December 2008

Job Search with Bidder Memories

Carlos Carrillo-Tudela and Eric Smith*

PRELIMINARY

Abstract

Suppose a job seeker forgoes a potential match and decides to continue searching. A worker who can recall this opportunity when subsequently paired with another firm has a distinct advantage over workers without such attachments. The worker can engage multiple firms in competitive Bertrand bidding rather than facing a monopolist. Accounting for this distinction, firms sacrifice some monopoly payoff when bidding for labour. The Diamond paradox does not occur. Moreover, this paper demonstrates that this wage determination mechanism can be introduced in a variety of environments to shed light on job-to-job transitions, wage dispersion and the scarring effects of unemployment.

1 Introduction

Economic rents arise when potential trading partners meet each other in the presence of matching frictions. The way in which traders allocate these rents has profound effects on economic outcomes. Diamond (1971), for example, demonstrates that in a model with homogeneous goods and price setting, the selling firm obtains all gains to trade. In this model - one that on the surface appears to be a natural framework - buyers have no incentive to participate. Market breakdown or unravelling can occur.

To resolve the so-called Diamond paradox in a labor market setting, the Diamond-Mortensen-Pissarides (DMP) framework adopts wage bargaining, either strategic offer-counter offer or axiomatic Nash, to divide the match specific surplus.[1] Although bargaining provides a useful resolution to rent sharing in many situations, a number of difficulties remain unresolved. Axiomatic Nash may be inconsistent with forward looking agents or may generally lack theoretical underpinnings (Coles and Masters, 2006). Strategic bargaining can be sensitive to bargaining protocols especially if agents are heterogeneous or have asymmetric information (Shimer, 2006).

To address these shortcomings, this paper revisits wage and price setting in search models with take-it-or-leave-it offers. The objective is to offer a sensible, tractable alternative that overcomes some of the limitations of the bargaining mechanism and then to explore its links with a variety of labor market regularities.

The Diamond (1971) matching model and those that followed specified that if the traders fail to agree to terms, they break-up, the match dissolves entirely, and the potential trading

[1] Heterogeneity among buyers can also alleviate the Diamond paradox. See for example Albrecht and Axell (1984). Small but positive search costs unravel these costs.

partners lose all contact. They forget the match occurred. Interpreting a firm's wage offer as a bid for worker services in an auction, the standard auction in the literature has a single monopolistic bidder.[2]

The approach adopted here relaxes this position and allows job seekers to remember and recall (at least to some extent) past encounters if they decide to continue searching for employment opportunities. This assumption appears plausible in several contexts. Another job seeker may eventually fill previously encountered opportunities that were left behind, but this will not occur immediately. As a result, a job seeker can potentially increase the number of firms bidding in the auction for his or her services. If workers have memories that allow them to recall previous encounters, they have the potential to alter the number of bidders for their services and hence the outcome of the auction.

A worker who is fortunate and finds two willing bidders engages them in a bidding war that results in Bertrand wages. In this case, market power switches sides and the worker captures all of the match rents. Continued job search can cause the firm to lose its lone bidder status. When making an acceptable offer even as the lone bidder, a firm must therefore take into account the fact that the worker's outside option of search is not the same as an unattached worker. Firms ultimately offer enough at the first encounter to avoid Bertrand wages so the workers will not exercise the continued search option. The essential point is that given the threat to recall bidders, firms who encounter job seekers have to pay more than the value of unattached search to secure the worker's services, even when they are the sole bidder. The outside option is now the value of attached search, a distinctly better

[2] Burdett and Mortensen (1998) and Burdett and Judd (1983) are both models with wage posting that can be interpretted as having auctions for workers with an unknown number of bidders. Despite its many useful insights, the Burdett - Mortensen wage dispersion breaks down with the addition of search costs.

position for the worker since the next encounter leads to Bertrand wages.

The contribution here, however, extends beyond providing a new, relatively plausible resolution of the Diamond paradox. The more general point is that this transparent, dynamically consistent wage determination process can be adapted and adopted into a variety of environments that have posed difficulties for modelling matching frictions. The proposed mechanism also applies outside the labor market setting into other markets with matching frictions, for example housing and durable goods where other resolutions of Diamond's paradox, for example on-the-job search or bargaining may not be appropriate.

Extending the model offers potential insights into wages across individuals and over time. To demonstrate these possibilities the paper presents extensions of the basic mechanism with on-the-job search and then with risk aversion and time varying unemployment insurance (UI) payments. On-the-job search leads to wage dispersion as employed workers eventually find a second bidder. Wage dispersion of a different nature obtains in models with risk aversion and time varying UI payments. Assets and time varying UI schedules induce time dependent payoffs which are matched by firms as they bid for workers. A scarring effect occurs as job seekers who are unfortunate and do not match early on become increasingly desperate. Unlike other models with heterogeneous agents, the results are not sensitive to unraveling with small search costs (Albrecht and Axell, 1984). Gaumont, Schindler and Wright (2005) examine a model with heterogeneous matches and show that the law of two prices obtains. Given time varying benefits, in this paper wages spread out continuously between those with the highest payoffs at the beginning of search to those who have searched for an extended period of time and have the lowest possible search payoffs.

2 Basic Framework

The economy closely follows the continuous time, steady-state Diamond-Mortensen-Pissarides (DMP) framework. A unit mass of risk neutral, infinitely lived workers are either unemployed or employed at each point in time. Unemployed workers receive z per unit of time. Employed workers' wages are determined below. The economy also contains a mass of risk neutral, infinitely lived firms. A firm is a job which at any point in time can be either vacant or filled. The flow cost, measured in units of productivity, of maintaining this vacancy open is $c > 0$. Firms and workers discount the future at rate $r > 0$.

Once a worker becomes employed at any given job, he or she produces strictly positive $x > z$ per period of time. Output is sold in a competitive market at a price normalised to 1, so that x denotes the flow revenue to a firm employing a worker. During production the job-worker match is subject to idiosyncratic shocks that render the match unproductive and lead to the dissolution of the match. In this case, the worker is displaced and becomes unemployed while the job becomes vacant. We assume that these shocks follow a Poisson process with rate $\delta > 0$.

Unemployed workers and vacant jobs search randomly for each other. To begin, there is no on-the-job search, an assumption relaxed below. When a worker and a vacancy see each other, an auction takes place for the worker's services in which the firm makes a wage offer to the worker. The worker decides whether to accept a bid or continue searching for a better offer.

An important departure from the DMP framework is that unemployed workers can recall previously met vacancies, if such vacancies are still available.[3] In particular, assume workers are able to store and recall a maximum of $N \in \mathbb{N}^+$ vacancies, where N is a finite but potentially very large number. Letting ϕ denote the rate at which vacancies become unavailable each period, dt, the worker is able to recall a vacancy with probability ϕdt.[4] Once employed, workers do not lose this information and if displaced they can recall available vacancies. We assume that firms cannot recall workers.

The ability to recall previous contacts allows for the possibility that a worker has more than one bidder in the wage auction. Given that the firm knows the number of other firms bidding for the worker, wage offers depend on the number of vacancies competing for the worker. If at any point in time the worker can engage two or more firms in an auction, these firms become engaged in Bertrand competition (second price auction). However, if a worker finds only one vacancy, the firm retains a monopoly position with all the bargaining power. This is the Mortensen rule of wage determination.[5] Let ω_i denote the firm's wage offer when $i \geq 0$ vacancies compete for the worker's services. A firm commits to pay ω_i to the worker for as long as the worker stays in the firm.

Let u_i denote the number of unemployed workers that can recall i vacancies and let v denote the number of vacant jobs in the economy. Total unemployment is given by $u = \sum_{i=0}^{N} u_i$. Free entry of firms determines the number of vacancies. The number of meetings, M, that take place at each point in time is governed by a meeting function that depends on the num-

[3] The possibility that the vacancy might not be available in the future implies that there is no perfect recall.

[4] At this stage ϕ remains undetermined. Suppose a worker and a firm meet for the first time and that the worker decides to continue searching. In the meantime the firm might encounter other workers that may accept the job straightaway or continue their search and recall the job at a future date. Hence, ϕ depends on the rate at which the firm encounters other applicants, the firm's wage policy and workers' search strategies.

[5] See Mortensen (1982) and Benoit, Kennes and King (2006)

ber of vacancies and the unemployment rates for each i. Namely, $M = m(v, u_0, u_1, ..., u_N)$. Assume that u_i for all i enter the meeting function additively and hence can be conveniently replaced by u. Assume further $m(v,u)$ is increasing and concave in v and u and exhibits constant returns to scale. Let $\theta = v/u$. Using this notation and the properties of $m(.,.)$, any worker's vacancy finding rate is given by $\lambda_w(\theta) = m(\theta, 1)$ and the rate at which vacancies find workers is $\lambda_f(\theta) = m(\theta,1)/\theta$.

2.1 Workers' problem

Given θ, the worker maximises expected lifetime utility. Let U_i denote the expected value of unemployment given that this worker can potentially recall $i \geq 0$ vacancies. Similarly, let $E_i(t)$ denote the expected lifetime payoff to a worker employed at a wage ω_i with current employment tenure t given that $i \geq 1$ vacancies last bid. Although there is no on-the-job search, this worker is able to recall previous vacancies after a displacement shock. This possibility implies the expected value of employment is duration dependent for $i \geq 2$.

Consider the search strategy of an unemployed worker. When meeting a vacant job this worker must decide whether to accept the job offer or to continue searching for another vacancy knowing that there is the possibility of recalling this vacancy in the future. Ignoring $O(dt^2)$ terms throughout and assuming that an unemployed job seeker knows immediately when a bidder becomes no longer available, dynamic programming arguments imply

$$U_0 = zdt + \frac{1}{1+rdt}\left[(1-\lambda_w dt)U_0 + \lambda_w dt \max\{E_1(0), U_1\}\right]$$

$$U_1 = zdt + \frac{1}{1+rdt}\left[(1-\lambda_w dt - \phi dt)\max\{E_1(0), U_1\} + \lambda_w dt \max\{E_2(0), U_2\} + \phi dt U_0\right]$$

$$U_i = zdt + \frac{1}{1+rdt}\left[\begin{array}{c} (1-\lambda_w dt - i\phi dt)\max\{E_i(0), U_i\} + \lambda_w dt \max\{E_{i+1}(0), U_{i+1}\} \\ +i\phi dt \max\{E_{i-1}(0), U_{i-1}\} \end{array}\right]$$

for all $i \geq 2$. An unemployed worker holding i vacancies stops searching and accepts a job offer if and only if that offer yields an initial payoff $E_i(0) \geq U_i$. Noting that a worker holding i vacancies rejected the same number of offers and continued searching for additional offers, U_i are the solutions to the following Bellman equations

$$(r + \lambda_w)U_0 = z + \lambda_w \max\{E_1(0), U_1\}$$

$$(r + \lambda_w)U_i = z + \lambda_w \max\{E_{i+1}(0), U_{i+1}\} - i\phi(U_i - U_{i-1}), \quad \text{for all } i \leq N - 1,$$

$$(r + \lambda_w)U_N = z + \lambda_w \max\{E_{N+1}(0), U_N\} - N\phi(U_N - U_{N-1}).$$

Now consider an employed worker. During any time interval, the probability of being displaced is δdt. If displaced and if only one vacancy had competed in the auction (the displaced firm), the worker becomes unemployed with $i = 0$. If more than one vacancy competed in the prior auction for the displaced worker, the worker is able to recall $j \leq i - 1$ of these vacancies. In this case, the worker becomes unemployed (holding j vacancies) and searches for additional offers. The corresponding expected values of employment solve the following Bellman equations

$$E_1(0) = \omega_1 dt + \frac{1}{1 + rdt}\left[(1 - \delta dt)E_1(0) + \delta dt U_0\right]$$

$$E_i(t) = \omega_i dt + \frac{1}{1 + rdt}\left[\begin{array}{c} (1 - \delta dt)E_i(t + dt) + \delta dt P_0(t + dt, i - 1)U_0 \\ +\delta dt \sum_{j=1}^{i-1} P_j(t + dt, i - 1)\max\{E_j(0), U_j\} \end{array}\right],$$

where $P_j(t, n)$ denotes the probability that an employed worker is able to recall j vacancies at time t, given that, when displaced, n vacancies had previously competed with the displacing employer in the auction for the worker's services. As ϕ describes the Poisson rate at which vacancies become unavailable to a worker,

$$P_j(t, n) = \binom{n}{j}(e^{-\phi t})^j(1 - e^{-\phi t})^{n-j}.$$

describes this probability. Noting that a displaced worker holding $i-1$ vacancies preferred unemployment U_j to employment $E_j(0)$ in the past for $j \leq i-1$, $E_i(.)$ are the solutions to

$$(r+\delta)E_1(0) = \omega_1 + \delta U_0, \tag{1}$$

$$(r+\delta)E_i(t) = \omega_i + \dot{E}_i(t) + \delta \sum_{j=0}^{i-1} P_j(t, i-1)U_j, \quad \text{for all } i \leq N+1, \tag{2}$$

2.2 Firms' problem

Taking as given θ, unemployed workers' search strategies, and competition faced at the time of recruiting a worker, a firm's chooses a wage offer to maximise the expected net return of posting a vacancy. Let V denote the expected value of a vacant job. Since a firm can potentially meet workers that can recall $n \geq 0$ other vacancies, let J_n denote the expected value of employing a worker that can engage this firm in Bertrand competition with n other firms. Standard dynamic programming arguments then imply

$$(r+\lambda_f)V = -c + \lambda_f \sum_{n=0}^{N} \gamma_n \max\{V, J_n\} \tag{3}$$

where γ_n denotes the probability that the contacted worker can engage the firm in Bertrand competition with n other firms. On the other hand, the firm's expected value of a filled job after competing with n other firms and having a winning bid of ω_{n+1} is

$$(r+\delta)J_n = x - \omega_{n+1} + \delta V, \quad \text{for all } n \geq 0. \tag{4}$$

The wage determination mechanism implies that when $n = 0$ firms will optimally set ω_1 such that $E_1 = U_1$ as in one to one meetings they retain all the bargaining power. In this case the worker is indifferent from accepting employment and continue searching for additional offers. On the other hand, when the firm faces competition for the worker the optimal wage ω_{n+1} is such that $V = J_n$ for all $n \geq 1$. Bertrand competition implies firms will optimally bid up to a wage that makes them indifferent between hiring the worker and continuing to search for another worker. Since all worker-firm pairs produce the same flow output x and firms do not recall workers, the Bertrand wage is independent of the number of bidders. Let $\omega_i = \omega^B$ for all $i \geq 2$.

2.3 Unemployment rate and vacancy creation

An important implication of the wage determination mechanism used here is that unemployed workers accept the first job offer they encounter. In turn, this implies $u_i = 0$ for all $i \geq 1$ and $u_0 = u$.[6] It is then straightforward to show that conditional on θ, the steady state rate of unemployment equals

$$u = \frac{\delta}{\delta + \lambda_w(\theta)}. \tag{5}$$

Immediate trade between a vacancy and an unemployed worker also implies that $\gamma_i = 0$ for all $i \geq 1$. Evaluating (4) at $n = 0$ and noting that profit maximisation and free entry renders $V = 0$, equation (3) implies that conditional on ω_1, θ solves

$$\frac{x - \omega_1}{(r + \delta)} = \frac{c}{\lambda_f(\theta)}. \tag{6}$$

Given u, the vacancy rate is then given by $v = u\theta$. As is standard in the DMP framework, equation (6) implies that firms create vacancies up to the point in which the expected value of employing a worker equals the expected cost of a maintaining a job vacant.

[6] Generally the unemployment rate is given by

$$u_i = \frac{\delta \sum_{j=i}^{N} \left[\int_{t=0}^{\infty} P_i(t,j)dt\right] e_{j+1}}{\lambda_w \Pr(E_{i+1} \geq U_{i+1})}.$$

for all $i \geq 0$, where e_j denotes the employment rate of workers hired holding j vacancies and Pr() describes the probability that the offered wage is accepted. Given immediate trade $\Pr(E_1 \geq U_1) = 1$ and $e_{j+1} = 0$ for all $j \geq 1$.

2.4 Wages

To analyse wages given θ, first note that due to free entry $V = J_n = 0$ for $n > 2$. The Bertrand wage therefore equals productivity, $\omega^B = x$. Next consider ω_1 and recall $E_1(0) = U_1$. Equation (1) implies that

$$\omega_1 = rU_1 + \delta(U_1 - U_0).$$

That is, the wage offered to unemployed workers not only compensates them for giving up U_1 but also for the expected loss $(U_1 - U_0)$ when a future displacement shock arrives. Furthermore, $\omega^B = x > z$ implies that $E_2(0) > U_2$. Using the workers' value function derived earlier, U_0 and U_1 can be expressed as

$$rU_0 = z + \lambda_w(\theta)[U_1 - U_0], \tag{7}$$

$$rU_1 = z + \lambda_w(\theta)[E_2(0) - U_1] - \phi(U_1 - U_0), \tag{8}$$

To fully characterise ω_1, however, we first have to obtain $E_2(t)$.

Claim 1: *Given* θ,

$$E_2(t) = \frac{x + \delta U_0}{r + \delta} + \frac{\delta(U_1 - U_0)}{r + \delta + \phi} e^{-\phi t}. \tag{9}$$

for all $t \geq 0$.

Proof: See Appendix.

Suppose a worker was employed when holding two vacancies. This worker will be paid productivity for as long as this employment lasts. Also note that this worker preferred unemployment holding one vacancy over immediate employment. To understand (9), note that without the possibility of recall after a displacement shock the value of employment would be given by $\hat{E}_2 = (x - \delta U_0) / (r + \delta)$. The assumption of recall, however, generates the second term in (9). Since there is a chance a displaced worker is able to recall a previous firm and become unemployed with $i = 1$, employees enjoy a capital gain of $(U_1 - U_0)/(r + \delta + \phi)$. As a worker's tenure increases the probability that the firm still has the job vacant, $e^{-\phi t}$, decreases and this capital gain becomes less likely.

Claim 2: *Given* θ,

$$\omega_1 = \frac{(r+\delta)(r+\phi)(r+\phi+\delta+\lambda_w)z + \lambda_w(r+\delta+\lambda_w)(r+\phi+\delta)x}{(r+\delta)(r+\delta+\phi+\lambda_w)(r+\phi+\lambda_w)+\phi\lambda_w^2}. \tag{10}$$

Proof: See Appendix.

The wage ω_1 is a weighted average of a worker's productivity and opportunity cost of employment. When setting ω_1 firms have to compensate the worker for their option of waiting for a second offer to arrive and engaging the firms into Bertrand competition.

2.5 Equilibrium

Definition: *A matching equilibrium is a triple* (u, v, ω_1) *such that (5), (6), (10) are simultaneously satisfied.*

Note that in equilibrium $\phi = \lambda_f(\theta)$.[7] Using (10) we have that

$$\omega_1(\theta) = \beta(\theta)x + [1 - \beta(\theta)]z, \tag{11}$$

for any θ, where

$$\beta(\theta) = \frac{\lambda_w(r+\delta+\lambda_w)(r+\delta+\lambda_f)}{(r+\delta)(r+\delta+\lambda_f+\lambda_w)(r+\lambda_f+\lambda_w)+\lambda_f\lambda_w^2}.$$

Given this useful result we now show existence of equilibrium.

[7] To see this suppose that the worker decided to wait for another vacancy to arrive given that everybody else trades immediately. In this case, it is easy to see that $\phi = \lambda_f(\theta)$ as the vacancy finds another worker and matches at rate $\lambda_f(\theta)$.

Claim 3: *There exists a unique matching equilibrium* (u^*, v^*, ω_1^*).

Proof: See Appendix.

To obtain further insights, we compare our model with a standard matching model used in the DMP literature in which Nash bargaining determines wages and workers cannot recall vacancies to bid up wages. (See Pissarides, 2001, Chapter 1). In the conventional model wages are given by

$$\omega_p(\theta) = (1 - \beta_n)z + \beta_n(x + c\theta), \tag{12}$$

where β_n denotes workers' exogenous bargaining power in the Nash product. Labour market tightness affects wages through its effect on the expected advertising costs of a vacancy. An increase in θ yields a higher job finding rate and a shorter unemployment duration, increasing the expected value of unemployment and hence workers' outside option. The bargaining solution implies that firms compensate workers by an exogenous proportion β_n of the reduction in the expected cost of holding a job vacant. In our model, on the other hand, the worker's effective bargaining power is fully endogenized and wages are determined by (11).

Although firms make take-it-or-leave-it offers in one-to-one meetings, they have to compensate workers for the option value of waiting until a second vacancy arrives and the subsequent Bertrand competition, knowing that when a displacement shock arrives the losing firm might still be available for recall. This effect is captured in $\beta(\theta)$, the worker's effective bargaining power. With recall, an increase in labour market tightness has an additional effect that increases this option value and hence the workers' bargaining power and ultimately wages.

The next result establishes the relationship between the standard model and the model with recall. In the standard matching model, the equilibrium v-u ratio, θ_p, varies with the Nash bargaining β_n. Wages and unemployment rates ω_p and u_p then follow from equations (12) and (5), respectively and $v_p = \theta_p u_p$. Claim 4 establishes the unique Nash bargaining power parameter, β_n^*, that corresponds to the v-u ratio found in the model with recall. Simply put, this result endogenizes the Nash bargaining parameter - if the Nash bargaining parameter equals β_n^*, the equilibrium outcomes in the two models are equivalent.

Claim 4: *There exists a unique β_n given by*

$$\beta_n^* = \frac{\beta(\theta^*)\lambda_f(\theta^*)(x-z)}{\lambda_f(\theta^*)(x-z)+\lambda_w(\theta^*)c},$$

such that $u_p = u^*$, $v_p = v^*$ and $\omega_p = \omega_1^*$.

Proof: See Appendix.

Implicit differentiation of (18) (in the Appendix) reveals that $d\theta_p/d\beta_n < 0$. Hence, $\beta_n > \beta_n^*$ implies $\theta_p < \theta^*$ and the standard matching model exhibits a higher unemployment rate, a lower vacancy rate and a higher wage than in our model. Also note that our model is generically not constrained efficient. Following Hosios (1991) it is easy to verify that the outcome of our model is efficient only when $\beta_n^* = \eta_f$, where η_f denotes the absolute value of the elasticity of λ_f.

To obtain a sense of the quantitative effects of the recall option we calibrate the model. The matching function is Cobb-Douglas: $M(v,u) = \mu u^a v^{1-a}$ so that $\lambda_w(\theta) = \mu\theta^{1-a}$, $\lambda_f(\theta) = \mu\theta^{-a}$ and $\eta_f = a$. The time period is a month and $r = 0.004$, which implies a yearly discount factor of 0.95. Following Hall and Milgrom (2008) we set $x = 1$, $z = 0.71$, $c = 0.43$, $a = 0.5$. As in Shimer (2005) we set $\delta = 0.034$ and set $\mu = 0.45$ to target a job find rate of $\lambda_w = 0.45$

and an average v-u ratio of $\theta = 1$. Finally, we set $\beta_n = a$ so that the standard matching model gives efficient outcomes.

Simulations show that $u^* = v^* = 0.07$, $\beta = 0.87$ and $\omega^* = 0.96$; while in the standard matching model the corresponding values are $\theta_p = 0.55$, $u_p = 0.09$, $v_p = 0.05$ and $w_p = 0.97$. The job finding rate in the latter case is $\lambda_w = 0.33$. These parameter values generate $\beta_n^* = 0.3443$. Since $\beta_n > \beta_n^*$, as suggested by Claim 4, our wage determination mechanism implies that firms offer lower wages and create more vacancies than in an a constrained efficient Nash Bargaining economy. This implies a higher job finding rate and lower unemployment. Given a symmetric bargaining power benchmark of $\beta_n = 0.5$, the option of recall is able to explain 68.9 percent of this benchmark bargaining power.

Now consider the impact of an increase in labour productivity on equilibrium outcomes. It is useful to compare the elasticity of θ with respect to $x - z$ in both models. In the standard matching model

$$\varepsilon^p_{\theta,x-z} = \frac{r + \delta + \beta_n \lambda_w(\theta)}{\lambda_w(\theta)\beta_n + (r+\delta)(1-\eta_w)},$$

where $\eta_w \in [0,1]$ is the elasticity of $\lambda_w(\theta)$ and under a Cobb Douglas matching function $1 - \eta_w = a$. Equation (17) implies this elasticity is given by

$$\varepsilon_{\theta,x-z} = \frac{1-\beta(\theta)}{\theta\beta'(\theta) + (1-\eta_w)(1-\beta(\theta))}.$$

For this comparison consider the case in which $\beta_n = \beta_n^*$. Using the parameter values described above we obtain that $\varepsilon^p_{\theta,x-z} = 1.1 > \varepsilon_{\theta,x-z} = 0.75$. The vacancy-unemployment ratio is more responsive to changes in net productivity when wages are determined by Nash bargaining since an increase in productivity yields a lower wage increase under bargaining, which induces firms to create proportionally more vacancies. As suggested by Shimer (2005), however, the responsiveness of θ to changes in x with Nash Bargaining is too small to explain the variation of θ over the business cycle. The calibration exercise suggests that the model with recall exhibits the same property.[8]

3 On-the-job Search

This and the following section extend the basic model in a number of directions. The purpose is two fold. First, the extensions generate insights into the behavior of wages across workers and over time. The results are interesting on their own. Moreover, these extensions illustrate that the proposed wage determination process provides a well-grounded, flexible approach that can be applied in a variety of situations that have caused technical and conceptual difficulties for other approaches to wage formation, Nash and strategic bargaining in particular. The appeal of this methodology lies not only with the merits of the process but also in its use for applied model building.

The first extension allows for on-the-job search job-to-job transitions.[9] With recall on-the-job search plays two roles: (i) workers engage in on-the-job search to increase their wages and (ii) to accumulate information about the location of jobs. The latter has a positive value as it allows a displaced worker to recall firms encountered while employed. Job destruction shocks can then lead to unemployment (if no vacancy is available for recall) or to immediate

[8] Hagedorn and Manovskii (2008) propose a different calibration strategy that uses a higher value of the opportunity cost of employment ($z = 0.95$), a higher values of the cost of posting a vacancy ($c = 0.584$) and a lower value of the Nash Bargaining parameter ($\beta_n = 0.052$). They argue that under their calibration strategy the standard matching model does reproduce the observed variation of θ at business cycle frequencies. We consider their parameter values to analyse if our model is able to generate a higher elasticity of θ with respect to net productivity. Under Hagedorn and Manovskii parametrization we have that $\varepsilon^p_{\theta,x-z} = 1.54$ and $\varepsilon_{\theta,x-z} = 0.85$, which clearly improves the empirical predictions of both models. However, for our model the elasticity is still insufficient to generate enough volatility in the v-u ratio.

[9] See Fallick and Fleishman (2004) and Nagypal (2008) for evidence documenting the importance of such behaviour.

re-employment and an apparent job to job transition (if a vacancy contact was made and remains open for recall). Jolivet, Postel-Vinay and Robin (2006) and Christensen, Lentz, Mortensen, Neumann and Werwatz (2005) find that reallocation shocks are pervasive in many labour markets and explain an important proportion of job-to-job transition. Allowing workers to recall vacancies gives a microfoundation for such transitions.[10]

To make the analysis as simple as possible, assume that workers are able to store only one vacancy to recall in the future, i.e. $N = 1$, so that employed workers stop searching after receiving their first outside offer.[11] Let e_i denote the number of workers holding $i \geq 0$ vacancies to recall and consider the matching function $M = m(v, u + \alpha e_0)$, where $\alpha > 0$ denotes the search intensity of employed workers. $\alpha = 0$ gives the model without on-the-job search, whereas $\alpha = 1$ implies employed workers search with equal intensity as unemployed workers. Empirical evidence (Jolivet, Postel-Vinay and Robin, 2006) suggests that $\alpha \in (0, 1)$. As before assume that the matching function is increasing, differentiable and concave in both of its arguments and exhibits constant returns to scale. Letting $\widehat{\theta} = v/(u + \alpha e_0)$ denote our new market tightness, the rate at which unemployed workers meet vacancies is $\lambda_u(\widehat{\theta}) = m(\widehat{\theta}, 1)$, while the meeting rate of employed workers is $\lambda_e(\widehat{\theta}) = \alpha m(\widehat{\theta}, 1)$. The rate at which vacancies meet workers is $\lambda_f(\widehat{\theta}) = m(\widehat{\theta}, 1)/\widehat{\theta}$.

Reconsider the worker's problem and note that the Bellman equations describing the expected value of unemployment remain unchanged. Moreover, the Bellman equation describing the expected value of employment for $i = 2$ also remains unchanged. The expected

[10] Postel-Vinay and Robin (2002) analyse a similar model. The main difference is that they did not allow workers' to recall past offers or endogeneized vacancy creation.

[11] We implicitly assume that the worker is not able to "swap" a new vacancy for an old one. Otherwise, workers will continue searching even when he or she has N vacancies. Since vacancies can dissappear, the worker will prefer to discard old ones for the new ones he encounter which "re-sets the clock" of a vacancy's life.

value of employment given that one vacancy bid for the worker's services, however, is now given by

$$rE_1(0) = \omega_1 + \lambda_e[E_2(0) - E_1(0)] - \delta[E_1(0) - U_0]$$

$U_1 = E_1(0)$ determines wages ω_1:

$$\omega_1 = rU_1 + \delta[U_1 - U_0] - \lambda_e[E_2(0) - U_1].$$

Given a $\theta = \widehat{\theta}$, the wage offered to unemployed workers, ω_1, is now smaller. With on-the-job search, workers are prepared to accept a lower wage as in investment for future wage growth. Given a $\widehat{\theta}$, substituting (7), (8) and (9) in the previous equation yields the following generalisation of (11)

$$\omega_1(\widehat{\theta}) = \beta(\widehat{\theta})x + (1 - \beta(\widehat{\theta}))z - \alpha\psi(\widehat{\theta})(x - z), \tag{13}$$

where

$$\psi(\widehat{\theta}) = \frac{\lambda_u(r + \lambda_f + \lambda_u)(r + \delta + \lambda_f)}{(r + \delta)(r + \delta + \lambda_f + \lambda_u)(r + \lambda_f + \lambda_u) + \lambda_f \lambda_u^2}$$

and we used $\phi = \lambda_f(\widehat{\theta})$ in equilibrium, $\omega^B = x$ and replaced λ_w with λ_u in the expression for $\beta(.)$.

Now consider the steady state measures of unemployed and employed workers. During each period dt, the flow out of unemployment is given by those worker that received an offer, $\lambda_u dtu$. The unemployment inflow is composed of those employed workers with $i = 0$ that did not receive an outside offer and got displaced, $(1 - \lambda_e dt)\delta dte_0$. Steady state turnover implies that $u = (\delta/\lambda_u)e_0$. In the case of employed workers with $i = 0$, during each period dt the flow out is given by those workers that did not get displaced and received an outside offer, $(1 - \delta dt)\lambda_e dte_0$ and those workers that did not receive an outside offer and got displaced,

$(1-\lambda_e dt)\delta dt e_0$. The inflow into employment e_0 is given by those unemployed workers that received an offer, $\lambda_u dt u$, and by those employed workers holding $i=1$ that got displaced and could recall their vacancy, $(1-\lambda_f dt)\delta dt e_1$ and those $i=1$ workers that did not get displaced but lost their vacancy, $(1-\delta dt)\lambda_f dt e_1$. Steady state turnover then implies that $e_0 = [\lambda_u u + (\lambda_f+\delta)e_1]/(\lambda_e+\delta)$. Noting that $u+e_0+e_1=1$, we have that

$$u = \frac{\delta(\delta+\lambda_f)}{(\delta+\lambda_f)(\delta+\lambda_u)+\alpha\lambda_u^2}, \tag{14}$$

$$e_0 = \frac{\lambda_u(\delta+\lambda_f)}{(\delta+\lambda_f)(\delta+\lambda_u)+\alpha\lambda_u^2}, \tag{15}$$

$$e_1 = \frac{\alpha\lambda_u^2}{(\delta+\lambda_f)(\delta+\lambda_u)+\alpha\lambda_u^2}. \tag{16}$$

With on-the-job search a matching equilibrium is a quintuple-tuple $\{u, e_0, e_1, v, \omega_1\}$ such that equations (13), (14), (15), (16) and the generalisation of (6)

$$\frac{x-\omega_1}{r+\delta+\alpha\lambda_u(\widehat{\theta})} = \frac{c}{\lambda_f(\widehat{\theta})\gamma_0(\widehat{\theta})}$$

are simultaneously satisfied. Using arguments as in the previous sections, the next claim shows that an equilibrium exists.

Claim 5: *There exists a unique matching equilibrium with on-the-job search* $(u^*, e_0^*, e_1^*, v^*, \omega_1^*)$.

Proof: See Appendix.

Numerical simulations contrast the models with and without on-the-job search. Consider a matching function of the form $M(v, u+\alpha e_0) = \mu(u+\alpha e_0)^a v^{1-a}$ so that $\lambda_u(\widehat{\theta}) = \mu\widehat{\theta}^{1-a}$, $\lambda_e(\widehat{\theta}) = \alpha\mu\widehat{\theta}^{1-a}$ and $\lambda_f(\widehat{\theta}) = \mu\widehat{\theta}^{-a}$. Let $\alpha = 0.25$ so that the meeting rate of employed workers is one quarter of that for unemployed workers. The rest of the parameter values are as before. In equilibrium, $\widehat{\theta} = 0.08$, which implies $\lambda_u = 0.13$ and $u = 0.20$, $e_0 = 0.78$, $e_1 = 0.02$ and $v = 0.03$. Vacancy creation is smaller with on-the-job search as expected profits are smaller given Bertrand competition for employed workers. In turn, this reduces the unemployed workers job finding rate and increases the unemployment rate.

On the other hand, workers anticipate a wage increase as soon as an outside offer arrives when employed. They are prepared to accept a lower wage to start employment. Given the above parameter values, wages are below the flow payoff while unemployed $\omega_1^* = 0.6 < z$. Moreover, increases in x yields a lower value of ω_1.

An interesting feature bourne from the simulations is the non-monotonic relation of $d\omega_1/dx$ with respect to α. For values of $\alpha \leq 0.18$ increases in x generate an increase in wages, while for higher values of α increases in x decreases wages ω_1. Moreover, as Shimer (2003) argues, this extension has the potential to generate a higher volatility in θ as an increase in x reduces the reservation wage of workers and gives firms greater incentives to create more vacancies.

4 Risk Aversion and Duration Dependent UI

This section returns to the initial case of no on-the-job search but allows for consumption smoothing with risk averse agents and potentially time varying UI payments. The aim is not to derive analytic solutions for this general formulation. The objective is to demonstrate that this approach yields familiar relationships that can be subsequently refined to evaluate consumption choices in models with uncertain search unemployment (Lentz and Tranaes, 2005; Browning, Crossley and Smith, 2007) with or without duration dependent UI payments.

Evaluating UI policy issues with both an insurance and moral hazard problem has proved challenging in the literature. See for example Coles and Masters (2005). In the search literature, characterizing optimal UI is often done with risk neutral agents so focuses on problems of moral hazard. The paper does not directly confront this issue but instead outlines the approach the proposed wage mechanism can take.

The paper does offer an exact solution for the analysis of effects of UI payments for the simpler case found in the literature in which there are risk neutral workers and hence there is no insurance motive for UI payments. The moral hazard problem is allowed. The outcome of wage setting with bidder recall is explicitly derived and shown to be consistent with several established phenomena. Wages are dispersed without the existence of on-the-job search. Wages vary by employment duration. A scarring effect from unemployment obtains in which wages fall with the duration of job search (Addison, Centeno and Portugal 2004). Alternatively, there is a direct relationship between wages and the length and magnitude of UI payments.

To keep things simple, suppose again that there are no layoffs $\delta = 0$ and that workers can recall at most one bidder, $N = 1$. The government does not observe wage offers, so it cannot condition UI payments on employment status and a moral hazard problem arises. Adjusting notation to account for time variation (t now represents unemployment duration) and asset levels A, the workers decision is given by

$$U_0(A,t) = \max_{c_0} \left\{ u(c_0)dt + \frac{1}{1+rd\tau}\left[(1-\lambda_w d\tau)U_0(A',t') + \lambda_w dt U_1(A',t')\right] \right\}$$

and

$$U_1(A,t) = \max_{c_1} \left\{ u(c_1) + \frac{1}{1+rd\tau} \left[\begin{array}{c} (1-\lambda_w d\tau - \phi d\tau)U_1(A',t') \\ +\lambda_w d\tau u(rA' + \omega^B)/r + \phi d\tau U_0(A',t') \end{array} \right] \right\}$$

where time and assets evolve in the standard ways

$$t' = t + dt$$

$$A' = (1 + rdt)[A + b(t)dt - c_i dt] \; i \in \{1, 2\}$$

Given no search while employed and no layoffs, the payoff to accepting a job is given by the wage which as before equals $rU_1(A, t)$. Standard procedures lead to the following characterization of consumption and the progression of payoffs and hence wages.

Claim 6: *The optimal consumption path satisfies*

$$-u"(c_0(A, t)) = -\lambda_w[u'(c_0(A, t)) - u'(c_1(A, t))]$$

$$-u"(c_1(A, t)) = -\lambda_w[u'(c_1(A, t)) - u'(rA + \omega^B)] - \phi[u'(c_1(A, t)) - u'(c_0(A, t))]$$

and the time progression of payoffs evolves according to

$$-\dot{U}_0(A, t) + (r + \lambda)U_0(A, t) = u(c_0(A, t)) + \lambda_w U_1(A, t) + u'(c_0(A, t))[rA + b(t) - c_0(A, t)]$$

$$-\dot{U}_1(A, t) + (r + \lambda + \phi)U_0(A, t) = u(c_1) + \lambda_w \omega^B / r + \phi U_1(t) + u'(c_1(A, t))[rA + b(t) - c_1(A, t)]$$

Proof: See Appendix

Solutions to these differential equations naturally depend on the timing of the benefits along with the specification of utility. For many specifications of risk aversion, numerical methods are further required. Given the relative familiarity of the consumption asset equation (Browning, Crossley and Smith, 2007), the analysis here focuses on the time dimension of payoffs (wages) and the dependence on benefits by assuming the simple case of linear utility.

Consumption becomes indeterminate and only the time evolution retains economic meaning. In this case, assets have no role. Set $A = 0$ and let consumption be per period income, $c_0 = c_1 = b(t)$. The differential equations reduce to

$$\dot{U}_0(\tau) + (r + \lambda)U_0(\tau) = z + b(\tau) + \lambda_w U_1(\tau)$$

$$\dot{U}_1(\tau) + (r + \lambda + \phi)U_0(\tau) = z + b(\tau) + \lambda_w \omega^B / r + \phi U_1(\tau)$$

These two linear differential equations can be solved using familiar techniques to yield general solutions that depend on the nature of the duration dependence in b. The solutions can be used to help the optimal UI payment schemes.

Rather than pursuing policy design issues, attention turns to the solution given a fixed payment for a duration of fixed length Γ. Given linear payoffs, the solutions are given by

$$U_0(\tau) = Ae^{-(r+\lambda_w+(\phi+s)/2]t} + Be^{-(r+\lambda_w+(\phi-s)/2]t} + U_0^b$$

and

$$U_1(\tau) = \frac{\phi + s}{2\lambda_w} Ae^{-(r+\lambda_w+(\phi+s)/2]t} + \frac{\phi - s}{2\lambda_w} Be^{-(r+\lambda_w+(\phi-s)/2]t} + U_1^b$$

where $s = (\phi^2 + 4\lambda\phi)^{1/2}$ and

$$\begin{aligned}
A &= \frac{(r + \lambda_w + \phi)2\lambda_w + (r + 2\lambda_w + \phi)(\phi - s)}{[(r + \lambda_w)^2 + r\phi]2s} b \\
B &= -\frac{(r + \lambda_w + \phi)2\lambda_w + (r + 2\lambda_w + \phi)(\phi + s)}{[(r + \lambda_w)^2 + r\phi]2s} b \\
U_0^b &= \frac{(r + 2\lambda_w + \phi)(z + b) + \lambda_w^2 \omega^B / r}{(r + \lambda_w)^2 + r\phi} \\
U_1^b &= \frac{(r + 2\lambda_w + \phi)(z + b) + (r + \lambda_w)\lambda_w \omega^B / r}{(r + \lambda_w)^2 + r\phi}
\end{aligned}$$

Algebra establishes that $\dot{U}_i(\tau) < 0$. Wages fall with the duration of unemployment over time. Wage dispersion occurs over the range $[U_1(\Gamma), U_1(0)]$ due to the scarring effect of finding employment later in the spell of joblessness. The scarring effect becomes more pronounced as the termination of payments approaches.

Unemployment insurance offers one obvious scenario of time dependence that generates wage dispersion associated with longer spells of unemployment. There are, of course, other possibilities that might arise from the matching process (stock-flow matching) or other limitation on resources (e.g. assets and risk aversion above). UI payments are an illustrative example of the way in which the wage mechanism presented here can incorporate time varying phenomena to deliver tractable, plausible and verifiable outcomes.

5 Discussion

Search and matching models have become an essential tools for evaluating not only individual labor market experiences but also aggregate economic performance. The Diamond-Mortensen-Pissarides framework with wage bargaining lies at the heart of much of these successes. (See Mortensen and Pissarides, 1999a,b; Rogerson, Shimer and Wright (2006) for reviews.) However successful, this framework and especially wage bargaining does not fit universally into all situations with search frictions.

To supplement the DMP model, this paper adopts auction bidding for worker services. A worker can control to some degree the number of bidders in the auction. The ability to recall previously eligible bidders provides the worker with a counter weight to the firm's advantages through price setting as highlighted by the well-known Diamond paradox. Workers who can recall bidders in order to engage them in Bertrand competition have a distinct advantage over workers without such attachments. Firms will account for this difference when trying to hire the worker and thereby sacrifice some monopoly payoff to the worker. In this case, the Diamond paradox on monopoly pricing does not occur nor do workers have the ability to fully extract all rents. Sharing occurs.

Allowing for recall is more than a simple alternative to bargaining for basic matching models. It is a plausible, dynamically consistent method of sharing the match surplus that can be introduced in a variety of environments to yield insights and potential links with observed behaviours. To demonstrate this flexibility, the paper introduces the recall mechanism into environments that present difficulties for wage bargaining. The first extension contains on-the-job search. The second adopts concave utility and time varying UI payments. The proposed wage mechanism is not only straightforward to use but the resulting equilibrium outcomes shed light on job-to-job transitions, wage dispersion and the scarring effects of unemployment.

References

Addison, J., M. Centeno, and P. Portugal (2004) "Reservation Wages, Search Duration and Accepted Wages in Europe", IZA Discussion Paper No. 1252, August

Albrecht, J. and B. Axell (1984) "An Equilibrium Model of Search Unemployment", *Journal of Political Economy,* 92: 824-840

Beniot, J., J. Kennes and I. King, (2006), "The Mortensen Rule and Efficient Coordination Unemployment", *Economics Letters*, Vol. 90(2), pp. 149-155.

Browning, M., T Crossley and E. Smith (2007) "Asset Accumulation and Short-Term Unemployment", *Review of Economic Dynamics*, 10:400-423.

Burdett, K.and K Judd (1983) "Equilibrium Price Dispersion", *Econometrica*, 51:955-970.

Burdett, K. and D. Mortensen (1998) "Wage Differentials, Employer Size and Unemployment", *International Economic Review*, 39:257-273.

Coles, M. and A. Masters (2006) "Optimal Unemployment Insurance in a Matching Equilibrium", *Journal of Labor Economics*, 24: 109-138.

Christensen, B., R. Lentz, D. Mortensen, G. Neumann and A. Werwatz, (2005) , "On the job Search and the Wage Distribution", *Journal of Labor Economics*, 23, 1, pp. 31-58.

Diamond, P. (1971) "A Model of Price Adjustment", *Journal of Economic Theory*, 3:156-168.

Fallick, B. and Fleischman, C. (2004) , "Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows", Finance and Economics Discussion Series, 2004-34, Board of Governors of the Federal Reserve System.

Gaumont, D., M. Schindler and R. Wright (2006) "Alternative Theories of Wage Dispersion", *European Economic Review*, 50: 831-848.

Hagedorn, M. and I. Manovskii, (2008), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited", *American Economic Review*, Vol. 98, No. 2, pp. 1692-1706.

Hall, R. and P. Milgrom, (2008), "The Limited influence of Unemployment on the Wage Bargain", *American Economic Review*, Vol. 98, No. 2, pp. 1653-1674.

Hosios, A., (1990), "On the Efficiency of Matching and Related Models of Search Unemployment", *Review of Economic Studies*, 57(2), pp. 279-98.

Jolivet, G., Postel-Vinay, F and Robin, J-M (2006) , "The Empirical Content of the Job Search Model: Labor Mobility and Wage Distributions in Europe and the US", *European Economic Review*, 50, pp. 877-907.

Lentz, R. and T. Tranaes (2005) "Job Search and Savings: Wealth Effects and Duration Dependence", J*ournal of Labor Economics*, 23: 467-490.

Mortensen, D., (1982), "Property rights and efficiency in mating, racing and related games", *American Economic Review*, Vol. 72, No. xx, pp. 968-979.

Mortensen, D. and C. Pissarides (1999a) "New Developments in Models of Search in the Labor Market", in *Handbook of Labor Economics*, O. Ashenfelter and D. Card (eds.), Amsterdam: NorthHolland.

Mortensen, D. and C. Pissarides (1999b) "Job Reallocation, Employment Fluctuations, and Unemployment Differences", in M. Woodford and J. Taylor (eds.) *Handbook of Macroeconomics*, Amsterdam, North-Holland.

Nagypal, E. (2008) , "Worker Reallocation over the Business Cycle: The Importance of Job-to-Job Transitions", mimeo, Northwestern University

Pissarides, C., (2001), *Equilibrium Unemployment Theory*, 2^{nd} ed. Cambridge, MA, MIT Press.

Rogerson, R., R. Shimer and R. Wright, (2005) "Search-theoretic Models of the Labor Market: A Survey" *Journal of Economic Literature*, 43: 959-988.

Shimer, R., (2003) "Dynamics in a Model of On-the-Job Search", mimeo, University of Chicago.

Shimer, R., (2005), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies", *American Economic Review*, Vol. 95, No. 1, pp. 25-49.

Shimer, R., (2006) "On-the-job Search and Strategic Bargaining", *European Economic Review*, 50: 811-830.

APPENDIX

Proof Claim 1: Fix θ and note that the workers' value functions for $i = 2$ is given by

$$(r+\delta)E_2(t) = x + \dot{E}_2(t) + \delta\left[P_0(t,1)U_0 + P_1(t,1)U_1\right].$$

Noting that $P_0(t,1) = 1-e^{-\phi t}$ and $P_1(t,1) = e^{-\phi t}$, we have that $E_2(t)$ solves the following linear ordinary differential equation

$$\dot{E}_2(t) - (r+\delta)E_2(t) = -[x + \delta U_0 + \delta e^{-\phi t}(U_1 - U_0)],$$

subject to the boundary condition

$$\lim_{t\to\infty} E_2(t) = \frac{x+\delta U_0}{r+\delta}.$$

Using $e^{-(r+\delta)t}$ as the integrating factor the general solution to this differential equation is

$$E_2(t) = Ae^{(r+\delta)t} + \frac{x+\delta U_0}{r+\delta} + \frac{\delta(U_1-U_0)}{r+\delta+\phi}e^{-\phi t},$$

where A is the constant of integration. Applying the boundary condition gives the equation stated in the Claim.$\|$

Proof of Claim 2: Fix a θ. Evaluating $E_2(t)$ at $t = 0$ and substituting this expression into (8) yields

$$rU_1 = z + \lambda_w\left[\frac{x}{r+\delta} + \frac{(r+\lambda_f)U_1}{r+\delta+\phi} + \frac{\phi U_0}{(r+\delta)[r+\delta+\phi]}\right] - \phi\left[U_1 - U_0\right].$$

Using (7) we have a system of linear equations in U_0 and U_1 such that the solution for U_0 and U_1 is given by

$$U_0 = \frac{(r+\delta)[(r+\delta+\phi)(r+\phi+\lambda_w)+\lambda_w(r+\phi)]z+\lambda_w^2(r+\delta+\phi)x}{r\left[(r+\delta)(r+\delta+\phi+\lambda_w)(r+\phi+\lambda_w)+\phi\lambda_w^2\right]},$$

$$U_1 = \frac{[(r+\delta)(r+\delta+\phi)(r+\phi+\lambda_w)+\delta\phi\lambda_w]z+\lambda_w(r+\lambda_w)(r+\delta+\phi)x}{r\left[(r+\delta)(r+\delta+\phi+\lambda_w)(r+\phi+\lambda_w)+\phi\lambda_w^2\right]}.$$

Noting that $\omega_1 = rU_1 + \delta(U_1 - U_0)$ and using the above expression for U_0 and U_1 yields the expression stated in the claim$\|$.

<u>Proof of Claim 3:</u> Substituting out $\omega_1(\theta)$ from (6) we obtain that the equilibrium θ solves

$$[1-\beta(\theta^*)](x-z) = \left[\frac{r+\delta}{\lambda_f(\theta^*)}\right]c. \tag{17}$$

Let $\Gamma(\theta) = (1-\beta(\theta))(x-z)$. Some algebra establishes Γ is decreasing in θ as $\beta'(\theta) > 0$. Similarly, define $\Psi(\theta) = (r+\delta)c/\lambda_f(\theta)$. Since $\lambda_f'(\theta) < 0$ by the properties of the matching function, Ψ is strictly increasing in θ. Continuity then implies that there exists a unique θ^* such that $\Gamma(\theta^*) = \Psi(\theta^*)$. The wage offered to unemployed workers is then given by $\omega_1^* = \omega_1(\theta^*)$. The equilibrium rate of unemployment u^* is obtained by using (5) and the vacancy rate is given by $v^* = u^*\theta^*$. $\|$.

<u>Proof of Claim 4</u>: Using the results in Pissarides (2001), it is easy to verify that in the standard matching model equilibrium θ_p is given by

$$(1-\beta_n)\lambda_f(\theta_p) - \frac{\beta_n c\lambda_w(\theta_p)}{x-z} = \frac{(r+\delta)c}{x-z}. \tag{18}$$

Similarly, using (17), in the recall equilibrium we have that θ^* solves

$$[1-\beta(\theta^*)]\lambda_f(\theta^*) = \frac{(r+\delta)\,c}{x-z}.$$

Since both equilibria always exist, it follows that θ_p and θ^* satisfy

$$(1-\beta_n)\lambda_f(\theta_p) - \frac{\beta_n c \lambda_w(\theta_p)}{x-z} = [1-\beta(\theta^*)]\lambda_f(\theta^*).$$

Solving for β_n^* yields the result stated in the Claim.‖

Proof of Claim 5: Given $\widehat{\theta}$, first note that the expected value of a vacancy is given by

$$(r+\lambda_f(\widehat{\theta}))V = -c + \lambda_f(\widehat{\theta})\gamma_0(\widehat{\theta})\max\{V, J_0\} + \lambda_f(\widehat{\theta})\gamma_1(\widehat{\theta})\max\{V, J_1\},$$

where $\gamma_0 = u/(u+\alpha e_0)$ and $\gamma_1 = \alpha e_0/(u+\alpha e_0)$. If a vacancy meets an employed worker earning ω_1, Bertrand competition implies that $J_1 = 0$. Moreover, since free entry implies that $V = 0$, we have that

$$J_0 = \frac{c(u+\alpha e_0)}{\lambda_f(\widehat{\theta})u}.$$

Now consider the expected value described by J_0. Since as soon as the worker receive an outside offer the firm obtains a continuation payoff of zero, we have that

$$(r+\delta+\alpha\lambda_u(\widehat{\theta}))J_0 = x - \omega_1.$$

Using the expression for u, the expression for ω_1 and the above two conditions we have the generalisation of (17) describing the equilibrium $\widehat{\theta}$,

$$\left[1-\beta(\widehat{\theta}^*) - \alpha\psi(\widehat{\theta}^*)\right](x-z) = \frac{\left[r+\delta+\alpha\lambda_u(\widehat{\theta}^*)\right]\left[(\delta+\lambda_f(\widehat{\theta}^*))(\delta+\alpha\lambda_u(\widehat{\theta}^*))\right]c}{\delta(\delta+\lambda_f(\widehat{\theta}^*))\lambda_f(\widehat{\theta}^*)}.$$

Let $\Gamma(\widehat{\theta}) = 1-\beta(\widehat{\theta}) - \alpha\psi(\widehat{\theta})$ and note it is decreasing in θ since $\beta' > 0$ and $\psi' > 0$. Similarly, let $\Psi(\widehat{\theta}) = [r+\delta+\alpha\lambda_u(\widehat{\theta})][(\delta+\lambda_f(\widehat{\theta}))(\delta+\alpha\lambda_u(\widehat{\theta}))]c/[\delta(\delta+\lambda_f(\widehat{\theta}))\lambda_f(\widehat{\theta})]$. Differentiation implies that $\Psi' > 0$. Continuity of Γ and Ψ then establishes that there exists a unique $\widehat{\theta}^*$ such that $\Gamma(\widehat{\theta}^*)$ and $\Psi(\widehat{\theta}^*)$. The wage offered to unemployed workers is then given by

$\omega_1^* = \omega_1(\widehat{\theta}^*)$. The equilibrium u, e_0 and e_1 are given by substituting $\widehat{\theta}^*$ in (14), (15) and (16), respectively. The equilibrium vacancy rate is then given by $v^* = [u^* + \alpha e_0^*]\,\widehat{\theta}^*$. ‖

Proof of Claim 11: The first order conditions are given by

$$u'(c_i) = U_i^A(A) \; i = \{1, 2\}$$

where the U^A denotes the partial derivative with respect to A. Total differentiation gives

$$u^{//}(c_i)dc_i = U_i^{AA}(A,t)dA + \dot{U}_i^A(A,t)dt \; i = \{1, 2\}$$

The envelope condition with respect to A yields

$$-U_0^{AA}(A,t)(rA + b(t) - c_0) - \dot{U}_0^A(A,t) = -\lambda_w[U_0^A(A,t) - U_1^A(A,t)]$$

$$\begin{aligned} -U_1^{AA}(A,t)(rA + b(t) - c_1) - U_0^{At}(A,t) &= -\lambda_w[U_1^A(A,t) - u'(rA + \omega^B)] \\ &\quad -\phi[U_1^A(A,t) - U_0^A(A,t)] \end{aligned}$$

Substitution gives the stated result for the consumption asset relationship.

$$-u^{//}(c_0(A,t)) = -\lambda_w[u'(c_0(A,t)) - u'(c_1(A,t))]$$

$$\begin{aligned} -u^{//}(c_1(A,t)) &= -\lambda_w[u'(c_1(A,t)) - u'(rA + \omega^B)] \\ &\quad -\phi[u'(c_1(A,t)) - u'(c_0(A,t))] \end{aligned}$$

Differentiation with respect to time and then using the consumption asset equations gives

$$-\dot{U}_0(A,t) + (r+\lambda)U_0(A,t) = u(c_0(A,t)) + \lambda_w U_1(A,t) + u'(c_0(A,t))[rA + b(t) - c_0(A,t)]$$

$$-\dot{U}_1(A,t) + (r+\lambda+\phi)U_0(A,t) = u(c_1) + \lambda_w\omega^B/r + \phi U_1(t) + u'(c_1(A,t))[rA + b(t) - c_1(A,t)].$$