FEDERAL RESERVE BANK *of* ATLANTA

Housing Tenure and Wealth Distribution in Life-Cycle Economies

Pedro Silos

Working Paper 2005-25
October 2005

WORKING PAPER SERIES

FEDERAL RESERVE BANK *of* ATLANTA | WORKING PAPER SERIES

Housing Tenure and Wealth Distribution in Life-Cycle Economies

Pedro Silos

Working Paper 2005-25
October 2005

Abstract: Common practice in the housing and wealth distribution literature has proceeded as if the modeling of housing rental markets was unnecessary due to renters' relative low levels of wealth and the small fraction they represent in the total population. This paper shows, however, that their inclusion matters substantially when dealing with wealth concentration over the life cycle. Renters are concentrated in the poorer and younger groups. This concentration results in a pattern of housing wealth concentration over an agent's life that is decreasing, with a slope as steep as that of nonhousing (or financial) wealth. The author constructs an overlapping-generations economy with a housing rental market that is consistent with this fact.

JEL classification: E21, D30

Key words: wealth concentration, life cycle, housing

This paper is part of my Ph.D. dissertation, completed while I was at the University of Iowa. I would like to thank Beth Ingram, Paul Weller, Steve Williamson, and especially B. Ravikumar and Charles Whiteman for their help. The views expressed here are the author's and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System. Any remaining errors are the author's responsibility.

Please address questions regarding content to Pedro Silos, Research Department, Federal Reserve Bank of Atlanta, 1000 Peachtree St. NE, 30309-4470, 404-498-8630 (phone), 404-498-8956 (fax), Pedro.Silos@atl.frb.org.

Housing Tenure and Wealth Distribution in Life-Cycle Economies

1 Introduction

The comparison between the characteristics of the wealth distribution that results from equilibrium models and its properties in US data has been the subject of an extensive literature in macroeconomics. Aside from a few exceptions, housing wealth has rarely been modeled explicitly but has been lumped together with other assets in the "capital" stock, following the tradition of one-sector stochastic growth models. Even when housing has been separated from other assets (such as in Díaz and Luengo-Prado (2003) or Gruber and Martin (2003)) the existence of a real estate rental market has been completely ignored. While it is true that renters are a minority in the total population, they represent an overwhelming majority among the poorer and younger age groups. As I show below, their inclusion in dynamic macroeconomic models has important implications for the wealth concentration over the life cycle: Gini coefficients for housing wealth are decreasing with age, with a slope almost as large as that of financial wealth. Previous work (Silos (2005)) has shown that the Gini coefficients of housing wealth when economies do not allow for renting housing services display little variation over the life cycle.

The modeling strategy is close to Huggett (1996), introducing some of the elements that he considers necessary for models to succeed in matching wealth distribution moments, but separating housing from the remaining capital stock and allowing agents to consume housing services through renting. It is an overlapping generations economy in which agents are subject to idiosyncratic income risk and death uncertainty. Income is taxed and the proceeds finance a pay-as-you-go Social Security system which provides pensions for retired workers.

In addition to the already mentioned work by Díaz and Luengo-Prado (2003) and Gruber and Martin (2003) in which they introduce durable goods into Aiyagari's (1994) model, there are two related studies that are worth mentioning for their close relation to the research presented here. The first is an article by Gervais (2002) on which the modeling of the housing rental market in this paper builds. He presents a deterministic economy in which agents have the choice of renting housing services or owning real estate capital, and analyzes the welfare implications of different taxation schemes. The second is a manuscript by Platania and Schlagenhauf (2000). They construct a life-cycle model where agents are subject to idiosyncratic risk and study the asset allocation problem between business capital and housing. These agents also have the choice between renting housing services in a rental market and owning housing capital. In their model, individuals are constrained to hold a fixed amount of housing and it is therefore inappropriate to study wealth distribution, or even portfolio choice, issues.

2 The Model

The economy consists of a continuum of agents with a total measure of one that live for at most I periods. Agents are born with zero wealth, and work during the first T periods of their lives. Retirement is mandatory at the end of period T and people live off their accumulated wealth for the remaining $I - T$ periods.

Individuals maximize their expected lifetime utility over non-housing consumption (c) and housing services (s):

$$U(c, s) = E \sum_{i=1}^{I} \beta^{i-1} u(c_i, s_i) \tag{1}$$

In the previous expression, the time-discount factor is denoted by β [1].

[1] All generations discount the future at the same rate. The model with uncertain lifetimes is presented below.

2.1 Social Security

The government runs a "pay-as-you-go" system that taxes the younger generations (workers) and partly subsidizes consumption and investment expenditures of the older generations. The retirement benefits (b) consist of a fraction (replacement rate) ϕ of the average wage.

2.2 Technology

There is an aggregate technology operated by a representative firm that produces output in this economy using capital K and labor N:

$$Y = F(K, N) \tag{2}$$

This production function satisfies the usual properties, increasing in both arguments, strictly concave and homogeneous of degree one. Output can be costlessly allocated to consumption, business capital investment and investment in residential capital.

The technology for transforming residential capital (h) into housing services (s) is linear: $s_i = h_i$, where s_i is the amount of services enjoyed by an age-i individual. These services can be obtained by either owning residential capital or by renting them in a housing rental market. Both renting and owning are mutually exclusive. In an attempt to mimic fiscal policy in several developed nations, homeownership is "subsidized" in the form of deductible mortgage interest rate payments. Hence, in the absence of any additional friction agents would rather own than rent. However, housing capital is not perfectly divisible: if an agent wants to own she needs to buy a house of at least size $\underline{h}$. If she can not afford it, she must enjoy housing services by renting. The financing of a home purchase is done by entering in to a one period mortgage. Agents are only allowed to borrow at most a fraction $1 - \gamma$ of the value of the new home. Hence, we can think of γ representing the downpayment fraction. There is no difference in depreciation between rented capital and owned capital, with all housing capital depreciating at a rate of δ_h.

Agents supply inelastically whatever amount of time they are endowed with. However, they are subject to productivity shocks that alter their level of efficiency. The structure of financial markets is such that agents can not trade directly contingent claims to hedge against shocks to labor productivity and the smoothing of income fluctuations is done by adjusting the holdings of capital and residential stocks.

2.3 The Agent's Problem

Aside from the usual choices of allocating consumption and savings, and allocating total investment between residential and business capital, agents have to decide whether they want to rent housing services or they want to own residential capital stock. As usual, to finance both types of consumption, individuals obtain income by inelastically supplying one unit of labor and from renting capital.

Denote by $V_i(k, h\chi_O, \xi)$ the value function of an agent belonging to generation i that enters the current period. The state variables for this optimization problem are the level of capital holdings (k), the previous housing status described by the indicator variable χ_O and the house holdings h (in case she was a home owner), and the value of the productivity shock in the previous period (ξ). The consumer will compare the value of becoming a renter versus the value of purchasing a home. Denote these two values by V^R and V^O respectively. Then, for an age i individual:

$$V_i = max\{V_i^R, V_i^O\} \tag{3}$$

V^R will be determined by solving the problem:

$$V_i^R = \max_{a',s,c} \left\{ u(c,s) + \beta \int V_{i+1}(a', 0, \xi') d\Phi(\xi') \right\} \tag{4}$$

s.t.

$$c + ps + a' \leq (1 + r(1 - \tau))a + y + (1 - \delta_h)\chi_O h \tag{5}$$

and

$$a' \geq 0 \tag{6}$$

The functional equation (4) states that agents choose housing services, consumption and asset holdings to maximize lifetime utility. They take into account that they will enter next period with zero housing wealth and having rented in the present, hence the value of zero for χ_O. Equation 5 is the budget constraint. The sum of expenditures on consumption, rented housing services and investment cannot exceed the sum of capital income, labor income and depreciated real estate capital. Analogously, the value of becoming a homeowner is the solution to:

$$V_i^O = \max_{a', h', c} \left\{ u(c, s) + \beta \int V_{i+1}(a', h', \xi') d\Phi(\xi') \right\} \tag{7}$$

s.t.

$$c + h' + a' \leq (1 + r(1 - \tau))a + y + (1 - \delta_h)\chi_O h \tag{8}$$

$$s = h' \tag{9}$$

$$a' \geq -(1 - \gamma)h' \tag{10}$$

$$h' \geq \underline{h} \tag{11}$$

Sources of income other than from asset holdings are denoted by y, which equals b, the pension benefits, if the agent is retired and $(1 - \tau)we(i)\xi$ if the agent is of working age. Notation is standard. Prime variables denote next period values, w is the wage, and $e(age)$ is an age-specific efficiency factor. Expectations are taken with respect to the distribution of the productivity values ξ, denoted by Φ. The borrowing constraint (10) for the homeowners' problem is a downpayment constraint: the agent can not borrow more than a fraction equal to $(1-\gamma)$ of the house she wants to buy. In addition, the indivisibility constraint (11) states that any house the individual buys must have a minimum size of $\underline{h}$.

2.4 The Rental Market

The rental market is run by a rental agency/financial institution. In addition, this rental agency also takes deposits from investors and loans assets in the form of mortgages to potential homeowners. I have followed Gervais (2001) in modeling the banks/rental agencies as two-period lived institutions. Their problem is relatively simple. Depositors hold assets and obtain an interest rate of r. Using these deposits, banks loan money (also at a rate r) to home buyers, and buy residential capital. This residential capital is in turn rented to individuals at a price p per unit. At the end of the second period they sell the undepreciated part of the residential stock to a new institution. Formally, their problem is simply:

$$\max_{\{B^b_{t+1},,S^b_{t+1},H^b_{t+1},,D^b_{t+1}\}} (pS^b_{t+1} - \delta_h H^b_{t+1} + rB^b_{t+1} - rD^b_{t+1}) \tag{12}$$

$$H^b_{t+1} + B^b_{t+1} \leq D^b_{t+1} \tag{13}$$

$$S^b_{t+1} \leq H^b_{t+1} \tag{14}$$

where H^b is the rented stock, D^b are deposits, B^b are loans and S^b are housing services provided through this rental agency. For this maximization problem to be well defined the following no-arbitrage condition needs to hold:

$$p = r + \delta_h \tag{15}$$

In words, this condition implies that faced with having one unit of resources to spend, an individual can take two different actions: the first, buy residential capital and rent it, paying a maintenance cost of δ_h and obtaining a revenue equal to p; and the second, open a deposit in the bank and obtaining a net revenue of r. These two actions must yield the same profit. This condition will guarantee that banks/rental agencies will be making zero profits in equilibrium.

2.5 Equilibrium

Denote by z the triple (a, h, ξ) and by μ_t the measure of agents of age t. A stationary equilibrium is a set of decision rules for consumption $c(z,t)$, financial asset holdings $a(z,t)$, real estate holdings $h(z,t)$ and housing services rented $x(z,t)$, prices w,r, age-dependent distributions across wealth and income levels $\Psi_1, \ldots, \Psi_I$, a tax rate τ, level of benefits b, and aggregate quantities K, N, D^b, H^b such that:

1. Decision rules are optimal.

2. Prices are determined competitively:

$$r = F_1(K, N) - \delta_k \tag{16}$$

$$w = F_2(K, N) \tag{17}$$

3. Asset markets clear:

$$K = \sum_{t=1}^{T+TR} \mu_t \int_Z a(z,t))d\Psi_t - H^b \tag{18}$$

$$D^b = \sum_{t=1}^{T+TR} \mu_t \int_Z a(z,t)d\Psi_t - K \tag{19}$$

4. Rental market clears:

$$H^b = \sum_{t=1}^{T+TR} \mu_t \int_Z x(z,t)d\Psi_t \tag{20}$$

5. Goods market clears:

$$F(K, N) = \sum_{t=1}^{T+TR} \mu_t \int_Z (c(z,t) + \delta_h h(z,t))d\Psi_t + \delta_k K + \delta_h H^b \tag{21}$$

6. The government balances its budget in every period.

The equilibrium definition is standard and most equations are straightforward. However, two equations deserve further explanation: the capital market clearing condition (18) and the rental market clearing condition (20). To compute aggregate capital K the stock of rented residential capital needs to be subtracted from the total amount of financial assets held by individuals. Notice that a can be negative therefore it includes the amount agents borrow from banks. The rental market clearing condition just states that the amount of residential stock owned by financial institutions H^b needs to be equal to the amount of services rented by individuals.

3 Parameterization

3.1 Demographics

The model period is set to be equivalent to 5 years. Agents are assumed to be born being 21 years old. Individuals live for $I = 12$ periods (60 years), and retire after 9 periods.

3.2 Social Security

The level of Social Security benefits is determined in equilibrium after specifying a rate taxed to workers. This rate, denoted ϕ is set at a value of 0.1. This number is taken from Huggett (1996) and it is consitent with the average of contributions to social security as a fraction of labor income, as reported in the Social Security Bulletin.

3.3 Preferences, Technology and Endowments

The utility function chosen is of the constant relative risk aversion class, standard in the wealth distribution literature, with a Cobb-Douglas aggregator between housing services and non-housing consumption:

$$u(c,s) = \frac{(c^{\theta} s^{1-\theta})^{1-\sigma}}{1-\sigma} \quad (22)$$

Parameters in the model were chosen to match some features of the United States economy during the last forty years. The discount factor β was chosen to match a steady-state interest rate of about 4% per year. This value was chosen by Fernández-Villaverde and Krueger (2002), although most likely the return to "equity" in the data is somewhat larger. However, it is important to notice that technology in this model is riskless.

The parameter θ - the share of non-housing consumption in the utility function - was set at 0.8. This value is consistent with housing expenditures being about 20% in the Consumer Expenditures Survey (see Peterson (2003)). I will present results with several values of σ, the coefficient of relative risk aversion, which in turn will imply different values for β.

3.3.1 The Earnings Process

When looking at the properties of the wealth distribution, it is important to have an accurate approximation to the earnings process. Huggett (1996) estimates an AR process for the logarithm of the labor endowment:

$$z_t = \rho z_{t-1} + \epsilon_t \quad (23)$$

The disturbance term ϵ is distributed normally with mean zero and variance σ^2_{ϵ}. Huggett sets $\sigma^2_{\epsilon} = 0.045$, with a persistence parameter chosen so that the unconditional variance is equal to 0.38, which in turn implies a Gini coefficient in earnings of 0.42. This resulted in a value for ρ of 0.96.

For computational purposes I have approximated this process as a seven state Markov chain. In the Appendix I present a comparison of the moments implied by the continuous and the discrete state processes. The transformation of the continuous state process into its five-year equivalent was done prior to its conversion into a discrete state process.

In addition to this idiosyncratic productivity shock agents face an age-dependent efficiency profile $\{\eta_i\}_{i=1}^{I}$ used in Huggett and Ventura (1999) [2]. Hansen (1993) estimated median wage rates from the Current Population Survey (CPS) for different age groups. Huggett and Ventura used Hansen's estimates, and set them to be the wage corresponding to the age in the center of the group and linearly interpolated to obtain values for all ages. A plot of this efficiency profile is shown in Figure 1.

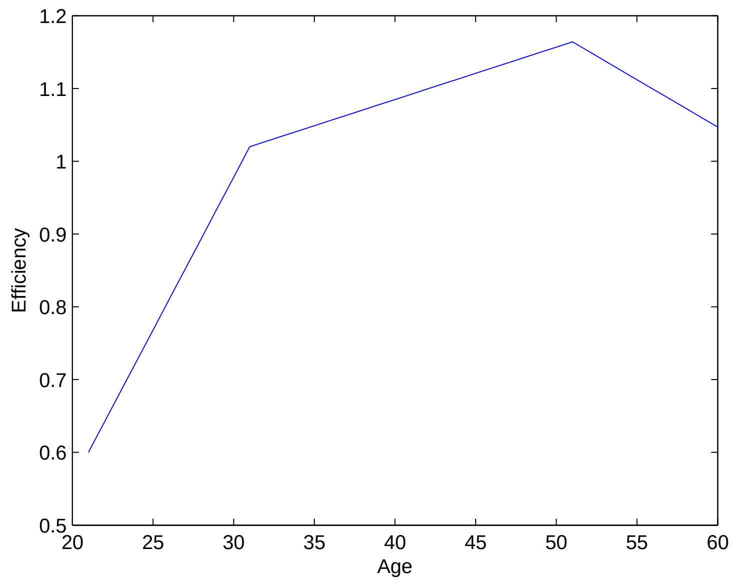

Figure 1: Efficiency Profile

3.4 Technology

Output is produced by combining capital and labor through a Cobb-Douglas production function:

[2] I thank Mark Huggett for making the data available.

$$Y = K^{\alpha} N^{1-\alpha} \tag{24}$$

The value chosen for α was 0.30 which implies a share of labor in total income of 0.70, roughly consistent with US data. The depreciation rates were obtained from the data using conditions relating investment, capital and output in the steady state. For both types of capital, business and residential, in a steady-state:

$$\frac{I_k/GDP}{K/GDP} = \delta_k \tag{25}$$

$$\frac{I_h/GDP}{H/GDP} = \delta_h \tag{26}$$

US data from 1964-2003 implies values for δ_h and δ_k of 4.3% and 9.4% per year. The equivalent five-year values were $\delta_h = 1-(1-0.043)^5 = 0.197$ and $\delta_k = 1-(1-0.094)^5 = 0.3895$.

The structure of the housing market implies that in order to be a home owner, the size of the purchase must be equal or larger than $\underline{h}$. The value for this parameter was chosen so that the model would deliver an aggregate homeownership rate of approximately 68%.

In the US economy a typical value for downpayment fractions of the house value at the time of the purchase is 20% (see Fernández-Villaverde and Krueger (2002)). For this reason the borrowing constraint is specified so that agents can borrow up to 80% of the house they want to buy: $1-\gamma = 0.8$.

Table 1 summarizes the parameterization for the model, stating values for parameters and their target/source. Details about the solution of the model are included in the Appendix, but the methods are fairly standard. Once decision rules are obtained, summary statistics are computed by simulating life-cycle paths for a large number of agents drawing shocks from the appropriate (discretized) distribution for productivity shocks.

Table 1: Summary of Parameter Values

Parameter / Variable	$\sigma = 1.5$	$\sigma = 2$	Target / Source
β	0.976	0.9677	r=4% per year
$\underline{h}$	0.276	0.276	Rate Owners=68%
ϕ	0.1	0.1	Huggett (1996)
θ	0.8	0.8	20% exp. in housing; Peterson (2003)
z	—	—	Huggett and Ventura (1999)
e	—	—	Huggett and Ventura (1999)
α	0.36	0.36	NIPA
δ_k	0.094	0.094	"
δ_h	0.043	0.043	"
$1-\gamma$	0.8	0.8	Fernández-Villaverde and Krueger (2002)

Table 2: US data (1964-2003)

Variable	Average
$(K+H)/GDP$	3.471
K/GDP	1.754
H/GDP	1.717
$K/(K+H)$	0.505
$H/(K+H)$	0.495
H_O/H_R	2.96

4 Results

Table 2 shows some aggregate annual statistics[3] for the United States economy during the period 1964-2003[4].

The table shows the importance of housing in the aggregate economy, with residential stocks representing half of the entire capital stock. The total capital to output ratio, $\frac{K+H}{GDP}$, seems somewhat larger than values previously reported, for example in Cooley and

[3]The appendix provides definitions of all variables used throughout the paper.
[4]The last entry of the table corresponds to data from 1987 to 2003.

Table 3: Model Output, Averages

Variable	Model Period, $\sigma = 1.5$	Model Year, $\sigma = 1.5$
$(K+H)/GDP$	0.837	4.186
K/GDP	0.594	2.519
H/GDP	0.330	1.667
$K/(K+H)$	0.554	0.554
$H/(K+H)$	0.446	0.446
H_O/H_R	4.652	4.652
Variable	Model Period, $\sigma = 2$	Model Year, $\sigma = 2$
$(K+H)/GDP$	0.846	4.230
K/GDP	0.508	2.539
H/GDP	0.338	1.697
$K/(K+H)$	0.553	0.553
$H/(K+H)$	0.447	0.447
H_O/H_R	4.673	4.673

Prescott (1995), but the definition of GDP does not include housing services. Housing services approximately represent 10% of total output. The last cell in Table 2 gives the ratio of the stock of residential capital that is owned to that which is rented. In the US economy the average for this ratio is 2.96.

Table 3 gives the model's values for some selected aggregate statistics, including those in Table 2. To facilitate the comparison I present the results also in their yearly equivalents. While the model overestimates the level of business capital accumulation (about 4.2 in the model versus 3.5 in the data), both the aggregate wealth composition and the ratio of residential stock to output are close to their empirical counterparts. Business capital is about 0.55 of total wealth and the residential stock is about 1.7 times GDP, roughly what is observed in the data.

Data on households' wealth come from the Survey of Consumer Finances (SCF) . The SCF has become the main source used by financial economists to address any question

Table 4: US Economy: Gini Coefficients

Variable	US Pop.	21-25	26-30	31-35	36-40	41-45	46-50
Non-Housing	0.750	1.069	0.902	0.834	0.747	0.720	0.720
Primary Residence	0.645	0.857	0.739	0.687	0.646	0.612	0.580
Variable	51-55	56-60	61-65	66-70	71-75	76-80	—
Non-Housing	0.700	0.696	0.704	0.655	0.648	0.675	—
Primary Residence	0.612	0.583	0.599	0.606	0.535	0.559	—

related to the composition of balance sheets in US households. I have used the 2001 version in which a total of 4,400 families were interviewed. The SCF gives great detail on the housing side of households' asset position. It provides responses about quantities owed from different mortgages, HELOCs, market values of primary residence, values of vacation homes and other real estate participations. Without a not so direct relation to the model presented in this paper, it gives information about types of mortgages (e.g. whether it is a mortgage from the Veterans' administration, the Federal Housing Administration, etc. . .), frequency of payments, real estate taxes, number of units in the lot, etc...

Table 4 presents data on Gini coefficients for financial assets net of housing and the value of the primary residence for the overall population and the different age groups. The Gini indices for non-housing wealth over the life cycle follow a decreasing pattern (a decreasing level of concentration), due to a large number of net borrowers in the younger age groups. It starts with a value larger than unity for the first age group (21-25), decreasing to about a value of 0.66 for the older generations. For a similar reason, the larger concentration of renters in the younger age groups, the Gini index for the value of the primary residence is also decreasing. Levels of concentration, however, are smaller than for non-housing wealth. The largest coefficient occurs also for the younger age group with a value of 0.86, decreasing to a value of about 0.55 for the older age groups. For the entire population the Gini for non-housing wealth is is 0.10 larger than for the value of the house (0.75 vs. 0.65). Figures 3 and 4 depict the Gini coefficients for the US economy and the model implied Gini indices for non-housing wealth and housing wealth respectively. The model is roughly consistent with the pattern of concentration of wealth over the life cycle for non-housing wealth, especially for the younger age groups. The Gini index for the 25-30 year-old individuals is about 0.985. It is decreasing over the first age groups as it is in the data, but understates the magnitude observed in the data for the agents of older generations by an average of 0.25-0.30. In addition, for the older generations it increases due to the larger proportion of renters, which increases the variance of housing wealth, hence increasing the Gini index. Finally, for the last age group the Gini coefficient is zero, given that everybody is a renter and everybody holds zero housing wealth.

Finally, the model replicates the pattern of homeownership rates observed in the data. This pattern is shown in Figure 2. In the data, the majority of renters is concentrated in the younger age groups with a proportion of only about 19% of homeowners. The fraction of homeowners increases steadily until age 55, remaining at a value of about 0.85 thereafter. The model replicates the lower value for the fraction of homeowners at the younger age groups and the steady increase until age 50, fluctuating about a value of 80%. In the last generation (not plotted) the percentage of renters is 100% given the absence of estate taxes and bequests.

4.1 Uncertain Lifetimes

In the previous section all generations discount the future at a constant rate β. However introducing mortality risk has been shown to have an important quantitative effect for the wealth distribution (Huggett (1996)) and the well known hump in life-cycle consumption (Feigenbaum (2005)). In this section I modify the model by introducing uncertain life-

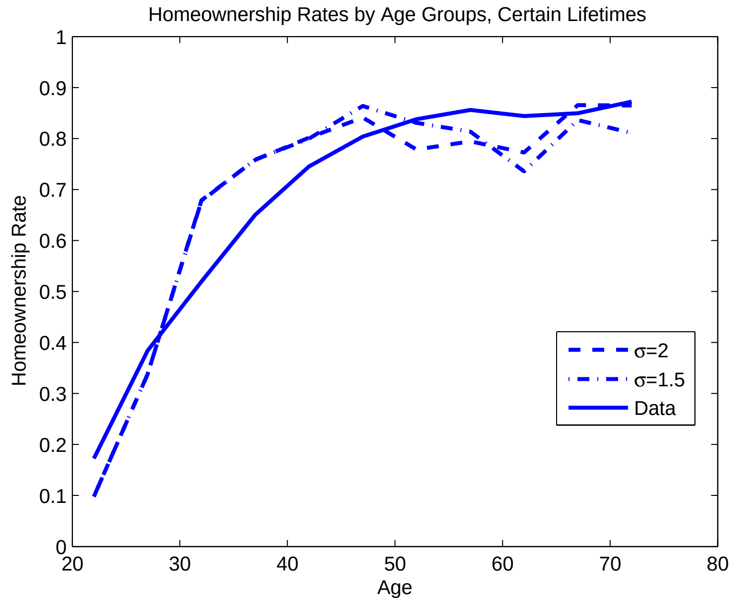

Figure 2: Homeownership Rates by Age Group

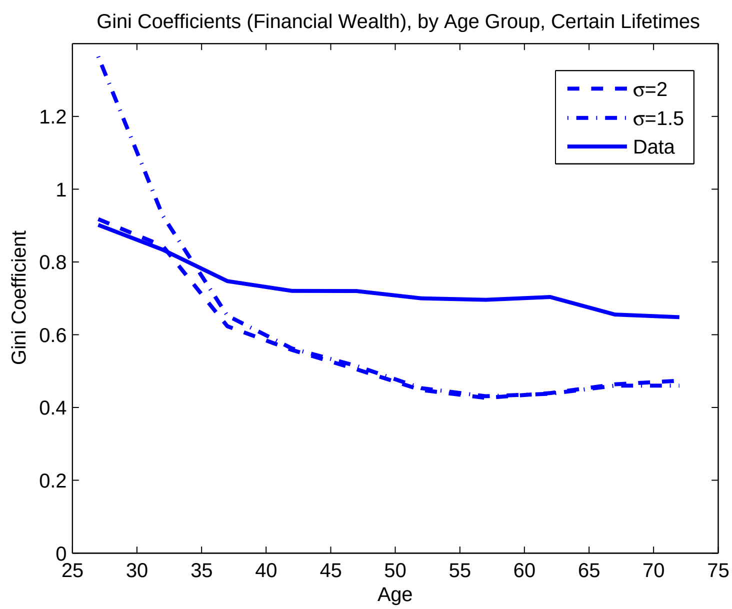

Figure 3: Gini Coefficients: Non-Housing Assets

times which decrease the discount factor for all agents, but proportionately more for the older generations. The survival probabilities are obtained from the US Census Bureau.

Figure 4: Gini Coefficients: Primary Residence

With this modification now individuals maximize:

$$U(c,s) = E\sum_{i=1}^{I} \beta^{i-1}\phi_{i+1}u(c_i, s_i) \tag{27}$$

In the above expression ϕ_{i+1} denotes the probability of surviving to age $i+1$ at the time of birth. The other characteristics of the model remain unchanged except for the fact that dead individuals leave accidental bequests that are distributed in a lump sum manner to the surviving fraction of the population. These bequests or transfers, denoted Tr, increase the level of income of individuals and therefore equations (5) and (8) now read:

$$c + ps + a' \leq (1 + r(1-\tau))a + y + Tr + (1-\delta_h)\chi_O h \tag{28}$$

and,

$$c + h' + a' \leq (1 + r(1-\tau))a + y + Tr + (1-\delta_h)\chi_O h \tag{29}$$

Table 5: Model Output (Uncertain Lifetimes), Averages

Variable	Model Period, $\sigma = 1.5$	Model Year, $\sigma = 1.5$
$(K+H)/GDP$	0.969	4.849
K/GDP	0.514	2.570
H/GDP	0.456	2.280
$K/(K+H)$	0.489	0.489
$H/(K+H)$	0.511	0.511
H_O/H_R	5.450	5.450
Variable	Model Period, $\sigma = 2$	Model Year, $\sigma = 2$
$(K+H)/GDP$	0.963	4.819
K/GDP	0.507	0.507
H/GDP	0.456	2.283
$K/(K+H)$	0.492	0.492
$H/(K+H)$	0.508	0.508
H_O/H_R	4.538	5.538

respectively for renters and homeowners. In addition, in equilibrium the aggregate transfers must equal the assets owned by the dead fraction of the population. Formally,

$$Tr = \sum_{t=1}^{T+TR} \mu_t(1-\phi_{t+1}) \int_Z (a(z,t)+h(z,t))d\Psi_t \tag{30}$$

The remaining parts of the equilibrium definition are identical to the ones in the previous section.

Parameter values are identical to the previous section, with two exceptions. To achieve an interest rate of about 4% the values for β are now 0.958 when $\sigma = 2$ and 0.964 when $\sigma = 1.5$. The other exception is the minimum house size that delivers the target homeownership fraction, which is set at 0.3.

The performance of the model with uncertain lifetimes, regarding matching the long-run averages in Table 2 is displayed in Table 5. The model performs worse than the one with certain lifetimes in almost all averages with the exception of the aggregate wealth composition. Uncertain lifetimes deliver a fraction of housing capital in the total wealth of about 0.49 which is roughly the value in the data. This comes at the cost of an increase in both the total capital to GDP ratio and the residential stock to GDP ratio: their values are now 4.8 and 2.28 respectively, farther away from the empirical counterparts shown in Table 2. It is worth noticing that in this economy GDP is considerably smaller that in the previous section. The reason is that aggregate employment is measured in efficiency units and relative to the model with certain lifetimes, we are giving more weight to the younger generations which are less productive. Mortality risk reduces business capital holdings and therefore business capital to output ratios remain roughly constant across the two models. However, housing consumption also decreases, but proportionately much less, and that accounts for the increase in total wealth to output and housing wealth to output ratios.

Figures 5 and 6 show the implications of the model for the level of wealth concentration over the life cycle. Death uncertainty increases the level of indebtedness of younger generations resulting in too large levels of wealth concentration at the initial stages of the life cycle. The behavior after age 40 (not many borrowers in any case) is similar to the certain death model.

Regarding housing wealth, larger amounts of housing are consumed in the first period, leading to an increase in the proportion of renters and decreasing the wealth concentration with respect to the certain death case. The difference in homeownership behavior in the first year is apparent by comparing Figures 7 and 2.

5 Conclusion

This paper has explored the properties of the wealth distribution and asset accumulation in a life cycle economy where housing is modeled separately from other types of assets. One important feature of the housing market that has been introduced in this study and

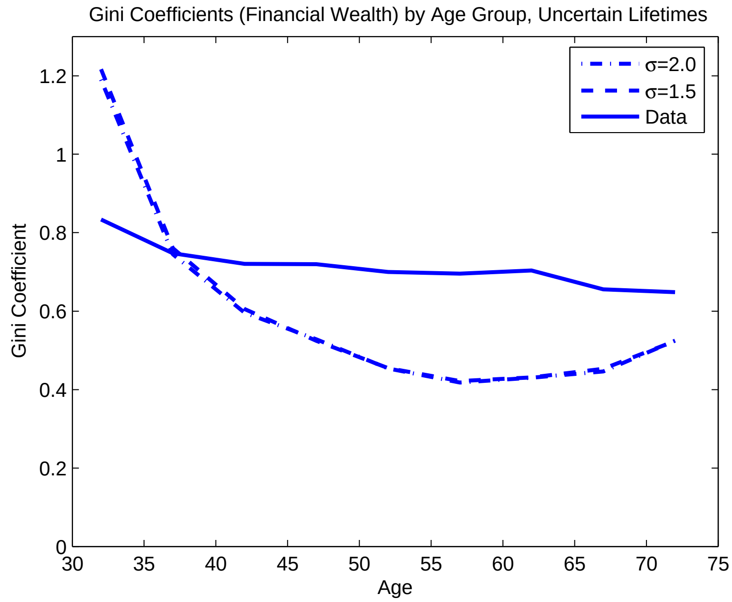

Figure 5: Gini Coefficients: Non-Housing Assets

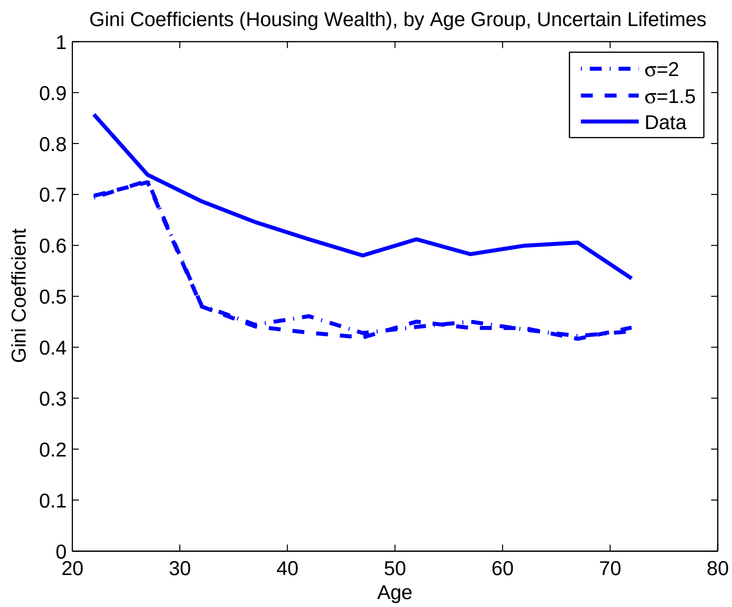

Figure 6: Gini Coefficients: Primary Residence

omitted in all other studies of the wealth distribution is the existence of a rental market for housing services. Housing is the most important asset for most US households, but some

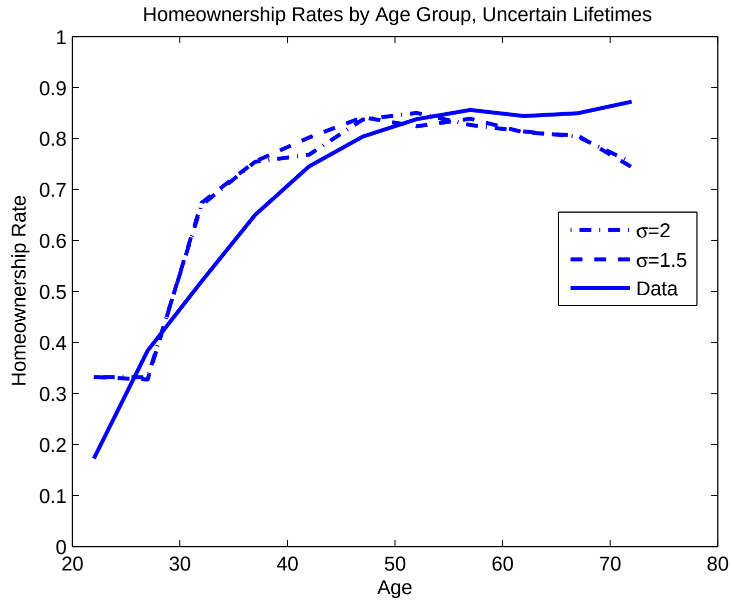

Figure 7: Homeownership Rates by Age Group

of those households choose to rent housing services investing their wealth in alternative assets. The presence of renters has implications for the life cycle concentration of wealth. Gini coefficients for housing wealth are decreasing, quite steeply, while they are rather flat for models that only consider homeowners. The model is also consistent with the pattern of homeownership rates that are observed for different age groups, as well as other aggregate features of the US economy.

References

[1] Aizcorbe, A., Kennickell, B. and Moore, K. (2003), "Recent Changes in U.S. Family Finances: Evidence from the 1998 and 2001 Survey of Consumer Finances", *Federal Reserve Bulletin*, vol. 89, pp. 1-32.

[2] Aiyagari, R (1994),"Uninsured Idiosyncratic Risk and Aggregate Saving", *Quarterly Journal of Economics*, Vol. 109, No. 3, pp. 659-684.

[3] Cooley, T. and Prescott, E. (1995), "Economic Growth and Business Cycles", in *Frontiers of Business Cycle Research*, Cooley, T. (ed.), Princeton University Press.

[4] Díaz, A. and Luengo-Prado, M., (2003), "Precautionary Savings and the Wealth Distribution with Durable Goods", manuscript, Northeastern University.

[5] Feigenbaum, J., (2005), "Can Mortality Explain the Consumption Hump?", manuscript, University of Pittsburgh.

[6] Fernández-Villaverde, J. and Krueger, D. (2002) "Consumption and Saving over the Life Cycle: How Important are Consumer Durables", Stanford Institute for Economic Policy Research Discussion Paper 01-34.

[7] Gervais, M., (2001), "Housing Taxation and Capital Accumulation", *Journal of Monetary Economics*, Vol. 49 (7), pp. 1461–1489

[8] Gruber, J. and Martin, R. (2003), "Precautionary Savings and the Wealth Distribution with Illiquid Durables", International Finance Discussion Paper Number 773, Board of Governors of the Federal Reserve System.

[9] Hansen, G. (1993), “The Cyclical and Secular Behavior of the Labor Input: Comparing Efficiency Units and Hours Worked”, *Journal of Applied Econometrics*, Vol. 8, pp. 71-80.

[10] Huggett, M. (1996), “ Wealth Distribution in Life-Cycle Economies”, *Journal of Monetary Economics*, 38(3), pp. 469-494.

[11] Huggett, M. and Ventura, G. (1999), “On the Distributional Effects of Social Security Reform”, *Review of Economic Dynamics*, Vol. 2, pp. 498-531.

[12] Peterson, B., (2003), “Individual Uncertainty, Aggregate Uncertainty and the Housing Market”, manuscript, Indiana University, Department of Economics.

[13] Platania, J. and Schlagenhauf, D., (2000), “Housing and Asset Holding in a Dynamic General Equilibrium Model”, manuscript, Florida State University.

[13] Silos, P. (2005a) “Housing, Portfolio Choice and the Macroeconomy”, Federal Reserve Bank of Atlanta Working Paper WP-2005-20.

6 Appendix

6.1 Computational Details

As it has become customary, the model is solved numerically, given the absence of closed form solutions. After discretizing the earnings process and fixing parameter values, I constructed grids for the variables of interest $\mathcal{A} = \{a_1, \ldots, a_{NK}\}$ and $\mathcal{H} = \{h_1, \ldots, \underline{h}, \ldots, h_{NH}\}$. Notice that the grid for residential stocks includes values that can not be chosen by individuals who want to own. Hence the housing variable needs to include information about tenure status, summarized by the variable χ_O. Although state variables (h and a need to be discretized, it is desirable to allow control variables (h' and a') to take on a continuum of values. Housing in the utility function, however, usually creates difficulties, both when either working directly through Bellman equations (optimization) or through Euler equations (solutions of non-linear systems). In this paper, although the number of possible values for a' is not discrete, for housing it is. Hence given a state summarized by the triple (a, h, χ_O) and for a given value of h', the optimal a' is found by using a golden section search method and linear interpolation. Repeating this for all possible values of h' will give different values for the expected lifetime utility for different combinations of a' and h'. The individual will pick the h' that yields the highest value. The procedure is summarized in the following steps:

Step 1: Guess a level of aggregate capital K.

Step 2: Given interest rates and wages implied by that level of capital and the technology assumption, solve for the policy functions for all ages, tenure status, income levels and house and capital holdings.

Step 3: Simulate life cycle paths for consumption, investment, etc..., for a large number of agents. Compute aggregate capital.

Step 4: If the level of capital is close to the one guessed in *Step 1*, an equilibrium has

Table 6: Earnings Approximation: Accuracy

Variable	Mean	Variance	Skewness	1st Autocorr.
z_t	0.003	0.584	0.054	0.815
z_t, discrete	-0.009	0.523	0.003	0.773
e^{z_t}	1.349	1.562	3.871	0.787
e^{z_t}, discrete	1.284	1.052	2.220	0.730

been found. Otherwise, update capital and return to *Step 2*.

6.1.1 Approximation of the Earnings Process

The accurate approximation as a discrete state process of the continuous state autoregression in Chapter 1 is important because the characteristics of the earnings process greatly affect the model's output regarding the wealth distribution.

The approximation involved two steps. The first step involves transforming a yearly model into the 5-year frequency. This was done by simulating the yearly model and sampling every fifth element to construct the five year equivalent. The second step involves the discrete state approximation to this 5-year model. The number of states used in the approximation is 7. Computational constraints precluded the number to be larger although it would clearly be desirable.

The Table 6 provides a comparison of moments for z_t and e^{z_t}, (the income shocks), and their discrete approximations:

6.2 Data

6.2.1 National Accounting Data

Almost all of the aggregate data comes from the Bureau of Economic Analysis website (*www.bea.gov*). The only exceptions are the United States population, the average weekly hours worked and the number of employees in the private sector, all of which come from the Bureau of Labor Statistics Website (*www.bls.gov*). The data are annual (except when extracting the Solow residual, see below) starting in 1964 and ending in 2003.

- *Gross Domestic Product*: Output is defined as Gross Domestic Product minus Consumption Expenditures in Durable Goods minus Expenditures in Housing Services minus Net Exports minus Government Consumption and Investment Expenditures. Output was transformed into *per capita* terms through dividing by the US population and transformed into real terms by deflating using the GDP deflator.

- *Investment*: Aggregate investment is Total Gross Private Domestic Investment. Business investment is the sum of non-residential investment in structures, equipment and software. Residential Investment is Total Investment minus Business Investment.

- *Consumption*: Consumption is defined as Personal Expenditures in Consumption minus Expenditures in Durable Goods minus Expenditures in Housing Services. Investment and Consumption were also deflated by the GDP deflator and transformed into *per capita* terms through dividing by the US population.

- *Capital Stocks*: The stocks of both residential and business capital come from the Fixed Assets Tables (Current Net-Cost). The definition of Residential Capital is Residential Structures. Business capital is defined as Total Private Fixed Assets minus Residential Structures. Data on residential stocks by tenure (renters vs. owners) also come from the Fixed Assets Tables.

6.2.2 Wealth Data

Data on the wealth distribution comes from the 2001 Survey of Consumer Finances (SCF). This survey provides information about the wealth composition, income, and demographic variables. It is sponsored by the Federal Reserve Board and collected by the National Organization for Research at the University of Chicago. It is conducted every three years and its sample size is relatively small, interviewing around 4,500 families.

The SCF oversamples wealthier families, given the high level of concentration of the wealth in the United States, therefore appropriate weights need to be used to compute statistics from this dataset. All calculations reported in this paper are weighted averages.

Definitions of Variables: I have defined variables in the same way as Aizcorbe, Kennickell, and Moore (2003).

- *Financial Assets*: Instruments in this category include checking accounts, savings accounts, money market accounts (including the ones in mutual funds), call accounts at brokerage houses, certificates of deposit, stocks (including stocks at mutual funds), government bonds (including mutual funds), tax free bonds, mortgage-backed bonds, corporate and foreign bonds , IRAs (and other quasi-liquid retirement accounts), account type pension plans (including 401(k)'s), life insurance and other financial assets (including among other things cash or royalties).

- *Non-Financial Assets*: Includes the value of all vehicles, the value of the primary residence and other real estate participations, vacation homes, net equity in business at market value, and other non-financial assets (such as jewelry, art, rare books, etc. . .).

- *Debt*: Housing debt (which includes debt on primary residence and all other residential property), credit card debt, other installment loans, loans against pensions, against life insurance, and any other miscellaneous loans.

- *Net Worth*: It is defined as Total Assets (financial and non-financial) minus Total Debt.