Uncertainty and Disagreement in Economic Forecasting

Stefania D’Amico and Athanasios Orphanides
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1 Introduction

Individual decisions, aggregate economic outcomes, and asset prices are often shaped by probabilistic assessments of future events, the uncertainty surrounding them, and disagreement across individuals regarding these assessments. The roles of disagreement and uncertainty are often confounded in economic analysis, however, in large part because the observation and measurement that would be needed for such investigations is lacking. For instance, it is unclear from the available evidence whether variations over time in the risk and term premia embedded in bond and equity prices primarily reflect changes in aggregate uncertainty or swings in the degree of the consensus in beliefs about economic fundamentals.

Surveys of expectations offer one potential source for information that could be useful in addressing such questions. Unfortunately, virtually all surveys of analysts, professional forecasters, businesses and households collect information on point predictions of future events. Aggregating information on the most likely outcome in each individual’s assessment offers some indication of the degree of consensus among individuals regarding the most likely outcome. It does not, however, provide meaningful information regarding the uncertainty that each individual may attach to his or her point forecast or the evolution of the associated individual and aggregate uncertainty over time, or any disagreements regarding individual assessments of uncertainty.

The Survey of Professional Foreplers presents a useful exception to the typical survey structure. Since its inception, this quarterly survey has asked respondents to provide probabilistic assessments of the outlook for inflation and for either nominal or real output. Focusing on these probabilistic responses allows construction and limited comparisons of aggregate proxies for uncertainty and disagreement. Starting with the important study by Zarnowitz and Lambros (1988) a number of authors have suggested various approaches to measure uncertainty as reflected in these survey responses.\footnote{Recent studies examining these density forecasts and their properties include Rich and Tracy (2003), Giordani and Soderlind (2003), Lahiri and Liu (2004), Wallis (2005), Engelberg, Manski and Williams (2006).} In this paper, we extend earlier work and present quarterly time series of various measures of uncertainty and disagreement that correct some apparent biases due to imperfections in the survey.

We focus our attention on the probabilistic inflation forecasts that cover an almost con-
tinuous span approaching forty years, from 1968Q4 to 2006Q1. Following earlier work, our starting point is the estimation of the characteristics of the probability densities for each forecaster in each period available in the survey. An important difficulty, however, is that the design of the survey and individual respondent practices, particularly a tendency to approximate and round the probabilistic responses, induces some errors in the estimation of individual uncertainty. These errors appear especially problematic when individual uncertainty is small and can result in systematic biases in estimates of uncertainty. To arrive at an aggregate characterization of uncertainty that mitigates this problem, we propose robust estimation of a cumulative density function approximating the distribution characterizing individual uncertainty in each quarter. Doing so provides direct estimates of both the average degree of uncertainty as well as the dispersion of individual uncertainty assessments in each quarter. This allows comparing and contrasting average uncertainty not only with disagreement about the mean outlook, (the most commonly discussed form of disagreement), but also with disagreement about the uncertainty to the outlook.

As an illustration of the potential usefulness of distinguishing uncertainty from various forms of disagreement we also examine their relationship with and predictive content for the evolution of the term premia embedded in the term-structure of government securities. We show that all three measures of uncertainty and disagreement we discuss are significantly correlated with term premia. However, their comovement in the sample makes it difficult to identify their relative importance with precision.

2 Description of the Survey Data

The first Survey of Professional Forecasters was conducted in November 1968 as a joint effort by the American Statistical Association and the National Bureau of Economic Research and quarterly surveys have been conducted since then. Currently, the Survey is maintained by the Federal Reserve Bank of Philadelphia that took over its administration in the summer of 1990. The number of respondents has changed over time and, especially in its early years, fluctuated considerably. As Figure 1 indicates, however, with the exception of the late

\(^2\)See Zarnowitz and Braun (1987) and Croushore (1993) for useful historical descriptions and assessments of the survey.
In the following, we highlight some special features relating primarily to the density forecasts that are of importance for our analysis. Since the inception of the survey, respondents have been asked to provide probabilistic forecasts for inflation, as measured by the concept of aggregate output most relevant at the time of each survey (that is GNP or GDP with fixed or chain weights). A complicating factor in the design of the survey is that in each quarter respondents are asked to forecast the annual percentage changes in the average output deflator (currently the GDP price index) between the previous and current year between the current and following year. When the survey started, there was one question referring to only one of these annual forecasts, but since the early 1980s, forecasts for both have generally been asked. Because the questions always refer to the average of the calendar year, their structure introduces a seasonal pattern in the quarterly horizons relevant for the forecast. For example, in the first quarter of the year, the current year forecast is a 4-quarter ahead forecast, counting from the last quarter of available data (the fourth quarter of the previous year) to the last quarter of the current year. The following year forecast reflects an 8-quarter ahead horizon, ending with the fourth quarter of the subsequent year. By contrast, in the second quarter of the year, these horizons become 3 and 7 quarters ahead, respectively. Similarly, in the third quarter the forecast horizons are 2 and 5 quarters ahead, and finally, in the fourth quarter of the year, they are down to 1 and 4 quarters ahead, respectively. As a result of this design in the survey the underlying expected uncertainty reflected in the individual responses as well as the expected degree of disagreement have seasonal patterns. An approximate year-ahead horizon can be constructed by using a time series that concentrates on the 4-quarter ahead horizon in first quarters, the 3-quarter ahead horizon in second quarters, the 6-quarter ahead horizons in third quarters and the 5-quarter ahead in fourth quarters. Nonetheless, an element of seasonality would certainly remain in this quarterly series.

The presence of this seasonality, in conjunction with the way the questions are posed and responses presented introduces some additional complications. In each quarter, the survey presents respondents with a number of bins denoting spanning a wide range of
outcomes for the annual rate of inflation and asks them to place probabilities in those bins, adding to 100%. In recent years, the survey has included 10 bins, with width equal to one percent, except for the edges that are open ended. Figure 2 shows two examples of individual responses from this period in the form of the histogram. The example on the left shows a respondent who placed probability mass in all available bins. This would be the expected outcome if the underlying distribution were continuous with wide support and the responses were provided in rather exact terms, say in multiples of 1 percent or fractions of 1 percent. This, however, is not the typical practice. Most of the time, most respondents place all their density in a few bins only. Some pertinent summary statistics for all responses since 1992 are presented in Figures 3–5. As can be seen in Figure 3, the distribution of the number of non-zero-bins depends sensitively on the forecast horizon. At one extreme, when the horizon is just one quarter, almost 60 percent of respondents place all probability mass in just one or two bins. At the other extreme, when the horizon is 8 quarters, about 60 percent of respondents use 4 or more bins in their answers.

An important reason for this pattern in the number bins employed is likely the manner in which respondents may be rounding the answers they provide. Examination of the individual entries reveals that quite often the probabilities in all non-zero bins are multiples of 10 percent or multiples of 5 percent. The breakdown of this practice, by horizon, is shown in Figure 4. The relationship of rounding with the number of bins employed is reported in Figure 5. In each case, responses are classified as reflecting 10% rounding if all non-zero bins are multiples of 10, 5% rounding if all non-zero bins are multiples of 5 and 1% rounding if they do not fall in either of the other two categories. As can be seen in Figure 4, almost 80 percent of two-bin responses are multiples of 10 percent, like the example on the right panel of Figure 2. By contrast, only half of responses that spread to 5 or more bins are reported in multiples of either 5 or 10 percent. This pattern suggests that answers are often provided in approximate form but also, since this approximations are more coarse when fewer bins are utilized, that it may disproportionately influence estimates of individual uncertainty when it is relatively low.

Some other issues also generate difficulties that need to be addressed in constructing quarterly estimates of disagreement and uncertainty that have a consistent interpretation
over time. One is the changing number and width of the intervals over time. Fewer and larger bins induce greater biases. The survey initially offered 15 bins with each interval having a width of 1%. But from 1981 to 1991 only 6 bins with a width of 2% were presented to respondents. Since 1992, the probabilistic questions offer 10 bins whose width equal to 1%.

Another complication relates to changes in the number and composition of the respondents, which was shown in Figure 1. In early surveys the average number of respondents was around 60, while in recent years the average has been closer to 30. In addition, at times the turnover of the SPF forecasters has been quite high. To eliminate possible biases from these variations, attention could be restricted to a subset of “regular” forecasters who contributed consistently over a certain number of years. Zarnowitz and Lambros, for example, restricted attention to forecasters who participated in at least 12 surveys. To check the robustness of our results against such participation criteria, we computed and compared our measures using samples including only respondents who participated in at least 4, 8 and 12 surveys, in addition to the sample of all respondents.

Finally, errors in the survey introduce missing observations in some quarters. For example, there is some uncertainty about what years respondents were providing probabilistic responses in 1985:Q1 and 1986:Q1 so these two quarters are excluded from the analysis.

3 Parameter-Free Analysis of the Probabilistic Beliefs

In this section we present some direct measures of uncertainty and disagreement that do not assume any specific continuous distributions for the probabilistic beliefs but are simply based on different ways of computing sample means and variances. The first step needed to compute these measures is to make some assumptions about the range over which the individual histograms are defined. We assume that the first and last intervals (that are open ended in the survey) are closed and have the same width as that of the central intervals. Results are generally not be very sensitive to this assumption since most of the times respondents place zero probability in the extreme intervals. When they do place some probability at the edges, a potential bias is introduced. The second step requires making a choice regarding the concentration of the probability mass within each interval. We adopt
two common alternatives, either that the frequencies are concentrated at the midpoints of the intervals, or that the probability mass is uniformly distributed within the interval (Zarnowitz and Lambros (1987)). Both alternatives entail a degree of approximation that also depends on the width of the intervals and bias upwards estimates of the individual variances. To compensate for this, we apply the Sheppard correction to variance estimates from these two methods (see Kendall and Stuart (1977)), although we recognize that this correction is much less accurate for the uniform assumption relative to the alternative that places all density within an interval at the midpoint of the interval.

To compute the mean ($\mu$) and variance ($\nu$) of the individual histograms for the mid-point ($M$) and the uniform ($U$) assumption we apply the following formulas, respectively:

$$
\mu^M_{i,h,t} = \sum_{j=1}^{n} x_{j,h,t} \cdot p_{i,j,h,t},
$$

$$
\nu^M_{i,h,t} = \left[ \sum_{j=1}^{n} \left( x_{j,h,t} - \mu^M_{i,h,t} \right)^2 \cdot p_{i,j,h,t} \right] - \frac{w_t^2}{12},
$$

$$
\mu^U_{i,h,t} = \sum_{j=1}^{n} \frac{(u_{j,t} - l_{j,t})}{2} \cdot p_{i,j,h,t},
$$

$$
\nu^U_{i,h,t} = \left[ \sum_{j=1}^{n} \frac{(u_{j,t}^3 - l_{j,t}^3)}{3(u_{j,t} - l_{j,t})} \cdot p_{i,j,h,t} - \left( \mu^U_{i,h,t} \right)^2 \right] - \frac{w_t^2}{12},
$$

where $p_{i,j,h,t}$ is the probability that the forecaster $i$ assigns to the $j$th interval in the survey conducted at time $t$ with horizon $h$; $u_{j,t}$ and $l_{j,t}$ are the upper and lower limits of the $j$th interval; and $w_t$ is the width of the central range, thus the entire term $\frac{w_t^2}{12}$ represents the Sheppard’s correction for the second moment.

To obtain aggregate measures of the mean expectation of inflation and uncertainty about inflation for a specific quarter, the individual means and variances are averaged across all $N_t$ respondents:

$$
\mu^M_{h,t} = \frac{1}{N_t} \sum_{i=1}^{N_t} \mu^M_{i,h,t},
$$

$$
\nu^M_{h,t} = \frac{1}{N_t} \sum_{i=1}^{N_t} \nu^M_{i,h,t},
$$
\[ \mu_{h,t}^U = \frac{1}{N_t} \sum_{i=1}^{N_t} \mu_{i,h,t}^U, \]

and

\[ v_{h,t}^U = \frac{1}{N_t} \sum_{i=1}^{N_t} v_{i,h,t}^U. \]

In order to compute the measure of disagreement, we calculate the cross-sectional standard deviation of the individual means obtained using both assumptions:

\[ s_{h,t}^M = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left( \mu_{i,h,t}^M - \mu_{h,t}^M \right)^2}, \]

and

\[ s_{h,t}^U = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left( \mu_{i,h,t}^U - \mu_{h,t}^U \right)^2}. \]

Having computed the variances of the individual histograms, it is also possible to quantify a further concept of disagreement, that is the dispersion of the second moments of the probabilistic beliefs rather than the first moments. This measure, which can be interpreted as disagreement about inflation uncertainty, can be computed as follows:

\[ \phi_{h,t}^M = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left( v_{i,h,t}^M - v_{h,t}^M \right)^2}, \]

and

\[ \phi_{h,t}^U = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left( v_{i,h,t}^U - v_{h,t}^U \right)^2}. \]

4 **Parametric analysis of the probabilistic beliefs**

Next we examine parametric methods for the analysis of the probabilistic beliefs. This can be divided in two parts. In the first part of the analysis we examine assumptions on the distribution of inflation faced by an individual, while in the second part we examine assumptions about the distribution of inflation uncertainty across individuals.

For the first part, we focus on the assumption that a normal distribution approximates the probabilistic beliefs of each respondent sufficiently well (Giordani and Söderlind (2003)).
As we discuss below, however, this assumption provides useful estimates only for forecasters that assign positive probability to three or more bins. Initially, we also experimented with fitting alternative distributions that accommodated deviations from normality. However, we concluded that it was difficult to distinguish between true deviations from normality and apparent deviations due to the approximation and rounding of individual responses. Consequently, we decided to focus on the first two moments of the individual probabilistic distributions, and limited attention to the normal density.

For each individual probabilistic response, we fit the Normal CDF so that it matches as closely as possible the individual empirical CDF obtained by accumulating the densities in individual bins. More precisely, we identify the mean and variance of the Normal that minimize the sum of the squared differences between the empirical CDF and the probabilities implied by the normal distribution over the same intervals. The resulting estimates of the mean and the standard deviation of the normal \( \mathcal{N} \) constitute the estimate of the expected inflation and of the individual uncertainty respectively, that is if

\[
X_{i,h,t} \sim \mathcal{N} \left( \mu_{i,h,t}, \sigma^2_{i,h,t} \right),
\]

then

\[
\mu^N_{i,h,t} = \hat{\mu}_{i,h,t}; \quad \sigma^2_{i,h,t} = \hat{\sigma}^2_{i,h,t}.
\]

Assuming that this procedure produces meaningful estimates for all individuals, we can aggregate the resulting individual results as in the previous section:

\[
\mu^{N}_{h,t} = \frac{1}{N_t} \sum_{i=1}^{N_t} \mu^N_{i,h,t}; \quad \sigma^2_{h,t} = \frac{1}{N_t} \sum_{i=1}^{N_t} \sigma^2_{i,h,t}.
\]

However, fitting normal densities to individual histograms provides meaningful estimates of the underlying distribution only when when forecasters assign positive probabilities to at least 3 bins. To illustrate the difficulty encountered with fewer bins, it is instructive to compare the estimation associated with the two example empirical densities shown in Figure 6. On the left, a unique best fitting normal distribution can be easily identified. On the right, however, the two bins reported may reflect approximate responses that could correspond to a wide range of possible distributions, including the two distributions shown. The wider distribution could be the correct one if the zero entries in the bins from 1 to 2 and from 4 to 5 are
merely the result of rounding down probabilities just under 5 percent, so that all reported bins could be in multiples of 10 percent. But the narrower distribution could be correct if no such rounding took place and uncertainty was indeed so small that the true probabilities in the bins from 1 to 2 and from 4 to 5 are virtually zero. A minimization procedure that matches the normal CDF to the empirical CDF, of course, cannot not recognize the first possibility and its implied rounding errors. Instead, for any probabilistic forecast where all the density is concentrated in two consecutive bins, it mechanically identifies the best fitting normal distribution to be the one with infinitesimal (zero) variance and a mean located at the point dividing the two consecutive bins. That is, if the approximate nature of the probabilistic responses, and associated rounding of answers results in a 2-bin distribution, fitting a normal density will result in a severe downward bias in the reported individual uncertainty. Using the mean of individual standard deviations computed in this manner would subsequently result in a significant bias in the estimate of average uncertainty.

One way to mitigate this problem is to model directly the distribution of the individual uncertainties and treat the uncertainties of forecasters that assign a positive probability only to one or two intervals as small but unobserved. Specifically, let us suppose that the individual uncertainty originates from a Gamma distribution:

\[ v_{i,h,t} \sim \text{Gamma}(\alpha_{h,t}; \beta_{h,t}) \]

then the values of the mean of inflation uncertainty and disagreement about this uncertainty are recovered from the parameters \( \alpha_{h,t} \) and \( \beta_{h,t} \). To the extent this distribution could be fit by treating the variances of responses with one or two bins as small but unobservable, the distortion to the estimation of the distribution could be avoided. This can be done by fitting a truncated distribution to the individual variances over the range of variances that are observable. We implement this by fitting a distribution to the empirical CDF of the individual variances that exceed some threshold, recognizing that the CDF has positive mass at the threshold, but treating variances below that threshold as unobservable. An example is presented in Figure 7. More precisely, define \( C \) the chosen cutoff, then \( v_{i,h,t}(c_\tau) \) for all \( c_\tau > C \) are all the individual variances above the threshold and \( F(v_{i,h,t}(c_1)), \ldots, F(v_{i,h,t}(c_{N_c})) \), where \( c_1, \ldots, c_{N_c} \) are the right endpoints of the intervals in which the range of uncertainty values has been discretized, are the empirical CDF of \( v_{i,h,t}(c_\tau) \) defined at these
endpoints. We then minimize over the distribution parameters $\theta_1$ and $\theta_2$:

$$
\min_{\theta_1, \theta_2} \sum_{\tau=1}^{N_c} \left( \text{CDF} \left( v_{i,h,t}(c_\tau); \theta_1, \theta_2 \right) - F(v_{i,h,t}(c_\tau)) \right)^2,
$$

where $\text{CDF} \left( v_{i,h,t}(c_\tau); \theta_1, \theta_2 \right)$ is the cumulative distribution function of the candidate model chosen to fit the distribution of the individual uncertainties. Since the uncertainty can only take positive values, the choice of candidate models is limited to those distributions that are defined only on a positive support. Among possible candidates we examined the Gamma, Chi-square and Lognormal but in the following we concentrate our attention on the Gamma, which provided the best fit among these. The resulting parameters of the distributions we estimate by quarter allow us to track the characteristics of the distribution of uncertainty over time. Since we are interested in the average of $\sigma_{i,h,t} = \sqrt{v_{i,h,t}}$ and we model $v_{i,h,t}$ to be Gamma distributed, we apply a change in variable to recover the mean and standard deviation of $\sigma_{i,h,t}$. Recall, that if $v_{G_{h,t}} = E_{G_{h,t}}(v_{i,h,t}) = \hat{\alpha}_{h,t} \cdot \hat{\beta}_{h,t}$ and $\text{Var}(v_{i,h,t}) = \hat{\alpha}_{h,t} \cdot \hat{\beta}_{h,t}^2$, where $E^G$ indicates the expected value under the Gamma distribution, then:

$$
\hat{\sigma}^G_{h,t} = E_{G_{h,t}}(\sigma_{i,h,t}) = \hat{\beta}_{h,t}^{1/2} \cdot \frac{\Gamma(\hat{\alpha}_{h,t} + 1/2)}{\Gamma(\hat{\alpha}_{h,t})},
$$

$$
\hat{\phi}^G_{h,t} = \sqrt{E_{G_{h,t}}(\sigma_{i,h,t} - \sigma^G_{h,t})^2} = \sqrt{\hat{\alpha}_{h,t} \cdot \hat{\beta}_{h,t} - (\hat{\sigma}^G_{h,t})^2},
$$

It follows that fitting directly the distribution of the individual uncertainties provides a way to summarize the quarterly uncertainty that improves upon taking a simple average of individual standard deviations by circumventing the problems associated with respondents with just one or two bins.

We apply this methodology to all the individual measures of uncertainty previously obtained, that is we assume:

\[
v_{i,h,t}^M \sim \text{Gamma}(\alpha_{h,t}; \beta_{h,t}); \quad v_{i,h,t}^U \sim \text{Gamma}(\alpha_{h,t}; \beta_{h,t}) \quad \text{and} \quad v_{i,h,t}^N \sim \text{Gamma}(\alpha_{h,t}; \beta_{h,t}),
\]

and derive the following measures:

\[
\hat{\sigma}^GM_{h,t} = E_{G_{h,t}}(\sigma_{i,h,t}^M) = \left( \hat{\beta}_{h,t}^M \right)^{1/2} \cdot \frac{\Gamma(\hat{\alpha}_{h,t}^M + 1/2)}{\Gamma(\hat{\alpha}_{h,t}^M)},
\]

\[
\hat{\phi}^GM_{h,t} = \sqrt{E_{G_{h,t}}(\sigma_{i,h,t}^M - \sigma^GM_{h,t})^2} = \sqrt{\hat{\alpha}_{h,t}^M \cdot \hat{\beta}_{h,t}^M - (\hat{\sigma}^GM_{h,t})^2},
\]
\[ \hat{\sigma}_{h,t}^{GU} = E_{h,t}^{G}(\sigma_{i,h,t}^{U}) = \left(\hat{\beta}_{h,t}^{U}\right)^{1/2} \cdot \frac{\Gamma(\hat{\phi}_{h,t}^{U} + 1/2)}{\Gamma(\hat{\phi}_{h,t}^{U})}, \]

\[ \hat{\phi}_{h,t}^{GU} = \sqrt{E_{h,t}^{G}\left(\sigma_{i,h,t}^{U} - \sigma_{h,t}^{GU}\right)^{2}} = \sqrt{\hat{\alpha}_{h,t}^{U} \cdot \hat{\beta}_{h,t}^{U} - \left(\sigma_{h,t}^{GU}\right)^{2}}, \]

\[ \hat{\sigma}_{h,t}^{GN} = E_{h,t}^{G}(\sigma_{i,h,t}^{N}) = \left(\hat{\beta}_{h,t}^{N}\right)^{1/2} \cdot \frac{\Gamma(\hat{\phi}_{h,t}^{N} + 1/2)}{\Gamma(\hat{\phi}_{h,t}^{N})}, \]

\[ \hat{\phi}_{h,t}^{GN} = \sqrt{E_{h,t}^{G}\left(\sigma_{i,h,t}^{N} - \sigma_{h,t}^{GN}\right)^{2}} = \sqrt{\hat{\alpha}_{h,t}^{N} \cdot \hat{\beta}_{h,t}^{N} - \left(\sigma_{h,t}^{GN}\right)^{2}}, \]

where the double superscript indicates that the Gamma (G) methodology has been applied to the individual uncertainties derived under the midpoint (M), uniform (U) and normal (N) assumptions.

5 Characteristics of the cross-sectional distributions over time

Figures 8–14 summarize the results of the quarter-by-quarter estimation in our sample. Each figure presents a seasonal time series of alternative estimates of \( \hat{\mu} \), \( \hat{\delta} \), \( \hat{\sigma} \) and \( \hat{\phi} \) corresponding to different estimates of the individual means and variances. The time series are seasonal in that the forecast horizons vary by quarter according to our “approximate” year-ahead convention. Thus, in Q1, the forecast horizon is 4 quarters, in Q2 it is 3 quarters, in Q3 it is 6 quarters and in Q4 it is 5 quarters.

The mean for the individual means as well as the dispersion of the individual means do not appear to be sensitive the the estimation assumption. The midpoint and uniform methods are, of course, identical. But in addition, as Figures 8 and 9 confirm, the normal density method provides essentially similar results.

The three methods do result in somewhat different estimates of the characteristics of the distributions of uncertainty, though the differences are small. Figure 10 compares the mean uncertainty obtained by fitting the truncated Gamma distribution by quarter to each of the three alternatives. Similarly, Figure 11 compares the disagreement regarding individual uncertainty. The mean estimates for uncertainty are generally higher for the
uniform method, as would be expected. But the midpoint and normal methods do not have a material difference in the mean.

Table 1 shows the correlations of the mean, uncertainty and disagreement measures for the midpoint-Gamma estimates. As can be seen, they are all positive but those between \( \mu \) and \( s \), and between \( \sigma \) and \( \phi \) are the highest. Comparing the top and bottom panels indicates that the estimates are essentially similar regardless of whether the aggregate inflation forecast attributes are based on only regular survey respondents or all respondents, including irregular ones.

To compare the evolution of uncertainty and disagreement over time Figures 12-14 collect the three measures corresponding to each of the three methods we used to estimate individual variances. The comovements of the three variables are essentially similar in each figure so we can focus attention on just one. As can be seen, disagreement and uncertainty co-move, but their evolution differs over time. To see more clearly the correlation of mean uncertainty with the mean outlook ad the two measures of disagreement, Figures 15-17 show scatterplots of these variables. As can be seen, the correlation is evident regardless of whether one concentrates attention on just one quarter each year (to circumvent seasonality concerns) or all quarters together.

6 Inflation and Term Premia

To illustrate the potential usefulness of distinguishing the various attributes of the inflation outlook, in this section we examine their relationship to term premia. For this analysis, we utilize estimates of the forward term premium at the 2-year, 5-year and 10-year ahead horizons obtained from an arbitrage-free three-factor model of the term structure (see Kim and Orphanides, 2005). The quarterly time-series for each premium selects the premium prevailing at the last business day of the middle month of the quarter, so the premia are always measured after the survey for that quarter has been conducted.

Table 2 presents the simple correlations of these three premia with the pooled (seasonal) measures of disagreement and uncertainty corresponding to the midpoint-Gamma estimation. (Results from the other two methods are essentially similar and are thus not reported.) As can be seen, the term premia at all three horizons shown are positively correlated with
the mean forecast for inflation as well as with mean uncertainty, and the disagreement about both the mean and the uncertainty. However, the correlation in this sample is strongest with the uncertainty.

To gauge the statistical significance of these correlations, and account for the seasonal nature of the inflation forecast variables, Table 3 presents linear regressions of the 2-year premium on each one of the various measures, allowing for quarter-specific intercepts and slope coefficients. The row noted with \( p \) denotes the p-values associated with the hypothesis that all slope coefficients equal zero, (based on a HAC-robust covariance). As can be seen, each of the four attributes of the inflation forecast individually significantly correlates with the 2-year premium, but the relationship is strongest for the mean uncertainty of the forecast. As before, the estimates are essentially similar regardless of whether the aggregate inflation forecast attributes are based on only regular survey respondents or all respondents, including irregular ones.
References


Table 1
Correlations of Attributes of Distributions of Inflation Forecasts
(Midpoint-Gamma, pooled quarters)

<table>
<thead>
<tr>
<th>Premium</th>
<th>( \hat{\mu} )</th>
<th>( \hat{\sigma} )</th>
<th>( \hat{s} )</th>
<th>( \hat{\phi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>All respondents</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\mu} )</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>0.47</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{s} )</td>
<td>0.66</td>
<td>0.39</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>( \hat{\phi} )</td>
<td>0.25</td>
<td>0.65</td>
<td>0.25</td>
<td>1.00</td>
</tr>
<tr>
<td>At least 12 surveys</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\mu} )</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>0.45</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{s} )</td>
<td>0.67</td>
<td>0.37</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>( \hat{\phi} )</td>
<td>0.24</td>
<td>0.63</td>
<td>0.23</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes: \( \hat{\mu} \) and \( \hat{s} \) are the mean and standard deviation of the distribution of the individual mean forecasts. \( \hat{\sigma} \) and \( \hat{\phi} \) are the mean and standard deviation of the distribution of the individual standard deviations. The entries report the correlations of the quarterly time-series obtained by pooling together the attributes of the 4-, 3-, 6-, and 5-quarter-ahead forecasts available in the first, second, third and fourth quarter, respectively.
Table 2  
Simple Correlations with Term Premia  
(Midpoint-Gamma, pooled quarters)

<table>
<thead>
<tr>
<th>Premium</th>
<th>$\tilde{\mu}$</th>
<th>$\tilde{\sigma}$</th>
<th>$\tilde{s}$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All respondents</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-year</td>
<td>0.40</td>
<td>0.59</td>
<td>0.25</td>
<td>0.34</td>
</tr>
<tr>
<td>5-year</td>
<td>0.51</td>
<td>0.66</td>
<td>0.34</td>
<td>0.38</td>
</tr>
<tr>
<td>10-year</td>
<td>0.59</td>
<td>0.68</td>
<td>0.42</td>
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<td>At least 12 surveys</td>
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<tr>
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<td>0.59</td>
<td>0.26</td>
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<td>0.66</td>
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<td>0.37</td>
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<tr>
<td>10-year</td>
<td>0.59</td>
<td>0.69</td>
<td>0.40</td>
<td>0.41</td>
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</tbody>
</table>

Notes: See notes to Table 1.
Table 3
Explaining Term Premia
(2-year premium, Midpoint-Gamma)

<table>
<thead>
<tr>
<th>Q</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\sigma}$</th>
<th>$\hat{s}$</th>
<th>$\phi$</th>
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<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td></td>
</tr>
</tbody>
</table>

**All respondents**

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<tr>
<td>1</td>
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<td>3.23</td>
<td>1.34</td>
<td>1.63</td>
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<td>0.17</td>
<td>4.65</td>
<td>1.07</td>
<td>2.32</td>
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<tr>
<td>3</td>
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<td>3.93</td>
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<td>3.64</td>
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<td>$\bar{R}^2$</td>
<td>0.16</td>
<td>0.38</td>
<td>0.13</td>
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<tr>
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<td>0.001</td>
<td>0.008</td>
<td>0.005</td>
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</table>

**At least 12 surveys**

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<td>3.21</td>
<td>1.34</td>
<td>1.44</td>
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<td>3.21</td>
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<td>3.79</td>
<td>-0.07</td>
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<tr>
<td>$\bar{R}^2$</td>
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<td>0.39</td>
<td>0.11</td>
<td>0.10</td>
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<tr>
<td>p</td>
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<td>0.001</td>
<td>0.028</td>
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</table>

Notes: Each column in each panel reports the slope parameters, $\gamma_{X,Q}$ of the regression:

$$\tau_t = \sum_{Q=1,4} (\gamma_0 + \gamma_{X,Q}X_t) \cdot Q_t + \eta_t$$

where $\tau$ is the two-year forward term premium, and $X$ is the attribute of the inflation forecast shown at the top of the column. The quarter-specific constants are not reported. $p$ denotes the p-value of the hypothesis that all the slope parameters are zero.
Figure 1

Number of Survey Respondents

- All respondents
- Regular (4+ surveys)
- Regular (8+ surveys)
- Regular (12+ surveys)
Figure 2

Two Examples of Individual Density Forecasts
Figure 3

Number of Non-Zero Bins by Forecast Horizon
Figure 4

Rounding by Forecast Horizon
Figure 5
Rounding by Number of Non-Zero Bins
Figure 6

Fitting a Normal Density: Two Examples
Figure 7

Gamma Distribution of Individual Variances
Figure 8

Inflation Outlook
Figure 9
Disagreement

Percent


Normal  Midpoint/Uniform
Figure 10

Uncertainty

![Graph showing uncertainty over time with different distributions: Normal, Midpoint, and Uniform. The x-axis represents years from 1969 to 2005, and the y-axis represents percent uncertainty. The graph displays variability in uncertainty across the years with peaks and troughs.]
Figure 11

Disagreement about uncertainty
Figure 12
Uncertainty and Disagreement
(Normal-Gamma)

Percent
Uncertainty
Disagreement about mean
Disagreement about uncertainty
Figure 13

Uncertainty and Disagreement
(Midpoint-Gamma)

![Graph showing uncertainty and disagreement over time](image-url)
Figure 14

Uncertainty and Disagreement
(Uniform-Gamma)
Figure 15

Inflation Outlook and Uncertainty
Figure 16

Disagreement about Outlook and Uncertainty

![Graph showing disagreement about outlook and uncertainty]
Figure 17

Disagreement about Uncertainty and Uncertainty
Figure 18

Term Premium

Percent

Two years
Five years
Ten years