Contractual Opportunism, Limited Liability, And the Role of Financial Coalitions

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Abstract: In this paper we investigate a stylized model whereby risk-averse entrepreneurs seek capital from risk-neutral agents for purposes of current consumption. By incorporating the fact that all agents have limited liability, the right to withhold effort, and the right to propose renegotiated contracts, we are able to show that it is not generally possible for competitive financial markets to achieve a first best allocation of risk. Conversely, we show that financial coalitions are capable of providing for the optimal allocation of risk-bearing in a renegotiation-proof fashion.

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I. Introduction

It is by now well understood that there exist a variety of models providing conditions sufficient to ensure that financial intermediaries or coalitions can improve social welfare when compared to competitive financial markets. An important feature of these models is that financial coalitions arise endogenously as the optimal trading arrangement in a constrained environment. At a general level these constraints can be viewed as ones where valuable resources are potentially lost in trade or in the costly production of some non-consumption good (e.g., information) which is valuable in terms of resolving ex ante or ex post uncertainty about outcomes and/or actions. Specific examples range from early work by Townsend (1978), who directly introduces fixed transactions costs into a production/exchange economy, to later papers, such as Diamond and Dybvig (1983), who utilize an imperfect secondary market (or early liquidation penalty) to introduce trading frictions. An alternative approach, rooted in the costly information and production/monitoring genre, has also developed to justify the existence of financial intermediaries. Diamond (1984), Ramakrishnan and Thakor (1984), and Boyd and Prescott (1986) have specific direct or opportunity costs associated with monitoring or screening.

One of the important reasons for studying the theory of intermediation involves a need to more fully understand the welfare characteristics, and associated policy implications, of alternative trading arrangements. However, as Townsend (1983) observes, both the welfare properties and associated policy prescriptions arising from the solutions to these models are sensitive to the nature of transactions costs or other frictions. Ideally, one would like to compare the welfare implications of alternative contractual organizations in a framework that imposes a minimal amount of excess structure.

The purpose of this paper is to examine whether or not financial coalitions will arise endogenosly in a world of symmetric information and no costs, either direct or opportunity.

What we consider is a simple production/exchange model where some of the participants are risk averse. Absent exogenous frictions, this is a standard risk sharing problem. However, we recognize that each participant still has a minimal number of property rights. These include corporate and/or personal limited liability, the right to renegotiate a contract, and the right to provide human capital on a strictly voluntary basis. We argue that, when combined, these three natural frictions are sufficient to demonstrate that financial coalitions can be used to implement a welfare maximizing solution to the risk sharing problem. Conversely, competitive financial markets generally fail to provide for Pareto-optimal allocations. The intuition for the second result lies in the fact that a competitive market system, by its nature, works to provide potential issuers of securities with the present value of future cash flows. Given limited liability, issuers of securities have every incentive to ask for a larger share of output after the initial agreement has been reached. Moreover, issuers have little to lose in such negotiations since their own share of output, absent renegotiation, is quite small at Pareto-optimal allocations. Thus, the ex post bargaining power of renegotiating agents is always strong enough to undo any Pareto-optimal allocation induced in a competitive market setting. On the other hand, when entrepreneurs contribute effort instead to intermediary coalitions, their ability to act opportunistically is greatly curtailed. First, they stand to lose in renegotiation their entire contractual payment from the coalition. Second, because the entire net present value of their project is not given to them up front, their ability to opportunistically consume early in order to reduce the viability of the threat of other claimants to seize personal assets is eliminated. Indeed, we are able to show that, when complete insurance is feasible, bilateral labor contracts can also achieve the optimal solution in a renegotiation-proof fashion. These contracts have lenders doling out a riskless cash flow stream to borrowers, the second period payment being contingent on a labor input. The contingency of the second period payment strengthens the ex post bargaining position of lenders both by lowering the payoff of the borrower in the event of disagreement and by preventing strategic consumption by borrowers. The only obstacle to bilateral arrangements is that the supply of riskless cash flows, combined with non-negativity of consumption restrictions, may render full insurance infeasible and, thus, render large risk-sharing coalitions optimal.

These observations lead us to believe that risk-sharing coalitions may dominate risk-sharing via markets even in very "simple" economic environments, where informational asymmetries are not significant. The idea that coalitions are a viable, possibly dominant, form of organization in such settings has found some support in field studies. Udry (1990), for example, provides evidence that credit markets in Northern Nigeria more closely resemble the coalitions described here than traditional financial markets. The importance of risk sharing in such situations is also documented by Townsend (1994). He shows that through a combination of small-scale financial intermediaries, crop storage, and family networks, the consumption of villagers in southern India is rendered nearly independent of idiosyncratic risk.

Our framework is related to that in a number of papers in the existing literature. Hart and Moore (1992), for example, show that what they call the "inalienability of human capital" and the ability to renegotiate are critical features in the design of optimal debt contracts, even in a world of certainty. Importantly, however, they do not consider the impact of this inalienability on risk sharing, either in a market or coalitional setting. Moreover, Hart and Moore do not consider the strategic impact of allowing agents the freedom to choose the timing of consumption, which is a critical feature of our framework. Another paper relevant to this discussion is the work of Atkeson (1991), which deals with international lending and the risk of repudiation. The lenders in his model are, however, essentially monopolists. In a multi-period setting, with no alternative options for borrowers, it is possible to show that under certain circumstances, market based financial contracts can be designed to guarantee performance on the part of borrowers. Our analysis, in contrast, considers competitive financial markets. In fact, because of the access to competitive capital markets, the reservation utility of issuers in the event negotiations fail is very high because the loss of the cash flows from their investment project has only a small effect on the value of their diversified asset portfolio.

Our paper can, in some sense, be viewed as a modification of the framework used by Townsend (1978). We have, in essence, modified his assumptions that there exist (a) fixed transactions costs and (b) a countably infinite number of individuals with homogeneous tastes who engage in current production but only future consumption. By considering a finite number of agents, whose unique contributions may be required in production, we allow each individual, acting alone, to have some bargaining power. Because this ex post bargaining power may hinder the implementation of some output allocations, it becomes relevant when there is a desire for current and future consumption but a subset of the population has only future production possibilities, with no current endowment. Coalitions arise as natural solutions to this simple implementation problem. Thus, there is no need to impose exogenous transactions costs in order to provide a *raison d' être* for financial coalitions.

The remainder of the paper is structured as follows. Section II includes the model set-up, basic definitions, and some propositions characterizing Pareto-optimal allocations. Section III establishes the renegotiaton proofness of financial coalitions. Section IV contains a proof that, absent a liquidation option, competitive markets are not renegotiation-proof. Section V provides a discussion of the circumstances under which even the most favorable liquidation technology (from an outside claimant's perspective) will not "solve" the renegotiation problem in a market setting. Section VI contains some concluding remarks and a discussion of how market mechanisms might dominate financial coalition arrangements in more complex economies.

II. FRAMEWORK AND BACKGROUND RESULTS

2.1 Economic Environment

The economy we consider is one with two dates and 2N agents. N of the agents are risk neutral. We call these agents "lenders" or L-type agents. Each of these agents has an initial (date 0) endowment of e and no date 1 endowment. The other N agents are identical, strictly risk-averse, expected-utility maximizers and possess the same time-additive preferences given by a continuously differentiable utility-of-wealth function, $U: [0, \infty) \rightarrow \mathbb{R}$, $U' > 0$, $U'' < 0$. For convenience, we also impose the standard Inada condition, $\lim_{x \downarrow 0} U'(x) = \infty$. These type-B or "borrower" agents have no initial endowment. Let $\mathbf{N}(N) = \{1, 2, \ldots, N\}$ represent the index set for agents. Hereafter, except where limit arguments are being developed, we suppress the dependence of $\mathbf{N}$ on N. Each of these type-B agents are "endowed" with a project which may be undertaken with a costless investment of effort.

Type-L agent utility is not affected either by the riskiness of the cash flows they receive or by the distribution of cash flows across dates. Total expected value is all that is relevant. Further, these agents hold large balances of liquid, storable, time 0 consumption. Thus, L-agents are well-suited to absorbing risk and smoothing the intertemporal consumption of the risk-averse B-agents. In this sense, they represent natural "financiers." B-agents have investment prospects. However, these prospects promise payoffs only in the future. B-agents are in this way in a situation similar to that entrepreneurs; they have future prospects but lack current consumption endowments. Further, because B-agents are risk-averse, they have an incentive to trade some of rights to their investment prospects in exchange for current consumption.

The project's random cash flows, which accrue at date 1, are distributed according to a two-point distribution, with equal probabilities, ½ each that the cash flows from project j, denoted by $\tilde{X}_j$ will be either zero or $\bar{x}$. The returns on the N projects are independently and identically distributed. Let $X = \{0, \bar{x}\}^N$ represent the set of all possible realizations of the random vector $\tilde{\mathbf{X}} = (\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_N)$ and define $\mu = ½\, \bar{x}$ as the (common) expected value of $\tilde{X}_j$, j = 1, ... N. Let $\tilde{X}$ represent a random variable with the same distribution as $\tilde{X}_j$, j = 1, ... N. Let $\bar{m}: X \rightarrow \mathbb{R}^+$, defined by $\bar{m}(x) = 1/N(\Sigma x_i)$, represent the average gross cash flow in output state x. Agents are restricted to investment/savings plans which induce non-negative consumption at both dates with probability 1. Thus, in our setting, the exogenous data for an economy is completely specified by the 4-tuple (U, e, μ, N) where U is the utility of wealth function of the B-agents, e is the date 0 endowment of the L-agents, μ is the expected date 1 cash flow to the projects of the B-agents, and N is the number of agents of each type. The set of economies, which we represent by E, is the set of 4-tuples (U, e, μ, N), where U is strictly concave and satisfies the Inada condition, e and μ are strictly positive, and N is a natural number greater than 2.

2.2. Financial Coalitions, Markets, and Renegotiation

We consider two alternative allocation schemes. The first involves a financial coalition, which is discussed in more detail below. In essence, it involves type-B agents trading in their projects for a debt claim on aggregate output. Type-L agents hold equal residual claims on time 1 output under this scenario. The second involves a simple stock market economy which is augmented by a collection of call option contracts. The option contracts are options on aggregate wealth. Thus, by buying a diversified portfolio of stocks and writing options, investors are able to construct debt-like claims on aggregate output. As we show later, the ability of investors to purchase debt-like claims on aggregate output is a necessary condition for the stock market economy to achieve the fully Pareto efficient allocation of risk.

Because agents of type-B must voluntarily contribute effort in order for their projects to produce output, they can always threaten to withhold their effort contribution if their share of the output of the project is not increased. This generates a classical bargaining problem. We model this bargaining problem by using a simplified bargaining mechanism: given the initial allocations (and security prices in the case of the stock market economy), any type-B agent (say (B, i)) is free to propose new sharing rules for the time 1 output of project j = i. If her offer is accepted, the new claims replace the old. If the proposal is rejected, a "coin-flip" occurs, whereby agent (B, i), or the remaining agents, get to propose an alternative final allocation rule for the cash flows of project i. When it is the shareholder's turn to make a proposal (or the coalition's turn in the intermediation economy), there is some question as to which shareholder will make the proposal.[1] However, the only time shareholders make proposals is at the last move in the renegotiation game. At this point, the unique equilibrium outcome, regardless of which shareholder makes the final proposal, would be for (i) the shareholder to propose that the shareholders (other than the renegotiating shareholder) receive all of the cash flows from the project and (ii) the renegotiating B-agent to accept this offer. Similarly, when a renegotiating B-agent is in a position to make the final offer, the unique equilibrium outcome would be for the B-agent to make a final offer which granted her all the project's cash flows. Thus, to simplify the somewhat cumbersome process of modeling contract renegotiations, we will simply assume that, when agent i's renegotiation proposal is rejected, there is a ½ chance that the renegotiating agent (B, i) will receive all of the output from project i and be freed from all contractual obligations to the negotiating party and a ½ chance that she will receive no share of the output of project i and lose the right to any contracted payments from the other negotiating party. Further particulars of the negotiations will depend on whether the intermediary coalition economy or the stock market economy is being considered. For example, in the stock market setting, we will consider the impact of allowing the personal security holdings of renegotiating agents to be seized by the firm's owners. Nevertheless, the above assumptions ensure that a maximal degree of similitude exists between the renegotiation process in the two economies. The bargaining specification closely resembles that of Hart and Moore (1992) and, as they point out, produces allocations resembling those produced in an extensive form bargaining game with "exit" options, that is, a bargaining game where, at the cost of losing contracted payments from the other negotiating party and the cash flows to the project, each of the negotiating parties can exit the negotiations.

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[1] This is particularly true if there are short sales in equilibrium. However, in the models in Sections 3 and 4, all equilibrium holdings of equity securities are non-negative.

Thus, a financial coalition is *not renegotiation-proof* if any L-agent can strictly gain from renegotiating her output share, given that she conjectures that all other L-agents accept the allocation dictated by the coalition. A competitive economy is *not renegotiation-proof* if there exists an L-agent who, taking market prices as given, can strictly gain from following an alternative consumption savings policy, feasible given market prices, and renegotiating her share of project revenues, given that other agents follow the competitive equilibrium consumption plan. To abstract away from corporate governance issues in defining renegotiation-proofness in the market setting, we assume, in the competitive market setting, that renegotiation offers are only accepted if all agents holding a positive fraction of the firm's shares, in the given equilibrium, prefer accepting the offer to playing the coin-flip bargaining game. This assumption is "conservative" in that it makes the renegotiation of contracts by B-agents more difficult than the majority-rule procedure which more closely tracks the stylized facts regarding actual governance procedures. Thus, the assumption makes it uniformly more difficult for us to prove that market equilibria are not renegotiation-proof. In the coalitional setting, the issue of unanimity is moot because, as will be seen, all coalition members have identical preferences over outcomes. However, for definiteness, we assume that renegotiation-proposals are accepted or rejected by majority vote.

2.3. Pareto-optimal Risk Allocations

Before proceeding to an analysis of the coalition, we first consider some properties of symmetric Pareto-optimal allocations. In the context of our analysis, an allocation is 5-tuple, $(q, \{C^0(B, i)\}_{i \in \mathbf{N}}, \{C^0(L, i)\}_{i \in \mathbf{N}}, \{C^1(B, i)\}_{i \in \mathbf{N}}, \{C^1(L, i)\}_{i \in \mathbf{N}})$, where q represents per-agent quantity of the time 0 endowment stored for consumption at time 1, and, for each $i \in \mathbf{N}$ and type $t \in \{B, L\} = \mathbf{T}$, $C^0(t, i)$ is a non-negative scalar and $C^1(t, i)$: X $\rightarrow [0, \infty)$ is the random variable representing the consumption of agent (t, i) as a function of the state $x \in X$ realized. A Pareto-optimal allocation is an allocation with the property that there exist positive weight vectors $\{\gamma(t, i)\}_{(t, i) \in \mathbf{T} \times \mathbf{N}}$ such that the allocation solves:

$$\text{Max} \sum_{i=1}^{N} \gamma(B, i)\,(U(C^0(B, i)) + E[U(\tilde{C}^1(B, i))]) + \sum_{i=1}^{N} \gamma(L, i)\,(C^0(L, i) + E[\tilde{C}^1(L, i)]) \quad \text{(POP)}$$

s.t.,

$$\sum_{i=1}^{N} C^1(B, i)(x) + C^1(L, i)(x) = N\,(\bar{m}(x) + q)\text{ , for all } x \in X \quad \text{(PO1)}$$

$$\sum_{i=1}^{N} C^0(B, i) + C^0(L, i) + N\,q = eN \quad \text{(PO2)}$$

$$0 \le q \le e, \quad \text{(PO3)}$$

In the above expressions $E(\cdot)$ represents expectations relative to the probability measure, π, induced by the realizations of the random variables $\tilde{\mathbf{X}} = (\tilde{X}_1, \tilde{X}_2, ... \tilde{X}_N)$. Thus, for all $x \in X$, $\pi(\{x\}) = \pi^o$, where $\pi^o = 2^{-N}$. The Pareto-optimal allocations are characterized in the next proposition.

Proposition 1. There exists a symmetric Pareto-optimal allocation. Furthermore, all such allocations, $(q, \{C^0(B, i)\}_{i \in \mathbf{N}}, \{C^0(L, i)\}_{i \in \mathbf{N}}, \{C^1(B, i)\}_{i \in \mathbf{N}}, \{C^1(L, i)\}_{i \in \mathbf{N}})$, satisfy the following characterizations:

(i) there exists $k^* \in [0, \bar{x}]$ such that for all $i \in \mathbf{N}$,

$$C^1(B, i)\,(x) = \text{Min}[q + \bar{m}(x), k^*]\text{ , } C^1(L, i)\,(x) = \text{Max}[\,q + \bar{m}(x) - k^*, 0];$$

(ii) Either

$$\forall\, i \in \mathbf{N}, C^0(B, i) = e \text{ and } \forall\, i \in \mathbf{N}, E[U'(\tilde{C}^1(B, i))] \le U'(C^0(B, i)), \text{ and } q = 0, \text{ or}$$

$$\forall\, i \in \mathbf{N}, C^0(B, i) = c_o^* < e \text{ and } \forall\, i \in \mathbf{N}, E[U'(\tilde{C}^1(B, i))] = U'(C^0(B, i)) \text{ and } q \in [0, e)$$

Proof. See Appendix.

Lemma 1. Let (U, e, μ, N) be an economy in which U exhibits decreasing absolute risk aversion. If $(q, \{C^0(B, i)\}_{i \in \mathbf{N}}, \{C^0(L, i)\}_{i \in \mathbf{N}}, \{C^1(B, i)\}_{i \in \mathbf{N}}$, and $\{C^1(L, i)\}_{i \in \mathbf{N}})$ is a Pareto-optimal allocation for (U, e, μ, N), then

$$E[\tilde{C}^1(B, i)] \ge C^0(B, i).$$

Proof. DARA implies the $U''' > 0$, i.e., U' is convex. Thus, by Jensen's inequality, $U'(E[\tilde{C}^1(B, i)]) \le E[U'(\tilde{C}^1(B, i))]$. Proposition 1 (ii) and $U'(E[\tilde{C}^1(B, i)]) \le E[U'(\tilde{C}^1(B, i))]$

imply that $U'(E[\tilde{C}^1(B, i)]) \leq U'(C^0(B, i))$. This implies, because U' is decreasing, that $E[\tilde{C}^1(B, i)] \geq C^0(B, i)$. □

Note that because B-type agents are risk-averse and L-agents are risk-neutral, there is a net social gain from L-agents absorbing some or all of the risk exposure of B-agents. Pareto-optimal allocations differ in how this gain is divided up between agents in the economy. Since L-agents can always receive a payoff of e by simply holding their initial endowment, no allocation can satisfy individual rationality unless the utility of the L-agents at least equals e. That is, individual rationality requires that

$$E[\tilde{C}^1(L, i)] + C^0(L, i) \geq e, \text{ for all } i \in \mathbf{N}. \tag{1}$$

Subject to this constraint, the best feasible allocations for B-agents are those in which

$$E[\tilde{C}^1(L, i)] + C^0(L, i) = e, \text{ for all } i \in \mathbf{N}. \tag{2}$$

Pareto-optimal allocations with this property will be called B-preferred outcomes. The next result shows that in all such outcomes, the value of type-B agents' expected second period consumption is at least equal to half their expected output.

Lemma 2. In any economy, symmetric B-preferred Pareto optima exist. Further, if type-B agents exhibit decreasing absolute risk aversion, in all such Pareto-optimal allocations,

$$E[\tilde{C}^1(B, i)] \geq \tfrac{1}{2} E[\tilde{M}] = (1/2)\mu$$

where

$$\tilde{M} = \frac{1}{N}\sum_{i=1}^{N} \tilde{X}_i$$

Proof. The existence of symmetric Pareto-optimal allocations follows from the regularity of the optimization problem and the fact that the feasible set is nonempty. The characterization follows because, in any borrower-preferred allocation, $E[\tilde{C}^1(L, i)] + C^0(L, i) = e$, for all $i \in \mathbf{N}$. This implies by the resource conservation constraints, and the symmetry of the allocation, that

$$E[\tilde{C}^1(B, i)] + C^0(B, i) = E[\tilde{M}].$$

The result is then immediate from Lemma 1. □

Proposition 1 states that, in all symmetric Pareto-optimal allocations, each of the risk-averse B type agents receives proportional shares of a debt-like claim on aggregate output while the risk-neutral type-B agents hold proportional shares of an option on aggregate output. The second result, stated in Lemma 2, follows because the transferability of date 0 income to date 1 implies that, for all $i \in \mathbf{N}$, $E[U'(\tilde{C}^1(B, i))] \leq U'(C^0(B, i))$. From the perspective of the future analysis, the most important consequence of Proposition 1 is that consumption depends only on aggregate output. Thus, in order to implement the first-best allocation, institutions must be designed which make consumption independent of the output of individual projects even though type-B agents have a natural bargaining power specific to the particular project which requires their labor. In the next section, we will show that a financial intermediary coalition is able to implement these first-best outcomes.

III. IMPLEMENTATION OF PARETO-OPTIMAL ALLOCATIONS VIA FINANCIAL COALITIONS

In this section we demonstrate that Pareto-optimal allocations can be implemented by financial coalitions. Our plan of attack is as follows: first we define an allocation mechanism. Next we provide, in Proposition 2, a sufficient condition for a coalitional allocation to be renegotiation-proof. All allocations satisfying our characterization of Pareto-optimal allocations satisfy the conditions of Proposition 2. Thus, Proposition 1 and 2 combined imply that financial coalitions can implement Pareto-optimal allocations.

The coalition operates as follows. All L-agents pool their capital endowments forming a lending coalition. They propose the following allocation of output. At time zero, the coalition pays b_0 units of time 0 consumption to each type-B agent, and l_0 units of time 0 consumption to each L-agent. The coalition stores $q = e - l_0 - b_0 \geq 0$ units of time 0 consumption per agent. Type-B agents surrender their investment to the coalition. In return, each type-B agent receives a debt claim on aggregate time 1 output with promised payment of b_1 per agent. All type-L agents hold equal residual claims on time 1 output. A coalition structure is thus a 4-tuple, (b_1, b_0, l_0, q), specifying payments at time 0, social storage, and the structure of claims on time 1 output. Note that consumption allocations from the coalition do not completely determine intertemporal consumption patterns. Both L and B-agents have one residual decision: how much of the time 0 consumption good to store for future consumption and how much to consume at time 0.

Two questions regarding coalition structures are salient. First, will agents gain from joining in such coalitions ex ante? Second, will they have ex post incentives to opportunistically renegotiate the terms of their relationship with the coalition after they have received their contractual time 0 payments? The Pareto-efficiency and individual rationality results of the previous section address the first question. The second question, regarding renegotiation-proofness, is addressed by Proposition 2. Proposition 2 demonstrates that coalition structures which ensure that B-agents receive a senior claim on aggregate output with an expected value at least equal to 1/2 the value of expected per capita output at time 1, ($\frac{1}{2}$ $E[\tilde{M}]$), are renegotiation-proof.

Proposition 2. In any economy (U, e, μ, N) in which B-agents exhibit decreasing absolute risk aversion, coalition structures (b_1, b_0, l_0, q) which satisfy

$$E[\mathrm{Min}[b_1,\ \tilde{M} + q] \geq \tfrac{1}{2}\ E[\tilde{M}],$$

are renegotiation-proof.

Proof. To prove that the coalition structure is renegotiation-proof, we need to show that no B-agent has an incentive to renegotiate her contract when she conjectures that no other B-agent will renegotiate. If no agents renegotiate, the period 1 cash flow to agent (L, i) is given by $\tilde{C}^1$(L, i| n), where

$$\tilde{C}^1(L, i|\ n) = l_0 - C^0(L, i) + \mathrm{Max}[\ \tilde{M} + q - b_1, 0], \tag{3}$$

while those for the i^{th} project holder can be written as

$$\tilde{C}^1(B, i|\ n) = b_0 - C^0(B, i) + \mathrm{Min}[\ \tilde{M} + q\ , b_1]. \tag{4}$$

Now suppose that a type-B agent, agent j, proposes a renegotiated contract. A renegotiation proposal by the j^{th} borrower is a division of the cash flows from project j between (B, j) and the coalition. By our assumption that the cash flows from each project are supported by

$\{0, \bar{x}\}$, the division is uniquely determined by the payment received by j when cash flow $\bar{x}$ is realized. Let "f" represent this payment. The remaining projects are unaltered. Therefore, the cash flow to the representative type-L agent, if she accepts renegotiation, is given by

$$\tilde{C}^1(L, i| y) = \tilde{C}^1(L, i, -j) + \frac{\mathrm{Max}[\tilde{X}_j - f, 0]}{N} \tag{5}$$

where

$$\tilde{C}^1(L, i, -j) = l_0 - C^0(L, i) + \mathrm{Max}[\ \tilde{M}^{-j} + q - b_1 + \tfrac{1}{N}b_1, 0], \tag{6}$$

and

$$\tilde{M}^{-j} \equiv \frac{1}{N} \sum_{\substack{k=1 \\ k \neq j}}^{N} \tilde{X}_k . \tag{7}$$

If this proposal is rejected, and nature draws the next proposal, the cash flows to type-L agents will be either $\tilde{C}^1(L, i, -j)$ (agent j gets to choose and $f = \bar{x}$) or $\tilde{C}^1(L, i, -j) + \tilde{X}_j/N$. (the coalition chooses and $f = 0$). Given equal probabilities, the expected cash flows to a representative type-L agent from rejecting the offer is

$$E[\tilde{C}^1(L, i| r)] = E[\tilde{C}^1(L, i, -j)] + \frac{\frac{1}{2} E[\tilde{X}_j]}{N}. \tag{8}$$

Therefore, the proposal will be accepted if and only if the expected value of the right hand side of equation (5) is greater than the right hand side of equation (8), or

$$E\{ \frac{\mathrm{Max}[\tilde{X}_j - f, 0]}{N} \} > \frac{\frac{1}{2} E[\tilde{X}_j]}{N}.$$

It follows that the proposal will be accepted if and only if $\frac{1}{2}(\bar{x} - f) \geq \bar{x}/4$, or $f \geq \bar{x}/2 = \mu$. Any $f < \mu$ will be rejected by all agents in the L-coalition. The period 1 consumption of agent (B, j) if a proposal of $f \leq \bar{x}/2$ is made (ensuring the acceptance of the proposal) is given by

$$\tilde{C}^1(B, j| y) = \mathrm{Min}[f, \tilde{X}_j] + b_0 - C^0(B, j). \tag{9}$$

The possible cash flows to j when $f > \mu$ are given by $b_0 - C^0(B, j) + \tilde{X}_j$ (agent j wins the coin flip and chooses $f = \bar{x}$) or $b_0 - C^0(B, j)$ (the coalition wins the coin flip and chooses $f = 0$).

We now show that the expected utility to agent j from not renegotiating exceeds that obtained via renegotiation. We first show that proposals of $f < \bar{x}/2$, are dominated. To show this, let b_1' be the solution to

$$E[\mathrm{Min}[b_1', \tilde{M} + q]] = \tfrac{1}{2} E[\tilde{X}_j]. \tag{10}$$

Now b_1' is unique since (a) the left hand side of equation (10) is strictly increasing and convex in b_1, and (b) because $q \geq 0$, $E[q + \tilde{M}] \geq \frac{1}{2} E[\tilde{M}] = \frac{1}{2} E[\tilde{X}_j]$. By the hypothesis of the proposition, $E[\mathrm{Min}[b_1, \tilde{M} + q]] > \frac{1}{2} E[\tilde{X}_j]$. Thus we have by (10) that

$$E[\mathrm{Min}[0, q + \tilde{M}]] < \tfrac{1}{2} E[\tilde{X}_j] < E[\mathrm{Min}[b_1, q + \tilde{M}]]. \tag{11}$$

Therefore, there exists a unique $b = b_1' \in (0, b_1]$ which solves equation (10).

Suppose, first, that the proposal is rejected. The expected utility to agent j from the coin flip is then given by

$$E[U(\tilde{C}^1(B, j|\, r))] =$$

$$\tfrac{1}{2} [U(b_0 - C^0(B, j))] + \tfrac{1}{2} E[U(b_0 - C^0(B, j) + \tilde{X}_j)] \leq E[U(\tilde{C}''(B, j))], \tag{12}$$

where $\tilde{C}^{1''}(B, j) = b_0 - C^0(B, j) + \frac{1}{2} \tilde{X}_j$. This last inequality follows from Jensen's inequality and the concavity of $U(\cdot)$. The consumption associated with not renegotiating is no smaller than

$$\tilde{C}^{1'}(B, j) = b_0 - C^0(B, j) + \mathrm{Min}[b_1', q + \tilde{M}]. \tag{13}$$

It follows from equation (3), (11), and (13) that

$$E[U(\tilde{C}^{1'}(B, j))] \leq E[U(\tilde{C}^1(B, j|\, n)]. \tag{14}$$

Now, from equation (10) and the definitions of $\tilde{C}''(B, j)$ and $\tilde{C}'(B, j)$, it follows that

$$E[\tilde{C}^{1'}(B, j)] = E[\tilde{C}^{1''}(B, j)]. \tag{15}$$

As shown in Proposition 1, the debt contract maximizes the utility of the risk-averse agents over all limited liability claims on output. Thus,

$$E[U(\tilde{C}''(B, j)] \leq E[U(\tilde{C}'(B, j)]. \tag{16}$$

It follows that equations (12), (14), and (16) yield the desired conclusion that (B, j) is no better off renegotiating her contract when her renegotiation proposal is rejected. However, in order to induce proposal acceptance, it must be the case that $f \leq \bar{x}/2$. In this case, the time one consumption to (B, j) from renegotiating, $\tilde{C}^1(B, j|\, y))$, is, from equation (9), weakly less than $\tilde{C}''(B, j)$ in all states of the world. It thus follows, from (14), (15), and (16), that

$E[U(\tilde{C}^1(B, j| y))] \leq E[U(\tilde{C}^1(B, j| n))]$. Thus, the coalition structure is renegotiation-proof.

□

The intuition behind Proposition 2, is that, given equal bargaining power, B-agents can capture no more that half of the returns on their project in post contracting renegotiations. The cash flows resulting from renegotiation are also riskier than the debt claim on average aggregate output provided by the coalition. This follows from the fact that the debt claim on aggregate output is the least risky claim on output consistent with limited liability. Thus, as long as the expected value of the claim received by B-agents at least equals 1/2 the total returns from the project, B-agents will have no incentive to renegotiate their contracts even if they are risk-neutral. Because they are risk-averse, they will strictly lose from such renegotiation. Thus, debt claims on aggregate output, b_1, satisfying the conditions of the proposition, are renegotiation-proof.

Our next task is to relate the renegotiation-proof coalitional structures to the Pareto-optima of the game. To accomplish this, note that the actual consumption pattern induced by a coalition structure depends on the time 0 per capita consumption of the lenders, which we denote by C_0^L, as well as b_1 and b_0. Knowing these three parameters, and knowing that B-agents will make an individually optimal storage decision for the endowment income they receive, allows us to uniquely determine allocations from coalition structures as follows: the allocation determined by coalition structure (b_0, b_1, C_0^L) is given by $(q, \{C^0(B, i)\}_{i \in \mathbf{N}}, \{C^0(L, i)\}_{i \in \mathbf{N}}, \{C^1(B, i)\}_{i \in \mathbf{N}}, \{C^1(L, i)\}_{i \in \mathbf{N}})$, where

$$q = e - C_0^L - b_0 + \phi(b_0, b_1, C_0^L),$$

and for all $i \in \mathbf{N}$,

$$C^0(L, i) = C_0^L$$

$$C^0(B, i) = b_0 - \phi(b_0, b_1, C_0^L)$$

$$C^1(B, i)(x) = \phi(b_0, b_1, C_0^L) + \mathrm{Min}[b_1, \bar{m}(x) + e - C_0^L - b_0])$$

$$C^1(L, i)(x) = \mathrm{Max}[\bar{m}(x) + e - C_0^L - b_0 - b_1, 0]$$

where

$\phi(b_0, b_1, C_0^L) = \text{argmax}\{ g \in [0, b_0] : U(b_0 - g) + E[U(g + \text{Min}[b_1, \tilde{M} + e - C_0^L - b_0]\}$.

The next result is a straightforward consequence of our earlier analysis. In Proposition 1 we showed that when agents exhibit decreasing absolute risk aversion, the optimal allocation calls for B-agents to receive a disproportionate share of the expected value of their consumption stream at time 1 (the time at which they must bear risk). This implies, by Proposition 2, that the Pareto-optimal allocations satisfy the sufficient conditions for renegotiation-proofness. This result is formalized in Proposition 3.

Proposition 3. Consider any economy (U, e, μ, N) such that U exhibits decreasing absolute risk aversion.[2] Then there exist coalition structures which induce Pareto-optimal allocations of risk.

Proof. Let $(q, \{C^0(B, i)\}_{i \in \mathbf{N}}, \{C^0(L, i)\}_{i \in \mathbf{N}}, \{C^1(B, i)\}_{i \in \mathbf{N}}, \{C^1(L, i)\}_{i \in \mathbf{N}})$ be a B-preferred symmetric Pareto-optimal allocation. Such an allocation exists by Proposition 1. Pick any $i \in \mathbf{N}$ and let $b_0 = C^0(B, i)$. Choose $b_1 = k^*$, where k^* is defined in Proposition 1. Let $C_0^L = e - q - b_0$ and note that $(q, \{C^0(B, i)\}_{i \in \mathbf{N}}, \{C^0(L, i)\}_{i \in \mathbf{N}}, \{C^1(B, i)\}_{i \in \mathbf{N}}, \{C^1(L, i)\}_{i \in \mathbf{N}})$ is the allocation induced by (b_0, b_1, C_0^L). This is immediate because the Pareto-optimality of the allocation, and the way b_0, b_1, and C_0^L were chosen, ensure that $\phi(b_0, b_1, C_0^L) = 0$. We need only show that (b_0, b_1, C_0^L) is renegotiation-proof. Lemma 2 shows that $E[\tilde{C}^1(B, i)] \geq \frac{1}{2}E[\tilde{M}]$. By definition

$$C^1(B, i)(x) = \text{Min}[b_1, \bar{m}(x) + e - C_0^L - b_0].$$

Thus, $E[\text{Min}[b_1, \tilde{M} + e - C_0^L - b_0]] \geq \frac{1}{2}E[\tilde{M}]$, implying that (b_0, b_1, C_0^L) is renegotiation-proof. □

[2] In fact, the actual restrictions of agent preferences required to ensure the implementability of Pareto-optimal risk sharing arrangements is somewhat weaker than decreasing absolute risk aversion. All that is actually required is that $U' > 0$, $U'' < 0$, and $U''' \geq 0$. These are weaker conditions since, for smooth preferences, $U''' > 0$ is a necessary condition for DARA.

We note that, for $e \geq \mu$, a bilateral labor contract will also serve to implement a Pareto-optimal allocation of the risk. This case, of complete insurance at a zero risk premium, highlights the fact that even the smallest coalitions (two) can provide for perfect risk sharing if the economy is well endowed relative to average future production opportunities. Moreover, this is true for any N since multiple coalitions of two will not change the bargaining position of any one type-B agent.

In the next section we show, conversely, that corporate limited liability is sufficient to result in market allocations that are not renegotiation-proof when $N > 2$. Moreover, eliminating the renegotiating type-B agent's property rights to her remaining claims cannot guarantee a renegotiation-proof allocation. We show that personal limited liability alone is a powerful force working to prevent market allocations from achieving first-best from a risk sharing perspective.

IV. STOCK MARKET ECONOMY

In this section we consider the attainability of Pareto-optimal allocations in a market economy. As before, each B-type agent is endowed with a project which pays off only at time 1 and each L-type agent is endowed with time zero consumption. In a market economy setting these endowments can be traded. To simplify notation, we assume that time 0 consumption is the numeraire good in the economy and, thus, the price of time 0 consumption is 1. We also normalize the ownership claims to the j^{th} project so that they can be represented by a single share of stock. That is, we assume that each "project" j has one share outstanding, with a market price of P_j. Let $C^0(t, i)$ be the current, time 0, consumption of an investor of type $t \in T \equiv \{B, L\}$ and index i, with $C^0(t) = (C^0(t, 1), C^0(t, 2), \ldots C^0(t, N))$. Let $S(t, i)$ be the stored funds of an investor of type $t = \{B, L\}$ and index i and $S(t) = (S(t, 1), S(t, 2), \ldots S(t, N))$. Funds are stored at a zero interest rate from period 0 to period 1. Let $Z(t, i, j)$ represent the fraction of the j^{th} firm's shares received by the i^{th} investor of type t. Let $Z(t, i)$ represent the vector of shares purchased by agent $Z(t, i)$. That is $Z(t, i) = (Z(t, i, 1), Z(t, i, 2) \dots Z(t, i, N))$, $Z(t, i) \in [0, 1]^N$. Let $Z(t) = (Z(t, 1), Z(t, 2) \dots Z(t, N))$ represent the vector of share purchases of all agents of type t, $Z(t) \in [0, 1]^{N^2}$. Finally, $Z = (Z(B), Z(L))$ represents the vector of all shareholdings of all agents, $Z \in [0, 1]^{2N^2} \equiv \mathbf{Z}$. We represent the price of the j^{th} firm's shares by P_j and define $P = (P_1, P_2, \dots P_N)$ as the vector of share prices, $P \in \mathbb{R}^N = \mathbf{P}$. We assume a complete set of options on aggregate output trade. Let $Z^c(i, t, w)$ represent the number of calls held by agent (t, i), with strike price $w = (k/N)\bar{x}$, where k varies between 1 and N. Let $Z^c(t) = (Z^c(t, 1), Z^c(t, 2) \dots Z^c(t, N))$ represent the vector of call purchases of all agents of type t, $Z^c(t) \in [0, 1]^{2N^2}$. Finally let $Z^c = (Z^c(B), Z^c(L))$ represent the vector of all holdings by all agents, $Z^c \in [0, 1]^{2N^2} \equiv \mathbf{Z}^c$. We define P^c_w as the price of a call option on aggregate output with strike price w. Finally, define $P^c = (P^c{}_1, P^c{}_2, \dots P^c{}_N)$ as a generic vector of call prices, $P^c \in \mathbb{R}^N \equiv \mathbf{P}^c$. Absent renegotiation, the time 1 claim of agent (t, i) is given by

$$C^1(t,i|\,n) = S(t,i) + \sum_{j=1}^{N} Z(t,i,j)\tilde{X}_j + \sum_{k=1}^{N} Z^C(t,i,k)\ \mathrm{Max}\left(\tilde{M} - \tfrac{k}{N}\bar{x}, 0\right). \tag{17}$$

The budget-feasible set of consumption and asset holding policies for agent $(t, i) \in \mathbf{T} \times \mathbf{N}$, given price vector $(P, P^c) \in \mathbf{P} \times \mathbf{P}^c$, is a 4-tuple, $(Z(t, i), Z^c(t, i), C^0(t, i), S(t, i)) \in \mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R}^+ \times \mathbb{R}^+$ which satisfies the following condition:

$$\sum_{j=1}^{N} P_j Z(t,i,j) + \sum_{w=\bar{x}/N}^{\bar{x}} P^c_w Z^c(t,i,w) \le \psi(t,i,P) - C^0(t,i) - S(t,i) \tag{18}$$

where,

$$\psi(t,i,P) = \begin{cases} e \text{ if } t = L \\ P_j \text{ if } t = B \end{cases}$$

$$P\{\tilde{C}^1(t, i|\, n) \ge 0\} = 1 \text{ for all } (t, i) \in \mathbf{T} \times \mathbf{N}. \tag{19}$$

Equation (18) embodies the budget feasibility conditions that the price of a bundle be no greater than the market value of the agent's endowment. Equation (19) incorporates the non-negativity constraint that no agent's consumption can be negative in any state of the world. A 4-tuple, $(Z^*(B, i), Z^{c*}(B, i), C^{0*}(B, i), S^*(B, i))$ for an agent i of type-B is a competitive market-optimal policy given prices (P, P^c), if there exists no other budget feasible policy $(Z'(B, i), Z^{c'}(B, i), C^{0'}(B, i), S'(B, i))$ such that

$$U(C^{0*}(B, i)) + E[U(\tilde{C}^*(B, i \mid n))] < U(C^{0'}(B, i)) + E[U(\tilde{C}^{1'}(B, i \mid n))]. \tag{20}$$

Similarly, for L-agents, the 4-tuple $(Z^*(L\ i), Z^{c*}(L, i), C^{0*}(L, i), S^*(L, i))$ is a competitive market-optimal policy for agent (L, i) given prices (P, P^c) if there exists no other budget feasible policy $(Z'(L, i), Z^{c'}(L, i), C^{0'}(L, i), S'(L, i))$ such that

$$C^{0*}(L, i) + E[\tilde{C}^{1*}(L, i \mid n)] < C^{0'}(L, i) + E[\tilde{C}^{1'}(L, i \mid n)]. \tag{21}$$

A market clearing demand vector (Z, Z^c) is an element of $Z \times Z^c$ which satisfies:

$$\sum_{i=1}^{N} Z(B,i,j) + \sum_{i=1}^{N} Z(L,i,j) = 1 \text{ and}$$

$$\sum_{i=1}^{N} Z^c(B,i,w) + \sum_{i=1}^{N} Z^c(L,i,w) = 0. \tag{22}$$

A competitive equilibrium of the economy is a triple $(Z^*, Z^{*c}, C^*, S^*, P, P^c) \in Z \times Z^c \times C \times S \times P \times P^c$ such that (Z^*, Z^{c*}) is market clearing and for each $(t, i) \in \mathbf{T} \times \mathbf{N}$, $(Z^*(t, i), Z^{c*}(t, i), C^{0*}(t, i), S^*(t, i))$ is optimal for (t, i) given (P, P^{c*}). The consumption pattern induced by the equilibrium portfolio decisions (Z^*, Z^{*c}, C^*, S^*) is called the equilibrium allocation associated with equilibrium $(Z^*, Z^{*c}, C^*, S^*, P, P^c)$, or just the competitive allocation when the equilibrium under consideration is clear.

Some characterizations of the competitive allocations are immediate. First, note that, because all the cash flows on all securities are independent and identically distributed, the equilibrium stock prices will be the same for all assets. This fact, combined with the strict concavity of the utility-of-wealth function for B-agents, implies that the allocation of all B-type investors will be the same. Further, the expected total consumption of all of the risk-neutral type-L investors will be the same. For this reason we can, and will, without loss of generality, assume that the competitive equilibrium allocations are symmetric. Because such allocations must be Pareto-optimal and individually rational (autarky is always an option) they must satisfy the necessary conditions for symmetric Pareto-optimal allocations given by Proposition 1 and be individually rational. These facts impose some strong structural restrictions on equilibrium allocations. Notably, if $e \geq E[\tilde{M}]$, then it is feasible, without violating the non-negativity condition (equation (19)) for all B-agents to receive a riskless consumption stream with a total expected value at least equal to the project with which they are endowed. Individual rationality on the part of L-agents implies that no allocation could provide B-agents more than the expected value of their consumption. Pareto-optimality implies that for a fixed distribution of expected value, the risk of B-agents is minimized. Thus, in any Pareto-optimal allocation it must be the case that B-agents bear no risk. This implies that, in any competitive equilibrium, $Z^*(B, i) = 0$ for all $i \in \mathbf{N}$. On the other hand, if $e < E[\tilde{M}]$, the non-negativity of personal consumption constraint (19) may prevent L-type agents from completely insuring the risk faced by B-agents. In this case, B-type agents may have positive share holdings. The Pareto-optimality conditions, in fact, ensure that, in this case, they will all hold an equally weighted portfolio of securities combined with a short position in call options on aggregate output. This follows because the combination of an equally weighted stock portfolio and a short position in a call on aggregate output is the only portfolio pattern which induces the Pareto-optimal consumption pattern.

For a given economy (U, μ, e, N), a competitive equilibrium, $(Z^*, Z^{*c}, C^*, S^*, P, P^c)$, is not renegotiation-proof if there exists a B-type agent, say j', taking market prices as given, who can generate a strictly higher level of expected utility from following consumption savings policy $(Z''(B\ j'), Z^{c}{}''(B, j'), C^{0}{}''(B, j'), S''(B, j'))$ feasible under (P, P^c) instead of $(Z^*(B\ j'), Z^{c*}(B, j'), C^{0*}(B, j'), S^*(B, j'))$ and renegotiating her share of the j' project, given that other agents follow their competitive equilibrium consumption plan. If there does not exist any competitive equilibrium for the economy (U, μ, e, N) which is renegotiation-proof, we will say that the stock market economy is not renegotiation-proof. To abstract away from corporate governance issues in defining renegotiation-proofness, we assume that renegotiation offers are only accepted if all agents holding a positive fraction of the firm's shares, in the given equilibrium, prefer accepting the offer to playing the coin-flip bargaining game. This assumption is "conservative" in that it makes the renegotiation of contracts by B-agents more difficult than the majority-rule procedure which more closely tracks the stylized facts regarding actual governance procedures. Thus, the assumption makes it uniformly more difficult for us to prove that market equilibria are not renegotiation -proof.

In this section, we demonstrate that, even under this conservative definition of renegotiation-proofness, when "corporate limited liability" obtains, that is when both B-and L-agents have the option of exiting negotiations while retaining their portfolio of investments in other assets, the stock market economy is not renegotiation-proof. Briefly, our argument runs as follows: suppose that the stock market allocations were renegotiation -proof. In this case, it would have to be the case that each B-type agent (B, j) prefers not to attempt to renegotiate her share of the j^{th} project's returns given that she conjectures that no other type-B agents will attempt to renegotiate their contracts. If this were the case, then allocations induced by security holdings in the economy in question would be identical to those in a standard competitive market economy. Thus, they would be Pareto-optimal. Now, Pareto-optimality requires that the payoffs to B-type agents be measurable functions of aggregate output. This will imply that (B, j) can own no more than 1/N shares of firm j. However, in this case, it can be shown that the fraction of firm j that B can obtain via renegotiation always exceeds 1/N. Thus renegotiation is optimal and the renegotiation-proofness of the market allocation is contradicted. This result is formalized in the next proposition.

Proposition 4. In the presence of corporate limited liability, the stock market economy is not renegotiation-proof.

Proof. See Appendix.

Proposition 4 immediately raises two natural questions: first, does the failure of renegotiation-proofness imply that the stock market economy cannot attain the first-best risk allocation? Second, what sort of allocations would prevail in the market economy in the presence of a renegotiation option? With regard to the first point, note that Proposition 4 does not directly address the issue of whether Pareto-optimal risk sharing could be attained in the market economy in which regeneration occurred along the equilibrium path. To address this issue would require a complete development of the game-form of the market/renegotiation game. However, because of the multiplicity of agents and the dynamic structure of the game, formalization is very tedious and, in the end, produces the obvious conclusion that Pareto-optimality cannot be obtained in Nash equilibria in which renegotiation takes place.[3] To see this, note that Proposition 4 shows that, if a (B, i) agent believes that other B- agents will not renegotiate, (B, i) can strictly gain from renegotiating her contract, regardless of the investment consumption pattern chosen at time 0. (B, i)'s renegotiation of her ownership share in firm i increases her share of firm i and does not itself impair her claim on any other asset. Thus, the cash flows after renegotiation stochastically dominate the cash flow to (B, i) in the absence of renegotiation, regardless of her asset holding pattern. The only effect the renegotiations of other players have on agent (B, i) is on the value of her portfolio. Thus, all B-agents have an incentive to renegotiate regardless of their conjectures regarding the actions of other agents. However, renegotiation implies that each (B, i) agent holds more than 1/N share of project i, and thus the equilibrium allocation of risk will not be Pareto-efficient.

It is also worthwhile to note that the above analysis simply implies that market economies cannot attain fully efficient risk sharing in the presence of opportunities for contractual opportunism. It does not imply that in a market game with renegotiation, no equilibrium would exist. We conjecture that, in the market game with renegotiation opportunities, one possible Nash outcome would be for all B-agents to renegotiate their contracts and for this fact to be reflected in the time 0 prices for claims on firm output. Such an outcome would be isomorphic to an economy in which entrepreneurs hold restricted

[3] A formal model in which this conclusion is established is available from the authors upon request.

non-alienable shares of their firms in proportion to their bargaining power and outside claimants hold unrestricted shares. Such an outcome indeed represents a solution to the market game with renegotiation. The point is, however, that this solution does not support efficient risk sharing.

Also worthy of note is the subtle role risk aversion plays in the above analysis. Risk aversion on the part of B-agents is in no way required in order either to ensure the renegotiation-proofness or the lack of renegotiation-proofness of any given consumption allocation. In fact, it makes the demonstration of these results much more complex. Strict risk aversion is only important in order to ensure that the failure of certain allocations to be renegotiation-proof has economic significance. For, in the absence of risk aversion, an autarkic economy in which each agent simply consumes her own personal endowment is (weakly) optimal. Such an allocation is clearly renegotiation-proof. The role of risk aversion is to ensure that only allocations which produce a strong separation between bargaining power and ownership structure are Pareto-optimal. This is the wedge which allows the formation of risk-sharing financial coalitions to have economic significance.

V. Competitive Market Economies without Corporate Limited Liability.

From the above discussion it is clear that, given corporate limited liability, it is impossible to implement Pareto-optimal allocations in a competitive market setting. The reason for the failure of the competitive market mechanism is straightforward. Pareto-optimality requires that an agent's income depend only marginally on the outcomes of any particular investment project, yet the bargaining power of B-type agents is concentrated on a single cash flow stream. This gives them an incentive to attempt to opportunistically withhold labor input from the project with which they are associated, if shareholders do not grant them a larger share of cash flows than called for in Pareto efficient allocations. Corporate limited liability implies that, if (B, j) attempts to negotiate with the owners of firm j, neither (B, j) nor the shareholders of firm j have "at risk" any cash flows other than the cash flows from the j^{th} project. That is, both parties can exit negotiations without impairing any claims other than their equity claims to project j. Since the Pareto-optimal allocation calls for (B, j) to hold only a negligible interest in firm j, this implies that (B, j) has little "at risk" in the negotiations in the market economy setting with limited corporate liability.

Clearly, if financial market implementations of Pareto-efficient risk allocations are to be renegotiation-proof, we need to increase the risk to renegotiation for type-B agents by lowering the value of their exit options. The obvious way to do this is to provide shareholders with a means of seizing the portfolios of managers who attempt opportunistic renegotiation. That is, we must allow shareholders to pierce the corporate veil by seizing the assets of borrowers who attempt to renegotiate their claims.

At this point, the question becomes one of how to incorporate this "liquidation" option into the analysis. A basic desideratum for the selection of a liquidation technology is that the game form which is adopted should not effectively allow shareholders to precommit to liquidation by making a final offer, which if not accepted results in automatic liquidation. Such an ability to precommit to a first-and-final-offer, even in the absence of any liquidation option, will eliminate any possible scope for contractual opportunism not only in our model but in any model for it grants one of the parties in the negotiations all of the bargaining power. Conversely, the liquidation technology cannot be one that is capable of extracting current consumption from the renegotiating agent. Otherwise, all of the bargaining power would rest with outside claimants.

Subject to this desideratum, a number of possible formulations that incorporate a liquidation option are conceivable. One possibility would be to utilize an approach similar to Hart and Moore (1992) and assume that the shareholders of firm i can, as an initial response to a managerial renegotiation proposal, either liquidate the project and seize all of the personal wealth of the renegotiating B-agent or continue negotiations with the manager. The liquidated value of the firm, i.e., the value without the manager's input, is zero by assumption. Thus, the value of liquidated assets equals the value of the manager's personal wealth. Because the liquidation decision is unconditional, this formulation satisfies our desideratum. Shareholders cannot precommit to liquidation conditional on a negative response to a take it or leave it proposal. Rather, they can only liquidate when liquidation is in their interest, at the point in the game at which they make a liquidation decision. We call this liquidation technology a "standard" one since there is no pre-commitment option to outside claimants.

In general, adding a liquidation option of the sort discussed above will not render Pareto-optimal allocations implementable in a competitive market economy setting. The inherent weakness of a threat to seize the wealth of an entrepreneur if she proposes renegotiation can be seen in the following proposition.

Proposition 5. There exists a renegotiation proposal that, to outside claimants, dominates liquidation of the Hart and Moore (1992) variety in a mean/variance sense.

Proof: To show this result, define consumption to outside agents if liquidation occurs as $\tilde{C}^1(t, i \mid \iota)$, where

$$\tilde{C}^1(t, i \mid \iota) = (\tilde{C}^1(t, i) \mid n)(N/(N-1)) - \tilde{X}/(N-1), \tag{23}$$

and consumption if the renegotiated contract is accepted is given by

$$\tilde{C}^1(t, i \mid y) = (\tilde{C}^1(t, i) \mid n) - (\tilde{X}\,\alpha/(N-1)), \tag{24}$$

where α is the additional share of output allocated to the entrepreneur and $\tilde{C}^1(t, i \mid n)$ is consumption for an agent of type t absent renegotiation or liquidation. It is straightforward to verify that there exists an $\alpha > 0$ but less than 1/2 such that the expected value of equation (24) is greater than that of equation (23). To show that the variance of equation (24) is lower than that of (23), note that the variance of equation (24) is decreasing in α at zero and over some positive range. Therefore, all that needs to be shown is that Var $(\tilde{C}^1(t, i \mid \iota)) \geq$ Var $(\tilde{C}^1(t, i \mid n)) = \sigma_c^2$, where Var $(\cdot)$ is the variance operator. Now Var $(\tilde{C}^1(t, i \mid \iota)) = \sigma_c^2(N/(N-1))^2 + \sigma_x^2 - 2\sigma_{cx}(N/(N-1)^2)$, where 1/N is the initial shareholdings of equity for type-B agents when $e < \mu$ (or type-L agents when $e \geq \mu$) and σ_x^2 and σ_{cx} are the variance of cash flows for an individual project and the project cash flow's covariance with consumption for a typical type t agent who does not renegotiate. If $\sigma_{cx} \leq 0$, we are finished since, for $N \geq 2$, $(N/(N-1))^2 \geq 1$. More generally, since the cash flows from the projects are i.i.d. and a risk-averse investor will buy a non-negative amount of insurance at a zero premium, it follows that $\sigma_{xc} \leq \sigma_x^2 / N$. Straightforward algebra shows that, since by assumption, $N \geq 3$, the variance of the cash flow given by equation (23) exceeds that of the cash flow given by equation (24). □

Proposition 5 shows that, for $e < \mu$, liquidation is not stochastically dominated by renegotiation for all risk-averse investors. However, type-B agents, who in fact hold the equity in this case, would have to have preferences such that mean/variance dominated portfolios are optimal in order for liquidation to be a viable strategy. Moreover, for the case where $e \geq \mu$, all risk is borne by type-L agents, who care only about expected value. Therefore, in this case, the market allocation is not renegotiation-proof, even in the absence of corporate limited liability.

From the above discussion it is clear that, unless the renegotiation-form violates the desideratum of no first-and-final offer, incorporation of the standard liquidation option into the analysis will not, except in very unusual cases, ensure renegotiation-proofness for market allocations. The rationale for liquidation decreases further as the number of outside claimants gets larger, since in this case they own almost all of the firm in all Pareto-optimal risk allocations. It is simply not going to be in their interest to liquidate this productive asset in order to be able to obtain the renegotiating agent's portfolio, which represents only a portion of the total value of the firm. It follows that, in order to obtain renegotiation-proofness, one requires a technology which will allow shareholders to liquidate the personal wealth of the B-agents without losing project returns.

From the point of view of discouraging renegotiation and thus ensuring the renegotiation-proofness of market outcomes, the best sort of liquidation technology of this sort would be one which stipulated that *any* proposal for contract renegotiation automatically triggers the liquidation of all personal wealth of the agent who proposes the renegotiated contract. In this case, the *only* source of second period income for a type-B agent who attempts renegotiation is the cash flows he can extract through the process of renegotiation itself. It is best to think of this liquidation technology as a limiting point of all possible liquidation technologies which could be employed in a market setting to prevent contractual opportunism. Thus, to contrast it with the earlier liquidation scheme, we refer to this technology as a perfectly efficient liquidation technology. However, as the next two propositions demonstrate, even with perfectly efficient liquidation technology, Pareto-optimal allocations cannot always be implemented in market economies. The proofs of the next two propositions require results from the following lemma.

Lemma 3. In any competitive market equilibrium of (U, e, μ, N), the equilibrium utility of the B-agents is no greater than $U(\xi(0)) + U(\xi(1))$, where $\xi(0) = \mathrm{Min}[\ \frac{1}{2}E[\tilde{X}], e]$ and $\xi(1) = E[\tilde{X}] - \mathrm{Min}[\ \frac{1}{2}E[\tilde{X}], e]$.

Proof. Let $(C^0(B, i), \tilde{C}^1(B, i))$ be the consumption pattern induced by a given competitive equilibrium. First, note that the competitive market outcomes are symmetric (because endowments are symmetric) and satisfy individual rationality for all agents. Symmetry implies that the time 0 consumption of each type-B agent must be less than e because the total time 0 consumption endowment is Ne. That is,

$$i \in \mathbf{N},\ C^0(B, i) \leq e. \tag{25}$$

The individual rationality of L-agents implies that their utility must at least equal utility in the absence of trade. This implies that the sum of the date 0 and date 1 expected cash flows to each L-agent must at least equal e, their utility from consuming their endowment. This implies, by the value conservation equations (PO1 and PO2), that the sum of the expected cash flow at dates 0 and 1 to a B-agent must not exceed μ. That is, for all $i \in \mathbf{N}$,

$$C^0(B, i) + E[\tilde{C}^1(B, i)] \leq \mu. \tag{26}$$

By Jensen's inequality,

$$U(\ C^0(B, i)) + E[U(\tilde{C}^1(B, i))] \leq U(C^0(B, i)) + U(E[\tilde{C}^1(B, i)])\,. \tag{27}$$

Equations (26) and (27) imply that

$$U(C^0(B, i)) + U(E[\tilde{C}^1(B, i)]) \leq$$

$$\text{Max}_{\alpha,\beta}\,[U(\alpha) + U(\beta):\ \alpha \leq e,\ \alpha + \beta \leq E[\tilde{X}]]$$

$$= U(\xi(0)) + U(\xi(1)). \tag{28}$$

Combining equations (25) and (28) yields the desired result. □

We now show that if there exists sufficient endowment to fully ensure the income of B-agents, market allocations will not be renegotiation-proof even in the presence of perfectly efficient liquidation technology. The intuition for this result is that, when shareholders can confiscate the personal wealth of agents who attempt renegotiation, an opportunity for "opportunistic consumption" by B-agents emerges. Such agents can sell of their ownership stake in their endowed project and consume the entire proceeds from the sale at time 0, thereby rendering the personal wealth liquidation option worthless. Type-B agents can then renegotiate with the firm's new shareholders. This strategy involves opportunistic consumption because the B-agent is distorting her pattern of consumption in a Pareto-inefficient manner in order to improve her subsequent bargaining position. Personal limited liability makes this strategy credible. It is physically impossible to extract wealth from an agent who has no wealth and, thus, the brute fact of personal limited liability puts an upper bound on the ability of shareholders to extract wealth from renegotiating agents. However, consumption opportunism is not costless. In particular, it involves a reallocation of consumption across dates and states in exchange for a fairly large increase in total expected consumption. A very risk-averse agent may not find this trade off attractive. However, as we will show in Propositions 6 and 7, moderately risk-averse agents will exploit opportunities to engage in opportunistic consumption and this will lead to a failure of renegotiation-proofness of market allocations. Since claim renegotiation always produces a Pareto-inefficient allocation of risk, this implies a failure of the competitive market system to produce efficient risk allocation.

Proposition 6: Suppose that a perfectly efficient personal wealth liquidation technology exists and $e \geq \mu$. If $U(\mu)/U(\mu/2) \geq 4/3$, then the stock market economy is not renegotiation-proof.

Proof: Suppose, to obtain a contradiction, that the stock market economy with the liqudidation option is renegotiation-proof. Then, by our definition of renegotiation-proofness, agents have no incentive to renegotiate their claims given the competitive allocations provided by the stock market. However, the liquidation option is worthless if the renegotiating agent, call her agent, say (B, 1), engages in opportunistic consumption, i.e., sells her security for P_1*, consumes the proceeds now, and attempts to obtain a fraction, $\gamma = \alpha + Z(B, 1, 1)$, $1/N \geq Z(B, 1, 1) \geq 0$, of her own project's cash flow. We have already shown, in the proof of Proposition 4 that, for N> 2, outsiders will accept a renegotiated contract if $\gamma \leq 1/2$. Therefore, by choosing $\gamma=1/2$, the expected utility to agent 1 from proposing a renegotiated contract is $U(P) + E[U(\tilde{X}/2)] = U(P) + U(\mu)/2$. Now, from Lemma 3, the maximum expected utility agent (B, 1) can obtain if she does not renegotiate is $2U(\mu/2)$ (if $e \geq \mu$.). Moreover, in this case, $P = \mu$, and the stock market economy is not renegotiation-proof if $U(\mu)/U(\mu/2) \geq 4/3$. □

Proposition 6 shows that the elimination of corporate limited liability and the addition of the "best" liquidation technology cannot render market allocations renegotiation-proof in an economy in which entrepreneurs display a high level of risk tolerance and there exists a large initial endowment. The issue becomes somewhat more complex when the economy is endowment poor. The analysis of endowment poor economies is more complex because market prices are difficult to characterize in general. For an intermediate range of endowments, i.e., when $\frac{1}{2}\mu \leq e < \mu$, simple limiting characterizations are still possible. For in this case, in the limit, as N increases to infinity, it is possible for B-agents to equate marginal utility at the first and second dates. However for very low levels of endowments, even limit marginal utilities cannot be equated and, thus, second period cash flows are discounted by the market. In this case, the characterization of even the limiting relationships is subtle. These results are summarized in the next proposition.

Proposition 7. Suppose that a perfectly efficient personal wealth liquidation technology exists and $e < \mu$. If

$$U(\xi(0)) + U(\xi(1)) < U\left(\frac{U'\left(\mathrm{Max}\left[[\mu - e]^{*}, e\right]\right)}{U'(e)}\mu\right) + \left[U(\mu)\right]/2 \tag{29}$$

then as the number of investors becomes arbitrarily large ($N \to \infty$), the financial market economy is not renegotiation-proof.

Proof: The first order condition for the maximization problem implies that all equilibrium prices, P, must satisfy

$$P = \frac{E[U'(\tilde{C}^1(B,1)\tilde{X}]}{U'(C^0(B,1))} = \frac{\mu E[U'(\tilde{C}^1(B,1))] + \rho\sigma_x\sigma_{U'}}{U'(C^0(B,1))} \tag{30}$$

where ρ, σ_x, and $\sigma_{U'}$ are the correlation between $\tilde{X}$ and $U'(\tilde{C}^1(B,1))$, the standard deviation of $\tilde{X}$ and the standard deviation of $U'(\tilde{C}^1(B,1))$, respectively. Now, $\rho \geq -1$ and $\mu = \sigma_x$. For specificity, let agent 1 be the agent who considers making a renegotiation offer. By passing to subsequences, we can assume, by Proposition 1, that the sequence of allocations satisfy one or the other of the two following conditions: $C^0(B, i) = e$ or $E[U'(\tilde{C}^1(B,1))] = U'(C^0(B,i))$. Along sequences of allocations in which $E[U'(\tilde{C}^1(B,1))] = U'(C^0(B,i))$,

$$P \geq \mu(1 - (\sigma_{U'}/U'(C^0(B,1)))). \tag{31}$$

By the law of large numbers $\sigma_{U'} \to 0$, in probability. It follows that, in this case, $P \geq \mu$. By similar reasoning, if investors display DARA, one can show that, along any sequence of economies such that $C^0(B, i) = e$, $P \geq U'(E(\tilde{C}^1(B,1))\mu(1 - \sigma_{U'})/U'(e)$. Substituting for $E(\tilde{C}^1(B,1)) = \mu - e$ and $e = C^0(B, i)$ and again noting that $\sigma_{U'} \to 0$, in probability, and $\tilde{C}^1(B,1) \to \mu - e$, in probability, we have, by the continuity of U', that $P \geq (\mu U'(\mu - e)/U'(e))$. Therefore, if Equation (29) is satisfied, we have that in both cases $U(P) + U(\mu)/2 \geq U(\xi(0)) + U(\xi(1))$ and the market allocation is not renegotiation-proof$\square$

In order to obtain more insight into this issue, consider the case of B-agents whose expected utility of wealth functions exhibit constant proportional risk aversion. That is, suppose that U: $\mathbb{R}^+ \rightarrow \mathbb{R}$ is given by

$$U_\lambda(w) = \frac{w^{(1-\lambda)}}{1-\lambda}, \lambda \in [0, 1).$$

For utility functions in this class it is easy to see that the satisfaction of the renegotiation-proofness condition given in Propositions 6 and 7 depends only on the ratio e/μ. Thus by varying λ and e/μ we can present, in a two-dimensional diagram, the region over which the market economy, even with a perfectly efficient liquidation technology, is not renegotiation-proof. Inspection of the diagram in Figure 1 shows that when the economy is very date-0-endowment poor, i.e. the ratio e/μ is very small, the failure of renegotiation-proofness is assured only at very low levels of risk aversion. For example, when $e/\mu = 0.05$ the failure of renegotiation-proofness is assured by the conditions of Proposition 7 only when relative risk aversion is less than approximately 0.256. The reason for this is that whenever the economy is very endowment poor, it is not technically feasible for B-agents to equate the marginal utility of date 0 and date 1 consumption. The higher marginal utility of date 0 consumption implies a lower market value for date 1 consumption and this, in turn, implies that the increased date 0 consumption from the opportunistic strategy is small. However, once e/μ reaches 0.50 it is possible in the limit B-types to equate their date 0 and date 1 consumption levels. This relative surplus of date 0 endowment implies that, in equilibrium, L-agents are the marginal holders of such capital. This fact, combined with the Law of Large Numbers, implies that, from this point onwards, risk-neutral pricing obtains in the limit. The range of risk aversion levels at which renegotiation-proofness thus stabilizes, with renegotiation-proofness being guaranteed to fail whenever relative risk aversion is less than approximately 0.585. Proposition 6 shows that when $e/\mu \geq 1$, it is not necessary to appeal to the Law of Large Numbers in order to establish that the market allocation is not renegotiation-proof for reasonably high levels of risk tolerance.

VI. Conclusion

In this paper, we provide a rationale for financial coalitions which, unlike much of the earlier literature, does not assume costs which can be spread over a large number of investors. We formally establish the superiority of coalitions over competitive market-based financial contracts by simply combining the facts that individuals possess limited liability and have the right to propose renegotiated contracts with other individuals who hold claims on their effort-dependent future cash flows.

Simple competitive markets fail to provide for the first-best allocation of risk when some individuals are risk averse precisely because securities markets operate in a fashion such that borrowers with valuable projects can and do receive the proceeds of their future cash flows now. Conversely, under a cooperative structure, entrepreneurs can be disciplined by other members of the coalition in the sense that they receive only a portion of the project's value up front. In this case, entrepreneurs find it in their interest to forego proposing new sharing rules since an acceptable (to the coalition) renegotiated contract will provide them with, at most, an average total payoff equal to that of the original agreement.

We have also shown that there exist certain parameters of the model such that simple bilateral labor contracts can achieve the same optimality properties as those obtained by the financial coalition. This is an avenue that we feel is worth investigating since this result shows that it is not simply the bilateral nature of market contracts that cause the ex post inefficiencies with respect to risk sharing arrangements.

One advantage of market mechanisms is obscured in our analysis. When viewed from a mechanism design perspective, one can show, absent the sort of opportunities for opportunism present in our analysis, that a market mechanism can implement Pareto-optimal allocations for a wide range of preference profiles. In our analysis we restricted our coalitional implementation schemes to settings in which agents were extremely homogeneous. It is an open question as to whether coalitional implementations of first-best risk sharing arrangements are possible when greater heterogeneity of preferences is allowed, especially if there is some informational asymmetry regarding agent preferences. Perhaps the transfers and subsidies required to ensure renegotiation-proofness would make coalition formation less attractive than the insurance arrangements obtainable in a market setting.

Thus, when taken as a whole, our results suggest that coalitions will be most effective in situations where participants are fairly homogeneous with respect to information, investment opportunities, and attitudes toward work vs. leisure. While we leave this to future research, it is easy to imagine how markets could dominate, or at least coexist, with financial intermediaries in more complex economies. For example, with heterogeneous investment opportunities, some entrepreneurs may be willing to forego the risk-sharing benefits of the coalition in order to extract higher expected consumption from selling their shares in the stock market. A similar argument could be made in situations where some agents have a very high disutility of effort. Their attempt to "free-ride" on the productive output of other agents could cause the coalition to be unattractive to other members of the group. Extending the model to the multi-period case and adding asymmetric information about borrower types, as in Diamond (1989), could also increase our understanding of the necessary conditions for the optimality of market based contracts. In any case, more complex models along these lines may ultimately shed some light on the empirical regularity found in almost every developed economy; there exist a multiplicity of financial contracts, some of them market based and others which are mutual in nature.

Appendix

Proof of Proposition 1. The first order necessary conditions characterizing the Pareto-optimal allocations can be expressed as follows:

$$\gamma^i\, U'(C^0(B, i)) - \lambda_1 = 0, \text{ for all } i \in \mathbf{N}, \tag{A1}$$

$$\gamma^i\, U'(C^1(B, i)(x))\, \pi^o - \lambda_2(x) = 0, \text{ for all } i \in \mathbf{N} \text{ and } x \in X, \tag{A2}$$

$$C^0(L, i)\, (\beta^i - \lambda_1) = 0,\ \beta^i - \lambda_1 \leq 0, \text{ for all } i \in \mathbf{N}, \tag{A3}$$

$$C^1(L, i)(x)\, (\beta^i\, \pi^o - \lambda_2(x)) = 0,\ \beta^i\, \pi^o - \lambda_2(x) \leq 0, \text{ for all } i \in \mathbf{N} \text{ and } x \in X, \tag{A4}$$

$$q\, \Big(\sum_{x \in X} \lambda_2(x) - \lambda_1 \Big) = 0, \quad \sum_{x \in X} \lambda_2(x) - \lambda_1 \leq 0. \tag{A5}$$

Further, for symmetric Pareto optima of the game, the multipliers are equal across agents, that is, $\gamma_i = \gamma$, $\beta_i = \beta$ for all $i \in \mathbf{N}$.

Lemma A1. Suppose that $(q, \{C^0(B, i)\}_{i \in \mathbf{N}}, \{C^0(L, i)\}_{i \in \mathbf{N}}, \{C^1(B, i)\}_{i \in \mathbf{N}}, \{C^1(L, i)\}_{i \in \mathbf{N}})$ is a Pareto-optimal allocation. Then, for all $i \in \mathbf{N}$, and $x', x'' \in X$,

$$\bar{m}(x') \underset{\leq}{\overset{\geq}{=}} \bar{m}(x'') \Rightarrow C^1(B, i)(x') \underset{\leq}{\overset{\geq}{=}} C^1(B, i)(x'').$$

Proof. If for any $i \in \mathbf{N}$, $C^1(B, i)(x') > C^1(B, i)(x'')$ then $\lambda_2(x') < \lambda_2(x'')$. This implies that for all $j \in \mathbf{N}$, $C^1(B, i)(x') > C^1(B, i)(x'')$. Let $\bar{\beta} = \mathrm{Max}_i\, \beta^i$. By (A4), $\bar{\beta}\, \pi^o - \lambda_2(x') \leq 0$. Thus $\bar{\beta}\, \pi^o - \lambda_2(x'') < 0$. This implies, by (A4), that $C^1(L, i)(x'') = 0$ for all $i \in \mathbf{N}$. Taken together, these results imply that

$$\sum_{i=1}^{N} \big(C^1(B,i)(x') + C^1(L,i)(x')\big) \geq \sum_{i=1}^{N} C^1(B,i)(x') > \sum_{i=1}^{N} C^1(B,i)(x'') =$$

$$\sum_{i=1}^{N} (C^1(B,i)(x'') + C^1(L,i)(x'')). \tag{A6}$$

The feasibility condition (PO1) then implies that $\bar{m}(x') > \bar{m}(x'')$. Thus, if for any $i \in \mathbf{N}$,

$$C^1(B, i)(x'') > C^1(B, i)(x') \text{ for any } i \in \mathbf{N} \Rightarrow \bar{m}(x'') > \bar{m}(x'). \tag{A7}$$

Similarly, one can show that

$$C^1(B, i)(x'') < C^1(B, i)(x') \text{for any } i \in \mathbf{N} \Rightarrow \bar{m}(x'') < \bar{m}(x'). \tag{A8}$$

Taking the contrapositive of both of these implications yields the desired conclusions. □

Lemma A2. Suppose that (q, $\{C^0(B, i)\}_{i \in \mathbf{N}}$, $\{C^0(L, i)\}_{i \in \mathbf{N}}$, $\{C^1(B, i)\}_{i \in \mathbf{N}}$, $\{C^1(L, i)\}_{i \in \mathbf{N}}$) is a Pareto-optimal allocation. Then there exists a unique $k^* \in \mathbb{R}$ such that if

$$(\forall i \in \mathbf{N})\ (\forall x \in X)[m(x) < k^* \Rightarrow C^1(L, i)(x) = 0],$$

and

$$(\forall i \in \mathbf{N})\ (\forall (x', x'') \in X \times X)[\mathrm{Min}[\, m(x''), m(x')] \geq k^*,\ C^1(B, i)(x'') = C^1(B, i)(x')].$$

Proof. First, suppose that, for all $x \in X$, and $i \in \mathbf{N}$, $C^1(L, i)(x) = 0$, then the result holds trivially with k* equal to any number greater than the highest possible level of average output $\bar{x}$. Suppose next that there exists $i' \in \mathbf{N}$ and $x' \in X$ such that $C^1(L, i')(x') > 0$. Let $X_0 = \{x \in X: \exists\, i \in \mathbf{N}, C^1(L, i)(x) > 0\}$. Let $k^* = \mathrm{Min}\{m(x) : x \in X_0\}$. Let $x'' \in \mathrm{argmin}\{m(x) : x \in X_0\}$. Let i" be any $i \in \mathbf{N}$ such that $C^1(L, i'')(x'') > 0$. Then $\lambda_2(x'') = \beta^{i'} \pi^o$ (by (A4)) and $\beta^{i'} = \bar{\beta}$, again by (A4), (here $\bar{\beta}$ is defined as in the proof of Lemma A1). Let $m'' = m(x'')$ and let $\lambda_2'' = \lambda_2(x'')$. If $m(x) \geq m''$, then by Lemma A1, $C^1(B, i)(x) \geq C^1(B, i)(x'')$ and thus, by (A2), $\lambda_2(x) \leq \lambda_2''$. Because (A4) implies that $\lambda_2(x) \geq \lambda_2'' = \bar{\beta}\pi^o$, we have $\lambda_2(x) = \lambda_2''$. This implies, by (A4), that $C^1(B, i)(x) = C^1(B, i)(x'')$. □

Proof of Proposition 1(continued). (i) Existence follows because the Pareto problem satisfies standard regularity conditions. The characterization in (i) follows because symmetry implies that, for all $(i, j) \in \mathbf{N} \times \mathbf{N}$, $C^1(B, i)(x) = C^1(B, j)(x) \equiv c^1(B)(x)$. Lemma 2 shows that there exists m* such that, if $\bar{m}(x) < m^*$, $C^1(L, i)(x) = 0$. Thus, by the total resource allocation condition,

$$N\, C^1(B)(x) = \Sigma x_i + N\, q,$$

i.e., for $\bar{m}(x) < m^*$, $C^1(B)(x) = \bar{m}(x) + q$. By Lemma A2, over all x such that $m(x) \geq m^*$, $C^1(B)$ is constant. Thus, by symmetry, we have for $\bar{m}(x) \geq m^*$, $C^1(B) = k^*$ for some $k^* \in \mathbb{R}$. Since consumption is monotone in $\bar{m}(x)$, we also have that $C^1(B, i)(x) = \mathrm{Min}[q + m(x), k^*]$. The resource allocation condition thus implies that $C^1(L, i)(x) = \mathrm{Max}[q + \bar{m}(x) - k^*, 0]$.

(ii) To prove (ii) first note that, in any Pareto-optimal allocation, (A1), (A2), and (A5) imply that

$$E[U'(\tilde{C}^1(B, i)))] \leq U'(C^0(B, i)), \text{ for all } i \in \mathbf{N}. \tag{A9}$$

Symmetry implies that there exist scalars $c^d(t)$, $t \in \mathbf{T}$, and $d = 0$ or 1 such that $C^1(t, i)(x) \equiv c^1(t)(x)$ and $C^0(t, i) \equiv c^0(t)$ for all $(t, i) \in \mathbf{T} \times \mathbf{N}$. Feasibility (PO2) thus implies that $c^0(B) \leq e$. To show that (ii) holds, we need only show that

$$E[U'(\tilde{C}^1(B, i))] < U'(C^0(B, i)) \Rightarrow c^o(L) = 0 \text{ and } q = 0. \tag{A10}$$

Feasibility (PO2) implies $c^o(L) = 0$ and $q = 0 \Rightarrow c^0(B) = e$. To establish (A10), let x^o be any state in which $c^0(B) = k^*$ (where k^* is defined as in Proposition 1) such that L-agents have positive consumption. Because $c^1(L)(x^o) > 0$, we have by (A4),

$$\beta = \lambda(x^o)/\pi^o. \tag{A11}$$

We also have, because k^* is the highest possible level of consumption at time 1 for B-agents

$$U'(k^*) = U'(c(B)(x^o)) \leq E[U'(c^1(B))] < U'(c^0(B)).$$

This implies, by (A1) and (A2), that $\lambda_1 > \lambda_2(x^o)/\pi^o$. Thus, by (A1), (A3) and (A4), $c^0(L) = 0$. Because $E[U'(c^1(B))] < U'(c^0(B))$, (A5) implies that $q = 0$. □

Proposition 4. Let $(Z^*, Z^{*c}, C^*, S^*, P, P^c)$ be a competitive equilibrium of the economy (U, e, μ, N); then $(Z^*, Z^{*c}, C^*, S^*, P, P^c)$ is not renegotiation-proof.

Proof of Proposition 4. For definiteness, suppose that $j = 1$. Let $\tilde{C}^1(t,i|y)$ be the consumption of type-L agents should they accept the proposal. It follows that

$$\begin{aligned}\tilde{C}^1(t,i|y) &= S^*(t,i) + \textstyle\sum_{j\neq 1} Z^*(t,i,j)\tilde{X}_j + \sum_w Z^{c*}(t,i,w)\mathrm{Max}(\tilde{M} - w, 0)\\ &\quad +(1-\gamma)(Z^*(t,i,1))\tilde{X}_1 / (1 - Z^*(B,1,1))\\ &= \tilde{C}^1(t,i,-1) + (1-\gamma)(Z^*(t,i,1))\tilde{X}_1 / (1 - Z^*(B,1,1))\end{aligned} \tag{A12}$$

$$\tilde{C}^1(t,i,-1) \equiv S^*(t,i) + \sum_{j\neq 1} Z^*(t,i,k)\tilde{X}_j + \sum_w Z^c{}^*(t,i,w)\,\mathrm{MAX}\left(\tilde{M} - w, 0\right) \tag{A13}$$

where $\gamma \equiv \alpha + Z(B,1,1)$ is the proposed fraction of output to the entrepreneur and α is defined in the text. Alternatively, suppose that the proposal is rejected. Then the cash flows to a representative type t agent will be either

$$\tilde{C}^1(t, i, -1) \text{ or } \tilde{C}^1(t, i, -1) + \frac{Z^*(t, i, 1)}{1 - Z^*(t, 1, 1)} \tilde{X}_1$$

depending on the allocation of bargaining power by the coin-flip. Thus, the expected payoff from rejecting the offer is

$$E[\tilde{C}^1(t, i, -1)] + \tfrac{1}{2}\,\frac{Z^*(t, i, 1)}{1 - Z^*(t, 1, 1)}\,\mu\,.$$

By the conditional form of Jensen's inequality, we see that the expected utility of rejecting the project for a type-B agent is less than $E[U(\tilde{C}''(B, i, -1)]$, where

$$\tilde{C}''(t, i, -1) \equiv \tilde{C}^1(t, i, -1) + \tfrac{1}{2}\,\frac{Z^*(t, i, 1)}{1 - Z^*(t, 1, 1)}\,\tilde{X}_1. \qquad \text{(A14)}$$

For type-L agents, the expected utility from rejecting the renegotiation proposal is

$$E[\tilde{C}''(L, i, -1)] = E[\tilde{C}^1(t, i, -1)] + \tfrac{1}{2}\,\frac{Z^*(t, i, 1)}{1 - Z^*(t, 1, 1)}\,\mu. \qquad \text{(A15)}$$

Consider a renegotiaton offer of $\gamma \leq 1/2$. Such an offer, if accepted, produces a cash flow to agent (t, i) of

$$\tilde{C}^1(t, i, -1 \mid A) \equiv \tilde{C}^1(t, i, -1) + (1 - \gamma)\,\frac{Z^*(t, i, 1)}{1 - Z^*(t, 1, 1)}\,\tilde{X}_1. \qquad \text{(A16)}$$

Because $\gamma \leq 1/2$, $\tilde{C}^1(t, i, -1 \mid A)$ stochastically dominates $\tilde{C}''(B, i, -1)$ in a first order sense. Thus, both L and B type agents will accept all renegotiaton offers in which $\gamma \leq 1/2$. The second period cash flow to (B, 1) from making such an offer and having the offer accepted is equal to

$$\tilde{C}^1(B, 1, -1 \mid A) \equiv \tilde{C}^1(B, 1, -1) + \gamma\tilde{X}_1. \qquad \text{(A17)}$$

The second period cash flow from not renegotiating is

$$(\tilde{C}^1(B, 1)n) = \tilde{C}^1(B, 1, -1) + Z^*(B, 1, 1)\,\tilde{X}_1. \qquad \text{(A18)}$$

Because competitive allocations are symmetric and Pareto-optimal, it must be the case that $Z^*(B, 1, 1) < 1/N \leq 1/3$, because, by assumption $N \geq 3$. Thus, there exists $\gamma > Z^*(B, 1, 1)$ which, if proposed, will be accepted. Because the second period cash flow to (B, 1) from having such a proposal accepted dominates that from not renegotiating in the sense of first order stochastic dominance, (B, 1) can always gain by renegotiating. Thus, the competitive equilibrium allocation is not renegotiation-proof. □

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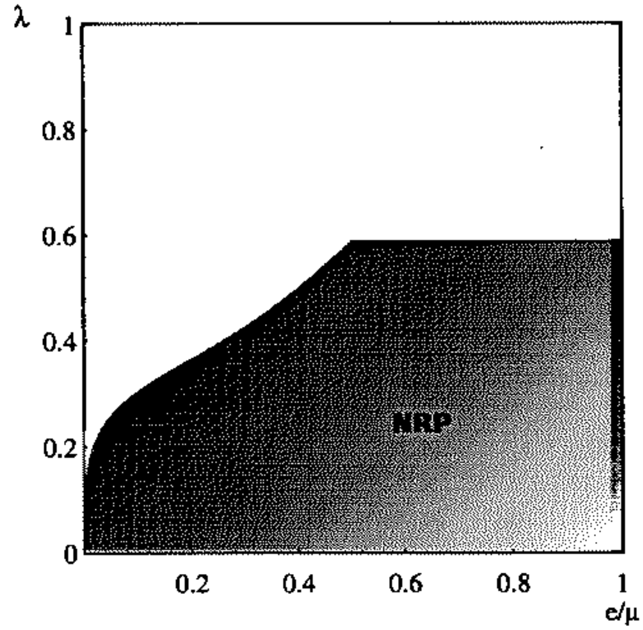

Figure 1. Renegotiation proofness with perfect liquidation technology.

Figure 1 depicts the set of parameters under which market allocations are not renegotiation-proof even when shareholders have recourse to a "perfectly" efficient liquidation technology, a technology which allows them to seize the personal wealth of any agent who attempts to renegotiate her contractual relations with the firm without incurring any dissipative costs. The diagram depicts the case of agents with constant proportional risk aversion represented by $\lambda \in [0, 1)$.The value of this parameter is represented along the vertical axis. Under constant proportional risk aversion, the only other relevant parameter for determining whether the conditions of Proposition 7 are satisfied is the ratio between the time 0 endowment of the L-agents (e) and the expected value of the time 1 cash flows to the B-agents (μ). The value of this ratio is represented along the horizontal axis. The shaded region labeled "NRP" represents the parameter values which satisfy the sufficient conditions of Proposition 7 for the failure of renegotiation-proofness.