Mortgage Contracts and Housing Tenure-Duration Decision*

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Abstract

The homeownership rate, and the housing tenure decision has changed drastically in the United States. In the last sixty years, the homeownership rate has increased from forty-four percent to sixty-seven percent. A similar change has occurred in housing size. A major factor in this change has been innovations in the mortgage market. Mortgage contracts evolved from being short duration with low LTV, to longer duration, high LTV ratios, as well as refinancing options. In this paper, we carefully analyze various mortgage contracts and their effects on tenure, duration, and investment decisions using a model with heterogeneous consumers and liquidity constraints. We find that different types of mortgage contracts have an important effect in the demand for housing over the life-cycle. For example, loans that allow for a lower LTV imply earlier entry in the market, larger homes in general, as well as a decreased frequency of duration. Finally, we reverse the question, and we explore the implications of the optimal mortgage contract.

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1 Introduction

The homeownership rate, and the housing tenure decision have changed drastically in the United States. In the last century, the United States has gone from being a country of renters (fifty six percent of housing units where rented in 1945) to a country of homeowners (thirty two percent are rental properties in 2005). The expansion in homeownership and the change in housing size during the postwar period is a result of the so-called “American Dream”. A major factor, as documented in Chambers, Garriga, and Schlagenhauf (2005) is innovations in the home financing market. Specifically, mortgage contracts have evolved from being short duration with low LTV, to longer duration, with higher LTV ratios, as well as refinancing options.

This paper explores the implications of innovations on housing financing markets for the investment in owner occupied housing. We investigate the wealth-portfolio implications of these financial innovations, as well as the ramifications for the tenure decision. (i.e. renting vs. owning), and the duration decision, (i.e. frequency of changing the housing investment decision). Our investigation examines a variety of mortgage contracts using a quantitative equilibrium model with heterogeneous consumers and liquidity constraints. Our model is in the tradition of the theoretical construct developed by Henderson and Ioannides (1983) and it has the following features: homeownership is part of the household’s portfolio decision; life-cycle effects play a prominent role; rental and ownership markets coexist; and households make the discrete choice of whether to own or rent.

We employ an overlapping generation framework where individuals face uninsurable labor income uncertainty, and mortality risk. Households make decisions with respect to the consumption of goods, the consumption of housing services, and saving which can be in the form of either (real) capital and/or housing. Hence, the model stresses the dual role of housing as a consumption and investment good. Investment in housing differs from real capital in that a mortgage contract is required, and changes in the housing investment position result in transaction costs. These latter costs associated with the adjustment of the housing position result in the infrequent changing of housing investment positions.

We employ techniques used in heterogeneous agent macroeconomic economies to solve our model. Once we have determined the (steady state) equilibrium, the empirical implications of the model are determined and compared with actual data.

We find that different types of mortgage contracts have an important effect on the demand for housing over the life-cycle. With lower LTV ratios we find that the model predicts that first-time buyers will enter into the housing market sooner, and with larger homes. For repeated buyers the model predicts decreased frequency of duration with an increased in their investment position in housing. These predictions are consistent with patterns that we document using a set of sample years from the American Housing Survey. We also find that mortgage contracts have important implications for households with different income levels.

Mortgage contracts have different implications for the mortgage debt structure of households. Contracts with lower downpayment requirements in the debt burden over the life-cycle, which has also been a pattern observed in the data.

This paper is organized into five sections. In the first section, we describe the properties of different mortgage contracts. In the second section we describe the model economy and define equilibrium. The third section discusses the estimation of the model to the US economy. The next section analyzes the performance of the model with a standard mortgage contract, while the final

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1 Some of the other research that examines housing in a general equilibrium general equilibrium setting are Berkover and Fullerton (1992), Díaz and Luengo-Prado (2002), Fernández-Villaverde and Krueger (2002), Gervais (2002), Nakajima (2003), and Plantania and Schlagenhauf (2002).
section examines the implications of alternative mortgage contracts.

## 2 Mortgage Contracts

A mortgage contract is a debt instrument which uses the dwelling unit to collateralize the loan. A variety of different mortgage products exist in the marketplace. In Table 1, we present types of primary mortgage contracts actually used.

### Table 2: Types of Primary Mortgage Contracts

<table>
<thead>
<tr>
<th>Type of Contract</th>
<th>1993</th>
<th>1999</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Payment (standard)</td>
<td>0.84</td>
<td>0.91</td>
<td>0.93</td>
</tr>
<tr>
<td>Adjustable rate mortgage (ARM)</td>
<td>0.11</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>Adjustable term mortgage</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Graduated payment mortgage</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Balloon</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Other</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Combination of the above</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Sample Size</td>
<td>33,367</td>
<td>34,747</td>
<td>39,038</td>
</tr>
</tbody>
</table>

Source: American Housing Survey (AHS)

Even though a number of mortgage contracts are available, these products actually vary in terms of three dimensions: the amortization schedule, the payment schedule, and the length of maturity.

To characterize the different mortgage contracts it is useful to introduce some notation. The decision to purchase a house of size $h$ and price $p$ requires a downpayment equal to $\psi$ percent of the value of the house. Consequently, households need take on debt equal to $D_0 = (1 - \psi)ph$. Let $r^m$ be the interest rate of a mortgage contract with maturity length $N$. At each period $s$, a household faces a mortgage payment that depends on the price of housing, the housing size, length of mortgage, downpayment fraction, the mortgage interest rate, as well as the type of mortgage contract. We denote the mortgage payment at time $s$ as being determined by the function $m_s(x)$ where $x$ is defined by the five-tuple $(p, h, \psi, N, r^m)$. This payment can be decomposed into an amortization term $A_s$ that depends on the amortization schedule and an interest payments $I_s$ which depends on the payment schedule. That is,

$$m_s(x) = A_s + I_s, \quad \forall s,$$

where the interest payments are calculated $I_s = r^mD_s$. The laws of motion for the level of debt $D_s$

$$D_{s+1} = D_s - A_s, \quad \forall s,$$

rearranging terms we have

$$D_{s+1} = (1 + r^m)D_s - m_s(x), \quad \forall s.$$

The law of motion for the level of (home) equity with respect to the loan $E_s$ is

$$E_{s+1} = E_s + A_s, \quad \forall s,$$

where $E_0 = 0$ denotes the home equity in the initial period.

We consider a variety of mortgage contracts which differ in their amortization and payment schedule. More precisely, we will consider a contract with constant amortization; a balloon payment
loan; a combo-loan with a finance downpayment; and a contract with payments that grow either arithmetically or geometrically. Each of these contracts are just special cases of the generalized contract we have discussed.

2.1 Mortgage with constant payments

This mortgage contract is the standard contract in the U.S., and is characterized by a constant mortgage payment over the length of the mortgage. The constant mortgage payment results in an increasing amortization schedule of the principal, and a decreasing schedule for interest payments. That is, the constant payment schedule is equal to

\[ m_s(x) = A_s + I_s, \]

and satisfies

\[ m_s(x) = \lambda D_0. \]

where \( \lambda = r^m[1 - (1 + r^m)^{-N}]^{-1} \). The contract front loads the interest rate payments and back loads the principle payments

\[ A_s = \lambda D_0 - iD_s. \]

The law of motion for the debt and home equity are

\[ D_{s+1} = (1+r^m)D_s - m, \quad \forall s. \tag{5} \]

The law of motion for the level of home equity (with respect to the loan) \( E_s \) is

\[ E_{s+1} = E_s + [\lambda D_0 - r^m D_s], \quad \forall s, \tag{6} \]

2.2 Combo loan

In the late 1990’s a new mortgage product became popular as way to avoid large downpayment requirements and mortgage insurance.\(^2\) This product is known as the combo loan and amounts to having two different loans. The first covers the standard loan \( D_1 = (1 - \psi)ph \), with mortgage payments \( m^1_s(x) \), and maturity \( N_1 \). The second loan partially or fully covers the downpayment amount \( D_2 = \kappa \psi ph \) (where \( \kappa \in (0,1] \) and represents the fraction of downpayment financed by the second loan). The second loan has an interest premium \( r^m_2 = r^m_1 + \zeta \) (where \( \zeta > 0 \)), a mortgage payment \( m^2_s(x) \), and a maturity \( N_2 \leq N_1 \).\(^3\) In this case

\[ m_s(x) = m^1_s(x) + m^2_s(x) \]

\[ = (A_{1s} + I_{1s}) + (A_{2s} + I_{2s}), \]

the laws of motion for both loans, and home equity are computed as in the mortgage with constant repayment.

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\(^2\)Government sponsored mortgage agencies initiated the use of this product in the late 1990’s and this product became popular in private mortgage markets between 2001 and 2002.

\(^3\)The combo-loan have been used to reduce the downpayment requirement and allowed for the avoidance of mortgage insurance. The "80-20" combo loan program corresponds to the traditional loan-to-value rate of 80 percent using a second loan for the 20 percent downpayment. The "80-15-5" mortgage product requires a 5 percent downpayment provided by the home purchaser with the remaining 15 percent coming from a second loan.
2.3 Mortgage with constant amortization

An alternative mortgage contract assumes a constant amortization of the principal $A_s = A_{s+1} = A$, with an interest payment schedule that depends on the size of outstanding level of debt $D_s$ and the length of the loan $N$. The constant amortization terms is calculated as

$$A = \frac{D_0}{N} = \frac{(1 - \psi)ph}{N}. \quad (7)$$

Under this contract mortgage payments decrease over time. The mortgage payment is

$$m_s(x) = \frac{D_0}{N} + r^m D_s, \quad (8)$$

while the law of motion for the outstanding level of debt and home equity are

$$D_{s+1} = D_s - \frac{D_0}{N}, \quad \forall s, \quad (9)$$

$$E_{s+1} = E_s + \frac{D_0}{N}, \quad \forall s, \quad (10)$$

equivalent to a standard mortgage with constant payment.

2.4 Balloon loans

A balloon loan is a very simple mortgage contract where all the principle borrowed is paid at maturity $N$. This product is popular in periods where mortgage rates are very high and home purchasers anticipate lower mortgage rates in the future. In addition, homeowners who expect to stay in the home a short duration may find this product of interest. The amortization schedule can be written as:

$$A_s = \begin{cases} 0 & \forall s < N \\ (1 - \psi)ph & s = N \end{cases}$$

All the mortgage payments, except the last one, reflect interest rate payments $I_s = r^m(x)D_0$. The mortgage payment for this contract is:

$$m_s(x) = \begin{cases} I_s & \forall s < N \\ (1 + r^m)D_0 & s = N \end{cases}$$

where $D_0 = (1 - \psi)ph'$. This contract has the advantage that if the interest is fully deductible, households only have to effectively pay the full loan in the last period. The evolution of the outstanding level of debt can be written as

$$D_{s+1} = \begin{cases} D_s, & \forall s < N \\ A_N, & s = N \end{cases}$$

2.5 Mortgage contract with growing payments

In an environment with high housing prices, another product that may help first time buyers is the graduated payment mortgage(GPM) where mortgage payments grow over time. This product could be attractive to first time buyers as mortgage payments track the increase in wage income over time. Of course, this product increases the lender’s risk exposure because the borrower builds equity in the home at a slower rate than the standard contract which may explain the lack of popularity of
this product. The repayment schedule depends on the growth rate of these payments. We consider two different cases.

1. **Geometric Growth:** In this type of contract mortgage payments evolve according to a constant geometric growth rate given by

\[ m_{s+1}(x) = (1 + g) m_s(x) \]

where \( g > 0 \). Consequently, the amortization term and interest payments are variable too. Formally,

\[ m_s(x) = A_s + I_s, \]

with the initial mortgage payments being,

\[ m_0(x) = \lambda_g D_0, \]

where \( \lambda_g = \left( r^m - g \right) \left[ 1 - (1 + r^m)^{-N} \right]^{-1} \). The law of motion for the level of debt satisfies

\[ D_{s+1} = (1 + r^m) D_s - (1 + g)^s m_0(x), \]

and the amortization term is \( A_s = \lambda_g D_0 - r^m D_s \).

2. **Arithmetic Growth:** In this case, the mortgage payment grows at a constant nominal amount \( \Delta = m_1(x) - m_0(x) \). The law of motion for the repayment schedule is

\[ m_s(x) = m_0(x) + s\Delta, \]

The initial repayment is calculated as usual, and is given by

\[ m_0(x) = \frac{[D_0 + \frac{\Delta N}{t}] r^m}{[1 - (1 + r^m)^{-N}]} - \Delta \left( \frac{1}{r^m} + N \right). \]

The law of motion for the outstanding debt is

\[ D_{s+1} = (1 + r^m) D_s - (m_0(x) + s\Delta). \]

In this case the amortization term is \( A_s = (m_0(x) + s\Delta) - r^m D_s \).

3 **The Model**

The model economy is comprised of households, a representative firm, a financial intermediary and a government sector. The household sector is populated by overlapping generations of *ex ante* identical households who live a finite period, \( J \), with certainty. The share of age-\( j \) households is denoted by \( \mu_j > 0 \) where \( \sum_{j=1}^{J} \mu_j = 1 \).

---

In 1974 Congress authorized an experimental FHA insurance program for GPM’s. In this program, negative amortization was permitted, but required higher downpayments so that the outstanding principle balance would never be greater during the life of the mortgage than would be permitted for a standard mortgage insured by FHA. Activity under this program and successor programs has been limited.
3.1 Households

In this economy, households have access to two assets to smooth out income uncertainty. Households can invest in a riskless financial asset we will call capital and denote by $a' \in \mathcal{A}$ with a net return $r$, and/or in a housing durable good denoted by $h' \in \mathcal{H}$ with a market price $p$. The prime is used to denote future variables. The housing asset generates shelter services according to the linear technology function $s = g(h') = h'$. Shelter services can be acquired in a rental market at the rental price, $R$, per unit of shelter.

Household preferences are given by the expected value of a discounted sum of momentary utility functions:

$$E \sum_{j=1}^{J} \beta^{j-1} \left( c_j^\gamma s_j^{1-\gamma} \right)^{1-\sigma} \left( \frac{1}{1-\sigma} - 1 \right),$$

where $\beta$ is the discount factor, $c_j$ is the consumption of goods at age $j$, and $s_j$ is the consumption of housing shelter services at age $j$. The utility function parameters are represented by $\sigma$ the coefficient of relatives risk aversion, and $\gamma$ the consumption share of non-durable goods.

Household income during working years, $j < j^*$, depends on a transitory and a persistent component. The transitory earnings component, $\epsilon \in \mathcal{E}$ is draw from a probability space, and the realization of the current period productivity component evolves according to the transition law $\Pi_{\epsilon', \epsilon}$. The persistent component $\nu_j$ has a deterministic trend over the life-cycle. After tax labor income is denoted by $(1 - \tau_p)\nu_j$, were $\tau_p$ represents a tax used to fund an old age transfer. During the retirement years, $j \geq j^*$, a household receives a retirement benefit from the government equal to $\theta$. The household’s sequential budget constraint depends on the exogenous labor income and wealth $(1 + r)a$, where $r$ denotes the net interest rate and $a$ is the asset holding position $a$. Formally we denote the period income by

$$y(\epsilon, j, a) = \begin{cases} 
(1 - \tau_p)\epsilon \nu_j + (1 + r)a, & \text{if } j < j^*, \\
\theta + (1 + r)a, & \text{if } j \geq j^*.
\end{cases}$$

Given the income level $y(\epsilon, j, a)$, the current housing position $h$, the length of the mortgage contract remaining $n$, households’ choose consumption $c$, amount of housing services to consume, $s$, asset position $a'$, and housing position $h'$.

We rule out the existence of annuity markets. Consequently, a household can only self insure against uncertainty. We also assume that households face a borrowing constraint. Finally, we assume that households are born with wealth.

We can think of the household as being in one of five situations with respect to their housing investment position that they enter the current period and the position they enter the next period.

1. **Renter yesterday** ($h = 0$) and **renter today** ($h' = 0$)

   Consider a household that does not enter the current period with a house, $h = 0$, and decides not to buy a house in the current period, $h' = 0$. In other words, the individual decides to remain a renter. The implied budget constraint is

   $$c + a' + Rs = y(\epsilon, j, a),$$

   where $Rs$ denote the cost of housing services purchased in the rental market. Their is no restriction on the size of housing services rented.

2. **Renter today** ($h = 0$) and **homeowner tomorrow** ($h' > 0$)
Next case, we have a households who rented in the previous period, \( h = 0 \), but decides to invest in housing, \( h' > 0 \). The purchase of a house face a downpayment requirement requires a downpayment \( \psi \in (0,1) \), and the payment of some transaction costs \( \phi_B \in (0,1) \). Hence, households need an initial investment \((\psi + \phi_B)ph'\) to enter in the housing market. The rest of the investment is financed with a mortgage contract that requires a mortgage payment denoted as \( m(p, h, \psi, N, r^m) \). The decision to take an investment position in housing gives the household another possible source of income if part of the services from this investment are rented to other households. This possibility is represented by the term \( R(g(h') - s) \) where the housing investment generates \( g(h') \) services. Owning a house also generates a maintenance expense which is complicated by the option of renting housing services to other households. The maintence expense depends on \( h' \) and the stock of home utilized by the homeowner \( h_c \), and is summarize by a function \( x(h', h_c) \). The budget constraint for this case is:

\[
c + a' + (\phi_B + \psi)ph' + m(p, h, \psi, N, r^m) + x(h', h_c) = y(\epsilon, j, a) + R(g(h') - s). \tag{14}
\]

3. **Homeowner today** \((h > 0)\) and **renter tomorrow** \((h' = 0)\)

A third possible situation is the household that enters a period with a positive housing investment position, \( h > 0 \), and decides to sell off their entire investment position and rent housing services, \( h' = 0 \).\(^6\) The budget constraint for this situation is:

\[
c + a' + Rs = y(\epsilon, j, a) + [(1 - \phi_S)ph - D_n]. \tag{15}
\]

The budget constraint indicates two important features of the housing investment position. First, if the initial housing position is sold, the individual must rent housing services equal to \( Rs \). Second, the sale of the house generates income, \( ph \), minus any selling costs, \( \phi_S \), and remaining principle which we denote as \( D_n \).\(^7\)

4. **Homeowner today** \((h > 0)\) and **homeowner tomorrow** \((h' > 0)\)

The last two cases deal with a household that enters the period with a housing investment position, \( h > 0 \), and decides to continue to have a housing investment position, \( h' > 0 \). The critical issue is whether the household decides to change their housing position.

(a) **Homeowner maintains housing size**

If the household decides to maintain their housing investment, \( h = h' \), then the budget constraint is:

\[
c + a' + m(p, h, \psi, n, r^m) + x(h', h_c) = y(\epsilon, j, a) + R(g(h') - s). \tag{16}
\]

\(^5\)There is an implicit moral hazard problem in renting housing services to other households - renters decide on how intensely to utilize a house, but may not actually pay the resulting costs. In order to calculate the appropriate amount of maintence investment, the amount of housing that is subject to owner depreciation, \( \delta_O \), and the amount of housing that is subject to renter depreciation, \( \delta_R \), must be known. Let \( h_c(s) \) correspond to the amount of housing required so that housing services of \( s \) can be generated. If this amount is equal or exceeds the amount of services generated by \( h' \), the depreciation costs are determined by the depreciation rate \( \delta_O \). If the household decides to consume less that the amount of services generated from the housing position, the part of the housing position that the household lives in, \( h_c(s) \), depreciates at the rate \( \delta_O \) while the remaining part of the house, \( (h' - h_c(s)) \), depreciates at the rate \( \delta_R \).

\[
x(h', h_c) = \begin{cases} 
\delta_Oph', & \text{if } h_c(s) \geq h' \\
\delta_Oph_c(s) + \delta_Rph[h' - h_c(s)], & \text{if } h_c(s) < h'.
\end{cases}
\]

\(^6\)In the last period, all households must sell \( h \), rent housing services and consume all their assets, \( a \), as a bequest motive in not in the model. In the last period, \( h = a' = 0 \).

\(^7\)As our analysis will be conducted at the steady state, we do not have to allow for differences in purchase and selling prices.
In this situation, the household must make a mortgage payment if \( n > 0 \).

(b) **Homeowner changes housing size**

If the household decides to either up-size or down-size their housing investment position, (i.e., \( h \neq h', h > 0, h' > 0 \)), the budget constraint is more cumbersome

\[
\begin{align*}
&c + a' + (\phi_B + \psi)ph' + m(p, h, \psi, N, \rho) + x(h', h_c) \\
&= y(c, j, a) + R(g(h') - s) + [(1 - \phi_S)ph - D_n],
\end{align*}
\]

This constraint accounts for the additional income from selling their home (net of transaction costs, \( \phi_S ph \), and remaining principle).

### 3.2 The Financial Intermediary

The financial intermediary is a zero profit firm. The firm receives the deposits of the household, \( a' \), offers mortgages to the household sector. These mortgages generate payments each period. In addition, financial intermediaries receive principle payments from those individuals who sell their home with an outstanding mortgage position. These payments are used to and pay a net interest rate on these deposits of \( r \). The balance sheet condition of the financial intermediary is:

**Financial Intermediary Balance Sheet**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans to firms</td>
<td>Deposits</td>
</tr>
<tr>
<td>Net mortgage loans</td>
<td></td>
</tr>
</tbody>
</table>

### 3.3 Market Equilibrium

This economy has three markets: the asset market, the rental of housing services market, and goods market. In the asset market the total amount of deposits from households has to be used to finance the mortgage market and capital used by firms. We assume a stand-in neoclassical firm that produces in a competitive market with a constant returns to scale Cobb-Douglas production function \( F(K, N) = K^\alpha N^{1-\alpha} \), where \( K \) denotes capital and \( N \) denotes effective labor input.

The second market in the model is the rental of housing services market. Equilibrium in this market requires that the total demand for housings services must be equal to the amount of housing services generated by the relevant housing stock. Finally, Walras Law ensures that the goods market clears. A formal definition of the recursive equilibrium used is provided in the appendix.

### 4 Calibration and Estimation

We calibrate and estimate the parameters of the model to reproduce some key properties of U. S. This strategy allows us to pick some parameter values, and estimate the remaining as an exercise in exactly-identified Generalized Method of Moments. With the parameterized model, we will evaluate the impact of different mortgage contracts in the investment and tenure decisions.

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8The spread between the mortgage rate and the return on capital is assumed to cover fixed costs.
4.1 Parameters Set in the Calibration

A period in the model is three years. Households start their life at age 20 and can live until age 80. In terms of our model, individuals live a maximum of 21 periods. Retirement is mandatory at age 65, (model period 16). We have to determine the discount rate, $\beta$, the coefficient of relative risk aversion, $\sigma$, and the consumption share of non-durable goods, $\gamma$, in household preferences. The values of $\beta$ and $\gamma$ will be estimated, but the curvature parameter, $\sigma$, is set to 2.0.

The specification of the stochastic income process is based on Storesletten, Telmer and Yaron (2001). We discretize this income process into a five-state Markov chain using the methodology presented in Tauchen (1986). The values we report reflect the three-period horizon employed in the paper. As a result, the efficiency values associated with each possible productivity values are $\epsilon = \{4.41, 3.51, 2.88, 2.37, 1.89\}$ and the transition matrix

$$
\pi = \begin{bmatrix}
0.47 & 0.33 & 0.14 & 0.05 & 0.01 \\
0.29 & 0.33 & 0.23 & 0.11 & 0.03 \\
0.12 & 0.23 & 0.29 & 0.24 & 0.12 \\
0.03 & 0.11 & 0.23 & 0.33 & 0.29 \\
0.01 & 0.05 & 0.14 & 0.33 & 0.47
\end{bmatrix}.
$$

The age-specific permanent component $\nu_j$ is estimated from earnings data in the PSID. We set $\theta$ to be equal to thirty percent of average income and let tax rate $\tau_p$ adjust to balance the budget.

In the housing market we calibrate the transaction costs associated with buying and selling housing, $\phi_B$ and $\phi_S$, to 3 and 6 percent. These figures are consistent with observed buying and selling fees. We allow for a wedge between the rate of return on capital and the mortgage rate of three percent which is close to the difference between the a fixed and floating rate mortgage mortgage rates. In the benchmark model where the calibrated target is 1999, we set the length of the mortgage, $N$, to 10 which corresponds to 30 years, and the downpayment requirement, $\Psi$, to ten percent. Other than social security taxes, we set taxes on all sources of income to be zero so as to focus on the "pure" effects of mortgage contracts.

Each household begins with an initial asset position. The distribution of initial assets was based on the asset distribution observed in 1999 Panel Study on Income Dynamics (PSID). Each income state was given their corresponding level of wealth to match the nonhousing wealth to earnings ratio.

4.2 Estimation Targets

The parameters that need to be estimated are the three depreciation rates, $\delta$, $\delta_O$, $\delta_R$, the relative importance of consumption goods to housing services, $\gamma$, and the discount rate, $\beta$. We identify these parameter values so that the statistics in the model economy are the same as five statistics observed in the actual economy. Our calibration or estimation of the five parameters is an exercise in exactly-identified Generalized Method of Moments. One calibration target is the ratio of capital to gross domestic product which is equal 3.00 over the period 1958-2001. We define the capital stock in the U.S. economy as total private fixed assets plus the stock of durable goods as defined by the Bureau of Economic Analysis. A second calibration target is the ratio of the housing capital stock to the nonhousing capital stock. The housing capital stock is defined as the value of fixed assets in owner and tenant residential property. If this measure of the housing stock is subtracted from the previously defined measure for the capital stock for the economy, we find ratio of the housing stock to nonhousing capital stock to be 0.60. This data also comes from the BEA.

The next estimation target is the fraction of output that goes to investment in capital goods. The ratio for this period is 0.043. The fourth target is the fraction of output that is allocated to
investment in housing. For the same period, this ratio is 0.032 where we define housing investment as investment in residential structures. The final target is the ratio of the number of square feet in owner-occupied housing to the number of square feet in rental housing. Data from the 1999 American Housing Survey indicates that this ratio is 4.25.

Using these calibration targets, the annualized estimates of the utility parameters $\beta$ and $\gamma$ are equal to 0.964 and 0.804, respectively. The depreciation rate of capital, $\delta$, is estimated to be 0.067. The depreciation rate on owner occupied housing, $\delta_0$, is 0.022 while the estimated depreciation rate on rental housing, $\delta_R$, is 0.090. The estimated parameters and calibration targets are summarized in Table 1. It is important to note that the estimation problem is not separate from the solution of the model. That is, we jointly solve estimation problem and model solution. In the appendix, we sketch the computational algorithm.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Target</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of wealth to gross domestic product ($K/Y$)</td>
<td>3.00</td>
<td>3.006</td>
</tr>
<tr>
<td>Ratio of housing stock to capital stock ($H/K$)</td>
<td>0.60</td>
<td>0.599</td>
</tr>
<tr>
<td>Housing Investment to Housing Stock ratio ($x_H/H$)</td>
<td>0.032</td>
<td>0.0319</td>
</tr>
<tr>
<td>Ratio owner-occupied to rental housing square feet</td>
<td>4.25</td>
<td>4.24</td>
</tr>
<tr>
<td>Ratio capital investment to GDP ($\delta K/Y$)</td>
<td>0.043</td>
<td>0.0431</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual Discount Rate</td>
<td>$\beta$</td>
<td>0.964</td>
</tr>
<tr>
<td>Share of consumption goods in the utility function</td>
<td>$\gamma$</td>
<td>0.804</td>
</tr>
<tr>
<td>Depreciation rate of owner occupied housing</td>
<td>$\delta_0$</td>
<td>0.022</td>
</tr>
<tr>
<td>Depreciation rate of rental housing</td>
<td>$\delta_R$</td>
<td>0.090</td>
</tr>
<tr>
<td>Depreciation rate of capital stock</td>
<td>$\delta_K$</td>
<td>0.067</td>
</tr>
</tbody>
</table>

5 Evaluation of The Baseline Model

In order to examine the implications of alternative mortgage contracts, the model needs to be evaluated to see if it is a credible instrument for normative analysis. We define the benchmark model as one where the mortgage function has constant payments with a ten percent downpayment requirement for every housing purchase. From an aggregate perspective, it is important to know whether the model generates reasonable housing statistics that we compare with 1999 data. Table 1 provides a summary of the aggregate performance of the model on certain key dimensions.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Home Own Rate</th>
<th>Home Own Rate</th>
<th>Ave House</th>
<th>Ave Apart. House</th>
<th>Ave Apart. House</th>
<th>Ave Apart. House</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data (AHS, 1999)</td>
<td>66.8%</td>
<td>39.7%</td>
<td>1973</td>
<td>1050</td>
<td>179</td>
<td>94</td>
</tr>
<tr>
<td>Standard Mortgage Contract</td>
<td>68.2%</td>
<td>30.0%</td>
<td>2082</td>
<td>825</td>
<td>144</td>
<td>75</td>
</tr>
</tbody>
</table>

The ownership rate measures participation in the housing market. In 1999, the AHS estimates that the homeownership rate in the United States was 66.8 percent, while the model predicts a rate of 68.2 percent. Another interesting dimension, and a focal point of current policy, is the participation of the younger households. The data indicates an ownership rate of 39.7 percent, and
the model predicts a smaller rate of 30 percent for all homeowners under age 35. Next, we want to consider whether the model generates housing and apartment units sizes consistent with the observed data. The observed average house size is 1973 square feet and the standard deviation is 179 square feet. The model predicts the average house size to be 2082 with a standard deviation of 144 square feet. As can be seen, the model slightly overpredicts average house size, and generates too little dispersion. In the rental market, the model underpredicts the mean and the variance of the apartment size. Given the restrictions implied by the assumptions, we are pleased with these aspects of the model.

In addition, we are particularly interested in determining how the model performs in terms of the age distribution. In Figure 1, we compare the distribution of the homeownership rate by age generated by the model with the distribution observed in the 1999 American Housing Survey.

As can be seen, the general pattern generated by the model is consistent the pattern observed in the data. However, as can be seen, the model underestimates the homeownership rate for households younger than age 35. After age 35, the seems to overpredict the homeownership rate. For example, at age 60, the homeownership rate is approximately eighty percent. The model generated homeownership rate is approximately ninety percent. However, it is important to note that some households are renting in each age-cohort.9

Another aspect of the model that needs to be examined is the allocation of housing in the household’s portfolio. We use the 1998 Survey of Consumer Finances to calculate household portfolio values which include the estimated value of the house adjusted for remaining principle to calculate the net housing investment position as well as stock and bond holding. Bonds are defined as bond funds, cash in life insurance policies, and the value of investment and rights in trusts or estates, while stocks are defined as shares of stocks in publicly held corporations, mutual funds, or investments trusts including stocks in IRA’s. In order to see if the model generates reasonable portfolio allocation, we calculated the share of housing in the portfolio. Housing is measured as

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9The overprediction of homeownership after age 35 is likely a result of two factors. First, the model abstracts from certain demographic features, (e.g., a change in martial status or household size) that can result in movements into the rental market. Second, discretizing the earnings process into a five state process allows too many households to afford positive investment positions in housing. We initially examined a three state version of the income process and found that all households older than age 55 owned housing.
net of mortgage principal, and the total portfolio is net housing plus other assets. In Figure 2 we present this ratio from the data and from the model.

The relative importance of housing in the portfolio increases rapidly for households of age 40 or less. After 40, the ratio slowly increases until age 65 when the relative importance of housing increases in the portfolio. This pattern associated with older households is a result of two factors. These households choose to maintain their housing position, and they consume their assets which increases the importance of housing in the portfolio. As can be seen, the model replicates this ratio very well. Flavin and Yamashita (2002), and Li (2004) have argued that the ratio of housing investment to total assets has a "U-shaped" pattern by age. Yet, the ratio we presented in Figure 2 does not show a pronounced "U-shaped" pattern. If the ratio is calculated conditional on homeownership the model is capable of generating such a figure.

The model has the attractive feature that the rental market is endogenous as households make a decision about their housing position and another decision on the amount of housing services to consume. In 1996 HUD, in conjunction with the American Housing Survey, surveyed rental housing owners in greater detail. This one time survey is known as the Property Owner and Manager Survey (POMS). We use this survey to assemble data on the characteristics of rental property owners. Given our model, we define a household as a rental property owner if they report that rental property is directly owned or owned in a partnership. The age characteristics of landlords are presented in Figure 3.
As can be seen, the percentage of those who rent property is very small under age 30. This ratio gradually increases in the data until the late fifties. At the peak, the percent of households who are landlords is approximately 9 percent. After age sixty, the ratio declines. The sample indicates an increase in the ratio after age seventy. This may be a result of the small number of households over age seventy in the data. The obvious question is how does the model perform. As can be seen, the model generates a very similar pattern, but smoother. For households under age fifty, the fraction of households is overstated. This suggests that households are taking larger housing positions that their housing service demands require with the thought of consuming the housing services at older ages. This is consistent with the fact that the model under forecasts the percent of households that are landlords between age fifty and age sixty-five.10

In sum, we believe the model performs very well when compared to actual data.

6 Evaluation of Various Mortgage Contracts

In this section, we will compare various mortgage contracts to the standard constant payment contract. We will focus on the implications of contracts for the investment in owner occupied housing with special interest in first-time buyers, the wealth-portfolio implications, as well as the ramifications for the tenure decision (i.e. renting vs. owning), and the duration decision (i.e. frequency of changing the housing investment decision).

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10This issue is studied in more detail in Chambers, Garriga, and Schlagenhauf(2005b).
Table 2: Summary of Aggregate Results by Mortgage Type

<table>
<thead>
<tr>
<th>Mortgage Type</th>
<th>Home Own Rate (over 25)</th>
<th>Home Own Rate (under 35)</th>
<th>Ave House Size</th>
<th>Ave Apart. House Size</th>
<th>Std House Size</th>
<th>Std Apart. House Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data (AHS, 1999)</td>
<td>66.8%</td>
<td>39.7%</td>
<td>1973</td>
<td>1050</td>
<td>179</td>
<td>94</td>
</tr>
<tr>
<td>Standard Mortgage Contract</td>
<td>68.2</td>
<td>30.0</td>
<td>2082</td>
<td>825</td>
<td>144</td>
<td>75</td>
</tr>
<tr>
<td>Standard-Combo Loan</td>
<td>64.1</td>
<td>31.1</td>
<td>2019</td>
<td>816</td>
<td>156</td>
<td>93</td>
</tr>
<tr>
<td>Balloon</td>
<td>65.4</td>
<td>33.6</td>
<td>2041</td>
<td>1011</td>
<td>166</td>
<td>151</td>
</tr>
<tr>
<td>Balloon-Standard</td>
<td>66.6</td>
<td>27.4</td>
<td>2060</td>
<td>818</td>
<td>143</td>
<td>78</td>
</tr>
<tr>
<td>Constant amortization</td>
<td>70.9</td>
<td>32.4</td>
<td>2148</td>
<td>818</td>
<td>140</td>
<td>60</td>
</tr>
<tr>
<td>GPM-Arithmetic</td>
<td>64.5</td>
<td>31.2</td>
<td>2025</td>
<td>815</td>
<td>155</td>
<td>91</td>
</tr>
<tr>
<td>GPM-Geometric (10 percent)</td>
<td>62.8</td>
<td>29.0</td>
<td>1993</td>
<td>832</td>
<td>158</td>
<td>101</td>
</tr>
</tbody>
</table>

1. Housing and rental units size are measured in terms of square feet.

In Table 2 we present a set of selected aggregate measures for various mortgage contracts. As can be seen, the type of mortgage contract can have important implications for these summary statistics. For the aggregate homeownership rate for 25 and older, a constant amortization mortgage will generate the highest participation rate among all mortgage contracts considered. In contrast, a GPM with a 10 percent geometric growth will generate the lowest ownership rate. If a policy maker desires to increase the ownership for the youngest cohorts, we find that pure balloon results in the highest participation rate for households under age 35. Again, the GPM with the geometric growth results in the lowest participation. The constant amortization contract has the property allowing more households to participate, but also purchasing bigger houses. The other mortgage contracts not only reduce participation, but the average home size. Changes in the equilibrium rental rate for apartment units will affect the individual incentives to supply rental services, and this partially determines the average size of the rental unit. The explanation for these different descriptive facts become clearer when the distribution implications of these contracts are examined.

6.1 Combo Fixed Payment Contracts

An alternative to the standard mortgage contract is the "combo-loan" product. The "80-20" and the "80-15-5" combo products have become popular. The former program corresponds to the traditional loan-to-value rate of 80 percent using a second loan for the 20 percent downpayment. The "80-15-5" mortgage product requires a 5 percent downpayment provided by the home purchaser with the remaining 15 percent coming from a second loan. The second loan has an interest rate approximately 2 percent higher than the interest rate on the primary mortgage. We will examine the implications of "90-10" combo loan where the second mortgage is for half the length of the primary mortgage and a two percent spread. For this contract, the overall homeownership rate is 64.1 percent which is somewhat lower than the standard contract. However, the homeownership rate for households under age 35 increases from 31.0 percent with the standard contract to 26.4 percent. As can be seen in the housing investment and distribution of buyers figures presented in Figure 4, the homeownership rate increases dramatically for the youngest individuals. This contract allows households to enter the housing market earlier. However, the percentage change in the distribution of buyers in the late twenties and early thirties actually fall suggesting lower income families are not benefiting from this product. Overall, we see that this product allows first time buyers to enter into the housing market and stresses the importance of the downpayment
constraint for policymakers who want to increase the homeownership rate of young households. The overall ownership rate declines as homeownership falls for households age 35 to 60. This is a result of the higher overall mortgage payment which forces less wealthy households into the rental market.

This product has implications for the amount of investment in housing as well as the role of housing in the financial portfolio. Except for the youngest households, the role of housing declines slightly until age 60. After this age, household’s portfolio seem to be identical to the portfolios in the standard contract. Households in the 45 to 60 age range play a larger buying role in the housing market and increase the size of their housing investment. This contract seems to cause the distribution of landlords to get older.

Figure 4: The Combo Loan

6.2 Balloon Type Mortgage Contracts

Balloon contracts have the property that household pay an interest payment each period, but do not make a contribution to outstanding principle until the last period when the entire outstanding principle balance must be paid. This type of mortgage contract is more popular in some European
countries. We will consider two types of balloon contracts. The first contract assumes a thirty year horizon so as to be consistent with the duration in all other contracts that are being studied. The other balloon contracts assume a four period or twelve year contract followed by a six period or eighteen year standard contract. With a simple balloon contract the monthly payment is lower than with a standard contract which results in a decrease in the overall homeownership rate to 65.4 percent. In contrast to the overall homeownership rate, the rate for households under the age of thirty-five increases to 33.6 percent. A decline in homeownership occurs around the time the balloon payment comes due. Some households are forced to sell and then can not afford to immediately to take a positive investment position in housing.

The results from the balloon contract are presented in Figure 5. The balloon mortgage contract does impact savings and portfolio allocations. As younger households take advantage of lower mortgage payments housing positions are greater than the standard contract positions are the younger ages. Because holders realize they are faced with a large balloon payment in the future, they begin to save. As a result, the percentage of net housing in the portfolio is much lower under the balloon contract. Asset holding, or savings, is much higher under this contract.

Figure 5: A Simple Balloon Contract
An alternative balloon mortgage contract is one where the balloon payment is replaced with a standard mortgage contract. We assume the balloon contract is four periods (twelve years) and the standard contract is for six periods (eighteen years). This type of mortgage product results in a lower overall homeownership rate as compared to the simple balloon contract and the standard contract. The homeownership rate for households under age thirty-five is lower than the homeownership rate under a standard contract. Under this contract households do not have to save as much, as they will be financing the balloon payment with a standard mortgage contract. In Figure 6, it is clear that the distribution of assets by age is identical to the distribution in the standard contract. The fraction of net housing in the portfolio is smaller under this mixed balloon contract until age sixty. After age sixty, the fraction of net housing in the portfolio is identical to the standard contract.

6.3 Mortgage with constant amortization

A mortgage with a constant amortization requires the household to pay a constant fraction of principle each period. As a result, mortgage payments decrease over time. With this mortgage
contract, the overall homeownership rate increase to 70.9 percent. The increase in the homeownership rate is a result of an increase in the homeownership rate for households under 30 and an increase in the homeownership rate for households in the 40 to 50 age cohort. The effect of this contract can be clearly seen by looking at financial portfolios under this contract and the standard contract. As can be seen in Figure 7, the decline in mortgage payments results in households entering into the housing market at a younger age. This translates into households under age 50 having a larger fraction of their portfolio in housing. Younger households skew their portfolios toward housing. In the late thirties, they begin to rebalance their portfolios toward assets. By age sixty, their portfolios look like the portfolios in a standard contract. The increase in the investment housing by the younger households partially translates into the distribution of landlords becoming younger.

![Figure 7: Mortgage Contract with Constant Amortization](image)

6.4 Graduated Payment Mortgages

With respect to this mortgage contract, we will consider GPM with an arithmetic payment schedule and a GPM with geometrically increase payment schedule. We have an increase of five percent each period in the arithmetic contract. In the geometric contract, we will assume the growth rate
is ten percent per period.

For the arithmetic schedule, the aggregate homeownership rate is 3.7 percent lower than the standard contract. The fact that the mortgage payment is lower for younger households means an earlier entry into the housing market for younger households who can afford to purchase a home. The homeownership rate for households under age thirty-five is 31.2 percent. The distributional implications of this contract are presented in Figure 8. With an arithmetic schedule, the homeownership rate is lower for households in the thirty-five to fifty-five cohort. In other words, some households never enter the housing market because the realization of rising mortgage payments while others sell and leave the housing market when payments become large. Saving behavior is not much different from the standard contract. With a lower homeownership rate, more households desire rental services. The additional demand for housing is supplied by older households with a positive investment position.

A geometrically increasing mortgage contract seems attractive at first glance as mortgage payments will be lower at younger ages and then increase at a rate corresponding to earnings growth. Because of a lower initial burden, this contract suggests a boost for homeownership. Our analysis
of the arithmentic contract suggests a less enthusiastic result. With the geometric contract, the aggregate homeownership rate is identical to the arithmetic rate and the homeownership rate for households under age 35 is actually lower. Some households realize they face geometrically increasing mortgage payment and stay out of the housing market. This is reflected in Figure 9 by the decline in the homeownership rate between ages 30 and 55. This contract leads to a slight increase in savings. The increase in the demand for rental services is provided by households over age sixty who take a greater position in the rental market.

Figure 9: Geometric ($g_y=.10$)

![Graphs showing homeownership rate, housing investment, distribution of buyers, distribution of landlords, percent of housing net portfolios, percent of housing gross portfolios, asset holdings, and loan to value ratio over household age.]

7 Conclusions

[To be completed]
8 Appendix:

8.1 Stationary Equilibrium

We restrict ourselves to stationary equilibria. The individual state of the economy is denoted by \((a, h, n, \epsilon, j) \in A \times H \times M \times E \times J\) where \(A \subset \mathbb{R}_+\), \(H \subset \mathbb{R}_+\), \(M \subset \mathbb{R}_+\), and \(E \subset \mathbb{R}_+\). For any individual, define the constraint set of an age \(j\) individual as \(\Omega_j(a, h, n, \epsilon, j) \subset \mathbb{R}_4\) as all four-tuples \((c, s, a', h')\) such that the budget constraint (9) is satisfied as well as the following nonnegativity constraints hold:

\[c > 0, \ s > 0, \ a' \geq 0, \ h' \geq 0.\]

Let \(v(a, h, n, \epsilon, j)\) be the value of the objective function of an individual with the state vector \((a, h, n, \epsilon, j)\) defined recursively as:

\[v(a, h, n, \epsilon, j) = \max_{(c, s, a', h') \in \Omega_j} \left\{ U(c, s, a'), \beta \Pi_j E[v(a', h', \max(0, n - 1), \epsilon', j + 1)] \right\}\]

where \(E\) is the expectation operator conditional on the current state of the individual.

**Definition 1 (Stationary Equilibrium):** A stationary equilibrium is a collection of value functions \(v(a, h, n, \epsilon, j): A \times H \times M \times E \times J \to \mathbb{R}; \) decision rules \(c(a, h, n, \epsilon, j), s(a, h, n, \epsilon, j), a'(a, h, n, \epsilon, j), \) and \(h'(a, h, n, \epsilon, j): A \times H \times M \times E \times J \to \mathbb{R}_+; \) prices \(\{r, p, R\}; \) government policy variables \(\{\tau, \theta\}; \) and invariant distribution \(\Gamma(a, h, n, \epsilon, j)\) such that

1. given prices, \(\{r, p, R\}\), the value function \(v(a, h, n, \epsilon, j)\) and decision rules \(c(a, h, n, \epsilon, j), s(a, h, n, \epsilon, j), a'(a, h, n, \epsilon, j), \) and \(h'(a, h, n, \epsilon, j)\) solve the consumer’s problem;

2. the price vector \(\{r, r^m\}\) is consistent with the zero-profit condition of the financial intermediary;

3. the asset market clears

\[\int_{A \times H} \sum_{E \times M \times J} \mu_j a'\Lambda(\Lambda) \Gamma(\Lambda) = K' + \int_{A \times H} \sum_{E \times M \times J} \mu_j m(h', n, i) \Gamma(\Lambda),\]

4. the rental market clears

\[\int_{A \times H} \sum_{E \times M \times J} \mu_j s(\Lambda) \Gamma(\Lambda) + \int_{A \times H} \sum_{E \times M \times J} \mu_j s(\Lambda) \Gamma(\Lambda) = \Pi,\]

5. The retirement program is self-financing

\[\int_{A \times H} \sum_{E \times M \times J} \mu_j \theta I_\omega \Gamma(\Lambda) = \int_{A \times H} \sum_{E \times M \times J} \mu_j (1 - I_\omega) \tau_p \epsilon v_j \Gamma(\Lambda).\]

6. letting \(T\) be an operator which maps the set of distributions into itself aggregation requires

\[\Gamma'(a', h', n - 1, \epsilon', j + 1) = T(\Gamma)\]

and \(T\) be consistent with individual decisions.

We will restrict ourselves to equilibria which satisfy \(T(\Gamma) = \Gamma\).
8.2 Computational Method

Our computation strategy allows us to jointly solve for the equilibrium and the estimation process. To compute the equilibrium we discretize the state space by choosing a finite grid. However, choices for both types of consumption are continuous. The joint measure over the state space $\Lambda$ (assets, $a$, housing, $h$, periods remaining on the mortgage, $n$, income shock, $\epsilon$, and age, $j$), is denoted by $\Gamma(\Lambda)$ and can be represented as a finite-dimensional array. The estimation method is a mix between non-linear least squares and an exactly identified generalized method of moments. The objective function to minimize can be written as the sum of two criteria:

$$L(\Theta) = \min_{\Theta} \{\lambda L_1(\Theta) + (1 - \lambda) L_2(\Theta)\},$$

The first criteria requires the estimate parameters to be consistent with market clearing in the asset market and housing market

$$L_1(\Theta) = \sum_{i=1,2} \gamma_i \left( \frac{\bar{p}_{j+1}(\Theta_{j+1})}{\bar{p}_j(\Theta_j)} - 1 \right)^2,$$

where $\bar{p}_{j+1}(\Theta_{j+1})$ represents the equilibrium price calculated with parameters $\Theta_{j+1}$ in iteration $j + 1$. The second criteria requires the implied aggregates in the model $F_n(\Theta)$ to match their counter part in the data $\bar{F}_n$

$$L_2(\Theta) = \sum_N \alpha_n (F_n(\Theta) - \bar{F}_n(\Theta))^2,$$

The indirect inference procedure proceeds as follows:

- Guess a vector of parameters $\Theta \equiv (\beta, \gamma, \delta, \delta_r, \delta_o)$ and a vector of prices $\bar{p} = (r, R)$.
- Calculate the social security transfers from the invariant age-distribution $\Pi$.
- Solve the household’s problem to obtain the value function $v(a, h, n, \epsilon, j)$, and the decision rules $a'(a, h, n, \epsilon, j), h'(a, h, n, \epsilon, j), c(a, h, n, \epsilon, j), s(a, h, n, \epsilon, j)$ starting with $v(\cdot, \cdot, \cdot, J + 1) = 0$.
- Given the policy functions, calculate the implied invariant distribution $\Gamma$, the implied aggregates $\{\bar{F}_n\}_{n=1}^N$ and market prices $\bar{p}$.
- Calculate $L(\Theta)$, and find the estimator of $\hat{\Theta}$ that solves

$$\min_{\Theta} L(\Theta).$$
References


