The Great Housing Boom of China

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Abstract: China’s housing prices have been growing nearly twice as fast as national income in the past decade despite (1) a phenomenal rate of return to capital and (2) an alarmingly high vacancy rate. This paper interprets such a prolonged paradoxical housing boom as a rational bubble that emerges naturally from China’s large-scale economic transition, featuring an exceptionally high rate of return to capital driven by massive resource reallocation. Because such primarily resource-reallocation-driven high capital returns are not sustainable in the long run, expectations of high future demand for alternative stores of value can induce even the currently most productive agents to speculate in the housing market, even if housing provides no rents or utilities. We show that such speculative investment behavior can create a self-fulfilling housing bubble that grows much faster than the national income during an economic transition, thus explaining China’s massive “ghost apartment” phenomenon and decade-long faster-than-income growth in housing prices despite high capital returns.

JEL classification: E22, E23, O11, O16, P23, P24, R31

Key words: housing bubble, resource misallocation, Chinese economy, development, economic transition
1 Introduction

Housing prices in China have experienced prolonged and rapid growth in the recent decade, increasing significantly faster than China’s spectacular aggregate income. Data based on 35 major Chinese cities show that average real housing prices have grown at an annual rate of around 17% for the past decade, far exceeding the nation’s 10% average gross domestic product (GDP) growth in the same period.\footnote{The average income growth rate for the 35 largest cities was 11% during that period.} Closely associated with such a housing boom is the growing number of empty or “ghost” apartments across Chinese cities. In 2013 the national urban housing vacancy rate reached 22.4%, far above the level of developed countries (e.g., the homeowner vacancy rate in the United States was only about 3% during the peak of the U.S. housing bubble around 2006). Of note, the majority of the ghost apartments are sold properties, which suggests an excessively strong (speculative) demand rather than an excess supply.\footnote{Many home buyers in China are upper-middle-income and high-income households and they often own multiple homes for investment purposes (see Section 2). Survey data also show that the majority (62% in 2012) of home buyers purchase houses for investment purposes.}

During the same period, China has also enjoyed a phenomenal rate of return to capital. For example, between 1998 and 2012, China’s real rate of return to capital was constantly around 20% or above. This rate of return is unprecedented even compared with the best-performing emerging economies. Yet housing investors in China consist not only of middle-income and high-income households but also firms, including the most productive and profitable firms.

The combination of these features—namely, (i) the decade-long faster-than-income growth of real housing prices, (ii) the exceptionally high vacancy rate, and (iii) the unprecedented high rate of return to capital—is puzzling. A standard neoclassical model, either with land as a production factor or with housing services in the utility function, predicts that housing prices can grow at most as fast as aggregate income and thus can hardly explain China’s phenomenal housing price growth and the alarmingly high vacancy rate. Alternatively, in the classical Samuelson-Tirole bubble economies, housing assets can serve as a store of value even if they provide no utilities. This framework can explain the massive ghost apartment phenomenon in China, but it requires the critical assumption that the rate of return to capital is excessively low in the economy, which seems at odds with the prolonged high rate of return to capital in China.
What economic forces are at work to generate (and sustain) the great housing boom in China? Why would entrepreneurs divert their rapidly rising wealth toward housing instead of productive capital? What are the economic and welfare consequences of such a paradoxical housing boom?

In this paper, we propose a theory to explain the great housing boom. The key element in our theory is a prolonged economic transition (after economic reform) featuring massive resource (such as labor) reallocation from a conventional less productive sector to an emerging sector consisting of productive but financially constrained entrepreneurs. The rate of return to capital in the emerging sector remains exceptionally high over the transition period because of the large pool of “surplus” labor gradually unleashed from the traditional sector. However, such high capital returns in the emerging sector—driven mainly by resource reallocation—are not sustainable in the long run. Thus, a sufficiently low rate of return to capital in the remote future generates expectations of high future demand for alternative stores of value (such as housing).

In a financially backward economy with capital controls and a limited supply of financial assets, such rational expectations can lead to great current demand for housing by entrepreneurs in the emerging sector, even if housing provides no rents or utilities—which means that a housing bubble will arise in China after the housing market reform in the late 1990s. More importantly, the growth rate of a housing bubble will be dictated by the rate of return to capital in the emerging sector. This implies that the bubble (if one emerges) is predicted to grow much faster than aggregate income during the economic transition.

The key mechanism behind this theory of a faster-than-income growing housing bubble hinges on the notion of “marginal investors” in the housing market: The rate of capital returns facing productive entrepreneurs will dictate the bubble’s growth rate. In other words, our theory implies that the participation of other agents facing lower capital returns or interest rates—the non-marginal investors—can affect only the level of housing prices, not the growth rate of housing prices. To support such a marginal investor theory, we present empirical evidence at both the city and firm levels. Among other things, we document an important empirical fact that the rate of private returns to capital across major Chinese cities is highly predictive of the city’s excessive housing price growth above its disposable income.

The implications of our model are not restricted to housing bubbles. Chinese investors have indeed speculated on various “valueless” assets and storable goods as alternative stores of value, such as stamps, garlic (similar to tulip bulbs), tea, salt, and art, among others. They have also speculated in the stock market, which resulted in stock market bubbles. But these bubbles have burst from time to time due to either the quite limited market size (such as garlic) relative to China’s astronomical stock of savings or the lack of critical regulations to mitigate risks (especially in the case of the stock market). Hence, the housing market has become the relatively more attractive and stable market for speculators, given its enormous market size and relatively higher transparency and lower volatility than the stock market.
We show that with such a mechanism, a calibrated model can quantitatively replicate China’s house price dynamics over the past decade fairly well and still be consistent with many other salient features of the Chinese economy. Our theory also predicts that such an abnormally fast-growing housing bubble will lose steam as the economy approaches the Lewis turning point when the surplus labor is exhausted. This prediction is consistent with the recent data from China.

Our paper fits into the fast-growing literature on economic development and resource misallocation under financial frictions. In such an environment, following policy reforms that remove important sources of resource misallocations, there exists a prolonged transition in which capital and labor are reallocated gradually from the less productive sector to the more productive (but financially constrained) new firms. While the bulk of the literature emphasizes the effects of resource reallocation on improving allocative efficiency and the associated saving-investment dynamics during the transition, we argue that such a transition may also be prone to asset bubbles, especially growing bubbles, even when the economy enjoys fast productivity growth and high returns to capital. This prediction is also supported by evidence from other emerging economies in Asia, such as Korea, Taiwan, and Vietnam, which experienced faster-than-income growth of housing prices during their respective economic transition periods featuring labor reallocation from traditional less productive sectors to the emerging and more productive sectors.

To incorporate asset bubbles into such a transition economy, we extend the framework of Song, Storesletten, and Zilibotti (SSZ, 2011) to a setting with an intrinsically valueless asset—housing. The SSZ model is attractive for our purposes because it can endogenously generate and quantitatively account for some important features of China’s economic transition, such as a persistently high rate of aggregate income growth and persistently high returns to capital in the emerging sector, which we argue are key to understanding China’s prolonged paradoxical housing boom. A nice property of the SSZ model is that it features an endogenous AK growth period during the economic transition, which can sustain a constant and exceptionally high rate of return to capital in the private sector for a prolonged period without diminishing returns, consistent with Chinese data on capital returns. Once the transition ends, however, the model reaches a Lewis turning point without surplus labor and starts to behave like a standard neoclassical model with declining returns to capital and GDP growth rate. Our contribution and value added is to show that such a development...
path can sustain *growing* bubbles—bubbles that grow much faster than aggregate income despite an exceptionally high rate of return to capital.

Our paper is one of the first to study *growing* bubbles, as opposed to static bubbles or bubbles that grow at or below the growth rate of the economy.\(^5\) In addition, our model sheds light on the economic and welfare implications of China’s housing bubble: It can significantly prolong China’s economic transition and reduce social welfare. Unlike many traditional bubble models where bubbles are welfare *improving* because of the existence of dynamic inefficiency, in our model bubbles can exist even when the economy is dynamically efficient, thanks to the disparity between social and private rates of return to capital. That is, our model economy is dynamically efficient from a social perspective even though it is dynamically inefficient from the private sector’s view point.\(^6\) Hence, by crowding out private capital formation and other productive activities, the growing bubble in our model creates a severe negative externality on the permanent income of all agents.\(^7\) Accordingly, the occurrence of the housing bubble generates a substantial degree of resource misallocation and welfare losses, prolonging economic transition and slowing aggregate economic growth. Such adverse welfare consequences offer an appealing explanation and rationale for the Chinese government’s concerns over the great housing boom and its policies to contain the bubble.

Our paper also contributes to the emerging literature on China’s high housing price puzzle. Most theoretical works in this area focus on why the housing price *level* is so high in China. For example, Wei, Zhang, and Liu (2012) provide a theory to link the high housing price levels in major cities in China to high household saving rates in these areas due to an unbalanced gender ratio.\(^8\) In sharp contrast, our paper focuses on why housing prices in China have been able to grow much faster than the economy over a prolonged period. Models that explain only the high housing price level from the demand side are insufficient to understand China’s growing housing bubble, which suggests that China’s housing price level is too low instead of too high relative to its long-run equilibrium level. Hence, by shifting the

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\(^5\)For the rapidly developing housing bubble literature, see Caballero and Krishnamurthy (2006); Kocherlakota (2009); Farhi and Tirole (2012); Giglio and Severo (2012); Martin and Ventura (2012); Ventura (2012); Burnside, Eichenbaum, and Rebelo (2013); Miao and Wang (2013); and Galí (2014), among many others.

\(^6\)See also Farhi and Tirole (2012) for a similar result. In both papers, agency frictions drive a wedge between the social rate of returns to capital and the equilibrium rate of returns. Accordingly, bubbles exist even in an environment with dynamic efficiency. However, the reason bubbles may reduce welfare differs between our paper and theirs. In their paper, the presence of bubbles raises the equilibrium interest rate, which reduces the price of other external liquid assets.

\(^7\)See the next section for empirical evidences on the housing bubble’s crowding-out effect on China fixed capital formation.

\(^8\)Wang and Wen (2012) argue that high housing prices cannot explain China’s high average household saving rate. Instead, the culprit lies in China’s rapid income growth in conjunction with precautionary saving under borrowing constraints due to market incompleteness and idiosyncratic consumption/income risks (Wen, 2009).
analysis from the level of housing prices to the growth rate of housing prices, our paper sheds light on China’s housing price dynamics, as well as why such a growing housing bubble may create resource misallocation and prolong China’s economic transition, an issue unaddressed in Wei, Zhang, and Liu (2012).

The remainder of the paper is organized as follows: Section 2 presents some institutional backgrounds and stylized facts about China’s housing market to frame the questions we raise and support the key assumptions in our theoretical model. Section 3 describes a simple two-period benchmark model to illustrate our essential explanations of the great housing boom, as well as the model’s qualitative implications. Section 4 extends the analysis to a multiperiod version of our model for a quantitative analysis. Section 5 concludes with remarks for further research.

2 Stylized Facts

2.1 Housing Price Growth and Vacancy Rate

It is well known that the official housing price indices published by the Chinese government suffer from many measurement problems and do not control for housing quality. Hence, they tend to severely underestimate the growing trend of China’s housing prices. To correct such problems, Wu, Deng, and Liu (2014) use independently constructed housing price indices based on sales of newly built housing units in 35 major Chinese cities. These city-level series are then aggregated into a national level indicator using a weighted average formula, with the total transaction volume during 2006-2010 in each city as the weight. The resulting national-level housing price index shows a much faster growth rate than the official housing price index. For example, the national-level real housing price index has increased at a rate of 17% per year between 2006:Q1 and 2010:Q4. If we ignore the negative impact of the 2008 financial crisis, the average growth rate of housing prices was about 20% per year during this period (see Figure 1, solid line with circles).

The increase in housing prices is also accompanied by rapidly rising land values in China. Figure 1 (dashed line with stars) shows that nationwide real constant quality land values have grown at an average rate of more than 16% per year between 2004:Q1 and 2013:Q2. Accordingly, land value has constituted an important and increasing share in housing prices.

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9 For example, the National Bureau of Statistics of China (NBSC) provides two major housing price indices. Based on these housing price indices, the average growth rate of housing prices in China is below the average growth rate of the economy. However, Wu, Deng, and Liu (2014) argue that these measures are severely biased downward because they fail to control both the complex-level quality changes (e.g., housing suburbanization) and unit-level quality changes (e.g., developers’ pricing strategies).
For example, according to Wu, Gyourko, and Deng (2012), in the city of Beijing the average share of land value in housing prices was 37% before 2008 and surpassed 60% after 2010.

The faster-than-income growth rate of housing prices is prevalent across almost all major cities in China. Figure 2 shows that most of the 35 major cities in China have experienced a significantly faster growth rate in housing prices than city-level aggregate disposable income, which takes into account population growth due to migration. For example, in large cities such as Shanghai and Beijing, the average real growth rate of housing prices during the same period is 2 to 3 times higher than the respective real growth rate of disposable income. The fact that house prices grow persistently faster than aggregate disposable income at both the national and the city levels casts doubt on the conventional wisdom that China’s housing price growth is driven mainly by the increased utilitarian demand for housing due to rural-to-urban migration, or solely by the rapidly increasing purchasing power of Chinese citizens.\(^\text{10}\)

In another important empirical study, Fang, Gu, Xiong and Zhou (2015) use an independent data source based on 120 Chinese cities to document the patterns of housing price growth and per capita aggregate income growth in the sample period of 2003-2013. Among other things, they found that housing prices grew persistently faster than per capita disposable income or gross regional product in the first- and second-tier cities in China. Such evidence is consistent with that presented by Wu, Deng and Liu (2014).

Along with the great housing boom is the continuously rising and alarmingly high housing vacancy rate. According to the China Household Finance Survey (2014), in 2013 the average vacancy rate in the first-, second-, and third-tier cities in China was 21.2%, 21.8%, and 23.2%, respectively.\(^\text{11,12}\) Among different groups of households, 35.1% of entrepreneurial households own vacant houses (or 29.9% of them have multiple apartments). Furthermore, the proportion of households with vacant houses increases with household income. In the top 10th percentile income group, for example, 39.7% of households have vacant (multiple) houses, which is about 22 percentage points higher than households in the lowest income quantile.

The faster-than-income growth of housing prices implies a rapidly rising price-to-income ratio for average wage earners. Ge and Yang (2014) use data from the China Household

\(^{10}\)See Garriga et al. (2014) for the migration view of China’s housing price boom.

\(^{11}\)The definition of the vacancy rate in China corresponds to the homeowner vacancy rate in the Housing Vacancy Survey conducted by the U.S. Census Bureau, computed as the proportion of the vacant self-owned housing units in total homeowner housing units. The definition of vacant housing units does not include housing units that are newly built but not yet sold.

\(^{12}\)China’s first-tier cities usually refer to Beijing, Shanghai, Guangzhou, and Shenzhen, which constitute “The Big 4.” Second-tier cities include the provincial capital cities and other municipalities directly under the central government. Third-tier cities include all other cities.
Income Survey (2014) and find that the growth rate of real wages has been increasing since the economic reform. Between 1998 and 2007, the average real wage growth reached 9.0% per year, almost as fast as real per capita GDP growth.\textsuperscript{13} However, housing prices have been growing much faster. The gap between real housing price growth and real wage growth is more than 8 percentage points. With a rising housing price-to-income ratio, it becomes increasingly difficult for the average Chinese household, especially low-income households who do not yet own a house or want to purchase additional houses, to use housing as a store of value. Nonetheless, data show that even those in the bottom income cohort of China’s urban population have engaged in housing investment despite the excessively high price-to-income ratio. The explanation could be simple: Suppose rational home buyers expect housing prices to grow faster than their disposable income; they would opt to jump into the housing market sooner rather than later if they want to become homeowners during their lifetime. Hence, the fact that China’s rapidly rising housing prices have not discouraged and prohibited households in the lowest income range to buy houses, despite severe borrowing constraints and a more than 30% down payment requirement, suggests that they either observe or expect housing prices to grow significantly faster than their incomes.

\subsection{2.2 Returns to Capital and Resource Reallocation}

It is well documented that the average real rate of return to capital in China has remained around 20\% over the past decade (see, e.g., Bai, Hsieh, and Qian, 2006). Following this literature, we reconfirm this finding here, using the marginal revenue product of capital as a proxy for the rate of return to capital (as in Bai, Hsieh, and Qian, 2006).\textsuperscript{14,15} Panel A of Figure 3 shows that the real rate of return to capital was on average 20\% between 1998 and 2012.\textsuperscript{16} In particular, it increased steadily from 18\% in 2001 to 26\% before the financial crisis year of 2008. Similarly, the measured average after-tax real rate of return to capital (excluding urban housing) was about 18.2\% between 1998 and 2012, approximately the same

\textsuperscript{13}According to the data from the NBSC, the national average growth rate of real per capita disposable income between 1998 and 2012 was 9.3\% per year.

\textsuperscript{14}Specifically, we measure the capital-to-output ratio at market prices and include any expected change in the price of capital as part of its returns. Our computed series of the marginal (revenue) product of capital between 1998 and 2005 are essentially the same as those of Bai, Hsieh, and Qian. (2006).

\textsuperscript{15}Ideally, we should use the income share of reproducible capital. In China, however, the data on reproducible capital income are not available, since there was no market for land in China before the mid-1990s and the market for leaseholds is very imperfect. Bai, Hsieh, and Qian (2006) find that after 1990 the measured average rate of return to capital is close to its counterparts in the non-agricultural and non-mining sectors.

\textsuperscript{16}Until 2005, the Chinese statistical authorities classified all self-employment income as labor income. Therefore, if anything, the reported capital share understated the true capital income at least before 2005.
As the estimated growth rate of the aggregate housing prices in real terms.\textsuperscript{17}

Underlying the enduring high rate of return to capital is the massive labor reallocation in China. Panel B of Figure 3 plots the evolution of the share of private employment in total employment. Following SSZ, we adopt two measures of the private employment share: (i) the share of domestic private enterprises (DPEs) in total employment (which equals employment in DPEs plus SOEs); and (ii) \((\text{DPE+FE})/(\text{DPE+FE+SOE+COE})\), where FE pertains to employment of foreign enterprises, COE that of collectively owned enterprises, and SOE that of state-owned enterprises. For both measures, the private employment share increased steadily for most years during the 1998-2011 period and surpassed 60\% in 2011. We believe the massive degree of labor reallocation from SOEs to the emerging private …rms and the associated low wage rate are key to sustaining the prolonged high rate of return to capital in China over the past decade despite a high investment rate.

2.3 Empirical Evidence Consistent with the “Marginal Investor” Hypothesis

A crucial premise in our theoretical model to simultaneously explain the three stylized facts of China’s great housing boom is the marginal investor hypothesis—that the faster-than-income growth rate of housing prices (despite high capital returns and a high vacancy rate) is driven mainly by speculative housing demand from agents (entrepreneurs) in the productive sector of the economy who have access to high capital returns. Hence, the higher the private rate of return to capital, the higher the rate of housing price growth above the aggregate income growth. Standard economic theories would find it puzzling and paradoxical that well-to-do entrepreneurs and productive firms with high capital returns would engage in speculative housing (or real estate) investment. Such theories would find it even more puzzling that private firms’ capital returns across different cities are positively correlated with or predictive of these cities’ housing price growth in excess of their aggregate income growth. In what follows, we document precisely such stylized facts from three different perspectives. First, we use household-level evidence to show the predictive power of the entrepreneurial status in the vacancy rate of a city’s housing market. Second, we conduct cross-city panel regression analysis to show a strong empirical linkage between excess housing price growth above disposable income growth and the rate of return to capital facing private firms across

\textsuperscript{17}The measured after-tax returns to capital excluding urban housing are computed by excluding the urban residential capital stock from the measured capital stock and by excluding its imputed rent (assumed as 3\% of the original value of the residential capital stock by the NBSC) and tax on output and enterprise income from the capital income.
different regions. Finally, we use firm-level data to reveal the extent of firm involvement in real estate investment and the linkage between their capital returns and ownership structure.

2.3.1 Household Evidence

As noted earlier, China’s average vacancy rate is at least as high as 22.4% across cities in 2011—nearly a quarter of privately owned housing units in China are unoccupied (by owners). What explains such a high homeowner vacancy rate? The China Household Finance Survey (2014), which conducts regression of housing vacancy status against an exhaustive list of both household-level and macro-level variables, shows that the homeowner’s entrepreneurial status (i.e., whether the homeowner owns a private business) has a strong predictive power on the vacancy status of the housing units in all Chinese cities. In other words, entrepreneurs with access to alternative asset (capital) returns are more likely to own multiple unoccupied housing units than other homeowners. This fact holds true even if the regressions control for household incomes, the education level of the household head, attitude for risky investment, housing price-to-rent ratio, urbanization rate, and whether the family has male members to be married (see Table 1 of the CHFS, 2014). Notice that entrepreneurs account for 17% of China’s urban population and that, conditional on holding vacant housing units, 25% of the homeowners are entrepreneurs.

2.3.2 Cross-City Evidence

The concept of the marginal investor in our model is borrowed from the asset pricing literature, where the excessively high rate of return to risky assets is determined by the marginal investor who is able to participate in such assets market without being borrowing constrained. In our model, the marginal investors in the housing market are the entrepreneurs who have access to the high capital returns and yet decide to also participate in the housing market. Such alternative asset returns to the marginal investors will dictate the rate of return to housing investment in a self-fulfilling housing bubble equilibrium by the no-arbitrage condition.

As far as we know, the best empirical approach to support the marginal investor theory in the asset pricing literature is to assess the predictive power of the investors’ marginal value of wealth (proxied by their leverage position) on the excess asset returns (see, for example, Adrian, Etula, and Muir, 2014). Here we follow a similar strategy by investigating the predictive power of private capital returns in different cities on the excess housing price growth (i.e., the growth rate of housing prices minus the growth rate of aggregate disposable income).
income). Our evidence suggests that (i) across major Chinese cities the private rate of returns to capital is a strong predictor of the city’s excess housing price growth and (ii) capital returns of private firms have a larger and more significant predictive power on excess housing returns than do capital returns of SOEs.

Specifically, we measure capital returns in a region as the ratio of total profit to the net value of fixed assets. We run the panel fixed-effect regression of excess housing price growth against the capital returns of different types of firms in different regions. Our panel data cover excess housing price growth and capital returns for 35 majors cities in China between 2006 and 2010. Table 1 reports the results from our panel regression with excess housing price growth as the dependent variable. Columns (1) and (2) suggest that capital returns are highly significant in predicting excess housing returns regardless of the firm type. Column (3) shows that when both types of firms are included as independent variables, the capital returns of POEs exhibit a more significant and stronger predictive power on excess housing price growth than those of SOEs. This evidence suggests that private enterprises are more likely to be the marginal investors in the housing market than are SOEs.

Quantitatively, Column (3) of Table 1 implies that a 1-percentage-point higher rate of capital return for private firms in a city at a particular time point would predict a 0.7-percentage-point higher housing price growth above income growth in that city. For example, Beijing’s private real rate of return to capital between 2006 and 2010 is 29.23% and its excess housing price growth rate is 18.78 percentage points in the same period, whereas in Lanzhou (the capital of a northwestern province) the aggregate capital returns was 9.76% per year in the same period. So there is a 19.47-percentage-point gap for capital returns between these two locations, which would predict Lanzhou’s excess housing price growth to be about 5.15 percentage points. In the data, Lanzhou’s excess housing price growth was 5.06 percentage points above its aggregate income growth, which is consistent with the prediction of our regressed results.18

2.3.3 Firm Evidence

Firm-level data show that a substantial fraction of non-real estate firms (including the very productive ones) in China engage in real estate investment that is unrelated to their original business. This stylized fact indicates not only a close link between capital returns and

18Although correlation is not causation, the reverse of the causal linkage (that high housing price growth causes a high profit ratio or rate of return to capital) is a less appealing theory because it fails to explain why housing price growth is excessively high in the first place and why it would necessarily lead to high capital returns.
housing returns, but also a possible source of the crowding-out effect of the housing bubble
on capital investment, as we show in the next section.

Here we use the data on publicly listed firms, based on the China Stock Market &
Accounting Research (CSMAR) database, to check the extent of involvement by non-real
estate firms in real estate investment and issues related to our marginal investor hypothesis.
We restrict our sample to firms that have been traded for at least two years on the China
A-share stock market over the period of 2007-2013. We exclude firms in the real estate
and construction sectors.

Table 2 reports the summary statistics of non-real estate firms with investment in real
estate that is unrelated to their original business. About 45 percent of firms have such
investment properties (purchased for rent and capital gain, instead of as a necessary input
or production factor in their own business). The share of real estate investment property
in the total physical assets of these firms is 15% on average and is stable over time.

We now examine the capital returns of these firms investing in the housing market. We
argue that if both private and SOE firms invest in the housing market, firms with higher
capital returns tend to be the marginal investors. Our evidence shows that among those
non-real estate firms involved in real estate investment, SOEs on average have lower capital
returns (productivity) than POEs.

Specifically, using the above sample of firms, we regress capital returns against the degree
of state ownership. We construct capital returns at the firm level using the ratio of operating
profit to the one-period lag of property, plant, and equipment (PPE), which have been
excluded from the value of investment property since 2007. We adopt three different measures
to gauge the degree of state ownership. The first is a direct measure of the state-owned stock
share; the second and third measures pertain simply to state-ownership dummies. For the
second measure, we let the state-ownership dummy have a value of 1 if its state-owned stock
share exceeds 50%. For the third, we let the dummy have a value of 1 if the state-owned
stock share exceeds 25%. To be consistent with our model’s assumption, we also add an
one-digit industry dummy. Table 3 reports the estimated coefficients on the measured

\[ KP_{it} = \text{cons} + \beta \times S_{it} + \sum_{j \in J} \gamma_j \times Ind\_dum_j^i + \varepsilon_{it} \]

\footnote{Since January 1, 2007, all Chinese listed firms have been required to disclose their real estate holdings
for investment purposes, which includes any land and buildings held for rental income and/or for capital
appreciation purposes.}

\footnote{As mentioned by Li, Shao, and Tao (2015), prominent examples of non-real estate firms diversifying into
real estate include Youngor (a leading garment company), Kweichow Moutai (a leading liquor company),
and Suning (a leading electronics retailer).}

\footnote{The empirical model is...}
state ownership. Clearly, for all three measures, the return to capital is indeed negatively correlated with the degree of state ownership among the firms investing in real estate.\textsuperscript{22} This, again, suggests that private enterprises tend to be the marginal investors in the housing market.

To sum up, the empirical evidence presented in this section supports our marginal investor hypothesis in that (i) entrepreneurs or productive firms are extensively involved in the housing market and are an important determinant of China’s high vacancy rate; (ii) private capital returns are highly predictive of the excess housing price growth above income growth across major cities in China; and (iii) on average, the capital returns of SOEs (that invest in the housing market) are lower than those of POEs and are less predictive of excess housing price growth across major cities.

\subsection*{2.4 Crowding-Out Effects on Capital Investment}

The rapid growth in housing prices is accompanied by a spectacular boom in the real estate sector. Data from the \textit{China Statistical Yearbook (CSY) 2012} show that the share of total real estate investment in GDP increased by more than threefold, from 4.2\% in 1999 to 13.2\% in 2011. Booming residential investment accounts for about 70\% of the real estate boom. The average nominal growth rate of residential investment is 25.5\% per year, compared with an average nominal GDP growth rate of 13.9\% per year. Accordingly, the share of residential investment in GDP rose from 2.4\% in 1999 to 9.5\% in 2011, a fourfold expansion.

On the other hand, the rapidly growing housing bubble has shown a strong crowding-out effect on China’s capital formation for both SOEs and POEs. We measure the crowding-out effects by estimating the correlation coefficients between housing price growth and non-real estate investment growth.\textsuperscript{23} To remove seasonal effects, all growth rates are on a year-to-year basis, which means the growth rate of a particular month is compared with the same month in the previous year. Table 4 presents the correlation between real housing price growth (deflated by the consumer price index) and real investment growth (deflated by the producer price index). The second and third columns show the correlations of aggregate housing price growth with growth in real estate investment and other types of investment, where $KP$ denotes the capital returns, $S_{it}$ is the measure of the degree of a firm’s state ownership, and $Ind\_dum$ is the industry dummy.

\textsuperscript{22}This does not rule out the possibility that in some industries monopolized by SOEs (e.g. petroleum), SOE firms investing in the property market can also enjoy very high revenue-based productivity.

\textsuperscript{23}Due to data availability constraints, we are able to decompose aggregate investment into only real estate investment and the rest.
respectively. In addition to reporting the contemporaneous correlations, we also report lead and lag correlations.

Table 4 shows that the growth of real estate investment is significantly and positively correlated with housing price growth, while non-real estate investment is significantly negatively correlated with housing price growth. More importantly, the results show that current growth in housing prices is a strong predictor of a future drop in non-real estate investment growth, with the peak correlation between housing price growth and investment growth reached at a 5-month lead. This crowding-out effect of housing price growth on non-housing investment is consistent with the predictions of our model.\textsuperscript{24}

Our findings of crowding-out effects of housing investment are also supported by independent empirical studies. For example, Li, Shao, and Tao (2015) find that firms with real-estate investment property tend to experience under-investment in fixed capital formation by 10\% compared with their industry benchmark. Wu, Gyourko, and Deng (2015) find that, for publicly listed firms, real estate value has no impact on fixed capital investment via the collateral channel, further suggesting that real estate investment crowds out fixed capital formation. Similarly, Chen, Liu, and Zhou (2013) provide empirical evidences that increases in real estate prices tend to crowed out firms’ fixed capital formation in China.

2.5 Other Facts Concerning Model Assumptions

Our model makes the following simplifying assumptions: Both the land supply and the interest rate are fixed. In addition, our model focuses on housing price dynamics over the past decade, which corresponds to a period of massive SOE privatization in China.

\textit{SOE Reform.} Under China’s planned economy, SOEs were the major employers in cities and played the pivotal role of maintaining low unemployment and ensuring social stability. By the mid-1990s, the Chinese government realized that its gradualist reform policy could no longer manage the mounting losses of SOEs. Beginning in 1997, China moved forward with more aggressive restructuring with large SOEs, accomplished through large-scale privatization. The reallocation of labor and capital from SOEs to POEs has been a key source of productivity growth in the past decade.

\textit{Land Supply.} In China the land available for home construction is strictly controlled by the government. During 1997-2000, land available for new construction was limited to 20.40 million acres; during 2001-2010, it was limited to no more than 30.72 million acres. This restriction on the size and new release of construction land was further strengthened by the

\textsuperscript{24} A wide class of models (e.g., Kocherlakota, 2009 and Martin and Ventura, 2012) predicts that housing bubbles, by serving as collateral, crowd in capital investment.
National Land Use Plan 2006-2020, passed by the State Council of China in August 2008. According to this regulation, the total land available for construction in urban and rural areas is limited to 506.25 million acres by 2010 and 558.6 million acres by 2020. The same plan requested that the amount of cultivated land in 2010 and 2020 be maintained at 1.818 billion acres and 1.805 billion acres, the so-called red line lower limit for the total amount of arable land. Figure 4 shows the total amount of arable land in China. It is clear that since 2003 the amount of arable land has been more or less stabilized, implying a de facto fixed supply of land for home and real estate construction.\footnote{A relatively inelastic supply of land is crucial for the existence of a sustained housing bubble.}

Financial Repression and Interest Rate Control. China has made significant progress since 1978 in opening its economy to the outside world, but financial reform significantly lags its economic reform in goods-producing sectors. China’s financial repression is easily seen in Figure 5 where interest rates are essentially flat with the deposit rate substantially below the lending rate. Funds are channeled through state-owned banks to the conventional sector occupied mainly by SOEs. There are few investment alternatives for household savings: Stock markets are poorly regulated and dominated by SOEs, interest rates are set by the government, the national capital account is closed, and the exchange rate is fixed or tightly managed. Through a system of strict capital controls where the state directly manages the banking sector and financial intermediation, the government has been able to maintain or suppress interest rates at below market-clearing levels. When the interest rate is fixed at a level below the market-determined rate, SOEs are able to survive despite productivity inefficiency.

3 The Benchmark Model

In this section, we develop a theory of China’s great housing boom consistent with the institutional background and stylized empirical facts about China and its housing market behaviors. In particular, we extend the SSZ model to a setting with an intrinsically valueless asset—housing—and prove that a housing price bubble that grows faster than GDP exists even if housing provides no rents or utilities to investors. For simplicity we exclude the access of low-income households (workers) to the housing market because their participation has only a level effect on the housing prices but no growth effects. We emphasize the growing nature of the bubble because the traditional bubble literature often focuses exclusively on static bubbles or bubbles that grow at most at the same rate as the economy, which is contradicted by the Chinese data. In this section we illustrate our main story in a two-
period overlapping-generations (OLG) model. We extend the model to a more realistic setting with multiperiod OLGs for the quantitative analysis in Section 4.

### 3.1 The Environment

The economy is populated by two-period-lived agents with overlapping generations. Agents work when young and consume the returns to their savings when old. Agents have heterogeneous skills. In each cohort, half of the population are workers without entrepreneurial skills and the other half are entrepreneurs. Entrepreneurial skills are inherited from parents; we do not allow transition of social classes (for simplification without loss of generality). The total population $N_t$ grows at an exogenous rate $\nu$.

Before the economy starts, the government owns $\overline{H}$ units of housing (land), which are in fixed supply. At the beginning of the first period, the government sells the housing stock to the market (if there is demand) and consumes all the proceeds.

#### 3.1.1 Technology

There are two production sectors and thus two types of firms. Labor is perfectly mobile across the two sectors, but capital is not. The first sector is composed of conventional firms—F-firms, which (for simplicity) are owned by a representative financial intermediary (e.g., a state-owned bank) and operated as standard neoclassical firms.

The second sector is a newly emerging private sector composed of unconventional firms—E-firms. The E-firms are operated by entrepreneurs. More specifically, E-firms are owned by old (parent) entrepreneurs, who are residual claimants on profits, and they hire their own children as managers. Workers can choose to work for either type of firm.

E-firms are more productive than F-firms but are severely borrowing constrained—they cannot borrow from each other or from any other sources. As a result, they must self-finance capital investment through their own savings. In contrast, F-firms can rent capital from their representative financial intermediary at a fixed interest rate $R$. Accordingly, F-firms can still survive in the short run despite inferior technology. Over time, however, labor will be gradually reallocated from F-firms to E-firms as the capital stock of E-firms expands. Thus, the economy features a transition stage during which F-firms and E-firms coexist but with the F-sector shrinking and the E-sector expanding. When the transition ends, only E-firms exist and the economy becomes a representative-agent growth model with neoclassical

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26We can assume that the F-firms have market power and our main results do not change qualitatively.
features. Our focus in this paper is on the transition stage.\textsuperscript{27}

The technologies of the two types of firms follow constant returns to scale

\[ y_t^F = \left( k_t^F \right)^{\alpha} \left( A_t n_t^F \right)^{1-\alpha}, \quad y_t^E = \left( k_t^E \right)^{\alpha} \left( A_t \chi n_t^E \right)^{1-\alpha}, \]  

(1)

where \( y^j, k^j, \) and \( n^j \) denote per capita output, capital stock, and labor, respectively, for a type-\( j \) firm, \( j \in \{E,F\} \). The parameter \( \chi > 1 \) captures the assumption that E-firms are more productive than F-firms. Technological growth in both sectors is constant and exogenously given by \( A_{t+1} = A_t (1 + z) \). However, during the economic transition, resource reallocation can generate endogenous growth faster than the growth in \( A_t \).

### 3.1.2 Worker’s Problem

Workers can deposit their savings into the representative bank and earn a fixed interest rate \( R \). Without loss of generality, we assume that workers do not speculate in the housing market. Allowing workers to invest in housing does not change our main results—although the housing price level would be much higher, the growth rate of housing prices is unaffected. This is because the equilibrium growth rate of housing prices in our model is determined by the rate of return to capital of the entrepreneurs, who are the marginal investors in the bubbly equilibrium.\textsuperscript{28}

The worker’s consumption-saving problem is

\[ \max_{c_{1t}^w, c_{2t+1}^w} \log c_{1t}^w + \beta \log c_{2t+1}^w \]  

subject to \( c_{1t}^w + s_t^w = w_t \) and \( c_{2t+1}^w = s_t^w R \), where \( w_t \) is the market wage rate, and \( c_{1t}^w, c_{2t+1}^w \), and \( s_t^w \) denote, respectively, consumption when young and consumption when old, and the worker’s savings.

### 3.1.3 The F-Firm’s Problem

In each period, an F-firm maximizes profits by solving the following problem:

\textsuperscript{27}Note that the concept of “transition” in this paper is different from the convention in the neoclassical growth model, where transition means the dynamic path from an initial point toward the steady state. This conventional transition phase shows up in our model after the F-sector disappears. To avoid confusion, we call this neoclassical transition period “post-transition.”

\textsuperscript{28}The proof is available upon request.
where the rental rate for capital is the same as the deposit rate \( R \). The first-order conditions imply
\[
w_t = (1 - \alpha) A_t \left( \frac{\alpha}{R} \right)^{\frac{\alpha}{1 - \alpha}}.
\]
Note that during the transition, the wage rate, scaled by the level of technology, \( w_t/A_t \), is constant, due to a constant rental rate for capital and, accordingly, a constant capital-to-labor ratio, \( k_t^F / (A_t n_t^F) = (\alpha/R)^{\frac{1}{1 - \alpha}} \). When the transition is completed, all F-firms disappear, so equation (4) no longer holds.

### 3.1.4 The E-Firm’s Problem

Following SSZ (2011), we assume that young entrepreneurs receive a management fee \( m_t \) from their parents, which is a fixed \( \psi \leq 1 \) fraction of the output produced, \( m_t = \psi \left( k_t^E \right)^\alpha \left( A_t \chi n_t^E \right)^{1 - \alpha} \).

Therefore, the old entrepreneur’s problem can be written as
\[
\max_{n_t^E} (1 - \psi) \left( k_t^E \right)^\alpha \left( A_t \chi n_t^E \right)^{1 - \alpha} - w_t n_t^E
\]

The first-order conditions imply a linear relationship between \( n_t^E \) and \( k_t^E \)
\[
n_t^E = [(1 - \psi) \chi]^{\frac{1}{\alpha}} \left( \frac{R}{\alpha} \right)^{\frac{1}{\alpha}} \frac{k_t^E}{\chi A_t}.
\]

Such a linear relationship is obtained because of a constant wage rate, which in turn results from the constant interest rate \( R \). Accordingly, labor is reallocated to E-firms at a speed equal to the growth of the E-firm’s capital stock. Substituting (6) into (5) gives the E-firm’s profit:
\[
\pi (k_t^E) = (1 - \psi)^\frac{1}{\alpha} \chi^{\frac{1 - \alpha}{\alpha}} R k_t^E \equiv \rho^E k_t^E,
\]
where the first equality is based on equation

\[\text{SSZ also provide a micro-foundation for a young entrepreneur’s management fee as a fixed fraction of output: There exists an agency problem between the manager and owner of the business. The manager can divert a positive share of the firm’s output for her own use. Such opportunistic behavior can be deterred only by paying managers a compensation that is at least as large as the funds they could steal, which is a share \( \psi \) of output. An alternative interpretation of \( \psi \) is that it reflects the government policy that transfers resources from the capital owners (the old entrepreneurs) to the managers (the young entrepreneurs). See Miao, Wang and Zhou (2014), who study housing bubbles based on firm-level policy distortions.}\]
Whenever F-firms exist, the return to capital for E-firms, $\rho^E \equiv (1 - \psi) \frac{1}{\alpha} \chi \frac{1}{1-\alpha} R$, is a constant because $n_t^E$ increases linearly in $k_t^E$. Similar to SSZ, we impose the following assumption about an E-firm’s relative productivity such that an entrepreneur’s return to capital is higher than the deposit rate $R$: $\chi > \chi \equiv \left( \frac{1}{1-\psi} \right)^{\frac{1}{1-\alpha}}$.

### 3.1.5 The Young Entrepreneur’s Problem

The young entrepreneur decides consumption and portfolio allocations in housing investment, bank deposits, and physical capital investment. The rate of return to capital investment is simply $\rho^E$. We assume that the balanced growth rate, which equals the rate of return to housing investment at steady state, is higher than the bank deposit rate—that is, $(1 + z)(1 + \nu) > R$. As a result, the entrepreneur will always prefer investing in housing to depositing funds in the bank.\(^{30}\) Given the housing prices, denoted as $P_t^H$, the young entrepreneur faces a two-stage problem.

In the first stage, a young entrepreneur’s consumption-saving problem is

$$\max_{s_t^E} \log \left( m_t - s_t^E \right) + \beta \log R_{t+1}^E s_t^E, \quad (7)$$

where $R_{t+1}^E \equiv \max \{ \rho^E, P_{t+1}^H / P_t^H \}$ is the rate of returns for entrepreneurial savings and depends on the entrepreneur’s portfolio choice. First-order conditions give the optimal savings of young entrepreneurs, $s_t^E = m_t / (1 + \beta^{-1})$.

In the second stage, the young entrepreneur chooses portfolio allocation given the total savings $s_t^E$. The fraction $\phi_t^E$ of savings is invested in capital such that $K_{t+1}^E = \phi_t^E s_t^E N_t$, where $K_{t+1}^E = k_{t+1}^E N_{t+1}$ is the total capital deployed by E-firms. The remaining $(1 - \phi_t^E)$ fraction of savings is invested in housing, such that $P_t^H H_{t}^E = (1 - \phi_t^E) s_t^E N_t$, where $H_{t}^E$ denotes the total housing stock purchased by young entrepreneurs in period $t$. Throughout this paper, we ensure there exists an interior solution for portfolio choice, such that the following no-arbitrage condition holds:

$$\frac{P_{t+1}^H}{P_t^H} = \rho_{t+1}^E, \quad (8)$$

\(^{30}\)We will show that the housing price growth rate is constant in the transition stage and declines toward the steady-state level after the transition.
where $\rho_{t+1}^E = \rho^E$ (a constant) during the transition. Hence, an old entrepreneur’s income is simply $\rho^E s_t^E$. The above condition simply says that the rate of return to housing must equal the rate of return to entrepreneurial capital in a bubbly equilibrium.

### 3.1.6 The Bank’s Problem

For easy of exposition, we assume that each period the bank simply absorbs deposits from young workers, rents them to F-firms at interest rate $R$, and invests the rest in foreign bonds with the same rate of return $R$ (as in SSZ, 2011). As mentioned earlier, the result would be similar if we instead allowed the bank to also invest in housing on behalf of the workers and the F-firms. This is so because the F-firm is not the marginal investor determining the growth rate of housing prices in the housing market.

### 3.1.7 Time Line

To summarize, in each period the economic events unfold as follows:

1. At the beginning of period $t$, production of E-firms and F-firms takes place. Each young worker gets paid a real wage $w_t$ regardless of which sector they work in. Each young entrepreneur gets $m_t$.

2. Both the young entrepreneur and young workers make consumption and saving decisions. In addition, the young entrepreneur makes a portfolio choice $\phi_t^E$.

3. The housing market opens. Old entrepreneurs sell housing stock held in the previous period, $H_{t-1}^E$. Young entrepreneurs make a portfolio decision $\phi_t^E$.

4. F-firms repay their capital rents to the bank.

5. The currently old workers consume and die, as do the currently old entrepreneurs.

### 3.1.8 Law of Motion

Since E-firm is self-financed, the law of motion for the E-firms’ capital stock follows

$$K_{t+1}^E = \phi_t^E \left( \frac{\rho_t^E \psi}{(1 - \psi)} \right) \frac{1}{1 + \beta^{-1}} K_t^E, \quad (9)$$

where $\rho_t^E = \rho^E$ for all periods during the transition. As shown later, in this simple economy, the entrepreneur’s portfolio share in physical capital, $\phi_t^E$, is a constant, which, together
with a constant $\rho^E$, implies that the dynamics of the model have an AK feature during the transition: The growth rate of E-firms’ capital is constant. Similarly, we can obtain the implicit law of motion for housing demand as

$$P_t^H = (1 - \phi_t^E) \frac{\rho_t^E \psi}{(1 - \psi)} \frac{1}{\alpha} \frac{1}{1 + \beta} K_t^E,$$  

(10)

where we have used the housing market-clearing condition $H_t^E = \overline{H}$.

3.1.9 Post-Transition Equilibrium

We now characterize the equilibrium in the post-transition stage. Since $n_t^E = 1$, the profit of the E-firm is

$$\pi(k_t^E) = \alpha (1 - \psi) (k_t^E)^{\alpha} (A_t \chi)^{1-\alpha}.$$  

(11)

Note that $\pi(k_t^E)$ features decreasing returns to scale at this stage. The rate of return to E-firms’ capital is simply $\rho^E_{t+1} = \alpha (1 - \psi) (k_{t+1}^E)^{\alpha-1} (A_{t+1} \chi)^{1-\alpha}$.

3.1.10 The Steady State

The steady state of the economy is reached only in the post-transition stage. Since all per capita variables (except labor inputs and housing) grow at the rate $A_t$, we detrend them as $\hat{x}_t = x_t / A_t$.

At the steady state, the law of motion for capital (9) implies

$$\hat{k}^E = \left[ \frac{\psi \phi^E \chi^{1-\alpha}}{(1 + \beta^{-1}) (1 + z) (1 + \nu)} \right]^{1/\alpha}. $$  

(12)

Since $\rho^E = \alpha (1 - \psi) \left( \hat{k}^E / \chi \right)^{\alpha-1}$, we have

$$\rho^E = \alpha (1 - \psi) \frac{(1 + \beta^{-1}) (1 + z) (1 + \nu)}{\psi \phi^E}. $$  

(13)

Equation (13) implies that the rate of return to capital is negatively related to the E-firms’ portfolio share in physical capital, $\phi^E$. 
The equilibrium portfolio allocation $\phi^{E*}$ can be solved by the no-arbitrage condition. Since the supply of housing is fixed, the growth rate of housing prices, denoted as $\rho^H_{t+1} \equiv P^H_{t+1}/P^H_t$, equals the balanced growth rate, $(1 + z)(1 + \nu)$, in the steady state. As a result, the no-arbitrage condition implies the E-firms’ steady-state portfolio share in physical capital is

$$\phi^{E*} = \alpha (1 - \psi) \left(1 + \beta^{-1}\right) / \psi. \quad (14)$$

Intuitively, the larger the rate of returns to E-firms’ capital, as captured by $\alpha (1 - \psi)$, the larger the share of entrepreneurial savings in physical capital. On the other hand, the larger $\psi$ and $\beta$, which imply, respectively, a larger income share and saving propensity of young entrepreneurs, the lower the return to physical capital and thus the lower the share of entrepreneurial savings in physical capital.

Note that, in our model, due to the presence of agency frictions (i.e., $\psi > 0$), there is a wedge between the private and the social rates of return to capital. The social rate of return to E-firms’ capital is simply the marginal product of E-firms’ capital, denoted as $MPK^E$.\footnote{Implicitly, the planner solves a constrained optimization problem without agency frictions but with financial market imperfectness as in the benchmark model.}

By the definition of $\rho^E$, we then have $\rho^E < MPK^E$. Hence, in contrast to the standard bubble theory, dynamic inefficiency is a sufficient, but not necessary, condition for housing bubbles to exist in the long run. This has dramatically different and important welfare implications from those of the traditional bubble literature, as we show below.

### 3.2 Characterizing the Bubbly Equilibrium

In this subsection, we explore the equilibrium with housing bubbles. We first discuss the necessary conditions for housing bubbles to exist. We then show under which condition the equilibrium path with housing bubbles can be achieved. Next, we derive the growth rate of housing prices relative to that of aggregate output. Finally, we explore the normative implications of bubbles.

#### 3.2.1 Existence of Bubbles

Note that there always exists an equilibrium without bubbles in our model—that is, all financial resources are invested in capital and $\phi^E_t = 1$ for all $t$. We call this equilibrium the “fundamental equilibrium.” To understand the conditions for the existence of a bubbly equilibrium, we must understand the nature of the fundamental equilibrium to know under...
which conditions in the fundamental equilibrium a bubbly equilibrium can emerge.

Consider the steady state first. The necessary condition for a housing bubble to exist in the steady state (i.e., $\Phi < 1$) is that in the fundamental equilibrium the rate of returns to E-firms’ capital is below the balanced growth rate. In other words, the economy is dynamically “inefficient” from the perspective of the entrepreneurs. Intuitively, when the returns to capital are so low in the fundamental equilibrium, it is optimal for entrepreneurs to divert savings into housing as an alternative store of value. This condition, together with (14), implies the following parameter restriction on the bubbly equilibrium:

$$\alpha (1 - \psi) \left(1 + \beta^{-1}\right) < \psi,$$

or $\psi > \frac{\alpha (1 + \beta^{-1})}{[1 + \alpha (1 + \beta^{-1})]}$. Intuitively, a larger $\psi$ makes the bubble more likely to occur in two ways: First, it directly reduces the entrepreneur’s rate of return to E-firms’ capital. Second, by increasing the output share of the young entrepreneur, it increases the capital stock accumulated by the young, thus lowering the marginal product of E-firms’ capital.32

We are now able to characterize the conditions for the existence of bubbles in both the transition and the post-transition stages. Assumption (15) together with the law of motion for capital, (9), implies that in the fundamental equilibrium, we must have

$$K_{t+1}^E > \rho_t^E K_t^E, \forall t.$$  

(16)

This is so because the wedge between $K_{t+1}^E$ and $\rho_t^E K_t^E$ is a constant, and this constant exceeds 1 in the fundamental equilibrium. Accordingly, given that (16) is satisfied at the steady state, it must be satisfied for all previous periods. Forwarding (16) by one period and noticing that, with full depreciation of capital, $K_{t+1}^E = I_t^E$, where $I_t^E$ is investment in physical capital, we have

$$I_{t+1}^E > \rho_{t+1}^E I_t^E, \forall t.$$  

(17)

The inequality (17) is analogous to the necessary condition for the existence of bubbles in the model of Abel et al. (1989) (AMSZ henceforth). Intuitively, housing bubbles are possible if there exists a sequence of investment with costs exceeding the income flow it generates in all periods.

32 On the other hand, the incentive for entrepreneurs to hold bubbles does not depend on whether there exists a wedge between the social and private returns to capital.
The inequality (17) implies that in a bubbly equilibrium, the young entrepreneurs would voluntarily reduce their investment and hold housing in their portfolios, with the expectation that the revenues from selling these bubbles will be no less than their forgone income from capital investment. To see this point, note equation (9) implies that in the fundamental equilibrium,
\[ I^E_{t+1} = (1 + \varepsilon) \rho^E_{t+1} I^E_t, \]  
where \( \varepsilon \equiv \psi / [(1 - \psi) \alpha (1 + \beta^{-1})] - 1 > 0 \). Take the total derivative with respect to (18), and let \( dI^E_t = -(P^H_t \bar{H} - 0) \); that is, the resources generated from a reduction in capital investment (the left-hand side) are invested in housing (the right-hand side). Then, we have the inequality \( P^H_{t+1}/P^H_t > \rho^E_{t+1} \). In other words, the rate of return to housing investment would be greater than the rate of return to capital if entrepreneurial savings were all invested in physical capital. As a consequence, expecting the inequality (17) to hold for all future periods and thus a positive future demand for housing, the young entrepreneurs in period \( t \) opt to divert savings into housing, which would raise the housing price \( P^H_t \) until the point where the no-arbitrage condition \( P^H_{t+1}/P^H_t = \rho^E_{t+1} \) holds.

Our result is in contrast to that in the traditional bubble literature in two aspects. First, the traditional bubble literature (e.g. the original AMSZ test) evaluates dynamic (in)efficiency based on the economy-wide rate of return to capital, rather than the rate of return to capital for the marginal investors (i.e., productive entrepreneurs in our model). Second, in the traditional bubble literature, the social and private returns to capital facing an agent are assumed the same. In contrast, they are not the same in our model because of a wedge between the two. Thus, condition (16) needs to hold only with respect to the private returns to E-firms’ capital. This implies that an bubbly equilibrium may exist in our model under dynamic efficiency.

We now explore further the existence of bubbles under dynamic efficiency. Dynamic efficiency implies that the steady-state \( MPK^E \) in the fundamental equilibrium is larger than the balanced growth rate
\[ MPK^E* |_{\phi^E=1} > (1 + z)(1 + \nu). \]  

---

33 As argued by Giglio and Severo (2012), looking at the average rate of return in the economy is not sufficient to judge the dynamic (in)efficiency of market allocations, because there might be over-accumulation among those who cannot park funds in productive assets (e.g., Martin and Ventura, 2012).
With (13), condition (19) requires the following parameter restriction:

\[ \psi < \alpha (1 + \beta^{-1}). \tag{20} \]

Intuitively, the smaller \( \psi \) is, the smaller is the steady-state capital and the higher its marginal product. Also, similar to standard OLG models, a higher \( \alpha \) or a lower \( \beta \) makes the economy less likely to be dynamically inefficient. A combination of (15) and (20) gives further parameter restrictions for bubbles to exist when the economy is dynamic efficient:

\[ \alpha (1 - \psi) (1 + \beta^{-1}) < \psi < \alpha (1 + \beta^{-1}). \tag{21} \]

### 3.2.2 The Equilibrium Path

Given the existence of the bubble equilibrium, what ensures that the rational entrepreneurs will choose their asset portfolios each period to reach such an equilibrium? Note that apart from the two equilibrium sequences of housing prices (or investment) mentioned above, there exist many other equilibrium paths along which the holding of the bubble asset per capita converges to zero.\(^{34}\) It can be shown that the young entrepreneur’s portfolio share of savings in housing assets is characterized by the following first-order difference equation

\[
h_{t+1}^E = \frac{h_t^E (1 - \psi) \alpha (1 + \beta^{-1})}{\psi}, \tag{22}
\]

where \( h_t^E \equiv 1 - \phi_t^E \). The first argument on the right side, \( \frac{h_t^E}{1 - h_t^E} \), is an increasing and convex function of \( h_t^E \). This equation implies that there are two steady states for \( h_t^E \). At one steady state, there is no bubble, \( h_t^E = 0 \) for all \( t \). At the other, \( h_t^E = h_t^{E*} = 1 - \frac{(1 - \psi)\alpha (1 + \beta^{-1})}{\psi} > 0 \), so there is a bubble and the bubbly steady state is a saddle point. Moreover, for any initial value \( h_0^E \in (0, h_t^{E*}) \), the economy will converge to a bubble-less steady state: \( \lim_{t \to \infty} h_t^E = 0.\(^{35}\)

Only when \( h_0^E = h_t^{E*} \), is the bubble sustainable in the sense that the equilibrium path will converge to the bubbly steady state in the long run.

Alternatively, we can show the dynamics of the system using a phase diagram of \( \hat{p}_t^H \) and

---

\(^{34}\)Such equilibrium paths, according to Tirole (1985), are called asymptotic bubble-less.

\(^{35}\)On the other hand, for any initial value \( h_0^E > 1 - \frac{(1 - \psi)\alpha (1 + \beta^{-1})}{\psi} \equiv h_0^{E} \), the system will explode and violate the transversality condition, so it cannot be an equilibrium.
Appendix A shows that in this economy, there is a unique saddle path for $\hat{p}_t^H$ and $\hat{k}_t^E$. For any initial level of an E-firm's capital $\hat{k}_0^E$, when $\hat{p}_0^H = \hat{p}_0^H \equiv h_0^{E*} \frac{\psi^{E*}}{(1+\beta^{-1})(1-\psi)} \hat{k}_0^E$, the economy will converge to the bubbly steady state at which $\rho_{E*} = (1 + z)(1 + \nu)$. For any other $\hat{p}_0^H = [0, \hat{p}_0^H)$, the economy converges to a steady state without bubbles. A unique feature of our model is that the saddle path is linear during the transition stage due to its AK feature. This generates a constant faster-than-income growth rate of the bubble (housing prices).

To achieve the bubbly equilibrium, it is crucial for entrepreneurs in each period to expect a particular sequence of housing prices that converge to the bubbly steady state. This expectation is self-fulfilled in each period by the expected high future demand for housing, which is rationalized by the fact that the future rate of capital returns, $\rho_{E}^t$, will be sufficiently low in the post-transition stage. Under such an expectation, holding housing today can yield high capital gains tomorrow even if housing has no intrinsic value. Accordingly, it is rational for the currently young entrepreneurs to invest in housing even if they live only for a finite number of periods.

Therefore, we assume that the economy starts with an initial portfolio share of housing assets $h_0^E = 1 - \frac{(1-\psi)(1+\beta^{-1})}{\psi}$, which gives the following optimal portfolio share in physical capital:

$$\phi_t^E = \frac{\alpha (1 + \beta^{-1})(1 - \psi)}{\psi}, \forall t.$$  

Equation (23) suggests that the entrepreneurs’ portfolio share in housing assets is constant along the transition path. This provides a testable implication of our model.

Although the short length of the household survey data in China (starting from 2011) prevents a direct test of this implication on the share of entrepreneurial saving in housing, the empirical evidence provided in Table 2 suggests that the share of real estate in total fixed assets for non-real estate firms that invest in housing property (unrelated to their original business) is very stable over time: about 14% to 15% between 2007 and 2013. Such a stable pattern of portfolio share is consistent with our model’s prediction about the dynamics of entrepreneurial portfolios in housing assets.
3.2.3 The Growth Rate of Housing Prices Relative to Output

**Lemma 1** The growth rate of housing prices is equal to the growth rate of the output of E-firms in both transition and post-transition stages.

\[
\frac{P^H_{t+1}}{P^H_t} = \frac{Y^E_{t+1}}{Y^E_t}, \forall t. \tag{24}
\]

The intuition for Lemma 1 is as follows. In both the transition and post-transition stages, the optimal portfolio choices by entrepreneurs will equalize the returns to capital investment and the returns to bubbles through arbitrage. In this simple economy, the equilibrium portfolio \( \phi^E_t \) is a constant, as the wedge between \( K^E_{t+1} \) and \( \rho^E_t K^E_t \) in the fundamental equilibrium is constant. This gives \( K^E_{t+1} = \rho^E_t K^E_t \) in the bubbly equilibrium. Accordingly, the growth rate of the total output of E-firm equals the rate of returns to capital for entrepreneurs, which in turn equals the growth rate of housing prices according to the no-arbitrage condition.\(^{36}\)

With Lemma 1, the following proposition captures the growth rate of housing prices relative to that of aggregate output.

**Proposition 1** Denoting \( \Delta X_t \) as the growth rate of \( X_t \), the growth rate of housing prices exceeds that of aggregate output during the transition, and it converges to that of aggregate output when the transition ends. Specifically,

\[
\Delta \log P^H_{t+1} = \Delta \log Y_{t+1} + \Delta \log \frac{Y^E_t}{Y^E_t + Y^F_t}. \tag{25}
\]

Equation (25) implies that the gap between the growth rate of housing prices and that of aggregate output equals the growth rate of the E-firms’ output share in aggregate output. During the transition, the aggregate output growth is a weighted average of the output growth of the E-firms and F-firms. Since F-firms keep downsizing because of labor reallocation, this sector’s output growth rate is less than that of E-firms, implying a lower growth

\(^{36}\)More formally, the growth rate of total E-firm output is given by

\[
\frac{Y^E_{t+1}}{Y^E_t} = \frac{Y^E_{t+1}}{K^E_{t+1}} \frac{K^E_{t+1}}{K^E_t} \frac{K^E_t}{Y^E_t} = \frac{\rho^E_{t+1}}{\rho^E_t} \frac{\rho^E_t (1 - \psi)}{\rho^E_t} \frac{1 - \psi}{\rho^E_t} = \rho^E_{t+1}.
\]
of the aggregate economy than that of E-firms.\textsuperscript{37} Therefore, housing prices, which increase at the rate equal to that of the E-firms’ output according to Lemma 1, will grow faster than the growth rate of the economy at this stage. In the post-transition stage, the economy becomes essentially neoclassical: The growth rate of aggregate output equals the growth rate of E-firms’ total output. As a result, the housing price grows at the same rate of aggregate output, even before reaching the steady state.

Furthermore, standard algebra shows that E-firms’ output share in total output is given by

\[
\log \frac{Y^E_t}{Y^E_t + Y^F_t} = \log \frac{N^E_t/N_t}{1 - \psi + \psi N^E_t/N_t}.
\] (26)

Therefore, together with Proposition 1, equation (26) implies that the growth rate of housing prices relative to that of aggregate output depends positively on the growth rate of E-firms’ employment share, \(N^E_t/N_t\). Note that during the transition, the growth rate of E-firms’ employment share is constant due to the constant growth rate of the E-sector’s capital stock as implied by the AK feature.\textsuperscript{38} This implies a persistently higher growth rate of housing prices than that of aggregate output. For a similar composition effect, the aggregate rate of return to capital increases during the transition, despite the constant capital returns in both E-firms and F-firms.\textsuperscript{39}

The key to delivering a prolonged faster-than-GDP housing price growth rate in our model economy is the presence of a prolonged transition stage featured by labor reallocation from unproductive to productive (but financially constrained) firms. During this transition, the entrepreneurs’ high rate of return to capital is sustained by the existence of surplus labor in the F-sector and is prolonged by borrowing constraints in the E-sector. Cheap surplus labor is expected to be exhausted only after the transition ends when the diminishing returns to

\textsuperscript{37}More formally, the growth rate of F-firms’ total output follows

\[
\frac{Y^F_{t+1}}{Y^F_t} = \frac{RK^F_{t+1}/\alpha}{RK^F_t/\alpha} = \frac{K^F_{t+1}}{K^F_t}.
\]

\textsuperscript{38}More formally, the growth rate of E-sector’s employment share is

\[
\frac{N^E_{Et+1}/N_{t+1}}{N^E_{Et}/N_t} = \frac{K^E_{Et+1}/N_{t+1}}{K^E_{Et} (1 + z) /N_t} = \frac{\rho^E_t}{(1 + z)(1 + \nu)}.
\]

\textsuperscript{39}More formally, the aggregate rate of return to capital is computed as

\[
\rho_t = \frac{\rho^E K^E_t + \rho^F K^F_t}{K^E_t + K^F_t} = \frac{R}{1 - N^E_t/\chi_t \left[ 1 - \chi ((1 - \psi) \chi)^{-1/\lambda} \right]}. \]
capital start to take effect. It is precisely this trajectory of returns to capital that entices the productive agents (entrepreneurs) to invest in the housing market and become the marginal buyers. The unproductive agents (workers or F-firms) can also invest in housing (which yields much higher returns than bank deposits), but they will not be the marginal buyers.

This is in contrast to the results of Martin and Ventura (2012). In their paper, the marginal buyers of bubble assets are the unproductive agents who face an extremely low rate of return to capital (as in the traditional bubble literature). Consequently, their model predicts a slower-than-GDP growth in the housing bubble, which is inconsistent with the Chinese data. Moreover, in their paper, bubbles improve allocative efficiency by crowding in investment of the productive firms—which is again inconsistent with the Chinese data in Section 2. In contrast, as the next section shows, bubbles in our model can worsen the allocative efficiency by crowding out investment of the productive firms—a serious concern of the Chinese government for many years.

We also check the robustness of our model’s predictions for housing price growth by extending our benchmark economy to alternative settings where (i) housing services are valued; (ii) the economy consists of both labor-intensive and capital-intensive sectors such that the labor share of SOEs does not converge to zero; (iii) entrepreneurs can borrow against housing, and (iv) housing bubbles are stochastic. We show analytically that in each of these alternative setups, the model robustly predicts that housing prices grow faster than aggregate income during the economic transition.\footnote{These results are available upon request.}

### 3.2.4 Economic Consequences of a Growing Bubble

An interesting issue is the normative implication of bubbles in our model. Because bubbles can exist in our model without dynamic inefficiency, they may reduce, rather than increase, social welfare. We now explore the welfare implications of growing bubbles in detail.

We first study the implications of bubbles for aggregate consumption in both the transition and post-transition stages. With condition (20), the law of motion for capital (9) implies that in the fundamental equilibrium, at each period \( t \),

\[
K_{t+1}^E < MPK_t^E \cdot K_t^E. \tag{27}
\]

In other words, investment in bubbles is not optimal for the social planner, despite the incentive of entrepreneurs to invest in housing. The reason is simple: Given the sufficiently
high marginal product of E-firms’ capital, housing bubbles reduce the resource available for aggregate consumption by crowding out productive investment.\footnote{A similar wedge between social and private rates of return for capital occurs in the endogenous growth models of Grossman and Yangawa (1993) and King and Ferguson (1993), in which the labor productivity of individual firms depends positively on the aggregate stock of capital.}

We now analyze the effect of bubbles on aggregate consumption. To allow bubbles to reduce total entrepreneurial consumption, we further assume

\[
\psi < \alpha (1 + \beta^{-1}) [\psi + \alpha (1 - \psi)].
\]  

(28)

Note that \(\psi + \alpha (1 - \psi)\) is the share of E-firms’ output accrued to young and old entrepreneurs. Since \(\psi + \alpha (1 - \psi) < 1\), the inequality (28) is sufficient for the condition of dynamic efficiency, (20), to hold.\footnote{A combination of (15) and (28) implies \((1 - \psi) (1 - \alpha) < \psi\), which is guaranteed by \(m_t \geq w_t\), the necessary condition for the young entrepreneur to work as a manager rather than a worker.} To derive the effects of bubbles on each type of agent, we define the period-\(t\) aggregate consumption of agent type-\(j\) \(\in \{w, E\}\) as \(\bar{c}_t^j \equiv \bar{c}_{1,t}^j + \bar{c}_{2,t}^j (1 + \nu)^{-1}\). We have the following proposition:

\textbf{Proposition 2} Given that (15) and (28) are both satisfied, a housing bubble reduces aggregate consumption and welfare of both entrepreneurs and workers.

The intuition is as follows. In addition to the forgone capital returns, entrepreneurial housing investment reduces the lifetime income of future entrepreneurs and, thus, generates a negative externality on their consumption.\footnote{Given the initial capital stock and constant returns to capital, the permanent incomes of the old and young entrepreneurs alive in the first period are unchanged when a housing bubble is introduced.} It is easy to show that during the transition, since the rate of return to capital is constant, a reduction in lifetime income reduces the entrepreneur’s lifetime utility. In Appendix B, we also show that bubbles reduce the entrepreneurial lifetime utility at the steady state. For entrepreneurs born during the post-transition stage, a sufficient condition for welfare loss is \(\alpha \left(1 + \beta^{-1}\right) > 1 - \alpha^2\).

Regarding the impact of housing bubbles on workers’ consumption, note that the wage rate, a constant along the transition, is unaffected by the presence of a bubble during the transition. Hence, the welfare of workers during the transition is unaffected by the bubble. However, when the transition ends, their lifetime utility decreases as a result of the housing bubble. This is because the workers’ wage income starts to depend positively on E-firms’ capital stock, while the rate of return to saving (the deposit rate) is still fixed.

The next question is whether it is desirable to burst a bubble once it exists, given that bubbles crowd out productive investment. The answer is no. In our economy, the housing
bubble serves as a store of value that enables the young entrepreneurs to finance retirement consumption when old. Eliminating the bubble will therefore erode the retirement wealth of the old entrepreneurs who happen to hold housing at the time the bubble bursts. To ensure that no household is left worse off after the housing bubble bursts, the policymaker needs to compensate these old entrepreneurs for their losses—say, by issuing government bonds to the current-period young entrepreneurs. But the resources needed to compensate old entrepreneurs are the same resources that would have been released by the young entrepreneurs for capital accumulation. This implies that the policymaker is simply substituting another form of a bubble for housing without crowding in productive investment. Such a policy dilemma may explain why the Chinese government has been so reluctant to levy aggressive property taxes to burst the housing bubble, despite its rapid growth and sheer size, as well as its apparent crowding-out effects on capital investment. To support our argument, in our quantitative exercise below, we show that a bursting housing bubble would reduce the welfare for the majority of cohorts alive at the time of the burst.

4 Quantitative Analysis

This section brings the model to the data. To facilitate calibrations, we first extend our two-period benchmark model to a multi-period model. In the model agents live for $J$ periods, are born with zero wealth, and cannot die with negative wealth. Workers supply one unit of labor each period. They retire after $J^R$ years of work. Young entrepreneurs work for the old entrepreneurs in the first $J^E$ periods of life. For simplicity, we assume that an age-$j$ ($j < J^E - 1$) young entrepreneur can only make deposits in the bank with a fixed interest rate $R$. From age $J^E - 1$ on, she can have a portfolio choice by purchasing housing or investing in her own business. In this economy, we assume the capital depreciation rate $\delta < 1$.

4.1 The Quantitative Multi-period Model

The F-firm’s problem is similar to that in the benchmark model:

$$
\max_{k_t^F, n_t^F} \left( k_t^F \right)^\alpha \left( A_t n_t^F \right)^{1-\alpha} - w_t n_t^F - R^l k_t^F + (1 - \delta) k_t^F.
$$

(29)

For calibration purposes, we assume that lending to an F-firm is subject to a constant iceberg cost $\xi$, which represents the intermediation cost. In equilibrium, the lending rate for F-firms is $R^l = R / (1 - \xi)$.
An age-$j$ old entrepreneur in time $t$ solves the following problem:

$$
\pi (k_{j,t}^E) = \max_{n_{j,t}^E} (1 - \tau_t^y) (1 - \psi) \left( k_{j,t}^E \right)^\alpha \left( (1 - \alpha) (1 - \psi) A_t x n_{j,t}^E \right)^{1 - \alpha} - w_t n_{j,t}^E + (1 - \delta) k_{j,t}^E,
$$

(30)

where $k_{j,t}^E$ and $n_{j,t}^E$ denote the capital and labor deployed by an age-$j$ old entrepreneur at period $t$. We can derive the rate of returns for E-firms’ capital as

$$
\rho_t^E \equiv \frac{\pi (k_{j,t}^E)}{k_{j,t}^E} = \alpha (1 - \psi) (1 - \tau_t^y)^\frac{1}{2} \left( (1 - \alpha) (1 - \psi) A_t x / w_t \right)^{\frac{1}{2 \alpha}} + 1 - \delta.
$$

(31)

Note that, despite the heterogeneity in capital stock, the rate of return to capital is the same for all entrepreneurs alive in period $t$ under the Cobb-Douglas production function.

For calibration purposes, we assume that the production of E-firms is subject to a time-varying output wedge $\tau_t^y$. The purpose of introducing this wedge is to target the time path of the private employment share in China.\textsuperscript{44} Such an output wedge may capture, in reality, the preferential or distortionary policy toward private firms. For example, in the early stage of privatization, the Chinese government provided various supports (e.g., credits, tax deductions) to private firms, which encouraged their fast growth.\textsuperscript{45} This would show up as an implicit output subsidy to E-firms ($\tau_t^y < 0$). Over time, however, such preferential policies have started to be replaced by various government policies that restrict the growth of private firms (e.g., entry barriers for private firms into “strategic” industries and a heavy tax burden), which had contributed to the so-called “Guo Jin Min Tui” (the state advances, and the private sector retreats).\textsuperscript{46} This would show up in our model as an increase in the value of $\tau_t^y$. Since, in reality, government policies affect the overall profitability of private firms, we assume that such an output wedge also applies to young entrepreneurs’ managerial compensation.

\textsuperscript{44} Quantitatively, without the output wedge, housing prices still grow faster than GDP during the transition stage. However, the increase of the private employment share will slow down. Accordingly, the simulated housing prices and housing price-to-GDP ratio grow at a slower rate than their benchmark counterparts.

\textsuperscript{45} For example, on June 29, 2002, the Ninth National People’s Congress Standing Committee passed the Law of the People’s Republic of China on Promotion of Small and Medium-Sized Enterprise, which was implemented on January 1, 2003. In 2005, the State Council issued “Several Opinions of the State Council on Encouraging, Supporting and Guiding the Development of Individual and Private Economy and Other Non-Public Sectors of the Economy,” also called the “36 items for the non-public economy,” to support the development of private enterprises via preferential credit and tax policies.

\textsuperscript{46} For example, in 2007, the state government issued a document (the 39th Decree), which requests a transition from preferential corporate income tax rates to legal tax rates. According to this document, those who enjoyed a 15% corporate income tax rate before 2008 would have tax rates of 18%, 20%, 22%, 24%, and 25% for each year between 2008 and 2012, respectively.
For a worker of age $i$ in period $q$, his problem for the remainder of his life is
\[
\max \sum_{j=i}^{J} \beta^{j-i} \log c_{j,t}^{w}
\] (32)
subject to
\[
c_{j,t}^{w} + s_{j,t}^{w} = w_{t} + R s_{j-1,t-1}^{w}, \quad \text{for } j < J^{R}
\] (33)
\[
c_{j,t}^{w} + s_{j,t}^{w} = R s_{j-1,t-1}^{w}, \quad \text{for } j \geq J^{R}
\] (34)
\[
s_{j,t}^{w} = 0, \quad s_{0,t-1}^{w} = 0,
\] (35)
where the subscript $t \equiv q + j - i$ is the calendar time for the age-$i$ agent to become age $j$.

An entrepreneur of age $i$ in period $q$ has the following consumption-saving problem
\[
\max \sum_{j=i}^{J} \beta^{j-i} \log c_{j,t}^{E}
\] (36)
subject to
\[
c_{j,t}^{E} + s_{j,t}^{E} = m_{t} + R s_{j-1,t-1}^{E}, \quad \text{for } j < J^{E} - 1
\] (37)
\[
c_{j,t}^{E} + s_{j,t}^{E} = \rho_{t} s_{j-1,t-1}^{E}, \quad \text{for } j \geq J^{E} - 1
\] (38)
\[
s_{j,t}^{E} \geq 0 \quad \text{for } j \geq J^{E} - 1
\] (39)
\[
s_{j,t}^{E} = 0, \quad s_{0,t-1}^{E} = 0.
\] (40)

Here again, for the no-arbitrage condition to hold, we assume that the inner solution of entrepreneurial portfolio choice exists. Given the savings, $s_{j,t}^{E}$, the age-$j$ entrepreneur at period $t$ then makes the portfolio choice $\phi_{j,t}^{E}$.

**Proposition 3** There exists an equilibrium in which all entrepreneurs alive in period $t$ will invest the same share of wealth in housing; that is, $\phi_{j,t}^{E} = \phi_{t}^{E}$, for $j \in [J^{E} - 1, J - 1]$.

Proposition 3 allows us to derive the following equations for aggregate capital and housing stock in equilibrium.
where $N_{j,t}^E$ denotes the number of entrepreneurs of age $j$ at period $t$. Therefore, we can solve the aggregate labor demand and portfolio choices by aggregation. Appendix C describes the numerical algorithm to solve for the calibrated economy.

### 4.2 Calibration

We use data from the NBSC to calibrate the model. The model economy starts in 1998, when China started to privatize its SOEs. Each period in our model corresponds to one calendar year.

Consider the first set of parameters, whose values are set exogenously. Agents in our model enter the economy at age 22 and live for 50 years. This is consistent with an average life expectancy for males and females of 71.4 years according to the 2000 Chinese Population Census. Workers retire after 30 years. The population growth rate is set to $\nu = 0.03$, consistent with the average urban population growth rate in China during 2002-2012.\(^{47}\)

In terms of technology parameters, the capital income share is set to $\alpha = 0.5$, consistent with Bai, Hsieh, and Qian (2006).\(^{48}\) The capital depreciation rate is set to $\delta = 0.1$, which is the average depreciation rate between 1998 and 2012, computed using the method of Bai, Hsieh, and Qian (2006). The land supply is normalized to $\overline{H} = 1$, and the bank deposit rate is set to $R = 1.0175$, following SSZ (2011). Finally, the initial assets of the workers and retirees are set to be the same as the corresponding wealth in an initial steady state with only F-firms.

Now we turn to the second set of parameters, whose values are set endogenously to target certain data moments. We calibrate $\beta = 0.994$ to match the average aggregate saving rate

\begin{align}
K_{t+1}^E &= \phi_t^E \sum_{j=j^E-1}^{J-1} N_{j,t}^E s_{j,t}, \\
P_t^H H_t^E &= (1 - \phi_t^E) \sum_{j=j^E-1}^{J-1} N_{j,t}^E s_{j,t},
\end{align}

\(^{47}\)Urban population growth could be endogenously driven by rural-urban migration, which in turn could be an outcome of labor reallocation from SOEs to POEs. We view this as an interesting extension for future research.

\(^{48}\)Since in our model housing does not provide services and is in fixed supply, our measured capital stock corresponds to the concept of total reproducible capital stock, which we constructed following the method of Bai, Hsieh, and Qian (2006).
between 1998 and 2012 of 40%. We then set $\psi = 0.53$ to match the aggregate rate of returns to capital at 1998, which is 20%. Following SSZ, we calibrate the productivity parameter of E-firms to be $\chi = 5.64$ to match the following moment: The capital-to-output ratio of Chinese SOEs is 2.65 times that of domestic private firms. The iceberg cost $\xi$ is set to 0.0693 to match the marginal product of capital (MPK) for SOEs to be 0.093. The rate of labor-augmented technological growth is set to $z = 3.8\%$ to match the average growth rate of GDP of 10% during 1998-2012.

The initial entrepreneurial wealth is set to match an initial employment share of private firms at 1998. According to NBSC data, the employment share of private firms (DPE + FE) in total employment is 17% in 1998. Accordingly, the initial life-cycle distribution of wealth for managers and entrepreneurs is a scaled-down version of the life-cycle distribution of wealth for workers in the initial steady state. For the time path of output wedges, we assume a linear pattern between $y_{1998}$ and $y_{2012}$, such that $y_t = y_{2012} - (2012 - t) \kappa$ for 1998 $\leq t \leq 2012$. For $t \geq 2013$, we assume $y_t = 0$. We then calibrate $y_{2012}$ and $\kappa$ to best fit the trajectory of the private employment share between 1998 and 2011. This gives $\kappa = 0.027$, $y_{2012} = 0.01$.

### 4.3 Main Results

To assess the model’s performance, we compare the simulated housing prices with both the annualized constant-quality housing price indices between 2006 and 2010 and the annual constant-quality national-level land price indices by Wu, Gyourko, and Deng (2012). The land price data cover the period of 2004-2012 and were updated on Gyourko’s webpage. Since the land price tracks the housing price closely (see Figure 1), we view the land price as a good proxy for the housing price for those years with missing housing price data. To illustrate the growth rate of the housing price relative to that of GDP, we also construct the ratio of housing prices to GDP in both the model and the data. We then normalize the simulated and actual housing prices and housing price-to-GDP ratio in 2004 to 1.

Figure 6 shows the main predictions of the model, together with their data counterparts. In Panel A, we see that the simulated housing prices replicate the actual data fairly well until 2011, with an average growth rate of 19% between 2004 and 2011. The over-prediction

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49The two most authoritative sources of housing price data are Wu, Gyourko, and Deng (2014) and Fang et al. (2015). Neither of these two data sources allow us to go back earlier than 2003. Other publicly available data for housing prices, such as the NBS average price index (total revenue of housing sales/floor area of housing sales), are severely biased downward (Wu, Deng, and Liu, 2014); hence, they are not reliable for our purposes. The year 2004 may be a good starting point for another reason: Public auctions or bidding for land prices were non-existent before 2002.
of housing prices in 2012 may be due to the fact that since 2010, the Chinese government has adopted several policies to control housing prices, from which our model is abstracted. Panel B shows that the simulated housing price-to-GDP ratio increases from 1 in 2004 to 1.77 in 2012, or an average of 7.43% per year. Hence, our model can replicate the magnitude of the increase in housing prices relative to GDP reasonably well. We view this result as further support of our mechanism for the housing price dynamics in China, since labor reallocation from F-firms to E-firms is key to delivering a sustained high return to E-firms’ capital, not only in absolute terms, but also relative to the growth rate of aggregate output.

Panel C of Figure 6 shows that our model can closely match the dynamics of the private employment share. Note that as time approaches the year 2012, the increase of the private employment share starts to slowdown. This is because entrepreneurs slow their accumulation of physical capital in anticipation of an increase in implicit output distortion.

In Panel D of Figure 6, the aggregate rate of returns to capital is persistently high between 1998 and 2012. It starts to decline around 2008, which coincides with the turning point of the data. Two opposite effects in the calibrated economy drive the dynamics. On the one hand, the increase of the private employment share increases the average returns to capital, thanks to a higher E-firm rate of return to capital. On the other, the increase in implicit output distortion on E-firms during this period tends to reduce their net return to capital.

Panel E of Figure 6 shows that the aggregate output growth in our model replicates China’s GDP growth reasonably well: It is sustained around 10% between 1998 and 2012. Again, the observed hump shape of output growth rates is due to the aforementioned two opposite effects.

Panel F of Figure 6 plots the dynamics of aggregate TFP. Between 1998 and 2006, the average TFP growth rate is 6.13 percent. This is in line with the estimation of Brandt, Van Biesbroeck, and Zhang (2009), who report an estimate for manufacturing sector TFP growth of 6.1% to 7.7% during 1998-2006. Resource reallocation contributes 4.23% to annual TFP growth. Therefore, 69% of the TFP growth between 1998 and 2006 in our model is due to resource reallocation, which is broadly consistent with the findings of Brandt, Van Biesbroeck, and Zhang (2009). A falling TFP growth rate in the second half of our sample period is due to an increase in the implicit output distortion toward E-firms, which, in turn, slows the pace of labor reallocation.

4.4 Counterfactual Experiments

We now conduct several counterfactual experiments to shed additional light on the structure of our model.
Heterogeneity. A standard representative-agent neoclassical growth model also features a transitional period of capital accumulation before reaching the steady state, along which the MPK declines gradually. Would this declining MPK along the neoclassical transition generate a faster-than-GDP growth in housing prices, everything else equal? The answer is no. To illustrate this, we exclude F-firms in our model so the counterfactual economy is essentially neoclassical with only E-firms and without the transition stage featuring “surplus” labor and resource reallocation—the focus of our model. In this counterfactual economy, the parameters $\chi$ and $\hat{k}_t^E$ measure the aggregate, rather than E-firm-specific, productivity and capital stock, respectively. Therefore, we set $\hat{k}_t^E$ to be the same as the initial aggregate capital stock in the benchmark economy. We then recalibrate $\chi$ to target an initial aggregate rate of return to capital of 0.20. We keep all other parameters the same as in our benchmark economy. Figure 7 shows the simulated results in this counterfactual economy, together with their counterparts in the original model. Without firm heterogeneity, the private employment share always equals 100% (Panel C). Panel A shows that in this counterfactual economy, housing prices grow significantly slower than the benchmark counterparts. Two reasons explain this: First, in this counterfactual economy, the growth rate of housing prices equals the aggregate rate of return to capital. Second, the aggregate rate of return to capital falls over time along with capital accumulation (Panel D). In contrast, in our benchmark economy, the rate of housing price growth equals the rate of return to E-firm’s capital, and the AK feature of E-firms helps to sustain the high capital returns during the transition. Accordingly, in this counterfactual economy, the housing price relative to GDP is essentially flat (Panel B), despite a declining output growth rate (Panel E). Intuitively, without firm heterogeneity, the dynamics of aggregate output growth closely track the dynamics of the aggregate return to capital, which implies that housing prices grow at a rate similar to that of aggregate output.50

The Crowding-Out Effects of the Housing Bubble. We now explore the normative implications of housing bubbles. To this end, we shut down entrepreneurs’ access to the housing bubble by setting $\hat{\phi}_t^E = 1$ for all $t$. Accordingly, without demand the housing prices will always equal zero. All parameters remain the same as in the original calibration. Figure 8 plots the transition path for both the counterfactual and the original economies. Panel A shows that without housing bubbles, the private employment share rises much faster. For example, by 2011, the private employment share is around 77%, 9% higher than the benchmark.

50 When physical capital is not fully depreciated, the growth rate of E-firms’ total output will be somehow less than the aggregate or entrepreneurs’ returns to capital, because the growth rate of entrepreneurs’ wealth is decreasing over time before the economy reaches the steady state.
counterpart. This implies that the presence of a housing bubble prolongs the economic transition. Panel B shows that without housing bubbles, aggregate output grows faster between 1998 and 2011, with an average growth rate of around 11%. Accordingly, the steady-state aggregate output is 6.23% higher than its counterpart in the bubbly equilibrium. Panel C shows a similar pattern for the TFP growth rate. Between 1998 and 2011, TFP grows at an average rate of 6.05% per year, in contrast to the rate of 5.51% in our benchmark economy. Such a disparity suggests that housing bubbles exacerbate resource misallocation by crowding out productive investment, slowing labor reallocation from unproductive firms (F-firms) to productive firms (E-firms), thus resulting in permanently lower aggregate productivity and efficiency. Panel D suggests a significant welfare loss due to the presence of a housing bubble. Between 1998 and 2012, aggregate consumption would be 6.35% higher without the housing bubble. Even at steady state, aggregate consumption is 3.75% higher in the fundamental equilibrium. Intuitively, by crowding out productive capital investment, housing bubbles reduce the permanent incomes of future cohorts via both managers’ compensation and the wage rate.

The Housing Bubble Burst. Since the bubble is welfare reducing, it is tempting to conclude that it is welfare improving to burst the bubble after it emerges. This intuition is not entirely correct. To show this, we conduct an experiment with an unexpected bubble burst in year 2012 in our model (the last period of our data sample) and compute the welfare effects on both workers and entrepreneurs alive in 2012. Implicitly, the welfare results in such a counterfactual experiment could be used in a political economy model under a majority voting scheme among existing cohorts regarding whether to burst the housing bubble. The results of the experiment are presented in Figure 9. Interestingly, all workers born before 1990 in our economy (which corresponds to cohorts born in year 1969 or earlier in the data) do not experience welfare change. This is because these cohorts of workers retire before the bubble burst. Hence, their lifetime income, which is the present value of the wage rate, is unchanged. Workers born after 1990 experience welfare gains because they enjoy an increase in the wage rate during their working periods when the economy enters the post-transition stage after the bubble bursts. The younger workers are, the higher the welfare gain due to their longer working period.

In contrast, all entrepreneurs alive in 2012 suffer welfare losses. Specifically, the old entrepreneurs (aged 26 and older in our model) not only suffer a loss of housing wealth, but also miss the increase in managerial compensation due to the burst of the bubble. For the young entrepreneurs, two offsetting welfare effects are present. On the one hand, the burst of the bubble increases the compensation of the young entrepreneurs for their remaining
working periods as managers, thus increasing their lifetime income. On the other hand, the event decreases the rate of return to E-firms’ physical capital in the post-transition stage (which starts in the year 2019 in our model). Our quantitative results suggest that the welfare losses dominate the welfare gains for the young entrepreneurs alive in 2012.

To sum up, the majority of agents who are currently alive (workers plus entrepreneurs) will not benefit from the bubble’s burst. This is despite the fact that in the long run, all newborn entrepreneurs and workers enjoy welfare gain. This welfare result provides a rationalization for why the Chinese government has been reluctant to burst the housing bubble, even though doing so is welfare improving in the long run.

4.5 Further Discussions of Model Implications

While the focus of our paper is China, our model can shed light on the occurrence of housing bubbles in other emerging economies during their rapid-growth transition periods. The highlight of our theory is that economic transitions driven by massive resource reallocation must eventually end and that the associated high capital returns are thus unsustainable in the long run. Such transition economies are prone to bubbles because such a dynamic path of capital returns can induce even the most productive agents in the economy to seek alternative stores of value for their rapidly growing wealth. In financially backward economies without sophisticated regulatory institutions, land or the housing market often becomes the natural target of speculative investment. Indeed, in other East Asian economies during their earlier stages of industrialization and rapid economic growth, such as South Korea and Taiwan in the 1980s, intersectoral labor reallocations from agriculture to manufacturing (or the service sector) contributed importantly to their fast economic growth, and the high capital returns sustained by resource reallocation have been accompanied simultaneously by faster-than-income growing housing prices.\footnote{SSZ (2011) emphasize the resource reallocation to financially constrained private firms within the manufacturing sector as a key reason for the coexistence of acceleration of productivity growth and a foreign surplus in Korea and Taiwan in the 1980s. We view the intersectoral labor reallocation as essential for the housing booms experienced in these two countries during the 1980s.}

Also, Vietnam has experienced a transition stage similar to that in China. Therefore, a brief analysis of these economies’ housing booms serves as a useful examination of the relevance of our theory for other emerging countries.

In South Korea, land/housing prices almost tripled during 1985-1991, with an annual growth rate of 21.5% on average.\footnote{See Koo and Park (1994, Table 3.9) and Kim and Lee (2000). Also, according to these two studies, between 1980 and 1985, land prices increased by only 60%. After 1992, both land and housing prices leveled off until the 1997-98 financial crisis.} This is in contrast to an average real GDP growth rate of 12.5% during the same period. Interestingly, this housing boom coincided with important
structural changes in the Korean economy. In the 1980s, Korea experienced a massive reallocation of labor from agricultural to other sectors. The share of agricultural employment dropped by 20% between 1980 and 1992, compared with a mere 6% drop between 1992 and 2007 (Lee, 2010). Labor costs had been rising significantly since the mid-1980s, reflecting a pending shortage of labor (Smith, 2000, Figure 3.3). While the rate of increase slowed by the early 1990s, the real average monthly earnings in manufacturing still grew by an average of 7.8 per year between 1992 and 1996, while productivity growth lagged. This increase in labor costs increasingly undermined Korean firms’ competitiveness, which led to a rapid increase in relative export prices during this period (Smith, 2000, Figure 3.2). Such an increase in labor costs is consistent with the dynamics of the rate of return to capital for Korea. As shown by Panel A of Figure 10, the rate of return to capital was very high and increased for most of the 1980s before it started to fall in the late 1980s.\footnote{Nugent and Yhee (2002, Table 6) obtained a similar finding for the dynamics of capital productivity, especially for small and medium-sized enterprises. Specifically, capital productivity, measured by the ratio of gross value added to total assets, is more than 35 percent for small and medium-sized enterprises and more than 24 percent for large enterprises during 1985-1991. This is in contrast to a declining pattern of capital productivity for both types of firms between 1992 and 1997.}

Similar to South Korea’s experiences, the housing boom in Taiwan occurred during a time of massive labor reallocation and fast economic growth. According to Koo and Park (1994), Taiwan witnessed a sharp increase in land prices in the second half of the 1980s. Accordingly, average housing prices in the Taipei area more than quadrupled from late 1986 to early 1990 (implying a more than 40% annual growth rate of housing price during this period).\footnote{See Chen (2001, Table 1) for the data on the real housing price growth rate for Taipei during the late 1980s, and Tsai and Peng (2011) for evidence on the housing market boom for other major cities in Taiwan during this period.} During the same period, real GDP grew at an annual rate of 9.1%. At the micro level, Taiwan experienced a fast reallocation of labor from agriculture to manufacturing and service sectors since the 1970s. The share of agricultural employment dropped from 25% in 1978 to 13% in 1989. A labor shortage had gradually become apparent in the manufacturing sector since the late 1970s and more so after the mid-1980s. Labor costs rose as a result of this labor shortage. The average monthly real wage, which increased by 6.5% per year during 1981-1986, increased by 11.4% per year between 1986 and 1990.\footnote{Similar to Korea, Taiwan also experienced a sharp increase in the relative export prices in the second half of the 1980s (Smith 2000, Figure 3.2).} Accordingly, Panel B of Figure 11 shows that Taiwan’s returns to capital peaked in 1986-87 and declined thereafter.

Similar to China, Vietnam has witnessed a massive labor reallocation from SOEs toward the private sector in the past decade following a series of reforms (e.g., the establishment of the 2000 Enterprise Law). For example, within the manufacturing sector, the share of
workers employed by SOEs has declined from 0.305 in 1999 to 0.089 in 2009, while the share of workers employed by foreign-owned firms has increased from 0.052 to 0.224.\textsuperscript{56} Associated with Vietnam’s structural change is its fast economic growth. According to the International Monetary Fund, Vietnam’s average GDP growth was 7.2\% between 2000 and 2010—one of the fastest-growing economies in the world during this period.

Along with its economic transition, Vietnam’s real estate market has undergone remarkable changes in recent years. In 2003, the Vietnamese government enacted the Law on Land. This law casts the most significant reform of legal property rights in Vietnam’s history and paves the way for market-driven real estate prices in Vietnam. The real estate market in Vietnam experienced a boom between 2007 and 2010. For example, housing prices increased by 200\% between 2007 and the first half of 2008, and medium- to high-quality condominiums became one of the hottest asset markets, second only to Vietnam’s booming stock market. The incredible rise in housing prices, nonetheless, was believed to be the result of significant speculation rather than by changes in the fundamentals. Since the beginning of 2011, the real estate market in Vietnam has suffered huge losses in market value. Housing prices have dropped by 40 percent on average since then.

Therefore, despite important cultural and institutional differences, these three emerging economies all shared some common features in their development paths similar to China’s recent experience. Specifically, all three economies experienced faster-than-GDP growth in housing prices despite high capital returns during their respective economic transitions. Moreover, for both Korea and Taiwan, the completion of the transition process eventually led to rising labor costs and the cooling of housing bubbles. These features are consistent with the predictions of our theory.

5 Conclusion

This paper provides a framework to explain the coexistence of three paradoxical features of the great housing boom in China—the persistently faster-than-income growth rates of housing prices, the phenomenal capital returns, and the exceptional vacancy rates across major Chinese cities (the ghost-town phenomenon). Our theory suggests that China’s unprecedented income growth is not the full story behind the great housing boom. The decade-long housing boom contains a rational bubble arising naturally from China’s economic transition, which is featured by labor reallocation from the traditional low-productivity sector to the

\textsuperscript{56}See McCaig and Pavcnik (2013, Table 5). According to them, another important structural change during this period is the acceleration of reallocation of labor from the agricultural sector to the manufacturing and service sectors.
newly emerging high-productivity sector. Such labor reallocation sustains a very high rate of returns to capital in the emerging sector. Yet, the high capital returns will eventually come to an end when the surplus labor is depleted. Hence, rational expectations of high demand for alternative stores of value in the future can induce even the most productive current agents to speculate in the housing market, creating a self-fulfilling housing bubble that can grow much faster than aggregate income despite high capital returns. The model’s predictions are thus consistent (qualitatively and quantitatively) not only with China’s broad pattern of economic growth but also with the three paradoxical features of the great housing boom. We also show that such a growing housing bubble can crowd out productive capital investment, thus prolonging the economic transition and reducing social welfare.

A number of simplifying assumptions make our model tractable. For example, in our model housing does not provide utility services and workers do not participate in the housing market. Although we have argued that such omissions should not affect our main results, a richer model with both workers and entrepreneurs speculating in the housing market would enrich the welfare implications of growing housing bubbles. For example, a growing housing bubble distorts homeowners’ life-cycle consumption patterns under borrowing constraints by forcing people to save excessively when young to enter the housing market. Furthermore, the rapidly rising housing prices tend to worsen wealth inequalities across income classes, as the housing price growth is driven largely by the high- and upper-middle-income classes that have enjoyed the most rapid income growth during China’s economic development. In contrast, under borrowing constraints more and more low-income households are excluded from the housing market because their income growth lags behind housing price growth. Moreover, our model abstracts from several institutional details of China’s housing and land markets (e.g., a local government’s heavy reliance on revenue from land sales), which, we believe, might also be a contributing factor to the size (but not growth rate) of China’s housing bubble. These are all important issues for our future research.

Despite its simplicity, a calibrated version of our model quantitatively matches the growth dynamics of housing prices and other salient features of the recent Chinese experience reasonably well. We therefore view our model as a useful starting point to study the macroeconomic implications of the growing housing bubble in China. For example, a growing housing bubble reduces the private sector’s incentive to innovate. Because of the relatively low risk, low entry costs, low technology, and high profits in housing investment, the housing bubble has enticed many productive and high-tech firms in China to reallocate resources from research and development to the real estate market. In an economy in transition from a labor-intensive economy to a capital-intensive economy, such resource misallocation can be very costly: It may substantially prolong China’s economic transition and reduce China’s
TFP growth, especially when its population is aging fast and labor costs are rapidly rising. We plan to empirically validate and quantify such resource misallocation within our framework in future works.

References


Tables and Figures

Table 1. Excess Housing Price Growth and Capital Returns

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private capital returns</td>
<td>0.8153***</td>
<td>0.7101***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1725)</td>
<td>(0.1737)</td>
<td></td>
</tr>
<tr>
<td>SOE capital returns</td>
<td>0.7573***</td>
<td>0.5091**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2676)</td>
<td>(0.2124)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>139</td>
<td>139</td>
<td>139</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.2349</td>
<td>0.1111</td>
<td>0.2819</td>
</tr>
</tbody>
</table>

Note: *** Significant at the 1% level. ** Significant at the 5% level. Numbers in parentheses are standard errors. This table reports the results from our fixed-effects panel regression with excess housing price growth as the dependent variable and private and/or SOE capital returns as the independent variables. The estimation uses the robust or sandwich estimator of variance. The data are a balanced panel covering 35 major cities in China between 2006:Q1 and 2010:Q4.

Source: The data for the housing prices of the 35 cities are from Wu, Deng, and Liu (2014). The data for disposable income growth for the 35 cities are from the statistical communiques, various issues. The data for capital returns for various provinces and municipalities directly governed by the central government are from the China Statistical Yearbook (CSY), various issues.

Table 2. Summary Statistics for Firms with Investment Property

<table>
<thead>
<tr>
<th>Year</th>
<th>Total Number of Firms</th>
<th>Firms with IP</th>
<th>% of Firms with IP</th>
<th>Average IP/IP/(IP+PPE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>1373</td>
<td>609</td>
<td>44.36%</td>
<td>16.03%</td>
</tr>
<tr>
<td>2008</td>
<td>1489</td>
<td>662</td>
<td>44.46%</td>
<td>15.64%</td>
</tr>
<tr>
<td>2009</td>
<td>1534</td>
<td>699</td>
<td>45.57%</td>
<td>15.54%</td>
</tr>
<tr>
<td>2010</td>
<td>1681</td>
<td>732</td>
<td>43.55%</td>
<td>15.57%</td>
</tr>
<tr>
<td>2011</td>
<td>2027</td>
<td>858</td>
<td>42.33%</td>
<td>14.46%</td>
</tr>
<tr>
<td>2012</td>
<td>2254</td>
<td>872</td>
<td>38.69%</td>
<td>13.86%</td>
</tr>
<tr>
<td>2013</td>
<td>2249</td>
<td>926</td>
<td>41.17%</td>
<td>13.35%</td>
</tr>
</tbody>
</table>

Note: IP denotes Investment Property. This table provides the summary statistics for non-real estate publicly listed firms holding property assets for investment purposes. We restrict our sample to firms that have been traded for at least two years on the China A-share stock market over the period of 2007-2013. We exclude firms in the real estate and construction sectors.

Source: The firm level data are from CSMAR and authors’ calculation.
Table 3. Returns to Capital and Ownership Structure

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>State ownership</td>
<td>-1.02734**</td>
<td>-0.13056*</td>
<td>-0.49989***</td>
</tr>
<tr>
<td></td>
<td>(-9.32)</td>
<td>(-1.64)</td>
<td>(-8.98)</td>
</tr>
<tr>
<td>Observations</td>
<td>10957</td>
<td>10957</td>
<td>10957</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.2573</td>
<td>0.2521</td>
<td>0.2547</td>
</tr>
</tbody>
</table>

Note: This table reports the estimated coefficient in a regression of capita returns against three measures of state ownership and four one-digit industry dummies. Measure 1 is the state-owned stock share, measures 2 and 3 are state-ownership dummies with value of 1 if a firm’s state-owned stock share exceeds 50% and 25%, respectively. *** Significant at the 1% level. ** Significant at the 5% level. * Significant at the 10% level.

Table 4. Correlation between Housing Price Growth and Fixed Investment Growth

<table>
<thead>
<tr>
<th>Time</th>
<th>Nationwide</th>
<th>Real Estate Investment</th>
<th>Other Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>0.5255**</td>
<td>-0.3212**</td>
<td></td>
</tr>
<tr>
<td>$t - 1$</td>
<td>0.4765**</td>
<td>-0.4046**</td>
<td></td>
</tr>
<tr>
<td>$t - 2$</td>
<td>0.4115**</td>
<td>-0.4499**</td>
<td></td>
</tr>
<tr>
<td>$t - 3$</td>
<td>0.3320**</td>
<td>-0.5025**</td>
<td></td>
</tr>
<tr>
<td>$t - 4$</td>
<td>0.2710**</td>
<td>-0.5467**</td>
<td></td>
</tr>
<tr>
<td>$t - 5$</td>
<td>0.2025</td>
<td>-0.5438**</td>
<td></td>
</tr>
<tr>
<td>$t - 6$</td>
<td>0.1288</td>
<td>-0.5171**</td>
<td></td>
</tr>
</tbody>
</table>

Source: The aggregate monthly house price data from January 2006 to December 2011 are from Wu, Deng, and Liu (2014). The corresponding monthly investment data are from the CSY (various issues). To remove seasonality, the growth rates for housing prices and investment are computed as the growth rate over the same month of the previous year. **Significant at the 5% level.
Figure 1. Housing/Land Prices in China

Source: The hedonic house price data are from Wu, Deng, and Liu (2014), and the hedonic land price data are from Wu, Gyourko, and Deng (2012, downloadable from http://real.wharton.upenn.edu/~gyourko/chineselandpriceindex.html.)
Figure 2. Growth Rate of Housing Prices and Aggregate Income across Major Cities in China

Source: The hedonic house price data for 35 major cities are from Wu, Deng, and Liu (2014). The growth of disposable income is computed by the authors based on the growth rate of real disposable income and the growth rate of the urban-residing population from various issues of the statistics communiqués for different cities.
Figure 3. Returns to Capital and Labor Reallocation

Note: DPE, domestic private enterprises; FE, foreign enterprises; SOE; state-owned enterprises; COE, collectively owned enterprises.

Source: In Panel A, the rate of return to capital for various years is computed by the authors using the approach of Bai, Hsieh, and Qian (2006) and data from the CSY (various issues). The line with circles refers to the gross rate of return to capital, and the dash-dotted line refers to the after-tax return to capital excluding urban residential housing. In Panel B, the private employment shares are computed by the authors using data from the CSY (various issues).
Figure 4. Total Amount of Arable Land

Source: CSY (various issues).
Figure 5. China’s One-Year Benchmark Nominal Interest Rates

Source: CEIC database.
Figure 6. Transition in the Multiperiod Calibrated Economy

Note: This figure shows the evolution of key variables during and after transition of the calibrated economy. The solid and dash-dotted lines refer to the simulated results from the model and the data, respectively. In Panel A, the plus sign refers to annualized housing price data from Wu, Deng, and Lin (2014), whereas the dash-dotted line refers to annual land price data from Wu, Gyourko, and Deng (2012). Both the simulated housing price data and actual land price data are normalized to 1 in 2004. The housing price data in 2006 are normalized to equal the normalized actual land price data in 2006.
Figure 7. The Role of Firm Heterogeneity in the Calibrated Economy

Note: The figure shows the evolution of key variables in the calibrated economy (solid line) and the counterfactual economy in which there are no F-firms (dash-dotted line).
Figure 8. The Welfare Effects of Housing Bubbles in the Calibrated Economy

Note: The figure shows the evolution of key variables in the calibrated economy (solid line) and the counterfactual economy in which there are no housing bubbles (dash-dotted line).
Figure 9. Welfare Effects of Bubble Burst in 2012

Note: This figure shows the welfare effects of an unexpected burst of a housing bubble in 2012 for all cohorts alive at the time of the burst.
Figure 10: The Return to Capital for Korea and Taiwan

Note: The figures show the rate of return to capital for Korea and Taiwan during housing bubbles in these two countries.

Source: The rate of return to capital for various years is computed by the authors as\( (1 - \text{labour share}_t) \times \frac{Y_t}{K_t} - \delta_t \). The original source of GDP, capital stock, and capital depreciation rate data is the Penn World Table (8.0). \( Y_t \) is output-side real GDP at current PPPs in millions 2005US$, \( K_t \) is capital stock at current PPPs in millions 2005US$, and \( \delta_t \) is average depreciation rate of the capital stock. We convert \( Y_t \) and \( K_t \) into the local currency when computing the return to capital for individual countries.
A. Phase Diagram of the Benchmark Economy

The dynamic system can be described by two difference equations in the detrended E-firm’s capital and housing price:

\[
\begin{align*}
\hat{k}_{t+1}^E (1 + z) (1 + \nu) + \hat{p}_t^H H &= \frac{\psi}{1 + \beta^{-1}} \left( \hat{k}_t^E \right)^{\alpha} \left( \chi n_t^E \right)^{1-\alpha} \quad (A1) \\
\hat{p}_t^H (1 + z) (1 + \nu) &= \rho_{t+1}^E \hat{p}_t^H \quad (A2)
\end{align*}
\]

where

\[
\rho_{t+1}^E = \begin{cases} 
\rho^E & \text{if } \hat{k}_t^E \leq \bar{k}_E \\
\alpha (1 - \psi) \left( \frac{\hat{k}_t^E}{\chi} \right)^{\alpha-1} & \text{if } \hat{k}_t^E > \bar{k}_E
\end{cases}
\]

and \( \bar{k}_E = \chi / \left\{ [(1 - \psi) \chi]^{\frac{1}{\alpha}} (R/\alpha)^{\frac{1}{1-\alpha}} \right\} \) is the minimum level of an E-firm’s capital under which \( n_t^E = 1 \). Accordingly, the constant \( \hat{k}^E \) locus (\( \hat{k}_{t+1}^E = \hat{k}_t^E \)) is characterized by the following step function:

\[
\hat{p}_t^H = \begin{cases} 
\left[ \frac{\rho^E \psi}{(1-\psi)\alpha (1+\beta^{-1})} - (1 + z) (1 + \nu) \right] \hat{k}_t^E & \text{if } \hat{k}_t^E < \bar{k}_E \\
\frac{\psi}{1 + \beta^{-1}} \left( \hat{k}_t^E \right)^{\alpha} \chi^{1-\alpha} - \hat{k}_t^E (1 + z) (1 + \nu) & \text{if } \hat{k}_t^E \geq \bar{k}_E
\end{cases} \quad (A3)
\]

Obviously, when \( \hat{k}_t^E < \bar{k}_E \), the constant \( \hat{k}_E \) locus is an upward-sloping straight line due to the AK feature of E-firms’ return to capital during the transition stage. When \( \hat{k}_t^E \geq \bar{k}_E \), the constant \( \hat{k}_E \) locus looks like their counterpart in the standard neoclassical economy and is hump-shaped. Moreover, from (A1) and (A2), the constant \( \hat{p}^H \) locus is characterized by

\[
\hat{p}_t^H = \frac{\psi}{1 + \beta^{-1}} \left( \hat{k}_t^E \right)^{\alpha} \chi^{1-\alpha} - (1 + z) (1 + \nu) \rho^{E-1} ((1 + z) (1 + \nu)) \quad (A4)
\]

where \( \rho^{E-1} \) is the inverse function of \( \rho^E \left( \hat{k}_t^E \right) = \alpha (1 - \psi) \left( \hat{k}_t^E / \chi \right)^{\alpha-1} \). Obviously, the constant \( \hat{p}_t^H \) locus is always upward sloping. Moreover, since \( \rho^E > 1 \), the whole constant \( \hat{p}_t^H \)
locus is on the right side of $\hat{k}_t^E = \tilde{k}_t^E$. Note that the condition for $\psi$ in equation (9) ensures that the constant $\hat{k}_t^E$ locus and the constant $\tilde{p}_t^H$ intersect at a point where $\tilde{p}_t^H$ is positive, which is the bubbly steady state.

Figure A-1 plots the phase diagram for $\{\hat{k}_t^E, \tilde{p}_t^H\}$. For any initial $\hat{k}_0^E$, there could be three cases for $\tilde{p}_0^H$. Point A, at which $\tilde{p}_0^H = \tilde{p}_0^H$ or $h_0^E = \hat{h}_0^E$, corresponds to the saddle path equilibrium. Point B, at which $\tilde{p}_0^H < \tilde{p}_0^H$ or $h_0^E < \hat{h}_0^E$, corresponds to the asymptotic bubbly equilibrium. Point C, at which $h_0^E > \hat{h}_0^E$, has an explosive path of housing bubble, hence is not sustainable.

**B. Proof of Propositions and Lemmas**

In this section, we prove the various lemmas and propositions.

**Proof of Lemma 1.** The growth rate of E-firms’ output is

$$
\frac{Y_{t+1}^E}{Y_t^E} = \frac{Y_{t+1}^E}{K_{t+1}^E} \frac{K_{t+1}^E}{K_t^E} \frac{K_t^E}{Y_t^E} = \frac{\rho_t^E}{K_{t+1}^E} \frac{K_{t+1}^E}{(1 - \psi) \alpha}{(K_t^E)^{1-\alpha}} = P_t^H \beta 
$$

where $K_{t+1}^E/K_t^E$ depends on the equilibrium portfolio share of entrepreneurs in physical capital, $\phi_t^E$. We now solve for the equilibrium portfolio share of entrepreneurial savings in housing. Using the housing market-clearing condition, $\bar{H} = H_t^E$, we have

$$
(1 - \phi_t^E) \frac{1}{1 + \beta^{-1}} \psi (K_t^E)^{\alpha} (A_t \chi n_t^E N_t)^{1-\alpha} = P_t^H \beta. \quad (A6)
$$

Forwarding (A6) by one period, and with $(K_{t+1}^E)^{\alpha} (A_{t+1} \chi n_{t+1}^E N_{t+1})^{1-\alpha} = K_{t+1}^E \rho_{t+1}^E / [\alpha (1 - \psi)]$, equation (A6) can be rewritten as

$$
(1 - \phi_{t+1}^E) \frac{1}{1 + \beta^{-1}} \alpha (1 - \psi) = P_{t+1}^H \beta. \quad (A7)
$$

With the law of motion for capital (9), (A7) can be rewritten as

$$
(1 - \phi_{t+1}^E) \frac{1}{1 + \beta^{-1}} \psi \rho_{t+1}^E (K_{t+1}^E) = P_{t+1}^H \beta. \quad (A8)
$$
Dividing (A8) by (A6) for all \( t \), we have

\[
\frac{1 - \phi^E_{t+1}}{1 - \phi^E_t} = \frac{\phi^E_t}{1 + \beta^{-1} \alpha (1 - \psi)} \frac{\psi \rho^E_{t+1}}{P^H_{t+1}} = \frac{P^H_t}{P^H_t} = \rho^E_{t+1},
\]

or simply

\[
\frac{1 - \phi^E_{t+1}}{1 - \phi^E_t} = \frac{\phi^E_t}{1 + \beta^{-1} \alpha (1 - \psi)}, \forall t. \tag{A9}
\]

Equation (A9) is a first-difference equation capturing the dynamics of \( \phi^E_t \). One solution to equation (A9) is that

\[
\phi^E_t = \frac{\alpha (1 + \beta^{-1}) (1 - \psi)}{\psi}, \forall t. \tag{A10}
\]

To solve for \( K^E_{t+1}/K^E_t \), we substitute (A10) into (9) and obtain

\[
K^E_{t+1} = \rho^E_t K^E_t. \tag{A11}
\]

Equation (A11) is a variant of the no-arbitrage condition. Comparing (16) in the fundamental equilibrium and (A11) in the bubbly equilibrium, we see that in the bubbly equilibrium the optimal portfolio choice by an entrepreneur equalizes the return to capital investment and the return to bubbles by crowding out E-firms’ capital investment.

Finally, substituting (A11) into (A5), we obtain \( Y^E_{t+1}/Y^E_t = \rho^E_{t+1} = \rho^H_{t+1}. \)

**Proof of Proposition 1.** We first decompose the ratio of housing value to aggregate output as

\[
\frac{P^H_t Y^H_t}{Y_t} = \frac{P^H_t Y^H_t}{Y^E_t + Y^F_t}. \tag{A12}
\]

The first argument on the right side of (A12), \( P^H_t Y^H_t/Y^E_t \), can be further expressed as

\[
\frac{P^H_t Y^H_t}{Y^E_t} = \frac{P^H_t Y^H_t K^E_{t+1}}{Y^E_t} = \frac{1 - \phi^E_t K^E_{t+1}}{\phi^E_t Y^E_t}. \tag{A13}
\]

Equation (A11) implies

\[
K^E_{t+1} = (1 - \psi) \alpha Y^E_t. \tag{A14}
\]

With both (A10) and (A14), it is straightforward that \( P^H_t Y^E_t \) is a constant. Therefore,
by log-differencing (A12), we obtain (25).

Finally, we derive $Y_t^E/(Y_t^E + Y_t^F)$. Using (6), $Y_t^E$ can be expressed as

$$Y_t^E = \frac{N_t^E}{N_t (1 - \psi)} \kappa_F A_t N_t,$$

where $\kappa_F \equiv k_{Ft}/(n_{Ft} A_t) = (\alpha/R)^{1-\alpha}$. Similarly, it is easy to show that

$$Y_t^E + Y_t^F = \left(1 + \frac{\psi N_t^E}{1 - \psi N_t} \right) \kappa_F A_t N_t. \quad (A16)$$

**Proof of Proposition 2.** To prove this proposition, consider the fundamental equilibrium—that is, $\phi_t^E = 1$ for all $t$. According to (12), introducing housing reduces the steady-state physical capital. Hence, we only need to show under which condition a marginal reduction in physical capital reduces total entrepreneurial consumption. Aggregating the budget constraints of the young and old entrepreneurs at period $t$, and using the capital market-clearing condition, we obtain

$$\left[ N_t c_{1,t}^E + N_{t-1} c_{1,t-1}^E + N_{t+1} k_{t+1}^E \right] / 2 = N_t \left[ m_t + \rho_t^E k_t^E \right] / 2. \quad (A17)$$

With the definition of $c_t^E$ and $m_t + \rho_t^E k_t^E = [\psi + (1 - \psi) \alpha] y_t^E$, a detrended version of (A17) is

$$\widehat{c}_t^E = [\psi + (1 - \psi) \alpha] \widehat{y}_t^E - \widehat{k}_{t+1}^E (1 + z)(1 + \nu). \quad (A18)$$

Taking the derivative of the right side of (A18) with respect to $\widehat{k}^E$ at the steady state, we can obtain the following sufficient condition for introducing bubbles to reduce aggregate consumption for entrepreneurs:

$$[\psi + (1 - \psi) \alpha] MPK^E \big|_{\phi=1} > (1 + z)(1 + \nu). \quad (A19)$$

With (13) and the definition of $MPK^E$, the inequality (A19) can be rewritten as

$$[\psi + (1 - \psi) \alpha] \alpha (1 + \beta^{-1}) (1 + \nu) / \psi > (1 + z)(1 + \nu). \quad (A20)$$
Reordering \((A20)\), we obtain \((28)\).

The proof of the welfare implications for entrepreneurs in both transition and post-transition stages is straightforward. Substituting the detrended version of \((9)\) into \((A18)\), we obtain

\[
\tilde{c}_t^E = \left[ \psi + (1 - \psi) \alpha - \frac{\psi}{\alpha (1 + \beta^{-1})} \right] \tilde{y}_t^E.
\]

Since E-firms’ capital increases monotonically, we need to prove only

\[
\frac{\partial \tilde{c}_t^E}{\partial \tilde{k}_t^E} = \left[ \psi + (1 - \psi) \alpha - \frac{\psi}{\alpha (1 + \beta^{-1})} \right] MPK_t^E \mid_{\phi^E=1} > 0. \quad (A21)
\]

By assumption \((28)\), \(\partial \tilde{c}_t^E/\partial \tilde{k}_t^E > 0\) for all period \(t\). Hence, housing bubbles, by crowding out physical capital, reduce total entrepreneurial consumption.

For entrepreneurs born during the transition, \((A22)\) becomes

\[
\log (m_t - s_t^E) + \beta \log \rho^E s_t^E = (1 + \beta) \log k_t^E + \tilde{C},
\]

where \(\tilde{C}\) is a function of parameters. Therefore, it is easy to see that a reduction in capital stock would reduce the welfare of entrepreneurs born during the transition.

For the entrepreneur born in the post-transition stage, but before reaching the steady state, the lifetime utility can be expressed as

\[
\log (m_t - s_t^E) + \beta \log \rho_{t+1}^E s_t^E = \log \left( \frac{\psi \rho_t^E k_t^E}{(1 + \beta) \alpha (1 - \psi)} \right) + \beta \log \frac{\rho_{t+1}^E k_{t+1}^E (1 + \nu)}{\phi_t^E}. \quad (A22)
\]

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At steady state, equation (A22), after being detrended, becomes

\[(1 + \beta) \log \rho E^* k^E - \beta \log \phi^E + C\]

\[= (1 + \beta) \log \frac{1 - \psi}{\psi} \left(1 + \frac{1 - \beta^{-1}}{\phi^E}(1 + z)(1 + \nu)\right)^{\frac{\psi \phi^E \chi^{1-\alpha}}{(1 + \beta^{-1})(1 + z)(1 + \nu)}\right]^{\frac{1}{1-\alpha}} - \beta \log \phi^E + C\]

\[= \left[\frac{\alpha (1 + \beta)}{1 - \alpha} - \beta\right] \log \phi^E + \tilde{C}, \quad (A23)\]

where both \(C\) and \(\tilde{C}\) are functions of parameters. Hence, introducing housing (i.e., a reduction in \(\phi^E\)) reduces the steady-state welfare if \(\frac{\alpha (1 + \beta)}{1 - \alpha} > \beta\) or \(\alpha (1 + \beta^{-1}) > 1 - \alpha\). Note that the joint participation and incentive constraints of the young entrepreneurs implies \(m = \psi y^E > w = (1 - \alpha)(1 - \psi)y^E\), which gives the following parameter restriction: \(\psi > (1 - \alpha)(1 - \psi)\), or equivalently, \(\frac{\psi}{\psi + \alpha(1 - \psi)} > 1 - \alpha\). Therefore, with assumption (28), introducing housing reduces the entrepreneurial lifetime utility at the steady state.

For entrepreneurs born during the post-transition stage, it is easy to show that equation (A22) becomes

\[\log (m_t - s_t^E) + \beta \log \rho_{t+1}^E s_t^E\]

\[= \alpha (1 + \alpha \beta) \log k_t^E - (1 - \alpha) \beta \log \phi_t^E + \overline{C}, \quad (A24)\]

where \(\overline{C}\) is a function of parameters. We would like to compare (A24) under the fundamental and the bubbly equilibrium. In the bubbly equilibrium, since \(k_t^E\) is smaller due to previous cohort’s investment in housing, a sufficient condition for welfare loss with a reduction in \(\phi_t^E\) is

\[\alpha (1 + \alpha \beta) > (1 - \alpha) \beta\]

or \(\alpha (1 + \beta^{-1}) > 1 - \alpha^2\).

**Proof of Proposition 3:** We consider the portfolio choice of an entrepreneur of age \(j\) at period \(t\). Suppose that all other entrepreneurs alive at period \(t\) hold the same share of savings in housing, \(\phi_{k,t}^E = \phi_t^E\) for \(k \neq j\), so that the no-arbitrage condition holds, \(\frac{P_{H,t+1}}{P_{H,t}} = \rho_t^E\). Accordingly, the age-\(j\) entrepreneur is indifferent between housing and physical capital. So
\( \phi_{j,t}^E = \phi_t^E \) is an equilibrium solution. The same logic applies for the portfolio choice of other entrepreneurs alive at period \( t \). Hence, there exists a solution that all entrepreneurs hold the same share of housings in their net worth.

C. Numerical Algorithm

Again, we detrend all per capita variables (except labor inputs and housing) as \( \hat{x}_t = x_t/A_t \). For total labor inputs on both the supply and demand sides, we detrend them by dividing them by the size of the population, \( N_t \). Denote \( n_{t}^E \equiv \sum_{j=1}^{J_{E}-1} n_{j,t}^E \) as the total detrended labor demand by E-firms. Since the aggregation holds, the following equation determines total labor allocated to E-firms:

\[
 n_{t}^E = \left( (1 - \psi) (1 - \tau_t^y) \chi \right)^{\frac{1}{\alpha}} \left( \frac{R^l - 1 + \delta}{\alpha} \right)^{-\frac{1}{1-\alpha}} \hat{k}_t^E / \chi. \tag{A25}
\]

Similarly, denote \( n_{t}^w \equiv \sum_{j=1}^{J_{R}-1} n_{j,t}^w \) as the total detrended labor supply by workers. If \( n_{t}^E > n_{t}^w \), we have

\[
 \hat{w}_t = (1 - \psi) (1 - \tau_t^y)(1 - \alpha) \left( \hat{k}_t^E / n_{t}^E \right)^{\alpha} \chi^{1-\alpha}, \tag{A26}
\]
\[
 n_{t}^E = n_{t}^w, \quad n_{t}^F = \hat{k}_t^F = 0. \tag{A27}
\]

Otherwise,

\[
 \hat{w}_t = (1 - \alpha) \left( \frac{\alpha}{R^l - 1 + \delta} \right)^{\frac{\alpha}{1-\alpha}} \tag{A28}
\]
\[
 n_{t}^F = n_{t}^w - n_{t}^E \tag{A29}
\]
\[
 \hat{k}_t^F = \left( \alpha / (R^l - 1 + \delta) \right)^{\frac{1}{1-\alpha}} n_{t}^F. \tag{A30}
\]
Also, we have the following equations for both the transition and post-transition stages:

\[
\rho_t^E = \alpha (1 - \psi) [(1 - \alpha) (1 - \psi) (1 - \tau_t^y)]^{\frac{1 - \alpha}{\alpha}} + 1 - \delta, \tag{A31}
\]

\[
\hat{m}_t = \psi (1 - \tau_t^y) \left( \hat{K}_t^E \right)^{\frac{1 - \alpha}{\alpha}} \left( \chi n_t^E \right)^{1 - \alpha} / \sum_{j=1}^{J_{t-1}} n_{j,t}^E, \tag{A32}
\]

\[
H_t^E = \overline{H}, \tag{A33}
\]

\[
\hat{p}_t^H = \hat{p}_{t+1}^H (1 + z) (1 + \nu) / \rho_{t+1}^E, \tag{A34}
\]

\[
\hat{p}_t^H \overline{H} = (1 - \phi_t^E) \sum_{j=J_{E-1}}^{J-1} n_{j,t}^E \hat{s}_{j,t}^E, \tag{A35}
\]

\[
\hat{k}_{t+1}^E = \phi_t^E \sum_{j=J_{E-1}}^{J-1} n_{j,t}^E \hat{s}_{j,t}^E. \tag{A36}
\]

We assume transition takes \( T \) periods. At period \( T \), the economy enters the steady state. The algorithm to solve for the transition takes the following steps:

1. Guess the sequence of \( \{ \phi_t^E, \hat{k}_{t+1}^E, \hat{p}_t^H \}_{t=1}^{T-1} \).

2. Given \( \hat{k}_1^E \), compute \( \left\{ n_t^E, \hat{w}_t, n_t^E, \hat{k}_{t+1}^E, \rho_t^E, \hat{m}_t, \hat{s}_{j,t}^E, \hat{s}^w_{j,t}, H_t^E \right\}_{t=1}^{T-1} \).

3. Check the following conditions for each period \( t = 1, 2, \ldots, T-1 \):

\[
\phi_t^E = 1 - \frac{\hat{p}_t^H \overline{H}}{\sum_{j=J_{E-1}}^{J-1} n_{j,t}^E \hat{s}_{j,t}^E}, \tag{A37}
\]

\[
\hat{p}_t^H = \hat{p}_{t+1}^H (1 + z) (1 + \nu) / \rho_{t+1}^E, \tag{A38}
\]

\[
\hat{k}_{t+1}^E = \phi_t^E \sum_{j=J_{E-1}}^{J-1} n_{j,t}^E \hat{s}_{j,t}^E / [(1 + z) (1 + \nu)], \tag{A39}
\]

and (since \( \rho_{T+1}^E \) is not known)

\[
\hat{k}_{T+1}^E = \hat{k}_{E^*}^E = \left[ \frac{\phi_{E^*} (1 - \tau) \chi^{1-\alpha}}{(1 + \beta^{-1}) (1 + z) (1 + \nu)} \right]^{\frac{1}{1-\alpha}}. \tag{A40}
\]
Figure A-1: Phase Diagram of the Benchmark Economy

\[ \begin{align*}
\hat{p}_t^H &= \hat{p}_t^H \\
\hat{k}_{t+1}^E &= \hat{k}_t^E = \hat{k}_t^E \\
\hat{k}_{t+1}^E = (\rho^E)^{-1}[(1+z)(1+\nu)] \\
\end{align*} \]