

Capital Adequacy and Basel II

by

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ABSTRACT

Using a one common factor Black-Scholes-Merton model, this paper compares unbiased portfolio-invariant capital allocations with Basel II IRB capital allocations for corporate exposures. The analysis identifies substantial biases in the June 2004 IRB framework. For a wide range of portfolio credit risk characteristics considered, the Advanced IRB rules drastically undercapitalize portfolio credit risks. Implied default rates under the Advanced IRB rule exceed 5 percent. In contrast, Foundation IRB capital requirements allocate multiple times the capital necessary to ensure the supervisory target solvency rate of 99.9 percent. The biases that are identified potentially raise a number of important issues including the potential for increased systemic risk as regulatory capital rules promote consolidation in weakly capitalized Advanced IRB banks.

Key words: economic capital, credit risk, Basel II, internal models

JEL Classification: G12, G20, G21, G28

[CFR research programs](#): risk measurement, bank regulatory policy

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1. INTRODUCTION

Economic capital fulfills a buffer stock loss-absorbing function. It is defined as the amount of equity financing in a capital structure that is necessary to ensure that the default rate on a bank's funding debt never exceeds a maximum target rate selected by management. In practice, economic capital allocations are often estimated using value-at-risk (VaR) measures. Given the widespread acceptance of VaR-based capital allocation methods, it may come as a surprise to learn that perfectly accurate VaR models may produce biased estimates of economic capital requirements.

Kupiec (2004) derives the general recipe for calculating unbiased economic capital measures. When used for economic capital allocation purposes, VaR must be measured relative to a portfolio's initial market value and augmented with an estimate of the equilibrium interest payments that must be made on the bank's funding debt. The second step—estimating equilibrium required interest payments—cannot be accomplished within a traditional VaR framework, but instead requires the use of a formal asset pricing model or an empirical substitute. For the most part, published methodologies for allocating economic capital are biased because they fail to recognize the need to pay interest and a credit spread premium to bank debt holders.

Basel II's Internal Ratings Based (IRB) capital requirements are set using mathematical rules that have been distilled from a well-known class of VaR models that are constructed to generate portfolio-invariant capital requirements. The IRB approaches of Basel II set capital requirements based on exposure type (corporate, sovereign, bank, retail, SME, equity, etc) and the probability of default (PD), loss given default (LGD), and exposure at default (EAD) characteristics of an individual credit. Vasicek (1991), Finger (1999), Schönbucher (2000), and Gordy (2003) develop statistical models of portfolio credit loss distributions that generate portfolio-invariant capital allocation rules. Gordy (2003) explicitly

argues for using a portfolio-invariant VaR methodology to calibrate IRB capital requirements.¹

An important issue that arises regarding the use of portfolio-invariant methods is the bias that arises because these capital allocation models do not properly account for funding debt holder compensation. To the extent that these industry models have guided the Basel II IRB calibration process, IRB capital requirements are biased. This source of potential bias is not widely recognized and its potential magnitude has not been documented in the literature.²

The magnitude of bias in Basel II IRB capital requirements can be analyzed in the context of a portfolio-invariant version of a Black-Scholes-Merton (BSM) equilibrium model of credit risk. The BSM model is the theoretical foundation for virtually all economic capital models of credit risk including portfolio-invariant methods. While the basic BSM model may not accurately predict all features of the historical data on credit spreads and losses, it represents a useful equilibrium benchmark. It is consistently formulated and facilitates the construction and comparison of alternative measures of risk and capital in a controlled environment. If IRB capital measures are significantly biased in the BSM equilibrium setting, IRB model performance must also be suspect in the more complicated and opaque setting that characterizes real world credit risks.

The results of the IRB model calibration comparison are surprising and have important regulatory and competitive implications. Compared to a true unbiased economic capital allocation, the June 2004 Basel II Advanced IRB approach requires less than 20 percent of the capital needed to achieve the 0.1 percent regulatory target default rate assuming that all bank capital is Tier 1 (equity) capital. Even if all bank capital is equity, the Advanced IRB capital rules are consistent with a true bank default rate of over 5 percent—or

¹ Basel II background papers highlight the influence that the portfolio invariant capital allocation literature has on the IRB calibration process. For example, footnote 26 of the Basel Committee on Banking Supervision (2001) references the working paper that subsequently has been published as Gordy (2003).

² Kupiec (2004) demonstrates the existence of these biases in VaR-based capital rules in the single credit setting, but does not consider the biases that may arise in a well-diversified portfolio.

more than 5000 times the regulatory target rate. Moreover, regulatory capital rules allow banks to use subordinated debt and qualified Tier 3 capital to satisfy regulatory requirements. Consequently, an Advanced IRB bank's true loss absorbing capacity is likely to be more limited than these estimates in this analysis suggest.

In contrast to the Advanced IRB approach, Foundation IRB banks will be required to hold many times the capital necessary to achieve a .001 default rate. For high quality portfolios, Foundation IRB capital requirements specify more than 7 times the level of capital needed to achieve the regulatory target rate. As the quality of the credits in a portfolio declines, the Foundation IRB rules are less aggressive in over-capitalizing positions. Estimates suggest that capital for one-year lower quality credits are overstated by more than 160 percent. For short-dated credits, Foundation IRB capital requirements provide capital relief relative the 1988 Basel Accord. As the maturity of credits lengthens, however, the 1988 Basel Accord provides more favorable capital treatment. Overall, the analysis demonstrates that relative to Advanced IRB banks, Foundation IRB banks will be held to a much stricter prudential standard.

The substantial differences in the regulatory capital requirements specified by the alternative Basel II approaches potentially raise important prudential and structural issues. To the extent that banks enjoy safety-net engendered subsidies that are attenuated by minimum regulatory capital requirements, Basel II may create a strong incentive for banks to petition their supervisors for Advanced IRB treatment. Banking system assets will be encouraged to migrate toward Advanced IRB banks, either through consolidation or through an increase in the number banks that are granted regulatory approval for the Advanced IRB approach. The capital relief granted under the Advanced IRB approach may raise prudential concerns as well. For example, should FIDICA prompt corrective action (PCA) minimum leverage requirements be relaxed once Basel II is implemented, regulations would allow Advanced IRB banks to operate with substantial reductions in their capital positions raising the potential for a material increase in systemic risk.³

³ Unless PCA requirements are relaxed (12 U.S.C. Section 1831), PCA may become the binding capital constraint on Advanced IRB banks.

An outline of this paper follows. Section 2 summarizes the portfolio-invariant capital allocation literature and relates it to IRB model calibration. Section 3 presents a general methodology for setting unbiased buffer stock capital requirements. Section 4 revisits unbiased credit risk capital allocation in the context of the Black-Scholes-Merton (BSM) model. Section 5 derives unbiased portfolio-invariant credit risk capital measures in an infinitely granular one-common factor BSM model specification. Section 6 reports calibration results and Section 7 concludes the paper.

2. PORTFOLIO INVARIANT CAPITAL REQUIREMENTS AND THE IRB APPROACH

When a new credit is added to a portfolio, the diversification benefits within the portfolio in part determine the amount of additional capital that is required to maintain a given solvency margin, where solvency margin is defined as 1 minus the probability of default on a bank's funding debt. The marginal capital required for the new credit depends on the characteristics of the credit that is added, as well as the characteristics of the credits in the existing portfolio.

VaR techniques recognize diversification benefits in their prescription for capital requirements. Because an IRB approach sets capital based on characteristics of a credit in isolation, such an approach is not generally capable of recognizing diversification benefits. Under certain conditions, however, IRB capital rules can mimic the capital allocations set by a VaR approach. Vasieck (1991), Finger (1999), Schönbucher (2000), Gordy (2003) and others, have established conditions under which a credit's contribution to a portfolio VaR measure is independent of the composition of the portfolio to which it is added. For example, Gordy (2003) establishes that contributions to VaR are portfolio-invariant if: (a) there is only a single systematic risk factor driving correlations across obligors; and (b) no exposure in a portfolio accounts for more than an arbitrarily small share of total exposure. Under these assumptions, capital allocations can be set using (only) information on the individual credits' risk characteristics and the resulting portfolio capital allocation can still be equivalent to a VaR-based capital allocation.

Because IRB capital requirements can only be accurate in a portfolio-invariant setting, we adopt the assumption of single common factor risk generation and consider only

well-diversified portfolios that satisfy the assumptions used in the portfolio-invariant capital literature. In contrast to earlier studies, this study derives the implications of these assumptions in the full BSM model setting and uses the unbiased capital allocation methodology prescribed by Kupiec (2004) to estimate the capital requirements for individual IRB credits held in a well-diversified portfolio.

3. UNBIASED BUFFER STOCK CAPITAL FOR CREDIT RISKS

The intuition that underlies the construction of an unbiased economic capital allocation is transparent when considering portfolios composed of long positions in traditional financial assets such as bonds or equities because the value of the portfolio cannot go below zero. While it has not been widely recognized in the literature, should losses have the potential to exceed the initial market value of a portfolio as they can for example when a portfolio includes short positions, futures, derivatives, or other structured products, then economic capital calculations must be modified from the techniques described herein. Modifications to the capital structure alone may not be able to ensure that the bank is able to perform on its liabilities with the desired level of confidence. Kupiec (2004) provides further discussion.

Defining an Appropriate VaR Measure

VaR is commonly defined to be the loss amount that could be exceeded by at most a maximum percentage $(1 - \alpha)$ of all potential future asset or portfolio value realizations at the end of a given time horizon. The VaR coverage rate, α , sets the minimum acceptable solvency margin

Assume T is the capital allocation horizon of interest for asset A, which has an initial market value A_0 , and a time T value of, \tilde{A}_T . \tilde{A}_T has a cumulative probability density function $\Psi(\tilde{A}_T, A_T)$. An (α) coverage VaR measure, $VaR^\mu(\alpha)$, $\alpha \in [0,1]$, is often defined as,

$$VaR^\mu(\alpha) = E(\tilde{A}_T) - \Psi^{-1}(\tilde{A}_T, 1 - \alpha) \quad (1)$$

where $E(\tilde{A}_T)$ represents the expected end-of-period value of \tilde{A}_T and $\Psi^{-1}(\tilde{A}_T, 1-\alpha)$ represents the inverse of its cumulative density function of \tilde{A}_T evaluated at $1-\alpha$. In expression (1), profit and loss (P&L) are calculated relative to the expected time T value.

When using VaR type measures for capital allocation purposes, Kupiec (1999) establishes the importance of measuring P&L relative to the initial market value of the asset. This measure, $VaR^{A_0}(\alpha)$, is defined as,

$$VaR^{A_0}(\alpha) = A_0 - \Psi^{-1}(\tilde{A}_T, 1-\alpha) \quad (2)$$

In very short horizon calculations (e.g., daily market risk VaR measures) the difference between $VaR^\mu(\alpha)$ and $VaR^{A_0}(\alpha)$ is inconsequential. As the time horizon lengthens, as it does in most interesting capital allocation problems, the difference between $VaR^\mu(\alpha)$ and $VaR^{A_0}(\alpha)$ can be substantial and use $VaR^\mu(\alpha)$ will bias VaR capital allocation estimates.

Unbiased Capital Allocation for Credit Risk

Consider the use of a 99.9 percent, 1-year measure, $VaR^{A_0}(.999)$, to determine the necessary amount of equity funding for a long bond or loan position. By definition, there is less than 0.1 percent probability the asset's value will ever post a loss that exceeds its $VaR^{A_0}(.999)$ measure. It is tempting (but incorrect) to conclude that should the level of equity financing be set equal to $VaR^{A_0}(.999)$, there would be at most a 0.1 percent probability that subsequent portfolio losses could cause the bank to default on its funding debt.

If VaR is measured from the asset's initial market value and that VaR measure is statistically accurate, VaR can never exceed A_0 . If the bank were to set the share of equity funding equal to $VaR^{A_0}(.999)$, the amount of debt finance required to fund the asset would be $A_0 - VaR^{A_0}(.999)$. If the bank borrows $A_0 - VaR^{A_0}(.999)$ it must promise to pay back more than $A_0 - VaR^{A_0}(.999)$ assuming that interest rates credit risk compensation are positive. An unbiased economic capital allocation rule for 0.1 percent target default rate is to set equity capital equal to $VaR^{A_0}(.999)$ and in addition include the interest that will accrue on

the funding debt issue. Kupiec (2004) establishes the validity of this capital allocation recipe for both market and credit risks.

Portfolio Capital

While the discussion of unbiased buffer stock capital has thus far been presented in terms of a single credit, the results generalize to the portfolio context. Let A_{i0} and \tilde{A}_{iT} represent, respectively, the initial and time T value of asset i . The initial and time T value of a portfolio of assets is given by, ${}_p A_0 = \sum_{\forall i} A_{i0}$ and ${}_p \tilde{A}_T = \sum_{\forall i} \tilde{A}_{iT}$ respectively. Let the cumulative probability density for ${}_p \tilde{A}_T$ be represented by $\Psi({}_p \tilde{A}_T, {}_p A_T)$. An unbiased portfolio economic capital allocation with a solvency rate of α can be estimated by first calculating $VaR^{pA_0}(\alpha)$,

$$VaR^{pA_0}(\alpha) = {}_p A_0 - \Psi^{-1}({}_p \tilde{A}_T, 1 - \alpha) \quad (3)$$

and adding to it, an estimate of the equilibrium interest cost on the funding debt issued to finance the portfolio. In order to estimate the equilibrium interest cost, one must go beyond the tools of value-at-risk and utilize formal asset pricing models or empirical approximations to price the funding debt. This topic is discussed in detail in the following section.

4. Unbiased Buffer Stock Capital Allocation in a Black-Scholes-Merton Model

If the risk-free term structure is flat and a firm issues only pure discount debt, and asset values follow geometric Brownian motion, under certain simplifying assumptions⁴, Black and Scholes (1973), and independently Merton (1974), (hereafter BSM) established that the market value of a firm's debt issue is equal to the risk free discounted value of the bond's par value, less the market value of a Black-Scholes put option written on the value of the firm's assets. The put option has a maturity identical to the debt issue maturity, and a strike price equal to the par value of the debt. More formally, if A_0 represents the initial

⁴ There are no taxes, transactions are costless, short sales are possible, trading takes place continuously, if borrowers and savers have access to the debt market on identical risk-adjusted terms, and investors in asset markets act as perfect competitors.

value of the firms assets, B_0 the bond's initial equilibrium market value, and Par the bond's promised payment at maturity date M , the BSM model requires,

$$B_0 = Par e^{-r_f M} - Put(A_0, Par, M, \sigma), \quad (4)$$

where r_f represents the risk free rate and $Put(A_0, Par, M, \sigma)$ represents the value of a Black-Scholes put option on an asset with an initial value of A_0 , a strike price of Par , maturity M , and an instantaneous return volatility of σ .

The default (put) option is a measure of the credit risk of the bond. Merton (1974), Black and Cox (1976), and others show that the model will generalize as to term structure assumptions, coupon payments, default barrier assumptions, and generalized volatility structures, but the capital allocation discussion that follows uses the simplest formulation of the BSM model.⁵

Incorporating Credit Risk into the BSM Model

In the original BSM model, the underlying assets exhibit market risk. To examine portfolio credit risk issues, it is necessary to modify the BSM model so that the underlying assets in the portfolio are themselves risky fixed income claims. Consider the case in which a bank's only asset is a risky BSM discount debt issued by an unrelated counterparty. Assume that the bank will fund this bond with its own discount debt and equity issues. In this setting, the bank's funding debt issue is a compound option.

Let \tilde{A}_T and Par_p represent respectively the time T value of the assets that support the purchased discount debt and the par value of the purchased discount bond. Let Par_f represent the par value of the discount bond that is used to fund the asset purchase. For purposes of this discussion we restrict attention to the case where the maturity of the bank's funding debt matches the maturity of the BSM asset (both equal to M).⁶ The end-of-period cash flows that accrue to the bank's funding debt holders are,

⁵ That is, it assumed that the term structure is flat, asset volatility is constant, the underlying asset pays no dividend or convenience yield, and all debt securities are pure discount issues.

⁶ Kupiec (2004) discusses the case where maturities differ.

$$\text{Min}\left[\text{Min}\left(\tilde{A}_M, \text{Par}_P\right), \text{Par}_F\right] . \quad (5)$$

Funding Debt Equilibrium Value

In the original BSM model, the firm's underlying assets evolve in value according to geometric Brownian motion.

$$dA = \mu A dt + \sigma A dW \quad (6)$$

where dW is a standard Weiner process. If A_0 represents the initial value of the firm's assets, A_T the value of the firm's assets at time T , equation (6) implies that the physical probability distribution for the value of the firm's assets is,

$$\tilde{A}_T \sim A_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T} \tilde{z}} \quad (7)$$

where \tilde{z} is a standard normal random variable.

Equilibrium absence of arbitrage conditions impose restrictions on these asset's drift rate, $\mu = r_f + \lambda\sigma$, where λ is the market price of risk. If $dA^n = (\mu - \lambda\sigma)A^n dt + A^n\sigma dz$ is defined as the "risk neutralized" process under the equivalent martingale measure, the underlying end-of-period asset value distribution under the equivalent martingale measure, \tilde{A}_M^n , is,

$$\tilde{A}_T^n \sim A_0 e^{\left(r_f - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T} \tilde{z}} \quad (8)$$

The initial market value of the bank's funding discount bond is the discounted (at the risk free rate) expected value of the end-of-period funding debt cash flows taken with respect to probability density \tilde{A}_M^n . In the held-to-maturity (HTM) case, when the maturity of the bank's funding debt matches the maturity of the purchased BSM bond (both equal to M), the initial market value of the bank's funding debt is,

$$E^n \left[\text{Min}\left[\text{Min}\left(\tilde{A}_M, \text{Par}_P\right), \text{Par}_F\right] \right] e^{-r_f M} \quad (9)$$

The notation $E^\eta[\]$ denotes the expected value operator with respect to the probability density for \tilde{A}_M^η . Kupiec (2004) derives an analogous expression for the case when the maturity of the purchased bond exceeds the maturity of the funding debt issued.

Unbiased Buffer Stock Capital

Assume that the bank is investing in a BSM risky discount bond of maturity M . At maturity, the payoff of the bank's purchased bond is given by $\text{Min}[Par_p, \tilde{A}_M]$. Let $\Phi(x)$ represent the cumulative standard normal distribution function evaluated at x , and let $\Phi^{-1}(\alpha)$ represent the inverse of this function for $\alpha \in [0,1]$. Using the general notation defined in Section 2, the quantile of the end-of-period value distribution is,

$\Psi^{-1}(\tilde{A}_M, 1 - \alpha) = A_0 e^{\left[\frac{\mu - \sigma^2}{2}\right]M + \sigma\sqrt{M}\Phi^{-1}(1 - \alpha)}$, and the VaR measure appropriate for setting an economic capital allocation is,

$$VaR^{A_0}(\alpha) = B_0 - \text{Min}\left[Par_p, A_0 e^{\left[\frac{\mu - \sigma^2}{2}\right]M + \sigma\sqrt{M}\Phi^{-1}(1 - \alpha)}\right] \quad (10)$$

B_0 is the initial market value of the purchased discount debt given by expression (4).

Notice that $A_0 e^{\left[\frac{\mu - \sigma^2}{2}\right]M + \sigma\sqrt{M}\Phi^{-1}(1 - \alpha)}$ is the upper bound on the par (maturity) value of the bank's debt under the target solvency margin constraint. The initial market value of this funding debt issue is given by,

$$E^\eta\left[\text{Min}\left[\text{Min}(\tilde{A}_M, Par_p), A_0 e^{\left[\frac{\mu - \sigma^2}{2}\right]M + \sigma\sqrt{M}\Phi^{-1}(1 - \alpha)}\right]\right] e^{-r_f M}. \quad (11)$$

Equation (11) implies that the initial equity allocation consistent with the target solvency rate α is⁷,

⁷ In many situations, $Par_p > A_0 e^{\left[\frac{\mu - \sigma^2}{2}\right]M + \sigma\sqrt{M}\Phi^{-1}(1 - \alpha)}$, and expression (11) simplifies to,

(continued)

$$B_0 - E^\eta \left[\text{Min} \left[\text{Min}(\tilde{A}_M, Par_p), A_0 e^{\left[\frac{\mu - \sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(1-\alpha)} \right] \right] e^{-r_f M} . \quad (12)$$

Similarly, in the so-called mark-to-market (MTM) setting when $T \leq M$, the unbiased economic capital allocation is⁸,

$$B_0 - E^\eta \left[\text{Min} \left[\left(Par_p e^{-r_f(M-T)} - \text{Put}(\tilde{A}_T, Par_p, M-T, \sigma) \right), A_0 e^{\left[\frac{\mu - \sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(1-\alpha)} \right] \right] e^{-r_f T} \quad (13)$$

Portfolio Capital

To generalize capital allocation to the portfolio setting, define Par_{iP} , to be the maturity value of the i^{th} discount debt instrument in a bank's portfolio. Assume all credits mature at date M . The end-of-period value of the portfolio is ${}_P \tilde{A}_M = \sum_{\forall i} \text{Min}(\tilde{A}_{iM}, Par_{iP})$; it has a cumulative distribution function represented by $\Psi({}_P \tilde{A}_M, {}_P A_M)$. The payoff on the portfolio's funding debt is,

$$\text{Min} \left[\sum_{\forall i} \text{Min}(\tilde{A}_{iM}, Par_{iP}), Par_F \right] = \text{Min} [{}_P \tilde{A}_M, Par_F] . \quad (14)$$

If $(1-\alpha)$ is sufficiently small, the expression for portfolio VaR, $VaR^{P A_0}(\alpha)$, is given by,

$$VaR^{P A_0}(\alpha) = \sum_{\forall i} B_{i0} - \Psi^{-1}({}_P \tilde{A}_M, 1-\alpha) . \quad (15)$$

$$B_0 - E^\eta \left[\text{Min} \left[\tilde{A}_M, A_0 e^{\left[\frac{\mu - \sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(1-\alpha)} \right] \right] e^{-r_f M} .$$

⁸ See Kupiec (2004).

The maximum par value of the funding debt consistent with the target solvency margin is equal to the portfolio VaR critical value, $Par_F = \Psi^{-1}(\tilde{A}_M, 1 - \alpha)$, and so the buffer stock capital necessary to satisfy this requirement is,

$$\sum_{\forall i} B_{i0} - E^{\bar{\eta}} \left[\text{Min} \left[\tilde{A}_M, \Psi^{-1}(\tilde{A}_M, 1 - \alpha) \right] \right] e^{-r_f M} \quad (16)$$

$E^{\bar{\eta}} [\]$ indicates the expectation is taken with respect to the risk neutralized multivariate distribution of asset prices of which the bond values in the portfolio are derivative.

Expression (16) represents the recipe for economic capital allocation in the so-called held-to-maturity (HTM) case, when the maturity of the purchased bonds coincides with the maturity of the funding debt issued by the bank. In the MTM case, when the maturity of the funding debt (T) is less than the maturity (M) of the bonds, in the bank's portfolio, the unbiased economic capital allocation that sets a solvency margin α ,

$$\sum_{\forall i} B_{i0} - E^{\bar{\eta}} \left[\text{Min} \left[\sum_{\forall i} \left(Par_{iP} e^{-r_f(M-T)} - Put(\tilde{A}_{iT}, Par_{iP}, M - T, \sigma_i) \right), \Psi^{-1}(\tilde{A}_T, 1 - \alpha) \right] \right] e^{-r_f T} \quad (17)$$

In general, expressions (14)-(17) require the evaluation of a high order integral that does not have a closed-form solution. Consequently, in many cases they must be evaluated using Monte Carlo Methods. Section 5 considers portfolio capital allocation under the simplifying assumptions that generate portfolio-invariant capital allocation rules. These assumptions reduce significantly the complexity of the capital calculations.

5. UNBIASED CAPITAL ALLOCATION IN A SINGLE COMMON FACTOR BSM MODEL

The BSM framework can accommodate any number of multiple factors in the underlying asset price dynamic specification. Vasicek (1991), Finger (1999), Schönbucher

(2000), and Gordy (2003) have established that capital allocation can be simplified when a portfolio is well-diversified and asset values are driven by a single common factor in addition to individual idiosyncratic factors.

Let dW_M represents a standard Wiener process common in all asset price dynamics, and dW_i represents an independent standard Weiner process idiosyncratic to the price dynamics of asset i . Assume that asset price dynamics are given by,

$$dA = \mu A dt + \sigma_M A dzW_M + \sigma_i A dW_i, \quad (18)$$

$$dW_i dW_j = \rho_{ij} = 0, \quad \forall i, j.$$

$$dW_i dW_M = \rho_{im} = 0, \quad \forall i.$$

Under these dynamics, asset prices are log normally distributed,

$$\tilde{A}_{iT} = A_{i0} e^{\left[r_f + \lambda \sigma_M - \frac{1}{2}(\sigma_M^2 + \sigma_i^2) \right] T + (\sigma_M \tilde{z}_M + \sigma_i \tilde{z}_i) \sqrt{T}}, \quad (19)$$

\tilde{z}_M and \tilde{z}_i are independent standard normal random variables. Under the equivalent martingale change of measure, asset values at time T are distributed,

$$\tilde{A}_{iT}^\eta = A_{i0} e^{\left[r_f - \frac{1}{2}(\sigma_M^2 + \sigma_i^2) \right] T + (\sigma_M \tilde{z}_M + \sigma_i \tilde{z}_i) \sqrt{T}}. \quad (20)$$

The correlations between geometric asset returns are given by,

$$\text{Corr} \left[\frac{1}{T} \ln \left(\frac{\tilde{A}_{iT}}{A_{i0}} \right), \frac{1}{T} \ln \left(\frac{\tilde{A}_{jT}}{A_{j0}} \right) \right] = \frac{\sigma_M^2}{(\sigma_M^2 + \sigma_i^2)^{\frac{1}{2}} (\sigma_M^2 + \sigma_j^2)^{\frac{1}{2}}}, \quad \forall i, j. \quad (21)$$

If the model is further specialized so that the volatilities of assets' idiosyncratic factors are identical, $\sigma_i = \sigma_j = \bar{\sigma}$, $\forall i, j$, the pairwise asset return correlations become,

$$\rho = Corr \left[\frac{1}{T} \ln \left(\frac{\tilde{A}_{it}}{A_{i0}} \right), \frac{1}{T} \ln \left(\frac{\tilde{A}_{jt}}{A_{j0}} \right) \right] = \frac{\sigma_M^2}{\sigma_M^2 + \bar{\sigma}^2} \quad \forall i, j. \quad (22)$$

Portfolio Invariant Buffer Stock Capital

In the single common factor case, the calculations necessary to estimate a portfolio capital allocation can be simplified if the end-of-period portfolio value is expressed in return form. The portfolio return distributions that must be measured differ, however, according to the capital allocation horizon and the maturity of the credits in the portfolio. The process for setting an unbiased equity capital allocation is conceptually the same regardless of the horizon but we will observe industry credit modeling practice and treat MTM and HTM calculations as separate cases.

Held-to-Maturity (HTM) Horizon Return

The T -year rate of return on a BSM risky bond that is held to maturity is,

$${}_{HTM} \tilde{U}_{iT} = \frac{1}{B_{i0}} \left(\text{Min}(\tilde{A}_{iT}, \text{Par}_{Pi}) \right) - 1. \quad (23)$$

For bonds or loans with conventional levels of credit risk, ${}_{HTM} \tilde{U}_{iT}$ is bounded in the interval $[-1, a]$, where a is a finite constant. In most applications, a typically is less than 1. When realizations are in the range, ${}_{HTM} U_{iT} < 0$, ${}_{HTM} U_{iT}$ represents the loss rate on the bond held to maturity. When $0 < {}_{HTM} U_{iT} < \frac{\text{Par}_{Pi}}{B_{i0}} - 1$, the bond has defaulted on its promised payment terms, but the bond has still realizes a positive return. A fully performing bond posts a return equal to $\frac{\text{Par}_{Pi}}{B_{i0}} - 1$ which is finite and typically less than 1, as rarely do credits promise a

yield-to-maturity in excess of 100 percent. Notice also that under the one common factor BSM specification (19), conditional on a realized value for the market factor, \tilde{Z}_M , ${}_{HTM}\tilde{U}_{iT}$ are mutually independent.

The physical rate of return distribution (23) has an equivalent martingale distribution counterpart,

$${}_{HTM}\tilde{U}_{iT}^{\eta} = \frac{1}{B_{i0}} \left(\text{Min}(\tilde{A}_{iT}^{\eta}, \text{Par}_{Pi}) \right) - 1. \quad (24)$$

By construction, expressions (23) and (24) have identical support.

Recall that the analysis has been restricted to instruments that require a positive initial investment and cannot subsequently post a loss greater than the initial investment. This is important because a credit derivative or guarantee could, for example, generate a loss rate (or a gain) well outside of the support of the assumed return distribution.

Mark-to-Market (MTM) Horizon Return

Consider the T -year return on an M -year BSM risky bond, with $T < M$. The T -year mark-to-market (MTM) return is,

$${}_{MTM}\tilde{U}_{iT} = \left(e^{r_f(M-T)} - \frac{1}{B_{i0}} \text{Put}(\tilde{A}_{iT}, \text{par}, \sigma, M - T) \right) - 1. \quad (25)$$

For bonds or loans with conventional levels of credit risk, ${}_{MTM}\tilde{U}_{iT}$ is bounded in the interval $[-1, a]$, where a is a finite constant which, in most applications, is than 1. Unlike the HTM return case, there is no return interpretation regarding default in this holding period because a BSM bond is a discount instrument that can only default at maturity. The equivalent martingale MTM return distribution corresponding to expression (25) is,

$${}_{MTM} \tilde{U}_{iT}^\eta = \left(e^{r_f(M-T)} - \frac{1}{B_{i0}} Put(\tilde{A}_{iT}^\eta, par, \sigma, M-T) \right) - 1. \quad (26)$$

Expressions (25) and (26) have identical support.

For simple BSM loan or bond positions, the return distributions in both the HTM and MTM cases (expressions (23-26)) are bounded from above and below. Boundedness and the single common factor assumption are sufficient conditions for the probability convergence theorem that will simplify the capital allocation calculations for a well-diversified portfolio.

Portfolio Return Distribution

The T -period return on a portfolio of n risky individual credits, ${}_k \tilde{L}_T$, is

$${}_k \tilde{L}_T \equiv \frac{\sum_{i=1}^n {}_k \tilde{U}_{iT} B_{i0}}{\sum_{i=1}^n B_{i0}}, \quad \text{for } k = HTM, MTM \quad (27)$$

The T -period equivalent martingale return on the portfolio, ${}_k \tilde{L}_T^\eta$, is given by,

$${}_k \tilde{L}_T^\eta \equiv \frac{\sum_{i=1}^n {}_k \tilde{U}_{iT}^\eta B_{i0}}{\sum_{i=1}^n B_{i0}}, \quad \text{for } k = HTM, MTM \quad (28)$$

Unbiased Portfolio Buffer Stock Capital

Consider now the inverse cumulative density function associated with credit portfolio's end-of-period return, $\Psi^{-1}(\tilde{L}_T, 1 - \alpha)$. For purposes of this analysis, the portfolio will be composed of equal investments in individual credit risk "names." This assumption, while convenient, is more restrictive than necessary to establish subsequent results. It can be demonstrated that, even if portfolio investment shares are not equal in value, provided the largest portfolio investments satisfy certain limiting bounds, as $n \rightarrow \infty$, under the one factor BSM model specification,⁹

$$\Psi^{-1}(\tilde{L}_T, 1 - \alpha) - \frac{\sum_{i=1}^n E[\tilde{U}_{iT} | \tilde{z}_M = \Psi^{-1}(\tilde{z}_M, 1 - \alpha)] B_{i0}}{\sum_{i=1}^n B_{i0}} \xrightarrow{a.s.} 0 \quad (29)$$

By varying α over its entire 0-1 range, expression (29) provides the mapping for the asymptotic portfolio's inverse cumulative return distribution function.

Expression (29) can be used to estimate economic capital allocations for a so-called "infinitely granular" portfolio in the one common factor case. As the form of expression (29) indicates, the $(1 - \alpha)$ critical value of the portfolio's return distribution is the weighed average of "stressed" conditional expected returns on individual credits. These conditional expected values are independent of the composition of the portfolio. Moreover, in a well-

⁹ Propositions 1-2 in Gordy (2003) formally state the sufficient conditions necessary to ensure convergence. Condition A-2 in Gordy (2003) provides the technical bounds that are required to satisfy the "infinite granularity" condition. The Appendix provides further details related to the converge result..

diversified portfolio, the investment shares are approximately equal. The upshot is that expression (29) implies that individual credit's contribution to portfolio VaR is independent of the composition of the portfolio.

Using expression (29), the par value of a funding debt issue with a solvency target rate α can be computed as,

$${}_k Par_F^\infty(1-\alpha) = \sum_{i=1}^n E\left[{}_k \tilde{U}_{iT} \mid \tilde{z}_M = \Psi^{-1}(\tilde{z}_M, 1-\alpha)\right] B_{i0}, \quad k = MTM, HTM \quad (27)$$

The superscript “ ∞ ” is used to identify the asymptotic or so-called “infinitely granular” nature of the portfolio. Under the single factor BSM model assumptions, the “stressed” conditional expected value, $E\left[\tilde{U}_{iT}^k \mid \tilde{z}_M = \Psi^{-1}(\tilde{z}_M, 1-\alpha)\right]$ of a credits' return or loss distribution can be directly evaluated using numerical integration.

The second step in estimating an unbiased economic capital allocation is pricing the portfolio's funding debt. Unbiased buffer stock capital for a solvency rate of α is given by expression (16) in the HTM case, and by expression (17) in the MTM case. The calculation of the market value of the portfolio's funding debt and its unbiased capital allocation can be simplified by using the asymptotic equivalent martingale return distribution. Recall that physical and equivalent martingale portfolio return distributions share the same support. This implies, for a well-diversified portfolio, as $n \rightarrow \infty$, under the one factor BSM model assumptions,

$$\Psi^{-1}\left({}_k \tilde{L}_T^\eta, 1-\alpha\right) - \frac{\sum_{i=1}^n E\left[{}_k \tilde{U}_{iT}^\eta \mid \tilde{z}_M = \Psi^{-1}(\tilde{z}_M^\eta, 1-\alpha)\right] B_{i0}}{\sum_{i=1}^n B_{i0}} \xrightarrow{a.s.} 0 \quad (28)$$

Using expression (28), the market value of the funding debt can be priced using an algorithm to numerically evaluate the integral in expressions (16) and (17). First set

$$\Psi^{-1}\left(\tilde{L}_T^\eta, 1 - \alpha^\eta\right) \sum_{i=1}^n B_{i0} = {}_k \text{Par}_F^\infty(1 - \alpha)$$

and invert the equation to solve for the risk neutral probability of default $(1 - \alpha^\eta)$. Set up a grid of probabilities in the range $[0, (1 - \alpha^\eta)]$ and

$$\text{evaluate } \Psi^{-1}\left(\tilde{L}_T^\eta, 1 - \alpha\right) \sum_{i=1}^n B_{i0}$$

over this set and retain the portfolio values associated with the selected coordinates. Average the portfolio values for each grid interval. Use these average values and the cumulative densities associated with selected grid intervals to numerically evaluate the integral.

6. CALIBRATION RESULTS

The formula for calculating Advanced IRB capital requirements for corporate credits are given on page 60 the revised Basel II framework (Basel Committee on Banking Supervision (2004)). Inputs for the IRB capital calculation are a credit's PD, EAD, LGD, and maturity of the credit. Foundation IRB capital requirements are calculated by using the Advanced IRB capital requirement formula after substituting the assumption that a credit's LGD is 45 percent.

For individual BSM credits, the physical probability of default can be determined analytically. After solving for the critical value of the lognormal distribution realization that determines a credit's probability of default, a credit's expected loss given default can be calculated by numerical integration.

In this calibration analysis, portfolio capital requirements are calculated for an infinitely-granular portfolio where credit risk is generated by a single common factor BSM

model. The maintained assumptions for the BSM model are given in Table 1. All individual credits are assumed to have identical firm specific risk factor volatilities of 20 percent. The market and firm specific factor volatilities imply an underlying geometric asset return correlation of 20 percent. All credits in the portfolio have the same initial value, and all share an identical *ex ante* credit risk profile that is determined by the par value and maturity of the credit. For a given maturity, the par values of individual credits are altered to change the credit risk characteristics of a portfolio. All other BSM model parameters are held constant in the simulations, equal to the parameter values given in Table 1.

Table 1: Black-Scholes-Merton Model Assumptions

risk free rate	$r_f = .05$
market price of risk	$\lambda = .10$
market factor volatility	$\sigma_M = .10$
Firm specific volatility	$\bar{\sigma}_i = .20$
initial market value of assets	$A_0 = 100$
correlation between asset returns	$\rho = .20$

Consistent with Basel II, we consider only a one-year capital allocation horizon, but calculate capital for two alternative credit maturities---one year (the HTM case, expression (16)), and three years (the MTM case, expression (17)).

The analysis includes 16 infinitely granular portfolios of one- and three-year credits. The credit risk characteristics of the one-year maturity portfolios are reported in Table 2.

Individual credit probabilities of default range from 26 basis points for a bond with a par value of 55, to 4.35 percent for a bond with a par value of 70. As is characteristic of all BSM models, loss given default (from initial market value) for these one-year bonds are modest relative to the physically observed default loss history on corporate bonds.¹⁰ The Advanced IRB capital allocation rule explicitly corrects for loss given default, so *a priori*, there is no reason to expect that any specific set of loss given default values may compromise the performance of the Advanced IRB approach.¹¹

The results for one-year bond portfolios are reported in Table 3 and plotted in Figure 1. The results show that capital requirements generated under the Basel II Advanced IRB capital rule are far smaller than the capital needed to achieve the regulatory target default rate of 0.1 percent. In all cases examined, the Advanced IRB approach sets capital requirements that are less than 17 percent of the true capital needed to achieve the 99.9 percent target solvency rate.

¹⁰ Some industry credit risk models include a stochastic default barrier such as in the Black and Cox (1976) model to increase the loss given default in a basic BSM model and improve correspondence with observed market data.

¹¹ Paragraph 407 of the Basel Committee on Banking Supervision (2004) discusses the minimum requirements for Advanced IRB bank LGD treatment "*There is no specific minimum number of facility grades for banks using the advanced approach for estimating LGD. A bank must have a sufficient number of facility grades to avoid grouping facilities with widely varying LGDs into a single grade. The criteria used to define facility grades must be grounded in empirical evidence.*" The Basel II guidelines also do not appear to put a lower bound on the LGDs that banks can use in the Advanced IRB approach.

Table 2: Credit Risk Characteristics of Individual Portfolio Credits

par value	maturity years	initial market value	probability of default in percent	expected value given default	in percent		yield to maturity
					loss given default from initial value	loss given default from par value	
55	1	52.31	0.26	52.01	0.57	5.44	5.142
56	1	53.26	0.34	52.88	0.71	5.57	5.145
57	1	54.20	0.43	53.75	0.82	5.70	5.166
58	1	55.15	0.53	54.62	0.96	5.83	5.168
59	1	56.10	0.66	55.48	1.10	5.96	5.169
60	1	57.04	0.82	56.34	1.23	6.10	5.189
61	1	57.98	1.00	57.19	1.36	6.24	5.209
62	1	58.92	1.21	58.04	1.49	6.38	5.227
63	1	59.86	1.45	58.89	1.62	6.53	5.246
64	1	60.80	1.73	59.73	1.76	6.67	5.263
65	1	61.73	2.05	60.56	1.89	6.82	5.297
66	1	62.66	2.42	61.40	2.02	6.98	5.330
67	1	63.59	2.82	62.22	2.15	7.13	5.362
68	1	64.51	3.28	63.04	2.28	7.29	5.410
69	1	65.43	3.79	63.86	2.40	7.45	5.456
70	1	66.34	4.35	64.67	2.52	7.62	5.517

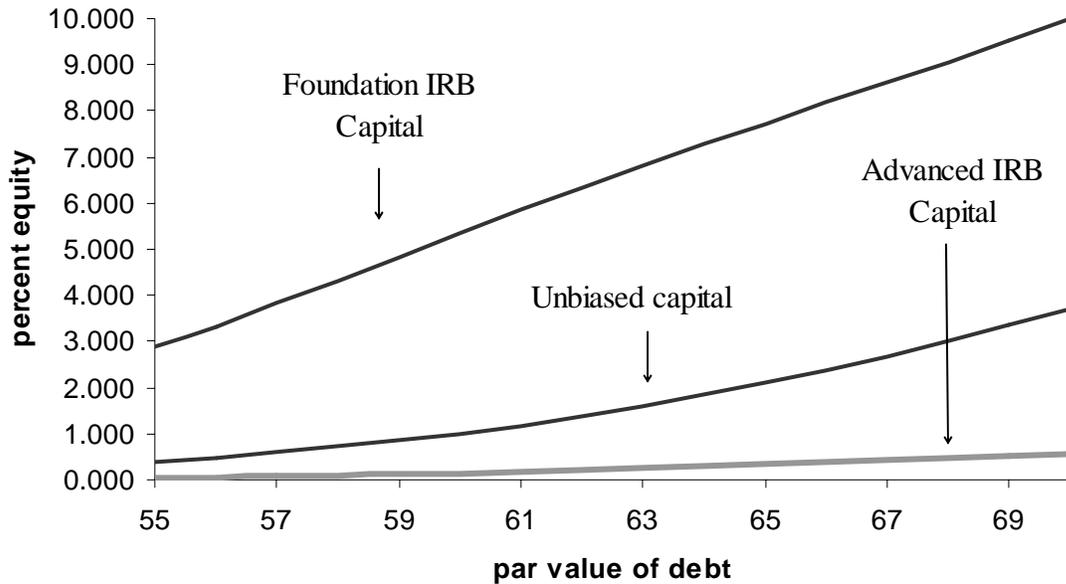
Table 3: Alternative Capital Allocation Recommendations

<i>capital requirement in percent of initial value</i>					
			99.9 percent		95 percent
			unbiased	June 2004	unbiased
			asymptotic	Foundation	Advanced
par	maturity	portfolio	IRB capital	IRB capital	portfolio
value	years	capital	requirement	requirement	capital
55	1	0.399	2.867	0.036	0.051
56	1	0.493	3.332	0.053	0.068
57	1	0.585	3.822	0.070	0.072
58	1	0.713	4.328	0.092	0.099
59	1	0.860	4.842	0.118	0.133
60	1	1.010	5.354	0.146	0.156
61	1	1.182	5.859	0.177	0.187
62	1	1.377	6.351	0.210	0.228
63	1	1.597	6.826	0.246	0.278
64	1	1.842	7.286	0.285	0.340
65	1	2.096	7.732	0.325	0.399
66	1	2.377	8.172	0.367	0.470
67	1	2.686	8.611	0.411	0.556
68	1	3.006	9.060	0.489	0.643
69	1	3.354	9.524	0.508	0.745
70	1	3.714	10.012	0.561	0.851

In principle, the unbiased capital allocation rule can be inverted to recover the Advanced IRB Approach's implied probability of default. In practice, inverting the relationship is a computationally intensive exercise. As an alternative, we calculate unbiased economic capital allocations for a 95 percent solvency margin (a 5 percent probability of default). The final column of Table 3 reports the economic capital associated with a 5 percent probability of bank default. The economic capital necessary to achieve a 95 percent solvency rate and the capital necessary under the Advanced IRB approach are plotted in Figure 2.

Advanced IRB capital requirements are consistently smaller than those that are needed to limit bank default rates to 5 percent. The results suggest that the implied target default rate consistent with the June 2004 Advanced IRB calibration is more than 5000 times the stated regulatory target. Such large difference in recommended capitalizations cannot be dismissed as an artifact of a dubious data given the strictly controlled nature of this analysis.

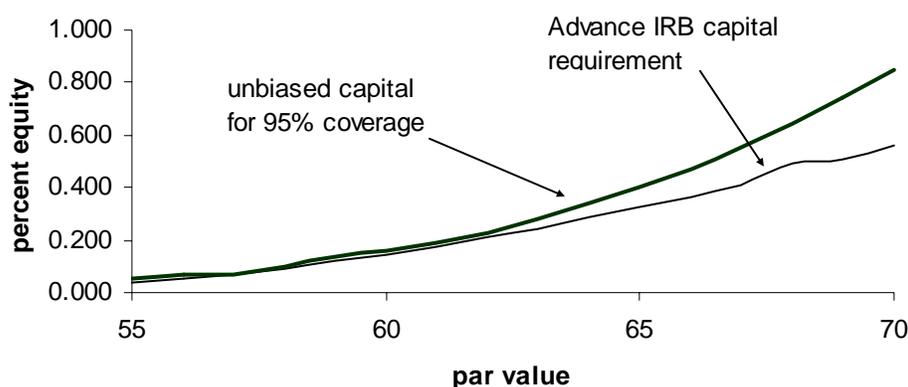
Figure 1: June 2004 IRB Capital Requirements



As a point of comparison, it should be noted that there are no published studies that document the accuracy of the Basel II IRB model calibrations. Notwithstanding Basel Committee representations of an intended prudential standard consistent with a 0.1 percent default rate, the Basel II Quantitative Impact Studies (QIS) and subsequent IRB model calibration adjustments have not focused on producing an IRB calibration consistent with any

specific target solvency margin. Rather, QIS results are reflected in updated calibrations that seemingly have been designed to achieve some measure of intuitive consistency among IRB credit classes (corporate, retail, etc) without creating a set of capital rules that will materially alter the regulatory capital requirements of an “average” internationally active bank. The results of this analysis suggest the possibility that the protracted Basel II IRB QIS studies and subsequent negotiations have produced an IRB calibration that is heavily skewed toward capital relief for Advanced IRB banks.¹²

Figure 2: Bias in Advanced IRB Solvency Target



The Foundation IRB approach uses the Advanced IRB capital allocation function with the added assumption that loss given default is 45 percent. As Figure 1 shows, this LGD assumption dramatically increases capital requirements over the Advanced IRB Approach.

¹² Some may object because the analysis ignores operational risk capital requirements. It should be noted that, by construction, there is no operational risk in this calibration exercise. Operational risk capital, moreover, is not intended as a buffer against a poorly designed credit risk regulatory capital rule.

For the portfolios examined herein, the Foundation IRB will set capital requirements that are many times larger than are needed to achieve the regulatory target default rate. Other things equal, these Foundation IRB banks will face a maximum default rate that is far less than the 0.1 percent regulatory target rate.

The economic capital allocation comparison is repeated for longer date credits. Unbiased economic capital allocations on three-year credits are calculated using the MTM unbiased capital allocation rule given in expression (17). The credit risk characteristics of the individual portfolio credits are reported in Table 4. Note that on these credits, expected values given default uniformly exceed the initial market value of the credit. If, in practice, banks are allowed to use these implied LGD values in the Advanced IRB approach, then these portfolios would not require any regulatory capital.

Table 5 reports the results for portfolios of three-year bonds. The results show that Foundation IRB capital requirements will be set much higher than is needed to achieve the regulatory target default rate. For all portfolios considered, the Foundation IRB approach will require substantially more capital on long-dated credit portfolios than would be required by an unbiased economic capital allocation rule. For high quality credit portfolios, the Foundation IRB capital requirements are almost 5 times too large; for lower quality credits, Foundation IRB capital is more than 3 times the amount needed to achieve the target solvency margin. Indeed, in all cases, the Foundation IRB Approach requires more than the 8 percent capital required under the 1988 Basel Accord.

Discussion

The calibration analysis suggests that there are significant shortcomings in the June 2004 IRB model calibrations. Advanced IRB banks are granted substantial regulatory capital

relief on all the portfolios that have been examined. While one might argue that the framers of Basel II had intended to include capital relief incentives to encourage banks to migrate toward more advanced risk measurement and capital allocation methodologies, the incentives that have been provided would substantially reduce prudential standards even for a bank that is perfectly diversified. Advanced IRB default rates may exceed 5 percent. In contrast, the Foundation IRB Approach overcapitalizes many portfolios, and for long-maturity credits, it requires capital in excess of 1988 Basel Accord.

The substantial capital relief granted under the Advanced IRB Approach may encourage banks to petition their supervisors for Advanced IRB approval. Absent liberal regulatory approval policies, there may be strong economic incentives that encourage industry consolidation into the banks that gain Advanced IRB regulatory approval. The calibration biases that have been identified suggest that the migration of assets into Advanced IRB banks could substantially increase systemic risk as banks that fully meet Advanced IRB minimum capital requirements may have default rates that are significantly in excess of 5 percent unless PCA minimum leverage regulations prohibit realization of the full capital relief granted by the Advanced IRB Approach.

Table 4: Credit Risk Characteristics of Individual Long-Maturity Credits

par value	maturity	initial market value	probability of default in percent	expected value given default	loss given default from initial value in percent	loss given default from par value in percent	yield to maturity in percent
55	3	47.07	4.05	48.76	-3.58	11.35	5.33
56	3	47.89	4.47	49.54	-3.43	11.54	5.35
57	3	48.71	4.92	50.31	-3.28	11.73	5.38
58	3	49.53	5.39	51.08	-3.14	11.92	5.40
59	3	50.34	5.89	51.85	-3.00	12.12	5.43
60	3	51.15	6.42	52.61	-2.86	12.31	5.46
61	3	51.95	6.97	53.37	-2.73	12.51	5.50
62	3	52.75	7.55	54.12	-2.61	12.70	5.53
63	3	53.54	8.16	54.87	-2.49	12.90	5.57
64	3	54.33	8.79	55.62	-2.38	13.10	5.61
65	3	55.11	9.45	56.36	-2.27	13.30	5.66
66	3	55.88	10.13	57.09	-2.16	13.50	5.70
67	3	56.65	10.83	57.82	-2.07	13.70	5.75
68	3	57.42	11.56	58.55	-1.98	13.90	5.80
69	3	58.17	13.22	59.27	-1.89	14.10	5.86
70	3	58.92	13.09	59.99	-1.81	14.30	5.91

7. CONCLUSIONS

Compared to unbiased capital requirements for an infinitely granular portfolio in a BSM single common factor setting, the Basel II Advanced IRB Approach substantially understates the capital that is required to achieve the regulatory target default rate of 0.1 percent on a bank's funding liabilities. Estimates suggest that banks with true default rates in excess of 5 percent could potentially meet the minimum risk-based regulatory capital requirements promulgated by the June 2004 the Advanced IRB Approach.

**Table 5: Alternative Capital Allocation Model
Recommendations for Long-Term Credits**

		<i>capital requirement in percent of initial value</i>		
		99.9 percent		
		unbiased asymptotic	Foundation IRB	Advanced IRB
par value	maturity	portfolio capital	capital requirement	capital requirement*
55	3	2.1204	10.5380	0
56	3	2.2936	10.9070	0
57	3	2.4835	11.2880	0
58	3	2.6806	11.6810	0
59	3	2.8851	12.0840	0
60	3	3.0963	12.4970	0
61	3	3.3145	12.9170	0
62	3	3.5387	13.3420	0
63	3	3.7690	13.7670	0
64	3	4.0049	14.1910	0
65	3	4.2462	14.6090	0
66	3	4.4926	15.0200	0
67	3	4.7435	15.4220	0
68	3	4.9987	15.8100	0
69	3	5.2579	16.6040	0
70	3	5.8431	16.5430	0

* Loss given default from initial value is negative, i.e., if the bond defaults, it is expected to pay off more than its initial market value. Under the Advanced IRB approach, such bonds are allocated 0 regulatory capital.

The simulation results reported in this paper strongly suggest that, as the IRB alternatives are currently calibrated, Basel II will result in alternative regulatory capital regimes which promulgate markedly different prudential standards. Under these regulatory alternatives, banks that adopt the Advanced IRB approach may gain substantial regulatory capital relief without a commensurate reduction in their potential risk profile. The Basel II system may create strong economic incentives for banking system assets to migrate into

Advanced IRB banks. Since the analysis suggests that Advanced IRB banks potentially carry higher default risk absent safety net support, the migration of system assets toward Advanced IRB regulatory capital treatment may not enhance financial stability. Given the prudential weaknesses that appear to be associated with the Advanced IRB approach, the adoption of Basel II in its current form may not reduce systemic risk in the international banking system.

Appendix

If the portfolio is well diversified in the sense that it is “infinitely granular” satisfying conditions A-2 in Gordy (2003), Gordy’s Proposition 3 establishes,

$$\Psi^{-1}\left({}_P\tilde{L}_T^k, 1-\alpha\right) - \Psi^{-1}\left[E\left({}_P\tilde{L}_T^k \mid \tilde{z}_M = z\right), 1-\alpha\right] \xrightarrow{a.s.} 0. \quad (\text{A1})$$

In the BSM single factor model, the individual credit return distributions are monotonic in the realized value of the single common factor \tilde{z}_M . Since we focus on long fully-funded credit risky positions that cannot lose more than their initial funded value, Assumptions A-3 and A-4 of Gordy (2003) are satisfied, and so Gordy’s Proposition 4 holds,

$$\Psi^{-1}\left[E\left({}_P\tilde{L}_T^k \mid \tilde{z}_M = z\right), 1-\alpha\right] - E\left[{}_P\tilde{L}_T^k \mid \tilde{z}_M = \Psi^{-1}(\tilde{z}_M, 1-\alpha)\right] \xrightarrow{a.s.} 0 \quad (\text{A2})$$

where,

$$E\left[{}_P\tilde{L}_T^k \mid \tilde{z}_M = \Psi^{-1}(\tilde{z}_M, 1-\alpha)\right] = \frac{\sum_{i=1}^n E\left[\tilde{U}_{iT}^k \mid \tilde{z}_M = \Psi^{-1}(\tilde{z}_M, 1-\alpha)\right] B_{i0}}{\sum_{i=1}^n B_{i0}}.$$

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