Stochastic Trends and Cointegration in the Market for Equities

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Abstract: We use a no-arbitrage, cost-of-carry pricing model to examine whether equity spot and futures markets are cointegrated. A stock index and its futures price should be cointegrated if the cost of carry is stationary. Otherwise, the appropriate cointegrating relationship is trivariate and includes the index, futures price, and cost of carry. We study the relationships among the Standard and Poor's 500 index, associated index futures price series, and interest rate for January 4, 1988, through June 30, 1995, and find that all three series are nonstationary. We further find that the index and futures price are not cointegrated unless the cost of carry is included in the cointegrating relationship. Our findings are consistent with the no-arbitrage pricing model and do not appear to be sensitive to the presence of structural breaks in the series.

JEL classification: G10, G12

Key words: cointegration, cost-of-carry model

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I. Introduction

Although stock and futures prices may wander widely, the two series may share the same stochastic trend. If so, the series are cointegrated and are not expected to drift too far apart. The relationship between stock index and stock index futures prices is critical because it has implications regarding predominant financial theory, including market efficiency and no-arbitrage pricing models. Many tests of financial models are inappropriate in the presence of cointegration (Engle and Granger (1987)). The theory of cointegration has been used to examine the temporal patterns in equity prices including dividends and stock prices (Campbell and Shiller (1987)), equity markets in different countries (Taylor and Tonks (1989), Wahab and Lashgari (1993), and Racine and Ackert (1996)), and other financial time series.[1]

While many empirical studies use the theory of cointegration, few attempt to provide a theoretical explanation for the existence of cointegration between financial time series. A recent exception is Brenner and Kroner (1995), who use a no-arbitrage, cost-of-carry asset pricing model to explain why some markets are expected to be cointegrated while others are not. They argue that cointegration depends critically on the time-series properties of the cost of carry. A stock index and its futures price will be cointegrated if the cost of carry, or the difference between the dividend yield and interest rate, is stationary. If this difference is not stationary, the correctly specified cointegrating regression would include the stock price, futures price, and spread between the dividend yield and interest rate.

The purpose of this paper is to address the issue of cointegration between equity spot and futures markets, a question not previously addressed in the literature. To do so, we study the time series relationship between the Standard and Poor's 500 index and index futures price series while recognizing that the appropriate cointegrating relationship may involve the cost of carry.

First the long-run behavior of each series is examined. The index and futures price series are nonstationary in levels but stationary when first differenced. That is, if the series is not first differenced, a shock in the current period will have an impact on all future periods. When nonstationary series move together over time they are cointegrated and a long-run relationship exists among them. The results indicate that if the cost-of-carry is excluded from the cointegrating relationship, stock and index futures markets are not cointegrated. However, when the cost-of-carry is included in the cointegrating equation, the S&P 500 index and index futures prices are cointegrated. Thus, a finding of cointegration in equity markets depends critically on the cost of carry and, as suggested by a no-arbitrage pricing model, equity spot and futures markets are cointegrated.

The remainder of this paper is organized as follows. Section II discusses cointegration and the no-arbitrage, cost-of-carry model. Section III describes our test of cointegration in spot and futures equity markets. Section IV presents the empirical results. The final section contains concluding remarks.

II. Cointegration in Equity Markets

The theory of cointegration is used extensively in studies of the relationship between futures or forward prices and spot prices. For example, Hakkio and Rush (1989) use cointegration theory to test whether the forward exchange rate is an unbiased predictor of the future spot rate. Current forward and spot rates should be "close together," and, therefore cointegrated, in an efficient market. More recently, Brenner and Kroner (1995) show that whether spot and futures or forward prices are cointegrated in an efficient market depends critically on the time-series properties of the cost of carry. If the cost of carry is stationary, spot and futures prices will be cointegrated.

Brenner and Kroner (1995) present a no-arbitrage, cost-of-carry asset pricing model which gives the futures price, $F_{t,t-k}$, as

(1) $$\text{Ln } F_{t,t-k} = \text{Ln } S_{t-k} + \text{Ln } D_{t,t-k} + Q_{t,t-k}$$

where S_{t-k} is the spot price at time t-k, $F_{t,t-k}$ is the futures price at time t-k for a contract that matures at time t, $D_{t,t-k}$ is the cost-of-carry or differential over the life of the futures contract, and $Q_{t,t-k}$ is an adjustment term which reflects marking-to-market. This no-arbitrage model assumes that the spot asset provides a continuous dividend yield and recognizes that an investor should be indifferent between 1) purchasing the asset and re-investing the dividends, and 2) purchasing a futures contract and investing in a risk-free bond.[2,3] If equation (1) failed to hold, an arbitrageur could devise a strategy which resulted in a riskless profit.

Because the marking-to-market adjustment ($Q_{t,t-k}$) is nonstochastic, equation (1) says that there is a linear relationship between the natural logarithm of the futures price and the spot price. The spot price and its futures price will be cointegrated if the natural logarithm of the differential is stationary. If not, the cointegrating relationship should include the spot price, futures price, and differential. The differential reflects interest rates, dividend yields, storage costs, and convenience yields. For equities that provide a constant dividend yield, the logarithm of the differential is $\text{Ln } D_{t,t-k} = k(r_{t,t-k} - q_{t,t-k})$ where $r_{t,t-k}$ and $q_{t,t-k}$ are the k-period interest rate and dividend yield at time t-k. The stock price and its futures price will be cointegrated if the natural logarithm of the cost of carry is stationary which is the case if the spread between the interest rate and dividend yield is stationary. The spread is stationary if the interest rate and dividend yield are each stationary or if the two series are themselves cointegrated.

III. A Test of the Model in Equity Markets

In order to examine cointegration in equity spot and futures markets we follow Brenner and Kroner (1995) and use a no-arbitrage, cost-of-carry asset pricing model to determine the appropriate cointegrating relationship. Our model differ, however, in that we do not assume that dividends are paid at a continuous rate. Instead we recognize that dividends are not constant and have been shown to exhibit distinct patterns. Harvey and Whaley (1991 and 1992) show that large estimation and asset pricing errors can result from the assumption that the dividend yield on a stock index is constant. If we define $d_{t,t-k}$ as the discounted value at time t-k of the non-constant dividend stream paid over the life of the futures contract, a no-arbitrage, cost-of-carry asset pricing model gives the futures price as

$$\text{Ln } F_{t,t-k} = \text{Ln } (S_{t-k} - d_{t,t-k}) + kr_{t,t-k} + Q_{t,t-k} \tag{2}$$

where $S_{t-k} - d_{t,t-k}$ is the dividend-adjusted value of the stock index.[4] If the interest rate is nonstationary then the differential has a stochastic trend and the dividend-adjusted index and its futures price should not be cointegrated. In this case, the appropriate cointegrating relationship contains the interest rate and the natural logarithms of the index and futures price. A test of the no-arbitrage model involves estimation of the cointegrating regression

$$\text{Ln } F_{t,t-k} = \alpha + \beta_1 \text{Ln } (S_{t-k} - d_{t,t-k}) + \beta_2 r_{t,t-k} + z_t \tag{3}$$

where the marking-to-market adjustment is contained in the constant term because it is nonstochastic.[5]

In order to examine the long-run relationships between the stock index, futures price on the index, and interest rate, we first test whether each series contains a unit root. Tests for cointegration are appropriate if the series are nonstationary but can be made stationary by differencing. If a linear combination of nonstationary assets is stationary, the assets are cointegrated. According to Brenner and Kroner (1995), the cost of carry or interest rate is likely nonstationary so that index and futures prices cannot be cointegrated. Tests for cointegration should then include the index and futures price, as well as the interest rate. In the following section we examine these long-run relationships and test whether equity spot and futures prices share a common stochastic trend.

IV. Empirical Results

The sample consists of daily observations for the S&P 500 stock index and associated futures contract closing prices for January 4, 1988 through June 30, 1995 resulting in 1,875 observations. The nearby futures contract with at least 15 days to maturity is used.[6] The S&P 500 index is a broad-based, market-value weighted index containing five hundred stocks traded on the New York Stock Exchange. Daily stock index price and dividend data are from the Standard and Poor's Corporation, closing futures price are from the Futures Industry Institute Data Center, and three-month treasury bill rates are from the Federal Reserve Bank of St. Louis. The three-month treasury bill rate is our measure of the riskfree rate, $r_{t,t-k}$. The daily dividend data was used to calculate the present value of the dividends ($d_{t,t-k}$) received over the remaining life of the relevant futures contract.

Prior to testing for cointegration, nonstationarity must be established. In order to determine the order of integration of each series, we used Augmented Dickey-Fuller (ADF) tests (Dickey and Fuller (1979 and 1981)) with MacKinnon's (1991) critical values. Table 1 reports the results of tests of the null hypothesis of a unit root in the natural logarithms of the S&P 500 index futures price series

($Ln F_{t,t-k}$) and dividend-adjusted price index ($Ln (S_{t-k} - d_{t,t-k})$), as well as the interest rate as measured by the three-month treasury bill rate ($r_{t,t-k}$). If the test statistic differs significantly from zero, the null hypothesis is rejected and we conclude that the series is stationary. We tested for unit roots in each series in levels and first differences. Lags specifies the number of lagged difference terms allowed in the test regression and constant indicates whether a constant term was or was not included.[7] The ADF tests indicate that all series are nonstationary and that they are integrated of order one.

Time series graphs also indicate a discontinuity in mean. Unit root tests are biased in favor of the unit root null hypothesis if there is a structural break in the series. We use Perron's (1990) test statistic which allows for a change in mean in order to ensure that the series are nonstationary after recognizing the structural break. Basically, Perron's approach involves using an ADF test with demeaned data and Perron's tabulated critical values to assess statistical significance. This approach assumes that the structural change is exogenous and occurs at a known date. Perron suggests that visual inspection of the series be used to indicate the date of the structural break.[8] Panel A of Table 2 reports the results of this unit roots test with an exogneously determined structural break. The results again suggest that the series are nonstationary.

Finally, to further examine the robustness of the nonstationarity conclusion, we use a unit root test procedure proposed by Zivot and Andrews (1992) that is not conditioned on an exogenously determined breakpoint. The test uses a series of modified Dickey-Fuller regressions and conservatively selects the breakpoint as that which is least favorable to the unit root null hyothesis. The results reported in Panel B of Table 2 are based on this endogenously determined breakpoint and again confirm that the series are nonstationary.

Because all the series are integrated of the same order, cointegration tests are appropriate.

Brenner and Kroner (1995) assert that if the cost of carry is nonstationary, equity spot and futures prices cannot be cointegrated. Table 3 reports the results for a test of this assertion. We test for cointegration using a framework proposed by Johansen (1988 and 1991). The Johansen test is essentially a multivariate Dickey-Fuller tests which determines the number of cointegrating equations, or cointegrating rank, by computing a likelihood ratio statistic for each added cointegrating equation in a sequence of nested models. If we cannot reject the hypothesis that the number of cointegrating equations (CE(s)) is none, the series are not cointegrated. If we cannot reject the hypothesis of at most one CE, there is one cointegrating vector and the series share a stochastic trend. The results reported in Table 3 suggest that the dividend-adjusted S&P 500 index and futures price series are not cointegrated which is consistent with Brenner and Kroner's assertion that the two series should not be cointegrated when the interest rate is nonstationary.[9]

Because the cost of carry is nonstationary, a test for cointegration in equity markets should include the dividend-adjusted index, futures price, and interest rate. To allow for the possibility that the cointegrating relationship, should one exist, is changing over time, we employ several estimation methods. First, we apply the Johanen (1991) method to the full sample and the results are presented in Table 4. Because the null hypothesis of at most one cointegrating equation (CE) cannot be rejected, there is evidence of cointegration in equity markets. As the no-arbitrage, cost-of-carry model suggests, there is a trivariate cointegrating relationship among equity spots, futures, and the cost of carry.

To allow for the possibility of changes over time in the cointegrating relationship, we use the Johansen framework to examine cointegration over two subsamples. The subsamples are selected using the exogenously determined structural break identified previously as October 13, 1989. The results reported in Panels A and B of Table 5 for each subperiod suggest that there is one cointegrating relationship between the three variables so that equity spot and futures markets are cointegrated.

To further examine the robustness of our results to structural breaks, we use a method proposed by Gregory and Hansen (1996) which is based on the Engle-Granger two-step procedure. This method includes dummy variables (DV_t) in the cointegrating regression to allow for a shift in the intercept or in the slope and intercept of the long-run relationship. In the second step, for each possible breakpoint, ADF regressions are run on the residuals from the corresponding cointegrating regression. The minimum ADF statistic endogenously determines the breakpoint and is compared to critical values supplied by Gregory and Hansen. Panel A of Table 6 reports a test of the null hypothesis of no cointegration versus an alternative of cointegration with shift in intercept. The null is rejected at the five percent significance level which supports a cointegrating relationship with change in intercept. The competing alternative for the test reported in Panel B of Table 6 is cointegration with shift in slope and intercept. In this case the null is maintained. Taken together, the Gregory and Hansen tests support a cointegrating relationship in equity markets with a possible shift in mean.

V. Concluding Remarks

Recent theoretical research conjectures that equity spot and futures markets are not cointegrated if the cost of carry is nonstationary and excluded from the cointegrating relationship. Although this relationship has important implications for tests of financial models, no empirical study has investigated the issue of cointegration in equity markets. This paper provides such an examination. We use a no-arbitrage, cost-of-carry pricing model to examine whether equity spot and futures markets are cointegrated. Using data for the Standard and Poor's 500 index and index futures price series for January 4, 1988 through June 30, 1995, we find that the index and futures price are not cointegrated if the cost of carry is excluded from the cointegrating relationship. In testing we recognize that there are distinct patterns in dividend payments so that a constant yield may not be a good representation of cash dividend payments from equities. In this case, the differential or cost of carry is the interest rate which we find is nonstationary. We conclude that the index, futures price, and interest rate are cointegrated, a result which is consistent with the no-arbitrage pricing model and suggests that a no-arbitrage assumption is reasonable in models of equity markets.

We also investigate the robustness of our results to structural breaks or regime shifts in the series. Although the cointegrating relationships may be changing over time, our results do not appear to be sensitive to the presence of structural breaks in the series. Thus, it seems that these regime shifts do not significantly impact the long-run equilibrium behavior of equity markets. Future research may examine the cause of these structural breaks and the short-run adjustment process in equity markets.

Endnotes

1. See, for example, Baillie and Bollerslev (1989), Hakkio and Rush (1989), Serletis and Banack (1990), Lai and Lai (1991), and Kroner and Sultan (1993).

2. Both portfolios result in one unit of the asset at the futures delivery date. To see this, note that portfolio 1) requires purchase of less than one unit of the asset which will grow to one unit as a result of the dividends that are paid at a continuous rate.

3. Forward contracts, rather than futures contracts, are actually priced using these no-arbitrage arguments. However, a relationship similar to (1) holds for futures given additional assumptions (Amin and Jarrow (1991) and Heath, Jarrow, Morton (1992)).

4. The model recognizes that an investor should be indifferent between 1) purchasing the stock index and borrowing at the risk-free rate and 2) purchasing an index futures contract and investing in a risk-free bond. Both portfolios result in one unit of the stock index at the futures delivery date.

5. See Brenner and Kroner (1995, page 28).

6. Some researchers such as Hein, Ma, and MacDonald (1990) suggest the use of nonoverlapping data. Brenner and Kroner (1995) recognize that their proposition assumes that the expiration date (t) is changing and the time to expiration of the futures contract (k) is fixed. Given our sampling procedure of rolling to the next contract as maturity approaches cointegration cannot, in theory, hold because the variance of the regression residual is time-varying. However, cointegration tests have low power in detecting changing variances, as our results suggest.

7. We began with a constant, trend, and 100 lags. The constant and trend terms were dropped if they were insignificant using conventional t-tests and the lagged terms were dropped if the t-statistic was less than one in absolute value. The trend term was excluded from the test regression in all cases because it was consistently insignificant.

8. A strong downward movement in U.S. stock markets was reported on the structural break date, October 13, 1989. Chow tests confirmed that the index and t-bill series exhibit structural breaks at this point. Further investigation revealed that a change in Federal Reserve policy occurred around the time of the discontinuity in mean. Whether or not the Fed's policy change resulted in the break is an interesting empirical question for future research.

9. The number of lagged difference terms was determined using a likelihood ratio test with a maximum of 100 lags (Enders (1996)).

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TABLE 1
Augmented Dickey-Fuller Tests for Unit Roots in Indexes and Index Futures Prices

The table reports the results of tests of the null hypothesis of a unit root in the natural logarithms of the S&P 500 index futures price series (Ln $F_{t,t-k}$) and dividend-adjusted price index (Ln (S_{t-k} - $d_{t,t-k}$)), as well as the interest rate as measured by the three-month treasury bill rate ($r_{t,t-k}$). If the test statistic differs significantly from zero, the null hypothesis is rejected and we conclude that the series is stationary. We tested for unit roots in each series in levels and first differences. Lags specifies the number of lagged difference terms allowed in the test regression and constant indicates whether a constant term was (yes) or was not (no) included. The tests are Augmented Dickey-Fuller tests and critical values are from MacKinnon (1991).

Series	Order	Lags	Constant	ADF Statistic	Critical Value (1%)
Ln $F_{t,t-k}$	levels	99	yes	-1.239	-3.437
	differences	98	yes	-4.410**	-3.437
Ln (S_{t-k} - $d_{t,t-k}$)	levels	99	yes	-1.236	-3.437
	differences	98	yes	-4.481**	-3.437
$r_{t,t-k}$	levels	100	no	-1.369	-2.567
	differences	90	no	-2.712**	-2.567

** denotes rejection of the hypothesis at 1% significance level

TABLE 2
Tests for Unit Roots in Indexes and Index Futures Prices with Structural Breaks

The table reports the results of tests of the null hypothesis of a unit root in the natural logarithms of the S&P 500 index futures price series (Ln $F_{t,t-k}$) and dividend-adjusted price index (Ln (S_{t-k} - $d_{t,t-k}$)), as well as the interest rate as measured by the three-month treasury bill rate ($r_{t,t-k}$). If the test statistic differs significantly from zero, the null hypothesis is rejected and we conclude that the series is stationary. Lags specifies the number of lagged difference terms allowed in the test regression. Panel A reports the results for unit root tests with an exogenously determined structural break and critical values are from Perron (1990). The tests reported in Panel B are based on an endogenously determined breakpoint with critical values from Zivot and Andrews (1992).

Panel A: Exogenously Determined Structural Break

Series	Lags	Perron Statistic	Critical Value (1%)
Ln $F_{t,t-k}$	94	-1.416	-3.23
Ln (S_{t-k} - $d_{t,t-k}$)	94	-1.433	-3.23
$r_{t,t-k}$	100	-1.596	-3.23

Panel B: Endogenously Determined Structural Break

Series	Lags	Zivot and Andrews Statistic	Critical Value (1%)
Ln $F_{t,t-k}$	99	-0.283	-5.34
Ln (S_{t-k} - $d_{t,t-k}$)	99	-0.229	-5.34
$r_{t,t-k}$	100	-0.191	-5.34

** denotes rejection of the hypothesis at 1% significance level

TABLE 3
Test for Cointegration in Equity Markets: No Cost of Carry

This table reports the results of a test for cointegration between the dividend-adjusted S&P 500 index and associated futures prices. If the null hypothesis of a unit root in the residuals of the cointegrating regression is rejected, there is evidence of cointegration. Lags specifies the number of lagged difference terms allowed in the test regression. The number of lagged difference terms was determined using a likelihood ratio test with a maximum of 100 lags (Enders (1996)). Critical values are from Johansen (1991). Standard errors are given below coefficient estimates.

Sample: January 4, 1988 - June 30, 1995			
Lags: 60			
Eigenvalue	Likelihood Ratio	1 Percent Critical Value	Hypothesized No. of CE(s)
0.0048	9.9006	16.31	None
0.0007	1.2085	6.51	At most 1
Normalized Coefficients			
$\ln F_{t,t-k}$	$\ln (S_{t-k} - d_{t,t-k})$		
1.0000	-1.0003 (0.0011)		

*(**) denotes rejection of the hypothesis at 5% (1%) significance level

TABLE 4
Test for Cointegration in Equity Markets: No Structural Break

This table reports the results of a test for cointegration between the dividend-adjusted S&P 500 index, associated futures prices, and interest rates. If the null hypothesis of at most one cointegrating equation (CE) cannot be rejected, there is evidence of cointegration in equity markets. Lags specifies the number of lagged difference terms allowed in the test regression. The number of lagged difference terms was determined using a likelihood ratio test with a maximum of 100 lags (Enders (1996)). Critical values are from Johansen (1991). Standard errors are given below coefficient estimates.

Sample: January 4, 1988 - June 30, 1995			
Lags: 70			
Eigenvalue	Likelihood Ratio	1 Percent Critical Value	Hypothesized No. of CE(s)
0.0133	33.4086	29.75	None**
0.0043	9.2723	16.31	At most 1
Normalized Cointegrating Coefficients			
Ln $F_{t,t-k}$	Ln (S_{t-k} - $d_{t,t-k}$)	$r_{t,t-k}$	
1.0000	-1.0000 (0.0001)	-0.0017 (0.0001)	

*(**) denotes rejection of the hypothesis at 5% (1%) significance level

TABLE 5
Test for Cointegration in Equity Markets: Exogenous Structural Break

This table reports the results of a test for cointegration between the dividend-adjusted S&P 500 index, associated futures prices, and interest rates for two sample periods. If the null hypothesis of at most one cointegrating equation (CE) cannot be rejected, there is evidence of cointegration in equity markets. Lags specifies the number of lagged difference terms allowed in the test regression. The number of lagged difference terms was determined using a likelihood ratio test with a maximum of 100 lags (Enders (1996)). Critical values are from Johansen (1991). Standard errors are given below coefficient estimates.

Panel A: Prior to Structural Break

Sample: January 4, 1988 - October 13, 1989			
Lags: 4			
Eigenvalue	Likelihood Ratio	1 Percent Critical Value	Hypothesized No. of CE(s)
0.0722	40.9906	29.75	None**
0.0140	7.8642	16.31	At most 1
Normalized Cointegrating Coefficients			
Ln $F_{t,t-k}$	Ln (S_{t-k} - $d_{t,t-k}$)	$r_{t,t-k}$	
1.0000	-0.9996 (0.0016)	-0.0021 (0.0012)	

Panel B: Subsequent to Structural Break

Sample: October 16, 1989 - June 30, 1995			
Lags: 30			
Eigenvalue	Likelihood Ratio	1 Percent Critical Value	Hypothesized No. of CE(s)
0.0820	125.9142	29.75	None**
0.0026	6.4487	16.31	At most 1
Normalized Cointegrating Coefficients			
Ln $F_{t,t-k}$	Ln (S_{t-k} - $d_{t,t-k}$)	$r_{t,t-k}$	
1.0000	-1.0001 (0.0001)	-0.0017 (0.0001)	

*(**) denotes rejection of the hypothesis at 5% (1%) significance level

TABLE 6
Test for Cointegration in Equity Markets: Endogenous Structural Break

This table reports the results of a test for cointegration between the dividend-adjusted S&P 500 index, associated futures prices, and interest rates. If the null hypothesis of a unit root in the residuals of the cointegrating regression is rejected, there is evidence of cointegration in equity markets. Lags specifies the number of lagged difference terms allowed in the test regression. We begin with 100 lags and lag terms with t-statistics less than one in absolute value are dropped. Critical values are from Gregory and Hansen (1996). Standard errors are given below coefficient estimates.

Panel A: Test for Cointegration with Shift in Intercept

Sample: January 4, 1988 - June 30, 1995				
Lags: 88				
ADF Statistic	1 Percent Critical Value	5 Percent Critical Value	10 Percent Critical Value	
-5.0871*	-5.44	-4.92	-4.69	
Cointegrating Coefficients				
Ln $F_{t,t-k}$	Constant	Ln (S_{t-k} - $d_{t,t-k}$)	$r_{t,t-k}$	DV_t
1.0000	-0.0207 (0.0083)	1.0034 (0.0013)	0.0019 (0.0001)	-0.0014 (0.0005)

Panel B: Test for Cointegration with Shift in Intercept and Slope

Sample: January 4, 1988 - June 30, 1995						
Lags: 88						
ADF Statistic	1 Percent Critical Value	5 Percent Critical Value	10 Percent Critical Value			
-4.5465	-5.97	-5.50	-5.23			
Cointegrating Coefficients						
Ln $F_{t,t-k}$	Constant	Ln (S_{t-k}-$d_{t,t-k}$)	$r_{t,t-k}$	DV_t	DV_t·Ln (S_{t-k} - $d_{t,t-k}$)	Dv_t·$r_{t,t-k}$

1.0000	-0.0164 (0.0094)	1.0027 (0.0016)	0.0019 (0.0002)	-0.0018 (0.0226)	0.0005 (0.0038)	-0.0004 (0.0003)

*(**) denotes rejection of the hypothesis at 5% (1%) significance level