Inflation Targeting in a Small Open Economy: The Colombian Case

Franz Hamann        Juan Manuel Julio
Paulina Restrepo    Alvaro Riascos

Banco de la República, Colombia
Monetary Policy in Colombia

- The Banco de la República conducts monetary policy using a “Flexible Forecast Inflation Targeting” strategy since September 1999.

- At the end of a given year, the Bank announces an inflation target for the next two years and publishes a Quarterly Inflation Report.

- The “logic” of the Quarterly Inflation’s Report is built upon a “core” model and its forecast (and yes, the interest rate forecast is also public).
“Chapulín Colorado-Disinflation“

Disinflation I

Sudden Stop

Disinflation II
The Current Core Model

- A semi-structural model, commonly used in many Central Banks.

- Very useful (within the Bank) to discipline the economic discussion around the elements of a model.

- We are in the process of building a structural model that captures the main frictions that characterize Emerging Market Economies.
The GE Model for IT in Colombia

• We see the GE model as a tool for improving consistency of our “story”.

• We have already used the GE model for policy analysis.

• Our aim is to “take the GE model bowling”.
What Kind of Model do we Want?

- Theoretically rich, tractable and explicable.

- Useful for contrasting alternative economic stories.

- Able to match the main facts about the Colombian economy.

- Flexible to the *formal* imposition of judgement.
Our General Strategy

- Identify the basic facts about the dynamics of the relevant Colombian macroeconomic series.

- Calibrate and solve a basic small open economy model.

- Compare the DGP of the model against the data (we use spectral analysis).

- Forecast a subset of variables (inflation, output and interest rates) using Bayesian techniques.
Main Features:

1. Small open economy: agents can freely borrow and lend in international financial markets.

2. Two sector model:
   - producers in perfect competition
   - retailers in monopolistic competition

3. Nominal rigidities: a fraction of retailers set prices based on past inflation and their last period’s price.
4. Money saves transaction costs (need money demand).

5. Imperfect pass-through.


7. Habit persistence to avoid the “price-consumption puzzle”.

8. Government:
   
   • Monetary policy rule: set nominal interest rate to target inflation.

   • Taxes/transfers & unproductive expenditure.
The Representative Household’s Problem:

\[
\max_{\{c,h,x,k,m,H\}} E_t \left( \sum_{t=0}^{\infty} \beta^t u(c_t, H_t, h_t, \mu^u_t) \right)
\]

subject to

\[
c_t + \phi + m^d_{t+1} + q_t x_t + b_{t+1} + q_t F_{t+1} = \frac{W_t}{P^c_t} h^s_t + \frac{R_t}{P^c_t} k^s_t + \frac{\Pi^R_t}{P^c_t} + \frac{\Pi_t}{P^c_t} + \frac{m^d_t}{(1 + \pi^c_t)} + \frac{b_t}{(1 + \pi^c_t)} (1 + i_t) + F_t q_t (1 + i^f_t) + \tau_t
\]
\[ k_{t+1} - (1 - \delta)k_t - f \left( \frac{x_t}{k_t} \right) k_t = 0 \]

\[ H_{t+1} - H_t - \rho(c_t - H_t) = 0 \]

\[ c_t = \left[ \int_0^1 c(z)^{\theta-1} \frac{1}{\theta-1} \right]^{\theta \over \theta-1} \]

and where

\[ (1 + i_t^f) = (1 + i_t^*) \left( 1 + \vartheta \left( \frac{F_t}{y_t} \right) \right) \]
The Producer’s Problem:

$$\max_{\{k,h\}} \Pi_t = P_t A_t (k_t^d)^\alpha (h_t^d)^{1-\alpha} - R_t k_t^d - W_t h_t^d$$
The Retailer’s Problem:

Each period retailer’s profits are given by:

\[ \Pi^R(z)_t = c(z)_t(p^c(z)_t - P_t) \]

If allowed to optimize:

\[ \max_{\{p^c(z)_t\}} E_t \sum_{j=0}^{\infty} (1 - \varepsilon) \varepsilon^j \Delta_{t+j} \frac{\Pi^R(z)_{t+j}}{P^c_{t+j}} \]

else:

\[ p^rule_t(z) = p^c_{t-1}(z)(1 + \pi^c_{t-1}) \]
Consolidated Monetary and Fiscal Authority

The government budget constraint:

\[ m_{t+1}^s - \frac{m_t^s}{1 + \pi_t^c} + \Phi (c_t, m_{t+1}, I_t) = \tau_t + g_t \]

Monetary policy rule:

\[ i_t = i + \zeta (\pi_t^c - \pi^c) + \xi (y_t - y^{ss}) \]
Competitive Equilibrium

**Definition:** A price system is a positive sequence

\[
\{W_t, R_t, p_t^{\text{rule}}, p_t^{\text{opt}}, P_t, e_t, i_t, i^f_t\}_{t=0}^\infty.
\]

**Definition:** Given the exogenous sequence \(\{A_t, \mu_t^g, \mu_t^u, g_t, P_t^*\}_{t=0}^\infty\) and \(m_0, k_0, b_0, F_0, H_0 > 0\), an equilibrium is a price system, a sequence of quantities \(\{c_t\}_{t=0}^\infty, \{x_t\}_{t=0}^\infty, \{k_t\}_{t=1}^\infty, \{h_t\}_{t=0}^\infty, \{H_t\}_{t=1}^\infty, \{b_t\}_{t=1}^\infty, \{F_t\}_{t=1}^\infty\) and a positive sequence of real money \(\{m_t\}_{t=1}^\infty\) in order that:

1. Given the prices and lump sum transfers, household’s optimal control problem is solved with \(\{m_t^d = m_t^s = m_t\}_{t=1}^\infty\),
\begin{align*}
\{k_t^d = k_t^s = k_t\}_{t=1}^{\infty}, \{b_t = 0\}_{t=1}^{\infty}, \{h_t^d = h_t^s = h_t\}_{t=0}^{\infty}, \{c_t\}_{t=0}^{\infty}
\end{align*}

and a level of \(\{F_t\}_{t=1}^{\infty}\) such that \((1 + i_t) = (1 + i_t^f) (1 + d_t)\)

2. The governments budget constraint and policy rule are satisfied for all \(t \geq 0\).

3. \(Y_t = C_t + I_t + G_t + F_{t+1} - (1 + i_t^f) F_t\) for all \(t\).
Nonlinear Dynamic System:

\[ c_t + x_t + g_t + F_{t+1} - (1 + i_{t+1}^f) F_t = A_t k_t^\alpha h_t^{1-\alpha} \]

\[ u_{ct} (c_t, H_t, h_t, \mu_t^u) + \eta_t \rho = \lambda_t 1 + \Phi_{ct} (c_t, m_{t+1}, x_t) \]

\[ u_{ht} (c_t, H_t, h_t, \mu_t^u) + \lambda_t q_t A_t (1 - \alpha) k_t^\alpha h_t^{-\alpha} = 0 \]

\[ \beta E_t \left( \lambda_{t+1} q_{t+1} A_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} + \gamma_{t+1} \left( c_0 - \frac{c_2 x_{t+1}^2}{k_{t+1}^2} \right) + \gamma_{t+1} (1 - \delta) \right) = \gamma_t \]
\[ \beta E_t \left( \frac{\lambda_{t+1}}{(1 + \pi_{t+1}^c) (1 + \Phi_{m_{t+1}}(c_t, m_{t+1}, x_t))} \right) = \lambda_t \]

\[ \beta E_t \left( \frac{\lambda_{t+1} (1 + i_{t+1}^f)}{(1 + \pi_{t+1}^c)} \right) = \lambda_t \]

\[ \beta E_t \left( \lambda_{t+1} (1 + i_{t+1}^f) q_{t+1} \right) = \lambda_t q_t \]

\[ \beta E_t \left( \eta_{t+1} + U_{H_{t+1}}(c_{t+1}, H_{t+1}, h_{t+1}, \mu_{t+1}^u) - \eta_{t+1}\rho \right) = \eta_t \]
\[ \lambda_t \left( \Phi_{x_t} \left( c_t, m_{t+1}, x_t \right) + q_t \right) = \gamma_t \left( c_1 + \frac{2c_2 x_t}{k_t} \right) \]

\[ k_{t+1} - (1 - \delta) k_t - f \left( \frac{x_t}{k_t} \right) k_t = 0 \]

\[ H_{t+1} - H_t - \rho (c_t - H_t) = 0 \]

\[ i_t = i + \zeta \left( \pi^c_t - \bar{\pi}^c \right) + \xi \left( y_t - \bar{y} \right) \]
\[(1 + i_t^f) = (1 + i_t^*) \left( 1 + \vartheta \left( \frac{F_t}{y_t} \right) \right)\]

\[P_t^c = \left[ \varepsilon (p_t^{rule})^{1-\theta} + (1 - \varepsilon) (p_t^{opt})^{1-\theta} \right]^{\frac{1}{1-\theta}}\]

\[(1 + \pi_t^c) = \left( \varepsilon (1 + \pi_{t-1}^c)^{(1-\theta)} + (1 - \varepsilon) \left( \frac{p_t^{opt}}{P_t^c} \right)^{(1-\theta)} (1 + \pi_t^c)^{(1-\theta)} \right)^{\frac{1}{1-\theta}}\]

\[p_t^{rule} = p_{t-1}^c (1 + \pi_{t-1}^c)\]
\[
\frac{p_t^{\text{opt}}}{P_t^c} = \frac{\theta \, \Theta_t}{\theta - 1 \Psi_t}
\]

\[
\Theta_t = \Delta_t c_t q_t + \varepsilon E_t (1 + \pi_{t+1}^c)^{\theta} \Theta_{t+1}
\]

\[
\Psi_t = \Delta_t c_t + \varepsilon E_t (1 + \pi_{t+1}^c)^{\theta-1} \Psi_{t+1}
\]
Solution

1. Calculate the steady state of the first order nonlinear dynamic system that characterizes the competitive equilibrium and log-linearize around the steady state.

2. Using KPR Method, the solution is of the form:

\[ Y_t = H x_t \]

\[ x_{t+1} = M x_t + R \eta_{t+1} \]

where \( Y \) is a vector of control, co-state and flow variables, \( x \) is a vector of states, \( H \) is the linearized policy function and \( M \) the state transition matrix. \( \eta_{t+1} \) is an innovation vector.
Calibration

- The model was calibrated to match some long run values of observed macro time series

- Some micro and macro studies were used also

- Simulations were performed with only one source of shocks (productivity shocks)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>1.06</td>
<td>To have $h = 0.33$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Approximately corresponds to the capital share in income</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.012</td>
<td>constructed capital time series</td>
</tr>
<tr>
<td>$a$</td>
<td>1.86</td>
<td>From the estimation of the demand for money</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.06</td>
<td>From the estimation of the demand for money</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.025</td>
<td>From the estimation of the demand for money</td>
</tr>
<tr>
<td>$\rho_{tech}$</td>
<td>0.83</td>
<td>Fitted time series of productivity</td>
</tr>
<tr>
<td>$\theta$</td>
<td>5</td>
<td>Corresponds to a markup of 25%</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.75</td>
<td>Prices changing every one year</td>
</tr>
<tr>
<td>$r$</td>
<td>6.81(a)</td>
<td>Estimated annual real interest rate for Colombia, Vasquez (2003)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.984(t)</td>
<td>Consistent with $r$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>5.5(a)</td>
<td>Target set for this year by the Central Bank</td>
</tr>
<tr>
<td>$i$</td>
<td>0.03(t)</td>
<td>Fixed according to $\pi$ and $r$</td>
</tr>
<tr>
<td>$i^*$</td>
<td>0.03(a)</td>
<td>International macro literature</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.5</td>
<td>Observed consumption’s volatility</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.8</td>
<td>Observed consumption’s persistence</td>
</tr>
</tbody>
</table>
Impulse Response to a Productivity Shock

- **Output**
- **Inflation**
- **Net Foreign Liabilities**
- **External Nominal Interest Rate**
- **Nominal Depreciation**
- **Consumption**
- **Nominal Interest Rate**
- **Productivity**
Impulse Response to a Government Expenditure Shock
Sensitivity Analysis: Productivity Shock when Prices are more Flexible
Assessing Population and Model Agreement

Diebold et al. Methodology

• Estimate Population Spectrum from Data

• Determine Salient Features of Data

• Assess Sampling Variability of Estimate

• Determine Model Theoretical Spectrum and Compare (test hypothesis)
The Hypothesis

When we compare sample and model spectrum and determine if last falls into confidence bands we are testing the null

\[ H_0 : F_Y(\omega) = F_M(\omega) \quad \text{for} \quad \omega \in [\omega_0, \omega_1] \]

which means that the true (but unknown) population spectrum equals model theoretical spectrum for frequencies between \( \omega_0 \) and \( \omega_1 \).
Estimating Population Spectrum

The estimated data (HP filtered) spectrum is:

\[ \hat{F}_Y^* (\omega_j) = \frac{1}{2\pi} \left\{ \sum_{\tau = -(t-1)}^{t-1} \Lambda (\tau) \hat{\Gamma}_\tau \exp (-i\omega_j \tau) \right\} \]

\( \Lambda (\tau) \) is a matrix of lag windows (kernel) that is:

- A Truncated, Symmetric and Positive weighing function of lags. Truncation lag defines the window size. Outside window weights = 0.

- The window size generates a trade-off between the smoothness and consistency of the estimated spectra and the bias-ness of the sample.
Assesing Sampling Variability

- Difficult to obtain analitically for general processes (phase and coherence - non linear)

- Can do Cholesky factor bootstrapping:
  - Stack observed sample vectors in \( Z = \begin{bmatrix} Y_1^T, Y_2^T, \ldots, Y_t^T \end{bmatrix}^T \)
  - For i-th iteration, randomly draw \( \varepsilon^{(i)} \) from an \( NT \) dimensional standard distribution
  - Compute \( z^{(i)} = \bar{z} + P^*\varepsilon^{(i)} \sim (1_T \otimes \mu, \Sigma^* = P^*P^{*T}) \) where \( \bar{z} = 1_T \otimes \bar{Y} \) and \( \Sigma^* \) is variance covariance matrix affected by \( \Lambda(\tau) \)
− Compute $\hat{F}^*(i)(\omega_j)$

− Obtain variance of spectrum, co-spectrum, coherence and phase functions.
Model Theoretical Spectrum

If model can be written in SSF

\[ Y_t = H \alpha_t \]
\[ \alpha_{t+1} = M \alpha_{t-1} + R \eta_{t+1} \]

where \( \eta_{t+1} \) is iid(0, \( \Omega \))

- We can use spectral arithmetic to compute model spectrum. Hamilton(1994, Ch. 6 and 10)

- The spectrum was calculated with just the productivity shock and applying the Hodrick and Prescott filtration.
Results
Salient Features of Data

• Inflation and output gap are dominated by periodic movements between 2 and 25 quarters. No long movements (data is HP filtered). Peak between 10 and 12 quarters, some degree of stickiness or persistence.

• Co-spectrum and coherence show results in the same direction.

• Coherence does not show significant frequency dominance.
Model Spectrum

- Some persistence both in the univariate spectra as well as in the cross spectra.

- Monotone spectra for output gap and cross spectra.

- Inflation spectra peaks for periodic movements of between 9 and 10 quarters.

- Higher correlation for periodic movements around five years.
Comparison

- Theoretical spectra and cross spectra fall into uncertainty bands for periodic movements of inflation and comovements of inflation and output gap of up to 5 years, and for periodic movements of output gap of up to 10 quarters.

- For longer periods the spectra and cross spectra of the model are significantly different from the population ones.

- Coherence falls into the uncertainty bands for most of the frequencies but the ones surrounding the model coherence peak and the long run periodic movements.
Final Remarks

- This is our first step in developing a consistent framework for policy analysis and forecast in Colombia.

- We hope this framework can improve the implementation of Inflation Targeting in terms of: consistency, transparency and accountability.

- We have seen how spectral analysis can help us to identify the scope and limitations of our core model.
The challenge

- Forecasting: we have performed some Bayesian forecasting exercises using the model as a prior. The results are promising (forthcoming).

- Theory: the model lacks several important properties of EMs
  - Liability dollarization
  - Sudden Stop of capital inflows
  - Financial / Housing sector frictions