conditional premia.

The unconditional foreign exchange risk premia are negative. Even if I am not using any equilibrium model to compute economic risk premia, this result is consistent with the predictions of Adler's and Dumas' international CAPM. They show that if the degree of risk aversion of investors is greater than one, foreign exchange risk premia should be negative. Nonetheless, the unconditional estimates of currency risk premia are not statistically significant. This result also supports Adler and Dumas (1995) and De Santis and Gerard (1998). I also find time variation in the estimates of the conditional premia statistically significant.

The unconditional global market risk premium is positive and statistically significant across estimation techniques. This result seems to be robust and does not support the analysis of Adler and Dumas (1995) or De Santis and Gérard (1998). They actually find that the world market unconditional risk premium is positive but insignificant. On the other hand, my findings are supported by the work of Hodrick, Ng, and Sengmüller (1999). Moreover, I find significant time variation in the global market conditional risk premium.

In addition, I estimated the unconditional and conditional premia implied by the set of economic and financial factors. Estimation results (not reported in the paper) do not provide any evidence of statistical significance of these premia. Even if the factors exhibit some non–trivial patterns of predictability for the first and second moment of asset returns, they do not seem to be priced.

In summary, I find that the signs of the risk premia associated with foreign exchange and inflation risks are largely consistent with the theoretical predictions of several models of international asset pricing. At the same time, foreign inflation unconditional and conditional risk premia and foreign exchange unconditional risk premia are imprecisely estimated. On the contrary, I document significant patterns of time variation of foreign exchange premia and find that constant and time-varying global market risk premia are precisely estimated. The remaining economic and financial factors are not priced, both unconditionally and conditionally.

The risk premia estimated above coincide with the Sharpe ratios of exact mimicking portfolios. But, in general, economic factors can be tracked only imperfectly by asset returns. Hence, in order to obtain Sharpe ratios on traded portfolios I need to standardize the estimates obtained above by the volatility of the approximate mimicking portfolio returns. The composition of the mimicking portfolios is estimated separately from the Sharpe ratios.

Table VII presents the Sharpe ratios on the eight hedging portfolios. I find that the volatilities of the mimicking portfolios cash flows are strongly significant. On the other hand, given the statistical insignificance of the mean estimates, the Sharpe ratios on the hedging portfolios are insignificant, with the exception of the global market portfolio.

VI. Hedging Demands and Tests of International Asset Pricing Models

In this section I discuss the results of tests of four explicit asset pricing models: IS-CAPM, I-CAPM (PPP), I-CAPM (SPOT), and II-CAPM.[24] Results of the tests are presented in Table VIII and in Table IX. The tests are performed using the full set of instruments $\mathbf{z}_t$ ("With conditioning information").

In Table VIII, I report the χ^2 statistic associated with a test of the overidentifying restrictions, the difference between the standard deviation of the candidate pricing kernel and the standard deviation of $q^{\star}_{t+1}$, the HJV statistic, and the standard deviation of the difference between the candidate pricing kernel and $q^{\star}_{t+1}$, the HJD statistic. I also report the p-values associated with the χ^2 test, and the t-ratios associated with the HJV and HJD statistics.

In the test of overidentifying restrictions, the coefficients of the candidate kernel are estimated by GMM, although the composition of the mimicking portfolios is estimated separately, outside of the GMM algorithm. In the other two tests, the coefficients of the candidate kernel are estimated separately.

I find that the χ^2 tests do not reject all five models conditionally. In particular, the

[24]The additional mimicking portfolios considered in the II-CAPM specification are obtained via an OLS regression of DINFLUS, EURO, USDIVYLD and DEFPREM on the augmented span of managed portfolio returns.

II-CAPM(PPP) can be weakly rejected at the 5% significance level but not at the 1% level.

On the contrary, tests of the HJ bounds and of the HJ distance measure indicate that all the models are rejected by the data – the standard deviation of the candidate kernels is always substantially lower than that of $q^{\star}_{t+1}$. Nonetheless, as shown in Table IX, there are differences in performance. The IS-CAPM generates the least volatile pricing kernel. The I-CAPM (SPOT) of Grauer, Litzenberger, and Stehle (1976) generates pricing kernels with somewhat higher volatility. The second most volatile pricing kernel is generated by the Adler and Dumas model. The most volatile kernel is generated by the II-CAPM (PPP). The difference between the volatility of the II-CAPM (PPP) and I-CAPM (SPOT) kernels is substantial (68 percent) while the difference between the volatility of the II-CAPM (PPP) and I-CAPM (PPP) kernels is close to 16 percent. Note that the standard deviation of a pricing kernel coincides with the average Sharpe ratio of the tangency portfolio constructed using the underlying set of assets. Hence, the HJV statistic can be interpreted as the difference between two average Sharpe ratios.

Overall, the evidence from these tests is that while all five models are not rejected by the Hansen and Singleton (1982) χ^2 test of overidentifying restrictions, the same models are formally rejected by the tests of HJ variance bounds and HJ distance. These results are in sharp contrast with the findings of Dumas and Solnik (1995) as well as De Santis and Gérard (1998). These authors do not reject the international CAPM in its conditional version on the basis of tests of overidentifying restrictions and cross-equations restrictions. The use of test statistics that do not reward the variability of alternative admissible pricing kernels allows me to reject the conditional international CAPM in all its specifications.

I also investigate the size and the significance of the unconditional demands induced by hedging against global market, inflation, and foreign exchange risks. The estimates of these hedging demands correspond to the coefficients of the mimicking portfolios that appear in the kernel specification described in Section II.D. The hedging demands are normalized to sum up to one. Standard errors are computed using the delta method.

As shown in Table X, the scaled hedging demands are precisely estimated. In particular, the coefficients associated with the cash-flows of the inflation hedging portfolios are large in magnitude and statistically significant. This result further explains why the international intertemporal CAPM in presence of deviations from PPP has better pricing implications than the other asset pricing models.[25]

VII. Conclusions

This paper presents a new approach for the estimation of risk premia associated with observable sources of risk, which is based on the moments of the minimum-variance kernel of Hansen and Jagannathan (1991). I also provide extensive evidence on the performance of four explicit asset pricing models: the IS-CAPM; the I-CAPM (PPP); the I-CAPM (SPOT); and the II-CAPM in presence of deviations from PPP.

In sharp contrast with Dumas and Solnik (1995) and De Santis and Gérard (1998), but in line with Hodrick, Ng, and Sengmüller (1999), I find the global market risk is priced both conditionally and unconditionally.

All international asset-pricing models are formally rejected by the data when the testing methodology is stringent enough. In addition, the II-CAPM (PPP) that I construct and test using mimicking portfolios outperforms the IS-CAPM. It generates a pricing kernel which is 72% more volatile than the one of the IS-CAPM. For these models, the differences is Hansen & Jagannathan variance bounds and distance measures are large and statistically significant. Hence, I add to the existing literature showing that, introducing deviations from PPP and dynamic hedging, the II-CAPM (PPP) is able to generate more accurate pricing implications than the other versions of the international CAPM. My findings show that the II-CAPM (PPP) outperforms the competitive international asset-pricing models because most of the economic and financial factors considered significantly affect the first and second conditional moments of asset returns. Finally, the result that the international CAPM in its alternative

[25] The inflation-mimicking portfolios make the II-CAPM(PPP) kernel more volatile and hence closer to the bounds of the minimum-variance kernel.

specifications does not hold is consistent with the evidence of at least mild segmentation of international equity markets and is supported by Hodrick, Ng, and Sengmüller (1999).

Future work should investigate the sensitivity of these results to the level of aggregation of asset returns, to the choice of the investment opportunity set of an investor, and to the measurement currency used in the analysis.

Appendix A

A. Unobservable Factors: Principal Components

One common approach to the study of asset returns is based on principal-component analysis. Let $\mathbf{S}_t$ denote the matrix whose columns are the orthonormal eigenvectors of $\mathbf{\Sigma}_{rrt}$. Since $\mathbf{S}_t^\top \mathbf{S}_t = \mathbf{I}$, I have

$$\mathbf{r}_{t+1} = \mathbf{S}_t^\top \mathbf{S}_t \mathbf{r}_{t+1} \equiv \mathbf{S}_t^\top \mathbf{r}_{p,t+1} \ , \tag{66}$$

where $\mathbf{r}_{p,t+1} \equiv \mathbf{S}_t \mathbf{r}_{t+1}$ is the vector of orthogonal factor-portfolio returns.[26] This makes it possible to rewrite the kernel $q_{t+1}^\star$ as

$$\begin{aligned} q_{t+1}^\star &= 1 - [\mathbf{r}_{t+1} - \mathrm{E}_t(\mathbf{r}_{t+1})]^\top \mathbf{S}_t^\top \mathbf{S}_t \mathbf{\Sigma}_{rrt}^{-1} \mathbf{S}_t^\top \mathbf{S}_t [\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] \\ &= 1 - [\mathbf{r}_{p,t+1} - \mathrm{E}_t(\mathbf{r}_{p,t+1})]^\top \mathbf{V}_t^{-1} [\mathrm{E}_t(\mathbf{r}_{p,t+1}) - r_{ft}\mathbf{S}_t^\top \mathbf{1}] \ , \end{aligned} \tag{67}$$

where $\mathbf{V}_t$ is a diagonal matrix whose v_{iit} element is the i-th eigenvalue of $\mathbf{\Sigma}_{rrt}$.

Three main insights can be developed based on the expressions above. First, the minimum-variance kernel is a linear combination of the factor-portfolio returns

$$q_{t+1}^\star = 1 - \sum_{i=1}^{N} \frac{r_{pi,t+1} - \mathrm{E}_t(r_{pi,t+1})}{\sqrt{v_{iit}}} S_{pit} \ , \tag{68}$$

where S_{pit} is the Sharpe ratio on the i-th factor-portfolio. Hence, the (standardized) risk premia associated with the factor portfolios are the coefficients relating the (standardized) innovations in the factor portfolio returns to the minimum-variance kernel. Moreover, equation (68) highlights how the variance of the minimum-variance kernel, and hence the squared Sharpe ratio of the tangency portfolio, can be decomposed according to the squared Sharpe ratios of the factor portfolios

$$\mathrm{Var}_t(q_{t+1}^\star) = S_{\tau t}^2 = \sum_{i=1}^{N} S_{pit}^2 \ . \tag{69}$$

[26] Note that the weights of the factor portfolios do not sum to one: the eigenvectors are normalized so that the sum of the squared elements is one.

Second, since the factor portfolio returns are orthogonal to each other, the covariance between $q^{\star}_{t+1}$ and any risk variable $y_{k,t+1}$ can be written as a linear combination of the sum of the covariances between each of the factor portfolio returns and $y_{k,t+1}$:

$$\lambda_{kt} = -\sum_{i=1}^{N} \frac{\mathrm{Cov}_t(r_{pi,t+1}, y_{k,t+1})}{\sqrt{v_{iit}}} S_{pit} \ . \tag{70}$$

This expression allows for a breakdown of the risk premium on an observable risk variable into the components due to the different factor portfolios.

Third, the ability of a subset of the factor portfolios to price all assets can be tested in the same way as any candidate pricing kernel.

B. Observable Factors: Multi-beta Models

A multi-beta model implies that expected excess returns are linear in the sensitivities of the returns to the risk variables, with coefficients given by the risk premia associated with the factors:

$$\mathrm{E}_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1} = \boldsymbol{\beta}_t \boldsymbol{\lambda}_t \ , \tag{71}$$

where $\boldsymbol{\beta}_t$ is an $N \times K$ matrix of projection coefficients of the returns on the risk variables. Hence, excess returns are described by the model

$$\mathbf{r}_{t+1} - r_{ft}\mathbf{1} = \boldsymbol{\beta}_t \boldsymbol{\lambda}_t + \boldsymbol{\beta}_t \mathbf{y}_{t+1} + \mathbf{e}_{t+1} \ , \tag{72}$$

where $\mathbf{e}_{t+1}$ is a vector of $N \times 1$ mean-zero perturbances orthogonal to the risk variables $\mathbf{y}_{t+1}$, with covariance matrix $\boldsymbol{\Sigma}_{eet}$.

Consider now the risk premia assigned by the minimum-variance kernel $q^{\star}_{t+1}$. From (16) I have

$$\begin{aligned} \boldsymbol{\lambda}^{\star}_t &= \boldsymbol{\alpha}_{yt} \boldsymbol{\beta}_t \boldsymbol{\lambda}_t \\ &= \boldsymbol{\Sigma}_{yrt} \boldsymbol{\Sigma}^{-1}_{rrt} \boldsymbol{\Sigma}_{ryt} \boldsymbol{\Sigma}^{-1}_{yyt} \boldsymbol{\lambda}_t \ . \end{aligned} \tag{73}$$

In the special case where $\mathbf{\Sigma}_{yyt} = \mathbf{I}$, i.e., the factors are orthogonal with unit variance, the equation above simplifies to

$$\boldsymbol{\lambda}_t^\star = \mathbf{\Sigma}_{yrt}\mathbf{\Sigma}_{rrt}^{-1}\mathbf{\Sigma}_{ryt}\boldsymbol{\lambda}_t \ . \tag{74}$$

It is straightforward to verify that the $\mathbf{\Sigma}_{yrt}\mathbf{\Sigma}_{rrt}^{-1}\mathbf{\Sigma}_{ryt}$ matrix equals the covariance matrix of the projections of the risk variables onto the span of asset returns, $\mathbf{\Sigma}_{y^\star y^\star t}$,[27] i.e., $\boldsymbol{\lambda}_t^\star = \mathbf{\Sigma}_{y^\star y^\star t}\boldsymbol{\lambda}_t$. Hence, the risk premia assigned by $q_{t+1}^\star$ are linear combinations of the multi-beta premia through coefficients, which depend on the covariance matrix of the hedging-portfolio cash flows. In the two-factor case, for example,

$$\lambda_{1t}^\star = \sigma_{y^\star 1t}^2\lambda_{1t} + \sigma_{y^\star 12t}\lambda_{2t} \ , \tag{75}$$

where σ_{y1t}^2 is also the R^2 of the projection of the first risk variable on the span of returns. In the special case where y_{1t} is perfectly tracked by the hedging portfolio, $\sigma_{y^\star 1t}^2 = 1$, $\sigma_{y^\star 12t} = 0$, and $\lambda_{1t}^\star = \lambda_{1t}$.

Appendix B

Equation (32) combines Merton's (1973) intertemporal CAPM with Adler and Dumas (1983) international asset pricing model in presence of deviations from PPP. The continuous-time portfolio selection problem of a representative investor can be stated as follows:[28]

$$\text{Max} \ \ E\int_t^T V(C,P,s) \ \ ds \ , \tag{76}$$

where $C = C(W, P, y_k, t)$ denotes nominal consumption expenditures, P is the price level index, V is a function homogeneous of degree zero in C and P expressing the instantaneous rate of indirect utility, and y_k is a state variable that affects utility through nominal consumption. Following Merton (1969), the wealth dynamics can be written as

$$dW = [\sum_{i=1}^N w_i(\mu_i - r_f) + r_f]Wdt - Cdt - \sum_{i=1}^N w_i\sigma_i dz_i \ , \tag{77}$$

[27] I have $\mathrm{Var}_\mathrm{t}(\mathbf{y}_{\mathrm{t+1}}^?) = \mathrm{Var}_\mathrm{t}(\boldsymbol{\alpha}_{\mathrm{yt}}\mathbf{r}_{\mathrm{t+1}}) = \mathbf{\Sigma}_{\mathrm{yrt}}\mathbf{\Sigma}_{\mathrm{rrt}}^{-1}\mathbf{\Sigma}_{\mathrm{rrt}}\mathbf{\Sigma}_{\mathrm{rrt}}^{-1}\mathbf{\Sigma}_{\mathrm{ryt}} = \mathbf{\Sigma}_{\mathrm{yrt}}\mathbf{\Sigma}_{\mathrm{rrt}}^{-1}\mathbf{\Sigma}_{\mathrm{ryt}}$.

[28] See Appendix in Adler and Dumas (1983) for a detailed explanation of the necessary assumptions.

where $\mathbf{w} = \{w_i\}$ is $(N+1) \times 1$ vector of weights, μ_i is the instantaneous expected nominal rate of return on security i expressed in a reference currency, σ_i is the instantaneous standard deviation of the nominal rate of return on security i, r_f is the risk-free rate expressed in a reference currency, and dz_i is the white noise of a standard Wiener process. Denoting with $J(W, P, y_k, t)$ the maximum value of (76) subject to (77), the Bellman principle states that total expected rate of increase of this function must be identically zero, so that

$$\begin{aligned} 0 \;=\; & \underset{(C,\mathbf{w})}{\text{Max}} \{V(C, P, y_k, t) + J_t + J_W[-C + W(\sum_{i=0}^{N} w_i(\mu_i - r_f) + r_f)] + J_{y_k}\alpha \\ & + J_P P\pi + \frac{1}{2} J_{WW} W^2 \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} + \frac{1}{2} J_{yy} s^2 + \frac{1}{2} J_{PP} \sigma_\pi^2 P^2 + J_{WP} W P \sum_{i=1}^{N} w_i \sigma_{i\pi} \\ & + J_{y_k W} W \sum_{i=1}^{N} w_i \sigma_{iy_k} + J_{y_k P} \sigma_{y_k \pi}\} \;, \end{aligned} \tag{78}$$

where π is the inflation rate in each country expressed in local units, σ_{ij} are the instantaneous covariances of the nominal rates of return on the various securities, σ_π^2 is the instantaneous variance of the inflation rate, α is the mean value of the state variable y_k, $\sigma_{i\pi}$ is the covariance between security i and the inflation rate π, and $\sigma_{y_k\pi}$ is the covariance between the state variable y_k and the inflation rate π.[29] Moreover, the homogeneity of degree zero of the function V implies that $J(W, P, y_k, t)$ and $C(W, P, y_k, t)$ that satisfy (78) must be homogeneous of degree zero in W and P: $J_P \equiv -(W/P)J_W$, $J_{PW} \equiv (-1/P)J_W - (W/P)J_{WW}$, $J_{PP} = 2(W/P^2)J_W + (W/P)^2 J_{WW}$. Hence, (78) can be rewritten as

$$\begin{aligned} 0 \;=\; & \underset{(C,\mathbf{w})}{\text{Max}} \{V(C, P, y_k, t) + J_t + J_W[-C + W(\sum_{i=0}^{N} w_i(\mu_i - r_f) + r_f)] + J_{y_k}\alpha - W\pi J_W \\ & + \frac{1}{2} J_{WW} W^2 \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} + \frac{1}{2} J_{yy} s^2 + W J_W \sigma_\pi^2 + \frac{1}{2}\sigma_\pi^2 W^2 J_{WW} - J_W W \sum_{i=1}^{N} w_i \sigma_{i_\pi} \\ & - W^2 J_{WW} \sum_{i=1}^{N} w_i \sigma_{i_\pi} + J_{y_k W} W \sum_{i=1}^{N} w_i \sigma_{iy_k} + J_{y_k P} \sigma_{y_k \pi}\} \;. \end{aligned} \tag{79}$$

Taking the first order conditions of (79) with respect to C and $\mathbf{w}$, I obtain

$$V_C \;=\; J_W \;, \text{ and} \tag{80}$$

$$0 \;=\; J_W(\mu_i - r_f) + W J_{WW} \sum_{j=1}^{N} w_j \sigma_{ij} - J_W \sigma_{i_\pi} - W J_{WW} \sigma_{i\pi} + J_{y_k W} \sigma_{iy_k} \;. \tag{81}$$

[29]See Chapter 13 of Ingersoll (1987) for a definition of the dynamics of the state variables.

Solving (79) for the optimal portfolio of risky assets of investor l directly in vector notation and reintroducing the time dimension, I obtain (32)

$$\mathbf{w}_t^l = \alpha^l \mathbf{\Sigma}_{rrt}^{-1}[E_t(\mathbf{r}_{t+1}) - r_{ft}\mathbf{1}] + (1-\alpha^l)\mathbf{\Sigma}_{rrt}^{-1}\mathbf{s}_{r\pi t}^l + \beta^l \mathbf{\Sigma}_{rrt}^{-1}\mathbf{s}_{ry_k t}^l \ , \tag{82}$$

where $\mathbf{s}_{ry_k t}^l$ and $\mathbf{s}_{r\pi t}^l$ are the vectors of time-varying covariances between each country's security returns and the k-th state variable and level of inflation respectively. Moreover, I define $\alpha^l \equiv -\frac{J_{Wt}^l}{J_{WWt}^l W_t^l}$ and $\beta^l \equiv (\alpha^l)^2 \times \frac{J_{Wy_k t}^l}{W_t^l}$.

Appendix C

While the minimum-variance pricing kernel, $q_{t+1}^\star$, satisfies the law of one price (equation (1)), in general it does not satisfy the no-arbitrage condition, $q_{t+1}^\star > 0$. Nonetheless, as in HJ, I can extend the analysis to take this restriction into account.

Let $\tilde{\boldsymbol{\alpha}}_t$ denote an $N \times 1$ coefficient vector, and define $\tilde{q}_{t+1} \equiv [1 - (\mathbf{r}_{t+1} - \mathrm{E}_t(\mathbf{r}_{t+1}))^\top \tilde{\boldsymbol{\alpha}}_t]^+ \equiv \max\{1 - (\mathbf{r}_{t+1} - \mathrm{E}_t(\mathbf{r}_{t+1}))^\top \tilde{\boldsymbol{\alpha}}_t, 0\}$. Assume

$$\mathrm{E}_t(\tilde{q}_{t+1}\mathbf{r}_{t+1}) = r_{ft}\mathbf{1} \ . \tag{83}$$

The random variable $\tilde{q}_{t+1}$ has the smallest variance among all nonnegative random variables satisfying restriction (83).

Consider the risk premium $\tilde{\lambda}_{kt}$ assigned by $\tilde{q}$. I can write

$$\begin{aligned} \tilde{\lambda}_{kt} &\equiv -\mathrm{E}_t[(\tilde{q}_{t+1} - 1)y_{k,t+1}] \\ &= -\mathrm{E}_t[(\tilde{q}_{t+1} - 1)y_{k,t+1}^\star] - \mathrm{E}_t[(\tilde{q}_{t+1} - 1)(y_{k,t+1} - y_{k,t+1}^\star)]\tilde{\lambda}_{kt} \\ &= \lambda_{kt}^\star - \mathrm{E}_t[(\tilde{q}_{t+1} - 1)(y_{k,t+1} - y_{k,t+1}^\star)] \ , \end{aligned} \tag{84}$$

where, in general, $\mathrm{E}_t[(\tilde{q}_{t+1} - 1)(y_{k,t+1} - y_{k,t+1}^\star)] \neq 0$. Hence, when the positivity restriction is imposed, the risk premium assigned by the minimum-variance kernel differs from the mean cash flow generated by the hedging portfolio by the quantity $-\mathrm{E}_t[(\tilde{q}_{t+1} - 1)(y_{k,t+1} - y_{k,t+1}^\star)]$. If $\tilde{q}_{t+1}$ is volatile, and if $y_{k,t+1}$ is mimicked poorly by its nearest hedge, then there is the potential for the discrepancy to be substantial.

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Table I
Summary Statistics of Asset Returns

I report summary statistics of monthly returns on the equity indices of four countries from MSCI. Indices include dividends. The sample covers the period April 1970 through October 1998 (343 observations). All returns are in percentage points per month and are denominated in U.S. dollars. "Corr_τ" denotes the autocorrelation coefficient of order τ. "Q_{36}" denotes the Ljung–Box Q statistics of order 36 (p–values in parenthesis).

Panel A: Means, Standard Deviations, and Autocorrelations

Country	Mean	Std. Dev.	Corr_1	Corr_2	Corr_3	Corr_4	Corr_{12}	Corr_{24}	Corr_{36}	Q_{36}
United States	1.1113	4.4171	0.002	-0.034	0.007	-0.018	0.048	0.005	-0.033	27.586 (0.842)
United Kingdom	1.3304	7.0525	0.085	-0.101	0.052	0.013	-0.023	0.047	-0.026	48.085 (0.086)
Japan	1.2296	6.6757	0.091	-0.022	0.078	0.043	0.032	0.006	0.033	46.582 (0.111)
Germany	1.2099	5.9069	-0.017	-0.013	0.052	0.067	-0.017	0.022	0.029	41.417 (0.246)

Panel B: Correlation Matrix

Country	United States	United Kingdom	Japan	Germany
United States	1.000	0.505	0.264	0.367
United Kingdom		1.000	0.358	0.425
Japan			1.000	0.366
Germany				1.000

Table II
Summary Statistics of Economic Variables

I report summary statistics of economic risk factors for the United States, the United Kingdom, Japan and Germany. The sample covers the period April 1970 through October 1998 (343 observations). $INFL$ denotes the unexpected rate of inflation (percentage points per month). $SPOT$ denotes the change in the logarithm of the spot exchange rate (percentage points per month). $WLDMK$ denotes the rate of return on the world market index from MSCI (percentage points per month). "Corr_τ" denotes the autocorrelation coefficient of order τ. "Q_{36}" denotes the Ljung–Box Q statistics of order 36 (p–values in parenthesis).

Panel A: Means, Standard Deviations, and Autocorrelations

Variable	Mean	Std. Dev.	Corr_1	Corr_2	Corr_3	Corr_4	Corr_{12}	Corr_{24}	Corr_{36}	Q_{36}
INFL_{US}	-0.0002	0.2406	0.160	-0.023	-0.112	-0.154	0.167	0.093	0.138	95.189 (0.000)
INFL_{UK}	-0.0001	0.6130	0.164	-0.019	-0.061	-0.119	0.475	0.418	0.387	295.03 (0.000)
INFL_{JAP}	0.0004	0.6770	0.100	-0.220	-0.060	-0.013	0.439	0.397	0.378	373.09 (0.000)
INFL_{GER}	-0.0007	0.3129	0.188	0.051	-0.019	-0.082	0.299	0.094	0.165	134.34 (0.000)
$\text{SPOT}_{\$/GBP}$	0.1049	3.0593	0.086	0.025	-0.019	0.012	-0.011	-0.038	-0.023	33.924 (0.568)
$\text{SPOT}_{\$/YEN}$	-0.3288	3.3611	0.080	0.033	0.048	0.039	0.073	-0.045	-0.085	37.342 (0.407)
$\text{SPOT}_{\$/DM}$	-0.2312	3.2740	0.036	0.070	0.014	-0.005	-0.007	0.004	0.017	32.180 (0.651)
WLDMK	1.0551	4.1392	0.068	-0.055	0.013	-0.021	0.053	0.045	-0.019	45.179 (0.140)

Panel B: Correlation Matrix

Variable	INFL_{US}	INFL_{UK}	INFL_{JAP}	INFL_{GER}	$\text{SPOT}_{\$/GBP}$	$\text{SPOT}_{\$/YEN}$	$\text{SPOT}_{\$/DM}$	WLDMK
INFL_{US}	1.000	0.057	0.141	0.157	0.016	0.013	0.095	-0.196
INFL_{UK}		1.000	0.337	0.224	-0.089	-0.015	-0.081	0.110
INFL_{JAP}			1.000	0.082	-0.009	0.029	-0.014	-0.008
INFL_{GER}				1.000	0.033	0.082	0.097	0.019
$\text{SPOT}_{\$/GBP}$					1.000	0.472	0.650	-0.261
$\text{SPOT}_{\$/YEN}$						1.000	0.578	-0.310
$\text{SPOT}_{\$/DM}$							1.000	-0.228
WLDMK								1.000

Table III
Summary Statistics of Instrumental Variables

I report summary statistics of the intruments used in the analysis. The sample covers the period March 1970 through September 1998 (343 observations). $DINFLUS$ denotes the lagged change in the U.S. rate of inflation. $EURO$ denotes the Eurodollar deposit rate. $USDIVYLD$ denotes the U.S. dividend yield in excess of the Eurodollar rate. $WLDMK$ denotes the world rate of return. $DEFPREM$ represents the U.S. default premium. All variables are expressed in percentage points per month. "Corr_τ" denotes the autocorrelation coefficient of order τ. "Q_{36}" denotes the Ljung–Box Q statistics of order 36 (p–values in parenthesis).

Panel A: Means, Standard Deviations, and Autocorrelations

Variable	Mean	Std. Dev.	Corr_1	Corr_2	Corr_3	Corr_4	Corr_{12}	Corr_{24}	Corr_{36}	Q_{36}
DINFLUS	-0.0012	0.2719	-0.355	-0.054	-0.033	-0.077	0.142	0.157	0.169	140.350 (0.000)
EURO	0.6503	0.2780	0.967	0.920	0.879	0.844	0.667	0.343	0.151	4051.0 (0.000)
USDIVYLD	-0.3353	0.2172	0.950	0.880	0.822	0.771	0.543	0.150	-0.088	2768.6 (0.000)
WLDMK	1.1672	4.1166	0.068	-0.035	0.015	-0.024	0.062	0.046	-0.016	46.196 (0.119)
DEFPREM	0.0141	1.1768	-0.198	-0.036	-0.005	0.013	-0.067	-0.054	0.028	65.046 (0.002)

Panel B: Correlation Matrix

Variable	DINFLUS	EURO	USDIVYLD	WLDMK	DEFPREM
DINFLUS	1.000	-0.008	0.000	-0.075	0.098
EURO		1.000	-0.952	-0.126	-0.046
USDIVYLD			1.000	0.131	0.077
WLDMK				1.000	-0.003
DEFPREM					1.000

Table IV
Instruments Selection and Heteroskedasticity

T-statistics, in parentheses, are adjusted for heteroskedasticity and serial correlation.

Panel A: Slope Estimates of Mean Equations

Var	CONST	JAN	DINFLUS	EURO	USDIVYLD	WLDMK	DEFPREM
United States	0.971 (4.099)	1.716 (1.696)	−0.408 (−1.759)	1.266 (1.453)	1.605 (1.893)	−0.101 (−0.349)	0.231 (0.882)
United Kingdom	1.076 (2.976)	3.114 (1.393)	−0.469 (−1.294)	2.828 (2.468)	3.408 (2.964)	0.086 (0.194)	0.084 (0.150)
Japan	1.193 (3.236)	0.448 (0.371)	−0.772 (−2.229)	2.273 (2.137)	2.835 (2.677)	0.629 (1.508)	0.154 (0.392)
Germany	1.243 (3.781)	−0.410 (−0.373)	−0.659 (−2.051)	0.457 (0.476)	0.863 (0.898)	0.161 (0.434)	0.574 (1.475)

Panel B: Slope Estimates of Variance Equations

Var	CONST	JAN	DINFLUS	EURO	USDIVYLD	WLDMK	DEFPREM
United States	3.192 (21.212)	1.062 (1.739)	0.105 (0.840)	0.039 (0.070)	−0.224 (−0.414)	−0.472 (−2.739)	−0.359 (−2.438)
United Kingdom	4.809 (20.170)	1.521 (0.847)	0.098 (0.427)	1.587 (1.848)	1.362 (1.578)	−0.079 (−0.256)	−0.375 (−0.887)
Japan	5.083 (22.131)	−0.528 (−0.709)	−0.364 (−1.595)	−1.262 (−1.956)	−1.194 (−1.867)	−0.016 (−0.057)	−0.056 (−0.221)
Germany	4.458 (21.111)	−0.252 (−0.351)	0.048 (0.238)	−0.825 (−1.361)	−1.028 (−1.712)	−0.216 (−0.932)	0.106 (0.423)

Panel C: Average Slope Estimates and Differences in Slope Estimates

Var	DINFLUS	EURO	USDIVYLD	WLDMK	DEFPREM
$\bar{\mu}_r$	−0.577 (−2.441)	1.706 (2.360)	2.178 (3.051)	0.194 (0.712)	0.261 (0.876)
$\bar{v}_r$	−0.028 (−0.222)	−0.115 (0.263)	−0.271 (−0.630)	−0.196 (−1.181)	−0.171 (−0.967)
$\bar{\mu}_r - \bar{v}_r$	−0.549 (−1.972)	1.821 (2.095)	2.449 (2.855)	0.390 (1.361)	0.432 (1.499)

Table V
Economic Risk Premia: $q^{\star}$ Approach

I report coefficients of the economic risk premia on the following eight economic factors: $INFL_{US}$, $INFL_{UK}$, $INFL_{JAP}$, $INFL_{GER}$, $SPOT_{\$/GBP}$, $SPOT_{\$/YEN}$, $SPOT_{\$/DM}$, $WLDMK$. T-statistics, in parentheses, are adjusted for heteroskedasticity and serial correlation. The t-statistics refer to the case where the composition of $q^{\star}$ is estimated outside and inside of the GMM algorithm, respectively. This table presents estimates of the conditional premia when the set of returns is augmented to include managed portfolios. In this case, note that the intercept can be interpreted as the unconditional risk premium with the larger set of securities.

Conditional Risk Premia

Risk Premia	$INFL_{US}$	$INFL_{UK}$	$INFL_{JAP}$	$INFL_{GER}$	$SPOT_{\$/GBP}$	$SPOT_{\$/YEN}$	$SPOT_{\$/DM}$	$WLDMK$
$CONST$	−0.0128 (−0.61) (−0.45)	−0.0198 (−0.94) (−0.85)	−0.0145 (−0.73) (−0.62)	0.0102 (0.45) (0.48)	−0.0317 (−1.52) (−0.95)	−0.0233 (−1.10) (−0.63)	−0.0275 (−1.12) (−0.75)	0.1171 (3.52) (2.28)
JAN	−0.0680 (−1.84) (−1.54)	0.0329 (0.81) (1.11)	0.0258 (1.16) (0.93)	0.0276 (0.64) (1.15)	−0.0035 (−0.17) (−0.12)	−0.0016 (−0.11) (−0.05)	−0.0222 (−0.87) (−0.57)	0.0895 (1.59) (1.56)
$DINFLUS$	0.0077 (0.26) (0.17)	0.0069 (0.30) (0.20)	0.0432 (1.49) (1.06)	0.0565 (1.59) (1.43)	0.0203 (0.89) (0.74)	0.0381 (1.41) (1.22)	0.0457 (1.28) (1.35)	−0.1249 (−2.37) (−2.20)
$EURO$	−0.0149 (−0.23) (−0.18)	0.0231 (0.34) (0.35)	0.0313 (0.53) (0.41)	−0.0948 (−1.25) (−1.13)	−0.0666 (−0.98) (−0.72)	−0.3286 (−3.45) (−2.09)	−0.1126 (−1.51) (−1.02)	0.2888 (2.65) (1.67)
$USDIVYLD$	−0.0570 (−0.89) (−0.72)	0.1027 (1.48) (1.48)	0.0433 (0.66) (0.52)	−0.0973 (−1.11) (−1.04)	−0.1472 (−1.92) (−1.50)	−0.3706 (−3.80) (−2.44)	−0.1827 (−2.16) (−1.56)	0.4617 (3.80) (2.73)
$WLDMK$	−0.0280 (−1.07) (−0.88)	0.0189 (0.64) (0.67)	−0.0542 (−1.58) (−1.57)	−0.0062 (−0.35) (−0.23)	−0.0416 (−1.42) (−1.36)	−0.0449 (−1.78) (−1.20)	−0.0420 (−1.31) (−1.20)	0.0331 (0.87) (0.60)
$DEFPREM$	0.0108 (0.32) (0.29)	−0.0254 (−0.70) (−0.84)	−0.0508 (−1.60) (−1.74)	−0.0344 (−0.84) (−0.70)	−0.0201 (−0.78) (−0.60)	−0.0589 (−1.88) (−1.34)	−0.0708 (−1.66) (−1.39)	0.0527 (0.82) (0.91)

Table VI
Economic Risk Premia: $y^\star$ Approach

I report coefficients of the economic risk premia on the following eight economic factors: $INFL_{US}$, $INFL_{UK}$, $INFL_{JAP}$, $INFL_{GER}$, $SPOT_{\$/GBP}$, $SPOT_{\$/YEN}$, $SPOT_{\$/DM}$, $WLDMK$. T-statistics, in parentheses, are adjusted for heteroskedasticity and serial correlation. The t-statistics refer to the case where the composition of $q^\star$ is estimated outside and inside of the GMM algorithm, respectively. This table presents estimates of the conditional premia when the set of returns is augmented to include managed portfolios. In this case, note that the intercept can be interpreted as the unconditional risk premium with the larger set of securities.

Conditional Risk Premia

Risk Premia	$INFL_{US}$	$INFL_{UK}$	$INFL_{JAP}$	$INFL_{GER}$	$SPOT_{\$/GBP}$	$SPOT_{\$/YEN}$	$SPOT_{\$/DM}$	$WLDMK$
$CONST$	−0.0128 (−0.57) (−0.45)	−0.0198 (−1.05) (−0.85)	−0.0145 (−0.98) (−0.62)	0.0102 (0.85) (0.48)	−0.0317 (−0.95) (−0.95)	−0.0233 (−0.65) (−0.63)	−0.0275 (−0.81) (−0.75)	0.1171 (2.28) (2.28)
JAN	−0.0198 (−0.59) (−0.46)	0.0244 (0.68) (0.65)	−0.0252 (−1.40) (−0.97)	0.0300 (1.79) (1.24)	0.0108 (0.34) (0.31)	0.0055 (0.20) (0.17)	−0.0013 (−0.04) (−0.03)	0.0859 (1.51) (1.49)
$DINFLUS$	0.0361 (1.09) (0.92)	−0.0358 (−1.32) (−1.16)	0.0075 (0.50) (0.33)	−0.0102 (−0.74) (−0.47)	0.0493 (1.51) (1.33)	0.0931 (2.41) (2.20)	0.0830 (2.55) (2.30)	−0.1249 (−2.24) (−2.23)
$EURO$	−0.0805 (−1.15) (−0.89)	0.0828 (1.22) (1.01)	0.0320 (0.62) (0.42)	0.0310 (0.80) (0.47)	−0.2296 (−2.31) (−2.21)	−0.2157 (−1.92) (−1.92)	−0.1636 (−1.58) (−1.52)	0.3384 (1.97) (1.97)
$USDIVYLD$	−0.1200 (−1.72) (−1.35)	0.1027 (1.52) (1.24)	0.0491 (0.90) (0.61)	0.0091 (0.23) (0.15)	−0.2806 (−2.79) (−2.72)	−0.2693 (−2.41) (−2.43)	−0.2259 (−2.22) (−2.11)	0.5189 (3.07) (3.07)
$WLDMK$	0.0089 (0.31) (0.27)	0.0298 (1.18) (1.06)	−0.0007 (−0.03) (−0.02)	0.0097 (0.68) (0.52)	−0.0501 (−1.23) (−1.28)	−0.0914 (−1.84) (−1.92)	−0.0444 (−1.06) (−1.06)	0.0364 (0.64) (0.63)
$DEFPREM$	−0.0674 (−1.83) (−1.52)	−0.0076 (−0.27) (−0.22)	−0.0231 (−1.22) (−0.80)	−0.0184 (−0.98) (−0.68)	−0.0439 (−0.96) (−0.93)	−0.0358 (−0.72) (−0.73)	−0.0675 (−1.28) (−1.26)	0.0600 (1.00) (1.00)

Table VII
Unconditional Risk Premia, Volatility of Hedging Portfolios and Sharpe Ratios

I report unconditional economic risk premia (λ_0), volatilities of the mimicking portfolios' excess cash flows (v_0) and Sharpe ratios ($S_{y_k^?}$) commanded by the following economic factors: $INFL_{US}$, $INFL_{UK}$, $INFL_{JAP}$, $INFL_{GER}$, $SPOT_{\$/GBP}$, $SPOT_{\$/YEN}$, $SPOT_{\$/DM}$, $WLDMK$. T-statistics, in parentheses, are adjusted for heteroskedasticity and serial correlation. The t-statistics refer to the case where the composition of $y^\star$ is estimated outside the GMM algorithm.

Sharpe Ratios	λ_0	v_0	$S_{y_k^?}$
$INFL_{US}$	−0.0128 (−0.56)	0.4279 (16.06)	−0.0301 (−0.56)
$INFL_{UK}$	−0.0198 (−1.03)	0.3563 (9.61)	−0.0558 (−0.98)
$INFL_{JAP}$	−0.0145 (−0.97)	0.2782 (17.60)	−0.0522 (−0.97)
$INFL_{GER}$	0.0102 (0.84)	0.2262 (16.17)	0.0450 (0.85)
$SPOT_{\$/GBP}$	−0.0317 (−0.93)	0.6336 (21.75)	−0.0501 (−0.93)
$SPOT_{\$/YEN}$	−0.0233 (−0.63)	0.6846 (20.55)	−0.0339 (−0.63)
$SPOT_{\$/DM}$	−0.0275 (−0.79)	0.6447 (18.63)	−0.0427 (−0.79)
$WLDMK$	0.1170 (2.18)	0.9947 (18.93)	0.1177 (2.10)

Table VIII
Tests of IS-CAPM, I-CAPM and II-CAPM

I perform conditional tests of the International Static CAPM (IS-CAPM), the International CAPM in presence of deviations from PPP (I-CAPM (PPP)), the International CAPM in presence of currency risk (I-CAPM (SPOT)) and the international intertemporal CAPM (II-CAPM(PPP) and II-CAPM(SPOT)) by using region subset tests (χ^2), Hansen-Jagannathan variance bounds (HJV), and Hansen-Jagannathan distance measures (HJD), respectively. The benchmark standard deviation of the scaled unrestricted normalized minimum–variance kernel is 0.3865. The standard deviations of the restricted normalized minimum–variance kernels with conditional information are reported in column two. With $q^\star_{IS}$, $q^\star_{I-PPP}$, $q^\star_{I-SPOT}$, $q^\star_{II-PPP}$, and $q^\star_{II-SPOT}$ I denote the normalized minimum–variance kernels with conditional information for the IS-CAPM, I-CAPM (PPP), I-CAPM(SPOT), II-CAPM (PPP), and II-CAPM(SPOT), respectively.

Models	With conditioning information		
Statistics	$\chi^2_{(dof)}$ $(p-value)$	HJV $(t-stat.)$ $std(q^\star_r)$ $r=IS,I-PPP,I-SPOT,II-PPP,II-SPOT$	HJD $(t-stat.)$
IS–CAPM	$33.09_{(27)}$ (0.19)	−0.2630 (−10.68) $std(q^\star_{IS})$=0.1231	0.3659 (12.62)
I-CAPM(PPP)	$31.16_{(23)}$ (0.12)	−0.2038 (−7.48) $std(q^\star_{I-PPP})$=0.1825	0.3402 (12.47)
I-CAPM (SPOT)	$31.17_{(24)}$ (0.14)	−0.2595 (−10.46) $std(q^\star_{I-SPOT})$=0.1266	0.3646 (12.43)
II-CAPM(PPP)	$30.37_{(19)}$ (0.05)	−0.1826 (−7.26) $std(q^\star_{II-PPP})$=0.2037	0.3280 (12.28)
II-CAPM(SPOT)	$30.42_{(20)}$ (0.06)	−0.2549 (−10.21) $std(q^\star_{II-SPOT})$=0.1313	0.3630 (12.35)

Table IX
Differences in Hansen-Jagannathan Variance Bounds and Distance Measures

I test whether the pricing implications delivered by the International Static CAPM (IS-CAPM), the International CAPM in presence of deviations from PPP (I-CAPM (PPP)), the International CAPM in presence of currency risk (I-CAPM (SPOT)) and the international intertemporal CAPM (II-CAPM(PPP) and II-CAPM(SPOT)) are statistically different from each other. The estimates of the differences in HJV and HJD measures are obtained by exactly identified GMM. T-statistics are obtained using the delta method.

Statistics	$(HJV_r - HJV_s)$ $(t-stat.)$ $std(q_i^?)$ $r,s = IS, I-PPP, I-SPOT, II-PPP, II-SPOT, r \neq s$	$(HJD_r - HJD_s)$ $(t-stat.)$
IS–CAPM – I-CAPM(PPP)	0.0592 (6.92)	−0.0256 (−2.26)
IS–CAPM – I-CAPM(SPOT)	0.0035 (1.85)	−0.0012 (−0.62)
IS–CAPM – II-CAPM(PPP)	0.0804 (9.23)	−0.0378 (−2.33)
IS–CAPM – II-CAPM(SPOT)	0.0081 (2.89)	−0.0028 (−0.79)
I-CAPM(PPP) – I-CAPM (SPOT)	0.0557 (6.32)	−0.0244 (−2.09)
I-CAPM(PPP) – II-CAPM(PPP)	0.0212 (2.89)	−0.0122 (−1.57)
I-CAPM(PPP) – II-CAPM(SPOT)	−0.0511 (−6.03)	0.0227 (2.00)
I-CAPM(SPOT) – II-CAPM(PPP)	0.0770 (8.52)	−0.0366 (−2.19)
I-CAPM(SPOT) – II-CAPM(SPOT)	0.0047 (2.20)	−0.0016 (−0.59)
II-CAPM(PPP) – II-CAPM(SPOT)	0.0723 (8.13)	−0.0350 (−2.11)

Table X
Hedging Demands

I report the unconditional normalized hedging demands for global market, inflation, and foreign exchange risks. T-statistics, in parentheses, are adjusted for heteroskedasticity and serial correlation.

Hedging Demands	$\boldsymbol{\alpha}$
$SPOT_{\$/GBP}$	0.129 (5.26)
$SPOT_{\$/YEN}$	0.038 (1.76)
$SPOT_{\$/DM}$	0.058 (2.38)
$WLDMK$	0.109 (10.04)
$INFL_{US}$	0.222 (8.91)
$INFL_{UK}$	0.364 (12.32)
$INFL_{JAP}$	0.173 (4.86)
$INFL_{GER}$	−0.093 (−3.04)