Mortgage Markets, Collateral Constraints, and Monetary Policy: Do Institutional Factors Matter?*

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Abstract

We study the role of institutional characteristics of mortgage markets in affecting the strength and timing of the effects of monetary policy shocks on house prices and consumption in a sample of industrialized countries. With frictionless credit markets, those characteristics should in principle be immaterial for the transmission of monetary impulses. We document three facts: (1) there is significant divergence in the structure of mortgage markets across the main industrialized countries; (2) at the business cycle frequency, the correlation between consumption and house prices increases with the degree of flexibility/development of mortgage markets; (3) the transmission of monetary policy shocks on consumption and house prices is stronger in countries with more flexible/developed mortgage markets. We then build a two-sector dynamic general equilibrium model with price stickiness and collateral constraints, where the ability of borrowing is endogenously linked to the nominal value of a durable asset (housing). We study how the response of consumption to monetary policy shocks is affected by alternative values of three key institutional parameters: (i) down-payment rate; (ii) mortgage repayment rate; (iii) interest rate mortgage structure (variable vs. fixed interest rate). In line with our empirical evidence, the sensitivity of consumption to monetary policy shocks increases with lower values of (i) and (ii), and is larger under a variable-rate mortgage structure.

Keywords: House prices, mortgage markets, collateral constraint, monetary policy.

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1 Introduction

The role of housing wealth on economic activity has recently attracted considerable attention among academic researchers, policy-makers and press commentators. This attention is partly explained by the sizeable rises in property prices and household indebtedness in several industrialized countries over recent years (Debelle (2004), Terrones and Otrok (2004)) and the need to understand both the determinants of such rises and their potential implications for monetary policy and financial stability. Beyond these policy considerations, there is growing interest in the effects of changes in property prices on consumption decisions, given the predominance of housing in total household wealth (Campbell and Cocco (2003)).

This paper studies the role of institutional characteristics of mortgage markets across the main industrialized countries, with particular focus on EU countries, in determining the channels of monetary policy transmission. We begin by establishing two facts on the relationship between mortgage markets, consumption and house prices. First, there is significant heterogeneity in the institutional characteristics of national mortgage markets across the main industrialized countries, and especially within the EU. Examples of such institutional characteristics include the typical duration of mortgage contracts, the required levels of down-payment, the existence (or lack thereof) of equity release products, and the interest-rate structure of mortgage contracts (e.g., variable vs. fixed rate). We interpret these indicators as measures of the degree of development/flexibility of mortgage markets. Second, the correlation between private consumption and house prices at the business cycle frequency is related to mortgage markets characteristics, with that correlation being larger in countries featuring more developed mortgage markets.

We then conduct a VAR-based analysis of the effects of monetary policy shocks on consumption and house prices in a sample of euro area countries, with the addition of Canada, the U.K. and the U.S.. We find significant heterogeneity in both the timing and strength of those effects across countries. In particular, we find that the size of the peak effect of a monetary policy shock on consumption and real house prices is positively related to indicators of development/flexibility in mortgage markets, such as the mortgage debt to GDP ratio, the loan-to-value (LTV henceforth) ratio, and the existence of equity release products.

The evidence that private consumption is more responsive to monetary impulses in economies with more developed mortgage markets is prima facie puzzling. In fact, in a baseline model of the monetary transmission with free borrowing and lending (usually labelled as New Keynesian,}

\[1\] For recent academic contributions see Aoki, Proudman and Vlieghe (2004), Davies and Heathcote (2005), Iacoviello (2005) and the literature review by Leung (2004); for contributions from a policy perspective see ECB (2003), Catte et al. (2004), Girouard and Blöndal (2001), BIS (2004) and IMF (2005); for a press account see The Economist (2003).
NK henceforth), the institutional features of credit markets should be immaterial for the effects of policy shocks. In general, then, less imperfect financial markets should allow agents to smooth consumption more efficiently. Hence accounting for our evidence requires a theoretical framework in which (at least) a fraction of agents do not act as permanent-income consumers.

We build a model that extends the baseline New Keynesian framework in three main directions. First, it allows for two sectors, respectively producing consumption goods and new housing. Second, it features heterogeneity of preferences between impatient consumers and patient consumers (in equilibrium, borrowers and savers respectively). The former do not act as standard permanent-income agents, but exhibit preferences tilted towards current consumption. The borrowers may be thought of as that share of the population for which acquiring a loan/mortgage requires providing an asset, and housing in particular, as a form of collateral. Third, private borrowing is constrained by the value of the collateral. That value is endogenously tied to the evolution of the nominal price of housing.

Thus, in a context where credit markets allow more easily to convert asset values into borrowing, and therefore spending, consumption should be more responsive to underlying shocks. In our framework, the relevant institutional features of the mortgage market are summarized by three main parameters: (i) the down-payment rate, (ii) the repayment rate (or rate of equity release), and (iii) the interest-rate structure of the contract. We calibrate and simulate the model based on our introductory evidence on the heterogenous characteristics of mortgage markets in industrialized countries. We find that the response of consumption to policy shocks is magnified in more flexible mortgage markets, symbolized by lower down-payment rates and lower rates of repayment. In addition, the prevalence of variable interest rate mortgages, and hence of a stronger pass-through of interest rate shocks to mortgage lending rates, also enhances the response of consumption to monetary policy shocks.

General equilibrium borrower-saver models build on the seminal analysis of Kiyotaki and Moore (KM) (1997) and Krusell and Smith (1998). In an important contribution, Campbell and Hercowitz (2004) extend this category of models to a real business cycle framework and explore the role of credit market innovations in contributing to the so-called Great Moderation. Iacoviello (2005) extends the KM framework to include features more typical of the New Keynesian monetary policy literature. Our paper differs from Iacoviello (2005) in three key respects. First, we document the cross-country heterogeneity of institutional characteristics of mortgage markets and relate those to the strength of the monetary policy effects on consumption. Second, we build a two-sector model in which both the production of new housing and asset price movements are endogenous. Third, we model a series of institutional details of credit markets, including a role for alternative typologies of interest-rate mortgage contracts. All these aspects are critical for our model to comply with the
results of our empirical analysis.

The paper is structured as follows. In Section 2 we document some key institutional differences in mortgage markets across industrialized countries. We then conduct some VAR-based empirical analysis in Section 3, focussing on the impact of a monetary policy shock on housing market-related variables. The structural model is developed in Section 4. Section 5 discusses the steady state of the model, which is then simulated in Section 6. Section 7 concludes.

2 Institutional Features of Mortgage Markets in the Industrialized Countries

In this section we document that mortgage markets differ significantly across industrialized countries in terms of both size and key institutional characteristics, such as the prevailing contractual arrangements and the available product range. This heterogeneity is particularly evident within the euro area, where mortgage lending remains a predominantly domestic business activity, largely reflecting national traditions and cultural factors as well as the institutional settings of the local banking sector.

Table 1 summarizes some of the institutional indicators that have been identified in the literature as most likely to have a bearing on the relationship between housing wealth and consumption, as well as on the channels of monetary policy transmission (see, e.g., MacLennan et al. (1998) and Debelle (2004)). We report data for a total of eighteen countries, including eleven euro area countries, Japan and the main Anglo-Saxon countries.

The indicators included in Table 1 are: (i) mortgage-debt to GDP ratio; (ii) extent of home ownership; (iii) typical LTV ratio; (iv) type of interest-rate structure; (v) typical mortgage contract duration, and (vi) diffusion of home equity release products.

Cross-country heterogeneity is pervasive in all indicators considered. Mortgage-to-GDP ratios vary widely across countries: values range between 15% in Italy and 111% in the Netherlands. Among the large countries, Italy and France have the lowest ratios, while the ratios in the U.K. and the U.S. are relatively high. Countries also differ in terms of home ownership ratios, with values ranging between 39% in Germany and 85% in Spain. With the exception of Germany, the majority of homes are owner-occupied in all countries. Also LTV ratios vary significantly across countries, ranging between 50% in Italy and over 110% in the Netherlands. Cross-country variations in these ratios partly reflect differences in legal and regulatory frameworks.\footnote{For instance, it has been argued (e.g. MacLennan et al., 1998, and Ahearne et al., 2005) that the reason why the LTV ratio has been historically low in Italy lies in the difficulty for the lender to enforce repossession in case of default of the borrower, given the country’s slow and costly judicial proceedings.} Hence, they reflect - at least to some extent - institutional factors which are largely exogenous.
The heterogeneity in terms of interest rate adjustment is also substantial across countries. Conceptually, mortgage contracts can be distinguished between variable and fixed rate mortgages: variable rate contracts are those in which the lending rate floats with, or is frequently adjusted to, a short-term market interest rate; fixed rate contracts are those in which the lending rate remains constant throughout the duration of the contract. In practice, contracts do not always fully conform to these conceptual types and often fall under intermediate categories (Borio (1996)). Among the EU countries, the U.K., Spain and Italy mainly have variable or adjustable rate mortgages, although for the latter two countries this reflects a relatively recent development.\(^3\) By contrast, Germany, France, Austria, Belgium, Denmark and the Netherlands are mainly characterized by fixed rate mortgages, similar to the U.S. and Canada.

Finally, an important element of divergence among national mortgage markets is the extent of the recourse to home equity release. Following changes in house prices and mortgage interest rates, collateral-constrained agents may wish to adjust their net borrowing positions or to re-finance the terms of their existing mortgages according to the changed conditions. For instance, following house prices rises, borrowers may increase the amount of their mortgage loans or apply for a second mortgage against the increased value of their collateral. The released mortgage equity may be subsequently used for a variety of purposes, such as debt refinancing, acquisition of durable goods, purchase of financial assets or home improvements. When mortgage interest rates decrease, agents may be willing to re-finance their mortgages to take advantage of lower interest payments in order to free liquidity for other expenditures or, alternatively, they may want to increase their borrowing to reflect their increased debt servicing capacity.

Overall, the use of home equity release remains limited in most countries as reported in Table 1, though mortgage equity extraction and refinancing have become significant at the aggregate level in a few of them (e.g. U.S., U.K. and the Netherlands). In some cases, the limited recourse to home equity release may reflect scarce availability of suitable mortgage contracts (e.g. due to regulatory constraints). However, in most countries borrowers are deterred from refinancing their contracts by administrative obstacles and prohibitive transaction costs.\(^4\) In such countries, mortgage lending is likely to interact with interest rate and house price developments only to a very limited extent (namely only for the new mortgage contracts and not for the existing ones, which mostly reflect market conditions prevailing at the time they were signed rather than current conditions). The U.S. has been historically one of the main exceptions to this pattern, with the exceptional nature of its national mortgage market becoming particularly evident in recent years as U.S. borrowers have taken advantage of low interest rates, rising house prices and a dramatic decline in transaction

\(^3\)Japan also has mainly variable rate mortgages.

\(^4\)For instance, Borio (1996) documents the penalties and administrative costs that borrowers willing to repay in advance their medium- and long-term (not necessarily mortgage) loans face in a number of countries.
costs to engage in a wave of mortgage refinancing and equity extraction commonly thought to have been large enough to influence aggregate spending.

2.1 House Prices and Consumption

In Table 3 we report the correlation between house prices and total private consumption measured at the business cycle frequency for a subset of countries with reliable house price data.\(^5\) While that correlation is generally positive, it is noticeable how it significantly varies across countries, ranging from 0.79 in the U.K. to almost zero in Italy.

A natural question is whether that correlation shows any significant pattern against the characteristics of mortgage markets. Figure 1 (1a to 1c) describes how the correlation between consumption and house prices varies with three indicators of development and flexibility of mortgage markets: (i) mortgage to GDP ratio, (ii) the degree of completeness in mortgage markets proposed by Mercer Oliver Wyman (2003) (MOW henceforth, which mainly measures the number of mortgage products available in a given market)\(^6\) and (iii) the typical LTV ratio. Notice that the correlation is significant and positive in all cases.

Table 4 shows how the correlation between house prices and consumption varies (on average across countries) with (i) the possibility of resorting to mortgage refinancing, and (ii) the interest-rate mortgage structure. The correlation is on average twice as large in those countries where mortgage refinancing is feasible, and is also higher in those countries with a prevalence of variable rate contracts.

2.2 Country Clustering

A further issue worth exploring is whether it is possible to identify “clusters” of countries on the basis of the institutional characteristics of their mortgage markets. In general, in countries where LTV ratios are high, the level of mortgage debt relative to GDP tends to be large. High LTV ratios and relatively large mortgage debts also tend to be accompanied by longer durations. In addition, countries where home equity release is common and households are able to borrow easily against their housing wealth tend to exhibit relatively high mortgage debt to GDP ratios. By contrast, there is no clear correlation between home ownership ratios and other characteristics, perhaps reflecting the prevailing role of public policies and cultural factors in determining the diffusion of home ownership in a country.\(^7\) Likewise, there is no obvious link between the prevailing type of

\(^5\)See Table 2 for a description of the house price data.
\(^6\)Note that this index is only available for EU countries.
\(^7\)Governments aiming to promote home ownership have historically intervened in a variety of ways, such as the establishment of public housing finance agencies, the provision of deposit insurance to institutions specialised in mortgage lending, regulation and direct provision by public authorities of rental housing, welfare support to mortgage...
interest rate adjustment and the relative size of the mortgage market or other institutional factors.

In general, mortgage markets tend to be larger and more flexible in the Anglo-Saxon economies than in Japan and continental Europe (with the exception of the Netherlands). In particular, mortgage equity release is more extensively used in the U.S., U.K., Australia and the Netherlands than in the other countries. This country split coincides with that between countries with market- and bank-based financial systems, suggesting that the extent to which households can borrow against their housing wealth partly depends on the availability of developed and well-functioning capital markets in which lenders can raise loanable funds and transfer risks. It should be also noted that countries with market-based financial systems are typically those in which mortgage markets have been longer and more extensively exposed to liberalization and deregulation. 8

Overall, within the EU there appears to be at least two clusters of countries:

- first, a group with less developed and more regulated mortgage markets (Italy, Germany, Austria, Belgium) where mortgage debt to GDP ratios tend to be low;
- second, a group of countries with deregulated mortgage markets and high mortgage debt to GDP ratios where home equity extraction is common (notably, the Netherlands, the U.K. and Denmark). This cluster of countries can be considered homogeneous to the U.S..

Other countries such as France and Spain fall under intermediate categories or may be undergoing structural adjustments that render their categorization more difficult (e.g. Spain which has been exposed to significant financial innovation in recent years).

3 The Transmission of Monetary Policy Shocks in EU Countries, the U.S. and Canada: a VAR Analysis

Institutional differences across mortgage markets are often cited as a likely source of cross-country differences in the speed and strength of the transmission of monetary policy impulses to the economy. The size and distribution of household mortgage debt, average maturity of contracts and type of interest rate adjustment are usually listed among the characteristics likely to determine the extent of the income and collateral effects induced by changes in interest rates (Debelle (2004)).

8 A more formal clustering exercise is pursued by Tsatsaronis and Zhu (2004), who group various national mortgage markets according to a set of institutional characteristics such as LTV ratios, the use of market or historical prices to value collateral and the extent of home equity release. The authors argue that most continental European countries are characterised by conservative lending practices and limited mortgage equity release, while Anglo-Saxon countries are exposed to more aggressive practices and more extensive mortgage equity release, particularly in countries where variable rate mortgages are predominant. The main exceptions to this classification are the Netherlands, Finland and Ireland among the continental European countries and Canada among the Anglo-Saxon countries.
BIS (1995) concludes that monetary policy could be expected to have comparatively stronger effects in Anglo-Saxon countries than in continental Europe (with the possible exception of Italy, where variable-rate mortgages predominate). Borio (1996) notes that this split coincides with that between countries with more or less developed financial structures, though this does not amount to conclusive evidence. Iacoviello (2002) relates variations in the magnitude of output responses to monetary policy shocks across European countries to differences in financial systems. Likewise, Angeloni et al. (2004) refer to institutional differences in housing finance as one possible explanation for the more muted response of private consumption to monetary policy shocks in the euro area compared with the U.S.. In recent years, the remarkable heterogeneity in private consumption developments between some continental European countries and most Anglo-Saxon countries at a time of (common) worldwide low interest rates has seemed to provide further confirmation about the importance of structural differences in mortgage markets across countries in determining the strength of the housing channel.

In this section we estimate VAR models for three Anglo-Saxon countries (Canada, the U.S. and the U.K.), seven euro area countries (Germany, Italy, France, Spain, the Netherlands, Belgium and Austria) plus a non-euro area EU member country with a highly developed mortgage market (Denmark). Given the more sophisticated nature of the Anglo-Saxon and Danish housing finance systems, they provide a natural benchmark against which to assess the potential implications of the existence of less flexible institutional settings in euro area countries.

We estimate the model on quarterly data over the sample period 1980:1-2004:4 (except from 1986 for Austria due to data availability). Each VAR model includes five endogenous variables: (i) real total private consumption, (ii) the consumer price index (CPI); (iii) real house prices (deflated using the CPI); (iv) the 3-month nominal interest rate, and (v) the real effective exchange rate. We include the real effective exchange rate to cater for open economy influences that, while arguably secondary for the U.S. economy, are likely to matter considerably for the European countries and Canada.

The VARs are specified in levels (hence long-run relationships are implicitly allowed for) and, with the exception of the interest rates, all variables are in logs. A constant and a linear trend are also added as exogenous variables. Based on the Schwartz information criterion, a lag order of two

9 Note that we include all major industrialised countries in our analysis with the exception of Japan, for which a measure of monetary policy shock may be particularly problematic due to the zero interest rate policy from the second half of the 1990s to the end of the sample period.

10 In particular, the US and the UK are characterised by relatively more developed and flexible mortgage markets, with the main contractual difference perhaps being the different type of mortgage lending rate adjustment (fixed in the US versus variable in the UK).

11 For the U.S., which is a large closed economy, we also estimate the model without the real effective exchange rate. Since this specification turns out to perform better than the one including the exchange rate according to standard information criteria and the statistical fit of the model, we select this one in the baseline exercise.
(in levels) is optimal for this model across all countries.

The VAR models include house prices since they are of direct relevance to the household sector and the housing market.\textsuperscript{12} However, the lack of harmonized data on house prices has to be emphasized. \textit{Table 2} reports a detailed description of the data used in this study, which indicates a certain degree of heterogeneity in the available house price data available. Even within the euro area house price data are not fully comparable. For this reason, the results on house prices have to be interpreted with some caution.

The identification of the monetary policy shock is achieved through a standard recursive procedure based on a Cholesky factorization of the estimated variance-covariance matrix. The policy-related variable - the 3-month nominal interest rate - is ordered after all other variables, except the exchange rate (changes in the ordering of the latter, however, do not affect the main results shown below).

\textit{Figure 2} reports the impulse responses of private consumption and of the real house price to a 100 basis points \textit{rise} in the policy interest rate, for all considered countries. Qualitatively, the impact of a policy shock is in line with previous studies for the U.S., Canada and euro area countries (Angeloni et al., 2004, Aoki et al., 2004, and Mojon and Peersman, 2003): both consumption and the real house price tend to \textit{fall}. However, a noticeable result of the VAR analysis is the significant \textit{heterogeneity} in the impact of a monetary policy shock across different countries. For example, there is a striking difference between the impact of a policy shock in France, where the effects are very small and almost statistically insignificant, and the impact in the Netherlands and the U.K., where the effects are very large (indeed larger than in the U.S.). In Germany, the effects are even of the "wrong" sign, although this may be partly due to the impact of the German reunification (see also Mojon and Peersman (2003) on this point) on the statistical properties of the model.

The finding that monetary policy transmission seems to be stronger in countries like the U.K., the Netherlands and the U.S., and weaker in France and Germany, may indeed suggest a link with the degree of development in mortgage markets. To further explore this issue, in \textit{Figures 3 and 4} we plot the estimated peak response of private consumption and the real house price to a standardized monetary policy shock in the cross-section of countries respectively against three indicators: (i) mortgage debt to GDP ratio; (ii) MOW index of completeness in mortgage markets; (iii) typical LTV ratio. In all cases, we find a clearly positive relationship. In particular, in the case of the MOW index the link appears to be quite strong, especially as regards the effects of monetary policy on real house prices.

In \textit{Table 5} we also relate the (cross-country) average estimated peak effects of a contractionary

\textsuperscript{12} Giuliodori (2004) conducts a similar analysis for several EU countries, finding similar results to this study. Note that, due to data limitations, we have not included another highly relevant variable in the VARs, i.e., mortgage debt.
monetary policy shock on private consumption and real house price to two dummy indicators: (i) the use of mortgage refinancing and (ii) the interest rate structure (predominantly fixed or variable interest rate). In line with the previous results, we find a comparatively stronger reaction of both consumption and the real house price to a policy shock in countries with a variable rate structure and, even more markedly, where mortgage refinancing is used. For example, the peak response of the real house price is 1.82 per cent where mortgage refinancing is allowed, and only 0.38 per cent where refinancing is not allowed or not practiced.¹³

**Summary of Empirical Evidence: Why is Consumption More Responsive in More Flexible Mortgage Markets?** Overall, the empirical analysis seems to convey a sufficiently robust general message: both the business-cycle link between private consumption and house prices, as well as the transmission of monetary policy shocks on consumption and house prices, seem to be significantly related to the characteristics of mortgage markets in different countries. In particular, house prices and private consumption co-move more strongly, and monetary policy seems more powerful (on consumer spending and house prices) in countries with more developed/flexible mortgage markets.

Two observations are relevant at this stage. First, a more structural investigation of the link between mortgage markets characteristics and the transmission of monetary policy shocks requires a modelling framework. Second, the fact that private spending is more responsive to monetary impulses in economies with more developed credit/mortgage markets may be perceived as a puzzle. In principle, in a standard representative-agent model of the monetary transmission with free borrowing and lending, the structure of credit/mortgage markets should be immaterial for the effects of policy. In addition, a priori, one may believe that more developed financial markets would allow households to smooth consumption more efficiently.

In the following, we present a model in which a fraction of agents, in equilibrium, do not choose to behave as permanent-income consumers. Rather, for these agents, it is optimal to *increase* consumption in light of any given rise in income. They can do this by increasing borrowing, although up to some endogenously determined limit. Thus, in a context where credit markets allow to convert asset values (e.g., housing) into borrowing and therefore consumption more easily,

¹³It is also interesting to note that our basic findings hold even when considering a broader measure of financial development (i.e., not exclusively linked to mortgage markets), namely the IMF Index of Financial Development (IMF 2006). Cross-country analysis (not reported here for brevity) confirms that house prices and consumption appear to be more correlated, and the impact of a standardized monetary policy shock larger, in countries with a higher reading of this index. This suggests that the nexus between financial development and effectiveness of monetary policy on consumption could be a generalized feature, which begs a structural explanation not necessarily (only) related to the structure of mortgage markets.
consumption itself should be in principle more responsive to underlying shocks. We describe our model in the next section.

4 The Model

The economy is composed of a continuum of households in the interval (0, 1). As in Iacoviello (2005) and Campbell and Hercowitz (2004), there are two groups of households, named borrowers and savers, that we assume of measure $\omega$ and $1 - \omega$ respectively. Each group of households is endowed with one unit of time, so that an individual borrower and an individual saver are endowed with a fraction $\frac{1}{\omega}$ and $\frac{1}{1-\omega}$ respectively. There are also two sectors, producing a durable good (identified as new housing) and non-durable goods respectively. In each sector there are competitive producers of a final good and monopolistic competitive producers of intermediate goods, with the latter hiring labour from the borrowers and savers. The two types of households feature heterogeneous preferences, with the borrowers being more impatient than the savers, implying that their marginal utility of consumption exceeds the marginal utility of saving.\(^{14}\) Both borrowers and savers derive utility from consumption of the non-durable final good and from housing services. Notice that debt accumulation reflects intertemporal equilibrium trading between the two agents. Borrowers are subject to a collateral constraint, with the borrowing limit tied to the value of the existing stock of housing.

4.1 Final Good Producers

In each sector ($j = c, d$) a perfectly competitive final good producer purchases $Y_{j,t}(i)$ units of intermediate good $i$. The final good producer in sector $j$ operates the production function:

$$Y_{j,t} = \left( \int_0^1 Y_{j,t}(i) \frac{\varepsilon_j^{-1}}{\varepsilon_j} \, di \right)^{\frac{\varepsilon_j}{\varepsilon_j-1}}$$

(1)

where $Y_{j,t}(i)$ is quantity demanded of the intermediate good $i$ by final good producer $j$, and $\varepsilon_j$ is the elasticity of substitution between differentiated varieties in sector $j$. Notice, in particular, that in the durable good sector $Y_{d,t}(i)$ refers to expenditure in the new durable intermediate good $i$ (rather than services). Maximization of profits yields demand functions for the typical intermediate good $i$ in sector $j$:

$$Y_{j,t}(i) = \left( \frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\varepsilon_j} Y_{j,t} \quad j = c, d$$

(2)

\(^{14}\)For previous examples of saver-borrower models, see Becker (1980), Becker and Foias (1987), Krusell and Smith (1998), Kiyotaki and Moore (1997).
for all \( i \). In particular, \( P_{j,t} \equiv \left( \int_0^1 P_{j,t}(i)^{1-\epsilon} \, di \right)^{\frac{1}{\epsilon}} \) is the price index consistent with the final good producer in sector \( j \) earning zero profits.\(^{15}\)

### 4.2 Borrowers

A typical borrower consumes an index of consumption services of housing and non-durable final goods, defined as:

\[
X_t \equiv \left[ (1-\alpha)^{\frac{1}{\eta}} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} D_t^{\frac{\eta-1}{\eta}} \right]^\eta
\]

where \( C_t \) denotes (non-durable) consumption services, \( D_t \) denotes housing services at the end of period \( t \), \( \alpha > 0 \) is the share of housing services in the composite consumption index, and \( \eta > 0 \) is the elasticity of substitution between consumption and housing services.\(^{16}\)

The borrower maximizes the following utility program:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(X_t, N_t) \right\}
\]

subject to the sequence of budget constraints (in nominal terms):

\[
P_{c,t} C_t + P_{d,t} (D_t - (1-\delta)D_{t-1}) + R_{t-1}^m B_{t-1} = B_t + W_t N_t + T_t
\]

where \( B_t \) is end-of-period \( t \) net nominal debt, and \( R_{t-1}^m \) is the nominal lending rate on debt contracts stipulated at time \( t-1 \) with maturity \( m \). Furthermore, \( W_t \) is the nominal wage earned by the borrower, \( N_t \) is labor supply, and \( T_t \) are net government transfers. Labor is assumed to be perfectly mobile across sectors, implying that the nominal wage rate is common across sectors.

In real terms (units of non-durable consumption), (5) reads

\[
C_t + q_t (D_t - (1-\delta)D_{t-1}) + \frac{R_{t-1}^m b_{t-1}}{P^c_{t-1}} = b_t + \frac{W_t}{P^c_{t-1}} N_t + \frac{T_t}{P^c_{t-1}}
\]

where \( q_t \equiv \frac{P_{c,t}}{P_{c,t}} \) is the relative price of housing, and \( b_t \equiv \frac{b_t}{P_{c,t}} \) is real debt. Notice that, as a consequence of debt being predetermined in nominal terms, variations in inflation affect the real ex-post cost of debt service, and therefore borrower’s net worth.

\(^{15}\) Hence the problem of the final good producer \( j \) is: \( \max P_{j,t} Y_{j,t} \) - \( \int_0^1 P_{j,t}(i)Y_{j,t}(i) \, di \) subject to (1).

\(^{16}\) To define a utility-based aggregate price index one needs to assume the existence of an additional final good producer, whose task consists in assembling housing and consumption services via the production function (3). The price index consistent with maximization of profits by this producer would read:

\[
P_t \equiv \left[ (1-\alpha) (P^c_{t})^{1-\eta} + \alpha (P_{d,t})^{1-\eta} \right]^\frac{1}{1-\eta}
\]
Later we will work with the utility specification:

\[ U(X_t, N_t) = \log(X_t) - \frac{v}{1 + \varphi} N_t^{1+\varphi} \quad (7) \]

where \( \varphi \) is the inverse elasticity of labor supply and \( v \) is a scale parameter.

**Variable vs. Fixed-Rate Contracts** The interest rate \( R_t^m \) on a mortgage contract of maturity \( m \) is related to the policy rates \( R_{t+k} \) \( (k = 0, 1, 2...) \) via the term-structure equation:

\[ R_t^m = \left( \sum_{k=0}^{m-1} \tau^k \right)^{-1} \sum_{k=0}^{m-1} \tau^k E_t \{ R_{t+k} \} \quad (8) \]

with \( \tau \in [0, 1] \).

In the case \( m = 1 \) the mortgage and policy rates coincide. Mortgage contracts are typically multi-period. Multi-period loan contracts can be defined as at variable rate (i.e., contracts tied to the short-term policy rate), or at fixed rate (tied to a long-term interest rate) depending on the value of \( \tau \). For \( \tau = 0 \) the mortgage rate is perfectly indexed to the policy rate, while for \( \tau = 1 \) it is fixed to the \( m \)-period interest rate. We assume that the decision on who bears the interest rate risk (either the borrower or the saver) mainly reflects institutional factors which lie outside the scope of our model.\(^{17}\)

**Collateral Constraint** Private borrowing is subject to a collateral constraint. We assume that the whole stock of debt is collateralized by the value of the accumulated stock of housing. By definition, if the collateral value depreciates at the same rate of physical depreciation \( \delta \), we would write the accumulated equity value at time \( t \) as:

\[ P_{d,t}D_t = \left[ \sum_{s=0}^{\infty} (1 - \delta)^s (D_{t-s} - (1 - \delta)D_{t-1-s}) \right] P_{d,t} \]

More generally, and as in Campbell and Hercowitz (2004), we allow for the collateral value to depreciate economically at a rate \( \xi \) higher than physical depreciation, and therefore write the collateral constraint as:

\[ B_t \leq (1 - \chi) \left[ \sum_{s=0}^{\infty} (1 - \xi)^s (D_{t-s} - (1 - \delta)D_{t-1-s}) \right] P_{d,t} \quad (9) \]

\[ = (1 - \chi)P_{d,t}(D_t - (1 - \delta)D_{t-1}) + (1 - \xi)B_{t-1}\frac{P_{d,t}}{P_{d,t-1}} \]

\(^{17}\)See Campbell and Cocco (2003) for a normative analysis of the optimal choice between a variable-rate and a fixed-rate mortgage contract based on household-level risk management.
where $\chi$ is the fraction of the housing value that cannot be used as a collateral, and where $\xi \geq \delta$. This type of constraint can be justified on the basis of limited enforcement.\textsuperscript{18} Since the borrower can run away with the assets in case of default, requiring a collateral ex-ante acts against that temptation. One can think of parameters $\chi$ and $\xi$ as being determined by institutional factors prevailing in the credit market. For one, $\chi$ can be defined as the down-payment rate (or inverse LTV ratio), and therefore represents a direct measure of the flexibility of the mortgage market (Jappelli and Pagano (1989)). As already discussed above, the value of $\chi$ may reflect legal and regulatory constraints changing across countries (see Table 1).

Parameter $\xi$ can be defined as the rate at which a good loses its value as collateral to the creditor. In the mortgage markets, $\xi$ may capture the effect of all those supply-side factors that influence the ability of households to refinance their existing mortgages or to use their housing wealth to release liquidity.\textsuperscript{19} For instance, lower values of $\xi$ closer to $\delta$ – and hence a better performance of the housing stock as a collateral in a lending relationship – may reflect technological, industrial and structural developments in the banking sector that render mortgage refinancing easier and less costly, thereby lengthening debt repayment. Bennett et al. (2001) argue that the increase in the propensity to mortgage refinancing observed in the U.S. in the 1990s was due to a combination of technological, structural and regulatory changes that rendered mortgage markets more competitive and efficient, thereby lowering the transaction costs associated with refinancing. An example may be developments in the information and banking technology available to lending institutions in order to process information on the creditworthiness of borrowers or to manage the risks associated with their mortgage portfolios (e.g., through the securitization of mortgage loans or the use of credit derivatives). In addition, the liberalization and deregulation of mortgage markets, with the ensuing product innovation and increase in competitive pressures, may also lower the value of $\xi$. Muellbauer and Murphy (1997) analyse the house price boom of the late 1980s in the U.K. and note that financial liberalization rendered illiquid assets more spendable and allowed households to increase their leverage ratios. Girouard and Blöndal (2001) and Debelle (2004) also describe the impact of financial liberalization and deregulation on the easing of borrowing constraints in more recent episodes in various OECD countries.

We will distinguish two alternative scenarios for the calibration of $\xi$:

- $\xi = \delta$ (baseline). In this case, the rate of repayment coincides with the rate of physical depreciation of housing. This scenario is akin to one of full mortgage refinancing.

\textsuperscript{18}Kiyotaki and Moore (1997).

\textsuperscript{19}See Krainer and Masquis (2003) for a model of optimal refinancing of a fixed-rate mortgage depending on house prices and interest rates. We leave for future research the task of embedding an explicit refinancing choice into the model.
\* \( \xi > \delta \). In this scenario \( \xi \) will assume alternative values depending on the typical average duration of the mortgage contract (see Table 1 and below for the parameterization).

Finally, notice that movements in real house prices affect the ability of borrowing. This assumption is consistent with the evidence that equity valuation effects have been important for the recent business cycle evolution in some OECD countries, in which the link between house price fluctuations and ability of borrowing has played a major role in supporting household consumption.\(^{20}\)

Assuming that, in a neighborhood of the deterministic steady state, equation (9) is always satisfied with the equality, we can rewrite the collateral constraint in real terms (i.e., in units of consumption) as follows:

\[
    b_t = (1 - \chi) q_t (D_t - (1 - \delta)D_{t-1}) + (1 - \xi) b_{t-1} \frac{q_t}{q_{t-1}}
\]

(10)

Notice that, in this specification, both the level and the rate of change of \( q_t \) affect the ability of borrowing.

Given \( \{b_{-1}, D_{-1}\} \) the borrower chooses \( \{N_t, b_t, D_t, C_t\} \) to maximize (4) subject to (6) and (10). By defining \( \lambda_t \) and \( \lambda_t \psi_t \) as the multipliers on constraints (6) and (10) respectively, and \( U_{i,t} \) as the marginal utility of variable \( i \), efficiency conditions read:

\[
    \frac{-U_{n,t}}{U_{c,t}} = \frac{W_t}{P_{c,t}}
\]

(11)

\[
    U_{c,t} = \lambda_t
\]

(12)

\[
    U_{c,t} Z_t = U_{d,t} + \beta (1 - \delta) E_t \{U_{c,t+1} Z_{t+1}\}
\]

(13)

\[
    \psi_t = 1 - \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{R^m_t}{\pi_{c,t+1}} \right\} + (1 - \xi) \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \psi_{t+1} \frac{q_{t+1}}{q_t} \right\}
\]

(14)

where

\[
    Z_t \equiv q_t [1 - (1 - \chi) \psi_t]
\]

can be defined as the "effective" relative price of housing. The latter depends directly on the real price of housing \( q_t \), and inversely on the shadow value \( \psi_t \) of relaxing the collateral constraint.

\(^{20}\)On the other hand, we are not explicitly allowing for the presence of home equity loans (otherwise defined as home mortgage loans). These are typically secondary loans for which accumulated equity (defined as the difference between the value of the outstanding housing stock and the debt principal still due) is used as a collateral. Allowing for home equity loans would not qualitatively alter our results.
4.2.1 Interpretation

Equation (11) governs the consumption/leisure margin, while (12) equates the marginal utility of consumption to the shadow value of the flow budget constraint (5). Equation (13) is an intertemporal condition driving the choice between housing and consumption. It requires the borrower to equate the marginal utility of current consumption (left-hand side) to the marginal gain of housing services (right-hand side). The latter depends on two components: (i) the direct utility gain of an additional unit of housing; and (ii) the expected utility stemming from the possibility of expanding future consumption by means of the realized resale value of a new unit of housing purchased in the previous period.

Equation (14) is a modified version of an Euler equation. Indeed it reduces to a standard Euler condition in the case of \( \psi_t = 0 \) for all \( t \). The shadow value of relaxing the collateral constraint \( \psi_t \) is tied to a payoff which has two components. The first is the current deviation from the standard Euler condition. When that component is positive the marginal utility of consumption exceeds the (expected) marginal utility of shifting consumption intertemporally. Hence the borrower has a marginal benefit from acquiring a unit of housing and purchase additional current consumption via a relaxation of the collateral constraint. The second term in (14) indicates that the shadow value of borrowing depends also on the ability of expanding future consumption, which is proportional to the rate at which the housing asset depreciates. The lower \( \xi \), the larger the rate at which borrowers can expand private borrowing at each time \( t \). In general, a unit of housing acquired in time \( t \) allows to expand future borrowing (and consumption) at a rate \( (1 - \xi)^j \) in period \( t + j \). In this respect, \( \xi \) can be thought of capturing (exogenous) variations in the rate of mortgage refinancing.

The Euler Gap Integrating both (13) and (14) forward, and combining, we can express the margin between consumption and housing in more compact form as:

\[
U_{c,t} = E_t \left\{ \sum_{j=0}^{\infty} [\beta(1-\delta)]^j U_{d,t+j} \right\} + (1-\chi)U_{c,t} q_t \psi_t \tag{15}
\]

\[
= E_t \left\{ \sum_{j=0}^{\infty} [\beta(1-\delta)]^j U_{d,t+j} \right\} + (1-\chi)E_t \left\{ \sum_{j=0}^{\infty} [\beta(1-\xi)]^j q_{t+j} \Delta_{t+j} \right\} \tag{16}
\]

where

\[
\Delta_t \equiv U_{c,t} - \beta U_{c,t+1} \frac{R_t^m}{\pi_{c,t+1}}
\]
is a term summarizing the deviation from the Euler condition in any given time $t$. We label $\Delta_t$ the Euler gap.

In (15), the marginal utility of consumption is equated to an alternative representation of the marginal utility of housing. The latter has two dynamic components. First, the current and expected future flow of utility of housing services. This term is standard in a framework with free borrowing. Second, the current and expected future benefits deriving from the possibility of expanding (current and future) consumption by means of increased borrowing. Indeed those benefits coincide with positive values of the Euler gap, which in turn reflect proportional variations in the tightness of the collateral constraint captured by the multiplier $\psi_t$. Notice that, in this interpretation, $(1 - \chi)(1 - \xi)^j$ is the effective rate at which the household can expand borrowing at any time $t + j$, with $j \geq 0$.

### 4.3 Savers

We assume that the savers are the owners of the monopolistic firms in each sector. A typical saver maximizes the utility program

$$
E_0 \left\{ \sum_{t=0}^{\infty} \gamma^t U(\bar{X}_t, \bar{N}_t) \right\}
$$

where

$$
\bar{X}_t \equiv \left[ (1 - \alpha)^{\frac{1}{\beta}} (\bar{C}_t)^{\frac{n-1}{\beta}} + \alpha^{\frac{1}{\beta}} (\bar{D}_t)^{\frac{n-1}{\beta}} \right]^{\frac{1}{n-1}}
$$

Importantly, the (im)patience rate $\gamma$ is such that $\gamma > \beta$. The saver’s sequence of budget constraints reads (in nominal terms):

$$
P_{c,t} \tilde{C}_t + P_{d,t}(\tilde{D}_t - (1 - \delta)\tilde{D}_{t-1}) + R_{m,t-1}^m \tilde{B}_{t-1} = \tilde{W}_t \tilde{N}_t + \tilde{B}_t + \tilde{I}_t + \sum_j \tilde{\Gamma}_{j,t}
$$

where $\tilde{W}_t$ is the nominal wage rate paid to the saver, and $\tilde{\Gamma}_{j,t}$ are nominal profits from the holding of monopolistic competitive firms in sector $j$.

Efficiency conditions for the saver’s program read:

$$
-\frac{\tilde{U}_{n,t}}{\tilde{U}_{c,t}} = \frac{\tilde{W}_t}{P_{c,t}}
$$

$$
\tilde{U}_{c,t} = \gamma E_t \left\{ \frac{\tilde{U}_{c,t+1} + R_{m,t}^m}{\pi_{c,t+1}} \right\}
$$
$$q_t = \frac{\tilde{U}_{d,t}}{U_{c,t}} + \gamma(1 - \delta)E_t \left\{ \frac{\tilde{U}_{c,t+1}}{U_{c,t}} q_{t+1} \right\}$$  \hspace{1cm} (22)

### 4.4 Production and Pricing of Intermediate Goods

Intermediate-good firm $i$ in sector $j$ hires labor to operate the following production function:

$$Y_{j,t}(i) = \Omega N_{j,t}^\omega(i) \tilde{N}_{j,t}^{1-\omega}(i) \hspace{1cm} \Omega \equiv \omega^\omega(1 - \omega)^{1-\omega}$$  \hspace{1cm} (23)

where $N_{j,t}$ and $\tilde{N}_{j,t}$ denote the amount of labor, respectively supplied by the borrower and the saver, employed in sector $j$. Each firm $i$ has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost proportional to output, and equal to:

$$\frac{\vartheta_j}{2} \left( \frac{P_{j,t}(i)}{P_{j,t-1}(i)} - 1 \right)^2 Y_{j,t}$$  \hspace{1cm} (24)

where the parameter $\vartheta_j$ measures the degree of sectoral nominal price rigidity. The higher $\vartheta_j$, the more sluggish the adjustment of nominal prices in sector $j$. For $\vartheta_j = 0$ prices are flexible.

The problem of each monopolistic firm is to choose the sequence $\{N_{j,t}(i), P_{j,t}(i)\}_{t=0}^\infty$ to maximize expected discounted nominal profits:

$$E_0 \left\{ \sum_{i=0}^\infty \Lambda_{j,t} \left( P_{j,t}(i) Y_{j,t}(i) - W_t \left( \omega N_{j,t}(i) + (1 - \omega) \tilde{N}_{j,t}(i) \right) \right) - \frac{\vartheta_j}{2} \left( \frac{P_{j,t}(i)}{P_{j,t-1}(i)} - 1 \right)^2 P_{j,t} Y_{j,t} \right\}$$  \hspace{1cm} (25)

subject to (23). In (25), $\Lambda_{j,t} \equiv \gamma E_t \left\{ \frac{\lambda_{j,t+1}}{\lambda_t} \right\}$ is the saver’s stochastic discount factor, and $\lambda_t$ is the saver’s marginal utility of nominal income.

Let’s denote by $\frac{P_{j,t}(i)}{P_{j,t}}$ the relative price of variety $i$ in sector $j$. In a symmetric equilibrium in which $\frac{P_{j,t}(i)}{P_{j,t}} = 1$ for all $i$ and $j$, and all firms employ the same amount of labor in each sector, the first order condition of the above problem reads:

$$((1 - \varepsilon_j) + \varepsilon_j mc_{j,t}) = \vartheta_j (\pi_{j,t} - 1) \pi_{j,t}$$  \hspace{1cm} (26)

$$-\vartheta_j E_t \left\{ \frac{\Lambda_{j,t+1}}{\Lambda_{j,t}} \frac{P_{j,t+1}}{P_{j,t}} \frac{Y_{j,t+1}}{Y_{j,t}} \left( \pi_{j,t+1} - 1 \right) \pi_{j,t+1} \right\} \hspace{1cm} (j = c, d)$$

where $\pi_{j,t} \equiv \frac{P_{j,t}}{P_{j,t-1}}$ is the gross inflation rate in sector $j$, and $mc_{j,t}$ is the real marginal cost in sector $j$.  

17
Optimal choice of labor inputs in sector \( j \) implies:

\[
\frac{W_t}{P_{j,t}} = mc_{j,t} \Omega \frac{Y_{j,t}}{N_{j,t}}
\]  \hspace{1cm} (27)

\[
\frac{\tilde{W}_t}{P_{j,t}} = mc_{j,t} \Omega (1 - \omega) \frac{Y_{j,t}}{N_{j,t}}
\]  \hspace{1cm} (28)

Finally, sectoral inflation and relative prices are related as follows:

\[
\frac{\pi_{d,t}}{\pi_{c,t}} = \frac{q_t}{q_{t-1}}
\]  \hspace{1cm} (29)

4.5 Market clearing

Equilibrium in the goods market of sector \( j = c, d \) requires that the production of the final good be allocated to total households’ expenditure and to resource costs originating from the adjustment of prices:

\[
Y_{c,t} = \omega C_t + (1 - \omega)\bar{C}_t + \frac{\vartheta_c}{2} (\pi_{c,t} - 1)^2 Y_{c,t}
\]  \hspace{1cm} (30)

\[
Y_{d,t} = \omega (D_t - (1 - \delta)D_{t-1}) + (1 - \omega) \left( \bar{D}_t - (1 - \delta)\bar{D}_{t-1} \right) + \frac{\vartheta_d}{2} (\pi_{d,t} - 1)^2 Y_{d,t}
\]  \hspace{1cm} (31)

where

\[
Y_{j,t} = \int_0^1 Y_{j,t}(i) \, di \quad (j = c, d)
\]

Equilibrium in the debt and labor market requires respectively

\[
\omega B_t + (1 - \omega)\bar{B}_t = 0
\]  \hspace{1cm} (32)

\[
N_t = N_{c,t} + N_{d,t}
\]  \hspace{1cm} (33)

\[
\tilde{N}_t = \tilde{N}_{c,t} + \tilde{N}_{d,t}
\]  \hspace{1cm} (34)
4.6 Monetary Policy

We assume that monetary policy is conducted by means of an interest rate reaction function, constrained to be linear in the logs of the relevant arguments:

\[
\ln \left( \frac{R_t}{R} \right) = (1 - \phi_r) \phi_x \ln \left( \frac{\pi_{j,t}}{\bar{\pi}} \right) + \phi_r \ln \left( \frac{R_{t-1}}{R} \right) + \zeta_t
\]

(35)

where \( R_t \) is the short-term policy rate, and \( \zeta_t \) is a policy shock evolving as:

\[
\zeta_t = \rho \zeta_{t-1} + u_t
\]

with \( u_t \sim i.i.d., \) with mean zero and variance \( \sigma_u^2 \). Our baseline is to employ a version of (35) in which \( \pi_{j,t} = \pi_{c.t} \), although the results will not be sensitive to employing rules in which the inflation index is the CPI.

4.7 Equilibrium

An (imperfectly) competitive allocation is a sequence for \( N_t, N_{j,t}, \tilde{N}_t, \tilde{N}_{j,t}, \tilde{w}_{c.t}, \tilde{w}_{d,t}, b_t, D_t, C_t, \tilde{C}_t, \tilde{D}_t, \pi_{j,t}, R_t, R_{q}^m, q_t, m_{c,j,t}, \) for \( j = c,d \), satisfying equations (6), (8), (10), (11)-(14), (20), (21),(22), (26), (27), (28), (29), (30), (31), (33), (34), (35).

5 Deterministic Steady State

In the deterministic steady state, as a result of heterogeneity in patience rates, the shadow value of relaxing the collateral constraint is always positive. This prevents the borrower from accumulating debt indefinitely (until labor income resources have been exhausted). The borrower will then always choose to hold a positive amount of debt. To show this we simply combine the steady-state version of (21), which implies \( R = \frac{\pi_{c}}{\bar{\pi}} \), with (14), obtaining:

\[
\psi = \frac{1 - \frac{\beta}{\gamma}}{1 - (1 - \xi)\beta} > 0
\]

(36)

Notice that, to insure a well-defined steady state, both heterogeneity in patience rates and a borrowing limit are required. In fact, if discount rates were equal, the steady-state level of debt would be indeterminate (Becker (1980), Becker and Foias (1987)). In this case, in fact, it would hold \( \beta \left( \frac{1}{\gamma} \right) = \beta RR = 1 \), where \( RR \) is the steady state real interest rate, and the economy would
display a well-known problem of dependence of the steady state on the initial conditions. With different discount rates, and yet still free borrowing, the consumption path of the borrower would be tilted downward, and the ratio of consumption to income would asymptotically shrink to zero. Hence a binding collateral constraint allows a constant consumption path to be compatible with heterogeneity in discount rates.

We assume that the price elasticity of demand is the same across sectors and that the relative price of housing is normalized to 1. By evaluating (13) in the steady state, and employing the utility specification (7), we obtain the borrower’s ratio between the stock of housing and consumption:

\[
\frac{D}{C} = \frac{\alpha}{1-\alpha} [Z (1 - \beta (1-\delta))]^{-\eta} \equiv \left( \frac{D}{C} \right) \tag{37}
\]

which is decreasing in the effective relative price of housing \(Z \equiv \left( 1 - \frac{(1-\chi)(1-\delta)}{(1-(1-\xi)(1-\delta))} \right)\).

The borrower’s steady-state leverage ratio reads:

\[
\frac{b}{D} = \frac{(1-\chi)\delta}{1-(1-\xi)} \tag{38}
\]

Notice that both a lower down-payment rate \(\chi\) and a lower repayment rate \(\xi\) increase the borrower’s leverage ratio.

To compute the steady-state level of debt we proceed as follows. We set parameter \(v\) in order to pin down a given level of hours worked in the steady state for each group of agents. By combining (6), (10) and (38) we can write:

\[
D = \frac{\omega\mathcal{N}}{\Psi \mu^c} \tag{39}
\]

where \(\Psi \equiv \left( \frac{\bar{b}}{\bar{c}} \right)^{-1} + \delta \left( 1 + \frac{(1-\gamma)(1-\chi)}{\eta \xi} \right)\), \(\mu^c \equiv \frac{\bar{c}^c}{\bar{c}^{\bar{c}^c}}\), and \(\mathcal{N}\) is aggregate labor supplied by the borrowers as a group (and assumed to be equal to the amount of labor collectively supplied by the savers, see the Appendix for more details).

Once obtained \(D\) from (39), using (38), one can solve for the unique steady-state level of borrower’s debt

\[
b = \frac{(1-\chi)\delta \omega\mathcal{N}}{(1-(1-\xi)) \mu^c \Psi} \equiv \bar{b} \tag{40}
\]

\[21\] In other words, under \(\beta = \gamma\), the economy would constantly replicate the initial (arbitrary) distribution of wealth forever.

\[22\] In this case the assumption \(\beta < \gamma\) is equivalent to \(\beta RR < 1\). In the absence of exogenous growth, this implies that the (gross) growth rate of consumption (\(\beta RR\)) is below the (gross) growth rate of income (which is 1). Hence, the ratio of consumption to output must shrink over time.
One can show that, under the assumption $\beta < \gamma$, the steady-state level of debt $\bar{b}$ is stable, i.e., the economy will converge to $\bar{b}$ starting from any initial value different from $\bar{b}$.

6 The Channels of Monetary Policy Transmission

In this environment the transmission of monetary policy shocks works primarily via three channels: (i) a nominal-debt channel, stemming from private debt being non-indexed and predetermined in nominal terms; (ii) a collateral-constraint channel, working via fluctuations in the shadow value of borrowing; and (iii) an asset-price channel, stemming from real house prices affecting the collateral value. It is important to emphasize that, conditional on monetary policy shocks, channel (i) and (ii) work independently of the presence of nominal price rigidity, although the strength of those channels can be affected by the degree of price stickiness.

Nominal Debt Channel With private debt being predetermined in nominal terms, fluctuations in current (non-durable) inflation affect the real ex-post cost of debt service. This is clear from the borrower’s budget constraint (6). This effect is akin to an income effect. For instance, a policy tightening, by rising the real cost of debt service, will induce the borrower to decrease spending in both consumption and housing.

Collateral-Constraint Channel Equilibrium fluctuations in the shadow value of borrowing $\psi_t$ are key to the transmission of policy shocks on consumption. To clarify this, notice that, because of durability, the term $\sum_{j=0}^{\infty} [\beta(1-\delta)]^j U_{d,t+j}$ in equation (15) can be thought of as being roughly constant. In fact, suppose $\delta$ were equal to 1 (i.e., no durability). In this case, variations in the shadow value of housing would be driven entirely by the current marginal utility of housing services. For values of $\delta$ sufficiently below 1, though, variations in the marginal utility of housing services in the distant future matter substantially for the current shadow value.23 This argument is particularly relevant in our environment, given the extremely low rate of physical depreciation of housing.

The above consideration allows to rewrite (15) as:

$$U_{c,t}q_t \simeq \text{const.} + (1-\chi)E_t \left\{ \sum_{j=0}^{\infty} [\beta(1-\xi)]^j q_{t+j} \Delta_{t+j} \right\}$$

(41)

Unlike a standard NK framework with free borrowing and lending, variations in the present discounted value of the Euler gap are the specific feature characterizing the monetary transmission

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23 See also Barsky et al. (2006).
under a collateral constraint. Consider a monetary policy contraction, in the form of an interest rate hike. This induces a tightening of the collateral constraint via two channels: first, and regardless of price stickiness, via an effect of debt inflation (see above); second, but only in the presence of price stickiness, via a rise in the real interest rate. Formally, as a result, \( \psi_t \) must rise, for the shadow value of relaxing the constraint is higher in the presence of a heightened service cost of debt. In this respect, \( \psi_t \) bears the genuine interpretation of an asset price. From (14), in fact, a rise in the shadow value \( \psi_t \) incorporates positive current and expected future variations in the Euler gap. Yet, in equation (41), a rise in the right-hand side implies that, for any given relative price \( q_t \), the marginal utility of consumption \( U_{c,t} \) must rise. Hence, in turn, consumption must fall.

The above interpretation clarifies the role of the institutional parameters \( \chi \) and \( \xi \). For the borrower, the policy contraction amounts to a negative shock to real income. A rise in the shadow value \( \psi_t \) signals exactly this effect. Recall that the borrower behaves in the opposite way to a standard permanent-income consumer. In fact, the borrower would like to decrease (increase) borrowing in light of a negative (positive) income shock (whereas the permanent-income consumer would instead obey to consumption-smoothing). A lower (higher) down-payment rate \( \chi \) and/or a lower repayment rate \( \xi \), both representative of a "more (less) flexible" mortgage market, entail that a larger (smaller) variation in consumption is needed to satisfy (41) for any given variation in \( \psi_t \) (i.e., for any given impact on the tightness of the collateral constraint). Intuitively, in times of negative (positive) shocks to real income, a more flexible mortgage market allows to decrease (increase) borrowing more rapidly, with this effect translating proportionally into a variation in consumption.

**Asset-Price Channel** Finally, movements in real house prices \( q_t \) also affect the transmission of monetary policy shocks, by affecting the value of the housing stock that can be used as a collateral. Fluctuations in that value affect the tightness of the collateral constraint. In our two-sector model, however, this effect is operative only in the case of asymmetric price stickiness. With prices flexible or equally sticky in both sectors, in fact, real house prices would remain unchanged in response to a monetary policy shock. Under our baseline assumption that house prices are flexible and non-durable prices sticky, however, a policy tightening will induce a fall in real house prices, thereby inducing (all else equal) a depreciation of the collateral value and a further tightening of the collateral constraint. In turn, this will induce a fall in the demand for borrowing, and therefore a fall in the demand for housing, which will further depress its relative price, all in a self-reinforcing fashion.

In this respect, the asset-price channel works by strengthening the impact of the collateral-constraint channel. In equation (41), in fact, a fall in \( q_t \) requires an even larger increase in the
marginal utility of consumption in order to match any given variation of the tightness of the collateral constraint represented by the right-hand side of (41).

7 Sensitivity to Policy Shocks and Institutional Factors

In this section we evaluate the transmission of monetary policy shocks. We begin by illustrating how the role of borrowers and of a collateral constraint affect the equilibrium dynamics relative to a baseline NK model. We then analyze how the transmission of monetary policy shocks is affected by three key institutional features:

- down-payment rate \( \chi \)
- repayment rate \( \xi \)
- mortgage structure (fixed vs. variable debt contract)

7.1 Calibration

We resort to the following calibration. Time is in quarters. We set the quarterly discount factor \( \gamma = 0.99 > \beta = 0.96 \). This value is in the range between values respectively chosen by Krusell and Smith (1998) and estimated by Iacoviello (2005). The annual real interest rate is pinned down by the saver’s patience rate and is equal to 4%. The annual physical depreciation rate for housing is generally low, and around 1% per year. Therefore we set \( \delta = 0.01/4 \) as a baseline value. The elasticity of substitution between varieties is 7.5, which yields a steady-state mark-up of 15%. We assume throughout that house prices are flexible, while we set the stickiness parameter for consumer prices equal to a benchmark value of \( \vartheta_c = 75 \). To pin down this value we proceed as follows. Let \( \theta \) be the probability of not resetting prices in the standard Calvo-Yun model. We parameterize \( \frac{1}{1-\theta} = 4 \), which implies \( \theta = 0.75 \), and therefore an average frequency of price adjustment of one year. This value is roughly in line with the micro-based evidence for European countries summarized in Alvarez et al. (2006) and Angeloni et al. (2006). Log-linearization of (26) around a zero-inflation steady state (in the consumption sector) yields a slope of the Phillips curve equal to \( \frac{(\varepsilon_c - 1)}{\vartheta_c} \), whereas the slope of the Phillips curve in the Calvo-Yun model reads \( \frac{(1-\theta)(1-\beta\theta)}{\theta} \). Setting the elasticity \( \varepsilon_c \) equal to 7.5, which implies a steady-state markup of 15 percent, the resulting stickiness parameter satisfies \( \vartheta_c = \frac{\theta(\varepsilon_c - 1)}{(1-\theta)(1-\beta\theta)} = 75 \).

\footnote{Our results do not hinge critically on the assumed relative degree of stickiness between house and consumption prices. See Monacelli (2006) for an analysis on this point. At the same time, the assumption that house prices are more flexible than consumption prices seems reasonable. For one, house prices tend to incorporate an asset-price behavior. In addition, as argued in Barsky et al. (2006), house prices, unlike consumption prices, are largely subject to negotiation upon transactions. Even the common perception that house prices are sticky downward is probably misguided.}
The current share of housing and housing-related expenditure is about 10% on average in the euro area. However, by adding owner-occupied housing that number would increase to 17.5%. Since we do not have rents in the model, we calibrate the share $\alpha$ in order to match the expenditure for owner-occupied housing. The latter value is estimated as being 7.5% in the euro area and 24% in the U.S., although statistical methodologies differ substantially. We choose to pick an intermediate value of $\alpha = 16\%$.

The down-payment rate is set at $\chi = 0.3$ in the baseline calibration, a value which is close to the euro area average, corresponding to a LTV ratio of about 0.7 (see Table 1). Below, however, we experiment with alternative values of this parameter.

As to the repayment rate $\xi$, in the baseline scenario we set $\xi = \delta$, and interpret this case as the one of full mortgage refinancing. Alternatively we link the quarterly repayment rate to the average duration of the loan. Table 1 shows that, within the European countries, the average duration ranges between 15 and 30 years. A duration of 30 years is also the typical one in the U.S.. In the table below we summarize how the value of $\xi$ changes depending on the specified loan duration:\footnote{All our results do not hinge on these assumptions in any significant way.}

<table>
<thead>
<tr>
<th>Mortgage duration</th>
<th>Quarterly repayment rate $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 yrs</td>
<td>0.0083</td>
</tr>
<tr>
<td>20 yrs</td>
<td>0.0125</td>
</tr>
<tr>
<td>15 yrs</td>
<td>0.0166</td>
</tr>
<tr>
<td>10 yrs</td>
<td>0.025</td>
</tr>
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</table>

Throughout we assume that (i) durable prices are flexible; (ii) the elasticity of substitution $\eta$ equals 1 (which implies Cobb-Douglas preferences in consumption and housing services); (iii) the monetary policy rule features a reaction to consumption price inflation.\footnote{For instance, the quarterly repayment rate for a 30-year loan is computed as $\frac{100}{120} = 0.83\%$.} We assume that the monetary policy innovation is a purely i.i.d. shock to the policy rule (35). The temporary nature of the shock helps to highlight how the transmission mechanism built in the model contributes to generate an effect of endogenous persistence in response to policy impulses.

### 7.2 The Role of Borrowers: Amplification and Persistence

We begin by describing the general features of the monetary transmission in our setup. Figure 5 depicts the effect on selected per capita variables of a 25 basis points rise in the nominal (policy) interest rate. Solid lines and dashed lines denote respectively the borrower’s and the saver’s choice variables. In order to isolate the role of down-payment we assume full mortgage refinancing, i.e., $\xi = \delta$, and a variable interest-rate mortgage structure.
In this exercise, we set the share of borrowers to a baseline value of \( \omega = 0.4 \), and defer the discussion on the effects of choosing alternative values of \( \omega \). Notice, first, that the monetary policy tightening induces a rise in the shadow value of borrowing \( \psi_t \). This signals a rise in current and expected future values of the Euler gap (see equation (41)), which in turn induces a contractionary effect on borrower’s consumption (collateral-constraint effect). Since house prices are flexible (and consumption prices sticky), the policy tightening induces also a fall in the real house price \( q_t \), which in turn reduces directly the collateral value, further contributing to a tightening of the borrowing conditions (asset-price effect). As a result, real debt falls, the demand for housing services drops on impact and then starts to gradually revert back towards the steady state.

To better understand why, despite prices being flexible in that sector, the demand for housing services falls, it is useful to notice that a policy tightening increases the user cost of housing. The relevant user cost for housing can be written, from (13), as:

\[
usc_t = Z_t - \beta(1-\delta)E_t \left\{ \frac{U_{c,t+1}}{U_{c,c}}Z_{t+1} \right\}
\]  

(42)

The user cost depends positively on the current effective relative price of housing and inversely on the future price. (Intuitively, expected capital gains on the holding of housing decrease the current user cost.) In turn, under a collateral constraint, the effective price of housing \( Z_t \) depends on the shadow value of borrowing \( \psi_t \). Hence the figure makes clear that fluctuations in the shadow value of borrowing (and therefore in the Euler gap) overwhelmingly drive the user cost. As a result, a policy tightening induces a rise in the user cost and a fall in the relative demand for housing services.

The figure shows also the response of consumption by a typical saver (dashed lines). Recall that the savers are standard permanent-income agents. Two competing effects drive their demand. For one, a positive income shock, which is the counterpart of the negative income shock for the borrowers. This effect leads the savers to increase both consumption and housing services. However, the rise in the real interest rate makes them substitute consumption intertemporally, so that, on balance, savers’ consumption is less responsive than borrowers’ consumption. At the same time, since the relative price of durables falls, the savers increase their demand for housing services. For these agents, in fact, the relevant user cost of housing is the one prevailing in the absence of any collateral constraint, and therefore it depends heavily on the behavior of the relative price \( q_t \) (and not on \( \psi_t \)).

Figure 6 illustrates the effect of varying the share of borrowers (impatient agents) on the response of aggregate consumption. We define aggregate consumption as:

\[
C_t \equiv \omega C_t + (1-\omega)\bar{C}_t
\]  

(43)
A tight calibration of the share of agents holding debt is particularly difficult, given the wide heterogeneity across countries and the within-country dispersion across income groups. Figure 7 shows that the share of households holding debt varies greatly across OECD countries, ranging from 10% and 20% in Italy and Germany to about 50% in the U.S. and the Netherlands. Noticeably, this pattern replicates the country-clustering identified in our empirical analysis, with the share of indebted household being higher in those countries featuring more developed/flexible mortgage markets (i.e., U.S., U.K. and European nordic countries). Generally, the share of indebted households rises with income. In the U.S., U.K., Canada, Finland, New Zealand and Sweden, the proportion of indebted households in the top income decile is about 80%, whereas in Germany is about 20% and in Italy is 30%.

One may argue that not all households holding debt are necessarily constrained. However, a comparison with a recent literature calibrating the numerical value of the share of so-called "rule-of-thumb" (ROT) consumers is instructive. ROT consumers are an extreme form of credit constrained agents who literally do not have access to financial markets. Mankiw (2000), Galf et al. (2006) calibrate this share as high as 50% for the U.S.. Forni et al. (2006) estimate this share to be about 40% for the Euro Area. The type of constraint the impatient agents are subject to in our framework is certainly less extreme. For one, the amount that these agents can borrow is endogenous to the value of the collateral. Furthermore, while it is true that not all households holding debt are constrained, it is definitely true that for most households acquiring a mortgage requires providing their house as a collateral. Since mortgage debt is the lion share of private debt in most industrialized countries, we believe that a fraction of impatient agents ranging between 30% and 50% to be a conservative estimate.

Consider now the effects on the response of consumption of alternative values of \( \omega \). With \( \omega = 0 \), and given the purely temporary nature of the shock, consumption falls and immediately reverts back to steady state. In this case, the response coincides with the one of savers’ consumption, which mimics the one of a representative-agent in a two-sector NK model, and is entirely driven by a standard intertemporal substitution effect. Notice that increasing the share of borrowers generates both an effect of amplification and persistence. On impact, in fact, consumption is more responsive for higher values of \( \omega \). Intuitively, for \( \omega > 0 \), all the additional channels of monetary transmission which are typical of our framework (nominal-debt, collateral-constraint and asset-price channel), and which affect only borrowers’ consumption, are now set in motion. In addition, though, the model generates an effect of endogenous persistence, in that consumption reverts back to steady state only after several periods beyond the life of the shock, with this effect being magnified for higher values of \( \omega \).

This effect of persistence depends on the form of the collateral constraint, namely on the ability
of borrowing being linked to an asset with high durability. In turn, the persistence depends on the value of \( \delta \), and tends to vanish for values of \( \delta \to 1 \) (not shown). Intuitively, when real income falls (rises) because of an increase (decrease) in real interest rates, the borrower optimally wishes to decrease (increase) real debt. But this requires depleting (increasing) the stock of housing. Since housing durability implies that the flow-stock ratio is low, it takes time to change the stock of housing, and therefore the demand for debt changes only gradually over time. In turn, this is reflected in a gradual effect on consumption spending.

### 7.3 Varying the Down-Payment Rate

*Figure 8* depicts the effect on selected variables of varying the *down-payment rate* \( \chi \). We continue to assume full mortgage refinancing, i.e., \( \xi = \delta \), and a variable interest-rate mortgage structure. We consider two variants to the baseline calibration: (i) a low down-payment rate \( \chi = 0.15 \), similar to the level prevailing, e.g., in Spain, and (ii) a high down-payment rate \( \chi = 0.5 \), close to the situation, e.g., in Italy (see *Table 1*). Most of the countries in our sample are comprised within this range for \( \chi \).

A smaller down-payment rate \( \chi \) leads to a more pronounced impact effect of the monetary policy shock on (borrower’s) consumption, real debt and the relative price of durables \( q \). As suggested above, the monetary tightening amounts to a negative shock to real income. In light of that, the borrower would like to decrease borrowing and therefore consumption. A lower down-payment \( \chi \) increases the effective rate at which the impatient agent can contract borrowing between any two periods in time. A more rapid contraction of borrowing leads to a more rapid contraction of both housing services and consumption. In addition, a lower down-payment rate increases, all else equal, the sensitivity of borrowing to changes in the value of the collateral, leading to a magnification of both the nominal debt channel and the collateral-constraint channel.

*Figure 9* illustrates how alternative values of \( \chi \) affect the response of aggregate consumption. In this simulation we set the share of borrowers to the baseline value \( \omega = 0.4 \). Notice that, in the limit case of \( \omega = 0 \), changing the down-payment rate would exert no effect on the response of consumption to the shock. Overall, we observe that the model exhibits aggregation properties in line with our empirical evidence. Aggregate consumption falls in response to the shock, with the impact response of consumption being magnified for lower values of the down-payment rate \( \chi \).

### 7.4 Shutting Down Stickiness: Decomposing the Channels

Next we evaluate the role of price stickiness. We compare the response of aggregate consumption under three scenarios: (i) flexible consumer prices; (ii) low stickiness and (iii) baseline stickiness. The first scenario, coupled with our maintained assumption that house prices are flexible, entails
that prices are fully flexible in both sectors. In the second scenario, the frequency of price adjustment is less than one quarter, in line with the empirical micro-based evidence of Bils and Klenow (2004) for the U.S.. In the third scenario, the frequency of price adjustment is at our baseline value of four quarters, which is considered realistic for the euro area countries based on the micro-based evidence discussed in Angeloni et al. (2006). Notice that in the flexible-price scenario the asset price channel is neutralized, since the relative price of housing is constant in response to a monetary policy variation, and hence does not affect the value of the collateral. As already argued above, though, abstracting from price stickiness in consumption prices alters also the strength of both the nominal-debt channel and of the collateral constraint channel.

Figure 10 depicts the effects on aggregate consumption of a 25 basis points i.i.d. increase in the nominal interest rate under alternative degrees of consumer price stickiness. Moving from the baseline case of four-quarter stickiness to the one of fully flexible prices substantially reduces the effect on consumption. On the other hand, though, the experiment shows that price stickiness is not a strictly necessary ingredient to the transmission mechanism of monetary policy shocks. Overall, under flexible prices and conditional on our parameterization, a 1% rise in the policy rate reduces aggregate consumption on impact by about 0.2%. Thus, the residual impact on consumption under flexible prices is still non-negligible and is due to the combination of the nominal-debt effect and of the collateral-constraint effect.

7.5 Varying the Repayment Rate

Figure 11 depicts the response of aggregate consumption to a temporary (i.i.d.) 25 basis-point rise in the nominal policy rate under alternative values of the repayment rate $\xi$. We do not report per-capita responses of selected variables because the picture is qualitatively similar to the one obtained above under alternative values for $\chi$.

The values chosen for $\xi$ are the ones reported earlier (see section 7.1), which correspond to alternative durations of the underlying mortgage contract. The baseline case, labelled full refinancing, corresponds to $\xi = \delta$. We think of this as a limit case in which continuous mortgage refinancing allows to make the rate of housing "economic" depreciation coincide with the physical rate of depreciation. Hence, implicitly, values of $\xi$ higher than $\delta$ can be thought of as capturing a reduced ability to refinance the mortgage. Notice that the effect of varying the repayment rate is qualitatively similar to the one of changing the down-payment rate, i.e., the peak response of consumption is magnified by lowering $\xi$. In fact, a lower $\xi$ rises the effective rate $(1 - \chi)(1 - \xi)^j$ at which the impatient agent can expand borrowing in any future period $t + j$. The latter point explains also why varying the repayment rate $\xi$ affects not only the impact response of consumption, but also its persistence, with a lower $\xi$ generating a more persistent decline of consumption below
baseline.

### 7.6 Varying the Interest-Rate Mortgage Structure

Figure 12 displays the effect of varying the interest-rate mortgage structure (which, in practice, corresponds to the degree of interest rate pass-through). We analyze three cases. The first case considers a debt structure in which the mortgage rate is freely linked to the short-term policy rate \( R^m_t = R_t \) for all \( t \), or alternatively \( \tau = 0 \) in equation (8)). The second case considers an intermediate possibility in which the mortgage interest rate is linked to a return on a ten-year bond \( m = 40 \), see equation (8)). The third case is a limit case of fixed-rate mortgage structure. This is approximated by considering the variant of the term structure equation (8) for \( \tau \rightarrow 1 \), with maturity \( m \) extending to a 30-year period.

A fixed-rate mortgage structure significantly dampens the dynamic effect on consumption relative to a case of flexible-rate structure. In particular, under fixed rates the impact effect on consumption is roughly half the one under variable rate mortgages. Notice, however, that a fixed-rate structure does not necessarily imply that consumption is unresponsive on impact. In this case, a policy tightening is still generating both a nominal-debt and a collateral-constraint effect (via a fall in the relative price of durables, which in turn depresses borrowing capability). With real house prices returning back to baseline, then, the effect on consumption is quickly reversed in the case of a fixed-rate mortgage structure, whereas it continues to persist under a variable rate structure.\(^{27}\)

### 8 Conclusions

We have studied the role of institutional characteristics of mortgage markets for the transmission of monetary policy on house prices and consumption in a sample of industrialized countries. We have provided evidence in support of three facts: first, there is significant divergence in the structure of mortgage markets across the main industrialized countries; second, at the business cycle frequency, the correlation between consumption and house prices increases with the degree of flexibility/development of mortgage markets; third, the transmission of monetary policy shocks on consumption and house prices is stronger in countries with more flexible/developed mortgage markets.

We have then built a DSGE model of the monetary transmission with three non-standard features: (i) two sectors; (ii) heterogeneity in patience rates; (iii) a collateral constraint on borrowing.\(^{27}\)

\(^{27}\)In our simulations, we found occasionally the model to be more easily prone to indeterminacy for values of \( \tau < 1 \). Adding a small degree of interest rate smoothing \( \phi_r > 0 \) solved the problem. This explains why the path of impulse responses in this case displays a more gradual adjustment.
We have analysed how the response of consumption to monetary policy shocks is affected by alternative values of three important institutional parameters of mortgage markets: (i) the down-payment rate; (ii) the mortgage-repayment rate (a proxy for the possibility of mortgage refinancing); (iii) interest-rate mortgage structure (variable vs. fixed interest rate). Consistent with our empirical evidence, the sensitivity of consumption to monetary policy shocks increases with lower values of the down-payment rate and of the mortgage repayment rate, and is larger under a variable-rate mortgage structure. Thus the model can rationalize the evidence that private consumption is more responsive to monetary impulses in economies with more developed/flexible mortgage markets, somewhat in contrast with the presumption that more developed mortgage (credit) markets should be conducive to more efficient consumption-smoothing.

There are several issues that have remained unexplored in this work and that it would be interesting to pursue in future research. First, providing a full estimation of the model. Second, introducing an endogenous choice by the households between variable and fixed-rate mortgage contracts. Third, studying how the optimal conduct of monetary policy varies according to the characteristics of mortgage markets, and in particular in the context of a currency area (such as the euro area) in which the heterogeneity of mortgage market institutions remains widespread.

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28 Iacoviello and Neri (2006) is an interesting step in this direction.
A Steady State

In this Appendix we provide further details on the derivation of the steady state. Notice first that labor mobility across sectors implies:

\[ W_c = W_d \quad ; \quad \tilde{W}_c = \tilde{W}_d \]  

(44)

Let \( N \) be the aggregate amount of labor supplied collectively by each group, which we assume to be equal across groups. Hence we have:

\[ \omega N = (1 - \omega)\tilde{N} = N \]  

(45)

From (27) and (28), using the fact that \( q = 1 \), we have

\[ \frac{N_c}{N_c} = \frac{N_d}{N_d} = \frac{1 - \omega}{\omega} \]  

(46)

Notice also, from (33) and (34), that:

\[ N = N_c \left( 1 + \frac{\tilde{N}_d}{N_c} \right) \quad \tilde{N} = \tilde{N}_c \left( 1 + \frac{N_d}{N_c} \right) \]  

(47)

Combining (47) and (46), we observe that:

\[ \frac{N}{N} = \frac{N_c}{N_c} = \frac{N_d}{N_d} \]  

(48)

Next, combining the steady-state version of (6) with (27), (46) and (45), we obtain the borrower’s steady-state stock of housing:

\[ D = \omega N \frac{\gamma - 1}{\gamma} \left[ Z \left( 1 - \beta(1 - \delta) \right)^\eta + \delta + \frac{(1 - \gamma)(1 - \lambda)}{\gamma} \right] \]  

(49)

Using (37) and (49), one can then obtain the borrower’s individual consumption \( C \).

In turn, using (23) and (46) one can write:

\[ Y_c = \Omega N_c^\omega N_d^{1-\omega} = \Omega N_c^\omega \left( \frac{\omega}{1 - \omega} N_c \right)^{1-\omega} = \omega N_c \]  

(50)

and similarly

\[ Y_d = \omega N_d \]  

(51)

The saver’s housing/consumption ratio reads
Using equilibrium conditions (30), (31), (33) and (34), as well as (50) and (51), one can obtain the individual saver’s housing stock as

\[
\left( \frac{\bar{D}}{C} \right) = \frac{\alpha}{1 - \alpha} \cdot \left( 1 - \gamma(1 - \delta) \right)^{-\eta} \tag{52}
\]

where \( C, D, \bar{N} \) and \( \bar{C} \) are all given. Finally, using (52) and (53), one can obtain individual saver’s consumption \( \bar{C} \).
References


<table>
<thead>
<tr>
<th>Country</th>
<th>Mortgage debt to GDP ratio (2004)</th>
<th>Home ownership ratio a</th>
<th>Loan to value ratio b</th>
<th>Interest rate adjustment c</th>
<th>Typical duration (years)</th>
<th>Equity release products</th>
</tr>
</thead>
<tbody>
<tr>
<td>BE</td>
<td>31%</td>
<td>72%</td>
<td>80-85%</td>
<td>F(75%), M(19%), V(6%)</td>
<td>20</td>
<td>No</td>
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<td>39%</td>
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<td>≤30</td>
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<td>70-80%</td>
<td>F(5%), M(15%), V(80%)</td>
<td>15-20</td>
<td>Very limited use</td>
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<td>ES</td>
<td>46%</td>
<td>85%</td>
<td>≈80%</td>
<td>V(≥75%), Rest mainly M</td>
<td>15-25</td>
<td>Very limited use</td>
</tr>
<tr>
<td>FR</td>
<td>26%</td>
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<td>F/M/Other(86%), V(14%)</td>
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<td>60-70%</td>
<td>V(70%), Rest mostly M</td>
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<td>15%</td>
<td>69%</td>
<td>50%</td>
<td>F(28%), Rest mainly M</td>
<td>10-25</td>
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<tr>
<td>LU</td>
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<td>NL</td>
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<td>AT</td>
<td>20%</td>
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<td>60%</td>
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<td>20-30</td>
<td>N.A.</td>
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<td>PT</td>
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<td>25-30</td>
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<tr>
<td>FI</td>
<td>38%</td>
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<td>75-80%</td>
<td>F(2%), V(97%), Other(1%)</td>
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<td>AU</td>
<td>74%</td>
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<td>CA</td>
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<td>UK</td>
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<td>69%</td>
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<tr>
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<td>61%</td>
<td>80%</td>
<td>F(36%), M and V(64%)</td>
<td>25-30</td>
<td>Limited use</td>
</tr>
</tbody>
</table>

Notes: a Share of owner-occupied dwelling.

b Estimated average loan-to-value ratio on new mortgage loans.

c Breakdown of new loans by type. Fixed (F): Interest rate fixed for more than five years or until expiry; Mixed (M): Interest rate fixed between one and five years; Variable (V): Interest rate renegotiable after one year or tied to market rates or adjustable at the discretion of the lender.

<table>
<thead>
<tr>
<th>Country</th>
<th>Source and definition</th>
<th>Availability</th>
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<tbody>
<tr>
<td>Spain</td>
<td>Banco de España and Bank of England: Residential property price per square meter, whole country</td>
<td>Annual data from 1980 to 1986 Quarterly data from 1987 Q1</td>
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<tr>
<td>France</td>
<td>Ministry of Equipment/ECLN and Bank of England: Residential property prices, new flats; good &amp; poor condition; whole country</td>
<td>Annual data from 1980 to 1984 Quarterly data from 1985 Q1</td>
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<td>Italy</td>
<td>Banca d'Italia: Residential property prices, new dwellings; good &amp; poor condition; whole country</td>
<td>Semiannual data from 1965 H1</td>
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<tr>
<td>The Netherlands</td>
<td>DNB: Residential property prices, existing dwellings; good &amp; poor condition; whole country</td>
<td>Monthly data from January 1976</td>
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<tr>
<td>Austria</td>
<td>ECB: Residential property prices, new and existing dwellings; good &amp; poor condition; whole country</td>
<td>Quarterly data from 1986 Q3</td>
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<td>Belgium</td>
<td>STADIM: Residential property prices, existing dwellings; good &amp; poor condition; whole country</td>
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<td>Denmark</td>
<td>NSI: New and existing one-family houses; whole country</td>
<td>Quarterly data from 1971 Q1</td>
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<td>Canada</td>
<td>BIS: residential property prices, existing dwellings, national average</td>
<td>Monthly data from January 1980</td>
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<td>United Kingdom</td>
<td>ONS: Residential property prices, new and existing dwellings; good &amp; poor condition; whole country</td>
<td>Quarterly data from 1968 Q2</td>
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<td>United States</td>
<td>BIS: residential property prices, existing single-family homes, per dwelling</td>
<td>Quarterly data from 1975 Q1</td>
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</tbody>
</table>

Note: Lower-frequency data have been converted to quarterly frequency by linear interpolation.
### TABLE 3. Correlation between Real House Prices and Consumption

<table>
<thead>
<tr>
<th>Country</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>United Kingdom</td>
<td>0.79</td>
</tr>
<tr>
<td>Spain</td>
<td>0.66</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.57</td>
</tr>
<tr>
<td>Canada</td>
<td>0.52</td>
</tr>
<tr>
<td>United States</td>
<td>0.52</td>
</tr>
<tr>
<td>France</td>
<td>0.45</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.40</td>
</tr>
<tr>
<td>Austria</td>
<td>0.23</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.15</td>
</tr>
<tr>
<td>Germany</td>
<td>0.12</td>
</tr>
<tr>
<td>Italy</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Notes: Quarterly data from 1980:1 to 2004:4. The real house price is deflated using the CPI. Consumption corresponds to total private consumption. Data are de-trended using the HP1600 filter.

### TABLE 4. Correlation between Real House Prices and Consumption

<table>
<thead>
<tr>
<th>Institutional feature</th>
<th>Average Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortgage refinancing</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>0.31</td>
</tr>
<tr>
<td>Yes</td>
<td>0.57</td>
</tr>
<tr>
<td>Interest rate structure</td>
<td></td>
</tr>
<tr>
<td>Fixed interest rate</td>
<td>0.37</td>
</tr>
<tr>
<td>Variable interest rate</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Notes: Quarterly data from 1980:1 to 2004:4. The real house price is deflated using the CPI. Consumption corresponds to total private consumption. Data are de-trended using the HP1600 filter. Countries where mortgage refinancing is practiced are the US, UK, the Netherlands and Denmark; it is not practiced in Canada, France, Germany, Spain, Italy, Austria and Belgium. Countries with predominantly variable rate mortgages are the UK, Spain and Italy; fixed rate mortgages are more common in the remaining countries.
### TABLE 5. Cross-country Average Absolute Response of Consumption to a Contractionary Monetary Policy Shock of 100 basis points

<table>
<thead>
<tr>
<th>Average response of consumption</th>
<th>Average response of the real house price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed interest rate</td>
<td>0.19</td>
</tr>
<tr>
<td>Variable interest rate</td>
<td>0.42</td>
</tr>
<tr>
<td>Mortgage refinancing</td>
<td>0.56</td>
</tr>
<tr>
<td>No mortgage refinancing</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed interest rate</td>
<td>0.64</td>
</tr>
<tr>
<td>Variable interest rate</td>
<td>1.61</td>
</tr>
<tr>
<td>Mortgage refinancing</td>
<td>1.82</td>
</tr>
<tr>
<td>No mortgage refinancing</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Note: Results are based on the VAR model estimated on quarterly data over the sample period 1980:1 to 2004:4. See text for further explanations. Countries where mortgage refinancing is practiced are the US, UK, the Netherlands and Denmark; it is not practiced in Canada, France, Germany, Spain, Italy, Austria and Belgium. Countries with predominantly variable rate mortgages are the UK, Spain and Italy; fixed rate mortgages are more common in the remaining countries.
FIGURE 1a. Correlation between Private Consumption and Real House Price and Mortgage-to-GDP Ratio

Sample period: Quarterly data from 1980:1 to 2004:4. The real house price is deflated using the CPI. Data are de-trended using the HP1600 filter.

FIGURE 1b. Correlation between Private Consumption and Real House Price and MOW Completeness Index

Note: the Mercer Oliver Wyman index is only available for EU countries.
FIGURE 1c. Correlation between Private Consumption and Real House Price and LTV Ratio

Sample period: Quarterly data from 1980:1 to 2004:4. The real house price is deflated using the CPI. Data are de-trended using the HP1600 filter.
FIGURE 2. VAR Impulse Responses to a 100 b.p. Shock to the Nominal Interest Rate (with 90% confidence bands)

Consumption

House prices

Canada

US

Netherlands

Spain
FIGURE 2 (continued).

Germany

France

Italy

United Kingdom
FIGURE 2. (continued)

Note: Results are based on the VAR model estimated on quarterly data over the sample period 1980:1 to 2004:4. See text for further explanations.
FIGURE 3. VAR Peak Responses of Total Private Consumption to a Contractionary Monetary Policy Shock and Indicators of Development and Flexibility of Mortgage Markets

\[ R^2 = 0.44 \]

\[ R^2 = 0.27 \]
Note: Results are based on the VAR model estimated on quarterly data over the sample period 1980:1 to 2004:4. See text for further explanations. The Mercer Oliver Wyman index is only available for EU countries.
FIGURE 4. VAR Peak Responses of the Real House Price to a Contractionary Monetary Policy Shock and Indicators of Development and Flexibility of Mortgage Markets

\[ R^2 = 0.44 \]

\[ R^2 = 0.56 \]
Note: Results are based on the VAR model estimated on quarterly data over the sample period 1980:1 to 2004:4. See text for further explanations. The Mercer Oliver Wyman index is only available for EU countries.
FIGURE 5. Model Impulse Responses to a Monetary Policy Tightening (i.i.d. shock)
($\omega = 0.4; \chi = 0.3; \xi = \delta$)
FIGURE 6. Model Impulse Response of Aggregate Consumption to a Monetary Policy Tightening (i.i.d. shock): Effect of Varying the Share of Borrowers $\omega$
FIGURE 7. Proportion of Households Holding Debt
(Source OECD Economic Outlook, November 2006)
FIGURE 8. Model Impulse Responses to a Monetary Policy Tightening (i.i.d. shock): Effect of Varying Down-Payment Rate $\chi$ (solid line $\chi = 15\%$, dashed line $\chi = 50\%$)
FIGURE 9. Model Impulse Response of Aggregate Consumption to a Monetary Policy Tightening (i.i.d. shock): Effect of Varying Down-Payment Rate $\chi$. 

![Graph showing the model impulse response of aggregate consumption to a monetary policy tightening. The graph plots Aggregate Consumption against time (0 to 10). Different lines represent varying down-payment rates: 15%, 20%, 30%, 40%, and 50%. The graph illustrates how each down-payment rate affects the response of aggregate consumption to a monetary policy tightening.](image-url)
FIGURE 10. Model Impulse Response of *Aggregate* Consumption to a Monetary Policy Tightening (i.i.d. shock): Effect of Varying Consumption *Price Stickiness*

Note: low-stickiness and baseline stickiness correspond respectively to 2-quarter and 4-quarter frequency of price adjustment in consumer prices.
FIGURE 11. Model Impulse Response of *Aggregate* Consumption to a Monetary Policy Tightening (i.i.d. shock): Effect of Varying Repayment Rate $\xi$
FIGURE 12. Model Impulse Response of Aggregate Consumption to a Monetary Policy Tightening (i.i.d. shock): Effect of Varying the Interest Rate Mortgage Structure