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Innovation and Growth with Financial, and other, Frictions

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Abstract: The generation and implementation of ideas, or knowledge, is crucial for economic performance. We study this process in a model of endogenous growth with frictions. Productivity increases with knowledge, which advances via innovation, and with the exchange of ideas from those who generate them to those best able to implement them (technology transfer). But frictions in this market—including search, bargaining, and commitment problems—impede exchange and thus slow growth. We characterize optimal policies to subsidize research and trade in ideas, given both knowledge and search externalities. We discuss the roles of liquidity and financial institutions, and show two ways in which intermediation can enhance efficiency and innovation. First, intermediation allows us to finance more transactions with fewer assets. Second, it ameliorates certain bargaining problems, by allowing entrepreneurs to undo otherwise sunk investments in liquidity. We also discuss some evidence suggesting that technology transfer is a significant source of innovation and showing how it is affected by credit considerations.

JEL classification: E4, G2, O3, O4

Key words: innovation, growth, ideas, technology transfer, liquidity, financial intermediation, search and bargaining

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1 Introduction

It is commonly argued that the generation and implementation of new ideas – i.e., the *evolution of knowledge* – is a major factor underlying economic performance and growth, and that financial development plays a role in this process.[1] This project is an attempt to better understand these issues. To this end, we build an endogenous growth model where productivity increases with knowledge and knowledge increases with research and development. We model the endogenous decision of agents to try to come up with and implement new ideas. Additionally, based on the premise that some agents are better than others at implementation, we explicitly model the exchange of ideas, or *technology transfer*. Our idea market is characterized by several explicit frictions, including search, bargaining and commitment problems that impede credit arrangements. These can all hinder the reallocation of ideas across agents, and hence the advance of knowledge. Financial intermediation can ameliorate these frictions, to some extent, and this is the channel through which it enhances economic growth.

We are interested in studying both the generation of new ideas, and the reallocation of these ideas from innovators to those with comparative advantage in implementation. On the former issue, we study how an economy might try to achieve a socially optimal level of innovative activity, which is interesting because knowledge is at least partly a public good, and hence ought to be subsidized. On the latter issue, we are interested in the exchange of ideas in the presence of frictions. The idea market is a thin market and agents are not generally price takers. Coming up with new ideas may involve fixed costs that cannot be recouped due to holdup problems in bargaining, leading to inefficiently low innovative activity. There is also the basic matching problem of getting innovators with good potential ideas together with the right entrepreneur to implement them. And there may be financial frictions that make it difficult to pay for ideas, which means it can be important to have institutions that help get liquid assets from those who need them less to those that need them more. Our goal is to model all of this explicitly.

[1] See any modern text on growth theory, such as Aghion and Howitt (1997) or Acemoglu (2009), and references therein, for extended discussions. An early proponent of the view that financial development is critical for growth is Goldsmith (1967). Recent work building on these ideas includes Greenwood, Sanchez and Wang (2008, 2010) and references therein. See also the survey by Levine (2005).

In our framework, individual producers have access to the frontier technology Z, which is in the public domain, but also come up with ideas for innovations that increase their own productivity z. Increases in z raise individual profits in the short run, then knowledge enters the public domain in the longer run. In the simplest case, an individual innovator i with an idea tries to develop it on his own, and only succeeds with probability σ_i, which is itself random (think of σ_i as indexing the quality of the match between an idea and the individual's expertise). Each innovation advances individual productivity by some amount, and these aggregate to give the evolution of the technology frontier. We show how the model generates a balanced growth equilibrium, where the growth rate depends on the number of innovators, their probabilities of success, the distance by which innovations move knowledge, and the way improvements in individual productivity affect the frontier technology. This benchmark, however, is only a stepping stone toward our study of economies where individual innovators do not necessarily implement ideas on their own, but instead may trade them.

This activity is described in the literature in terms of the following trade-off: When innovators come up with new ideas, should they try to implement them themselves? Or should they try to trade their ideas to others, say entrepreneurs, who may be better at development, marketing and related activities? If agents are heterogeneous in their ability to come up with ideas and to extract their returns, it makes sense for some to specialize in research and others in development. In this way, the exchange of ideas leads to a more efficient use of resources and increases the incentive to innovate. As Katz and Shapiro (1986) put it "Inventor-founded startups are often second-best, as innovators do not have the entrepreneurial skills to commercialize new ideas or products." As a special feature in *The Economist* (2005) on the market for ideas reports: "as the patent system has evolved, it ... leads to a degree of specialization that makes business more efficient. Patents are transferable assets, and by the early 20th century they had made it possible to separate the person who makes an invention from the one who commercializes it. This recognized the fact that someone who is good at coming up with ideas is not necessarily the best person to bring these ideas to market." Lamoreaux and Sokoloff (1999) argue that the "The growth of the U.S. economy over the nineteenth century was characterized by a sharp acceleration of the rate of inventive activity and a dramatic rise in the relative importance of highly specialized inventors as generators of new technological knowledge.

Relying on evidence compiled from patent records, we argue that the evolution of *a market for technology* played a central role in these developments" (emphasis added).

Financial intermediation can affect development by facilitating the redirection of resources from less productive to more productive uses. Here the resources in question are ideas. Of course, direct technology transfers are but one mechanism by which innovators and entrepreneurs interact to share knowledge and develop ideas – e.g., they can alternatively enter into longer-term partnerships, as in the venture capital market (Gompers and Lerner 1999). Our entrepreneurs are not in search of money to start a business; they have money, if sometimes not enough, plus skills, and what they need is to find someone with a good idea for sale. We focus on situations where an innovator wants to sell his idea outright, rather than enter a joint venture. One very important advantage of direct technology transfer is that it avoids strategic problems with joint implementation, as we discuss below. Another is that it allows innovators to get "back to the drawing board" in an effort to come up with more new ideas, which is their specialty, rather than getting tied up in development. Because this seems interesting, and is somewhat neglected in growth theory, we focus on direct technology transfers and model the market where this happens as one in which frictions play a role.[2]

Another factor we emphasize is liquidity, which determines the ability of entrepreneurs to pay up front. This can be motivated by limited commitment problems that impede credit. Commitment issues are important in this context because knowledge is difficult to collateralize – if you give someone your idea in exchange for promised future payments, and they renege, it is hard to repossess the information, depending of course on intellectual property rights, patent protection, etc. Other concerns include adverse selection (how does one know your idea is any good before one buys it) and

[2] Why search-and-bargaining frictions? One reason is generality: one can think of a standard competitive market as the special case where these frictions vanish (the large literature on this topic is summarized in, e.g., Mortensen and Wright 2002). Another reason is that this is how people who study the market say it works. Sakakibara (2010), e.g., claims that "since there is no public market for patents, the price of patents is determined by a private negotiation between a licensor and a licensee." Using 661 patent licensing contracts between 1998-2003 the author documents that "once the *matching process* is completed, the *terms of the contract are negotiated* between a licensor and a licensee" (emphasis added). Also, using data on U.S. patent records and patent lawyers over the nineteenth century, Lamoreaux and Sokoloff (1999) argue that "it was evident patent agents and lawyers often perform the functions of intermediaries in the market for technology, matching inventors seeking to to sell new technological ideas with buyers eager to develop, commercialize, or invest in them." We do not model this role for patent agents and lawyers explicitly, but this speaks to the importance of matching in this market.

moral hazard (how does one know you will carry your weight in a joint project). The first principle of Contract Theory is that the first best can be achieved if you sell your idea outright, internalizing the incentive problems. For Contract Theory to be relevant, there needs to be some appeal to liquidity constraints. We model this explicitly, and, moreover, we make liquidity endogenous. This allows us to discuss the impact of financial development on the idea market, and hence on growth, by introducing intermediaries whose role is to channel liquidity from those that have more than they need to those that have less.

Before proceeding, we clarify why we call the objects being traded in the model ideas, and not simply some generic factor of production, even if many of the insights apply more generally. Ideas here are indeed factors of production, but of a particular type that expands knowledge and raises productivity. Importantly, ideas are nonrival goods, at least in the long run, when knowledge enters the public domain. This means there is typically inefficient investment in innovation, from a social perspective, and we characterize the optimal corrective tax-subsidy schemes, which is made all the more interesting by search and bargaining frictions. Also, at least to some extent, ideas are indivisible (either you tell someone or you don't), although this is more of a technical consideration than a critical component of our theory. And, as we said, ideas are difficult to collateralize, making credit problematic in the presence of limited commitment, and motivating the consideration of liquidity. Lastly, the idea market is rife with information problems, including adverse selection and moral hazard, as mentioned above, making technology transfer preferable to joint ventures.[3]

The rest of the paper is organized as follows. Section 2 presents the basic model, without trade in ideas, and shows how it generates a unique balanced growth equilibrium. Even in this simplest

[3] Another response to frictions in the idea market, along the lines of Coase's (1937) theory of the firm, is to bring R&D in house (similar to, but not the same as, joint ventures). This is not inconsisent with our general view, although we do think the model applies more directly to technology purchases from the outside – i.e., to *arm's length transactions*, descibed by Investopedia as follows: "The concept of an arm's length transaction is to ensure that both parties in the deal are acting in their own self interest and are not subject to any pressure or duress from the other party. ... For example, if two strangers are involved in the sale and purchase of a house, it is likely that the final agreed-upon price will be close to market value (assuming that both parties have equal bargaining power and equal information about the situation). This is because the seller would want a price that is as high as possible and the buyer would want a price that is as low as possible. This contrasts with a situation in which the two parties are not strangers. For example, it is unlikely that the same transaction involving a father and his son would yield the same result, because the father may choose to give his son a discount."

context, one can endogenize the growth rate by introducing a free entry condition for potential innovators. We show that there is generally underproduction of knowledge in equilibrium, because it is to some extent a public good, and derive the optimal corrective policy. Section 3 studies trade between innovators and entrepreneurs without credit frictions, and again gives conditions for a unique balanced growth equilibrium. In this model we can allow entry into either, or both, innovative and entrepreneurial activity, and again derive the optimal corrective policy in the presence of both knowledge and search externalities. Section 4 introduces frictions that make it difficult to trade ideas on credit, leading to a role for liquidity. In this model innovation can be hindered by a shortage of liquid assets, but even if liquidity is plentiful and we implement the optimal policy, we do not generally get efficiency due to bargaining problems. Section 5 introduces financial intermediaries that serve to reallocate liquidity across agents. We show how intermediation allows the economy to finance more transactions with a given quantity of assets. Moreover, it helps get around the above-mentioned bargaining friction. This novel effect arises, intuitively, by allowing entrepreneurs to undo otherwise sunk investments in liquidity, thereby alleviating holdup problems. Section 6 sketches some evidence suggesting that technology transfers spur innovation and showing how credit imperfections hinder this process. Section 7 concludes. Technical results are relegated to the Appendices.[4]

2 The Basic Model

A $[0, 1]$ continuum of agents live forever in discrete time. Each period, there convenes a frictionless centralized market where agents trade consumption c, labor hours h, and an asset a. We take c as numeraire, w as the wage and ϕ as the asset price. We think of a as claims to Lucas (1978)

[4] Our approach is related to Holmes and Schmitz (1990, 1995), who also assume individuals differ in implementation ability, but they only study perfectly competitive markets while we allow frictions to play a role, endogenize growth, and study the effects of taxation and liquidity/intermediation. Many people have studied credit frictions and entrepreneurship; for a recent source of references, again see Greenwood et al. (2010), as well as Chatterjee and Rossi-Hansberg (2010) or Silveira and Wright (2010), who focus especially on the idea market. We also mention a recent paper that develops a similar model by Berentsen, Roja Breu and Shi (2009). Also related is work by Lucas (2009), Alvarez, Buera and Lucas (2008), and Lucas and Moll (2011). These papers all study similar issues, but our approach is different. In Lucas and Moll, e.g., while there is search/matching, agents learn nonrival ideas passively from those they meet; by contrast, our agents must buy ideas at negotiated prices; knowledge is rival in the short run but a public good in the long run; we explictly consider ex ante investment (entry) into R&D; we have labor, consumption and idea markets, and we explicitly model the role of liquidity/intermediation.

trees, in fixed supply A and bearing dividend δ, except here the dividend is not consumption but an intermediate good that is transformed into c according to technology $c = Z\delta a$ where Z is the aggregate state of knowledge (productivity). Thus, Z is the price of intermediate goods in terms of numeraire. The value function for agents in the centralized market is

$$\begin{aligned} W(a, z; Z) &= \max_{c,h,a'} \{u(c) - \chi h + \beta V(a', Z')\} \\ \text{st } c &= (\phi + Z\delta)\, a + wh - \phi a' + \pi(z), \end{aligned} \tag{1}$$

where $u(c)$ satisfies the standard assumptions, $V(a', Z')$ is the continuation value, and $\pi(z)$ is profit as a function of individual productivity z, distinguished from Z. There is no reproducible physical capital in the benchmark model, but in Appendix 1 we show how to include it.

We interpret each individual as an owner/operator of his firm, although it is equivalent to engage a manager to operate it. In either case, their problem is

$$\pi(z) = \max_{H} \{zf(H) - wH\}, \tag{2}$$

where $f(H)$ satisfies the usual assumptions and H is labor demand. Individuals may work at their own firms, but additionally work for (hire) others when $h > H$ ($h < H$). Output $f(H)$ is in units of the intermediate good, which is transformed into $zf(H)$ units of c. Individual productivity z may differ from the aggregate Z, depending on whether an agent innovates. There are for now only two types of agents: a fraction $\bar{n}_i$ have an opportunity to innovate, while the remaining $1 - \bar{n}_i$ do not. Each period, all agents start with the same aggregate knowledge Z, but those with an opportunity to innovate come up with an idea. Not all ideas come to fruition: the success probability is σ, where σ is a random draw from CDF $F_i(\sigma)$.[5] Each success increases individual productivity from $z = Z$ to $z = (1 + \eta)Z$. Thus, an innovator's individual productivity is given by:

$$z = \begin{cases} Z(1+\eta) & \text{with prob } \sigma \\ Z & \text{with prob } 1 - \sigma \end{cases}$$

One can think of σ as capturing the quality of an idea combined with the skill that any individual has at implementing it, to motivate the analysis below where agents trade ideas, although for now

[5] All innovators draw from the same F_i; the subscript merely indicates that this distribution is associated with innovators, as later we introduce a different type, entrepreneurs, who draw from another distribution. Also, we proceed as if types are permanent, but the results are exactly the same if every agent realizes an opportunity to innovate each period with probability $\bar{n}_i$.

the idea market is shut down and agents must try to develop ideas on their own. The number of successful innovations is $N = \bar{n}_i \int \sigma dF_i(\sigma) = \bar{n}_i \mathbb{E}\sigma$. Note that although the probability of success is random, each successful innovation advances productivity by a deterministic amount η; we also solved the case where η is random, and it did not add much other than notation. The aggregate state of knowledge evolves from one period to the next according to $Z' = G(N)Z$. Ideas are public goods in the long run, in the sense that after they are put into production, they enter the public domain and yield an advance in aggregate productivity after one period (it is not hard to extend this to many periods). Knowledge in the public domain is higher next period if more ideas are implemented successfully in the current period, $G'(N) \geq 0$.

As an example, consider knowledge evolving according to

$$Z' = \rho \left(\int_0^1 z_i^{\varepsilon} di \right)^{1/\varepsilon} \tag{3}$$

where ρ is an exogenous component and ε is a parameter affecting the substitutability of individual innovations in generating aggregate knowledge. As special cases, before adjusting for ρ, we have the following: $\varepsilon = 1$ implies productivity next period is given by average productivity this period (we all contribute equally to the frontier); $\varepsilon = +\infty$ implies it is given by maximum productivity (we stand on the shoulders of those giants with the very best knowledge); and $\varepsilon = -\infty$ implies it is given by minimum productivity (we are dragged down by the worst, as in "O-ring" theory).[6] It is easy to see (3) implies $Z' = \rho \left[N(1+\eta)^{\varepsilon} + (1-N) \right]^{1/\varepsilon} Z$. However, except for constructing examples, we do not need any particular functional form, and the growth rate generally is written

$$1 + g = Z'/Z = G(N). \tag{4}$$

We seek a *balanced growth equilibrium*, where c, w and ϕ grow at the same rate as Z, while h is constant. To pursue this, first eliminate h and π from the budget constraint to rewrite (1) as

$$W(a, z; Z) = \max_{c, a', H} \left\{ u(c) - \frac{\chi}{w} \left[c - (\phi + \delta Z)a + \phi a' \right] + \frac{\chi}{w} \left[z f(H) - wH \right] + \beta V(a', Z') \right\}$$

[6] Notice that an agent with productivity z who fails to innovate this period uses the fontier Z' next period, and it is possible that $Z' < z$; one can always raise the exogenous component ρ in (3), however, if one wants to avoid this.

where it is understood that $Z' = G(N)Z$ with $N = \bar{n}_i \mathbb{E}\sigma$. This conveniently separates into

$$\begin{aligned} W(a,z;Z) &= \frac{\chi}{w}(\phi + \delta Z)a + \max_c \left\{ u(c) - \frac{\chi}{w} c \right\} + \frac{\chi}{w} \max_H \{ z f(H) - wH \} \\ &\quad + \max_{a'} \left\{ \beta V(a', Z') - \frac{\chi}{w} \phi a' \right\}, \end{aligned} \tag{5}$$

showing that W is linear in wealth, and in particular $W_a = \chi(\phi + \delta Z)/w$.[7] Taking FOC we get

$$\begin{aligned} u'(c) &= \chi/w \\ z f'(H) &= w \\ \phi\chi/w &= \beta V_a(a', Z'). \end{aligned} \tag{6}$$

The continuation value depends on whether an agent has an opportunity to innovate: for those that do not $V(a,Z) = W(a,Z;Z)$; and for those that do

$$\begin{aligned} V(a,Z) &= \int_0^1 \{ \sigma W[a, Z(1+\eta); Z] + (1-\sigma) W(a,Z;Z) \} \, dF_i(\sigma) \\ &= W(a,Z;Z) + \mathbb{E}\sigma \{ W[a, Z(1+\eta); Z] - W(a,Z;Z) \}, \end{aligned} \tag{7}$$

which is the payoff from entering the centralized market with $z = Z$, plus the expected surplus from innovation. Given this, plus the linearity of W in a, we can insert the derivative $V_a = W_a$ into the FOC for a' to get

$$\frac{\chi}{w}\phi = \beta \frac{\chi}{w'}(\phi' + \delta Z'). \tag{8}$$

It is easy to verify that $\phi = Z\delta\beta/(1-\beta)$ is the unique bounded and non-negative solution to (8) (it is easiest if one writes it as a difference equation in ϕ/Z). Hence, the asset must be priced fundamentally – i.e., by the present value of its dividend stream, $\phi = Z\delta\beta/(1-\beta)$.

In addition to the asset price ϕ, the price of intermediate goods Z, and the price of consumption normalized to 1, we need to determine the wage. By Walras' Law we can find w either from goods- or labor-market clearing, and we use the former. In terms of supply $S = S(w)$, we have

$$S = \int_0^1 z_i f(H_i) di + A\delta Z = N(1+\eta) Z f(H_1) + (1-N) Z f(H_0) + A\delta Z,$$

[7] These simplifications are due to quasi-linear utility. As in models following Lagos and Wright (2005), this facilitates the analytics by reducing the dimensionality of the state space, since we do not have to track the distribution of wealth. It is not hard to generalize this, in principle, using numerical methods.

where H_1 solves (2) for successful innovators and H_0 solves it for the rest. From the FOC, $Z(1+\eta)f'(H_1) = w$ and $Zf'(H_0) = w$, clearly, H_0 and H_1 depend only on w/Z. Given Z, supply is decreasing in w – i.e., increasing in the relative price of consumption goods $1/w$ – because

$$S'(w) = \frac{N(1+\eta)f'(H_1)}{f''(H_1)} + \frac{(1-N)f'(H_0)}{f''(H_0)} < 0.$$

In terms of demand $D = D(w)$, the relevant FOC is $u'(c) = \chi/w$. In general, demand is increasing in w – i.e., decreasing in the relative price of goods $1/w$ – because

$$D'(w) = \frac{-\chi}{w^2 u''(c)} > 0.$$

To get balanced growth we need $u(c) = \log(c)$, which means $D = w/\chi$ and an increase in Z does not affect demand.[8] Setting $S(w) = D(w)$ yields the market clearing condition

$$\frac{w}{Z} = \chi\left[N(1+\eta)f(H_1) + (1-N)\,f(H_0) + A\delta\right], \tag{9}$$

which depends only on the normalized wage, $\bar{w} = w/Z$ since H_1 and H_0 are functions only of $\bar{w}$. It is obvious that this has a unique solution for $\bar{w}$, from which we easily determine the rest of the endogenous variables.

As an example, suppose $f(H) = 1 - \exp(-H)$. Then profit maximization implies $f(H_1) = 1 - w/Z(1+\eta)$ and $f(H_0) = 1 - w/Z$. This makes supply linear, $S(w) = Z\,(1 + N\eta + \delta A) - w$, so we can solve explicitly for the normalized wage

$$\bar{w} = \frac{\chi(1 + N\eta + A\delta)}{1+\chi}.$$

From this we get $c = w/\chi$, and the rest of the endogenous variables. Although this example is particularly easy, due to linear supply, for any increasing and concave $f(H)$ the results are basically the same. In general, the growth rate g is given by (4), which depends on technological parameters plus the number of ideas successfully implemented, $N = \bar{n}_i \mathbb{E}\sigma$, which in this simplest benchmark depends only on $\bar{n}_i$ and the distribution F_i. As the average match between ideas and implementation skills, parameterized by F_i, improves, g increases, along with w and c. Improvement in the overall

[8] As is standard, balanced growth requires either $U = \log(c) + v(h)$ or $U = c^{\varsigma} v(h)$, where $v(h)$ satisfies the usual assumptions. We have already assumed U is separable, so we need $u(c) = \log(c)$. See Waller (2010) for a recent discussion of balanced growth in related models.

quality of ideas, captured by the distance they move knowledge η, has similar effects. An increase in the effective stock of assets, $A\delta$, raises c and w, through a wealth effect, but not g.

This basic framework can be put to work even before introducing technology transfer. Consider giving potential innovators a choice over whether to participate in research-related activity at cost κ_i. Let the number of *active* innovators be $n_i \in [0, \bar{n}_i]$. The probability of success is $\mathbb{E}\sigma$, and the gain normalized by Z is $\Delta = (\pi_1 - \pi_0)/Z$, with $\pi_1 = Z(1+\eta)f(H_1) - wH_1$ and $\pi_0 = Zf(H_0) - wH_0$. Since (5) implies that W is linear in wealth with slope χ/w, the expected gain from a successful innovation is $\bar{\kappa}_i = \Delta\chi\mathbb{E}\sigma/\bar{w}$, and the number of innovators involved in active research satisfies

$$n_i = \begin{cases} 0 & \text{if } \kappa_i > \bar{\kappa}_i \\ [0, \bar{n}_i] & \text{if } \kappa_i = \bar{\kappa}_i \\ \bar{n}_i & \text{if } \kappa_i < \bar{\kappa}_i \end{cases} \tag{10}$$

Equilibrium is characterized by two curves shown in $(n_i, \bar{w})$ space in Figure 1, one representing entry (10) and the other representing market clearing (9), where now $N = n_i\mathbb{E}\sigma$ since only $n_i \leq \bar{n}_i$ potential innovators are active. Here the entry condition gives a horizontal line at $\bar{w} = \Delta\chi\mathbb{E}\sigma/\kappa_i$, while the market clearing curve is strictly increasing. Equilibrium exists uniquely, and it is easy to see how it varies with parameters. Assuming an interior equilibrium, $n_i \in (0, \bar{n}_i)$, an increase in κ_i shifts the entry curve down, reducing research-related activity n_i and hence growth. So does an increase in $A\delta$, this time through a shift in the market clearing condition. In terms of employment, it is easy to check that an increase in κ_i raises both H_0 and H_1, but not necessarily $H = NH_1 + (1-N)H_0$ because N falls. These and several other results are summarized in Table A.

	H_0	H_1	$\bar{w}$	n_i	c	ϕ	g	N
$\kappa_i \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	0	$\downarrow$	$\downarrow$
$A \uparrow$	0	0	0	$\downarrow$	0	0	$\downarrow$	$\downarrow$
$\delta \uparrow$	0	0	0	$\downarrow$	0	$\uparrow$	$\downarrow$	$\downarrow$
$\eta \uparrow$	$\downarrow$	?	$\uparrow$	?	$\uparrow$	0	?	?
$\chi \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	?	?	0	?	?
$\beta \uparrow$	0	0	0	0	0	$\uparrow$	0	0

Table A: Effects of Parameters in Basic Model

In terms of welfare, there is no presumption that equilibrium is efficient, since knowledge is at

least partially a public good. Consider the planner's problem:

$$\begin{aligned} J(Z) &= \max_{c,H_0,H_1,n_i} \{u(c) - \chi\left[NH_1 + (1-N)H_0\right] - \kappa_i n_i + \beta J\left[G(N)Z\right]\} \\ \text{st } c &= NZ(1+\eta)f(H_1) + (1-N)Zf(H_0) + A\delta Z, \\ n_i &\in [0, \bar{n}_i],\ N = n_i \mathbb{E}\sigma. \end{aligned} \tag{11}$$

The FOC for n_i implies

$$\kappa_i = \left\{u'(c)\left[Z(1+\eta)f(H_1) - Zf(H_0)\right] - \chi(H_1 - H_0) + \beta V'(Z')G'(N)Z\right\}\mathbb{E}\sigma. \tag{12}$$

The RHS of (12) is the marginal social benefit of innovative activity: the gain due to higher short-run output $u'(c)[Z(1+\eta)f(H_1) - Zf(H_0)]$, net of the change in labor cost $\chi(H_1 - H_0)$, plus the discounted benefit of better knowledge in the future $\beta V'(Z')G'(N)Z$, all multiplied by the probability a representative innovator is successful $\mathbb{E}\sigma$. The EC is

$$J'(Z) = \frac{1}{Z} + \beta J'(Z')G(N) = \frac{1}{Z} + \frac{\beta G(N)}{Z(1+g)} + \frac{\beta^2 G(N)^2}{Z(1+g)^2} + ... = \frac{1}{Z(1-\beta)}. \tag{13}$$

Note how (13) takes account of the fact that knowledges last forever. Combining (13) and (12), assuming an interior solution, the optimal number of active innovators satisfies

$$\kappa_i = \left[u'(c)Z\Delta + \frac{G'(N)}{rG(N)}\right]\mathbb{E}\sigma \tag{14}$$

where $r = 1/\beta - 1$ is the rate of time preference. The analogous equilibrium condition is $\kappa_i = u'(c)Z\Delta\mathbb{E}\sigma$, which generates too little entry, because in their private calculus innovators ignore the permanent external impact of knowledge. To correct this one can introduce a subsidy τ_i that reduces the cost of innovative activity to $\kappa_i - \tau_i$, financed by a lump-sump tax which (with quasi-linear utility) affects leisure but no other interesting variables. The following is now obvious from the discussion.

Proposition 1 *There exists a unique equilibrium with balanced growth at rate g given by (4), either with n_i fixed or with entry. Equilibrium is generally inefficient without intervention. The optimal policy, which yields the same outcome as the planner's problem, involves a research subsidy given by*

$$\tau_i = \frac{G'(N)\mathbb{E}\sigma}{rG(N)} > 0.$$

3 Technology Transfer with Perfect Credit

Having described existence, uniqueness, the effects of parameter changes and welfare in the baseline model, we now consider technology transfer by introducing entrepreneurs, who do not come up with ideas on their own, but may have a comparative advantage in implementation. The measures of innovators and entrepreneurs are $\bar{n}_i$ and $\bar{n}_e \leq 1-\bar{n}_i$; we can also have $1-\bar{n}_i-\bar{n}_e$ agents who work and consume but do not get involved in either innovative or entrepreneurial activity. Now, each period, just before the opening of the frictionless centralized market, there convenes a decentralized market for ideas where entrepreneurs and innovators meet bilaterally according to a standard Pissarides (2000) matching technology, giving the number of meetings $\mu(\bar{n}_i, \bar{n}_e)$ as a function of the measures of agents. There can be gains from trade because e may be better at implementing some ideas than i. Thus, suppose i has an idea that succeeds with probability σ_i drawn from $F_i(\sigma_i)$, and he meets e; if e takes over implementation, he succeeds with probability σ_e drawn from $F_e(\sigma_e|\sigma_i)$, where by assumption i and e both observe (σ_i, σ_e).

A meeting occurs for e in the idea market with probability $\alpha_e = \mu(\bar{n}_i, \bar{n}_e)/\bar{n}_e$, and similarly for i. Thus, with probability $1-\alpha_e$, e cannot trade, because he meets no one, but we can alternatively interpret this in terms of informational frictions by assuming e may not know (σ_i, σ_e). Suppose that sometimes e meets someone with an idea outside his area of expertise, whence he may *choose* to not trade, lest he get a bad idea (a lemon). If anyone can come up with a bad idea for free, then, as in Lester et al. (2011), e will never trade for something he cannot evaluate. Thus we reinterpret search in terms of private information, with σ_e being the probability e meets i times the probability he can evaluate the idea. This story is simplistic, but more sophisticated versions in related search-and-bargaining models are studied in recent papers by Li and Rocheteau (2009) and Rocheteau (2009), who also provide references to much earlier work. While more should be done modeling information explicitly, here we proceed as if (σ_i, σ_e) is known in every meeting.

In any case, at the beginning of each period, all agents see the aggregate state of knowledge Z and then innovators come up with new ideas – i.e., they draw σ_i. Then the matching begins. In any meeting where there are gains from trade, $\sigma_e > \sigma_i$, the parties i and e bargain over a payment

p that the latter will make to the former in the next centralized market. For now we abstract from liquidity considerations by assuming that e can commit to any payment in the relevant range. To determine p, we use the generalized Nash solution, with θ denoting the bargaining power of e. The linearity of W reduces this to

$$p = \arg\max \left[\sigma_e \Delta - p\right]^{\theta} \left[p - \sigma_i \Delta\right]^{1-\theta}.$$

This is easily solved for

$$p = p(\sigma_e, \sigma_i) = \Delta[\theta\sigma_i + (1-\theta)\sigma_e], \tag{15}$$

indicating that p is an average of the success probabilities, times the value of innovation defined above as $\Delta = (\pi_1 - \pi_0)/Z$.

Whoever takes the idea out of the market then tries to implement it, improving individual productivity from $z = Z$ to $z = Z(1+\eta)$ if successful. To reduce notation we assume that ideas are rival goods, in the short run, in the sense that if i trades an idea he cannot also try to implement it (one can easily allow ideas to be nonrival, or partially nonrival, even in the short run without changing the qualitative results in these kinds of models; see Silveira and Wright 2010). After the idea market closes, agents enter the centralized market and solve the problem in Section 2, summarized by (5), except for two new features. First, we have to add or subtract from income any payment an agent is owed or owes from the previous idea market, but just like a lump sum tax only this effects leisure. Second, we have to index the value functions by type. Using (15), for entrepreneurs and innovators, resp., we have

$$V^i(a,Z) = W^i(a,Z;Z) + \mathbb{E}\sigma_i \frac{\chi}{\bar{w}}\Delta + \alpha_i(1-\theta)\frac{\chi}{\bar{w}}\Delta\hat{\mathbb{E}}(\sigma_e - \sigma_i) \tag{16}$$

$$V^e(a,Z) = W^e(a,Z;Z) + \alpha_e\theta\frac{\chi}{\bar{w}}\Delta\hat{\mathbb{E}}(\sigma_e - \sigma_i). \tag{17}$$

where

$$\hat{\mathbb{E}}(\sigma_e - \sigma_i) = \mathbb{E}(\sigma_e - \sigma_i | \sigma_e > \sigma_i)\Pr(\sigma_e > \sigma_i) = \int_0^1 \int_{\sigma_i}^1 (\sigma_e - \sigma_i) dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i).$$

Compared with (7), i can still try to implement his own idea, but he may sell it to e, the expected surplus from which is the last term in (16); meanwhile, e only gets ideas via trade, although one could alternatively allow him to come up with some on his own, too.

Other than opening up trade in ideas, everything is the same as the benchmark model in Section 2. The only equilibrium condition that changes is the number of successful innovations,

$$N = \bar{n}_i \mathbb{E}\sigma_i + \bar{n}_e \alpha_e \hat{\mathbb{E}}(\sigma_e - \sigma_i),$$

where the first term is the number of success when ideas are implemented by innovators and the second captures the additional successes gained by technology transfer in any match where $\sigma_e > \sigma_i$. The growth rate is still $1 + g = G(N)$, although N is different. Thus, growth now depends on the measures $\bar{n}_i$ and $\bar{n}_e$, the distributions F_i and F_e, and the matching function μ. Notice that g is independent of θ and δA here, although this will change when we introduce credit frictions.[9]

As above we can consider endogenous entry. In fact, since $f(H)$ is concave, we can have two-sided entry, which is not the case in the typical search model (e.g., Pissarides 2000) that has a linear technology. Thus, both i and e choose whether to participate in the idea market, at costs κ_i and κ_e, resp. The measure of active innovators n_i still satisfies (10) from Section 2, except that now $\bar{\kappa}_i = u'(c) Z \Delta [\mathbb{E}\sigma_i + (1-\theta)\frac{\mu(n_i,n_e)}{n_i} \hat{\mathbb{E}}(\sigma_e - \sigma_i)]$, and the measure of active entrepreneurs n_e satisfies

$$n_e = \begin{cases} 0 & \text{if } \kappa_e > \bar{\kappa}_e \\ [0, \bar{n}_e] & \text{if } \kappa_e = \bar{\kappa}_e \\ \bar{n}_e & \text{if } \kappa_e < \bar{\kappa}_e \end{cases} \tag{18}$$

with $\bar{\kappa}_e = u'(c) Z \Delta \theta \frac{\mu(n_i,n_e)}{n_e} \hat{\mathbb{E}}(\sigma_e - \sigma_i)$. Equilibrium is characterized by (18) and (10), plus market clearing (9) with $N = n_i \mathbb{E}\sigma_i + \mu(n_i, n_e) \hat{\mathbb{E}}(\sigma_e - \sigma_i)$. In Appendix 2 we show there exists a unique interior equilibrium, $n_e \in (0, \bar{n}_e)$ and $n_i \in (0, \bar{n}_i)$, as long as κ_i and κ_e are neither too high nor low.

In terms of efficiency, the generalized version of the planner's problem in (11) is

$$\begin{aligned} J(Z) &= \max_{c, H_0, H_1, n_i, n_e} \{u(c) - \chi [N H_1 + (1-N) H_0] - \kappa_i n_i - \kappa_e n_e + \beta J [G(N) Z]\} \qquad (19) \\ \text{st } c &= N Z (1+\eta) f(H_1) + (1-N) Z f(H_0) + \delta Z A, \\ n_i &\in [0, \bar{n}_i],\ n_e \in [0, \bar{n}_e],\ N = n_i \mathbb{E}\sigma_i + \mu(n_i, n_e) \hat{\mathbb{E}}(\sigma_e - \sigma_i). \end{aligned}$$

Here the planner takes as given the matching process, and that agents trade ideas iff $\sigma_e > \sigma_i$, as

[9] It is easy to work out examples generalizing those in Section 2, with $G(N) = \rho [N(1+\eta)^\varepsilon + 1 - N]^{1/\varepsilon}$ and $f(H) = 1 - \exp(-H)$. For F_i and F_e, suppose that $\sigma_e = 1$ with probability 1 while σ_i is uniform on $[0, 1]$. Then we have: (i) if $\varepsilon = 1$ then $g = \rho\eta[n_i + \mu(n_i, n_e)]/2 - (1-\rho)$; (ii) if $\varepsilon = \infty$ then $g = \rho(1+\eta) - 2$; and (iii) if $\varepsilon = -\infty$ then $g = \rho - 1$.

they should. He also takes as given that the payment p is determined by bargaining with parameter θ, and can only choose participation in the idea market (plus consumption and employment in the centralized market). Assuming an interior solution, we get the FOC for n_e and n_i:

$$\begin{aligned}\kappa_e &= \left[u'(c)Z\Delta + \frac{G'(N)}{rG(N)}\right]\mu_e(n_i,n_e)\hat{\mathbb{E}}(\sigma_e - \sigma_i)\\ \kappa_i &= \left[u'(c)Z\Delta + \frac{G'(N)}{rG(N)}\right][\mathbb{E}\sigma_i + \mu_i(n_i,n_e)\hat{\mathbb{E}}(\sigma_e - \sigma_i)]\end{aligned}$$

Comparing these with the relevant equilibrium conditions, we can find the optimal corrective subsidies. Summarizing the results, we have:

Proposition 2 *As long as κ_i and κ_e are neither too high nor too low, there exists a unique interior equilibrium with two-sided entry. Equilibrium is generally inefficient without intervention. The optimal policy, which yields the same outcome as the planner's problem, involves subsidies:*

$$\begin{aligned}\tau_e &= \frac{G'(N)\mu_e(n_i,n_e)\hat{\mathbb{E}}(\sigma_e - \sigma_i)}{rG(N)} - u'(c)\,Z\Delta\hat{\mathbb{E}}(\sigma_e - \sigma_i)\left[\theta\frac{\mu(n_i,n_e)}{n_e} - \mu_e(n_i,n_e)\right]\\ \tau_i &= \frac{G'(N)[\mathbb{E}\sigma_i + \mu_i(n_i,n_e)\hat{\mathbb{E}}(\sigma_e - \sigma_i)]}{rG(N)} - u'(c)\,Z\Delta\hat{\mathbb{E}}(\sigma_e - \sigma_i)\left[(1-\theta)\frac{\mu(n_i,n_e)}{n_i} - \mu_i(n_i,n_e)\right]\end{aligned}$$

To explain the policy results, note than in addition to inefficiencies due to knowledge externalities discussed in Section 2, there are now also search externalities, depending on idea-market tightness n_e/n_i. Knowledge externalities are captured by the first terms in the optimal subsidies, as in Proposition 1. Search externalities are captured by the second terms, which build on the usual Hosios (1990) conditions requiring agents' bargaining power to be commensurate with their contribution to the matching process. For entrepreneurs this means $\theta = \mu_e n_e/\mu$, and for innovators $1-\theta = \mu_i n_i/\mu$. Constant returns in the matching function implies that one holds iff the other holds, so the Hosios condition generates efficient participation by both i and e. Even if θ satisfies the Hosios condition, we still want to subsidize participation due to knowledge externalities; if the Hosios condition fails, policy has to balance search versus knowledge externalities.

Although two-sided entry is interesting, consider the special case where $\bar{n}_e$ is fixed – say, all entrepreneurs participate because $\kappa_e = 0$ – so that we can more easily compare results with those in Section 2 where the idea market was shut down. The difference can be seen in Figure 1, where the participation condition (10) is given by a horizontal line at $\bar{w} = \Delta\chi\mathbb{E}\sigma/\kappa_i$ when the idea market

is shut down, but now slopes downward because entry causes congestion, reducing e's idea-market arrival rate and hence the return to innovation. The other equilibrium condition, market clearing (9), still generates a strictly increasing curve. We again have the existence of a unique equilibrium, and one can show that the qualitative effects of the parameter changes shown in Table A are exactly the same in this model. However, there are also new effects when the idea market is open, related to search-and-bargaining parameters.

An increase in entrepreneurs' bargaining power θ lowers the return to innovation, shifting down the curve defined by i's entry condition. This reduces n_i, and hence N, g, $\bar{w}$ and c. Additionally, increasing the matching rate, either because the technology $\mu(n_i, n_e)$ improves or we increase the measure of entrepreneurs, shifts up both curves. This increases $\bar{w}$, which lowers H_0 and H_1, and then (9) implies N unambiguously increases. Hence we have higher growth, despite the fact that n_i may go up or down when we increase the matching rate. When $\theta = 1$, e.g., innovators get no return from idea trade, due to a standard holdup problem: at the time of the bargaining, κ_i is a sunk cost, and does not affect p. Of course this holdup problem occurs for any $\theta > 0$, but in the extreme case $\theta = 1$ the entry curve is again horizontal, as in Figure 1, even though the idea market is open. An increase in the matching rate in this case implies n_i must fall, while $\bar{w}$ does not change, and so N does not change, by (9). This complete-crowding-out effect, with the fall in n_i exactly offsetting the improvement in matching, requires $\theta = 1$, but it illustrates how holdup problems in bargaining generally affect the return to and hence the amount of innovative activity.

The optimal subsidy τ_i in the one-sided entry model is still given by Proposition 2. Again the knowledge externality implies n_i is too low and should be subsidized, but if θ is below the value given by the Hosios condition we may want to tax entry to reduce congestion, with the optimal policy balancing knowledge and search externalities. One result we highlight is that increasing the efficiency of the matching technology – a reduction in search frictions – necessarily improves the allocation emerging from the idea market, and hence the implementation of new technologies, even though it may reduce the initial generation of ideas by crowding out n_i. The extent to which this happens depends on interactions between search and other frictions, including the holdup problem. For the record we summarize the main results with one-sided entry as follows:

Proposition 3 *As long as κ_i is neither too high nor too low, there exists a unique interior equilibrium with one-sided entry. Equilibrium is generally inefficient and the optimal policy, which yields the same outcome as the planner's problem, involves a subsidy τ_i as in Proposition 2.*

4 Technology Transfer with Imperfect Credit

To begin, assume that n_i and n_e are fixed, and as a preliminary step consider an exogenous credit constraint: when e meets i in the idea market, we impose $p \leq x$. There are at least two interpretations. One is that i insists on *quid pro quo*, e is holding transferable assets worth x, and he cannot hand over more than he has (as in many monetary models; see Williamson and Wright 2010 for a survey). Another is that e can secure a loan from i – trade credit – to be paid off in the next centralized market, but only up to the value x of his assets that he can pledge as collateral (as in many imperfect credit models; see Gertler and Kiyotaki 2010 for a survey). On the first interpretation there is final settlement when ideas are traded. On the second interpretation there is deferred settlement, with e either paying off his debt in the next centralized market, or, equivalently, surrendering collateral of the same value. Other than the timing of settlement, nothing of substance depends on which interpretation one adopts here.

In any case, for an idea-market trade to occur, once i and e have met two conditions now also have to be met: e must have comparative advantage in terms of implementation, $\sigma_i \leq \sigma_e$; and x must be big enough to cover i's reservation price $\sigma_i \Delta$. Thus, we need $\sigma_i \leq \min\{\sigma_e, x/\Delta\}$. If the bargaining solution derived without liquidity constraints in Section 3 satisfies $p \leq x$, then we set $p = \Delta\left[\theta\sigma_i + (1-\theta)\sigma_e\right]$, as before. It is easy to check that p satisfies the constraint iff

$$\sigma_e \leq B\left(\sigma_i, \frac{x}{\Delta}\right) \equiv \frac{1}{1-\theta}\left(\frac{x}{\Delta} - \theta\sigma_i\right). \tag{20}$$

When $\sigma_e > B(\sigma_i, x/\Delta)$, the unconstrained p is infeasible, and we have the following: if $x/\Delta \geq \sigma_i$ they close the deal with e paying $\bar{p} = x < p$; and if $x/\Delta < \sigma_i$ there is no trade because x does not cover i's reservation price. This is illustrated in Figure 2. There is no trade in the region labeled $\mathcal{A}_0$ because $\sigma_i > \sigma_e$ means there are no gains from trade. There is no trade in $\mathcal{A}_3$ because e cannot meet i's reservation price. There is unconstrained trade in $\mathcal{A}_1$, where e pays p. And there is constrained

trade in $\mathcal{A}_2$, where e pays $\bar{p} = x$.[10]

Market supply and demand for goods are the same as before, given N, but now

$$N = \bar{n}_i \mathbb{E}\sigma_i + \bar{n}_i \alpha_i \bar{\mathbb{E}}(\sigma_e - \sigma_i; x), \tag{21}$$

where

$$\bar{\mathbb{E}}(\sigma_e - \sigma_i; x) = \mathbb{E}\left(\sigma_e - \sigma_i | \min\{\sigma_e, x/\Delta\} > \sigma_i\right) \Pr\left(\min\{\sigma_e, x/\Delta\} > \sigma_i\right).$$

We still write supply and demand as in Section 2, but now there is an additional effect on supply coming through N, since Δ generally depends on x and $\bar{w}$. To see this, after simplification, one can derive

$$S'(w) = N\frac{f'(H_1)}{f''(H_1)} + (1-N)\frac{f'(H_0)}{f''(H_0)} + Z[(1+\eta)f(H_1) - f(H_0)]\frac{dN}{dw}. \tag{22}$$

The first two terms capture the standard result that, holding N fixed, higher w lowers hours and output. The final term is positive, however, because higher w relaxes the liquidity constraint, spurring trade and hence innovation, which can potentially lead to multiple equilibria.[11] In Appendix 3 we provide an explicit example to show multiplicity can arise, but also note that $S'(w) < 0$, and hence equilibrium must be unique, if η is not too big, as we assume for the present analysis.

Having described the outcome for a fixed x, we now want to make it a choice. First, from the total stock A, assume that a fraction $A_1 = \gamma A$ of assets are *liquid* in the precise sense that they can

[10] Here we simply impose a particular bargaining protocol: use Nash if the buyer can afford it; else, have him offer all he has. One could in principle try to adapt the axioms in cooperative bargaining theory to generate this type of outcome as a result (see, e.g., the survey in Thomson 1994). Or one can write down simple strategic models where it emerges as an equilibrium. Also, we note that the results here are in part due to the assumption that an idea is indivisible: i can neither trade part of it, nor trade it with probably less than 1 using a lottery (which would, by the way, reduce the problem to standard Nash bargaining by convexifying payoff space). But as in Silveira and Wright (2010) one can show that the main results go through if one relaxes these assumptions, albeit at the expense of simplicity and notation. With lotteries, i gets paid first and then transfers the idea with some probability; this allows i to get more out of the market, but there are still some meetings where he inefficiently keeps idea to himself even though $\sigma_e > \sigma_i$. Similarly, if ideas were divisible, i does not transfer enough information to e; the difference here is merely whether the inefficiency occurs on the intensive or extensive margin.

[11] The economics is as follows. When w is higher, individual innovators have less to gain from improving productivity. We are saying more than the obvious result that profit falls with w, we are saying the *difference* between profit at innovative and uninnovative firms $\pi_1 - \pi_0$ falls. This lowers i's reservation price, other things equal, making it more likely that e has enough liquidity to buy him out, thus increasing the probability of successful implementation. Through this channel higher wages might lead to more innovation, and since more innovation also leads to higher wages, multiplicity can arise.

be used to facilitate trade in the idea market – i.e., they are transferable, or pledgeable., depending on the interpretation as discussed in the first paragraph of this Section. While the stock A_1 may be exogenous, the price and hence the value of liquid assets is endogenous, and this is what matters for trade, since we now constrain p by $x = (\phi + Z\delta)\, a_1'/Z$. Other than A_1, the remaining $A_0 = (1-\gamma)A$ assets are illiquid, and do not facilitate idea-market trade, although they can always be traded in the frictionless centralized market. While it is certainly interesting, and for many issues, essential, to ask *why* certain assets can or cannot be traded in certain markets, much good work for all intents and purposes simply assumes this is the case (e.g., Kiyotaki and Moore 1997 or Holmstrom and Tirole 2010). While one can try to model this at a deeper level, based on intrinsic properties of assets like portability and recognizability, this is not the place to go into that.[12]

The dividend on both A_0 and A_1 is still δ (with no loss of generality), and if the price of A_j is ϕ_j, its gross return is

$$1 + r_j = \frac{\phi_j' + Z'\delta}{\phi_j}. \tag{23}$$

As is standard, the illiquid asset A_0 must trade at the fundamental price $\phi_0 = \beta\delta Z/(1-\beta)$, which means $1 + r_0 = (1+g)/\beta$. This is not necessarily true for the liquid asset A_1, however, as we shall soon see. Therefore, we define the *spread* or *liquidity premium* by

$$s \equiv \frac{r_0 - r_1}{1 + r_1} = \frac{(1+g)\phi_1}{\beta(\phi_1' + Z'\delta)} - 1, \tag{24}$$

which is the cost of being liquid: it is the rate of return one sacrifices by holding A_1 rather than A_0.

The bargaining outcome is still described by Figure 2, with $x = (\phi + Z\delta)\, a_1'/Z$ now endogenous, but predetermined at the time of the meeting. In equilibrium the price of intermediate goods is still Z, the price of the illiquid asset A_0 is still $\phi_0 = Z\delta\beta/(1-\beta)$, and goods market clearing is still described by (9) with N given by (21). The new equilibrium condition concerns the market for A_1, which clears when the spread s equates demand and supply for liquid assets. In terms of demand, consider first agents who are *not* entrepreneurs (i.e., they are buyers in the next idea market with probability 0). Such agents are willing to hold any amount of A_1 if the spread is $s = 0$, which

[12] Again, see Lester et al. (2011), Rocheteau (2009), and Rocheteau and Li (2009) for recent papers that study this issue using information theory; to be clear, however, we do *not* think this a closed problem.

means $\phi_1 = \phi_0$ is the fundamental price; they demand 0 if $s > 0$; and they want an arbitrarily large position if $s < 0$. In other words, demand coming from these agents is horizontal at $s = 0$.

For entrepreneurs, integrating across the regions in Figure 2, the payoff in the idea market is

$$V^e(a_0, a_1, Z) = W^e(a_0, a_1, Z; Z) + \alpha_e \theta \frac{\chi}{w} \int_{\mathcal{A}_1} (\sigma_e - \sigma_i) Z\Delta + \alpha_e \frac{\chi}{w} \int_{\mathcal{A}_2} [\sigma_e Z\Delta - a_1(\phi_1 + Z\delta)] \quad (25)$$

(see Appendix 4 for details). Notice a_1 affects the area of the different $\mathcal{A}_j$ regions, and hence the probability of trade, as well as the terms of trade when the constraint binds, as seen in the integrand of the last term. It is convenient to redefine e's centralized market choice as $x = a_1' (\phi_1 + Z\delta)/Z$, rather than a_1', analogous to using real rather than nominal balances in monetary theory. Also, the choice of a_0' is actually irrelevant for e's payoff, given illiquid assets are priced fundamentally, so we can ignore it. Hence, we can rewrite the relevant part of e's problem (5) as (again, see Appendix 4)

$$\begin{aligned} \max_{a_1'} \left\{ \beta V^e(0, a_1', Z') - \frac{\chi}{w} \phi a_1' \right\} &= \max_x \left\{ -sx + \alpha_e \theta \int_0^{\frac{x}{\Delta}} \int_{\sigma_i}^{B(\sigma_i, \frac{x}{\Delta})} (\sigma_e - \sigma_i) \Delta dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i) \right. \\ &\quad \left. + \alpha_e \int_0^{\frac{x}{\Delta}} \int_{B(\sigma_i, \frac{x}{\Delta})}^1 (\sigma_e \Delta - x) \, dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i) \right\}. \quad (26) \end{aligned}$$

Maximizing wrt x, using Leibniz Rule and a little algebra, we get the FOC $s = \ell(x)$ where

$$\ell(x) \equiv \alpha_e F_i' \left(\frac{x}{\Delta} \right) \int_{\frac{x}{\Delta}}^1 \left(\sigma_e - \frac{x}{\Delta} \right) dF_e \left(\sigma_e | \frac{x}{\Delta} \right) - \alpha_e \int_0^{\frac{x}{\Delta}} \left\{ 1 - F_e \left[B \left(\sigma_i, \frac{x}{\Delta} \right) | \sigma_i \right] \right\} dF_i(\sigma_i). \quad (27)$$

In words, $\ell(x)$ is e's marginal benefit of liquidity: the first term gives the increase in his expected payoff from not losing a deal because he cannot meet the reservation price, $x < \sigma_i/\Delta$; the second gives the decrease from paying a higher price when he could have done the deal at $\bar{p} = x$. The FOC equates this to marginal cost s, subject to some details concerning the SOC, or the concavity of the objective function in (26), which we deal with in the next footnote.

It is now straightforward to describe the (inverse) market demand curve for the liquid asset, say $L(x)$, in (x, s) space. If $s > 0$ then entrepreneurs want to hold the x that solves $s = \ell(x)$, and other agents want 0. When $s = 0$, e is satiated in liquidity at $x(0)$, at which point the second term in (27) dominates the first (this must be the case, e.g., for any $x \geq \Delta$, since the highest possible reservation price for an innovator i obtains when $\sigma_i = 1$). In this case entrepreneurs in aggregate hold $n_e x(0)$ units of the liquid asset, and, if there is any left, others hold the rest $A_1 - n_e x(0)$, which

they are happy to do at $s = 0$. And if s is big enough there is no $x > 0$ satisfying $s = \ell(s)$, so even entrepreneurs demand $x = 0$. Summarizing, one can show that demand by entrepreneurs is strictly downward sloping in (x, s) space, except possibly for some horizontal segments; it hits the s-axis at s_0; and it hits the x-axis at $x(0)$.[13] The result that entrepreneurs may be satiated in liquidity at $x(0)$, because any additional x would only increase the price in some meetings, can be attributed to a well-known property of Nash bargaining that buyers' payoffs may decrease when we relax their constraints (see, e.g., Aruoba et al. 2007). This happens when the real value of liquidity is more than entrepreneurs demand, in which case the market drives the spread to $s = 0$.

Moving to the supply side of this market, all we have to do is some accounting. Using the definitions of x and the returns r_0 and r_1, and setting $\bar{n}_e a_e = A_1$ for all $s > 0$, we can write

$$s = s(x) = \frac{\phi_1 a_e}{\beta x} - 1 = \frac{x - \delta A_1/\bar{n}_e}{\beta x} - 1. \tag{28}$$

This (inverse) supply relation is the spread (equivalent, the asset price ϕ_1) required to make the real value of A_1 equal to x. Notice $s(0) = -\infty$, $s'(x) > 0$, $s''(x) < 0$ and $s(\infty) = r$, where again r is the rate of time preference. Combining supply and demand, the asset market clears uniquely when $AM(x, \bar{w}) = 0$, where

$$AM(x, \bar{w}) \equiv s(x) - L\left[x/\Delta(\bar{w})\right]. \tag{29}$$

From (29) we get a unique x for any $\bar{w}$, with $\partial x/\partial \bar{w} < 0$.

Asset market equilibrium is shown in Figure 3, for different values of θ that translate into different demand, and different values of $A\delta$ that translate into different supply, with parameter values given in the Figure.[14] Notice that $\ell(x)$ can become negative, but market demand $L(x)$ is truncated by the

[13] A detailed proof can be found in Wright (2010). Note that the argument does not use the concavity of the objective function, which is difficult to imagine verifying in a model with generalized Nash (rather than linear) pricing. The proof proceeds by noting that while there may be multiple solutions to the FOC when the objective function is not concave, generically there is a unique global maximizer. The horizontal segments in the (inverse) demand curve obtain at nongeneric values of s that yield multiple global maximizers. In this case, as is standard, we can make market demand continuous by assigning any measure we like of entrepreurs to different global maximizers.

[14] As an aside, if one knows search-and-bargaining models in monetary theory, one may be perplexed that we can have $s > 0$ when $\theta = 0$, as is the case in the Figure if we reduce supply a little more. Why is e willing to pay a liquidity premium when he has no bargaining power, and hence, presumably, gets no surplus from trade in the idea market? The resolution of this obstensive puzzle is that he actually *does* get positive surplus from some trades in the idea market – those where he is constrained, and pays only $\bar{p} = x < p$.

horizontal axis, since $s < 0$ always implies excess demand. Clearly, from the Figure, e can be satiated below the value of x that guarantees he can close the deal in every idea-market meeting, which in this example is $x = 1$. This can only occur if $\theta < 1$, however; if $\theta = 1$ then $s = 0$ implies e chooses x so that he has enough liquidity to close the deal with probability 1. Intuitively, this is because liquidity is free at $s = 0$, and the price does not depend on x when $\theta = 1$. These findings are related to some results in pure monetary theory (e.g., Lagos and Wright 2005) that can be understood as follows. When e holds liquid assets he is making an investment, with sunk cost s, his loss in rate of return. If $\theta < 1$, he has to share the surplus generated by this investment with i, another hold up problem. Hence, he under invests unless $\theta = 1$. Notice that $\theta = 1$ does not generally satisfy the Hosios condition, however, so with endogenous entry there is no way to achieve full efficiency simply by picking θ, without even accounting for knowledge externalities.

It is clear that if the supply of liquid assets is above some threshold, $A_1 > A_1^*$, say, where the exact value of A_1^* is given in Proposition 4 below, then $s = 0$; and if $A_1 < A_1^*$, then liquidity commands a premium, $s > 0$. This is also similar to results in monetary theory (e.g., Geromichalos et al. 2007 or Lagos and Rocheteau 2008); the contribution here is more about deriving the implications for innovation and growth. To pursue this, recall the usual goods market clearing condition (9), which we reproduce as $GM(x, \bar{w}) = 0$ with

$$GM(x, \bar{w}) \equiv \frac{\bar{w}}{\chi} - N(1 + \eta) f\left[H_1(\bar{w})\right] - (1 - N) f\left[H_0(\bar{w})\right] - A\delta. \tag{30}$$

One can check that, as long as η is not too big, this delivers x as a function of $\bar{w}$, with $\partial x / \partial \bar{w} \geq 0$. Equilibrium is characterized by $(x, \bar{w})$ satisfying asset- and goods-market clearing, (29)-(30), from which we can easily find c, g etc. As shown in Figure 4, existence and uniqueness are apparent, at least as long as η is not too big, as mentioned above.

	H_0	H_1	$\bar{w}$	x	s	c	g	N
$\gamma \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
$A_0 \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	?	?
$\eta \uparrow$	$\downarrow$	?	$\uparrow$	?	?	$\uparrow$	?	?
$\chi \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	?	?	?
$\beta \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
$\theta \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

Table B: Effects of Parameters with Imperfect Credit

We report the effects of parameters in Table B, for the case $A_1 < A_1^*$, where liquidity is scarce. An increase in θ, e.g., shifts the AM curve up while GM is unaffected. This increases x, $\bar{w}$ and, consequently, growth. Intuitively, low θ makes e try to economize on liquidity, since he gets less of the idea-market surplus, as mentioned above, and this means he more frequently cannot meet the reservation price, which reduces technology transfer, innovation and growth. One can similarly show growth increases if matching frictions are reduced, or n_i increases, but not necessarily if n_e increases. The key here is that there are two channels at work. Consider a rise in $\bar{n}_i$. This shifts GM and AM up, promoting growth via two effects. First, there are simply more meetings in the idea market, so we get more ideas into the hands of those best able to develop them. Second, since the increase in n_i raises the matching probability α_e, the demand for and price of liquid assets goes up. There is now more real liquidity in the system, making the constraint $p \leq x$ less severe. An increase in n_e, however, while still increasing meetings, reduces rather than increases α_e, which has a negative effect on liquidity via the second channel. In general, whenever there are direct effects on the idea market, there are general equilibrium effects in the asset market that should also be taken into account.

To close this Section we mention that, as in the previous models, one can again consider a participation decision by i. Here the effects depend a lot on bargaining power. In Appendix 5 we show that an equilibrium exists, although we cannot show uniqueness in general. There we also solve for the optimal subsidy, as in previous Sections, and we argue that liquidity actually does not promote growth when θ is too big. We summarize all of these results for this version of the model as follows:

Proposition 4 *With imperfect credit and fixed participation $(\bar{n}_i, \bar{n}_e)$, there exists a unique equilibrium as long as η is not too big. With imperfect credit and entry by i, equilibrium exists and is interior, $n_i \in (0, \bar{n}_i)$, if κ_i is also not too big or too small. Equilibrium is generally inefficient unless three conditions are satisfied: entrepreneurs have bargaining power $\theta = 1$; the supply of liquid assets is abundant, $A_1 \geq A_1^*$ where*

$$A_1^* \equiv (\pi_1 - \pi_0)(1 - \beta)\bar{n}_e/\delta;$$

and, if there is entry by i, the subsidy is set to

$$\tau_i = \frac{G'(N)[\mathbb{E}\sigma_i + \mu_i \hat{\mathbb{E}}(\sigma_e - \sigma_i)]}{rG(N)} - u'(c)Z\Delta\hat{\mathbb{E}}(\sigma_e - \sigma_i)\mu_i.$$

5 Technology Transfer with Intermediation

It is commonly believed that financial development facilitates innovation and growth. To investigate this, we follow Berentsen, Camera and Waller (2007) and Chiu and Meh (2011) by introducing *banks* that operate while the idea market is open. They accept deposits at interest rate r_d and making loans at r_l, although in equilibrium competition yields $r_l = r_d$. Borrowers can commit to repay loans, and banks can commit to repay depositors, in numeraire goods in the next centralized market (one can endogenize repayment, as in Berentsen et al. 2007). After meeting and observing the realization (σ_i, σ_e) in the idea market, e can choose to deposit his assets in, or borrow from, banks to facilitate trade with i. Lack of commitment between e and i means that claims on liquid assets are still needed to trade in the idea market, even with commitment between e and his bank.[15]

[15] By *banks* here we mean any institution generally that facilitates credit and the reallocation of liquidity. If one interprets this as a bank, narrowly, one can think of it issuing liabilities that serve as a payment instrument (inside money) fully backed by deposits of liquid assets. Also, Chiu and Meh (2010) allow a fixed cost ξ to banking, potentially generating a loan-deposit spread $r_l > r_d$, and capture financial development as a reduction in ξ. Here we set $\xi = 0$, implying $r_l = r_d$, and financial development is captured by the emergence of banking, not a reduction in the cost. An alternative way to model financial development implicitly is used in Silveira and Wright (2010), where it is assumed that when e is short of liquidity he can try to raise additional funds, but this only succeeds with probability $1 - \varsigma$, and with probability ς there is an exogenous breakdown and the deal falls through. In this setup, financial development is captured as a reduction in ς, and we approach perfect credit when $\varsigma \to 0$. Here we prefer to model financial activity more explicitly.

For e in the centralized market, we now have

$$\begin{aligned} W(a_1, d, z; Z) &= \frac{\chi}{w}(\phi_1 + \delta Z)a_1 + \max_c \left\{ u(c) - \frac{\chi}{w} c \right\} + \frac{\chi}{w} \max_H \{ zf(H) - wH \} \\ &\quad + \max_{a_1'} \left\{ \beta V^k(a_1', Z') - \frac{\chi}{w}\phi_1 a_1' \right\} - \frac{\chi}{w} Z d(1 + r_d), \end{aligned}$$

which is the same as (5) in the baseline model except for the last term, which gives the real value of debt obligations to a bank d (if one has deposits in the bank then $d < 0$). Without loss in generality, given quasi-linear utility, bank loans are settled every period in the centralized market. Also, as discussed in the previous Section, we set $a_0 = a_0' = 0$ since holdings of illiquid assets are irrelevant for e's payoff when they are priced fundamentally.

In the idea market, after observing (σ_i, σ_e), the parties bargain under the recognition that e can always obtain a loan, which means that he is never literally liquidity constrained, although the intermediary will charge him interest r_d. The outcome is

$$p = \arg\max \left[\sigma_e \Delta - p(1 + r_d) \right]^{\theta} \left[p - \sigma_i \Delta \right]^{1-\theta},$$

since a payment to i of p now entails a cost to e of $(1 + r_d)p$. The solution is

$$p(\sigma_e, \sigma_i) = \Delta \left[\theta \sigma_i + (1 - \theta) \frac{\sigma_e}{1 + r_d} \right].$$

It it easy to show: if $\sigma_e < \sigma_i(1 + r_d)$ then e will deposit x and not trade, because the expected gain does not cover the interest cost; and if $\sigma_e \geq \sigma_i(1 + r_d)$ then e trades, depositing any excess liquidity $x - p(\sigma_e, \sigma_i)$ if $\sigma_e < \bar{B}(\sigma_i, x)$, and borrowing $p(\sigma_e, \sigma_i) - x$ if $\sigma_e > \bar{B}(\sigma_i, x)$, with $\bar{B}(\sigma_i, x)$ generalizing (20) in the previous section and shown in Figure 5:

$$\bar{B}(\sigma_i, x) \equiv \frac{1 + r_d}{1 - \theta} \left(\frac{x}{\Delta} - \theta \sigma_i \right).$$

Now e's choice of x can be written as the generalization of (26):

$$\max_x \left\{ -sx + x r_d + \alpha_e \frac{\chi}{\bar{w}} \theta \Delta \int_0^{\frac{1}{1+r_d}} \int_{\sigma_i(1+r_d)}^{1} \left[\sigma_e - \sigma_i(1 + r_d) \right] dF_e(\sigma_e | \sigma_i) dF_i(\sigma_i) \right\}.$$

Since banking relaxes liquidity constraints, the last term does not depend on the entrepreneur's assets. Market clearing for liquid assets is simply $r_d = s$, where the spread here is the same as the previous Section. Goods market clearing is also the same as before, with

$$N = \bar{n}_i \mathbb{E}(\sigma_i) + \bar{n}_i \alpha_i \mathbb{E} \left[\sigma_e - \sigma_i | \sigma_e > \sigma_i(1 + r_d) \right] \Pr \left[\sigma_e > \sigma_i(1 + r_d) \right],$$

since trade happens iff $\sigma_e > \sigma_i(1+r_d)$. Finally, deposits and loans have to net out, which requires

$$\alpha_e \int_{\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3} p(\sigma_e, \sigma_i) \leq x, \text{ with } = \text{ when } r_d > 0.$$

Summarizing, equilibrium now consists of $(x, r_d, \bar{w})$ satisfying the following: asset market clearing

$$\frac{x - \gamma\delta A/\bar{n}_e}{\beta x} = 1 + r_d; \tag{31}$$

goods market clearing

$$\bar{w} = \chi \left[N(1+\eta) f(H_1) + (1-N) f(H_0) + A\delta \right]; \tag{32}$$

with N given above; and the netting of deposits and loans, which after inserting p can be written

$$\Delta\alpha_e \int_0^{\frac{1}{1+r_d}} \int_{\sigma_i(1+r_d)}^1 \left[\theta\sigma_i + (1-\theta)\frac{\sigma_e}{1+r_d} \right] dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i) = x \text{ for } r_d > 0. \tag{33}$$

We can write goods market clearing (32) as $GM(r_d, \bar{w}) = 0$ in $(r_d, \bar{w})$ space, with $\partial\bar{w}/\partial r_d < 0$. Similarly, (31) and (33) can be written $BM(r_d, \bar{w}) = 0$ with

$$BM(r_d, \bar{w}) \equiv \frac{\gamma\delta A}{\bar{n}_e \left[1 - \beta(1+r_d)\right]} - \Delta\alpha_e \int_0^{\frac{1}{1+r_d}} \int_{\sigma_i(1+r_d)}^1 \left[\theta\sigma_i + (1-\theta)\frac{\sigma_e}{1+r_d} \right] dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i),$$

defining another negative relationship between r and $\bar{w}$. Given these two downward sloping curves, we can show an equilibrium always exists but not that it is unique. There are two types of equilibria. An equilibrium with $r_d = 0$ arises when there is a sufficient supply of liquid assets, in which case ideas are traded in every meeting where $\sigma_e > \sigma_i$, and an equilibrium with $r_d > 0$ arises when liquid assets are scarce. What is important to note is that the relevant threshold for sufficient liquidity is now A_1^{**}, which is below the threshold A_1^* required for efficiency in the economy without banking (see the Proposition 5 below for details).

We emphasize that banking enhances technology trade and hence innovation in two distinct ways. The first and more obvious function concerns the sharing of liquidity, similar to Diamond-Dybvig (1983) banking: a given quantity of liquid assets can be reallocated to those who need it most from those that do not need it, which entrepreneurs cannot do on as effectively on their own, without banks, because they do not know how much liquidity they need before the centralized market closes. When the arrival rate α_e is low, this function is all the more important, because with a low

probability of needing it, e wants to economize all the more on liquidity. This is relevant to the extent that, as some people argue, a shortage of assets is a real problem in the real world (e.g., Caballero 2006). The second and more novel function of banking is that it helps get around the holdup problem associated with investments in liquid assets by allowing entrepreneurs to undo these investments. Intuitively, without banks, when i bargains for a high price, e would like to be able to claim that he shouldn't have to pay so much because he needs to cover his cost, the spread s; but i counters that this is a sunk cost, which leads to a high price and *ex ante* underinvestment in liquidity. When banks are open, however, e has the outside option of depositing his assets, which in equilibrium earns $r_d = s$, and therefore the cost is, in fact, *not* sunk!

Of course, not everyone can do this, since deposits can exceed loans only if $r_d = 0$, but since each individual behaves competitively with respect to banking, the threat by e of putting his money in the bank and earning the going rate is credible in bilateral negotiations. When θ is low, this is all the more important, because then the holdup problem is all the more severe. This effect has not been discussed, to our knowledge, before in this context, since the related papers on intermediation and liquidity in similar models assume competitive markets without holdup problems. We think that bargaining is especially pertinent for trade in the idea market, which is sufficiently specialized and thin that the competitive price-taking hypothesis seems less than compelling. Therefore, this effect of financial intermediation may be especially significant in the context of technology transfer, and hence, in the context of innovation and growth.

	H_0	H_1	$\bar{w}$	r	s	c	g	N
$\gamma \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
$A_0 \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
$\chi \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	?	$\uparrow$	$\uparrow$
$\beta \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
$\theta \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$

Table C: Effects of Parameters with Intermediation

We report the effects of parameter changes in Table C, assuming an equilibrium with $r_d > 0$ exists uniquely. For instance, an increase in γ, the fraction assets that are liquid, shifts the BM curve down while GM is unaffected, reducing r_d, increasing $\bar{w}$ and, in general equilibrium, raising N and

g. We are left with generalizing the calculations in the previous model to endogenize participation and solve for the optimal subsidy. We summarize the results below (see Appendix 6 for a proof). Comparing the efficiency conditions here with those in Section 4, one can see explicitly the two functions of banks: it allows us to get by with a smaller quantity of liquid assets; and we do not need $\theta = 1$, because banking eliminates the holdup problem associated with investment liquidity.

Proposition 5 *There exists an equilibrium with financial intermediation and fixed participation* $(\bar{n}_i, \bar{n}_e)$. *With entry by* i, *an equilibrium with* $r_d = 0$ *exists if* $\mu(\bar{n}_i, \bar{n}_e)/\bar{n}_e$ *is not too big. Equilibrium is generally inefficient unless two conditions are satisfied: the supply of liquid assets is abundant,* $A_1 \geq A_1^{**}$, *where*

$$A_1^{**} \equiv \Omega\left(\pi_1 - \pi_0\right)(1-\beta)\bar{n}_e/\delta = \Omega A_1^*,$$

with A_1^* *being the analogous threshold without banking and* $\Omega \in (0,1)$ *given by*

$$\Omega = \alpha_e \int_0^1 \int_{\sigma_i}^1 [\theta\sigma_i + (1-\theta)\sigma_e] dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i);$$

and, if there is entry by i, *the subsidy is set as in Proposition 4. Note in particular that efficiency here does not require* $\theta = 1$, *as it did in the economy without banking.*

6 A Little Evidence

Here we report some evidence to support the case that technology transfer can be an important part of the innovation process, and that credit imperfections can hinder this process. Our empirical analysis makes use of the firm level data obtained from the World Bank Enterprise Surveys conducted between 2002 and 2005. The whole sample includes 4059 firms across 33 countries. We follow closely the statistical analysis in Carluccio and Fally (2009), but appropriately modify the sample and choice of variables to address our own research questions. Before going to detail, we highlight two findings: (i) in some countries (e.g., Germany), direct technology transfers from outside parties are an important way for firms to acquire new technology; (ii) firms' use of technology transfer is positively correlated with the financial development in a country, particularly for small firms.

Using survey responses, we can determine whether a firm has acquired a new technology in the period 2002-2005. Given our interest in direct technology transfer, we restrict attention to arm's

length (recall footnote 7) transfers from outside parties. In particular, firms in our sample are asked to report the most important way that they acquired new technology in the last 36 months. We focus on transfers through new licensing or turnkey operations obtained from international sources, domestic sources, universities and public institutions. We do not include transfers resulting from hiring, transfers from parent companies, internal development, and development in cooperation with other partners. In Table 1 (all data tables are at the end of the paper), we report cross-country summary statistics regarding the fraction of firms using direct technology transfers, and its relationship to financial development and firm size. Direct transfers are an important source of technology acquisition in some countries. In Germany, 12.6% of firms in the survey reported that the most important way they acquire technology is through new licensing or turnkey operations from international sources, domestic sources, universities and public institutions.

To study the effects of intermediation on technology transfer, we follow the literature and proxy financial development of a country by the ratio of private credit to GDP, taken from Beck, Demirg-Kunt and Levine (1999). Table 2 indicates that, overall, a higher level of financial development is associated with higher rates of technology transfer. The positive correlation is more significant for smaller firms, and tends to become smaller or even reversed as firm size increases. Tables 3-5 report results from three regressions to uncover the effects of financial development. Other control variables in the regression include market size, price of investment, openness, investment level, firm size, presence of foreign capital and industry dummies.[16] Table 3 reports results from a simple OLS regression. This yields a positive relationship between private credit to GDP and technology transfer, significant at the 10% level. This positive relation is strongly strengthened when the square of private credit to GDP is introduced, significant at the 1% level, when we control for firm and country specific variables.

To deal with endogeneity issues, in Table 4, we follow Djankov, McLiesh and Shleifer (2007) and instrument for private credit over GDP by legal origin and perform a 2SLS regression. This leads to considerably larger coefficients on private credit to GDP than the OLS regressions. Technology

[16] Variable definitions accompany the Tables; See Carluccio and Fally (2009) for a more detailed discussion of the statistical approach.

transfer is positively affected by private credit to GDP, with significant results at the 1% level in all six specifications. The strong positive and significant effects still exists when controls for country and firm specific characteristics are excluded. Table 5 shows results from a probit regression. The results are similar in terms of economic conclusions. The general pattern over all the different specifications is that the level of financial development has positive but diminishing effects on technology transfer, and the effect is greater for smaller firms. This is all broadly consistent with our theory.

While the above analysis focuses on how technology transfer depends on the level of financial development in a country, there is also an empirical literature that studies how the decision to acquire technology depends on a firm's own liquidity and financial constraints. Montalvo and Yafeh (1994), e.g., examine investment in foreign technology by Japanese firms in the form of licensing agreements. They conclude that "liquidity is an important consideration in the firm's decision to invest in foreign technology." In particular, they find that "Cash flow has a positive impact, and $REALCF$ (cash flow of firms with limited access to main bank loans) is always positive and significant. Furthermore, the coefficient of $REALCF$ is much higher than that of cash flow, implying that non-keiretsu firms are more liquidity constrained than group-affiliated firms". Also, Gorodnichenko and Schnizter (2010) study Business Environment and Enterprise Performances Surveys from 2002 to 2005, covering a broad array of sectors and countries, and containing direct measures of innovation and financial constraints. They find evidence that innovative activity is strongly influenced by financial frictions.

These results are all consistent with the implications of our model. This discussion of the evidence here is brief, and in the future more empirical work should be done to uncover just how important technology transfer might be, how it is related to liquidity and financial intermediation, and what are the implications for growth. The goal here has been primarily to lay out a theoretical framework within which one can organize such empirical work; we intend this Section to be mainly an illustration of how some simple facts support the general approach.

7 Conclusion

We conclude, as we began, by suggesting that the generation and implementation of new ideas are major factors underlying economic performance and growth, and that liquidity and intermediation play an important role in this process. We developed a novel endogenous growth model, where productivity increases with knowledge and knowledge increases with research and development. This process is aided by exchange, since those who come up with new ideas are not necessarily the best at implementing them. Our idea market incorporated explicit frictions, including search, bargaining and credit problems that hinder trade. The extent to which these matter depends of course on many institutional realities, including intellectual property rights, patent protection laws, contract enforcement, the ability of innovators and entrepreneurs to find each other in the first place, perhaps through third parties like patent agents or lawyers, and so on. We did not model all of these institutional factors in detail, but tried to capture market frictions at a more abstract level using search-and-bargaining theory. We also studied liquidity issues, and developed several interesting interactions between credit and other frictions in the model.

There are good theoretical reasons to think that liquidity might matter for the issues at hand, including the fact that knowledge is hard to collateralize. There is also much precedent for simply assuming that liquidity is crucial, as in most imperfect credit models, and basically of all contract theory (where, without liquidity constraints, the first best can typically be achieved by an agent simply buying out the principal). There is also a wealth of empirical work on liquidity, far too much to survey here (again, see the sources cited in the Introduction and the references therein). A goal here was to study how intermediaries ameliorate frictions, and thus affect technology transfer, innovation and growth. One result is that they allow the economy to get by with fewer liquid assets, by facilitating the reallocation of liquidity from those that have more than they need to those that have less. This helps get around the basic search/matching problem that implies entrepreneurs do not always have sufficient liquidity when they contact an innovator.

This result is perhaps not too surprising, but still worth formalizing, especially since some people argue that a shortage of liquid assets is a real problem (again, see Caballero 2006). A result that

was more surprising, at least it was to us, before we saw it, is that intermediaries also mitigate holdup problems in bargaining, by allowing entrepreneurs to undo otherwise sunk investments in liquidity. Even without intermediation we think the framework provides useful insights, e.g., how to optimally subsidize participation by innovators and/or entrepreneurs in the presence of search and knowledge externalities. We studied existence, uniqueness/multiplicity, efficiency and comparative statics for a series of increasing intricate models, although the framework is still quite tractable, and can potentially be extended in several directions, both in terms of theory and obviously in terms of empirical work. This is left to future research.

Appendix 1: The Model with Capital [For Online Publication]

Consider a CRS technology $f(K, ZH, ZT)$, where K is capital and T is the talent of the owner, assumed a fixed input. We subsume depreciation in the notation f. Here we study the planner's problem (equilibrium is similar):

$$\begin{aligned} V(Z,K) &= \max_{c,H_0,H_1,K_0,K_1,K',n_i} \left\{u(c) - \chi\left(H_0+H_1\right) - \kappa_i n_i + \beta V\left[G(N)Z, K'\right]\right\} \\ \text{st } c &= Nf\left[K_1, Z(1+\eta)H_1, Z(1+\eta)T\right] + (1-N)f(K_0, ZH_0) - K' + \delta ZA, \\ K &= NK_1 + (1-N)K_0,\ n_i \in [0,\bar{n}_i],\ N = n_i \mathbb{E}\sigma_i \end{aligned}$$

After eliminating the constraints, we take the FOC to get:

$$\begin{aligned} H_0 &: u'(c) Z f_H^0 = \chi \\ H_1 &: u'(c)(1+\eta) Z f_H^1 = \chi \\ K_1 &: f_K^1 = f_K^0 \\ K' &: u'(c) = \beta V_K(Z', K') \\ n_i &: \kappa_i = \left(u'(c)[f^1 - f^0 - f_K^1 K_1 + f_K^0 K_1] - \chi(H_1 - H_0) + \beta V_Z(G(N)Z, K') G'(N) Z\right) \mathbb{E}\sigma_i \end{aligned}$$

where $f_H^0 = f_H(K_0, ZH_0, ZT)$, etc. The envelope conditions are

$$V_Z(Z,K) = \Phi/c + \beta V_Z(Z', K') G(N) \text{ and } V_K(Z,K) = (f_K^0 + 1 - \delta)/c$$

where $\Phi = dc/dZ = N(1+\eta)\left(f_H^1 H_1 + f_T^1 T\right) + (1-N)\left(f_H^0 H_0 + f_T^0 T\right) + \delta A$.

We seek a balanced growth path where Z, c, K, f^1 and f^0 grow at $G(N)$ while H_0, H_1 and n_i are constant. By CRS, Φ is also constant, implying $V_Z = \Phi/c(1-\beta)$. Then

$$\begin{aligned} N &= n_i \mathbb{E}\sigma_i \\ G(N) &= \beta(f_K^0 + 1 - \delta) \\ \kappa_i &= \left[u'(c)\left(f^1 - f^0 - f_K^1 K_1 + f_K^0 K_1\right) - \chi(H_1 - H_0) + \beta \frac{\Phi G'(N) Z}{c(1-\beta)}\right] \mathbb{E}\sigma_i \\ c &= f_H^0/\chi Z \\ c &= Nf^1 + (1-N)f^0 + (1-\delta)K - G(N)K + \delta ZA \\ f_H^0 &= (1+\eta) f_H^1 \\ f_K^0 &= f_K^1 \\ K &= NK_1 + (1-N)K_0 \end{aligned}$$

solve for $(H_0, H_1, K_0, K_1, n_i, c, K, N)$. It is straightforward to study this model following the analysis in the text without capital.

Appendix 2: Equilibrium with Two-Sided Entry [For Online Publication]

Here we show that there exists a unique equilibrium in the two-sided participation model of Section 3, where $n_i \in (0, \bar{n}_i)$ and $n_e \in (0, \bar{n}_e)$, as long as κ_i and κ_e are neither too high nor too low. The equilibrium conditions are

$$\frac{\bar{w}}{\chi} = N(1+\eta) f(H_1) + (1-N) f(H_0) + A\delta \tag{34}$$

$$\kappa_i = \frac{\chi}{\bar{w}} \Delta(\bar{w}) [\mathbb{E}\sigma_i + (1-\theta)\frac{\mu(n_i, n_e)}{n_i} \hat{\mathbb{E}}(\sigma_e - \sigma_i)] \tag{35}$$

$$\kappa_e = \frac{\chi}{\bar{w}} \Delta(\bar{w}) [\theta \frac{\mu(n_i, n_e)}{n_e} \hat{\mathbb{E}}(\sigma_e - \sigma_i)] \tag{36}$$

where $N = n_i \mathbb{E}\sigma_i + \mu(n_i, n_e)\hat{\mathbb{E}}(\sigma_e - \sigma_i)$. Define $\zeta = n_e/n_i$, and write (34)-(36) as

$$\kappa_i = \frac{\chi}{\bar{w}} \Delta(\bar{w}) [\mathbb{E}\sigma_i + (1-\theta)\mu(1, \zeta)\hat{\mathbb{E}}(\sigma_e - \sigma_i)] \tag{37}$$

$$\kappa_e = \frac{\chi}{\bar{w}} \Delta(\bar{w}) \left[\theta \mu\left(1/\zeta, 1\right) \hat{\mathbb{E}}(\sigma_e - \sigma_i)\right]. \tag{38}$$

In $(\bar{w}, \zeta)$ space, the former gives a strictly increasing curve and the latter a strictly decreasing curve. The unique intersection determines equilibrium $(\bar{w}, \zeta)$. Denote this wage by $\bar{w}(\kappa_i, \kappa_e)$, where $\partial \bar{w}/\partial \kappa_i < 0$ and $\partial \bar{w}/\partial \kappa_e < 0$. Also, $\bar{w}(\kappa_i, \kappa_e)$ gets arbitrarily large for entry costs sufficiently small.

The $(\bar{w}, \zeta)$ pair still needs to satisfy goods market clearing

$$\frac{\bar{w}}{\chi} = n_i[\mathbb{E}\sigma_i + \mu(1,\zeta)\hat{\mathbb{E}}(\sigma_e - \sigma_i)][(1+\eta)f(H_1) - f(H_0)] + f(H_0) + A\delta.$$

and we need to check the implied (n_i, n_e) is interior,

$$n_i = \frac{\bar{w}/\chi - f(H_0) - A\delta}{[\mathbb{E}\sigma_i + \mu(1,\zeta)\hat{\mathbb{E}}(\sigma_e - \sigma_i)][(1+\eta)f(H_1) - f(H_0)]} \in (0, \bar{n}_i) \tag{39}$$

$$n_e = \zeta n_i \in (0, \bar{n}_e). \tag{40}$$

The numerator in (39) is a strictly increasing function of $\bar{w}$ and is 0 for an unique $\bar{w}$. So we can find $\hat{\kappa}_i$ and $\hat{\kappa}_e$ such that $\bar{w}(\hat{\kappa}_i, \hat{\kappa}_e)/\chi - f(H_0) - A\delta = 0$, implying $n_i = n_e = 0$. By continuity, we can then find κ_i and κ_e close to but bigger than $\hat{\kappa}_i$ and $\hat{\kappa}_e$ such that(39)-(40) are satisfied.

The above discussion establishes $(\bar{w}, \zeta)$ is unique. To see that (n_i, n_e) is unique, note that equilibrium is given by an intersection of two curves in the (n_i, n_e) space. One is the strictly negative relationship between n_i and n_e implicitly defined by (34) given $\bar{w}$; the other is the strictly positive relationship defined by (40) given ζ. Then (n_i, n_e) is determined by the unique intersection.

Appendix 3: Multiple Equilibria [For Online Publication]

Here we provide an example to show supply can be nonmonotone, and hence we can get multiplicity, in the model of Section 4 without the assumption made in the text that η is not too big. Set $A\delta = 0$. Letting $f(H) = 1 - \exp(-H)$, it is easy to solve for:

$$f\left[H_0(\bar{w})\right] = \begin{cases} 1 - \bar{w} & \text{if } \bar{w} \leq 1 \\ 0 & \text{if } \bar{w} > 1 \end{cases} \quad \text{and } f\left[H_1(\bar{w})\right] = \begin{cases} 1 - \bar{w}/(1+\eta) & \text{if } \bar{w} \leq 1+\eta \\ 0 & \text{if } \bar{w} > 1+\eta \end{cases}$$

Given N, supply is

$$S = \begin{cases} Z\left[N(1+\eta-\bar{w}) + (1-N)(1-\bar{w})\right] & \text{if } \bar{w} \leq 1 \\ Z\left[N(1+\eta-\bar{w})\right] & \text{if } \bar{w} \in (1, 1+\eta) \\ 0 & \text{if } \bar{w} \geq 1+\eta \end{cases}$$

To describe $N(\bar{w})$, first compute:

$$\Delta(\bar{w}) = \begin{cases} \eta - \bar{w}\log(1+\eta) & \text{if } \bar{w} \leq 1 \\ 1 + \eta - \bar{w}\left[1 - \log\left(\frac{\bar{w}}{1+\eta}\right)\right] & \text{if } \bar{w} \in (1, 1+\eta) \\ 0 & \text{if } \bar{w} \geq 1+\eta \end{cases}$$

Since $\Delta'(\bar{w}) < 0$ for $\bar{w} < 1+\eta$ and $\Delta(1+\eta) = 0$, $x/\Delta(\bar{w})$ is strictly increasing and approaches ∞ as $\bar{w} \to 1+\eta$. So $\eta > x$ implies there is a $\bar{w}' \in (0, 1+\eta)$ such that

$$\min\left\{\frac{x}{\Delta(\bar{w})}, 1\right\} = \begin{cases} \frac{x}{\Delta(\bar{w})} & \text{if } \bar{w} \leq \bar{w}' \\ 1 & \text{if } \bar{w} > \bar{w}' \end{cases}.$$

Moreover, we have

$$\bar{w}' = \begin{cases} \in (0, 1] & \text{if } x > \eta - \log(1+\eta) \\ (1, 1+\eta) & \text{if } x < \eta - \log(1+\eta) \end{cases}$$

and $N = n_i \mathbb{E}(\sigma_i) + n_i \alpha_i \int_0^{\frac{x}{\Delta}} \int_{\sigma_i}^1 (\sigma_e - \sigma_i) dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i)$. Then, after simplification,

$$S'(\bar{w}) = Z\left(-1 - \Delta'(\bar{w})\eta n_i \alpha_i \frac{x}{\Delta(\bar{w})^2} \int_{\frac{x}{\Delta(\bar{w})}}^1 \left[\sigma_e - \frac{x}{\Delta(\bar{w})}\right] dF_e(\sigma_e|\frac{x}{\Delta(\bar{w})}) f_i(\frac{x}{\Delta(\bar{w})})\right),$$

where $\Delta'(\bar{w}) < 0$ for $\bar{w} < 1 + \eta$.

Therefore supply can have a positive slope when the distribution is sufficiently concentrated over the relevant region, as shown in Figure 6. Then it is easy to specify demand so that we get multiplicity. Note that the above construction uses $\bar{w} < 1 + \eta$ as well as $\eta > x$. The restriction made in the text that η is not too big rules this out and allows us to prove uniqueness.

Appendix 4: The Entrepreneur Problem [For Online Publication]

Here we formulate e's maximization problem as in Section 4. Start with the intuitive expression

$$\begin{aligned} V^e(a, Z) &= (1 - \alpha_e) W^e(a, Z; Z) + \alpha_e \int_{A_0} W^e(a, Z; Z) \\ &\quad + \alpha_e \int_{\mathcal{A}_1} \left\{ \sigma_e W^e \left[a - p \frac{Z}{\phi_1 + Z\delta}, Z; Z \right] + (1 - \sigma_e) W^e \left[a - p \frac{Z}{\phi_1 + Z\delta}, Z; Z \right] \right\} \\ &\quad + \alpha_e \int_{\mathcal{A}_2} \left\{ \sigma_e W^e \left[0, Z(1 + \eta); Z \right] + (1 - \sigma_e) W^e(0, Z; Z) \right\} + \alpha_e \int_{\mathcal{A}_3} W^e(a, Z; Z). \end{aligned}$$

The first term is e's payoff when he does not meet anyone. The second is his payoff when he meets i but there are no gains from trade. The third is his payoff from (unconstrained) trade at p. The fourth is his payoff from (constrained) trade at $\bar{p}$. The final term is his payoff to not trading because he cannot meet i's reservation price. Now algebra leads to (27).

Given this, let $x = a\left(\phi_1 + Z\delta\right)/Z$, and write $\tilde{V}^e(x, Z) = V_t^e(a, Z)$ where

$$\begin{aligned} \tilde{V}^e(x, Z) &= \text{constant } + \chi/\bar{w} x + \alpha_e \theta \chi/\bar{w} \int_0 \int_{\sigma_i} (\sigma_e - \sigma_i) \Delta dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i) \\ &\quad + \alpha_e \chi/\bar{w} \int_0 \int_{B(\sigma_i, x)} \left(\sigma_e \Delta - x\right) dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i). \end{aligned}$$

Then

$$sx' = \left[\frac{(1 + g)\phi_1}{\beta(\phi_1' + Z'\delta)} - 1\right] x' = \frac{(1 + g)\phi_1 a'}{\beta Z'} - x',$$

implying $\phi_1 a' = \frac{\beta}{1+g} s x' Z' + \frac{\beta}{1+g} x' Z'$. Then we can rewrite the control variable in e's maximization problem as x, and the objective function as in (26).

Appendix 5: Entry with Credit Frictions [For Online Publication]

Here we substantiate some claims made in Section 4**.** Equilibrium $(n_i, x, \bar{w}, N)$ satisfies

$$(1 - \beta) x \bar{n}_e - \delta A_1 - \beta x \bar{n}_e L\left[\frac{x}{\Delta(\bar{w})}; n_i\right] = 0 \tag{41}$$

$$\bar{w}/\chi - N(1 + \eta) f\left[H_1(\bar{w})\right] - (1 - N) f\left[H_0(\bar{w})\right] - A\delta = 0 \tag{42}$$

$$N - n_i \mathbb{E}\sigma_i - n_i \alpha_i \int_0^{\frac{x}{\Delta}} \int_{\sigma_i}^1 (\sigma_e - \sigma_i) dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i) = 0 \tag{43}$$

plus the entry condition

$$\kappa_i > \bar{\kappa}_i(0) \text{ if } n_i = 0;\ \kappa_i = \bar{\kappa}_i(n_i) \text{ if } n_i \in (0, \bar{n}_i) \text{ and } \kappa_i < \bar{\kappa}_i(\bar{n}_i) \text{ if } n_i = \bar{n}_i, \tag{44}$$

where

$$\bar{\kappa}_i(n_i) = \frac{\chi}{\bar{w}}\Delta\mathbb{E}\sigma_i + \frac{\chi}{\bar{w}}\Delta(1-\theta)\frac{\mu(n_i,n_e)}{n_i}\int_0^{\frac{x}{\Delta(\bar{w})}}\int_{\sigma_i}^1(\sigma_e-\sigma_i)dF_e(\sigma_e|\sigma_i)dF_i(\sigma_i).$$

As n_i increases, both the upward sloping GM curve and the downward sloping AM curve shift up in $(x, \bar{w})$ space. Therefore, (41)-(43) define an increasing and continuous function $\bar{w} = \varsigma_w(n_i)$ from $[0, \bar{n}_i]$ onto $[\bar{w}(0), \bar{w}(\bar{n}_i)]$. Moreover, $(x, \bar{w})$ pairs that satisfy (41)-(43) define a function $x = \varsigma_x(\bar{w})$ with range $[\bar{w}(0), \bar{w}(\bar{n}_i)]$. We now need to check the entry condition. First, since $\bar{\kappa}_i(n_i)$ is strictly decreasing in n_i, for any $\bar{w} \in [\bar{w}(0), \bar{w}(\bar{n}_i)]$ and $x = \varsigma_x(\bar{w})$, there is a unique $n_i \in [0, \bar{n}_i]$ satisfying (44). So we can construct a continuous mapping from $\bar{w} \in [\bar{w}(0), \bar{w}(\bar{n}_i)]$ to $[0, \bar{n}_i]$. Together with the continuous increasing function $\varsigma_w(n_i)$, this ensures an equilibrium exists.

Next we show that n_i can decrease with A_1. Given $n_i \in (0, \bar{n}_i)$, we derive

$$\begin{aligned}
\left(\frac{\delta A_1}{x} - \beta x\bar{n}_e\frac{L'}{\Delta}\right)dx + \beta\bar{n}_e x L'\frac{x}{\Delta^2}\Delta' d\bar{w} - \beta x\bar{n}_e\frac{dL}{dn_i}dn_i &= \delta dA_1 \\
\bar{G}d\bar{w} - \left[(1+\eta)f(H_1) - f(H_0)\right]dN &= 0 \\
-\mu\Omega\frac{1}{\Delta}dx + \mu\Omega\frac{x}{\Delta^2}\Delta' d\bar{w} + dN - (\mathbb{E}\sigma_i + \mu_i\Phi)dn_i &= 0 \\
-\left[\frac{\chi}{\bar{w}}\Delta(1-\theta)\frac{\mu}{n_i}\frac{\Omega}{\Delta}\right]dx + \bar{E}d\bar{w} - \left[\frac{\chi}{\bar{w}}\Delta(1-\theta)\frac{d(\frac{\mu}{n_i})}{dn_i}\Phi\right]dn_i &= 0
\end{aligned}$$

where

$$\begin{aligned}
\Phi &= \int_0^{\frac{x}{\Delta}}\int_{\sigma_i}^1(\sigma_e-\sigma_i)dF_e(\sigma_e|\sigma_i)dF_i(\sigma_i) \\
\Omega &= \int_{\frac{x}{\Delta}}^1\left(\sigma_e - \frac{x}{\Delta}\right)f_i\left(\frac{x}{\Delta}\right)dF_e(\sigma_e|\sigma_i) \\
\bar{G} &= \frac{1}{\chi} - N(1+\eta)f'(H_1)H_1'(\bar{w}) - (1-N)f'(H_0)H_0'(\bar{w}) \\
\bar{E}(\theta) &= \frac{\chi}{\bar{w}^2}\Delta\mathbb{E}\sigma_i - \frac{\chi}{\bar{w}}\mathbb{E}\sigma_i\Delta' + \frac{\chi}{\bar{w}^2}\Delta(1-\theta)\frac{\mu}{n_i}\Phi - \frac{\chi}{\bar{w}}\Delta'(1-\theta)\frac{\mu}{n_i}\Phi + \frac{\chi}{\bar{w}}\Delta(1-\theta)\frac{\mu}{n_i}\Omega\frac{x}{\Delta^2}\Delta'
\end{aligned}$$

Note that $\bar{E}(\theta) > 0$ at least for $\theta \approx 1$. Then we have

$$\boldsymbol{\Upsilon}\begin{bmatrix} dx \\ d\bar{w} \\ dN \\ dn_i \end{bmatrix} = \begin{bmatrix} \delta dA_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where

$$\boldsymbol{\Upsilon} = \begin{bmatrix}
\frac{\delta A}{x} - \beta x\bar{n}_e\frac{L'}{\Delta} & \beta\bar{n}_e x L'\frac{x}{\Delta^2}\Delta' & 0 & -\beta x\bar{n}_e\frac{\partial L}{\partial n_i} \\
0 & \bar{G} & -(1+\eta)f(H_1) + f(H_0) & 0 \\
-\mu\Omega\frac{1}{\Delta} & \mu\Omega\frac{x}{\Delta^2}\Delta' & 1 & -(\mathbb{E}\sigma_i + \mu_i\Phi) \\
-\frac{\chi}{\bar{w}}\Delta(1-\theta)\frac{\mu}{n_i}\frac{\Omega}{\Delta} & \bar{E} & 0 & -\frac{\chi}{\bar{w}}\Delta(1-\theta)\frac{\partial(\frac{\mu}{n_i})}{\partial n_i}\Phi
\end{bmatrix}$$

One can show $\det(\boldsymbol{\Upsilon}) > 0$ at least for $\theta \approx 1$. Then

$$\det(\boldsymbol{\Upsilon})\frac{\partial\bar{w}}{\partial A_1} = \delta\left[(1+\eta)f(H_1) - f(H_0)\right](1-\theta)\Omega\frac{\chi}{\bar{w}}\frac{\mu}{n_i}\left(\mathbb{E}\sigma_i + \mu_i\Phi\right) - \mu\frac{\chi}{\bar{w}}\frac{\partial(\frac{\mu}{n_i})}{\partial n_i}\Phi$$

So $\partial\bar{w}/\partial A_1 = 0$ when $\theta = 1$ and $\partial\bar{w}/\partial A_1 > 0$ when $\theta \in (\theta_0, 1)$ for some $\theta_0 < 1$. Since $\partial N/\partial\bar{w} = \bar{G}/\left[(1+\eta)f(H_1) - f(H_0)\right] > 0$, we have $\partial N/\partial A_1 = 0$ when $\theta = 1$ and $\partial N/\partial A_1 > 0$ when $\theta \in (\theta_0, 1)$. Then we have

$$\begin{aligned}\det(\boldsymbol{\Upsilon})\frac{\partial n_i}{\partial A_1} &= \delta\left\{\frac{\chi}{\bar{w}}\Delta(1-\theta)\frac{\mu}{n_i}\frac{\Omega}{\Delta}\bar{G} - \left[(1+\eta)f(H_1) - f(H_0)\right]\mu\Omega\frac{1}{\Delta}\bar{E}\right. \\ &\quad \left. + \left[(1+\eta)f(H_1) - f(H_0)\right]\mu\Omega\frac{x}{\Delta^2}\Delta'\frac{\chi}{\bar{w}}\Delta(1-\theta)\frac{\mu}{n_i}\frac{\Omega}{\Delta}\right\}\end{aligned}$$

Therefore, $\partial n_i/\partial A_1 < 0$ when $\theta \in (\theta_0, 1]$.

Appendix 6: Equilibrium with Intermediation [For Online Publication]

We prove existence in the model of Section 5**.** First consider fixed participation. Then $GM(r_d, \bar{w}) = 0$ defines $\bar{w}$ as a decreasing function of r_d in $(r_d, \bar{w})$ space, with intercept $\bar{w}_0$ given by the solution to (32) with

$$N = \bar{n}_i\mathbb{E}(\sigma_i) + \bar{n}_i\alpha_i\int_0^1\int_{\sigma_i}^1(\sigma_e - \sigma_i)dF_e(\sigma_e|\sigma_i)dF_i(\sigma_i).$$

As $r_d \to \infty$, $\bar{w}$ converges monotonically to $\bar{w}_1 > 0$, defined as the solution to (32) with $N = \bar{n}_i\mathbb{E}(\sigma_i)$. As regards the $BM(r_d, \bar{w})$ curve, first, $r_d = 0$ when $\bar{w} \geq \bar{w}_2$, with $\bar{w}_2$ solving

$$\frac{\gamma\delta A}{\bar{n}_e(1-\beta)} = \Delta(\bar{w}_2)\alpha_e\int_0^1\int_{\sigma_i}^1[\theta\sigma_i + (1-\theta)\sigma_e]dF_e(\sigma_e|\sigma_i)dF_i(\sigma_i).$$

Second, the $BM(r_d, \bar{w})$ curve hits $r_b = r$ as $\bar{w} \to 0$, and it is strictly decreasing for $r_d \in [0, r)$. These observations ensure an intersection (interior or not), so equilibrium exists. There are two types of equilibria: (i) $r_d = 0$ and $\bar{w} = \bar{w}_0$; and (ii) $r_d \in (0, r)$ and $\bar{w} \in (\bar{w}_1, \bar{w}_0)$. When equilibrium with $r_d > 0$ exists uniquely, $\bar{w}_2 > \bar{w}_0$ and the BM curve crosses the GM curve from above. We conclude that when $r_d = 0$, $A\delta$ has no effect; and when $r_d > 0$ a rise in $A\delta$ or θ lowers r_d and increases N and g. This completes the case without entry. The case with entry is similar.

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Empirical Variable Definitions

Dependent

Technology Transfer: *Firm-specific variable.* Binary variable equal to one if the firm's (self reported) most important source of technology is any of: "new licensing or turnkey operations from international sources," "new licensing or turnkey operations from domestic sources," "new licensing or turnkey operations from domestic sources," "obtained from universities or public institutions." [2005:Q61b]

Independent - Explanatory

Private credit/GDP: *Country-specific variable.* The ratio of private credit by deposit money banks to GDP, used as a proxy for a country's level of financial development. Taken from Beck et al (1999).

Private credit/GDP: *Country-specific variable.* The previous term squared.

Independent - Instruments

Legal origin: *Country-specific variable.* A set of three dummy variables, French-civil, German-civil, and common law, indicating the origin of a country's legal system. A country's legal code can have multiple influences. Taken from Djankov et al (2007), and the CIA World Factbook.

Independent - Controls

Market size: *Country-specific variable.* The population of the country in which a firm operates. Taken from Penn World Tables 6.3.

Price level of investment: *Country-specific variable.* PPP over investment level, divided by exchange rate with US$, multiplied by 100. Taken from Penn World Tables 6.3.

Openness: *Country-specific variable.* Exports plus imports, divided by GDP. Taken from Penn World Tables 6.3.

Investment level: *Country-specific variable.* Investment as a share of GDP. Taken from Penn World Tables 6.4.

Firm size: *Firm-specific variable.* Number of permanent, full-time employees employed at a firm, self reported. [2005:Q66a]

Presence of foreign capital: *Firm-specific variable.* Dummy variable equal to one if a positive percentage of a firm is owned by foreign individuals or businesses, self reported. [2005:S5b]

Industry dummies: *Firm-specific variable.* A set of seven dummy variables designating a firm's industry. A firm belongs to a certain industry if the majority of its operations are in the specified field. Industries are: mining, construction, manufacturing, transport, wholesale, real estate, hotel and restaurant services, and "other" if none of these are applicable. [2005:Q2a-g; 2002:q2a-g]

Table 1: Summary of Country Statistics

	Technology Transfer			Private Credit to GDP
	Number of Observations	Mean	Std. Dev.	Mean
Albania	82	0.024	0.155	0.118
Armenia	182	0.005	0.074	0.069
Azerbaijan	164	0.110	0.314	
Belarus	93	0.011	0.104	
Bosnia	89	0.011	0.106	0.391
Azerbaijan	83	0.048	0.215	0.378
Croatia	94	0.000	0.000	0.563
Czech Republic	78	0.077	0.268	0.330
Estonia	40	0.048	0.158	0.619
Georgia	56	0.054	0.227	0.113
Germany	277	0.126	0.333	1.109
Greece	206	0.024	0.154	0.715
Hungary	91	0.099	0.300	0.475
Ireland	191	0.037	0.188	1.421
Kazakhstan	182	0.033	0.179	0.276
Korea	94	0.128	0.335	0.894
Kyrgyzstan	86	0.093	0.292	0.072
Latvia	51	0.098	0.300	0.549
Lithunia	57	0.053	0.225	0.328
Macedonia, FYR	63	0.032	0.177	0.226
Moldova	136	0.044	0.206	0.208
Poland	326	0.058	0.235	0.277
Portugal	126	0.016	0.125	1.403
Romania	247	0.045	0.207	0.166
Russia Federation	178	0.039	0.195	0.227
Serbia & Montenegro	110	0.018	0.134	0.229
Slovak Republic	50	0.060	0.240	0.314
Slovenia	65	0.015	0.124	0.530
Spain	185	0.016	0.127	1.301
Tajikistan	70	0.014	0.120	
Turkey	162	0.025	0.156	0.184
Ukraine	181	0.028	0.164	
Uzbekistan	64	0.016	0.125	

Note: Source: World Bank Enterprise Survey 2005.

Table 2: Percentage of Firms Engaging in Technology Transfer by Firm Size

	Below Mean Private Credit to GDP (%)	Above Mean Private Credit to GDP (%)
Firm Size (number of employees)		
2-10	2.25	4.76
11-50	4.06	5.60
51-100	5.47	6.47
101-250	5.60	2.84
251-500	5.16	4.21
501-1000	10.17	7.50
>1000	4.08	7.32
All Firms	4.16	5.13

Note: Source: World Bank Enterprise Survey 2005.

Table 3: OLS Regression of Technology Transfer on Private Credit, Uninstrumented

Independent Variables	Dependent Variable: Technology Transfer					
	(1)	(2)	(3)	(4)	(5)	(6)
Private credit to GDP	0.0139*	0.0287*	0.0276*	0.1308***	0.1607***	0.1649***
	(0.0080)	(0.0158)	(0.0159)	(0.0381)	(0.0464)	(0.0468)
Private credit to GDP^2				-0.0794***	-0.0839***	-0.0870***
				(0.0253)	(0.0774)	(0.0279)
Log market size		0.0191***	0.0181***		0.0158***	0.0148***
		(0.0039)	(0.0039)		(0.0041)	(0.0041)
Price level of investment		-0.0005	-0.0004		-0.0005	-0.0004
		(0.0004)	(.0004)		(.0004)	(.0004)
Openness		0.0006***	0.0005***		0.0005***	0.0004***
		(0.0002)	(0.0002)		(0.0002)	(0.0002)
Investment level		-0.0010*	-0.0010*		-0.0016***	-0.0016***
		(0.0005)	(0.0005)		(0.0006)	(0.0006)
Log firm size			0.0061***			0.0062***
			(0.0022)			(0.0022)
Presence of foreign capital			0.0050			0.0049
			(0.0110)			0.0110)
Industry dummies	No	No	Yes	No	No	Yes
Intercept	0.0395***	-0.1504***	-0.1730***	0.0153	-0.1211**	-0.1443
	(0.0055)	(0.0476)	(0.0508)	(0.0095)	(0.0485)	(0.0515)
Number of observations	3587	3509	3509	3587	3509	3509

Note: * ≡ Significant at 10% level, ** ≡ Significant at 5% level, and *** ≡ Significant at 1% level. Standard deviations are in parentheses.

Table 4: Two-Stage Least Squares Regression of Technology Transfer on Private Credit

Independent Variables	Dependent Variable: Technology Transfer					
	(1)	(2)	(3)	(4)	(5)	(6)
Private credit to GDP	0.0645***	0.3366***	0.3202***	0.5517***	0.4168***	0.4073***
	(0.0137)	(0.0608)	(0.0585)	(0.0764)	0.0764)	(0.0755)
Private credit to GDP^2				-0.3209***	-0.0768*	-0.0802*
				(0.0495)	(0.0448)	(0.0447)
Firm size × Private credit to GDP	-0.0000**	-0.0001***	-0.0001***	-.0000**	-0.0001***	-0.0001***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Log market size		0.0263***	0.0255***		0.0223***	0.0215***
		(0.0043)	(0.0043)		(0.0049)	(0.0048)
Price level of investment		-0.0067***	-0.0064***		-0.0059***	-0.0056***
		(0.0013)	(0.0012)		(0.0013)	(0.0013)
Openness		0.0010***	0.0010***		0.0009***	0.0008***
		(0.0002)	(0.0002)		(0.0002)	0.0002
Investment level		-0.0044***	-0.0041***		-0.0046***	-0.0043***
		(0.0009)	(0.0008)		(0.0009)	(0.0008)
Firm size	0.0000**	0.0001***	0.0000***	0.0000**	0.0000***	0.0000*
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0002)
Presence of foreign capital			0.0138			0.0140
			(0.0112)			(0.0111)
Industry dummies	No	No	Yes	No	No	Yes
Intercept	0.0121**	0.0350	0.0085	-0.0932***	0.0378	0.0121
	(0.0081)	0.0612)	(0.0622)	(0.0182)	(0.0605)	(0.0616)
Number of observations	3587	3509	3509	3587	3509	3509

Note: Private credit is instrumented by legal origin, * ≡ Significant at 10% level, ** ≡ Significant at 5% level, and *** ≡ Significant at 1% level. Standard deviations are in parentheses.

Table 5: Probit Regression of Technology Transfer on Private Credit

Independent Variables	Dependent Variable: Technology Transfer		
	(1)	(2)	(3)
Private credit to GDP	0.5640***	0.8845***	0.8934***
	(0.1147)	(0.3044)	(0.3002)
Firm size×private credit to GDP	-0.0004**	-0.0003	-0.0003
	(0.0002)	(0.0002)	(0.0002)
Log market size		0.2618***	0.2544***
		(0.0434)	(0.0438)
Price level of investment		-0.0208***	-0.0211***
		(0.0073)	(0.0072)
Openness		0.0090***	0.0089***
		(0.0018)	(0.0018)
Firm size	0.0003**	0.0002*	0.0002***
	(0.0001)	(0.0001)	(0.0001)
Industry dummies	No	No	Yes
Intercept	-1.9511***	-4.2743***	-4.3427***
	(0.0640)	(0.5348)	(0.5723)
Number of observations	3587	3509	3467

Note: Private credit is instrumented by legal origin, * ≡ Significant at 10% level, ** ≡ Significant at 5% level, and *** ≡ Significant at 1% level. Standard deviations are in parentheses.

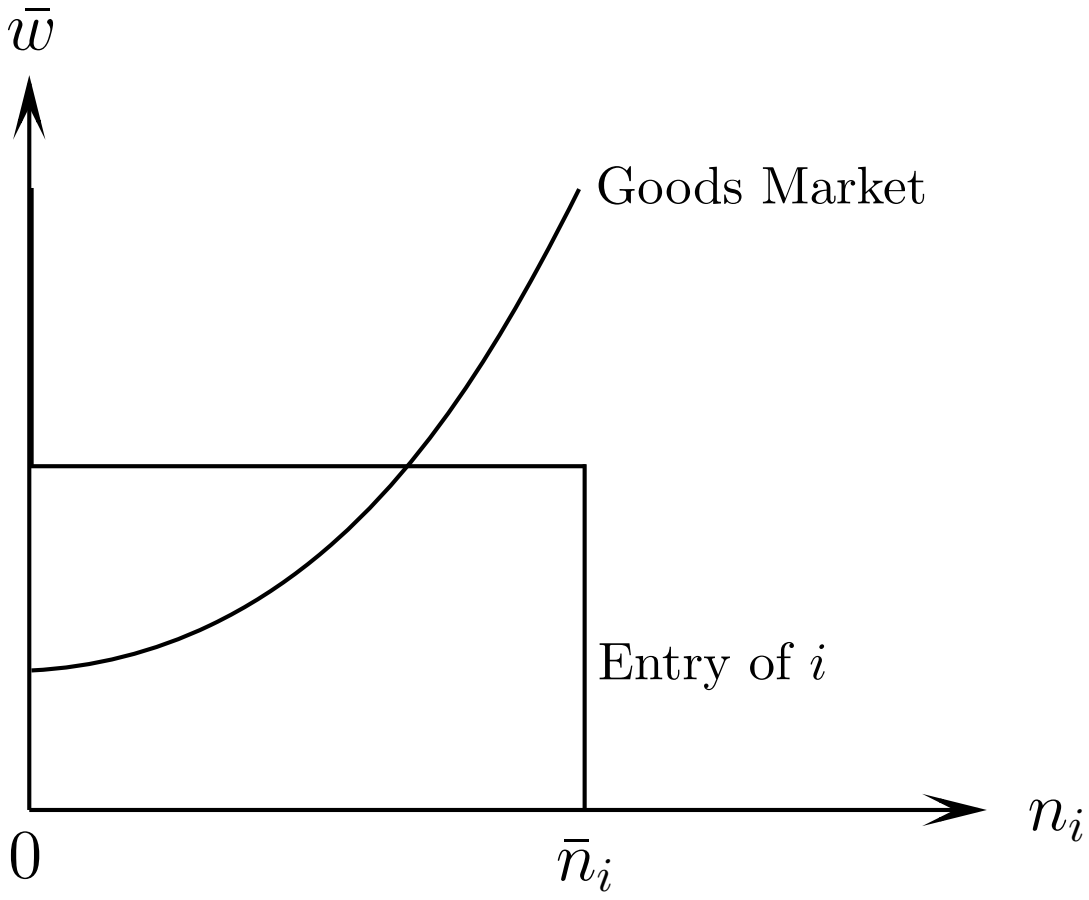


Figure 1: Equilibrium of Basic Model with Endogenous Innovation

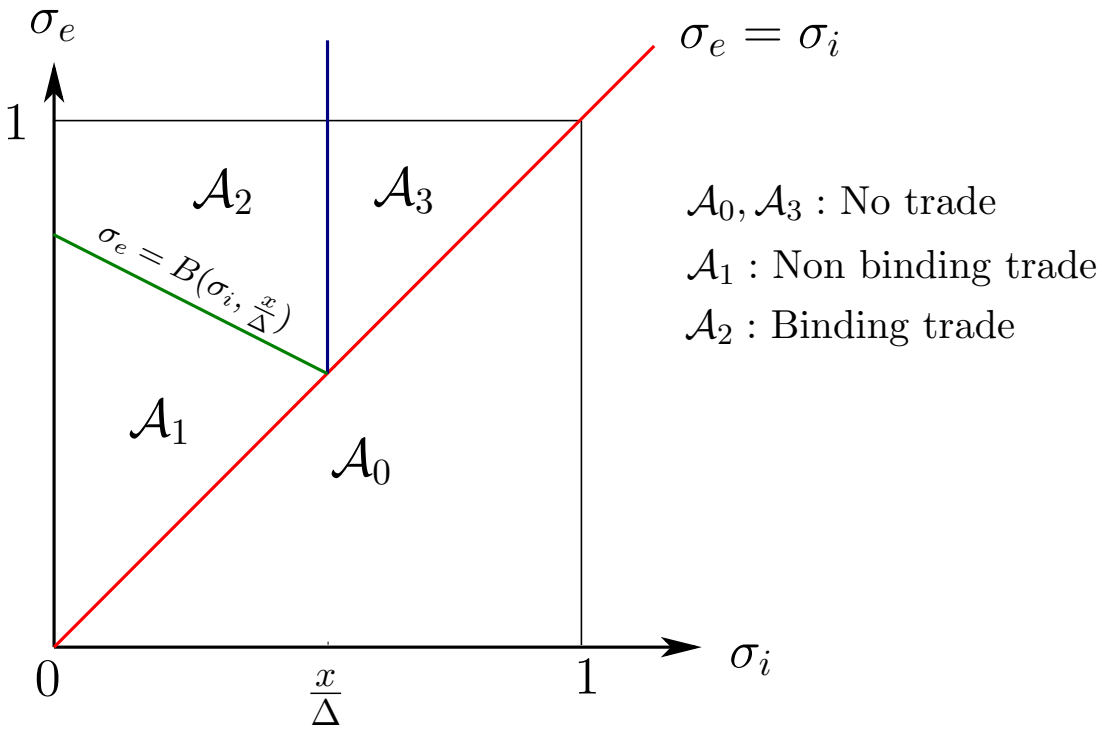
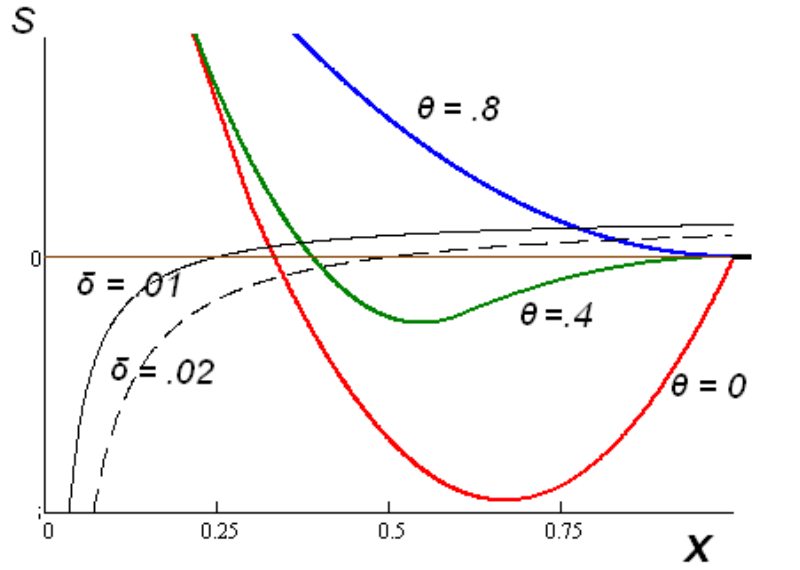


Figure 2: Bargaining Outcome with Credit Frictions



$\beta = .96 \quad A = 1 \quad \gamma = n = .5. \quad F_i \sim U[0,1] \quad F_e \sim U[.3,1]$

Figure 3: $S(x)$ and $\ell(s)$

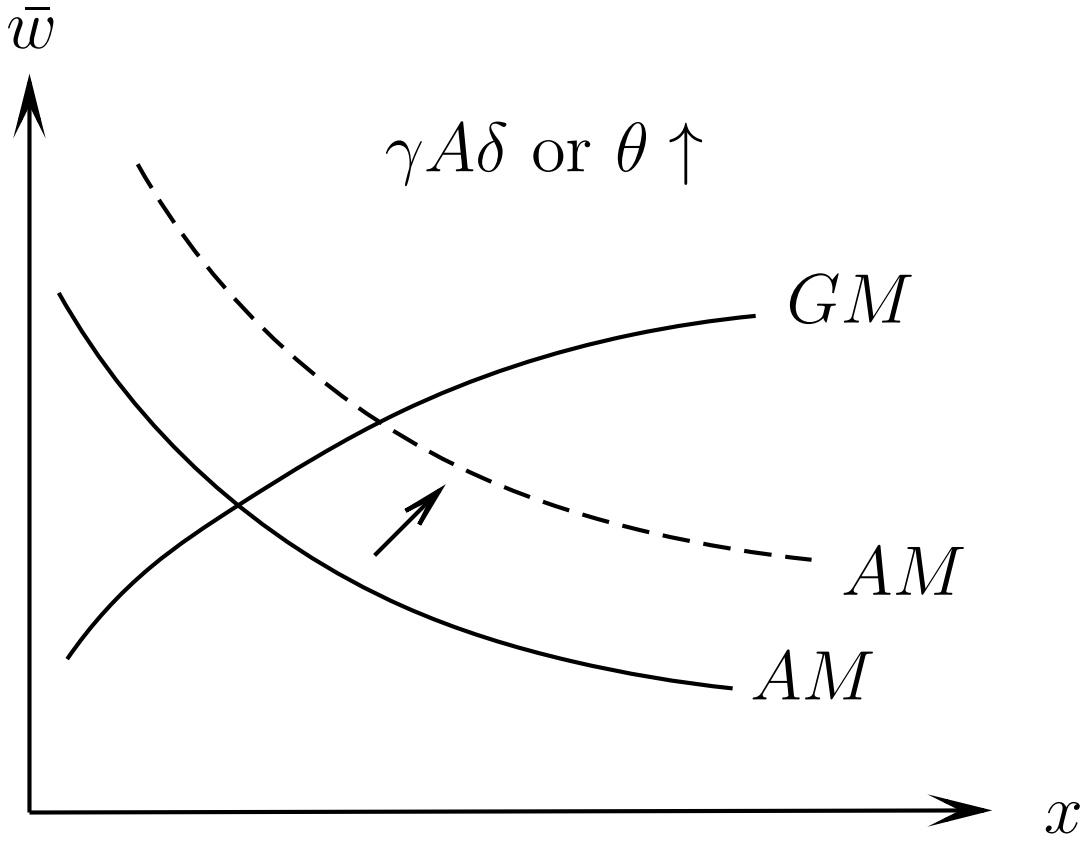


Figure 4: Effects of Increasing Liquidity or Bargaining Power

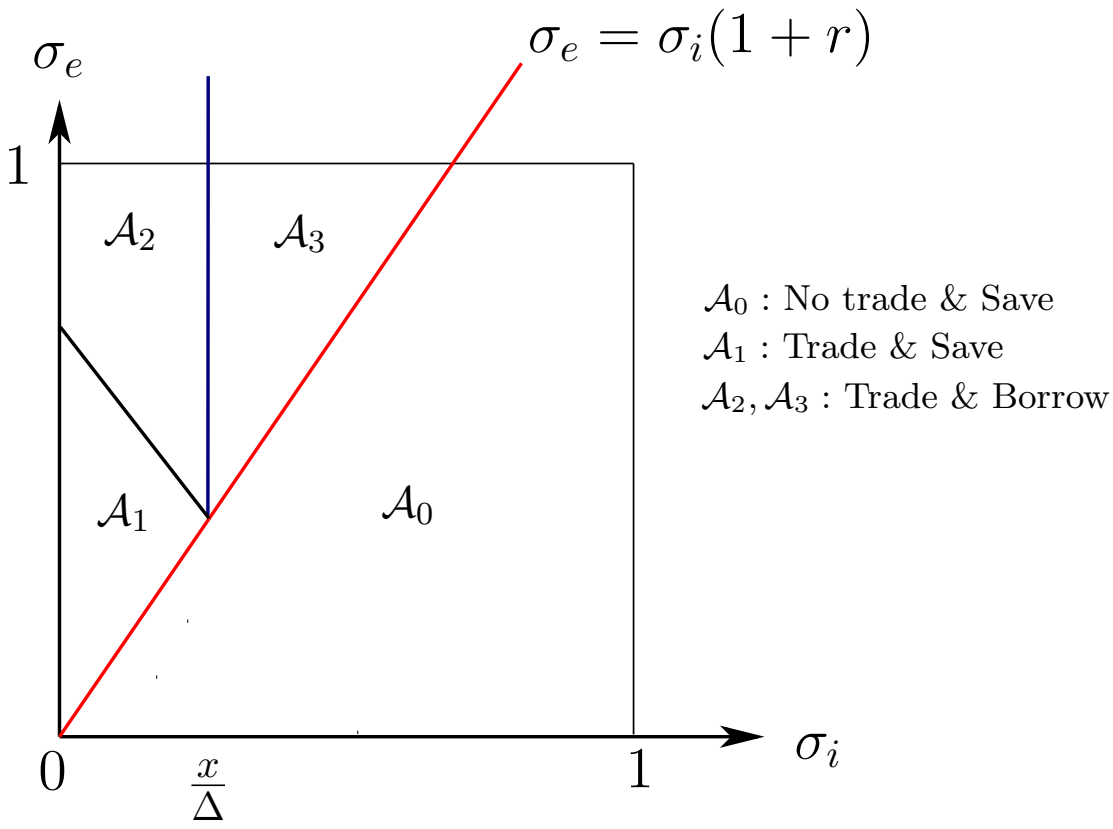


Figure 5: Bargaining Outcome with Intermediation

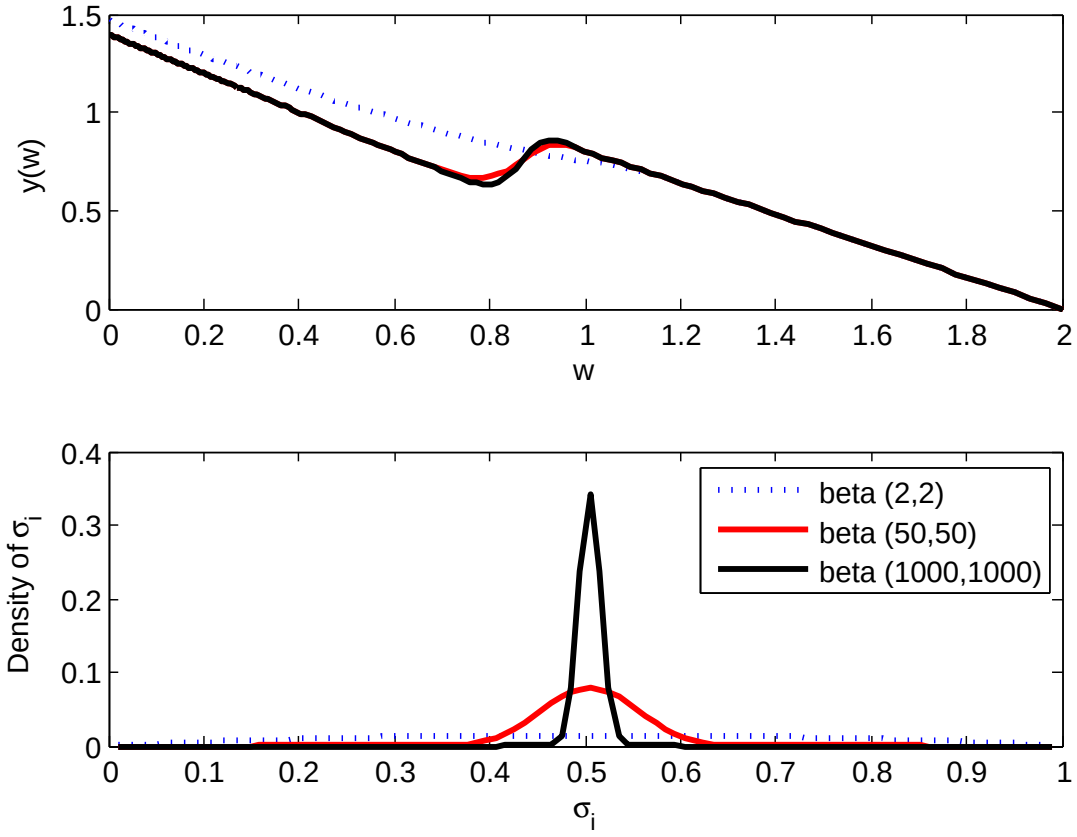


Figure 6: Example: $\sigma_e = 1$, $\sigma_i \sim beta(a, b)$