A Transitional Analysis of The Welfare Cost of Inflation

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Abstract: This paper applies new computational methods for studying nonstationary dynamics to reevaluate the welfare cost of inflation. A dynamic stochastic general equilibrium model with heterogeneous agents is studied. Incomplete markets induce agents to hold a fiat currency as insurance against idiosyncratic income fluctuations. Rather than comparing steady state equilibria, I measure the welfare cost of inflation by explicitly modeling the transitional dynamics that arise following a change in monetary policy. Transitional dynamics are shown to increase the welfare cost of inflation substantially. Also, contrary to conventional wisdom, transitional dynamic effects are shown to increase the benefits of reducing the inflation rate.

JEL classification: E4, E5

Key words: transitional dynamics, welfare cost of inflation, computable DSGE model

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1 Introduction

Traditionally, quantitative analysis of monetary policy has focused only on the long-run or steady state impact of monetary policy. This is especially true of efforts to measure the welfare cost of inflation. Bailey (1956), Fischer (1981), Lucas (1981, 1994), Cooley and Hansen (1989), Kehoe, Levine, and Woodford (1989) İmrohoroğlu and Prescott(1991), İmrohoroğlu (1992), Gomme (1993), and Dotsey and Ireland (1996) all estimate the welfare cost of inflationary monetary policy by comparing the steady states of economies exhibiting different inflation rates. These authors find the welfare cost of a steady, fully anticipated 10% inflation rate to be as high as 1.73% of annual GDP.[1] According to these estimates, the long-run impact of inflation on the welfare of consumers is substantial, nearly $120 billion in 1996.[2]

In addition to substantial long-run effects, however, monetary policy also has important short-run effects. Understanding the quantitative significance of these short-run effects is important because monetary policymakers seldom, if ever, face a choice between two or more steady states. Rather they find themselves and the economy in one particular set of circumstances and must evaluate the consequences of changing some element of monetary policy. Cooley and Hansen (1989) note that there may be adjustment costs associated with these changes in monetary policy that must be considered in addition to any long-run effects. Lucas (1981) also recognized the impor-

[1] Feldstein (1996) and Bullard and Russell (1997) find larger welfare cost estimates due to the interaction of the inflation tax with other forms of distortionary taxation.

[2] This figure is based on a 1996 real GDP of $6928.4 billion in chain weighted 1992 dollars.

tance of these short-run adjustments, noting that "the actual policies we try to evaluate are generally erratic in various systematic or unsystematic ways."

This paper extends recent work in monetary economics to reexamine the welfare costs of monetary policy. Following İmrohoroğlu (1992), we study an economy populated by heterogeneous agents and a government. There is no aggregate uncertainty in the model, but agents face idiosyncratic income uncertainty. Because income realizations are private information, there is no market for state contingent debt, and markets for private loans and other insurance instruments are precluded exogenously as in the work of Lucas (1980), Scheinkman and Weiss (1986), and Kehoe, Levine, and Woodford (1989). Incomplete markets induce the agents to hold nonnegative quantities of a fiat currency as a means of smoothing their consumption over time as discussed by Schechtman (1976) and Bewley (1977).

Inflation arises because the government prints money and distributes the seignorage revenue to the agents via lump sum transfers. These transfers increase the lifetime utility of the poorest agents and can be interpreted as a form of government insurance against low wealth. The inflation generated by these transfers, however, causes wealthier agents to economize on their holdings of real balances, hindering their ability to smooth consumption over time and lowering their lifetime utility. Typically, these competing effects generate a welfare loss in the aggregate.

Because money holdings must be nonnegative, the optimization problem faced by consumers involves a binding inequality constraint. This constraint precludes analytical solution of the model, generating a need for computational solution methods. A discrete state dynamic programming algorithm

is used to solve for the steady state equilibrium of the model. A similar algorithm, modified to accommodate nonstationary dynamics, is used to solve for the dynamic rational expectations equilibrium adjustment path following a change in the economic policy parameters of the model. These numerical procedures characterize both the long-run and short-run transitional response of the economy to monetary policy decisions.

Transitional dynamic analysis is used to reevaluate the welfare cost of inflation. The lifetime utility of agents is evaluated under varying assumptions about monetary policy. A compensating variation in income, which measures both the long and short-run effects of monetary policy decisions, is used to measure the true welfare cost of monetary policy decisions. The results of a transitional welfare analysis show that a simple analysis of steady states can dramatically misrepresent the true welfare costs of a monetary policy decision. For some parameterizations of the model the transitional welfare impact of inflationary monetary policy can exceed the steady state impact by more than 120%. In some extreme cases, transitional dynamic analysis can even lead to welfare cost estimates qualitatively different from those implied by an analysis of steady states.

The second section of this paper describes the economic environment being studied and defines both stationary and transitional equilibrium for this environment. Section three describes several different parameterizations of the model. Section four develops a measure of the transitional welfare cost of inflationary monetary policy and compares this measure with more traditional steady state welfare cost measures for several different policy changes. Section five concludes and an appendix contains all figures.

2 The Model

We consider a simple economic environment populated by a continuum of utility maximizing agents and a government. Time in the model is discrete, and all agents live forever. At any point in time t, there are two goods in the economy: a consumption good and an unbacked fiat currency. The consumption good is perishable and must be consumed during the current period. The fiat currency, on the other hand, is durable and can be used to transfer wealth between the current period and the next.

All agents in the economy have identical preferences described by a standard time separable utility function

$$U(\{c_t\}_{t=0}^{\infty}) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad 0 < \beta < 1. \tag{1}$$

The period utility function $u(\cdot)$ is assumed to be isoelastic

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}. \tag{2}$$

The restriction $0 < \sigma < \infty$ ensures that $u(c_t)$ satisfies the usual conditions: $u'(c_t) > 0$, $u''(c_t) < 0$, and the Inada condition $\lim_{c_t \to 0} u'(c_t) = \infty$. The discount factor β, and the parameter σ are the same for all agents. Expectations are computed over realizations of stochastic process affecting the agents and are conditional upon information known at time $t = 0$.

The income of each agent is determined by stochastic employment opportunities. We assume that an agent's employment status can be represented by a two-state Markov chain. In state one, an individual agent is employed and earns an income y_1. In state two, an agent is considered unemployed an earns an income y_2. The transition probabilities for these Markov employment opportunities are time invariant and are the same for every agent.

However, the employment status, and hence the income, of an individual agent is assumed to be independent of the income of all other agents. Thus each agent's current income is an independent realization of a common two state Markov chain with time invariant transition probabilities.[3] Furthermore, it is assumed that an agent's real income is unobservable to all other agents.

Because the realizations of the income process are unobservable, there is no market for state contingent debt that could insure the agents against the idiosyncratic uncertainty of real income. The only markets that do exist in this economy, at date t, are a market for the single time t consumption good and a market for an unbacked fiat currency.

The government performs monetary policy by making lump sum transfer payments $\tilde{T}_t$ to agents in each period t.[4] The agents behave competitively, taking prices and transfer payments as given. A typical agent solves

$$\max_{\{c_t\},\{\tilde{m}_t\}} \left\{ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \tag{3}$$

subject to the sequence of budget constraints

$$c_t + m_t \leq \frac{p_{t-1}}{p_t} m_{t-1} + y_t + \frac{\tilde{T}_t}{p_t} \quad \forall t = 0, 1, 2, \ldots \quad , \tag{4}$$

the liquidity constraints

$$m_t \geq 0 \quad \forall t = 0, 1, 2, \ldots \quad , \tag{5}$$

[3]The assumption of a continuum of agents, each receiving an independent realization of a common stochastic process, implies that aggregate real income is constant across periods; *cf.* Green (1987). There is no aggregate uncertainty in the model.

[4]Variables covered by a tilde denote nominal quantities, while the corresponding variables without tildes denote real quantities.

and the law of motion for income.

In solving their optimization problem, agents take their initial real balances m_{-1} as given and receive their initial income from some unconditional probability distribution on the state space of the income process. Because income realizations are idiosyncratic and stochastic, agents are heterogeneous with respect to their holdings of real balances. The distribution of agents entering period t is defined by a probability measure $\lambda(m_{t-1}, y_t)$ that describes the fraction of the population with real balances m_{t-1} and current income y_t. The ordered pair (m_{t-1}, y_t) evolves as a two dimensional Markov chain with transition probabilities determined by the transition probabilities of the exogenous income process and the optimal decision rule of a typical agent. These transition probabilities govern the evolution of the population distribution and under certain circumstances will determine a unique invariant distribution for the population.

If the nominal money stock, measured in per capita terms, evolves according to the deterministic difference equation $\tilde{M}_{t+1} = \tau \tilde{M}_t$ for any $1 < \tau < \infty$, then there exists a stationary equilibrium with a constant level of real balances and a constant gross inflation rate equal to τ. Given constant real balances M and a constant gross inflation rate of τ, we can write a typical agents optimization problem as a stationary discounted dynamic programming problem

$$V(m_{t-1}, y_t) = \max_{m_t} \left\{ u\Big(\frac{m_{t-1}}{\tau} + y_t + \frac{(\tau - 1)}{\tau} M - m_t\Big) + \beta E_t V(m_t, y_{t+1}) \right\} \tag{6}$$

where the maximization is subject to the constraints

$$0 \leq m_t \leq \frac{m_{t-1}}{\tau} + y_t + \frac{(\tau - 1)}{\tau} M \quad \forall t = 0, 1, 2, \ldots \quad , \tag{7}$$

and the law of motion for income.

Bellman's functional equation **(6)** can be solved numerically[5] for the value function $V(\cdot,\cdot)$. Numerical solution of Bellman's functional equation also provides the policy function

$$g(m_{t-1}, y_t) = \arg\max_{m_t} \left\{ u\Big(\frac{m_{t-1}}{\tau} + y_t + \frac{(\tau-1)}{\tau}M - m_t\Big) + \beta E_t V(m_t, y_{t+1}) \right\} \tag{8}$$

where the maximization is again subject to the constraints **(7)** and the law of motion for income. Given the time invariant policy function $g(\cdot,\cdot)$ we can define the Markov transition probabilities of the two dimensional state (m_{t-1}, y_t) using the stochastic transition function

$$Q((m,y),(A,B)) = \begin{cases} Q_y(y,B), & \text{if } \ g(m,y) \in A; \\ 0, & \text{if } \ g(m,y) \notin A \end{cases} \tag{9}$$

where $Q_y(y,B)$ represents the transition probabilities for the exogenous Markov income process. Given this stochastic transition function, a stationary distribution over the state space exists and satisfies

$$\lambda(A,B) = \int_{R_+}\int_Y Q((m,y),(A,B))\lambda(dm,dy) \quad \forall A \in \mathcal{B}_+, B \in \mathcal{Y}. \tag{10}$$

The stationary distribution $\lambda(\cdot,\cdot)$ represents the distribution of the population of agents over the state space $\mathbf{R}_+ \times Y$.

The distribution of the population combines with the optimal policy function to determine the per capita demand for real balances, denoted $\bar{m}$.

$$\bar{m} = \int_{R_+}\int_Y g(m,y)\lambda(dm,dy). \tag{11}$$

[5]The inequality constraints preclude an analytical solution.

A *stationary equilibrium* for this economy consists of values for τ and M, value and policy functions $V(\cdot,\cdot)$ and $g(\cdot,\cdot)$, and a probability measure $\lambda(\cdot,\cdot)$ such that:

1. The function $V(\cdot,\cdot)$ solves Bellman's functional equation **(6)**.

2. The function $g(\cdot,\cdot)$ satisfies equation **(8)**.

3. The probability measure $\lambda(\cdot,\cdot)$ satisfies equation **(10)** where the stochastic transition function $Q((m,y),(A,B))$ is induced by the policy function $g(\cdot,\cdot)$ according to equation **(9)**.

4. The market for real balances clears

$$M = \int_{R_+} \int_Y g(m,y)\lambda(dm,dy). \tag{12}$$

The parameter τ represents a monetary policy instrument, and for each $1 < \tau < \infty$ a stationary equilibrium exists. The definition of a stationary equilibrium for this economy depends, however, on a constant value of the parameter τ. Following a change in the value of τ (a change in monetary policy) the economy will not be in stationary equilibrium. Following a change in τ, there will be a period of adjustment as the economy moves between steady states. This period of transition will be characterized by nonstationary dynamics as prices and real balances adjust to their new steady state equilibrium values.[6]

[6]For an in-depth discussion of nonstationary dynamics and transitional equilibrium the interested reader should see Burdick (1994).

If we assume a finite transition horizon of T periods,[7] then following a change in monetary policy from τ_0 to τ_{T+1}, we can define a *transitional equilibrium* to be a sequence $\{\pi_t\}_{t=0}^{T+1}$ of gross inflation rates, a sequence $\{M_t\}_{t=0}^{T+1}$ of associated real balances, sequences $\{V_t(\cdot,\cdot)\}_{t=0}^{T+1}$ and $\{g_t(\cdot,\cdot)\}_{t=0}^{T+1}$ of value and policy functions, and a sequence $\{\lambda_t(\cdot,\cdot)\}_{t=0}^{T+1}$ of probability measures such that:

1. For $i = 0, T+1$ $\pi_i = \tau_i$ and τ_i, M_i, $V_i(\cdot,\cdot)$, $g_i(\cdot,\cdot)$, $\lambda_i(\cdot,\cdot)$ represent a stationary equilibrium.

2. For all $t = 1, 2, \ldots, T$, the value functions $V_t(\cdot,\cdot)$ satisfy a nonstationary version of Bellman's equation:

$$V_t(m_{t-1}, y_s) = \max_{m_t}\Big\{u\Big(\frac{m_{t-1}}{\pi_t} - m_t + y_t + \frac{(\tau_{T+1}-1)}{\pi_t}M_{t-1}\Big) + \beta E_t V_{t+1}(m_t, y_{t+1})\Big\} \tag{13}$$

   subject to the constraint

$$0 \leq m_t \leq \frac{m_{t-1}}{\pi_t} + y_t + \frac{(\tau_{T+1}-1)}{\pi_t}M_{t-1}. \tag{14}$$

3. For all $t = 1, 2, \ldots, T$, the policy function $g_t(\cdot,\cdot)$ satisfies:

$$g_t(m_{t-1}, y_s) = \arg\max_{m_t}\Big\{u\Big(\frac{m_{t-1}}{\pi_t} - m_t + y_t + \frac{(\tau_{T+1}-1)}{\pi_t}M_{t-1}\Big) + \beta E_t V_{t+1}(m_t, y_{t+1})\Big\} \tag{15}$$

4. For all $t = 1, 2, \ldots, T$, the distribution of agents $\lambda_t(\cdot,\cdot)$ satisfies:

$$\lambda_{t+1}(A, B) = \int_{R_+}\int_Y Q_t((m,y),(A,B))\lambda_t(dm, dy) \quad \forall t = 1, 2, 3, \ldots, T. \tag{16}$$

[7] Although the transition horizon can be infinite theoretically, assuming a finite transition horizon causes no problems in practice. A transition horizon of $T = 150$ months was used for the calculations in this paper.

Where $Q_t((\cdot,\cdot),(\cdot,\cdot))$ is the stochastic transition function induced by the policy function $g_t(\cdot,\cdot)$ according to a suitably modified version of equation **(9)**.

5. For all $t = 1, 2, \ldots, T$, the market for real balances clears

$$M_t = \int_{R_+} \int_Y g_t(m, y)\lambda_t(dm, dy) \quad \forall t = 1, 2, \ldots, T. \tag{17}$$

3 Calibration

The economy studied in this paper can be fully parameterized by the values for the preference parameters σ and β, the state space Y and transition probabilities $Q_y(y', y)$ for the exogenous income process, and the value for the parameter τ which determines the annual inflation rate. The values for the preference parameters $\sigma = 1.5$ and $\beta = .9957$ are the same for all economies studied in this paper. The value $\sigma = 1.5$ is common for this literature and $\beta = .9957$ was chosen to imply an annual discount rate of 5% when the length of a time period is taken to be one month. Three different specifications for the exogenous income process are investigated, and, for each of these three specifications, the parameter τ is varied from $\tau = 1.0$ to $\tau = 1.0117$, implying an annual inflation rate from 0% to 15% with the one month time period interpretation.

The first economy, referred to as economy one, has a Markov chain generating the idiosyncratic real income realizations with state space $Y = \{y_1, y_2\} = \{1.00, 0.25\}$ and transition probabilities described by the matrix

$$Q_y = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.9565 & 0.0435 \\ 0.5000 & 0.5000 \end{bmatrix}. \tag{18}$$

This parameterization of the exogenous income process corresponds roughly to recent U.S. experience, *cf.* İmrohoroğlu (1992). The average duration of an unemployment spell (the average length of time spent in the low income state) is two model periods or two months, and the aggregate unemployment rate is 8%. The income of an employed person is normalized to one and an unemployed person earns 25% of their employed income. Although these numbers do not reflect the unemployment statistics usually reported for the U.S. economy, in this model there is no distinction between in and out of the labor force. The income of unemployed persons in the economy is higher than that provided by U.S. unemployment insurance reflecting the opportunities of home production.

Table 1 contains the summary statistics for economy one for varying levels of inflation.[8] The effect of inflation on the steady state equilibria of economy one can be seen clearly in Table 1. As the annual inflation rate increases, the quantity of real balances held per capita, measured by Em, falls as agents attempt to avoid the inflation tax. As agents economize on their holdings of real cash balances, their ability to smooth consumption is reduced, and the standard deviation of consumption, measured by σ_c, increases.[9] The higher standard deviation of consumption causes agents to realize lower levels of utility as measured by the variable $\tilde{u}$.

[8]The values appearing in Table 1 are all numerical. The fact that average income and the standard deviation of income are equal to their theoretical values and that the market for the consumption good clears attests to the accuracy of the numerical solution.

[9]Because the economy is populated by a continuum of ex-ante identical agents, the variable σ_c measures both the standard deviation of consumption across agents at a point in time and also the standard deviation of individual consumption over time.

Annual Inflation Rate

	0.0%	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%
Ey	0.9400	0.9400	0.9400	0.9400	0.9400	0.9400	0.9400
σ_y	0.2035	0.2035	0.2035	0.2035	0.2035	0.2035	0.2035
Ec	0.9400	0.9400	0.9400	0.9400	0.9400	0.9400	0.9400
σ_c	0.0711	0.0800	0.0871	0.0932	0.0985	0.1023	0.1067
Em	2.4573	2.0207	1.7531	1.5537	1.4001	1.2886	1.1838
σ_m	0.7783	0.6413	0.5530	0.4894	0.4383	0.4076	0.3701
$\tilde{u}$	-0.1336	-0.1378	-0.1415	-0.1445	-0.1483	-0.1510	-0.1540

Table 1: Summary Statistics for Economy One

The variable $\tilde{u}$ appearing in Table 1 is defined as

$$\tilde{u} = (1-\beta)\int_{R_+}\int_Y V(m,y)\lambda(dm,dy). \tag{19}$$

For each inflation rate, this variable measures the average lifetime utility realized by agents, normalized to a per period level. The variable $\tilde{u}$ represents a social welfare function in which all agents utility is weighted equally. The welfare effects of inflationary monetary policy will be investigated more closely in the next section of the paper.

The second economy, economy two, is identical to economy one except that the parameters of the exogenous income process are changed. The income levels associated with the employed and unemployed states are kept the same so that $Y = \{y_1, y_2\} = \{1.00, 0.25\}$, but the transition probabilities of the Markov chain are changed. For economy two we let the transition

Annual Inflation Rate

	0.0%	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%
Ey	0.7425	0.7425	0.7425	0.7425	0.7425	0.7425	0.7425
σ_y	0.3561	0.3561	0.3561	0.3561	0.3561	0.3561	0.3561
Ec	0.7425	0.7425	0.7425	0.7425	0.7425	0.7425	0.7425
σ_c	0.0465	0.0518	0.0559	0.0593	0.0623	0.0649	0.6700
Em	1.0893	0.9347	0.8387	0.7694	0.7165	0.6747	0.6428
σ_m	0.5969	0.5252	0.4821	0.4516	0.4294	0.4119	0.3968
$\tilde{u}$	-0.4255	-0.4251	-0.4242	-0.4220	-0.4215	-0.4210	-0.4194

Table 2: Summary Statistics for Economy Two

probabilities for the exogenous income process be given by

$$Q_y = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.5000 & 0.5000 \\ 0.9565 & 0.0435 \end{bmatrix}. \tag{20}$$

This parameterization of the income process reflects an economy dominated by a large amount of temporary employment. An unemployed person is very likely to find a job quickly, $p_{21} = .9565$, but periods of employment last an average of only two months, and the aggregate employment rate for this economy is 66%. Although these statistics do not reflect aggregate U.S. experience, they may reflect conditions in some sectors of the U.S. economy. Summary statistics for economy two appear in Table 2.

Economy three is again identical to the previous two except for the parameterization of the exogenous real income process. The values of employed income and unemployed income are maintained at $Y = \{y_1, y_2\} = \{1.00, 0.25\}$,

Annual Inflation Rate

	0.0%	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%
Ey	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250
σ_y	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750
Ec	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250
σ_c	0.0625	0.0694	0.0748	0.0792	0.0830	0.0864	0.0893
Em	2.3380	1.9632	1.7300	1.5740	1.4483	1.3492	1.2732
σ_m	1.2359	1.0651	0.9588	0.8846	0.8292	0.7857	0.7486
$\tilde{u}$	-0.5417	-0.5446	-0.5472	-0.5493	-0.5514	-0.5532	-0.5549

Table 3: Summary Statistics for Economy Three

but the transition probabilities are changed to

$$Q_y = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.5000 & 0.5000 \\ 0.5000 & 0.5000 \end{bmatrix}. \tag{21}$$

This corresponds to an economy in which employment opportunities are white noise. There is no persistence in either employment or unemployment, and the aggregate employment and unemployment rates are both 50%. Table 3 contains summary statistics for this economy.

4 Welfare Analysis

Bailey (1956) pointed out that one measure of the welfare loss due to inflation is the area of the triangle under the long-run demand curve for real balances. Using a formula of Lucas (1981), we can express the area of this triangle as

a fraction C of real GDP

$$C = \frac{1}{bv}[1 - (1 + b\Pi)e^{-b\Pi}] \approx \frac{1}{2}\frac{b}{v}\Pi^2. \tag{22}$$

In equation **(22)**, b represents the interest semi-elasticity of the demand for real balances, v is the velocity of circulation, and Π is the annual inflation rate. Using this measure of welfare loss, Lucas (1981) found the welfare cost of 5% and 10% inflations to be 0.13% and 0.45% of GDP respectively. Using a similar measure of welfare loss, Fischer (1981) finds the welfare cost of a 10% inflation to be 0.3% of GDP.

Equation **(22)** can be used to estimate the steady state welfare cost of inflation in this model. Considering the case of a 10% annual inflation rate in economy one, we find the semi-interest elasticity of the demand for real balances to be

$$b = \left|\frac{1.4001 - 2.4573}{0.10 - 0.0}\right| \frac{1}{\frac{1.4001+2.4573}{2}} = 5.4814. \tag{23}$$

Velocity in this economy is defined as average annual income per capita, $12 \times 0.9400 = 11.28$, divided by average real balances per capita at zero inflation yielding $v = 4.5904$. Using equation **(22)**, these parameter values imply that the steady state welfare loss due to a 10% annual inflation rate is 0.59% of GDP. Similar calculations can be performed for the other economies and for different inflation rates.

Unfortunately, the long-run demand for money is not well defined following a change in monetary policy and hence this measure of the welfare cost of inflation cannot be used to determine the transitional welfare cost of inflation. One measure of the welfare cost of inflation that can be applied

for both the steady state and transitional dynamic approach is based upon the social welfare function

$$\tilde{u} = (1-\beta)\int_{R_+}\int_Y V(m,y)\lambda(dm,dy). \tag{24}$$

This social welfare function represents the average lifetime utility of an agent, normalized to a per period level of utility.[10] Because the measure $\lambda(\cdot,\cdot)$ represents both the distribution of agents over the state space $\mathbf{R}_+ \times Y$ and the relative amount of time that an individual agent spends in any measurable subset of the state space, $\tilde{u}$ measures both average utility computed over all agents, and average lifetime utility of an individual agent computed over time into the infinite future. Hence, $\tilde{u}$ represents a very intuitive social welfare function.

The problems with utility comparisons are well known, and averaging utilities over agents or over time certainly does nothing to alleviate these problems. We need to develop a measure of deadweight loss that is invariant to monotonic transformations of the period utility function $u(c)$.[11] One approach, advocated by İmrohoroğlu (1992), is to compute a compensating variation in income.

To compute a compensating variation in income, let the initial steady state represent the point of reference. The initial steady state of the economy will imply some value $\tilde{u}_0$ for the social welfare function. Once the

[10] The values $\tilde{u}$ for a number of steady state equilibria of our three economies appear in Tables 1, 2, and 3.

[11] All of the results reported so far are unique only up to a monotonic transformation of the utility function $u(c)$ and the corresponding transformation of the value function $V(m,y)$.

value $\tilde{u}_0$ is established we can examine the welfare effect of some change in monetary policy. Let the sequence of transitional value functions be given by $\{V_t(\cdot,\cdot)\}_{t=0}^{T+1}$ so that V_{T+1} represents lifetime utility in the new steady state equilibrium, and V_1 represents lifetime utility immediately following the change in policy. From the value functions V_1 and V_{T+1} we can compute

$$\tilde{u}_1 = (1-\beta)\int_{R_+}\int_Y V_1(m,y)\lambda_1(dm,dy), \tag{25}$$

$$\tilde{u}_{T+1} = (1-\beta)\int_{R_+}\int_Y V_{T+1}(m,y)\lambda_{T+1}(dm,dy), \tag{26}$$

where λ_1 represents the distribution of agents immediately following the policy change, and λ_{T+1} represents the distribution of agents in the new steady state.

In addition to changing monetary policy, we can also simultaneously alter the income process by changing the state space $Y = \{y_1, y_2\}$ to $Y(\theta) = \{(1+\theta)y_1, (1+\theta)y_2\}$ for $\theta \in R_+$. Each value of θ will imply values for $\tilde{u}_1(\theta)$ and $\tilde{u}_{T+1}(\theta)$. If we define the values θ_t and θ_s according to $\tilde{u}_1(\theta_t) = \tilde{u}_0$ and $\tilde{u}_{T+1}(\theta_s) = \tilde{u}_0$, then θ_t represents the variation in income required to compensate agents for the transitional impact of the policy change and θ_s is the variation in income required to compensate agents for the steady state impact of the change in policy. Unfortunately, this measure of the deadweight loss due to inflation is computationally expensive, requiring repeated solution of the transition path for different values of the variable θ.

A related, but computationally less expensive, measure of the welfare cost of inflation has been used by Cooley and Hansen (1989) and Gomme (1993). These authors compute the consumption subsidy required to raise the steady state utility of a representative agent to an appropriate reference

level. Cooley and Hansen compare the utility of a representative agent in an inflationary environment to the level achieved under the Pareto optimal allocation. Gomme compares the utility of a representative agent in an inflationary environment to the level achieved under an optimal monetary policy in which the cash in advance constraint does not bind. In this model there is no representative agent, no cash in advance constraint, and the Pareto optimal allocation cannot be achieved in this economy, but we can compute an analogous measure of the welfare loss by considering average utilities before and after a policy change.

Using the values $\tilde{u}_0, \tilde{u}_1$, and $\tilde{u}_{T+1}$ defined above we can define the values $\tilde{c}_0, \tilde{c}_1$, and $\tilde{c}_{T+1}$ implicitly as

$$u(\tilde{c}_0) = \tilde{u}_0; \tag{27}$$

$$u(\tilde{c}_1) = \tilde{u}_1; \tag{28}$$

$$u(\tilde{c}_{T+1}) = \tilde{u}_{T+1}; \tag{29}$$

where $u(c)$ is the period utility function.[12] Using the values $\tilde{c}_i \quad i = 0, 1, T+1$ define $\tilde{c}_t = \tilde{c}_1 - \tilde{c}_0$ and $\tilde{c}_s = \tilde{c}_{T+1} - \tilde{c}_0$. The variable $\tilde{c}_t$ represents the consumption subsidy that must be paid to compensate agents for the transitional impact of a change in policy, while the variable $\tilde{c}_s$ represents the consumption subsidy that must be paid to compensate agents for the steady state impact of the policy change. We can express these consumption subsidies as a fraction of real GDP by dividing the subsidy by per capita real income. We define $\omega_t = \frac{\tilde{c}_t}{Ey}$ and $\omega_s = \frac{\tilde{c}_s}{Ey}$ where Ey is aggregate income per capita

[12] Given the period utility function assumed throughout this paper, the variables $\tilde{c}_i$ can be defined explicitly as $\tilde{c}_i = ((1-\sigma)\tilde{u}_i + 1)^{\frac{1}{1-\sigma}} \quad i = 0, 1, T+1$.

computed from the ergodic distribution of the exogenous income process.[13]

The variables ω_s and ω_t are used to measure the welfare cost of inflation. The variable ω_s is a simplified measure of the compensating variation in income required to compensate agents for the steady state impact of particular policy decisions. As such, ω_s will misrepresent the true welfare cost of any policy decision since it ignores any short-run effects of the decision. The variable ω_t on the other hand is a simplified measure of the compensating variation in income required to compensate agents for *all* of the effects of any policy decision. The welfare measure ω_t considers both the short run and the long-run effects of policy decisions in determining the compensation required to leave social welfare unchanged. Thus it is the variable ω_t which should be used by policymakers to determine whether a particular course of action is desirable from a social welfare point of view.

The steady state welfare loss due to inflation in economy one appears in Table 4. The values reported in this table are the values ω_s described above. For example, the steady state welfare loss due to a 2.5% annual inflation rate rather than an inflation rate of 0% is 0.366% of real GDP, and this value appears in the (1,2) cell of Table 4 while the welfare loss due to a 5% annual inflation rate rather than a 2.5% inflation rate is 0.323% of real GDP and appears in the (2,3) cell of Table 4. Notice that the steady state welfare cost of inflation is increasing in the final inflation rate, but decreasing in the initial inflation rate.[14] Negative entries in Table 4 represent the welfare gain from

[13]Notice that $\omega_i > \theta_i \quad i = s, t$ since agents could always choose to consume the compensating variation in income.

[14]The designations initial and final are motivated by the policy experiments in which the inflation rate is changed from the initial rate to the final rate. These policy experiments

eliminating the deadweight loss due to inflation. Notice also that Table 4 is skew symmetric.[15] This skew-symmetry of Table 4 implies that if only the steady state effects of inflationary monetary policy are considered then the welfare gain from eliminating a certain amount of inflation is exactly offset by the welfare cost associated with causing the inflation in the first place.

Despite recent U.S. inflation experience, few other studies have reported the welfare cost of inflation rates below 5%.[16] Most estimates of the welfare cost of inflation found in the literature have concentrated on inflation rates between 5% and 10% annually. The steady state welfare cost of 5% and 10% annual inflation rates reported in Table 4 are consistent with the larger estimates already found in the literature.[17] Also absent from the literature are comparisons between positive rates of inflation such as those that appear in Table 4.

The rows of Table 4 are presented graphically in Figure 1, with each curve representing a particular initial inflation rate. The vertical axis in Figure 1 is a percentage of real GDP. Notice again that the steady state welfare cost of inflation is increasing in the final inflation rate but decreasing in the initial inflation rate. This fact indicates that the steady state welfare cost of raising the inflation rate from 0% to say 5% is greater than the welfare cost of raising

will be analyzed shortly.

[15] A matrix of elements (a_{ij}) is skew symmetric if $a_{ij} = -a_{ji}$.

[16] Feldstein (1996) is a notable exception although his results differ substantially from those of this paper due to his emphasis on the interactions between inflation and other forms of distortionary taxation.

[17] Bullard and Russell (1997) find the welfare cost of a 10% inflation rate to be an order of magnitude larger when the inflation tax interacts with other forms of distortionary taxation.

Final Inflation Rate

	0.0%	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%
0.0%		0.366	0.689	0.991	1.272	1.507	1.764
2.5%	-0.366		0.323	0.625	0.906	1.141	1.398
5.0%	-0.689	-0.323		0.302	0.583	0.818	1.075
7.5%	-0.991	-0.625	-0.302		0.281	0.516	0.773
10.0%	-1.272	-0.906	-0.583	-0.281		0.235	0.492
12.5%	-1.507	-1.141	-0.818	-0.516	-0.235		0.257
15.0%	-1.764	-1.398	-1.075	-0.773	-0.492	-0.257	

Table 4: Steady State Welfare Costs for Economy One

the inflation rate from 5% to 10%, but is less than the steady state welfare cost of increasing the inflation rate from an annual rate of 0% to 10%. The skew symmetry of Table 4 is not immediately obvious in Figure 1.

The transitional welfare cost of inflation in economy one is presented in Table 5. The entries in Table 5 are the values ω_t described earlier. It remains true that the welfare cost of inflation is increasing in the final inflation rate and decreasing in the initial inflation rate, but notice that the transitional welfare cost is much larger than the steady state welfare costs reported in Table 4. Table 5 indicates that the welfare cost of *raising* the inflation rate from an annual rate of 0% to 2.5% is 0.621% of real GDP. This transitional welfare cost is nearly double the steady state impact of 0.366% reported in Table 4. Clearly the transitional dynamics associated with changes in monetary policy have a very large impact on the economy and on the welfare of agents. Notice also that unlike Table 4, Table 5 is not skew symmetric. In

Final Inflation Rate

	0.0%	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%
0.0%		0.621	1.055	1.407	1.718	1.958	2.219
2.5%	-0.729		0.490	0.885	1.217	1.490	1.757
5.0%	-1.370	-0.546		0.426	0.784	1.061	1.352
7.5%	-1.959	-1.051	-0.463		0.380	0.673	0.972
10.0%	-2.514	-1.532	-0.885	-0.399		0.307	0.622
12.5%	-2.983	-1.918	-1.249	-0.740	-0.320		0.324
15.0%	-3.505	-2.370	-1.642	-1.105	-0.668	-0.337	

Table 5: Transitional Welfare Costs for Economy One

fact Table 5 indicates that the welfare gain attributable to the elimination of some amount of inflation actually exceeds the welfare cost of producing the inflation in the first place.[18]

The rows of Table 5 are presented graphically in Figure 2 with the vertical axis measuring the percentage of real GDP lost to inflation. As in Figure 1, Figure 2 shows clearly that the transitional welfare cost of inflation is increasing in the final inflation rate and decreasing in the initial inflation rate.

A comparison of Tables 4 and 5 reveals a number of interesting features. The difference, $\omega_t - \omega_s$, between the transitional welfare cost of inflationary

[18]It should be noted that were the government to attempt to exploit this fact to increase aggregate welfare repeatedly, then the assumption of rational expectations would be called into question in this model. It should also be noted that any potential benefits to reducing inflation in this context must be discounted back T periods, implying that the present value of the benefit will not exceed the original cost.

monetary policy decisions and the steady state welfare cost of inflation is plotted in Figure 3. For all possible policy experiments, the steady state welfare cost of higher inflation understates the transitional welfare cost of inflationary monetary policy by a significant amount, and similarly the steady state welfare benefits from lower inflation drastically understates the transitional benefits of reducing inflation. This shows definitively that an analysis of the steady state implications of economic policy is an inadequate guide to policy decisions. For example a policy maker who would consider reducing the inflation rate from 2.5% annually to 0% only if the welfare gain associated with the change in policy was at least 0.5% of GDP would incorrectly conclude that inflation should not be decreased by analyzing only the steady state implications of the policy decision. Once the transitional dynamics associated with the policy change are taken into account, the true welfare gain of such a policy change is revealed to be 0.729% of real GDP rather than the 0.366% of GDP indicated by an analysis of steady states.

Notice that transitional dynamics increase the benefits of reducing the annual inflation rate from 2.5% to zero by $\omega_t - \omega_s = 0.363\%$ of real GDP. Having the transitional impact of disinflationary monetary policy reinforce the steady state benefits to reducing inflation is contrary to most conventional wisdom. Economists have traditionally assumed that the costs of transition would operate against the long-run benefits of reducing inflation. Based on the computation of sacrifice ratios, Ball (1994) estimates the cost of reducing inflation to be between 2% and 3% of real GDP per percentage point decrease in the inflation rate. Based upon these results, Feldstein (1996) estimates the cost of lowering the inflation rate from 2% annually to zero to be about

5% of real GDP.

To understand why transitional dynamics reinforce the long-run benefits to reducing inflation in this model, consider the following. Reducing the inflation rate to zero in this model induces agents to hold a higher level of real balances at the same time that the supply of nominal balances is being held constant. The resultant drop in the price level increases the wealth of all agents not subject to the liquidity constraint. This increase in wealth makes everybody better off and reinforces the long-run benefits to reduced inflation. Conversely, raising the inflation rate has the opposite effect. Higher rates of inflation-cum-money growth induce agents to reduce their holdings of real balances at the same time that the quantity of nominal balances is increasing. The resultant rise in the price level reduces the wealth of all agents in the economy not subject to the liquidity constraint.

Notice also in Figure 3 that the difference $\omega_t - \omega_s$ grows with the final inflation rate but the rate of increase of this difference between the transitional welfare cost and the steady state welfare cost declines as the final inflation rate increases. Furthermore, these differences seem to be converging to a constant independent of the initial inflation rate. Figure 4 plots the relative difference between the two measures, $\frac{\omega_t - \omega_s}{\omega_s}$. Figure 4 shows again that an analysis of steady states misrepresents the true welfare cost of inflationary monetary policy by between 30% and 100% depending upon the final inflation rate. Figure 4 also shows, however, that the importance of transitional dynamics is declining in the final rate of inflation for economy one. Unfortunately, although Figure 4 seems to indicate a stable, well-behaved relationship between the steady state and transitional welfare costs, this is

not a general property, as we will see when we analyze the welfare cost of inflation in economy two.

Table 6 presents the steady state welfare loss due to inflation for economy two. Once again this welfare loss is measured by ω_s. The steady state welfare costs for economy two are quite different from those of economy one. In economy one, the steady state result of an increase in the inflation rate was a reduction in social welfare, while in economy two the steady state result of an increase in the inflation rate is an increase in social welfare. This is due to the fact that inflation benefits the relatively large fraction of the steady state population that is both unemployed and cash poor. Since these are the poorest agents in the economy, they have the highest marginal utility of income and benefit greatly from the transfer payments implied by the inflation. Because all agent's utilities are weighted equally by our social welfare function, the benefit derived from inflation by these poorest of agents more than offsets the much more moderate distress caused to the smaller fraction of relatively wealthier agents by the inflation tax.

The rows of Table 6 are presented graphically in Figure 5. Notice again that inflation is "good" in economy two. For a fixed initial inflation rate welfare gains are increasing (welfare costs are decreasing) in the final inflation rate, and, for a fixed final inflation rate, welfare gains are increasing (welfare costs are decreasing) in the initial inflation rate. Again, the reason behind this unusual result lies in the distribution of agents. Economy two is populated by relatively large numbers of "poor" agents and relatively small numbers of "rich" agents compared with economy one. Consequently, a relative large fraction of the population enjoys the tremendous benefits

Final Inflation Rate

	0.0%	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%
0.0%		-0.027	-0.080	-0.209	-0.242	-0.272	-0.367
2.5%	0.027		-0.053	-0.182	-0.215	-0.245	-0.340
5.0%	0.080	0.053		-0.129	-0.162	-0.192	-0.287
7.5%	0.209	0.182	0.129		-0.032	-0.063	-0.158
10.0%	0.242	0.215	0.162	0.032		-0.030	-0.125
12.5%	0.272	0.245	0.192	0.063	0.030		-0.095
15.0%	0.367	0.340	0.287	0.158	0.125	0.095	

Table 6: Steady State Welfare Costs for Economy Two

of the government transfer payments, while only a relatively small fraction suffer under the burden of the inflation tax. Because the poor agents have much higher rates of marginal utility, this generates a welfare increase in the aggregate.

The transitional impact of changes to the annual inflation rate are given in Table 7. Notice that the signs on the transitional welfare cost of inflationary monetary policy decisions are the opposite of those for the steady state welfare cost of inflation! This is perhaps the most damning evidence against the use of steady state analysis to evaluate the consequences of economic policy decisions. Steady state analysis might not only underestimate the consequences of changing the inflation rate, it might not even have the sign of the welfare impact of economic policy correct and can lead policymakers to qualitatively erroneous conclusions. In economy two, a policy maker might on the basis of a steady state analysis decide to increase the inflation

Final Inflation Rate

	0.0%	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%
0.0%		0.133	0.225	0.302	0.369	0.428	0.476
2.5%	-0.150		0.102	0.185	0.257	0.319	0.371
5.0%	-0.275	-0.110		0.089	0.163	0.228	0.282
7.5%	-0.386	-0.209	-0.092		0.078	0.145	0.201
10.0%	-0.490	-0.299	-0.177	-0.081		0.069	0.126
12.5%	-0.580	-0.380	-0.253	-0.154	-0.071		0.058
15.0%	-0.655	-0.450	-0.318	-0.218	-0.132	-0.059	

Table 7: Transitional Welfare Costs for Economy Two

rate in an attempt to raise social welfare. The results of a transitional analysis show, however, that such a policy decision would decrease rather than increase social welfare.

The transitional impact of inflation in economy two is presented graphically in Figure 6. Notice that Figure 6 looks much the same as Figure 2 did. The welfare cost of raising the inflation rate is positive despite what is indicated by an analysis of steady states. Also, the transitional welfare cost of raising the inflation rate increases with the final inflation rate and is declining in the initial inflation rate.

The difference, $\omega_t - \omega_s$, between the transitional welfare cost of inflationary monetary policy decisions and the steady state welfare cost of inflation for economy two is plotted in Figure 7. Unlike economy one, the rate of increase of the difference between the welfare costs for economy two does not appear to be declining for higher final rates of inflation but rather seems

relatively constant between different rates. With a relatively constant slope, this difference cannot be converging to any constant level. As was the case in economy one, however, the transitional impact of inflationary monetary policy exceeds that of the steady state impact.

Because of the marked differences between Figures 7 and 3, it should not be surprising that the relative welfare cost differences are also dissimilar. Figure 8 plots the relative difference between the two welfare measures, $\frac{\omega_t - \omega_s}{\omega_s}$. The seemingly stable and well behaved relationship that existed between the steady state and transitional welfare costs in economy one has disappeared. The reason for the drastic change in this graph is due to the fact that in economy one, the transitional dynamic effect of inflation tended to reinforce the steady state welfare impact of inflation, while in economy two, the transitional dynamic effects of inflation move social welfare in the opposite direction from the steady state impact.

Having analyzed the aggregate welfare effects of inflation for economies one and two, we now turn our attention to economy three. The steady state welfare cost of inflation ω_s for economy three is given in Table 8. Notice that for economy three, as for economy one, inflation is "bad." That is, higher inflation rate steady state equilibria are characterized by lower values of our social welfare function. In economy three, the fraction of the steady state population that is both unemployed and cash poor is not sufficiently reduced by higher rates of inflation for the benefits of inflation to outweigh the burden of the inflation tax on wealthier agents. As was the case in economy one, the steady state welfare cost of inflation is increasing in the final inflation rate and declining in the initial inflation rate. The steady state welfare cost of

Final Inflation Rate

	0.0%	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%
0.0%		0.151	0.282	0.394	0.498	0.594	0.678
2.5%	-0.151		0.131	0.243	0.347	0.443	0.527
5.0%	-0.282	-0.131		0.111	0.216	0.312	0.396
7.5%	-0.394	-0.243	-0.111		0.105	0.200	0.285
10.0%	-0.498	-0.347	-0.216	-0.105		0.096	0.180
12.5%	-0.594	-0.443	-0.312	-0.200	-0.096		0.085
15.0%	-0.678	-0.527	-0.396	-0.285	-0.180	-0.085	

Table 8: Steady State Welfare Costs for Economy Three

inflation in economy three is shown graphically in Figure 9.

The transitional welfare cost of inflation ω_t for economy three is given in Table 9. Here again, as was the case for economy one, the sign of the transitional welfare cost of inflation is the same as the sign of the steady state welfare cost. Also, the transitional welfare cost of inflation is substantially greater than the steady state welfare cost. The transitional welfare cost of inflationary monetary policy decisions is increasing in the final inflation rate and decreasing in the initial inflation rate as it was in all previous cases. Table 9 is plotted in Figure 10.

The difference, $\omega_t - \omega_s$, between the transitional welfare cost of inflation and the steady state welfare cost is plotted in Figure 11. Notice that these differences are again concave and seem to be converging to a constant that is independent of the initial inflation rate. As in economy one, transitional dynamics seem to be a progressively less important factor for aggregate welfare

Final Inflation Rate

	0.0%	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%
0.0%		0.289	0.487	0.629	0.752	0.859	0.955
2.5%	-0.345		0.224	0.386	0.522	0.642	0.741
5.0%	-0.634	-0.245		0.178	0.325	0.453	0.558
7.5%	-0.881	-0.452	-0.192		0.156	0.290	0.401
10.0%	-1.104	-0.641	-0.368	-0.164		0.143	0.256
12.5%	-1.317	-0.816	-0.527	-0.318	-0.148		0.119
15.0%	-1.495	-0.978	-0.666	-0.449	-0.274	-0.124	

Table 9: Transitional Welfare Costs for Economy Three

as the final inflation rate is increased. The relative difference between the two welfare cost measures is shown in Figure 12. Notice that the seemingly stable and well behaved relationship between the two welfare cost measures has reappeared. Notice also, however, that the steady state welfare cost of inflation still dramatically underestimates the true welfare cost of inflation by between 40% and 120%.

5 Conclusion

The purpose of this paper has been to study the impact of transitional dynamics on the welfare cost of inflation. Transitional dynamics were explicitly modeled in a class of dynamic stochastic general equilibrium models with heterogeneous agents. Both *stationary* and *transitional* equilibria were defined and these equilibria were seen to depend on solutions to dynamic program-

ming problems that had to be solved numerically. By employing fast and efficient computational methods, we were able to analyze a wide variety of inflationary monetary policies for several different parameterizations of the model.

Transitional dynamics were shown to have a dramatic impact on the welfare cost of inflation. For a common parameterization of the model, transitional dynamics were shown to increase the welfare cost of a 10% annual inflation rate (relative to no inflation) by 35%, from 1.27% of real GDP to 1.72%. For other monetary policy experiments, the welfare impact of transitional dynamics is even greater. A plot of the relative importance of transitional dynamics revealed that transitional dynamics have the greatest quantitative impact at lower rates of inflation.

The quantitative importance of transitional dynamics at lower rates of inflation is particularly interesting in light of recent U.S. inflation experience. An analysis of steady states suggests that reducing the annual inflation rate from 2.5% to 0% is equivalent to increasing annual real GDP by 0.36%. Conventional wisdom suggests that the effect of transitional dynamics should be to erode much of the welfare benefits associated with such a disinflation. The results of this paper indicate that in fact the opposite may be true. With transitional dynamics modeled explicitly, the welfare benefit of reducing the annual inflation rate from 2.5% to 0% is equivalent to increasing annual real GDP by 0.73%. Transitional dynamics double the welfare gain from eliminating a low annual inflation rate.

Alternative parameterizations of the model provided other interesting results. In some cases, transitional dynamics were shown not just to have an

important quantitative impact, but to have to qualitatively different implications. Such results point out the dangers inherent in relying too heavily on steady state analysis and highlight the need for explicit consideration of transitional dynamics when making policy decisions.

Transitional dynamic analysis has applications beyond the study of the welfare cost of inflation. The computational methods employed in this paper can be used to study the dynamic equilibrium response of the economy to *any* change in the exogenous economic environment. One can even use transitional dynamic analysis to study phenomena that have no impact on the steady state of the economy. One could for instance use transitional dynamic analysis to study the effect of one time changes to the nominal stock of fiat currency, even though the long-run neutrality of money implies that such a change has no real impact on the steady state of the economy. Alternatively, transitional dynamic analysis could also be used to study the effect of changes to preference parameters, the discount rate, and technology. Transitional dynamic analysis is even capable of analyzing the effect of simultaneously changing several different elements of the exogenous economic environment.

There are a number of interesting extensions which could be made to the model studied in this paper. The introduction of a market for physical capital or other stores of value would alter the results by providing agents with additional consumption-smoothing instruments. With the addition of physical capital one could also examine the effects of monetary policy on economic growth as in Gomme (1993). As inflation reduces the attractiveness of currency, one would expect agents to substitute into physical capital

generating an increase in output per capita. Other forms of distortionary taxation could also be added,[19] allowing the analysis of fiscal policy and its interaction with monetary policy.

Another interesting extension of the model would be the relaxation of the rational expectations assumption. Static or adaptive expectations, or various forms of learning could replace the assumption of rational expectations. Comparing the time paths of economic variables under various types of learning might yield important insights into how agents perceive and respond to policy decisions. One could also relax the assumption that policy changes are completely unanticipated. If policy changes were announced before they actually took effect, the dynamic response of the economy to the policy change could be much different. Along these same lines would be the addition of aggregate uncertainty to the model. Unfortunately, the addition of aggregate uncertainty to models with heterogeneous agents is technically complicated.

[19]The interaction between the inflation tax and other forms of distortionary taxation is the key factor in the findings of Feldstein (1996) and Bullard and Russell (1997).

A Figures

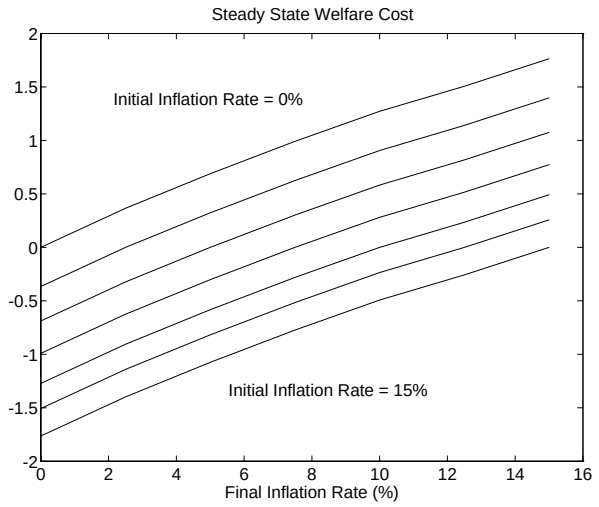

Figure 1: Steady State Welfare Costs for Economy One

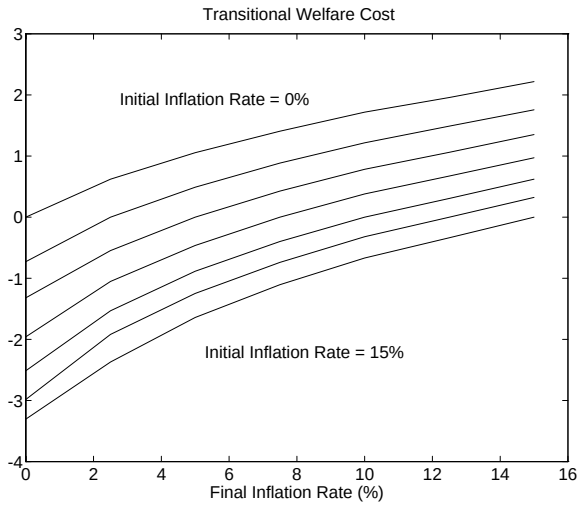

Figure 2: Transitional Welfare Costs for Economy One

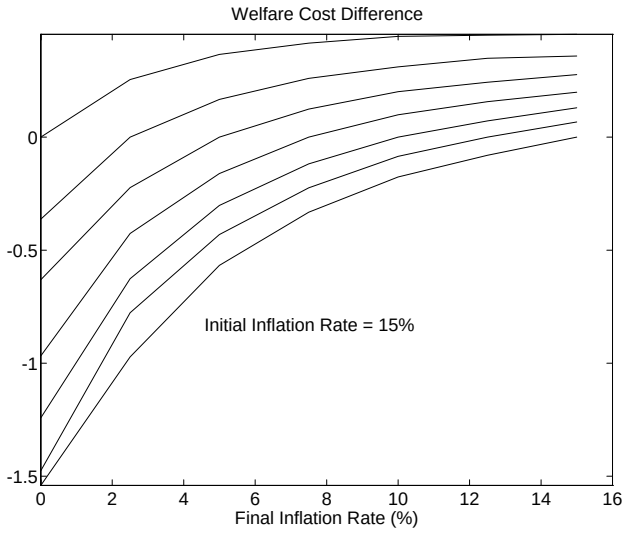

Figure 3: Welfare Cost Difference for Economy One

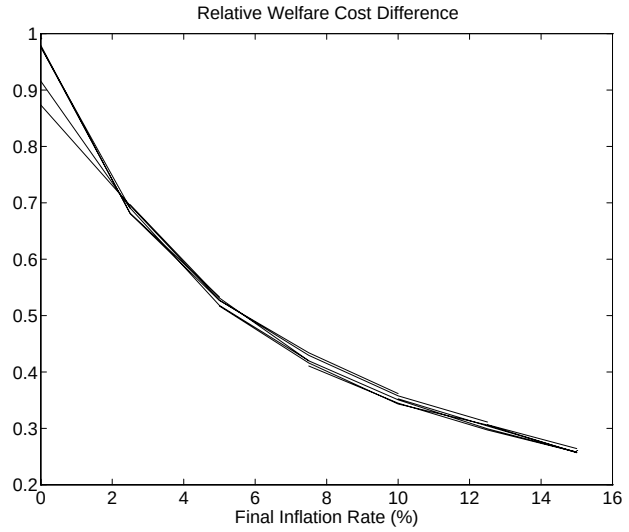

Figure 4: Relative Welfare Cost Difference for Economy One

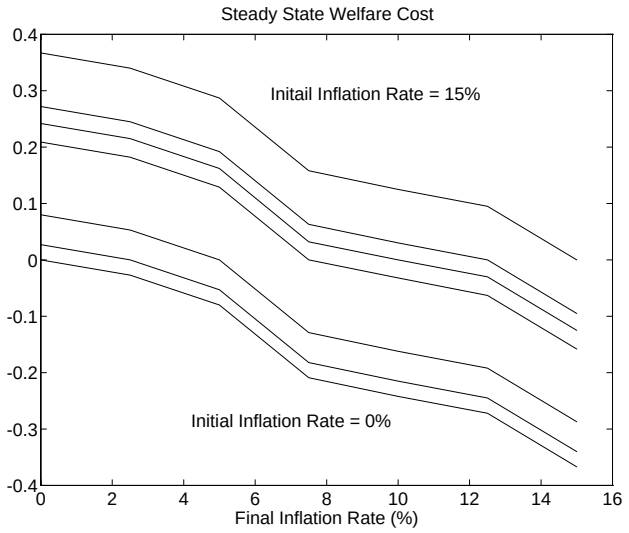

Figure 5: Steady State Welfare Costs for Economy Two

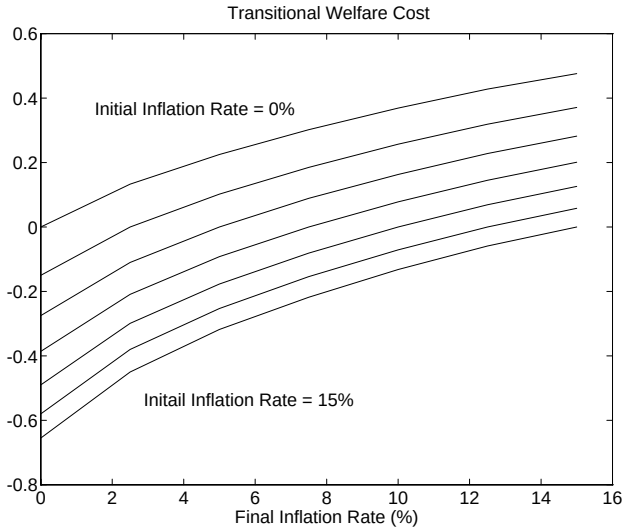

Figure 6: Transitional Welfare Costs for Economy Two

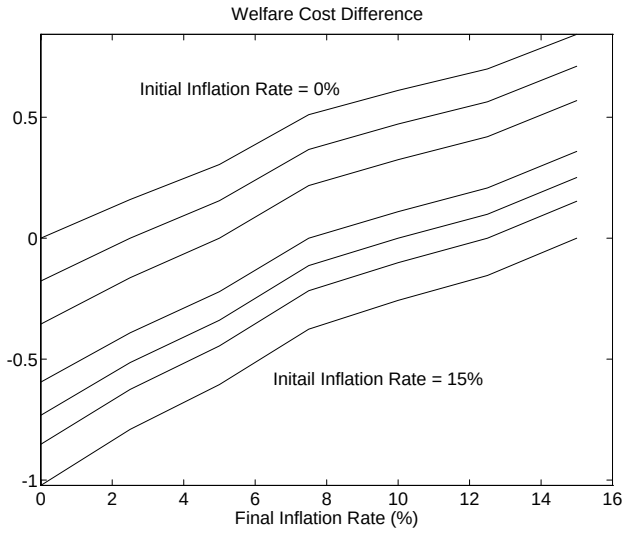

Figure 7: Welfare Cost Difference for Economy Two

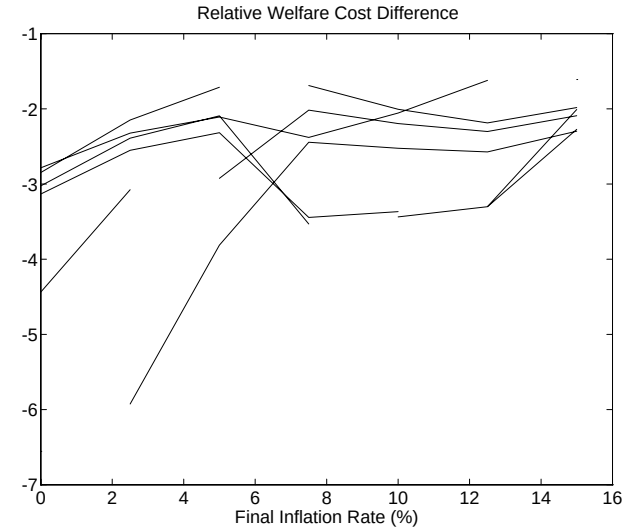

Figure 8: Relative Welfare Cost Difference for Economy Two

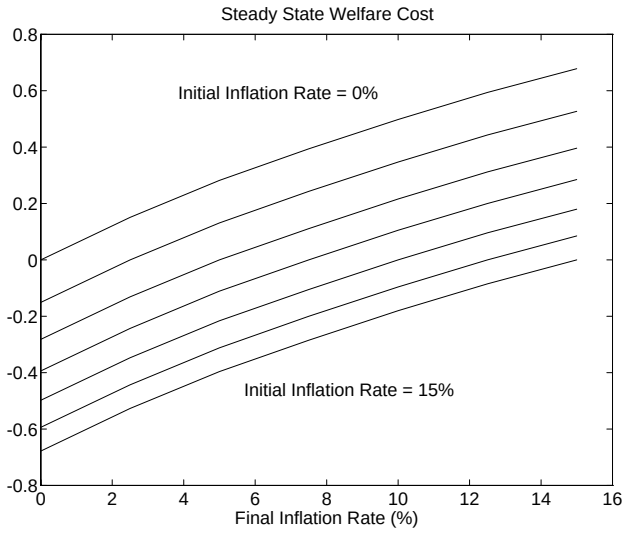

Figure 9: Steady State Welfare Costs for Economy Three

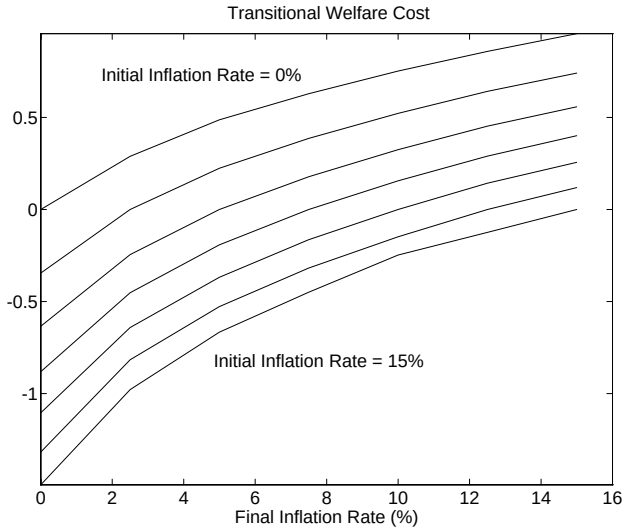

Figure 10: Transitional Welfare Costs for Economy Three

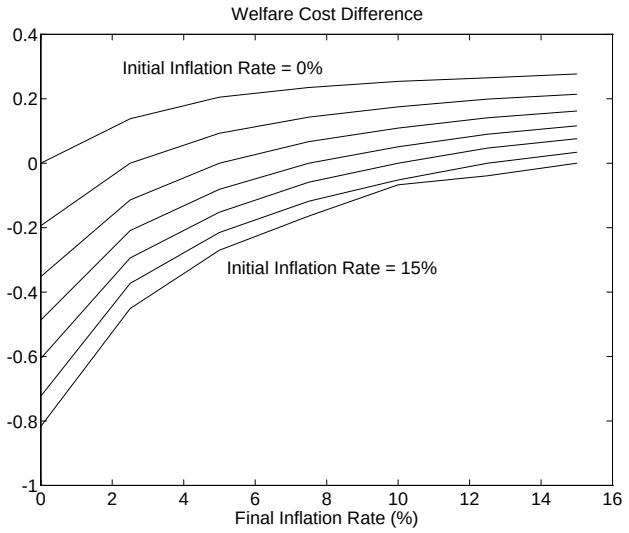

Figure 11: Welfare Cost Difference for Economy Three

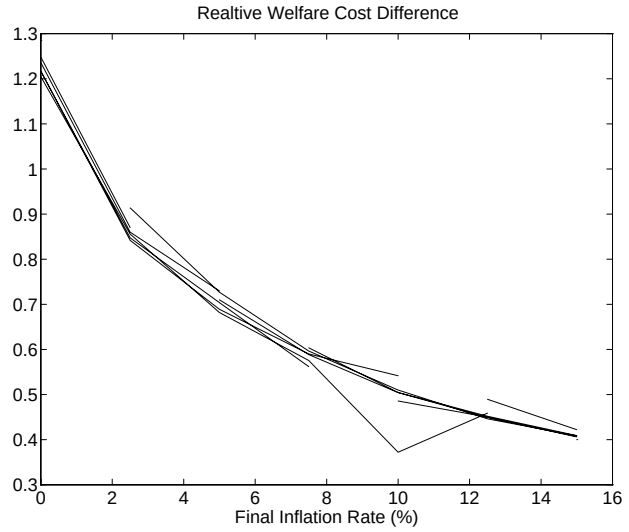

Figure 12: Relative Welfare Cost Difference for Economy Three

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