Venture Capitalists as Benevolent Vultures: The Role of Network Externalities in Financing Choice

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Abstract

This paper studies how externalities between entrepreneurial projects affect investment decisions and investors’ involvement with the companies they invest in. In the presence of externalities between projects, venture capitalists (VCs) can potentially increase the total value of their investment portfolio through better coordination of investment and control. We study a model, where VCs compete with individual investors (angels) to finance entrepreneurs’ projects. Entrepreneurs would prefer VCs to angels only if the former maximize the entrepreneurs’ company value. We find conditions, when this is the case. Using these findings, we make predictions about VCs’ portfolio characteristics, consistent with known empirical observations, e.g., presence in new industries with few players, high growth, high profitability and relatively similar size of portfolio companies. Surprisingly, high externality at the R&D stage does not give VCs as much advantage as one would expect. We also make a prediction that in industries with negative externalities at the market stage, one should observe lower entrepreneur’s ownership and higher returns to VCs’ investment into surviving firms. This happens, because entrepreneurs want to ensure continuation of their project and prefer smaller share of ownership to the project’s termination.

JEL classification: G24, G32
1. Introduction

A recent trend in entrepreneurial finance has been the emergence of networks of related start-up companies funded by a common venture capitalist. Both the academic and practitioner literatures provide evidence of this growing phenomenon (see Sorenson and Stuart (1999), Stuart and Robinson (1999) and articles in *Red Herring Magazine*\(^1\)), and it appears evident that strategic network considerations impact upon individual firm decisions: Intel invested in start-ups developing 3-D graphics chips, leading to an explosion in the market for 3-D PC games and higher demand for PCs using Intel chips.\(^2\) Excite was chosen instead of Infoseek as a provider of web searches on Netscape, largely because both Excite and Netscape are funded by Kleiner Perkins.\(^3\)

In spite of the abundance of anecdotal evidence, to our knowledge a comprehensive theory of how network externalities affect the financing decision and future values of firms is missing.

In this paper we develop a theoretical framework to study how network externalities between portfolio companies affect investment decisions of different groups of investors. These interpretations rest on the observation that venture capitalists take a portfolio approach and make decisions that maximize the value of their entire portfolio of companies. They do so by carefully screening which companies to invest in and by controlling these companies’ activity at later stages.

When entrepreneurs search for an investor to fund their projects, they choose the financier, who will maximize the entrepreneur’s wealth. When this wealth increases with the wealth of the entire venture capital portfolio, entrepreneurs choose venture capital financing to everybody’s benefit. A conflict arises, when venture capitalists make investment decisions that maximize the value of their overall portfolio, but would be sub-optimal or even value-destroying from a particular entrepreneur’s point of view. This contrasts with the typical situation in business angel finance, where projects are financed on a stand alone basis and the financier makes investment decisions to maximize the individual project’s payoff. In this case the entrepreneur does not risk having his project sacrificed for the common good of a portfolio even though the rational business angel will be taking account of other projects over which he has no control.

Our results explain the entrepreneurs’ choice of investors for their projects and why venture capitalists may finance start-ups that otherwise would not find a source of funds. Furthermore, we show that for some projects, venture capitalists are the *only* possible investors, because without coordination between investments, the NPV of such projects would be negative.

Since we consider entrepreneurial projects with a high degree of uncertainty about the required investment and about the payoff, for the purposes of this paper we consider only active investors (business angels and venture capitalists), as the potential investors for these projects. Active investors are those, who not only provide capital, but are also actively involved in mon-

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\(^1\) See, for example an article by Peter D. Henig "And now, EcoNets" in *Red Herring Magazine*, February 2000


\(^3\) Toby E. Stuart, Alliance networks: View from the hub, Mastering Strategy, FT supplement, November 15, 1999
itoring, advising and formulating a business strategy — activities that business angels and venture capitalists are well known for (see Ehrlich et al. (1994), Gorman and Sahlman (1989), Prowse (1998)). Another characteristic that is important to our results is that active investors have control rights disproportionately greater than the financial investments they make, and this allows them to influence the strategic decisions of portfolio firms without having majority share ownership of these firms (see discussion below).

From the empirical literature, starting from Sahlman (1990), we know that most VCs specialize in a particular stage of development and in a particular industry. They are diversified and focused at the same time. Experience is a key resource that the venture capitalist brings to the table in helping to nurture a developing firm. VCs and angels are involved in similar types of activities, namely, funding, monitoring, advising, formulating business strategy. Consistent with the existent literature (for example, Hellmann (1994), Kaplan and Stromberg (2000b) and Prowse (1998)) we define venture capitalists (VCs) as investors who finance and actively support a portfolio of several companies, and we define angels as investors who are actively involved with only one company, probably because of cash constraints, or because their other resources, such as personal time etc., are more limited than VC’s and do not allow active participation in several companies. Ehrlich et al. (1994) find that in comparison with business angels, VCs are more involved in the management of portfolio companies. Gorman and Sahlman (1989) report that venture capital firms in their sample have an average portfolio of nine entrepreneurial companies.

Our model is related to those of Bhattacharya and Chiesa (1995) and of Cabral (1998). Cabral (1998) develops a model in which different parties can engage themselves into a potentially rewarding joint venture. However, the free-rider problem hinders innovation — if the project is successful, the discovery (technological innovation) becomes a public good, and so the parties have an incentive to deviate (underinvest) at the beginning in order to free-ride. Bhattacharya and Chiesa (1995) have a similar model with implications closer to the current paper. They study the interaction between financial decisions and the disclosure of interim research results to competing firms. Technological knowledge revealed to a firm’s financier(s) need not also flow to its R&D and product market competitors. The authors show that the choice of financing source can serve as a precommitment device for pursuing ex-ante efficient strategies in knowledge-intensive environments. Hellmann (1998b) and Ueda (2000) have models where the entrepreneur chooses investors taking into account the possibility that investors can steal the entrepreneur’s idea. Unlike Ueda’s paper, where stealing by VC is bad, in our model information spillover is two-directional and can bring more value to the entrepreneur, not only because it could be him, who benefits from stealing other’s ideas, but also because of the externalities effect, which increases future payoff, a situation similar to the one studied by Economides (1996), where an incumbent firm subsidizes the entry of other firms.

The model features two entrepreneurs, each with a different risky innovative project that requires a two-stage financing from outside investors, angels or VCs. These investors provide not only financial capital, but also their advice, expertise and other forms of human capital. Important to our paper is an assumption that these active investors have control rights to make
decisions, crucial to the firm existence. As many authors show, these rights, disproportional to the cash flow rights, result from the complementarity between the financing and advisory roles of venture capitalists. For example, Repullo and Suarez (1998) find that the optimal contract between venture capitalists and an entrepreneur has the characteristics of convertible preferred stock. This outcome results from a double-sided moral hazard problem that arises at the second stage of investment, when both the entrepreneur’s and the advisor’s (i.e., VC’s) efforts are crucial for increasing the proportion of potential non-zero payoffs. Hellmann, in a series of papers (Hellmann (1994), Hellmann (1998a) and Hellmann (1998b)), studies why and under what circumstances entrepreneurs would voluntarily relinquish control to VCs. This happens, for example, when VCs have better expertise in decisions affecting the value of the firm. Casamatta (2000) shows that when VCs’ investment (of cash and effort) is high, it is optimal to give him convertible bonds, and when it is low, VCs should get common stock. Finally, Kaplan and Stromberg (2000a) provide empirical evidence that VC financing contracts separately allocate cash flow rights and control rights (board seats, voting rights, liquidation rights). We use these results in the paper, by simply assuming that after the investment is made, investors always have the control rights.

How do VCs create more value than active investors, who can invest only in one project? This happens due to the externalities between portfolio companies and VC’s ability to internalize these externalities to the interest of the entire population, rather than individual species. Network externalities are modeled in the following way: the results of the first R&D stage of one project affect the success/failure of the second stage of both projects. In addition, the final projects’ payoffs depend on how many projects continued to the market stage. The entrepreneurs aim to maximize their own profits, while the investors get a competitive zero expected return on their investments. Since these investments include investors’ time and efforts, zero return means that their efforts are fairly rewarded, and investors prefer invest more than less, except for the case when they get abnormal positive returns.

In the paper we show, that when the network externalities are positive, coordinated investment by VCs guarantees profitable investment into some projects that would otherwise have ex-ante negative NPV and fail to attract funding. When the network externalities are negative, coordinated investment allows early termination of some projects. Some positive NPV projects may even be sacrificed, to the total benefit of overall value of the VC portfolio. The entrepreneurs know that there is a probability that it is their project that will be terminated, yet they are willing to allow for this eventually because, if their project continues the payoff will be much higher, than with non-coordinated individual investment.

The main results of the paper are: first, we establish conditions, under which VCs successfully compete with angel investors, and, second, we make predictions about characteristics of the firms in the VCs’ portfolio, like high growth potential, high profitability and relatively similar size of portfolio companies. These predictions are consistent with known empirical observations. We also predict that VCs are dominant investors in new industries with few existing companies and high market externalities (both negative and positive).
Surprisingly, when the market externality is positive, high R&D externality does not give VCs as much advantage as one would expect. We also make a prediction that in industries with negative market externalities, one should observe lower entrepreneur’s ownership and higher returns to VCs’ investment into surviving firms. This happens, because entrepreneurs want to ensure continuation of their project and prefer smaller share of ownership to the project’s termination.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 establishes the main results, namely, how venture capital financing increase projects’ value for both positive and negative externalities. Section 4 concludes.

2. Model

We use a framework similar to that of Bhattacharya and Chiesa (1995) with each project’s payoffs being affected by the other project’s outcome.

We consider a two-period model with three dates, \( t = 0, 1 \) and \( 2 \). We have two entrepreneurs, \( E_1 \) and \( E_2 \), each endowed with his individual project and potential investors. All agents are assumed to be risk neutral and the riskless interest rate is normalized to zero.

The projects

Each project has two stages, the development stage, which we call the R&D stage, and the commercialization stage called the market stage at which technology is actually developed and production takes place. The R&D stage investigates the feasibility of a potentially promising technology for a given project giving an answer to the question, whether or not this technology works. Technology is very broadly defined here and can include, but is not limited to, new business models, distribution channels, markets, products, services etc.

At the R&D stage a firm can make a full scale investment of financial and human capital \( I \) into project \( i \) or it can make a nominal start-up investment \( \varepsilon \), \( 0 < \varepsilon \ll I \), which allows it to enter into business and continue to the market stage. If zero investment is made at \( t = 0 \), the firm is not in business and it cannot proceed to the market stage.

When \( I \) is invested at the R&D stage, the probability of this stage success is \( \beta \). Hence, if both projects get full-scale funding for the R&D stage, then the probability of success of at least one R&D stage is \( (1 - (1 - \beta)^2) \). If at least one project has a successful R&D stage, meaning that this project’s technology is feasible, then this technology can potentially be adopted by both projects at the market stage. In other words, only failure of R&D stages for both projects renders it impossible to go to the market stage.

Firms study different technologies, but all firms in business can adopt and develop successful technology at the market stage, even if their own R&D stage failed or if they didn’t make a full-scale investment at all. We interpret the results of the first stage as a costly answer to the
question "Will this technology work". Investing \( I \) gives the right to pose the question and get the answer, while investing \( \varepsilon \) allows only to observe the other’s research result.

A project continuation/termination decision is made contingent on information available at \( t = 1 \). We make a simplifying assumption that at the market stage the investment costs for all firms in business are the same. The market stage gives the following net payoffs, net of the second stage investment

\[
V_i = \begin{cases} 
\lambda_i V & \text{if both projects get stage 2 financing,} \\
V & \text{if only project } i \text{ gets stage 2 financing,} \\
0 & \text{if project } i \text{ is discontinued.}
\end{cases}
\]

For simplicity, we assume that \( \lambda_1 \leq \lambda_2 \). For computational simplicity, we also assume that \( \varepsilon \) is very small and in our calculations can be assumed to be equal to zero.

**Assumption 1.** \( \varepsilon \) is very small. \( \varepsilon \ll \min \left( \frac{1-\beta^2}{1-\beta}, I; \beta V; \beta^2 \lambda_1 V \right) \).

**Investors**

Initial investment includes both financial and human capital, which only active investors — business angels and VCs — can provide. Due to the capital constraints, each angel can invest only in a single project, while the VC can invest into both projects. Each project can have only one active investor.

If an investor provides \( I_i \), he gets a stake \( \alpha_i \) in project \( i \). Parties negotiate \( \alpha_i \) at \( t = -1 \) and perfect competition between investors should lead to a zero expected return on provided investment. Nevertheless, investors prefer involvement to no involvement into the project, because \( I_i \) includes investment of human capital, which is fairly rewarded.

**Contracts**

Investors offer contracts at time \( t = -1 \). We model investors’ cash-flow rights as simple equity stakes. Investors offer their participation for share \( \alpha \) of the firm. That is, for each project \( i \) they submit competitive bids \( \alpha \) and \( E_i \) chooses an active investor, who offers the most attractive terms. VCs can submit bids contingent on their participation in another project. Shares \( \alpha_{VC,1} \) and \( \alpha_{VC,2} \) are verifiable by both entrepreneurs, that is, \( E_i \) knows both \( \alpha_{VC,1} \) and \( \alpha_{VC,2} \), while the investment levels, \( I_j \), of competitors are not verifiable, although they are revealed in equilibrium. With angel (separate) investors, entrepreneurs and investors know only investment \( I_i \) and investor’s share \( \alpha_{A,i} \), but do not observe \( I_j \) and share \( \alpha_{A,j} \) of the rival project.

We assume that an active investor does not need majority cash flow rights to have control of the firm. Such separation of cash flow rights and control rights is very common for VCs, see, e.g., Kaplan and Stromberg (2000a).
Information structure

Projects’ characteristics $\beta$, $\lambda_i$, $V$, $I$, and $\varepsilon$ are common knowledge. At $t = -1$, entrepreneurs know shares $\alpha_i$ of "their" investor in all projects. For angel investment this is only one project, for VC investment these are the VC shares in both projects. At $t = 0$, entrepreneur $E_i$ observes investment into his own project, $I_i$, but not investment into another project, $I_j$. At $t = 1$, entrepreneurs and investors of the firms that got non-zero financing at the R&D stage learn results of the R&D stage of each project. If the R&D stage of project $j$ is successful, then the stakeholders of project $i$ can take technology $j$ and implement it in their own project.

The timeline

The timeline is summarized in the table below.

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = -1$</td>
<td>investors offer $\alpha_i$</td>
<td>$I_i$ are invested</td>
</tr>
<tr>
<td></td>
<td>entrepreneurs choose investors $i = 1, 2$</td>
<td>$I_i = {0; \varepsilon; I}$</td>
</tr>
<tr>
<td></td>
<td>$I_i$ observed</td>
<td>$t = 1$</td>
</tr>
<tr>
<td></td>
<td>- R&amp;D results are observed</td>
<td>- if $I_i$, success with probability $\beta$ are realized</td>
</tr>
<tr>
<td></td>
<td>- projects continued/terminated</td>
<td></td>
</tr>
<tr>
<td>$t = 0$</td>
<td>investments $I_i, I_i = {\varepsilon; I}$ are made.</td>
<td>$t = 2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>payoffs</td>
</tr>
<tr>
<td></td>
<td>At $t = -1$, the entrepreneurs announce their projects.</td>
<td>investors decide whether to participate in projects or not and submit their bids in the form of shares $\alpha_i$ they want for participation in project $i$. Each angel can participate only in one project and submit bids $\alpha_{A,i}$. Each VC can participate in both projects and can submit bids $\alpha_{VC,i}$. Entrepreneurs choose their investors (if any). Firms with non-zero funding continue, firms without investment die.</td>
</tr>
<tr>
<td></td>
<td>At $t = 0$ investments $I_i, I_i = {\varepsilon; I}$ are made.</td>
<td>Amount of investment into rival firms is not contractual.</td>
</tr>
<tr>
<td></td>
<td>At $t = 1$ the success or failure of the R&amp;D stage of projects, which received investment $I_i$, is observed by stakeholders of all firms in business. Investor of a failed project can use technology of the successful project, if he (and no entrepreneur) decides so. If none of the projects is successful at the R&amp;D stage, then both projects are discontinued.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>At $t = 2$, the net payoffs are realized.</td>
<td></td>
</tr>
</tbody>
</table>

Externalities

This model has two sources of externalities here. The R&D externality is created by the transferability of R&D results to the other project and is always positive. It is characterized by the probability of success of an individual project and is at its highest level, when this probability is equal to 0.5. The payoff (market) externality is related to the interdependence of projects payoffs and can be either negative or positive. We call the externality “positive” when $\lambda_1 + \lambda_2 > 1$ and we call it “negative” when $\lambda_1 + \lambda_2 \leq 1$. 
3. Equilibria

In this section we take as a benchmark case the first best outcome, which maximizes the total expected NPV of both projects, and compare it with equilibria achieved with angel’s (separate) investment and equilibria achieved using VC’s (portfolio) investment. We study positive and negative payoff externalities separately.

3.1. Positive payoff externality

If $\lambda_1 + \lambda_2 > 1$, then the payoff externalities are positive. The first best outcome is then determined as

$$E[\text{NPV}] = \max \{0; \beta (\lambda_1 + \lambda_2) V - I - \varepsilon; \beta (2 - \beta) (\lambda_1 + \lambda_2) V - 2I\},$$

where 0 corresponds to $I_1 = I_2 = 0$, the second element to the full-scale investment into one project with nominal investment into another and the third element to the full-scale investment into both projects.

Proposition 3.1 (First best investment with positive externalities). If $\lambda_1 + \lambda_2 > 1$, then the first best investment decision rule is

1. Invest $I$ both into project 1 and project 2 whenever

   $$\beta (1 - \beta) (\lambda_1 + \lambda_2) V \geq I - \varepsilon$$

2. Invest $I$ either in project 1 or in project 2 whenever

   $$\begin{cases} 
   \beta (1 - \beta) (\lambda_1 + \lambda_2) V < I - \varepsilon, \\
   \beta (\lambda_1 + \lambda_2) V \geq I + \varepsilon
   \end{cases}$$

3. Don’t invest whenever

   $$\beta (\lambda_1 + \lambda_2) V < I - \varepsilon.$$ 

Proof. Proof follows from finding which element in Expression (3.1) provides the maximum.

The R&D externality is bounded and reaches its maximum when $\beta = \frac{1}{2}$, while the payoff externality is monotonically increasing with $\lambda_1 + \lambda_2$. Both externalities have a positive sign, meaning that the greater they are, the higher the project payoffs are. The closer is $\beta$ to $\frac{1}{2}$, the more attractive is the full-scale investment in both projects.
Payoff matrix with separate investment

Under separate investment (by angel investors) the first best outcome may or may not be attainable. The projects’ net payoff matrix is

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>(\varepsilon)</th>
<th>(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(0; 0)</td>
<td>(0; (-\varepsilon))</td>
<td>(0; (\beta V - I))</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>(-(\varepsilon); 0)</td>
<td>(-(\varepsilon); -(\varepsilon))</td>
<td>((\beta \lambda_1 V - \varepsilon; \beta \lambda_2 V - I))</td>
</tr>
<tr>
<td>(I)</td>
<td>((\beta V - I; 0))</td>
<td>((\beta \lambda_1 V - I; \beta \lambda_2 V - \varepsilon))</td>
<td>((\beta (2 - \beta) \lambda_1 V - I; \beta (2 - \beta) \lambda_2 V - I))</td>
</tr>
</tbody>
</table>

where the investment made at the R&D stage in project 1 is given in the first column and in project 2 — in the first row. Depending on values of \(\beta, \lambda_1\) and \(\lambda_2\) (and \(V\) and \(I\) as well) different strategies can be equilibrium outcome.

Equilibrium \((I; I)\)

Equilibrium \((I; I)\) is a pure strategy equilibrium if and only if \(\beta (1 - \beta) \lambda_1 V > I - \varepsilon\). When \((I; I)\) is an equilibrium, it is the unique pure strategy equilibrium. Notice that in this case, separate investment decisions lead to the same outcome as the coordinated investment and is the first best outcome. Even without coordinating their actions, investors do not overinvest, because they invest only if they get enough profit from their own project, and the positive network externalities preclude overinvestment in that case.

What stakes do angel investors have in the projects in that case? Zero expected return for the investors is achieved when their shares in the projects are, respectively

\[
\alpha_{A,1} = \frac{I}{\beta (2 - \beta) \lambda_1 V}; \text{ and } \alpha_{A,2} = \frac{I}{\beta (2 - \beta) \lambda_2 V}.
\]

In order to be able to compete with angel investors at \(t = -1\), VCs have to offer the entrepreneurs individual contracts no worse than the ones offered by the angel investors, and they cannot offer better offers without suffering losses, so the outcome of VC financing is the same. Effectively, in this equilibrium the VC’s facility to “play the portfolio” is of no marginal benefit to him, and entrepreneurs choose angel investors to finance their projects.

Place Figure 3.1 here

Figures 3.1-3.5 provide an illustration of the positive externalities case when \(I/V = 0.5\), \(\varepsilon = 0\) and \(\lambda_1 = \lambda_2 = \lambda > 0.5\). The horizontally shaded areas on Figure 3.1 are where the VC and angels would make the same investment decisions. When both types of externalities are high,\(^4\) then all investors would fund both projects.

\(^4\)The R&D externality is the highest when \(\beta = 0.5\). It is the lowest, when \(\beta = 0\) or \(\beta = 1\).
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Positive Externality, \( I/V = 0.5 \)

Figure 3.1: VC investment decisions with positive externalities. The horizontally shaded area is where the VC funding decision coincides with angels’ funding decisions and does not increase the total NPV. \( I/V = \kappa = 0.5 \), \( \lambda_1 = \lambda_2 = \lambda \geq 0.5 \), \( \varepsilon = 0 \).

Equilibrium (\( \varepsilon; I \))

Results can be quite different when \( \beta(1-\beta)\lambda_1 V \leq I - \varepsilon \). With angel financing, investment into both projects is not an equilibrium outcome anymore. If

\[
\begin{align*}
\beta\lambda_2 V - I &> 0, \\
\beta\lambda_1 V - I &< 0,
\end{align*}
\]

then the only pure Nash equilibrium is the outcome, when project 2 gets full-scale funding for the R&D stage and firm 1 just enters the business investing \( \varepsilon \). If the R&D stage of project 2 is successful, then both projects will use its technology at the second stage. The following proposition shows that sometimes VC financing helps to achieve a better result in terms of the total expected payoff.
Proposition 3.2. If $\lambda_1 + \lambda_2 > 1$ and

\[
\begin{align*}
\beta \lambda_2 V - I &> 0, \\
\beta \lambda_1 V - I &< 0, \\
\beta (1 - \beta) (\lambda_1 + \lambda_2) V &> I, \\
(1 - \beta)^2 \beta (\lambda_1 + \lambda_2) V &> I - (3 - 2\beta) \varepsilon.
\end{align*}
\]

then at the R&D stage angel investors would invest $I$ only into project 2 and $\varepsilon$ into project 1, while a VC would invest at the R&D stages into both projects, thus increasing the total net payoff in comparison with the angel investment into separate projects. The expected total net payoff with angel investors is

\[
\beta (\lambda_1 + \lambda_2) V - I - \varepsilon > 0.
\]

Compared with angel financing, VC financing creates additional expected value

\[
E[\Delta V] = \beta (1 - \beta) (\lambda_1 + \lambda_2) V - (I - \varepsilon) > 0.
\]

The VC’s shares in projects 1 and 2 $\alpha_{VC,1}$ and $\alpha_{VC,2}$ satisfy

\[
\begin{align*}
\alpha_{VC,1} \lambda_1 + \alpha_{VC,2} \lambda_2 &= \frac{2I}{\beta (2 - \beta) V} \\
(\alpha_{VC,1} \lambda_1 + \alpha_{VC,2} \lambda_2) &= \frac{I - \varepsilon}{\beta (2 - \beta) V}, \\
\alpha_{VC,1} &\leq \frac{1 - \beta}{1 - \beta} + \frac{\varepsilon}{\beta (2 - \beta) \lambda_1 V}, \\
\alpha_{VC,2} &\leq \frac{1 - \beta}{1 - \beta} + \frac{I}{\beta \lambda_2 V}.
\end{align*}
\]

Proof. The proof is in Appendix A.

Corollary 3.3. If $\lambda_1 + \lambda_2 > 1$ and

\[
\begin{align*}
\beta \lambda_2 V - I &> 0, \\
\beta \lambda_1 V - I &< 0, \\
\beta (1 - \beta) (\lambda_1 + \lambda_2) V &> I, \\
(1 - \beta)^2 \beta (\lambda_1 + \lambda_2) V &< I - (3 - 2\beta) \varepsilon.
\end{align*}
\]

then the first best outcome is never achieved by investors.

Proof. The proof follows from Proposition 3.2.

Proposition 3.2 demonstrates that some projects can get full-scale funding at the R&D stage only from VCs, simply because individual project’s returns are not high enough to attract investment from angel investors. Competitive VC needs to get zero expected return on his total investment portfolio, but not necessarily on every component project.
Mixed equilibrium

If the following system of inequalities hold

\[
\begin{align*}
\frac{1}{2} \beta \lambda_1 V - I > 0, \\
\beta (1 - \beta) \lambda_2 V - I \leq 0,
\end{align*}
\] (3.2)

then two Nash equilibria exist: \((\varepsilon; I)\) and \((I; \varepsilon)\). Each entrepreneur would prefer to invest only \(\varepsilon\), if he knew that the other invests \(I\) at the R&D stage.

A mixed equilibrium exists in which entrepreneur \(i\) decides to invest \(\varepsilon\) the R&D stage with probability

\[
\rho_i = \frac{I - \lambda_j \beta (1 - \beta) V - \varepsilon}{\lambda_j \beta^2 V}
\]

and decides to invest \(I\) with probability

\[
1 - \rho_i = \frac{\lambda_j \beta V - I + \varepsilon}{\lambda_j \beta^2 V},
\]

where \(j = 2 - i\). The entrepreneurs’ expected payoffs are

\[
E[NPV_i] = \frac{\lambda_i \beta V - I + \varepsilon}{\beta} - \varepsilon
\] (3.3)

If entrepreneur \(i\) decides to invest \(\varepsilon\), then he will choose an angel investor, who would invest \(\varepsilon\) for shares, \(\alpha_{A,i,\varepsilon}\), such that

\[
\alpha_{A,i,\varepsilon} \lambda_i \beta V (p_j + (1 - p_j) (2 - \beta)) = \varepsilon
\]
or

\[
\alpha_{A,i,\varepsilon} = \frac{\beta \varepsilon}{\lambda_i \beta V - (1 - \beta) (I - \varepsilon)}.
\]

If entrepreneur \(i\) decides to invest \(I\), then the angel investor gets shares, \(\alpha_{A,i,I}\),

\[
\alpha_{A,i,I} = \frac{\beta I}{\lambda_i \beta V - (1 - \beta) (I - \varepsilon)}.
\]

Exact investment levels, \(I_i\), and shares offered, \(\alpha_{A,i}\), are not observed by rival entrepreneurs and cannot be contracted on, that is, \(\alpha_{A,j}\) cannot be a function of \(I_j\) and \(\alpha_{A,j}\).

When entrepreneurs play mixed strategies, the lack of coordination potentially leads to expected suboptimal investment. Can VC investment help to avoid this problem? The following proposition derives the conditions when VC investment gives a better outcome.

Proposition 3.4. For the positive externality case \((\lambda_1 + \lambda_2 \geq 1)\), if system of inequalities (3.2) holds, then the VC’s coordinated investment in both projects generates a better outcome, than separate financing by angel investors, who play a mixed equilibrium strategy.
1. If $\beta(1-\beta)(\lambda_1 + \lambda_2)V - I < 0$, then the VC invests $\varepsilon$ in one project and $I$ into the other project, taking an $\alpha_{VC,1}$ share of project 1’s payoff and an $\alpha_{VC,2}$ share of project 2’s payoff such that

$$\alpha_{VC,1}\lambda_1 + \alpha_{VC,2}\lambda_2 = \frac{I + \varepsilon}{\beta V},$$

$$\alpha_{VC,i} \leq \frac{I - (1-\beta)\varepsilon - \beta(1-\beta)\lambda_i V}{\beta^2 \lambda_i V}$$

Compared with angel financing, VC financing creates additional expected value

$$E[\Delta V] = \frac{2-\beta}{\beta}(I - \varepsilon) - (1-\beta)(\lambda_1 + \lambda_2)V > 0.$$  

2. If $\beta(1-\beta)(\lambda_1 + \lambda_2)V - I > 0$, then VC invests $I$ into each of projects 1 and 2, taking an $\alpha_{VC,1}$ share of project 1’s payoff and an $\alpha_{VC,2}$ share of project 2’s payoff such that

$$\alpha_{VC,1}\lambda_1 + \alpha_{VC,2}\lambda_2 = \frac{2I}{\beta(2-\beta)V},$$

and

$$\alpha_{VC,i} \leq \frac{I - (1-\beta)\varepsilon - (1-\beta)^2}{\beta^2 (2-\beta)\lambda_i V}$$

Compared with angel financing, VC financing creates additional expected value

$$E[\Delta V] = \frac{2(1-\beta)}{\beta}(I - \varepsilon) - (1-\beta)^2(\lambda_1 + \lambda_2)V > 0.$$  

Proof. The proof is given in Appendix.A

The intuition for the proof is very simple. VC has to create incentives for both entrepreneurs to seek funding from him, rather than from business angels. If these incentive compatibility, VC financing allows us to achieve the first best outcome.

Place Figure 3.2 here

Figure 3.2 illustrates Propositions 3.2 and 3.4, when the VC’s coordinated investment into both projects creates a strictly better outcome than separate angels’ investments. The horizontally shaded area is where the VC invests $I$ into each project, because the R&D externality is relatively high ($\beta$ is close to 0.5), and the vertically shaded area is where the VC invests $\varepsilon$ and $I$, when the probability of success is very high and funding the second project does not add much value.
Figure 3.2: VC investment decisions with positive externalities. Shaded areas are where VC funding decisions increase the total NPV. In the horizontally shaded area $I$ is invested into each of the projects. In the vertically shaded area, $I$ is invested in one project and $\varepsilon > 0$ in the other. $I/V = \kappa = 0.5$, $\lambda_1 = \lambda_2 = \lambda \geq 0.5$, $\varepsilon = 0$.

**Equilibrium $(0; 0)$**

If

$$\beta \lambda_2 V - I < 0,$$

then $(0; 0)$ is the only pure Nash equilibrium outcome with separate investment. No firm goes into business and both of them die. Sometimes this is an optimal decision, but sometimes, according to Proposition 3.1, the first best outcome would be to finance at least one project. Angel financing at stage 1 becomes impossible, whereas a VC can potentially lead to the first best outcome. The exact share of the VC’s ownership and the decision about the projects’ continuation at stage 2 depend on the direction of the inequality

$$\beta (1 - \beta) (\lambda_1 + \lambda_2) V - I \leq 0$$

This leads to the following proposition:
Proposition 3.5. When the system of inequalities
\[
\begin{align*}
&\beta \lambda_2 V < I, \\
&\beta(\lambda_1 + \lambda_2) V > I - \varepsilon,
\end{align*}
\]
holds, the VC is the only possible investor at the R&D stage.

1. If \(\beta(1 - \beta)(\lambda_1 + \lambda_2) V > I - \varepsilon\) then the VC’s investment of \(I\) into both projects at stage 1 leads to the first best outcome. His shares in projects 1 and 2, \((\alpha_{VC,1}; \alpha_{VC,2})\), satisfy
\[
\alpha_{VC,1,\lambda_1} + \alpha_{VC,2,\lambda_2} = \frac{2I}{\beta(2 - \beta)V},
\]
\[0 < \alpha_{VC,i} \leq 1.\]

2. If \(\beta(1 - \beta)(\lambda_1 + \lambda_2) V < I - \varepsilon\), then the VC invests at the R&D stage \(I\) into one project and \(\varepsilon, \varepsilon < \beta(\lambda_1 + \lambda_2) V - I\), into another. His shares in projects 1 and 2, \((\alpha_{VC,1}; \alpha_{VC,2})\), satisfy
\[
\alpha_{VC,1,\lambda_1} + \alpha_{VC,2,\lambda_2} = \frac{I + \varepsilon}{\beta V},
\]
\[0 < \alpha_{VC,i} \leq 1.\]

Proof. The proof follows from Proposition 3.1 and from the fact that no alternative financing is possible.

As before, the exact values \(\alpha_{VC,1}\) and \(\alpha_{VC,2}\) are determined by bargaining between both entrepreneurs and the VC and multiple equilibria exist.

Place Figure 3.3 here

Unlike shaded areas in Figure 3.2, where angel investors could provide funding for at least one project, for parameters represented by shading in Figure 3.3, angel investors would not provide funding for either project, whereas VC investors “resuscitate” them. As Proposition 3.5 shows, VC investment breathes new life into these projects which otherwise would not find investment. VC investment allows them to become viable and achieve a positive NPV outcome. When both projects should receive funding (the R&D externality is relatively high), then VC investment provides the first best outcome, which is shown as a horizontally shaded area. The vertically shaded area corresponds to the investment of \(I\) into one project and \(\varepsilon\) into the other project, while the optimal strategy would be to invest only into one project.

Notice that for \(\lambda_i < 1\) entrepreneur \(i\) prefers an outcome in which his rival would get a zero finding and entrepreneur \(i\) would get \(\beta V - I\), which is greater than what he actually gets with VC financing, namely, \(\beta \lambda_i V - I\).
Figure 3.3: VC investment decisions with positive externalities. Shaded areas are where VC funding resuscitates the projects with total positive NPV. In the horizontally shaded area I is invested into each of the projects. In the vertically shaded area I is invested in one project and ε > 0 in the other. $I/V = \kappa = 0.5$, $\lambda_1 = \lambda_2 = \lambda \geq 0.5$, $\varepsilon = 0$.

With both types of externalities being positive, whenever the VC is able to achieve a better outcome than angels, it is only because of his coordinated portfolio investment. At the second stage both projects continue without his interference. At this stage, it is not important, whether the VC has control or not. The only things that matter are the VC’s expertise and the technology transfer. One possible implication is that in industries with positive externalities, VC investors would exercise their control rights less than in the industries with negative externalities. As we are about to show, with the negative payoff externalities the VC’s ability to exercise control becomes crucial.

3.2. Negative payoff externality

When $\lambda_1 + \lambda_2 < 1$, the R&D externality and the payoff externality influence the projects’ outcomes in opposite directions. Whilst at the market stage, the externalities are purely negative and it is better to have only one project at this stage, earlier existence of two R&D projects
increases the chances of ultimate success for both. The first best outcome becomes

\[
E[NPV] = \max \{0; \beta V - I; \beta(2 - \beta)V - 2I\}
\]  

(3.4)

and the investment decision rule is now given in the following proposition

**Proposition 3.6 (First best investment with negative externalities).** If \(\lambda_1 + \lambda_2 < 1\), then the investment decision rule is

1. **Invest** \(I\) into both project 1 and project 2, but continue not more than one project at stage 2 whenever

\[
\beta(1 - \beta)V - I > 0
\]

2. **Invest** \(I\) into either project 1 or project 2 and \(\varepsilon\) into another. Continue not more than one project at stage 2 whenever

\[
\begin{cases}
\beta(1 - \beta)V - I < 0 \\
\beta V - I > 0
\end{cases}
\]

3. **Don’t invest at all** whenever

\[
\beta V - I < 0
\]

**Proof.** The proof follows from finding which element in Expression (3.4) provides the maximum.

Again, whether or not the first best outcome is achievable will depend on parameters of the problem.

The investment rule for separate investments by angels into projects 1 and 2 remains the same as in the case with positive externalities. The VC investor again plays a coordination role, potentially being able to provide a better outcome, because 1) as with positive externalities, he helps to optimize investment at the R&D stage, and, 2) he better coordinates the projects’ continuation at the market stage by (randomly) terminating one of them. Of course, the entrepreneurs rationally anticipate that their project may be terminated and require their expected payoffs to be at least as good as with angel investment. Ability to curtail one project to the advantage of another project can, ex ante, raise the expected value of all projects. This gives a rationale for why entrepreneurs may be willing *ex ante* to hand over to VCs the power over their firms, even though ex post this power may be used against the entrepreneurs best interests. We examine now the different possible investment equilibria.
Equilibrium \((I; I)\)

\((I; I)\) is a pure strategy equilibrium if and only if \(\beta(1 - \beta)\lambda_1 V - I > 0\). Furthermore, when \((I; I)\) is an equilibrium, it is unique. However, in a contrast to the positive externalities case, separate angel investment leads to an outcome very different from the coordinated investment case. Angel investors overinvest at the market stage, continuing both projects. The respective project ownership fractions taken by the angels are:

\[
\alpha_{A,1} = \frac{I}{\lambda_1 \beta (2 - \beta)V}; \quad \text{and} \quad \alpha_{A,2} = \frac{I}{\lambda_2 \beta (2 - \beta)V}
\]

For VC investment, the situation changes significantly. If the VC funds the R&D stage of both projects, then he will be able to terminate one of them at stage 2. We reiterate that “termination” does not necessary mean “liquidation,” it could equally be interpreted as forcing premature IPO, early commercialization of an underdeveloped product etc.

**Proposition 3.7.** If \(\lambda_1 + \lambda_2 < 1\), \(\lambda_2 < \frac{1}{2}\) and \(\beta(1 - \beta)\lambda_1 V - I > 0\), then VC invests in both projects, reaching the first best outcome. The fraction of ownership in projects 1 and 2 which gives VC investor the fair return is determined by

\[
\alpha_{VC,1} = \alpha_{VC,2} = \frac{2I}{\beta (2 - \beta)V}.
\]

**Proof.** The proof is in Appendix.A

**Corollary 3.8.** If \(\lambda_1 + \lambda_2 < 1\), \(\lambda_2 > \frac{1}{2}\) and \(\beta(1 - \beta)\lambda_1 V - I > 0\), then no VC financing is possible

Intuition behind Proposition 3.7 and its Corollary is very simple. VC would randomly continue one of the projects only if his stakes in both projects are equal, otherwise, he would always continue a project, in which his stakes are higher. At the same time, each entrepreneur chooses the VC investment if and only if he expects to get from it payoff higher than from the angel investment, which gives us the conditions of the proposition.

Condition \(\lambda_2 < \frac{1}{2}\) of Proposition 3.7 indicates that for the case of negative externalities, portfolio companies should be affected by competition in a relatively similar way, meaning that these companies cannot be very different from each other.

Equilibrium \((\varepsilon; I)\)

If

\[
\begin{align*}
\beta \lambda_1 V &< I, \\
\beta \lambda_2 V &> I,
\end{align*}
\]

(3.5)
then, with angel financing, the only pure Nash equilibrium is $(\varepsilon; I)$ — the outcome when only project 2 gets full-scale funding for the R&D stage. If the first stage is successful, then both projects will use its technology at the second stage, reducing the total profits below that could be achieved by a single firm.

The following proposition shows that VC financing helps to achieve a better result.

**Proposition 3.9.** If $\lambda_1 + \lambda_2 < 1$ and system of inequalities (3.5) holds, then

1. for $\beta (1 - \beta) V - I > 0$, and

   \[
   \begin{cases}
   \lambda_1 \leq \frac{1}{2} + \frac{1-\beta}{2} \frac{I-\varepsilon}{\beta V}, \\
   \lambda_2 \leq \frac{1}{2} + \frac{1-\beta}{2},
   \end{cases}
   \]

   at the R&D stage the VC invests $I$ into each project and in the case of success continues only one project. His share in project $i$ is

   \[\alpha_{VC,i} = \frac{2I}{\beta (2 - \beta) V}.\]

   In comparison with angel investors, who would invest $\varepsilon$ into the first project, $I$ into the second project and would continue both projects, VC investment generates a net expected gain

   \[E[\Delta NPV] = \beta (1 - \lambda_1 - \lambda_2) V + \{\beta (1 - \beta) V - (I - \varepsilon)\},\]

   where the first term is the gain from optimal project termination at the second stage and the second term is the gain from better investment coordination at the first stage.

2. for $\beta (1 - \beta) V - I < 0$ and

   \[
   \begin{cases}
   \lambda_1 < \frac{1}{2} - \frac{I-\varepsilon}{2\beta V}, \\
   \lambda_2 < \frac{1}{2} + \frac{I-\varepsilon}{2\beta V},
   \end{cases}
   \]

   at the R&D stage the VC invests $\varepsilon$ and $I$, and in the case of success continues only one project. His share in project $i$ is

   \[\alpha_{VC,i} = \frac{I + \varepsilon}{\beta V}.\]

   In comparison with angel investors, VC investment generates a net expected gain

   \[E[\Delta NPV] = \beta (1 - \lambda_1 - \lambda_2) V,\]

   from optimal project termination.

**Proof.** The proof is in Appendix A

The necessary condition for $\lambda_1$ to be positive is that $\frac{I}{V} < \beta$ meaning that the VC invests into highly profitable projects.
Mixed equilibrium

If
\[
\begin{cases}
\beta \lambda_1 V - I > 0, \\
\beta (1 - \beta) \lambda_2 V - I \leq 0,
\end{cases}
\]
then, as in the positive externalities case, entrepreneurs can play mixed strategies using separate investment by angels. VC investment sometimes allows us to achieve better outcome, as is shown in the following proposition.

**Proposition 3.10.** If
\[
\begin{cases}
\beta \lambda_1 V - I > 0, \\
\beta (1 - \beta) \lambda_2 V - I \leq 0,
\end{cases}
\]
\[\lambda_1 + \lambda_2 < 1,
\]
then the VC’s coordinated investment of both projects may generate a better outcome than would separate financing by angels playing mixed equilibrium strategies.

1. If
\[
\begin{cases}
\beta (1 - \beta) V - I < 0, \\
\beta (2 \lambda_2 - \beta) V - I (2 - \beta) + \varepsilon < 0,
\end{cases}
\]
then the VC invests \(\varepsilon\) in one project and \(I\) into the other project, taking a share, \(\alpha_{VC,i}\), of project \(i\) payoff such that
\[
\alpha_{VC,i} = \frac{I + \varepsilon}{\beta V},
\]
Compared with angel financing, VC financing creates additional expected value of
\[
E[\Delta V] = \frac{2 - \beta}{\beta} I + (\beta - \lambda_1 - \lambda_2) V + \varepsilon > 0.
\]

2. If
\[
\begin{cases}
\beta (1 - \beta) V - I > 0, \\
\beta (2 (1 - \lambda_2) - \beta) V + 2 (1 - \beta) (I - \varepsilon) > 0,
\end{cases}
\]
then the VC invests \(I\) into each of projects 1 and 2, taking a share, \(\alpha_{VC,i}\), of project \(i\) payoff such that
\[
\alpha_{VC,i} = \frac{2I}{\beta (2 - \beta) V},
\]
At the market stage only one project is continued. Compared with angel financing, VC financing creates additional expected value of
\[
E[\Delta V] = \frac{2(1 - \beta)}{\beta} I + (\beta(2 - \beta) - \lambda_1 - \lambda_2) V - 2 \frac{1 - \beta}{\beta} \varepsilon > 0.
\]
Figure 3.4: VC investment decisions with negative externalities. Shaded areas are where VC funding decision increases the total NPV. In the horizontally shaded area $I$ is invested in each of the projects. In the vertically shaded area $I$ is invested in one project and $\varepsilon$ in the other. $I/V = \kappa = 0.05$, $\lambda_1 = \lambda_2 = \lambda \leq 0.5$, $\varepsilon = 0$.

We omit the proof, which follows the logic of the proof of Propositions 3.4 and 3.9.

Figure 3.4 illustrates Propositions 3.7 - 3.10, when VC’s coordinated investment into both projects creates a strictly better outcome than separate angels’ investments. $I/V = 0.05$, $\varepsilon = 0$ and $\lambda_1 = \lambda_2 = \lambda \leq 0.5$. The horizontally shaded area is where VC investment provides the first best outcome and the vertically shaded area is where the VC achieve an outcome which can be made arbitrary close to the first best. In the former the VC invests $I$ into each project, because the R&D externality is relatively high. Notice that unlike the positive externalities case, the VC investment offers a strict improvement even when single investors would also invest into both projects at the R&D stage. This is, because, with negative payoff externalities, the VC is able to terminate one of the projects at the market stage, thus increasing the total NPV of investment. Other results are similar to the positive externalities case, but only one project gets to the market stage.
Equilibrium \((0; 0)\)

If
\[
\begin{align*}
\beta \lambda_2 V - I &< 0, \\
\beta V - I &> 0,
\end{align*}
\]
then \((0; 0)\) is the unique pure Nash equilibrium outcome under separate investment. Recalling Proposition 3.6, we see that the first best outcome is to finance at least one project at the R&D stage and, in the case of success, continue with one project at the market stage. As in the positive payoff externalities case, in equilibrium \((0; 0)\) angel financing at stage 1 becomes impossible, whereas VC financing can potentially lead to the first best outcome or very close to it. We get the following proposition:

**Proposition 3.11.** When the system of inequalities, \((3.6)\), holds, the VC is the only possible investor at the R&D stage.

1. If \(\beta(1 - \beta)V - I > 0\), then the VC invests \(I\) into each of projects 1 and 2, taking a share, \(\alpha_{VC,i}\), of project \(i\) payoff such that
\[
\alpha_{VC,i} = \frac{2I}{\beta(2 - \beta)V},
\]

2. If \(\beta(1 - \beta)V - I < 0\), then the VC invests \(\varepsilon\) in one project and \(I\) into the other project, taking a share, \(\alpha_{VC,i}\), of project \(i\) payoff such that
\[
\alpha_{VC,i} = \frac{I + \varepsilon}{\beta V}.
\]

**Proof.** The proof follows from Proposition 3.1 and from the fact that no alternative financing is possible.

Figure 3.5 shows the zone of VC projects’ resuscitation, which corresponds to Proposition 3.11.

**Place Figure 3.5 here**

4. Conclusion

The results of this paper allow us to make the following conjectures. First, VCs are more likely to invest into companies, which are related to each other, for example, belong to the same industry or related industries with high inter-industry externalities, when success or failure of
one company affects the future of other portfolio companies. They also invest into companies of relatively similar size.

Second, we should observe a relatively higher proportion of VC investment into industries with high network externalities that have the following features: 1) transferability of early stage R&D results between portfolio projects (low barriers for information spillovers or when VCs are able to lower these barriers for their portfolio companies), 2) investment into several projects which develop similar new technologies or new business models, with probabilities of success being neither very high nor very low, 3) industries with negative externalities and high entry barriers for second-movers.

Third, we should observe more information spillovers between VCs’ portfolio companies, than between companies financed independently. Stand-alone valuation analysis of some portfolio companies might even show that ex-ante they would not have been attractive as individual investments, perhaps having negative NPV.

Finally, in industries with negative externalities we should observe a higher proportion of
projects’ “termination” in the form of a premature IPOs, “strategic refocusing” etc. One possible indication of a premature IPO would the VC’s early sale of his shares at post-offering. Another interesting implication for these industries is that in VC-funded companies we should observe lower degree of entrepreneurs' ownership resulting in VC’s abnormally high returns. This happens, because entrepreneurs would like to increase the probability of their project continuation, thus sacrificing some share of their ownership, essentially ”bribing” the VC.

When coordinated investment and forceful project continuation/termination decisions are necessary because of a strong network externalities effect, VC investment into a portfolio of projects can potentially lead to the first best outcome, whereas angel investment into separate projects may generate suboptimal outcome.

Unfortunately, the first best outcome may not always be achievable even with VC investment. This is because each entrepreneur pursues his individual interests. To the extent that the VC must yield to the interests of individual entrepreneurs, the VC may be precluded from the first best coordination of projects. Clearly, contractual powers and contingent control terms written ex ante are vitally important in this regard and this explains the significant resources expended by VCs in legal fees and the design of state contingent securities, such as convertible shares (Kaplan and Stromberg (2000a)). Another hurdle to efficient project coordination may be the self interests of the VC. In the present paper we have assumed that they act competitively ex ante, however we identify the circumstances when their ability to hold-up entrepreneurs prevents the optimal outcome.

We believe that the results of this paper are applicable not only to the comparative analysis of venture capital and angel investment, but also to the internal funding of several projects by one individual firm and to multilateral versus bilateral lending, as in Bhattacharya and Chiesa (1995).
References


A. Appendix

Proof. Proposition 3.2. Investors expect ex-ante to get a fair (zero) return on their investment. It is easy to see that with separate (angel) investment, \((\varepsilon; I)\) is the equilibrium strategy.

Therefore, in return for providing capital, \(I\), for the R&D stage of project 2 a business angel requires share, \(\alpha_{A,2} = \frac{I}{\beta^{2}V}\). Another business angel gets share, \(\alpha_{A,1} = \frac{\varepsilon}{\beta^{2}V}\) for participating in project 1.

If VC invests \(I\) into both projects at the R&D stage, he expects to get
\[
(\alpha_{VC,1}\lambda_{1} + \alpha_{VC,2}\lambda_{2}) \beta (2 - \beta)V - 2I = 0
\]
Investment into another project is not contractible. Therefore, incentive compatibility constraints for the VC are
\[
(\alpha_{VC,1}\lambda_{1} + \alpha_{VC,2}\lambda_{2}) \beta V - (I + \varepsilon) < (\alpha_{VC,1}\lambda_{1} + \alpha_{VC,2}\lambda_{2}) \beta (2 - \beta)V - 2I
\]
or
\[
I - \varepsilon < (\alpha_{VC,1}\lambda_{1} + \alpha_{VC,2}\lambda_{2}) \beta (1 - \beta)V.
\]
Entrepreneurs are interested in VC, only if their profit is higher than with the angel’s investment. Participation constraints for entrepreneurs \(E_{1}\) and \(E_{2}\) are, respectively

\[
\begin{cases}
(1 - \alpha_{VC,1}) \beta (2 - \beta) \lambda_{1}V > \beta \lambda_{1}V - \varepsilon, \\
(1 - \alpha_{VC,2}) \beta (2 - \beta) \lambda_{2}V > \beta \lambda_{2}V - I,
\end{cases}
\]
or
\[
\begin{cases}
\alpha_{VC,1} \leq \frac{1 - \beta}{2 - \beta} + \frac{\varepsilon}{\beta (2 - \beta)\lambda_{1}V}, \\
\alpha_{VC,2} \leq \frac{1 - \beta}{2 - \beta} + \frac{I}{\beta (2 - \beta)\lambda_{2}V}.
\end{cases}
\]
Both incentive compatibility and participation constraints hold only if the following inequality is satisfied:
\[
\left[\frac{1 - \beta}{2 - \beta} (\lambda_{1} + \lambda_{2}) + \frac{I + \varepsilon}{\beta (2 - \beta)V}\right] \beta (1 - \beta)V > I - \varepsilon,
\]
\[
(1 - \beta)^2 \beta (\lambda_{1} + \lambda_{2})V > I - (3 - 2\beta)\varepsilon.
\]
With VC financing, entrepreneurs get total \((\lambda_{1} + \lambda_{2}) \beta (2 - \beta)V - 2I\) versus \((\lambda_{1} + \lambda_{2}) \beta V - (I + \varepsilon)\) that they would get with angel financing. \(\blacksquare\)

Proof. Proposition 3.4.

1. If \(\beta (1 - \beta) (\lambda_{1} + \lambda_{2})V - I < 0\), then according to Proposition 3.1 it is optimal to invest \(I\) in only one project. From the competitive zero expected return we get
\[
\alpha_{VC,1}\lambda_{1} + \alpha_{VC,2}\lambda_{2} = \frac{I + \varepsilon}{\beta V}.
\]
Entrepreneurs will prefer to obtain financing from the VC instead of angel investors, if their expected payoffs with VC financing will be greater or equal to their expected payoffs with angel financing, defined by (3.3). For Entrepreneur $i$ this constraint is

$$(1 - \alpha_{VC,i}) \beta \lambda_i V \geq \frac{\lambda_i \beta V - I + (1 - \beta) \varepsilon}{\beta}$$

or

$$\alpha_{VC,i} \leq 1 - \frac{\lambda_i \beta V - I + (1 - \beta) \varepsilon}{\beta^2 \lambda_i V} = \frac{I - (1 - \beta) \varepsilon - \beta (1 - \beta) \lambda_i V}{\beta^2 \lambda_i V}.$$  \hspace{1cm} (A.1)

We have to ensure that $0 < \alpha_{VC,i} \leq 1$:

$$\beta (1 - \beta) \lambda_1 V - I + (1 - \beta) \varepsilon \leq \beta (1 - \beta) \lambda_2 V - I + (1 - \beta) \varepsilon \leq 0$$

and

$$\frac{I - (1 - \beta) \varepsilon - \beta (1 - \beta) \lambda_2 V}{\beta^2 \lambda_2 V} < 1,$$

then

$$\beta^2 \lambda_2 V > I - (1 - \beta) \varepsilon - \beta (1 - \beta) \lambda_2 V,$$

$$\beta \lambda_2 V > I - (1 - \beta) \varepsilon.$$

Both conditions hold whenever system of inequalities (3.2) holds. We also have to check that participation constraints for the VC investor are satisfied, i.e., the zero-profit condition does not contradict (A.1), i.e., that

$$\frac{I + \varepsilon}{\beta V} \leq 2 \frac{I - (1 - \beta) \varepsilon - (1 - \beta) \beta (\lambda_1 + \lambda_2)}{\beta^2 V} \text{ does hold.}$$

This inequality is equivalent to

$$\beta (1 - \beta) (\lambda_1 + \lambda_2) V \leq (I - \varepsilon) (2 - \beta),$$

which follows immediately from

$$\beta (1 - \beta) (\lambda_1 + \lambda_2) V < I$$

and Assumption 1.

2. If $\beta (1 - \beta) (\lambda_1 + \lambda_2) V - I > 0$, then according to Proposition 3.1 it is optimal to invest in both projects. The VC invests $I$ into each of project 1 and 2, taking a share, $\alpha_{VC,1}$, of project 1’s payoff and a share, $\alpha_{VC,2}$, of project 2’s payoff. From the zero expected return we get

$$\lambda_1 \alpha_{VC,1} + \lambda_2 \alpha_{VC,2} = \frac{2I}{\beta (2 - \beta) V}.$$
Entrepreneur $i$ prefers VC investment to the angel investment if and only if the following participation constraint is satisfied

$$(1 - \alpha_{VC,i}) \beta (2 - \beta) \lambda_i V \geq \frac{\lambda_i \beta V - I + (1 - \beta) \varepsilon}{\beta}$$

or

$$\alpha_{VC,i} \leq 1 - \frac{\lambda_i \beta V - I + (1 - \beta) \varepsilon}{\beta^2 (2 - \beta) \lambda_i V} = \frac{I - (1 - \beta) \varepsilon - \beta (1 - \beta)^2 \lambda_i V}{\beta^2 (2 - \beta) \lambda_i V}.$$ 

or

$$\alpha_{VC,i} \leq \frac{I - (1 - \beta) \varepsilon}{\beta^2 (2 - \beta) \lambda_i V} - \frac{(1 - \beta)^2}{\beta (2 - \beta)}.$$  \hspace{1cm} (A.2)

Such pair always exists whenever inequalities (3.2) hold.

We also have to check that the VC does not have an incentive to deviate and invest only $\varepsilon$ into the second project. Sufficient condition for this is that both entrepreneurs prefer having $I$ rather than $\varepsilon$ being invested into their project, that is

$$(1 - \alpha_{VC,i}) \beta (2 - \beta) \lambda_i V \geq \lambda_i \beta V - \varepsilon,$$

or

$$\alpha_{VC,i} \leq \frac{1 - \beta}{2 - \beta} + \frac{\varepsilon}{\beta (2 - \beta) \lambda_i V}.$$  

These inequalities always hold, whenever $\beta (1 - \beta) \lambda_1 V < I - \varepsilon$ and inequalities (A.2) hold.

Finally, we check that participation constraints for the VC investor are satisfied, i.e., the zero-profit condition does not contradict (A.2), i.e., that

$$\frac{2I}{\beta (2 - \beta) V} \leq \frac{I - (1 - \beta) \varepsilon}{\beta^2 (2 - \beta) V} - \frac{(1 - \beta)^2}{\beta (2 - \beta)} (\lambda_1 + \lambda_2)$$

does hold. This inequality is equivalent to

$$\beta (1 - \beta) (\lambda_1 + \lambda_2) V \leq 2 (I - \varepsilon),$$

which follows immediately from

$$\beta (1 - \beta) \lambda_2 V < I - \varepsilon.$$
Proof. Proposition 3.7. From Proposition 3.6 we know that with \( \beta(1 - \beta) \lambda_1 V - I > 0 \) the first best outcome is achieved by both projects being financed at the R&D stage, but only one project being continued at the market stage. Zero expected return gives us

\[
\frac{1}{2} (\alpha_{VC,1} + \alpha_{VC,2}) \beta (2 - \beta) V - 2I = 0
\]

\[
\alpha_{VC,1} + \alpha_{VC,2} = \frac{4I}{\beta (2 - \beta) V}.
\]

On the other hand, \( \alpha_{VC,1} = \alpha_{VC,2} \), because otherwise the project with smaller \( \alpha_{VC, i} \) will be terminated at the market stage for sure. Therefore

\[
\alpha_{VC,i} = \frac{2I}{\beta (2 - \beta) V}
\]

When VC’s shares are equal in both projects, the one which will be terminated will be chosen randomly with equal probability. In order to agree to be financed by a VC instead of an angel investor, both entrepreneurs need to get an expected return at least as high as with angel investment. Thus, we get entrepreneur’s incentive compatibility constraints

\[
\frac{1}{2} (\beta (2 - \beta) V - 2I) \geq \lambda_i (\beta (2 - \beta) V - I),
\]

\[
\lambda_i < \frac{1}{2}.
\]

We assumed that \( \lambda_1 < \lambda_2 \). Therefore, for the negative externalities case \( \lambda_1 < \frac{1}{2} \), \( \lambda_2 < \frac{1}{2} \) according to the statement of the proposition. \( \blacksquare \)

Proof. Proposition 3.9. From Proposition 3.6 we know that with \( \beta(1 - \beta) \lambda_1 V - I > 0 \), the first best outcome is achieved by investing into both projects. To have a positive probability of project continuation, entrepreneurs must offer the VC equal shares, \( \alpha_{VC,1} = \alpha_{VC,2} \). Then zero expected return gives us

\[
\frac{1}{2} \alpha_{VC,1} \beta (2 - \beta) V + \frac{1}{2} (\beta (2 - \beta) \alpha_{VC,2} \beta V = 2I,
\]

\[
\alpha_{VC,1} = \alpha_{VC,2} = \frac{2I}{\beta (2 - \beta) V}.
\]

In order to agree to be financed by a VC instead of an angel investor, both entrepreneurs need to get an expected return at least as high as with angel investment. We get from the incentive compatibility constraints

\[
\left\{ \begin{array}{l}
\frac{1}{2} (\beta (2 - \beta) V - 2I) \geq \beta \lambda_1 V - \varepsilon, \\
\frac{1}{2} (\beta (2 - \beta) V - 2I) \geq \beta \lambda_2 V - I.
\end{array} \right.
\]
or

\[
\begin{align*}
    \lambda_1 & \leq \frac{1}{2} + \frac{1-\beta}{2} - \frac{I-\varepsilon}{\beta V}, \\
    \lambda_2 & \leq \frac{1}{2} + \frac{1-\beta}{2};
\end{align*}
\]

which are the conditions stated in the proposition.

Applying similar logic for \(\beta(1-\beta)\lambda_1 V - I < 0\), when it is optimal to invest \(\varepsilon\) and \(I\), we get

\[
\frac{1}{2} \alpha_{VC,1} \beta V + \frac{1}{2} \alpha_{VC,2} \beta V = I + \varepsilon,
\]

\[
\alpha_{VC,1} = \alpha_{VC,2} = \frac{I + \varepsilon}{\beta V}
\]

and inequalities for the entrepreneurs’ incentive compatibility constraints

\[
\begin{align*}
    \lambda_1 & \leq \frac{1}{2} - \frac{I-\varepsilon}{\beta V}, \\
    \lambda_2 & \leq \frac{1}{2} + \frac{I-\varepsilon}{\beta V}.
\end{align*}
\]