# Housing Markets and Labor Markets: Time to Move and Aggregate Unemployment*

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**Abstract**

A model is developed that allows for interaction between the labor market and the housing market. A job location has an associated commuting time that may affect the job acceptance decision. Obstacles to mobility, such as regulations in the housing market, will affect the reservation strategy of workers. Thus, aggregate unemployment will depend, at least partly, on the functioning of the housing market.

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# 1 Introduction

This paper starts with three facts. First, Table 1 reveals that 15.5% of American residents move yearly for one reason or another. In comparison, data for fifteen European Union countries show that less than 5% of residents move yearly.[1]

Table 1: Mobility in the U.S. and E.U.

| | **US** | **EU15** |
|---|---|---|
| Mobility rate | 15.5% | 4.95% |
| Share within county / area | 0.58 | 0.83 |
| Share between county / area | 0.42 | 0.17 |

Second, Table 2 provides an overview of reasons for moving for those that moved. In the U.S., most of the intra-county mobility is house related (65.4%), and only a small fraction (5.6%) move within a county for job related reasons. In contrast, work related mobility is approximately 33% of inter-county moves. Family related mobility is a fairly constant fraction of all moves. In the E.U., perhaps surprisingly, there is a very similar pattern. Although residents in the U.S. move about three times the rate of those in the E.U., the reasons why they move are roughly similar. This is our second fact. This observation suggests that the shocks that affect mobility are similar across the countries; yet, there is substantially less overall mobility in the E.U. [2]

Third, there are large geographical mobility differences between countries. Scandinavian countries report mobility rates which are close to the US rates, while Southern European countries

[1] Data for the U.S. comes from Geographic Mobility, Current Population Reports, P20-538, U.S. Census, May, 2001. Data for the E.U. comes from the European Community Household Panel 1999-2001. Mobility data for the U.S. come from the U.S. Census 2000. The two data sources cover essentially the same time period and ask very similar questions, making for a useful comparison of mobility patterns across countries. Although the paper does not address short vs. long distance moves (this is addressed in a companion paper Rupert and Wasmer 2009), we include those data for completeness. About 58% of the moves are within county. In contrast, mobility in the EU15 is about 1/3 of that in the U.S. The share of moves within an area/locality is higher, although at this level of disaggregation it is difficult to strictly compare to U.S. counties.

[2] Why People Move: Exploring the March 2000 Current Population Survey, Current Population Reports, P23-204, May, 2001, U.S. Department of the Census. The Census questions are actually more precise about the exact reasons for mobility, while no detailed questions are available in the ECHP. Table 9 in Appendix gives the details for the U.S.

Table 2: Reasons for Moving, US and EU15

| | Proportions, US | | | | Proportions, EU15 | | |
|---|---|---|---|---|---|---|---|
| All pop. (1+) | intra-county | inter-county | all | | intra-area | inter-area | all |
| Work related | 5.6% | 31.1% | 16.2% | Job related | 7.61% | 40.0% | 14.3% |
| Family related | 25.9% | 26.9% | 26.3% | Personal Reason | 31.6% | 29.8% | 31.3% |
| House related | 65.4% | 31.9% | 51.6% | House Related | 59.1% | 28.1% | 52.7% |
| Others | 3.0% | 10.1% | 6.0% | Not Available | 1.7% | 2.11% | 1.8% |
| All reasons | 100% | 100% | | All reasons | 100% | 100% | 100% |

exhibit the lowest of all rates. Part of the differences may be due to socio-cultural factors, such as family attachment, or to differences in regions leading to higher mobility costs. But objective factors can be identified too: for example rental housing market regulations as shown by Djankov et al. (2002). This is a composite index based on the difficulty to evict a tenant, reflecting the complexity and the length of the procedure at various stages (pre-trial, process of trial, execution of the court decision). In Table 3 we report the numbers for 13 European countries and contrast the indices with mobility rates and with average unemployment rates. Figure 1 plots the data in Table 3 and displays a strong, negative correlation between housing market regulations and mobility rates across European countries. This is our third fact.

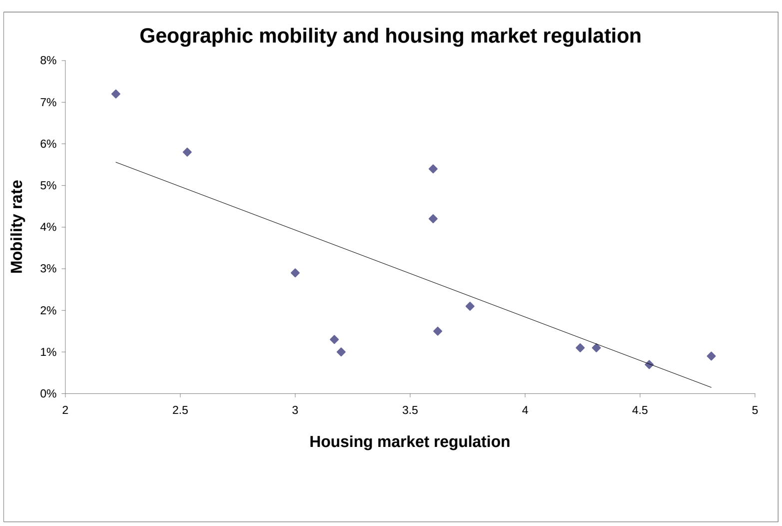
We use these three facts to develop a model that allows for interaction between the labor market and the housing market, both being markets with *frictions*, to address three questions: Can the cross-country differences in mobility be accounted for by a frictional parameter in the housing market as fact 3 suggests? Is the level of unemployment in a country affected by imperfections in the housing market; and if yes, through which mechanisms? We ask this question because of the large difference in observed unemployment rates. In July of 2007 the unemployment rate in the United States was 4.6 % while in the four large continental countries the unemployment rates were substantially higher: The Spanish unemployment rate was over 8%, Italy around 7%, Germany over 8% and France above 9%. Finally, quantitatively, would a reform of the housing market lead to a dramatic improvement in labor market performance in the US or in Europe?

Table 3: Mobility, Regulations and Unemployment

| | Mobility rate outside the area within 3 yrs. | Housing market regulation index | Average unemployment rate 1995-2001 |
|---|---|---|---|
| Denmark | 0.054 | 3.6 | 5.3 |
| Netherlands | 0.029 | 3 | 4.2 |
| Belgium | 0.013 | 3.17 | 8.5 |
| France | 0.042 | 3.6 | 10.4 |
| Ireland | 0.010 | 3.2 | 7.9 |
| Italy | 0.011 | 4.24 | 10.7 |
| Greece | 0.011 | 4.31 | 10.5 |
| Spain | 0.009 | 4.81 | 14.5 |
| Portugal | 0.007 | 4.54 | 5.6 |
| Austria | 0.015 | 3.62 | 4.02 |
| Finland | 0.058 | 2.53 | 11.9 |
| Germany | 0.021 | 3.76 | 8.3 |
| UK | 0.072 | 2.22 | 6.5 |

Source: Mobility data from ECHP. Regulation indices from Djankov et al. (2002). Unempoyment rates from Eurostat.

Figure 1: Housing Regulations and Mobility



In our model, a job location has an associated commuting time that may affect the job acceptance decision. Obstacles to mobility, such as regulations in the housing market, will affect the reservation strategy of workers. Thus, aggregate unemployment will have an effect on the functioning of the housing market. The model can be thought of as the "dual" of typical search models of the labor market. In particular, in the standard model (MP, RSW) there may be a non-degenerate distribution of wages but distance is degenerate. In our model, the distribution of wages is degenerate but there exists a non-degenerate distribution of distance from one's job. The theory predicts that greater frictions in the housing market make individuals more choosy about the jobs they take. The model finds that reducing frictions in the housing sector has small effects on the labor market in the U.S. but reduces unemployment in the E.U. by several percentage points.

Section 2 presents the model with labor market and housing frictions.Section 3 describes the optimal strategies and equilibrium. Section 4 describes how frictions in the housing market affect mobility and unemployment rates. Section 5 lays out the calibration strategy and parameters. Section 6 presents an extension that describes the effect of housing market regulations on arrival rate of housing offers. Section 7 concludes.

# 2 Model

## 2.1 Preferences and Search in the Housing Sector

Time is continuous and individuals discount the future at rate $r$. Individuals live in dwellings, defined as a bundle of services generating utility to individuals or a household. The defining characteristic of a dwelling, however, is that the services it provides are attached to a fixed location. The services can, of course, depend on the quality of the dwelling and its particular location. Amenities such as space, comfort, proximity to theaters, recreation, shops and the proximity to one's job increase the utility of a given dwelling. The dwelling may also be a factor of production of home-produced goods. In addition, the dwelling could be a capital asset. For these services, individuals pay a rent or a mortgage.

In this paper we focus on one particular amenity, distance to work. Because a dwelling is fixed to a location, the commuting distance to one's job, $\rho$, becomes an important determinant of both job and housing choice. We assume that space is symmetric, in the sense that the unemployed have the same chance of finding a job wherever their current residence, implying that $\rho$ is a sufficient statistic determining both housing and job choice. We call this property the isotropy of space: wherever an individual is, space looks the same.

Agents face two types of housing shocks. First, they may receive a family shock that arrives according to a Poisson process with parameter $\delta$. The shock changes the valuation of the current location, necessitating a move. This shock can be thought of as a marriage, divorce, the arrival of children, deterioration of the neighborhood, and so on. Upon the arrival of the shock they make one draw from the existing stock of housing vacancies, distributed as $G_S(\rho)$.[3] Note that agents may sample from the existing stock of houses at any time.

Second, agents randomly receive new housing opportunities that (possibly) allows them to relocate closer to their job. These arrivals are assumed to be Poisson with parameter $\lambda_H$. The distribution of *new* vacancies is given as $G_N(\rho)$.

We make the simplifying assumption that the efficiency of the housing market can be captured in a single variable, $\lambda_H$, the parameter determining frictions in the housing market. An increase in $\lambda_H$ means there are more arrivals of opportunities to find housing. As $\lambda_H$ approaches infinity, housing frictions go to zero. Obviously, if $\lambda_H = 0$ there is no mobility. The main idea behind $\lambda_H$ is that agents may not move instantaneously to their preferred location. Such restrictions might arise from length of lease requirements or eviction policies. In section 6 we discuss the relationship between housing market regulations and housing offers, $\lambda_H$, but now just assume it represents housing market frictions. This implies that the rent or mortgage is such that utility across dwellings will be equalized to reflect any differences in amenities, a fact that results from the assumption that space (distance) is isotropic.

[3]The one draw assumption is not very strong. It is equivalent to making up to $N$ independent draws, in which case it is like one single draw from a distribution $(G_S)^N$. See Lemma 1 in Quentin et al. (2006).

## 2.2 Labor Market

Individuals can be in one of two states: employed or unemployed. While employed, income consists of an exogenous wage, $w$. There is no on-the-job search, yet a match may become unprofitable, leading to a separation, which occurs exogenously with Poisson arrival rate $s$.

Unemployed agents receive income $b$, where $b$ can be thought of as unemployment insurance or the utility from not working. While unemployed, job offers arrive at Poisson rate $p$, indexed by a distance to work, $\rho$ , drawn from the cumulative distribution function $F_J$.

Let $E(\rho)$ be the value of employment for an individual residing at distance $\rho$ from the job. Let $U$ be the value of unemployment, which does not depend on distance, given the symmetry assumption made above. We can now express the problem in terms of the following Bellman equations:

$$
\begin{aligned}
(r+s)E(\rho) &= w-\tau\rho+sU+\lambda_H\int \max\left[0,(E(\rho')-E(\rho))\right]dG_N(\rho') \qquad (1)\\
&\quad +\delta\int \max[U-E(\rho),E(\rho'')-E(\rho)]dG_S(\rho'')\\
(r+p)U &= b+p\int\int \max[U,E(\rho'),E(\rho'')]dF_J(\rho')dG_S(\rho''), \qquad (2)
\end{aligned}
$$

where $\tau$ is the per unit cost of commuting and $\rho$ is the distance of the commute. Eq. 1 states that workers receive a utility flow $w-\tau\rho$; may lose their job and become unemployed – in which case they stay where they are; they receive a housing offer from the distribution of new vacancies $G_N$, which happens with intensity $\lambda_H$, in which case they have the option of moving closer to their job; and finally may receive a family shock $\delta$, and need to relocate.

Eq. 2 states that the unemployed enjoy $b$; receive a job offer with Poisson intensity $p$, at a distance $\rho'$, from the distribution $F_J(\rho)$. They have the option of rejecting the offer if the distance is too far, but also have the option to move instantaneously if they find a residence in the stock of existing vacant units. To the extent that $\rho'$ and $\rho''$ are independent draws, this means that there is a distribution, $F$, combining $F_J$ and $G_S$ such that the integral terms can be rewritten as

$\int \max[U, E(\rho)]dF(\rho)$, where $\rho$ is the minimum of the two draws: $\rho = Min(\rho', \rho'')$.[4]

# 3 Optimal Behavior, Equilibrium and Steady-states

## 3.1 Reservation Strategies

We now derive the job acceptance and moving strategies of individuals. Observe that $E$ is downward sloping in $\rho$, with slope

$$\frac{\partial E}{\partial \rho} = \frac{-\tau}{r+s+\lambda_H P_W + \delta P_\delta}, \tag{3}$$

where $P_W$ is the probability of moving conditional on receiving a housing offer and $P_\delta$ is the probability of moving conditional on receiving a family shock. Note that $0 < P_W < 1$ and $0 < P_\delta < 1$ and possibly depend on $\rho$. The function $E(\rho)$ is monotonic so that there exists a well-defined reservation strategy for the employed, with a reservation distance denoted by $\rho^E(\rho)$, below which a *housing offer* is accepted. Note that there is state-dependence in the reservation strategy of the employed, $\rho^E(\rho)$, with presumably $d\rho^E/d\rho > 0$. The further away the tenants live from their job, the less likely they will be to reject a housing offer, therefore,

$$P_W = G_N(\rho^E(\rho)).$$

Similarly, for a family shock, we have

$$P_\delta = G_S(\rho^E(\rho)).$$

After some intermediate steps (described in Appendix), we can show that the slope of $E(\rho)$ is given by

$$\frac{\partial E}{\partial \rho} = \frac{-\tau}{r+s+\lambda_H G_N(\rho) + \delta G_S(\rho^U)}. \tag{4}$$

Next, in the absence of relocation costs (this case is studied in the Appendix), tenants move as

[4] We prove in the appendix that $1 - F(\rho) = (1 - F_J(\rho))(1 - G_S(\rho))$.

soon as they get a dwelling offer closer to their current one, implying

$$\rho^E(\rho) = \rho.$$

Denote by $\rho^U$ the reservation distance for the unemployed, below which any *job offer* is accepted, it is defined by

$$E(\rho^U) = U.$$

Using the fact that $E(\rho^U) = U$,

$$\begin{aligned} b + p\int_0^{\rho^u} &[E(\rho') - U]dF(\rho') \\ &= w - \tau\rho^U + \lambda_H \int_0^{\rho^U} [E(\rho') - U]dG_N(\rho') + \delta \int_0^{\rho^u} [E(\rho'') - U]dG_S(\rho''). \quad (5) \end{aligned}$$

Integrating Eq. 5 by parts gives:

$$\rho^U = \frac{w-b}{\tau} + \int_0^{\rho^U} \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) - pF(\rho)}{r + s + \lambda_H G_N(\rho) + \delta G_S(\rho^U)} d\rho. \quad (6)$$

The determination of $\rho^U$ is shown in Figure 2.

Note that a job separation can occur in two ways. First, as mentioned above, there is an exogenous shock, $s$, inducing a separation from the match. Second, workers may receive a family shock, $\delta$, requiring them to redraw from the vacant housing stock distribution, $G_S$, but are unable to find a sufficiently close dwelling to the current job and (optimally) quit. That is, job separations are given by

$$\sigma = s + \delta(1 - G_S(\rho^u)). \quad (7)$$

With this specification the model is quite parsimonious, since a single variable, $\rho$, determines several dimensions of choice:

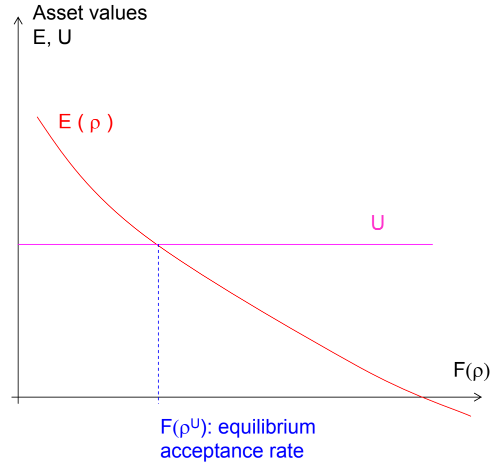
1. *Job acceptance*: $F(\rho^U)$ ;

2. *Residential mobility rate*: $\int \lambda_H G_N(\rho)$ over the distribution of commute distance of employed

workers $d\Phi$;

3. *Quit rate due to a family shock*: $\delta(1-G_S(\rho^U))$.

Figure 2: Determination of $\rho_u$




## 3.2 Free Entry

Assuming free entry of firms, and defining $\theta = \frac{V}{U}$ as labor market tightness, we have

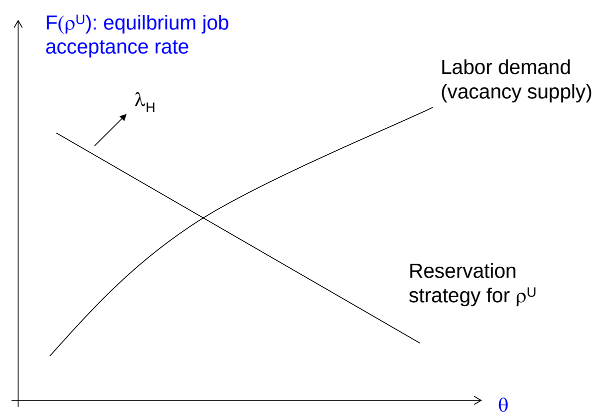
$$\frac{y-w}{r+\sigma} = \frac{c}{q(\theta)P_F},$$

where $P_F$ is the rate of acceptance of job offers by the unemployed, as expected from the viewpoint of the firm. We assume, still by symmetry, that the distribution of contacts between the firm and unemployed workers is given as $F(\rho)$, so that $P_F = F(\rho^U)$. This generates a positive link between $\theta$ and $\rho^U$ since $q'(\theta) < 0$, characterized by:

$$q(\theta)F(\rho^U) = \frac{c(r+\sigma)}{y-w}. \tag{8}$$

The intuition is quite simple. The firm's iso-profit curve at the entry stage depends negatively on both $\theta$ (as a higher $\theta$ implies more competition between the firm and the worker) and on $\rho^U$ (as more of their offers will be rejected because of distance). The zero-profit condition thus implies a positive link between $\theta$ and $\rho^U$. Note that this relation is independent of $\lambda_H$. On the other hand, $\rho^U$ is determined through (Eq. 6). It is decreasing in $p(\theta)$ and thus in $\theta$, as can be seen in (Eq. 19). When there are more job offers (higher $\theta$) workers can wait for offers closer to their current residential location; they are pickier. The two curves are represented in $(\rho^U, \theta)$ space in Figure 3.

Figure 3: Vacancies and Reservation Strategy




## 3.3 Unemployment and the Beveridge Curve

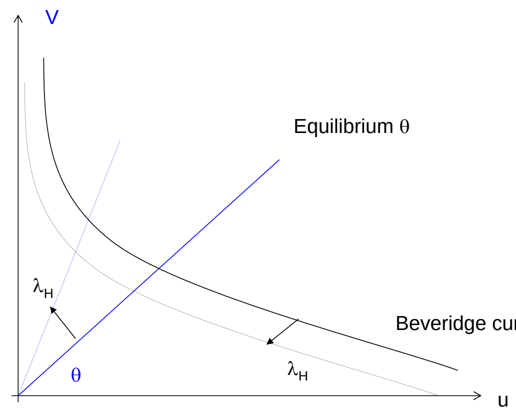
Recapitulating, an increase in $\lambda_H$, the efficiency of the housing sector, raises the acceptance rate of job offers, increasing $\theta$ and thus increasing job offers by firms.

Letting $p(\theta) = \theta q(\theta)$, the steady state unemployment rate is given as

$$u = \frac{\sigma}{\sigma + p(\theta)F(\rho^U)}. \tag{9}$$

In terms of a Beveridge Curve representation (vacancy and unemployment space), increasing $\lambda_H$ shifts the Beveridge curve inward (less structural mismatch) and also leads to a counter-clockwise rotation of $\theta$. A graphical representation of this result is shown in Figure 4.

Figure 4: Beveridge Curve




# 4 Housing Frictions and Mobility

## 4.1 The Effect of Regulations in the Housing Market

It is now possible to determine how housing frictions affect the decisions of workers and firms.

**Proposition 1** *An increase in $\lambda_H$ makes the unemployed less choosy about jobs:* $\partial \rho^U / \partial \lambda_H > 0$.

**Proof.** See Appendix. ■

Next, differentiating (Eq. 8) and using Proposition 1, we can determine the effect of housing frictions on job creation:

**Proposition 2** *An increase in $\lambda_H$ increases job creation:* $\partial \theta / \partial \lambda_H > 0$.

**Proof.** Same as Proposition 1. ■

This is an indirect effect caused by more job creation through the higher job acceptance rate of workers. Another interpretation of this effect is that *firms don't like to create jobs where workers have no place to live.*

Using these results it is now possible to determine the effect of housing market frictions on unemployment.

**Proposition 3** *An increase in $\lambda_H$ has three effects on unemployment:*

- *it reduces the quit rate in case of a $\delta$-shock;*
- *it raises the job acceptance rate of workers (through a higher thresehold $\rho^U$);*
- *it raises $\theta$ (Proposition 2) and thus job creation.*

**Proof.** See Appendix. ■

## 4.2 Distribution of commute distance

Let $\Phi(\rho)$ be the steady-state distribution of employed workers living at a location closer than $\rho$. $\Phi$ is governed by the following law of motion:

$$\begin{aligned}(1-u)\frac{\partial\Phi(\rho)}{\partial t} =& upF(\rho)+(1-u)\lambda_H G_N(\rho)\left[1-\Phi(\rho)\right]+(1-u)\delta G_S(\rho)\left[1-\Phi(\rho)\right] \\ &-(1-u)\Phi(\rho)s.\end{aligned} \tag{10}$$

Eq. 10 states that the number of people residing in a location at a distance less than $\rho$ from their job changes (either positively or negatively) due to:

- (+) the unemployed, $u$, receiving a job offer at rate $p$ with a distance closer to $\rho$ with probability $F(\rho)$;
- (+) the employed, $1-u$, who are further away from the current distance $\rho$ (a fraction $1-\Phi(\rho)$), who receive an offer in the housing market with intensity $\lambda_H$ closer to $\rho$ with probability $G_N(\rho)$;
- (+) the employed, $1-u$, who are further away from the current distance $\rho$ (a fraction $1-\Phi(\rho)$) and need to relocate due to a demographic shock, $\delta$, and receive a new distance smaller than $\rho$ with probability $G_S(\rho)$;
- (-) $s$ exits per unit of time from people losing their job.

In steady state and for all $\rho < \rho^u$

$$\Phi(\rho) = \frac{pF(\rho)\frac{u}{1-u} + \lambda_H G_N(\rho) + \delta G_S(\rho)}{s + \lambda_H G_N(\rho) + \delta G_S(\rho)} \tag{11}$$

$$= \frac{\frac{F(\rho)}{F(\rho^u)} s + \lambda_H G_N(\rho) + \delta G_S(\rho)}{s + \lambda_H G_N(\rho) + \delta G_S(\rho)} \leq 1 \tag{12}$$

with equality for $\rho = \rho^U$: $\Phi(\rho^U) = 1$ as no unemployed individual ever accepts a job offer farther away from a job than $\rho^U$. In addition, no employed worker moves farther away from his/her job, only closer to the current job. The second line above is obtained by replacing $u$ with its steady-state expression in (9). All workers have a house closer than distance $\rho^U$. We briefly remark on two extreme cases:

- If firms receive no applicants, ($\lambda_H G_N = \delta G_S(\rho) = 0$), the distribution of distance of the employed, $\Phi$, converges to $\frac{F(\rho)}{F(\rho^u)}$, that is, the distribution of $\rho$ conditional on a job being accepted.

- If jobs are not destroyed, $s = 0$, the distribution of distance of the employed, $\Phi$, converges to 0, that is, $\Phi(\rho) = 1$ for all $\rho > 0$: all workers eventually find a house infinitely close to their job.

## 4.3 Log Linearization

Let us first consider the special case: $\lambda_H \to \infty$. In the case where housing frictions go to zero, the model collapses to $\Phi(\rho) = 1$, meaning that all workers will be located infinitely close to their job. The job acceptance decision is indeterminate since we now have

$$\rho^U = \frac{w-b}{\tau} + \int_0^{\rho^U} d\rho.$$

The intuition is straightforward: if $w > b$, all job offers are accepted, meaning that $\rho^U$ goes to infinity. Therefore, we obtain the standard Pissarides value for tightness: $q(\theta^*) = \frac{c(r+\sigma)}{y-w}$ with

$\theta^* > 1$. In addition,

$$\frac{q(\theta^*)}{q(\theta)} = F(\rho^U) < 1$$

and therefore, with $q(\theta + d\theta) = q(\theta) + q'(\theta)d\theta = q(\theta)(1 + \eta_q d\theta/\theta)$, we have

$$\frac{q(\theta^*)}{q(\theta)} = 1 + \eta_q d\theta/\theta = F(\rho^U)$$

hence

$$\frac{d\theta}{\theta} = \theta^* - 1 = \frac{1 - F(\rho^U)}{-\eta_q} > 0$$

We can also decompose unemployment into two parts:

$$u = \frac{s}{s+p}$$

implying

$$\frac{du}{u} = -\frac{dp}{s+p} = -(1-u)\frac{dp}{p} = -(1-u)\frac{d\theta}{\theta}\eta_p$$

Thus

$$\frac{du}{u} = -(1-u)\frac{1-F(\rho^U)}{-\eta_q}\eta_p.$$

For example, with $\eta_q = -0.5$ and $\eta_p = 0.5$, the variation in unemployment purely due to imperfect housing markets is of the order of magnitude of the fraction of rejected offers $1 - F(\rho^U)$.

# 5 Calibration

## 5.1 Calibration strategy

In this section we will match the model to the data, in particular the mobility rates. We therefore need to calculate the mobility rate from the model. Denote by $M_K^S$ the number of movers of status $S = (U, E)$ (unemployed, employed) and for reason $K = (J, D)$ (job-related or family-related), we have:

1. Job-related mobility of the employed (those with a job but relocate once they sample a better

housing location):

$$M_J^E = (1-u)\lambda_H \int_0^{\rho^U} G_N(\rho) d\Phi(\rho) \tag{13}$$

$$= (1-u)\lambda_H \left[ G_N(\rho^U) - \int_0^{\rho^U} g_N(\rho)\Phi(\rho) d\rho \right], \tag{14}$$

where the second line is found by integrating by parts and noticing that $\Phi(\rho^U) = 1$.

2. Job-related mobility of the unemployed (those who have a job offer, accept it with probability $G(\rho^U)$ and may relocate if they drew a location from $G_S$ closer from their current $\rho$):

$$M_J^U = up \int_0^{\rho^U} G_S(\rho) dF_J(\rho)$$

3. Family-related mobility:

$$\begin{aligned} M_D^U &= u\delta \\ M_D^E &= (1-u)\delta \\ M_D^{E+U} &= \delta \end{aligned}$$

Note that in $M_D^E$, some workers quit their job (a fraction $1 - G_S(\rho^U)$) since they did not find acceptable housing in the current stock.

We first calibrate the model setting $\delta$ to zero, that is, shutting down the family shocks and exclusively focusing on labor market shocks in order to obtain a simple intuition of the mechanisms at work.

Next, we extend the calibration to take into account of the positive $\delta$ case which allows us to capture the impact of family shocks on labor turnover.

## 5.2 Calibration

The time period is one month and the interest rate, r, is set to 0.0033, corresponding to an annual rate of 0.04. We calibrate to the mobility rate of the employed, 17% annually between March 1999 and March 2000, so the target is (17/12)%. The number for the employed that move comes from the Bureau of the Census.[5] Of the roughly 31 million persons who moved during that year, 22.3 million of them were employed, 1.5 million unemployed and 7.8 million out of the labor force.

We begin by setting $\delta = 0$ (no demographic shock) and $G_S = 0$ (no vacant houses in the stock of housing) to gauge how well a model without these features might do in explaining the mobility phenomenon. Therefore, we are left with two distributions: $G_N$, new housing offers, and $F$, job offers. We assume that these distributions are represented by the same exponential function with parameter $\alpha$: $F = G_N = 1 - e^{-\alpha\rho}$.

To find a value for $\alpha$ we use data on the distribution of commute times from the Census 2000. Figure 5 shows that the distribution is close to an exponential. The mean commute time is about 24 minutes. Letting $\rho$ be the distance expressed in monthly commuting time, we write the distribution as $F(\phi\rho)$, where $\phi$ is a scale parameter reflecting the change in units since the data is for one-way commuting time. Assuming that people commute twice a day and 20 days a month, $\phi = 1/40$. When estimating the slope of log(1-F($\phi\rho$)) which, under the assumption of an exponential distribution with coefficient $\alpha$, is $\alpha\phi$, we obtain 0.0548 (or 0.065 if we drop commute times of zero) as the estimate. It follows that we should fix a value of $\alpha = 0.0548 * 40 = 2.19$ (or 2.6=0.065*40 under the second estimate).

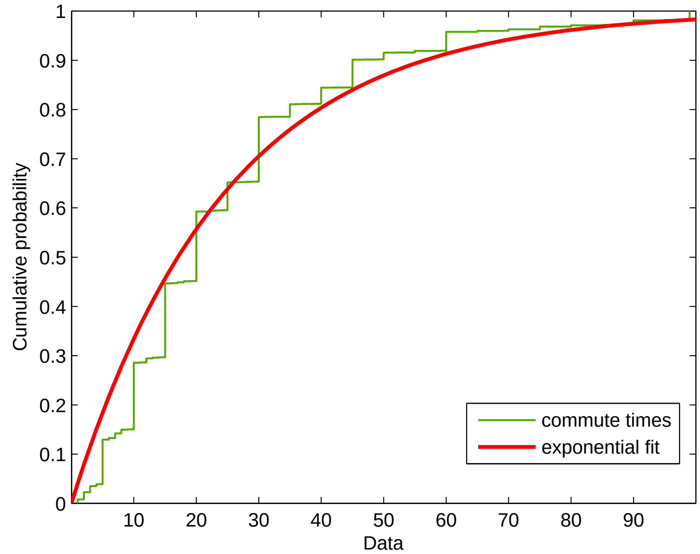
To find the cost of commuting, $\tau$, we use data from tax authorities in the U.S. and France. The Internal Revenue Service (IRS) in the U.S. and the tax authority BO Impôts in France provide standard mileage rates when using a car for business. For 2007, the allowance was $0.485 per mile (0.20 euro per mile) in the U.S. and 0.514 euro per kilometer for a 6CV car.[6]

The program finds the parameters of the model given a target unemployment rate of 4.2% in the U.S. (the average between March 1999 and March 2000), and a target job hiring rate of $p = 1/2.4$ monthly. The latter implies an average duration of unemployment of 2.4 months and therefore imposes a value for $s$ given that $u = s/(s+p)$ when $\delta = 0$: $s = p(u/(1-u)) = 0.0183$.

[5] Why People Move: Exploring the March 2000 Current Population Survey, P23-204, Bureau of the Census, May, 2001.

[6] The rate is progressive with power in France, ranging from 0.37 to 0.67 euro per kilometer.

Figure 5: Distribution of Commute Times in the U.S.




We match the mobility rate to a target value of 17% annually (17/12% monthly) with (Eq. 14). The program finds the values of $\alpha$ and $\lambda_H$ that are consistent with the target for mobility, given $\rho^U$, obtained from (Eq. 6).

We set $p(\theta) = A\theta^{0.5} * F(\rho^U)$. Setting $\theta = 1$ gives $A = 0.46$. Together with the free-entry condition, (Eq. 8), this fixes a value for recruiting costs $c$ after normalizing $y = 1$, and assuming that wages are 80% of output, $w = 0.8$. Note also that $q(\theta) = A\theta^{-0.5} * F(\rho^U)$.

We then repeat the exercise using values for the European Union. In particular, we set the mobility target and unemployment duration to one third of the U.S. value, unemployment benefits to 0.55 and the unemployment target to 10%.

## 5.3 Findings

The findings for the benchmark economy are given in Table 4 where each column represents a different arrival rate of housing offers. The value of the scale parameter of the matching function, $A$, is 0.4657, and the recruiting cost, $c$, is 3.858. These values do not change as $\lambda_H$ changes.

In the benchmark case workers reject about 10% of job offers and this decreases rapidly as housing frictions are reduced (that is, multiplying the speed of arrival of housing offers by 2, 3,

Table 4: U.S. Calibration

| | $\lambda_H = 0.0533$ | $2 * \lambda_H$ | $3 * \lambda_H$ | $10 * \lambda_H$ |
|---|---|---|---|---|
| $\theta_H$ | 1.000 | 1.152 | 1.215 | 1.249 |
| $\rho^U$ | 1.028 | 1.474 | 1.952 | 5.624 |
| $F^U$ | 0.895 | 0.960 | 0.986 | 1.000 |
| Reject | 0.105 | 0.040 | 0.014 | 0.000 |
| Unemp | 0.042 | 0.037 | 0.035 | 0.034 |
| Mobility | 0.014 | 0.022 | 0.027 | 0.045 |

Table 5: European Calibration

| | $\lambda_H = 0.0221$ | $2 * \lambda_H$ | $3 * \lambda_H$ | $10 * \lambda_H$ |
|---|---|---|---|---|
| $\theta_H$ | 1.000 | 1.188 | 1.366 | 2.348 |
| $\rho^U$ | 0.340 | 0.388 | 0.434 | 0.745 |
| $F^U$ | 0.525 | 0.572 | 0.614 | 0.805 |
| Reject | 0.475 | 0.428 | 0.386 | 0.195 |
| Unemp | 0.100 | 0.086 | 0.075 | 0.045 |
| Mobility | 0.004 | 0.008 | 0.011 | 0.026 |

or 10). Reducing frictions in the housing market by increasing the value of $\lambda_H$ ten times the benchamark value results in a tripling in mobility. The model converges rapidily to the ”frictionless” rate of unemployment which is 0.034. In short, housing frictions increase unemployment by a little more than half a percentage point.

We now calibrate the model to a “typical” European situation where mobility is a third of that of the US, and unemployment duration is also three times higher than in the US. The findings are shown in Table 5. We obtain a lower value of $\lambda_H$, 0.0221, so that there are more than twice as many housing offers in the U.S. as compared to Europe, $\lambda_H^{US}/\lambda_H^{EU} = 2.41$. We find $A = 0.2645$, that is, the scale parameter of the matching function is about half that of the US. The program also finds lower vacancy costs in Europe ($c = 1.4803$) which implies that total setup costs of firms in Europe are less than that of the US. In particular, hiring costs are equal to $c/q = 3.8579/0.4657 = 8.28$ in the US and $1.4803/0.2645 = 5.6$ in Europe.

In the European case, since the housing market was calibrated with a lower mobility rate,

Table 6: Counterfactuals for the E.U.

| Europe | *Benchmark* | $\lambda_H^{US}$ | A$^{US}$ | $A^{US}$ , $c^{US}$ | $b^{US}$ |
|---|---|---|---|---|---|
| $\theta$ | 1.000 | 1.2630 | 2.4385 | 0.6122 | 1.4177 |
| $\rho^U$ | 0.340 | 0.4072 | 0.2862 | 0.4278 | 1.0535 |
| $F^U$ | 0.525 | 0.5900 | 0.4657 | 0.6082 | 0.9005 |
| rej. rate | 0.475 | 0.4100 | 0.5343 | 0.3918 | 0.0995 |
| unemployment | 0.1000 | 0.0809 | 0.0436 | 0.0651 | 0.0727 |
| mobility | 0.0042 | 0.0093 | 0.0030 | 0.0037 | 0.0051 |

$\lambda_H$ is smaller. Hence, workers reject more job offers since they find it more difficult to move subsequently. The rejection rate is 0.475, nearly half of all job offers are rejected. Further, reducing frictions in the housing market, now delivers a quite significant improvement in the labor market as unemployment is reduced significantly.

### 5.3.1 More Counterfactuals for Europe

We now rerun the model using European values and then change the various parameters to US levels: in order, $\lambda_H$, $A$, A and c together and $b$. The findings are presented in Table 6.

The findings suggest that the European calibrated economy could gain almost as much in raising the efficiency of the housing market to US levels as in cutting unemployment benefits to the (very low) US levels. The largest effect, however, comes from increasing the effectiveness of matching in the labor market in Europe to that in the U.S.

## 5.4 Including the Demographic Shock

In the data, the total mobility rate is the result of shocks to jobs and housing. Therefore, we now extend the model to the case where $\delta > 0$. For the U.S., the model now finds vacancy costs to be higher, $c = 4.4657$ and the efficiency of the matching technology to be lower, $A = 0.4182$.

The calibration for Europe finds $c = 1.6662$ and $A = 0.1507$.

Table 7: U.S. Calibration, $\delta > 0$

| | $\lambda_H = 0.0247$ | $2 * \lambda_H$ | $3 * \lambda_H$ | $10 * \lambda_H$ |
|---|---|---|---|---|
| $\theta_H$ | 1.000 | 1.005 | 1.007 | 1.008 |
| $\rho^U$ | 0.478 | 0.574 | 0.672 | 1.385 |
| $F^U$ | 0.996 | 0.999 | 1.000 | 1.000 |
| Reject | 0.004 | 0.001 | 0.000 | 0.000 |
| Unemp | 0.042 | 0.040 | 0.039 | 0.036 |
| Mobility | 0.002 | 0.004 | 0.006 | 0.015 |

Table 8: Europe, $\delta > 0$

| | $\lambda_H = 0.0143$ | $2 * \lambda_H$ | $3 * \lambda_H$ | $10 * \lambda_H$ |
|---|---|---|---|---|
| $\theta_H$ | 1.000 | 1.023 | 1.043 | 1.132 |
| $\rho^U$ | 0.218 | 0.230 | 0.242 | 0.337 |
| $F^U$ | 0.922 | 0.932 | 0.941 | 0.981 |
| Reject | 0.078 | 0.068 | 0.059 | 0.019 |
| Unemp | 0.100 | 0.098 | 0.095 | 0.086 |
| Mobility | 0.001 | 0.002 | 0.003 | 0.009 |

### 5.4.1 Additional Experiments

The model also displays complementarities. In particular, the relationship between $\lambda_H$ and $u$ varies depending on the value of the utility from not working, $b$. Alternatively it is possible to vary $b$ and look at the link between the rejection rate, $(1 - F_U)$, and unemployment, $u$. Figure 6 and Figure 7 show the findings of these experiments.

# 6 Housing Market Regulations and Housing Offers

We now provide a simple explanation for the link between housing market regulations mentioned in the introduction and the parameter $\lambda_H$, the arrival of housing offers, that is we now describe an extension that endogenizes $\lambda_H$.

Landlords post vacancies and screen applicants. They offer a lease to the "best applicants", in a sense defined below. In case of a default on the rent by the tenant, however, landlords incur a loss

Figure 6: Unemployment and Housing Market Frictions

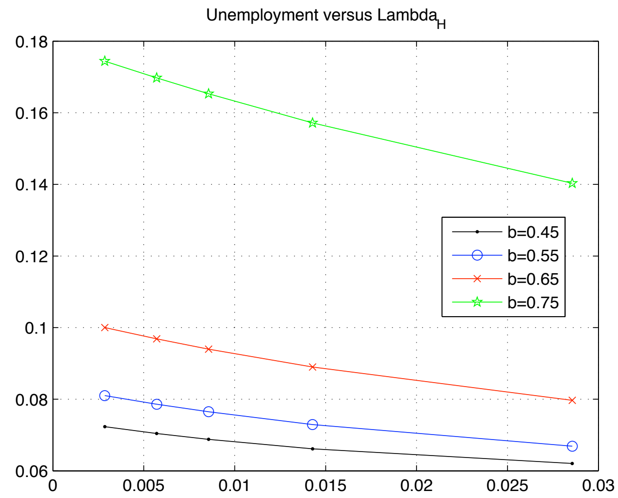
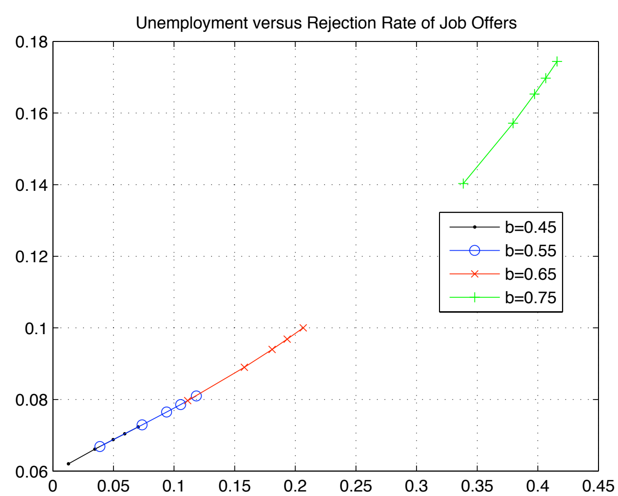



Figure 7: Unemployment and the Rejection Rate



due to the length of litigation and eviction procedures. The asset value of owning a dwelling with a tenant defaulting on the rent is denoted by $\Lambda$. Therefore, $\Lambda$ will decline with more regulation in the rental housing market, due to the length of the procedures to recover the unpaid rents and the dwelling.

Potential tenants (applicants) are all ex-ante homogenous but ex-post may represent a default risk for the landlord. More precisely, at the time of the contact between the applicant, the landlord gets a random signal on the tenant and postulates that the particular applicant has a specific default rate $\delta_i$ (a Poisson rate in continuous time). It follows that for such a tenant, the value of a filled vacancy for the landlord is:

$$rH_F = R + \delta_i(\Lambda - H_F)$$

where $R$ is the rent and $H_F$ is the value of a filled vacancy.[7] We therefore have:

$$H_F = \frac{R + \delta_i \Lambda}{r + \delta_i} \tag{15}$$

The derivative of $H_F$ with respect to $\delta_i$ is given as $\Lambda r - R$, which is negative since the landlord's value of a default, $\Lambda$, cannot be higher than the capitalized value of the rent $R/\delta_i$. Therefore, $H_F$ is decreasing in $\delta_i$, from $R/r$ to $\Lambda$. This implies that there will be a well defined reservation strategy by landlords given the signal they receive.

To derive this reservation strategy, we denote by $H_V$ the value of a vacant housing unit prior to screening. This value is exogenous and is given, in the long-run, by the cost of construction of new housing units. Therefore, it is independent of rental regulations and in particular of $\Lambda$. At the time of a contact with a tenant, landlords decide to offer a lease if the perceived value of default $\delta_i$

[7] Recall that we assumed in the model that the flow of service of the dwelling to the tenant was exactly compensating the rent so we did not need to include the rent in the Bellman equations of workers. We also assume that workers do not benefit from defaulting on the rent: in case of default, they may have a disutulity exerted by landlords or their lawyers so that, after default, the value of being in a dwelling is not higher than it was when paying. Therefore, the expression for the Belmann equations of tenants derived previously is unchanged .

is below a cut-off value $\overline{\delta}$ with

$$\begin{aligned} \frac{R+\overline{\delta}\Lambda}{r+\overline{\delta}} &= H_V \\ \text{or } \overline{\delta} &= \frac{R-r\Lambda}{H_V-\Lambda} \end{aligned}$$

Note that $\overline{\delta}$ is increasing in $\Lambda$: the higher is $\Lambda$, the easier it is to accept a tenant since the risk of default is lower.

Hence, the screening rate, $\alpha$, of tenants is prob $(\delta_i > \overline{\delta})$. Denoting by $L(\delta_i)$ the c.d.f. of default-rates, we have

$$\alpha = 1 - L\left(\frac{R-r\Lambda}{H_V-\Lambda}\right)$$

The screening rate is therefore increasing when $\Lambda$ is lower, that is, when rental housing market regulations are higher.

Finally, denote by $\phi$ the Poisson rate at which tenants receive dwelling offers. We assume that this contact rate is exogenous. It then follows that

$$\lambda_H = \phi\alpha$$

Therefore:

**Proposition 4**: $\lambda_H$ *is decreasing in the amount of housing market regulations*.

# 7 Concluding Comments

In this paper we have taken seriously the idea that labor market frictions, and in particular the reservation strategies of unemployed workers when they decide whether or not to accept a job offer, depend strongly on the functioning of the housing market. This interconnection between two frictional markets (housing and labor) can be used to understand differences in the functioning of labor markets.

This paper has offered such a model, based on decisions to accept or reject a job offer, given the commuting distance to jobs. The model is relatively parsimonious, thanks to simplifying assumptions such as the isotropy of space, an unrealistic assumption but which conveniently provides closed form solutions and makes it possible to explain quit, job acceptance and geographic mobility decisions with a decision rule based on a single dimension. The model is yet rich enough and can be extended to deal with new issues such as discrimination in the housing market, mobility allowances or "moving toward opportunity" schemes, spatial misatch issues and so on, as in the urban economics literature.

We have attempted to explain differences in mobility rates between Europe and the US. A calibration exercise of "Europe" and the US is able to account for differences in unemployment and mobility thanks to a parameter which captures the speed of arrival of housing offers to households.

In our calibration, we find that housing frictions account for only a small portion of US unemployment but several percentage points (out of 10) of European unemployment. However, policy-inducing mobility might never be able to reduce unemployment by 2 percentage points. Taking the $\lambda_H$ parameter to infinity is not an available option, given the nature of frictions. Tripling the European value of $\lambda_H$ to reach the US value is already an ambitious objective. Doing so reduces unemployment in Europe by 0.6 percentage points, which is large already, but suggests that after all, housing market imperfections may not be a major cause of aggregate unemployment. Of course, the lack of mobility is a more specific problem for outsiders in the labor market (young workers in particular) and relatively low effects in aggregate could hide massive effects for some categories of the population.

Finally, this conclusion of large effects of $\lambda_H$ on mobility and positive but not massive effects on unemployment is coherent with intra-European differences in housing market regulations. Based on indices of rental housing market regulation, we find a strong negative correlation of the indices with geographical mobilty and a small and positive correlation with unemployment rates.

Future work should attempt to enrich the model to introduce more specific urban features such as anisotropy of space and the existence of centers in cities and suburbs, as well as different groups of the labor force. Our work is a first step in integrating housing and labor markets in a coherent macroeconomic model.

# 8 Appendix

## 8.1 More on the reasons for moving

Table 9 gives some additional information on reasons for moving.[8]

Table 9: Reasons for Moving, U.S.

| | | | |
|---|---|---|---|
| **Work** | | **House** | |
| new job or job transfer | 60.5% | wanted own home, not rent | 22.2% |
| look for work or lost job | 9.7% | wanted new or better home/apartment | 35.8% |
| closer to work / easier commute | 19.6% | wanted better neighborhood/less crime | 8.6% |
| retire | 2.4% | wanted cheaper housing | 10.8% |
| other job related reason | 7.7% | other housing reason | 22.6% |
| All work related reasons | 100% | All house related reasons | 100% |
| **Family** | | **Other** | |
| change in marital status | 23% | attend or leave college | 38.2% |
| establish own household | 27.2% | change of climate | 11.0% |
| other family reason | 49.8% | health reason | 17.2% |
| All family related reasons | 100% | other reasons | 33.6% |
| | | All other related reasons | 100% |

Finally, Table 10 shows that the reasons for work-related mobility are quite different within and across counties. As expected, intra-county work-related moves bring workers closer to their job, while 70% of inter-county work-related moves are related to a new job.

## 8.2 More on the reasons for not commuting

Distance from home to a job, i.e., commuting time, certainly seems relevant to a decision about job and housing choices. In fact, Kahneman and Krueger (JEP, 2006) present evidence that for 909 women in Texas commuting time was the least enjoyable of the activities mentioned. Table 11 shows that intimate relations provided the highest level of happiness and commuting to work the

[8] Why People Move: Exploring the March 2000 Current Population Survey, Current Population Reports, P23-204, May, 2001, U.S. Department of the Census.

Table 10: Work-related Moves, U.S.

| Work related | All | Intra | Inter |
|---|---|---|---|
| new job or job transfer | 60.5% | 24.5% | 69.4% |
| look for work or lost job | 9.7% | 9.4% | 7.7% |
| closer to work / easier commute | 19.6% | 53.5% | 13.6% |
| retire | 2.4% | 1.4% | 3.0% |
| other job related reason | 7.7% | 11.1% | 6.3% |
| total | 100% | 100% | 100% |

lowest. It seems reasonable to construct a model that incorporates commuting time into decisions regarding job choice and housing location.

Table 11: Happiness Index

| | Index (0-5) | Time (hours) |
|---|---|---|
| Sex | 4.8 | 0.2 |
| Socialising after work | 4.1 | 1.1 |
| Relaxing | 4.0 | 0.8 |
| Dinner | 3.9 | 2.2 |
| Lunch | 3.9 | 0.6 |
| Exercising | 3.8 | 0.2 |
| Praying | 3.8 | 0.5 |
| Socialising at work | 3.8 | 1.1 |
| Shopping, Cooking | 3.2 | 3.1 |
| Computer at home | 3.2 | 0.9 |
| Housework | 3.0 | 1.1 |
| Childcare | 3.0 | 1.1 |
| Evening commute | 2.8 | 0.6 |
| Working | 2.7 | 6.9 |
| Morning commute | 2.0 | 0.4 |

## 8.3 Proof of the slope of E($\rho$)

We need to rewrite Eq. 1 and Eq. 2 as:

$$(r+s)E(\rho) = w - \tau\rho + sU + \lambda_H \int_0^{\rho} [E(\rho') - E(\rho)] dG_N(\rho') \tag{16}$$

$$+\delta \int_0^{\rho^u} [E(\rho') - E(\rho)] dG_S(\rho'') + \delta \int_{\rho^u}^{+\infty} [U - E(\rho)] dG_S(\rho'')$$

$$(r+p)U = b + p \int_0^{\rho^u} [E(\rho')] dF(\rho') + pU(1 - F(\rho^u)). \tag{17}$$

## 8.4 Proof of the determination of $\rho^U$

Rewrite the Bellman equations as:

$$\begin{aligned} rE(\rho) &= w - \tau\rho + s(U - E(\rho)) + \lambda_H \int_0^{\rho^{E}(\rho)} (E' - E) dG_N(\rho') \\ rU &= b + p \int_0^{\rho^U} (E' - U) dF(\rho') \end{aligned}$$

Take the value of $E$ in $\rho^U$, we have:

$$\begin{aligned} rE(\rho^U) &= w - \tau\rho^U + s(U - E(\rho^U)) + \lambda_H \int_0^{\rho^U} (E' - E(\rho^U)) dG_N(\rho') \\ &= rU = b + p \int_0^{\rho^U} (E' - U) dF(\rho') \end{aligned}$$

so

$$\rho^U = \frac{\lambda_H}{\tau} \int_0^{\rho^U} (E' - U) dG_N(\rho') - \frac{p}{\tau} \int_0^{\rho^U} (E' - U) dF(\rho') + \frac{w-b}{\tau} \tag{18}$$

This equation simplifies a bit after integrating by parts. Noting that

$$\frac{\partial E}{\partial \rho} = \frac{-\tau}{r + s + \lambda_H G_N(\rho)}$$

$$\int_0^{\rho^U} (E' - U) dH(\rho') = \int_0^{\rho^U} \frac{\tau H(\rho)}{r + s + \lambda_H G_N(\rho)} d\rho$$

where $H$ is any distribution such that $H(0) = 0$, we can thus rewrite $\rho^U$ as in the text.

## 8.5 Link between $F$, $F_J$ and $G_S$

We start from the integrand $A = \int\int \max[U, E(\rho'), E(\rho^{''})] dF_J(\rho') dG_S(\rho'')$. Noting that

$$\frac{dE}{d\rho} = \frac{-\tau}{r+s+\lambda_H G_N(\rho)}$$

we can rewrite $A$ as

$$\begin{aligned} A &= \int\int \{I[\rho' > \rho_U] I[\rho'' > \rho_U] U \\ +I[\rho' &< \rho''] I[\rho' < \rho^U] E(\rho') \\ +I[\rho'' &< \rho'] I[\rho'' < \rho^U] E(\rho'')\} \\ & dF_J(\rho') dG_S(\rho'') \end{aligned}$$

or

$$\begin{aligned} A &= U \int_{\rho^U} dF_J(\rho') \int_{\rho^U} dG_S(\rho'') \\ &+ \int_0^{\rho^U} E(\rho') \left( \int_{\rho'} dG_S(\rho'') \right) dF_J(\rho') \\ &+ \int_0^{\rho^U} E(\rho'') \left( \int_{\rho''} dF_J(\rho') \right) dG_S(\rho'') \end{aligned}$$

or

$$\begin{aligned} A &= U(1-F_J(\rho^U))(1-G_S(\rho^U)) \\ &+ \int_0^{\rho^U} E(\rho') \left(1-G_S(\rho')\right) dF_J(\rho') \\ &+ \int_0^{\rho^U} E(\rho'') \left(1-F_J(\rho'')\right) dG_S(\rho'') \end{aligned}$$

Denote by $F = 1-(1-F_J(\rho))(1-G_S(\rho))$. We have that

$$dF = (1-F_J) dG_S + (1-G_S) dF_J$$

and we can thus rewrite

$$
\begin{aligned}
A &= U(1-F_J(\rho^U))(1-G_S(\rho^U)) \\
&\quad + \int_0^{\rho^U} E(\rho)dF(\rho) \\
&= \int \max(U, E(\rho))dF(\rho)
\end{aligned}
$$

The last equality comes from the observation that $1-F(\rho^U) = (1-F_J(\rho^U))(1-G_S(\rho^U))$.

## 8.6 Proof of Proposition 1

Fully differentiating $\rho^u$ gives

$$
d\rho^U \left( \frac{r+s+pF(\rho)}{r+s+\lambda_H G_N(\rho)+\delta G_S(\rho^U)} \right) = \left( \int_0^{\rho^U} \frac{G_N(\rho)(r+s)+pF(\rho)G_N(\rho)}{[r+s+\lambda_H G_N(\rho)+\delta G_S(\rho^U)]^2} d\rho \right) d\lambda_H \quad (19)
$$

$$
-\left( \int_0^{\rho^U} \frac{F(\rho)d\rho}{r+s+\lambda_H G_N(\rho)+\delta G_S(\rho^U)} \right) dp + d\left( \frac{w-b}{\tau}. \right)
$$

Equation (19) indicates that $\rho^U$ depends positively on $\lambda_H$, negatively on $p$ and positively on $w-b$. An increase in $\lambda_H$ (more housing offers) will shift the curve in Figure 2 upward, raising $\rho^U$ and thus the acceptance rate of the unemployed. The intuition is simply that they take the job offer knowing that they will be able to find a better location in the near future because housing offers arrive very frequently.

## 8.7 Proof of Proposition 3

Differentiating $u$, and letting $\omega_\theta$ represent the partial derivative of $\theta$ to $\lambda_H$. gives:

$$
\begin{aligned}
\frac{du}{d\lambda_H} &= \frac{-\delta g_S(\rho^U)\omega_\rho(\sigma+p(\theta)F(\rho^U)) - \sigma[-\delta g_S(\rho^U)\omega_\rho + p(\theta)f(\rho^U)\omega_\rho + p'(\theta)F(\rho^U)\omega_\theta]}{[\sigma+p(\theta)F(\rho^U)]^2} \\
&= \frac{-\delta g_S(\rho^U)p(\theta)F(\rho^U)\omega_\rho - \sigma\left[p(\theta)f(\rho^U)\omega_\rho + p'(\theta)F(\rho^U)\omega_\theta\right]}{[\sigma+p(\theta)F(\rho^U)]^2}.
\end{aligned}
$$

Next, note that

$$
\frac{d\sigma}{d\lambda_H} = -\delta g_S(\rho^U)\frac{\partial\rho^U}{\partial\lambda_H} = -\delta g_S(\rho^U)\omega_\rho < 0,
$$

where $\omega_\rho$ is simply a convenient notation for the partial derivative of $\rho^u$ to $\lambda_H$ and

$$\begin{aligned}
\frac{d[p(\theta)F(\rho^U)]}{d\lambda_H} &= p'(\theta)F(\rho^U)\frac{\partial\theta}{\partial\lambda_h} + p(\theta)f(\rho^U)\frac{\partial\rho^U}{\partial\lambda_H} \\
&= p(\theta)f(\rho^U)\omega_\rho + p'(\theta)F(\rho^U)\omega_\theta > 0.
\end{aligned}$$

## 8.8 Adding up moving costs

Let $C$ be a relocation cost paid by workers. We ignore here $G_S$ assumed to be degenerate and fix $\delta = 0$. We have thus:

$$\begin{aligned}
rE(\rho) &= w - \tau\rho + s(U - E(\rho)) + \lambda_H \int \max\left[0, (E' - E - C)\right] dG_N(\rho') \\
rU &= b + p \int (E' - U) dF(\rho')
\end{aligned}$$

$E$ is downward sloping in $\rho$ with slope

$$\frac{dE}{d\rho} = \frac{-\tau}{r + s + \lambda_H P_W}$$

where $P_W \geq 0$ is the probability to move conditional on receving a housing offer, with $0 < P_W < 1$ possibly depends on $\rho$. The function $E(\rho)$ is thus monotonic and there is thus a well-defined reservation strategy, with a reservation distance denoted by $\rho^E$ for the employed above which a **housing offer** is rejected. Note in addition that there is NOW state-dependence in the reservation strategy of the employed: we have that $\rho^E(\rho)$ with presumably $d\rho^E/d\rho > 0$: the further away the tenants live from her job, the less likely they will reject an offer. (NB: to be shown in the general case). e can rewrite

$$P_W = G_N(\rho^E(\rho))$$

and obtain the Bellman equations as:

$$\begin{aligned}
rE(\rho) &= w - \tau\rho + s(U - E(\rho)) + \lambda_H \int_0^{\rho^E(\rho)} (E' - E - C) dG_N(\rho') \\
rU &= b + p \int_0^{\rho^U} (E' - U) dF(\rho')
\end{aligned}$$