Institutions of Foreign Exchange Settlement
in a Two-Country Model

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Abstract

Fujiki (2003) extends the Freeman (1996) model to show that when combined, an elastic money supply in the foreign exchange market and an elastic money supply in the domestic credit market yield efficiency gains in monetary equilibrium. This paper discusses whether three other institutional designs could achieve the same improvement in efficiency: (1) a private arrangement based on a payment versus a payment settlement standard supported by central banks’ free intraday credit, (2) a financial institution that provides a negotiable certificate of deposit, and (3) a currency union.

Key words: Central Bank, Foreign Exchange Market, CLS, Payment Versus Payment, Eurodollar, Currency Union.

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1. Introduction
Fujiki (2003) demonstrates the possibility of efficiency gains in monetary equilibrium from the combination of an elastic money supply in the foreign exchange market, to clear the exchange of fiat monies at gold standard parity, and an elastic money supply in the domestic credit market. This paper employs the Fujiki (2003) model to analyze three other institutional designs that seem to achieve the same improvement in efficiency: (1) a private arrangement based on a payment versus payment settlement standard supported by central banks’ free intraday credit, (2) a financial institution that provides a negotiable certificate of deposit, and (3) a currency union.

Since the model here is an extension of the Freeman (1996) model, this paper begins by reviewing Freeman’s model. Freeman considers an economy where agents are spatially separated, and private debt incurred between two parties can only be redeemed with fiat currency in a central clearing area. Suppose that the departure rate of creditors from the central clearing area is higher than the arrival rate of debtors. In this case, the amount of currency available at the central clearing area is less than the par value of debt, and late-departing creditors can buy the risk-free assets of early-departing creditors at discounted prices in exchange for fiat money. A central bank can issue additional fiat money to purchase the IOUs of early-departing creditors, and can receive fiat money from the debtors in the central clearing area. Then the IOUs of early-departing creditors can be cleared at par value, and the money stock remains constant as long as this central bank takes the money received from the debtors in the second-hand debt market out of circulation. Freeman (1996) shows that such a central bank intervention leads to an optimal allocation of resources.

Fujiki (2003) considers a two-country version of the Freeman model under a gold standard and assumes that old domestic creditors want to consume young foreign debtors’ goods in their second stage of life with a small probability. Suppose that old creditors know their preference for foreign goods only after their debt is settled in their domestic central clearing area. However, old domestic creditors must pay foreign currency to obtain goods from young foreign debtors. Imagine the “turnpike” of Townsend (1980) that connects the central clearing areas of two countries. Old creditors with taste shocks travel this turnpike, and they meet old creditors coming from the other country at a trading post. They exchange their fiat money for the fiat money of the other country. Suppose that the rates at which old creditors with taste shocks arrive at the trading post are not equal. For example, if one country is inhabited by many bankers (or late-departing creditors), only
a small fraction of old creditors comes to the foreign exchange market in the early stages of the market transaction. Then, the currency of a country with a large banking sector might be in short supply, compared with the gold standard parity. In such a situation, the fiat money of the other country, though its value is backed by gold, might be exchanged at a discount. This could happen even though the central banks intervene to clear all domestic debt at par value.

Fujiki (2003) shows that a central bank intervention in the foreign exchange market can prevent the market exchange rate departing from the gold standard parity, and can improve the ex-ante expected utility of agents. However, such an operation could increase the overall average utility of one country's creditors at the expense of the other country's creditors with a taste shock. To avoid the need for such an operation, it was shown that the central bank should be subject to a gold standard or price level target.

This paper considers three other institutional mechanisms that seem to replicate the welfare improvement suggested by Fujiki (2003). The first is a foreign exchange market that ensures that a final transfer of one currency occurs if and only if a final transfer of the other currency or currencies takes place (i.e., a so-called “payment versus payment standard”, hereinafter, PVP), supported by the central banks’ credit in the foreign exchange market. The second is the provision of a negotiable certificate of deposit that can be withdrawn in terms of foreign fiat money. The third is the introduction of a currency union. Before moving on to the details of the model, let us consider why those institutions are of interest.

First, observe that the overall supplies of fiat monies in the foreign exchange market are known ex ante. Therefore, if early-departing creditors whose currencies are in excess supply could wait until their counterparties are ready to make payment, all creditors could exchange their fiat money at the gold standard parity. However, the segmentation of the foreign exchange market among early-departing creditors and late-departing creditors results in a deviation of the market exchange rate from the gold standard parity. Thus, intraday credit provided by the central banks in the foreign exchange market should be beneficial in preventing fluctuations in the spot foreign exchange rate, since it essentially has the same effect as the central bank intervention proposed by Fujiki (2003). One might think that such an arrangement could be of little practical interest today. However, until only recently, foreign exchange transactions generally exposed market participants to the risk of paying the funds they owed on a trade but not receiving the funds due to them in
return.\textsuperscript{1} Time-zone differences heightened the risk of one party defaulting.\textsuperscript{2} In early 1996, the Committee on Payment and Settlement Systems (CPSS) prepared a report titled “Settlement Risk in Foreign Exchange Transactions” and set out a strategy to reduce foreign exchange settlement risk. One of the achievements of this initiative was the launch of the Continuous Linked Settlement (CLS) Bank in September 2002.\textsuperscript{3} The CLS Bank is a private, intraday, multi-currency, clearing bank that eliminates settlement risk through the world’s first simultaneous and irrevocable global multi-currency settlement system. As of February 2003, on a normal settlement day, the CLS Bank settled, on average, 60,000 payment instructions derived from foreign exchange deals submitted each day by its settlement members, with a gross value of approximately SUS600 billion. Thus, PVP is still an important policy concern.

The second institutional setup shows that the provision of a negotiable certificate of deposit similar to the Eurodollar achieves efficiency gains in monetary equilibrium.\textsuperscript{4} Note that young debtors need fiat money at hand by the end of the first period of their lifetime, since they repay their debts in the next period. Consider a financial institution that provides a negotiable certificate of deposit for old creditors with taste shock, in exchange for their domestic fiat money. This negotiable certificate of deposit can be withdrawn at the end of the period as foreign fiat money. Old creditors with taste shock transfer their deposits to foreign young debtors and obtain foreign young debtors’ goods. Foreign young debtors make withdrawals of foreign fiat money, and repay their debts in the next period. This financial institution has enough fiat money to distribute to the young debtors in exchange for their negotiable certificates of deposit at the gold standard parity, because withdrawals occur only at the end of the period. This financial institution helps to resolve temporary liquidity shortages in the foreign exchange market by changing the timing of transactions.

\textsuperscript{1} This discussion follows http://www.bis.org/cpss/cpssinfo02.htm. The Bank for International Settlements (BIS) provides a forum to facilitate cooperation between central banks on policy issues. The forum includes the Committee on Payment and Settlement Systems (CPSS), which serves as a forum for the central banks of the Group of Ten countries (G10) to monitor and analyze developments in domestic payment, settlement and clearing systems as well as in cross-border and multicurrency settlement schemes.

\textsuperscript{2} A famous example is the closure of Herstatt bank by the Bundesbank in June 1974. The timing of the bank’s closure led to a situation in which many banks that had paid DM to Herstatt bank could not receive US dollars in return.

\textsuperscript{3} For more information on CLS Bank, see http://www.cls-group.com. See also Kahn and Roberds (2001a) for an analysis of CLS Bank based on an economic model.

\textsuperscript{4} Eurodollar is a US dollar that has been put in a European bank or lent to a European customer to help trade and provide an international money system.
Third, thanks to the successful launch of the euro in 1999, policymakers do not have to worry about fluctuations in foreign exchange rates within the euro area, and thus it is tempting to argue that a currency union replaces an elastic money supply in the foreign exchange market, to clear the exchange of fiat monies at gold standard parity, accompanied by an elastic money supply in the domestic credit market. The analysis in this paper shows that the idea of “one money, one market” leads to more efficient resource allocation since the common currency equates the terms of trade faced by early-departing creditors and late-departing creditors with a taste shock. However, the proportion of fiat money circulated in two countries could change over time, even though the total money supply in the two countries remains constant. This could happen if the number of old creditors with a taste shock who bring an economy’s fiat money to the other economy is not the same in the two countries, and thus the price levels in the two countries are not constant. Thus, if the model in this paper is any guide to the welfare implications of a currency union, member countries should reach a consensus on income distribution across generations of creditors and debtors, and the degree of goods-market integration measured by the probability of old creditors' desire to consume goods in the other economy in addition to the level of the nominal exchange rate.

This paper is organized as follows. Sections 2 and 3 restate the model shown in Fujiki (2003) and the stationary equilibrium. Sections 4, 5 and 6 discuss other institutions that can improve the efficiency of equilibrium, and Section 7 concludes.

2. A Two-country model with a liquidity constraint
This section restates a model proposed by Fujiki (2003). There are two types of agent, called creditors and debtors, in the domestic country (hereafter Switzerland) and the foreign country (hereafter Germany). In both countries, creditors and debtors are scattered and live in small villages. Their populations are normalized to one, and their lifetime is divided into two periods. Between Switzerland and Germany, there is Lake Boden (Bodensee). Agents cannot swim, thus they first climb up the central hills in each country and then go through the tunnels connecting the two central hills to get to the other country. Swiss and German creditors and debtors are endowed with goods specific to their villages in their first period of life, in the amounts $y$, $x$, $Y$, and $X$, respectively. Lowercase letters represent Swiss variables and uppercase letters stand for German variables. Creditors consume in both periods of their lives, while debtors only consume in the first
period. Both debtors and creditors in each country travel during their lifetimes as summarized in Figures 1 and 2. The details of their consumption, borrowing, and debt settlements, which are summarized in Table 1, will be explained in the following sections.

Travel by debtors
Swiss debtors born at time \( t \) have a common, additively separable, continuous, and continuously differentiable utility function \( u = u(d_{xt}) + u(d_{yt}) \) that is strictly increasing and concave in each argument, and that has indifference curves that do not cross the axes. The argument \( d_{xt} \) is consumption from the debtor’s own endowment, and \( d_{yt} \) stands for consumption of a Swiss creditor’s endowment. The thick solid arrows labeled (1) in the top panel of Figure 1 show that after consuming \( d_{xt} \), a young Swiss debtor visits a young Swiss creditor’s village. The young Swiss debtor consumes \( d_{yt} \) of the creditor’s good, whose nominal price is \( p_{yt} \) at time \( t \). Since this young Swiss debtor does not have fiat money, he goes into debt with a promise to pay \( h_t = d_{yt}p_{yt} \) francs at time \( t+1 \). Afterwards, the young Swiss debtor comes back to his village (see the thick solid arrows labeled (3) in the top panel of Figure 1). He sells \( x - d_{xt} \) of his endowment at the market price \( p_{xt} \) to an old Swiss creditor or an old German creditor born at time \( t-1 \) who is visiting his village, in exchange for \( m_t \) francs. He will use fiat money to repay his debt at time \( t+1 \). The Swiss debtor born at time \( t \) chooses \( h_t \) to maximize \( u(d_{xt}) + u(d_{yt}) \) subject to \( h_t/p_{yt} = d_{yt} \), \( h_t = m_t \), and \( x - (h_t/p_{xt}) = d_{xt} \). The first order condition is Equation (1),

\[
u_x(x - \frac{h_t}{p_{xt}}) = \frac{p_{xt}}{p_{yt}} u_y(\frac{h_t}{p_{yt}}),
\]

(1)

where \( u_x \) and \( u_y \) are the derivatives of utility function \( u \). At time \( t+1 \), old Swiss debtors, born at time \( t \), climb up the Swiss central hill to repay their debt in francs (the thick solid arrow labeled as (1) in the upper panel of Figure 2). However, only fraction \( \lambda \) of old Swiss debtors (hereafter called early-bird debtors) arrive at the Swiss central hill before the arrival of old Swiss creditors. Old Swiss debtors die after their debt settlement. The trips made by German debtors are mirror images of those of Swiss debtors, as illustrated by the thick dotted arrows in the bottom panel of Figures 1 and 2.
Creditors’ travel and debt settlement

Swiss creditors born at time \( t \) consume \( c_{yt} \) units of their endowment at time \( t \). They exchange what remains of their endowments for the loan certificates issued by young Swiss debtors born at time \( t \), who promise to pay \( l_{i} \) francs at time \( t+1 \). The Swiss creditors’ budget constraints at time \( t \) are \( y_{pt} = c_{yt}p_{yt} + l_{i} \), and \( l_{i} \) depends on their consumption patterns, to be known at time \( t+1 \), as summarized in Table 1.

At time \( t+1 \), all old Swiss creditors visit the central hill to receive francs from the old Swiss debtors (the thin solid arrow labeled (1) in the upper panel of Figure 2). Upon arrival at the Swiss central hill and just before their debt settlement, old Swiss creditors realize their taste regarding the timing of debt settlement. With probability \( (1–\alpha) \), they are “early-departing creditors,” who can wait only for the arrival of the early-bird debtors to the central hill. With probability \( \alpha \), they are “late-departing creditors,” who can wait for the arrival of all old debtors. Therefore, at the central hill, debt settlement between all creditors and early-bird debtors occurs first, and then the remaining debts are cleared after the departure of early-departing old creditors and the arrival of the rest of the old debtors. Specifically, fraction \( 1–\lambda \) of the old Swiss debtors find that their debt is traded in the second-hand debt market, and the other creditors wait to receive their payment from them. Suppose that the number of early-departing creditors exceeds the number of early-bird debtors. Although early-departing creditors will offer to sell their debt to late-departing creditors, the total amount of liquidity, \( \lambda l_{i} \), becomes the upper bound for the nominal value of debt in the secondary debt market. Let the discount rate be \( \rho_{t+1} \) and let \( q_{i} \) be the par value of nominal debt purchased by late-departing Swiss creditors. Then, the nominal value of debt purchased by late-departing creditors, \( \rho_{t+1} q_{i} \), is less than or equal to the value available to creditors from early-bird debtors, \( \lambda l_{i} \). Hence, late-departing Swiss creditors face a domestic liquidity constraint, \( \lambda l_{i} \geq \rho_{t+1} q_{i} \).

Old Swiss creditors face another taste shock after debt settlement, immediately before their departure from the central hill. With probability \( \gamma \), they scatter randomly to young Swiss debtors’ villages (the solid arrow labeled (3) in the upper panel of Figure 2). If they depart early, their budget constraint at time \( t+1 \) is \( \bar{c}_{xt+1}p_{xt+1} = \rho_{t+1}(1–\lambda)l_{i} + \lambda l_{i} \), where \( \bar{c} \) indicates variables for early-

\(^{5}\) The trips made by German creditors born at time \( t \) and their debt settlement at the German central hill are mirror images of the trips to, and debt settlement at, the Swiss central hill (Figure 2, lower panel). Let \( A, \zeta \) and \( Q \) stand for
departing creditors. If they depart late, their budget constraint is \( \hat{\tilde{c}}_{xt+1} P_{xt+1} = (1 - \rho_t) q_t + l_t \), where \(^\wedge\) indicates variables for late-departing creditors, and they face the liquidity constraint \( \lambda_l \geq \rho_t q_t \).

With probability \( 1 - \gamma \), old Swiss creditors find that they want to consume young German debtors’ goods. More specifically, old creditors subject to a taste shock visit the tunnel connecting the two countries (the arrows labeled (4) in Figure 2).

Suppose that there are guardians in the middle of the tunnel, who are honest and can live without any consumption. The guardians receive francs from the old Swiss creditors and marks from the old German creditors, and exchange currencies. This is because, by law, Swiss (German) agents are forced to use marks (francs) for all transactions in Germany (Switzerland) (point (5) in Figure 2). Suppose also that for security reasons the guardians in the tunnel force agents to go in one direction through the tunnel without stopping. Then, it is not possible for early-departing old creditors and late-departing old creditors to trade their currencies with each other. Under those assumptions, the number of marks that early-departing old Swiss creditors with a taste shock can obtain is determined by the number of marks obtained from early-departing old German creditors with a taste shock. Early-departing old creditors with a taste shock face the nominal franc/mark exchange rate \( \tilde{e}_{t+1} \) in the middle of the tunnel at time \( t+1 \):

\[
(1 - \gamma)(1 - \alpha)[\rho_{t+1}(1 - \lambda) q_t + \lambda_{t+1}] = \tilde{e}_{t+1} (1)(1 - \Lambda)[\Xi_{t+1}(1 - \Lambda) L_t + \Lambda_{t+1}]. \quad (2)
\]

Late-departing old creditors with a taste shock face the nominal franc/mark exchange rate \( \hat{\tilde{c}}_{t+1} \) in the middle of the tunnel at time \( t+1 \):

\[
(1 - \gamma)\alpha[(1 - \rho_{t+1}) q_t + l_t] = \hat{\tilde{c}}_{t+1} (1 - \Gamma)[(1 - \Xi_{t+1}) Q_t + L_t]. \quad (3)
\]

After obtaining marks, old Swiss creditors with a taste shock go to the German central hill (see the thin solid arrow labeled (6) in the lower panel of Figure 2). Then they scatter randomly to young German debtors’ villages (see the solid arrow labeled (7) in the lower panel of Figure 2). Old German creditors with a taste shock go to the Swiss central hill, and then scatter randomly to the young Swiss debtors’ villages.

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the population of German late-departing creditors, the population of German early-bird debtors, the German discount rate, and the par value of nominal debt in the German secondary debt market.

6 Similarly, after the debt settlement at the German central hill, with probability \( \Gamma \), old creditors scatter to German debtors’ villages. Early-departing old creditors face the budget constraint \( \tilde{C}_{xt+1} P_{xt+1} = \Xi_{t+1}(1 - \Lambda) L_t + \Lambda_{t+1} \). The budget constraint for late-departing old creditors is \( \tilde{C}_{xt+1} P_{xt+1} = (1 - \Xi_{t+1}) Q_t + L_t \), and they also face the liquidity constraint \( \Lambda_{t+1} \geq \Xi_{t+1} Q_t \). With probability \( 1 - \Gamma \), old German creditors wish to consume young Swiss debtors’ goods.
In summary, suppose that Swiss creditors born at time \( t \) have the expected utility function 
\[
v(c_{yt}) + \gamma(1-\alpha)v(\tilde{c}_{xt+1}) + \gamma\alpha v(\tilde{c}_{xt+1}) + (1-\gamma)(1-\alpha)v(\tilde{c}_{xt+1}) + (1-\gamma)\alpha v(\tilde{c}_{xt+1}).
\]
Using the budget constraint and liquidity constraint at times \( t \) and \( t+1 \) yields the following optimization problem for the young Swiss debtors with respect to \( l_t \) at time \( t \):

\[
\max_{l_t} v(y - \frac{l_t}{p_{yt}}) + \gamma(1-\alpha)v\left(\frac{\rho_{t+1}(1-\lambda)l_t + \lambda l_t}{P_{yt+1}}\right) + \gamma\alpha v\left(\frac{(1-\rho_{t+1})(\lambda l_t/\rho_{t+1}) + l_t}{P_{yt+1}}\right) + (1-\gamma)(1-\alpha)v\left(\frac{\rho_{t+1}(1-\lambda)l_t + \lambda l_t}{P_{yt+1}}\right) + (1-\gamma)\alpha v\left(\frac{(1-\rho_{t+1})(\lambda l_t/\rho_{t+1}) + l_t}{P_{yt+1}}\right).
\]

Therefore, the first-order condition of this problem becomes:

\[
v'(y - \frac{l_t}{p_{yt}}) = \gamma(1-\alpha)v'(\frac{\rho_{t+1}(1-\lambda)l_t + \lambda l_t}{p_{yt+1}})[\rho_{t+1}(1-\lambda) + \lambda] \frac{p_{yt}}{p_{yt+1}} + \gamma\alpha v'(\frac{(1-\rho_{t+1})(\lambda l_t/\rho_{t+1}) + l_t}{p_{yt+1}})[(1-\lambda) + (\lambda/\rho_{t+1})] \frac{p_{yt}}{p_{yt+1}} + (1-\gamma)(1-\alpha)v'(\frac{\rho_{t+1}(1-\lambda)l_t + \lambda l_t}{p_{yt+1}})[\rho_{t+1}(1-\lambda) + \lambda] \frac{p_{yt+1}}{p_{yt+1}} \frac{p_{yt}}{p_{yt+1}} + (1-\gamma)\alpha v'(\frac{(1-\rho_{t+1})(\lambda l_t/\rho_{t+1}) + l_t}{p_{yt+1}})[(1-\lambda) + (\lambda/\rho_{t+1})] \frac{p_{yt+1}}{p_{yt+1}} \frac{p_{yt}}{p_{yt+1}},
\]
where primes indicate first derivatives.

3. Stationary equilibrium and benchmark allocation

3.1 A stationary equilibrium

This section restates a stationary equilibrium shown in Fujiki (2003). The market clearing conditions in the goods markets are equations (6) through (9):

\[
\bar{x} = d_{xt} + \gamma(1-\alpha)\tilde{c}_{xt} + \gamma\alpha \tilde{c}_{xt} + (1-\Gamma)(1-A)\tilde{C}_{xt} + (1-\Gamma)A\tilde{C}_{xt}, \tag{6}
\]
\[
\bar{y} = c_{yt} + d_{yt}, \tag{7}
\]
\[
\bar{X} = D_{xt} + \Gamma(1-A)\tilde{C}_{xt} + \Gamma A\tilde{C}_{xt} + (1-\gamma)(1-\alpha)\tilde{c}_{xt} + (1-\gamma)\alpha \tilde{c}_{xt}, \quad \text{and} \tag{8}
\]
\[
\bar{Y} = C_{yt} + D_{yt}. \tag{9}
\]
The currency markets’ clearing conditions are $\overline{m} = m_t = h_t$ in Switzerland and $\overline{M} = M_t = H_t$ in Germany, where $\overline{m}$ and $\overline{M}$ are money supplies exogenously determined by the central bank in each country based on their gold reserve. The equilibrium conditions in the loan markets are $l_t = h_t$ in Switzerland and $L_t = H_t$ in Germany. The liquidity constraints and the equilibrium of the secondary debt markets will yield $\lambda l_t = p_{t+1} q_t$ and $\alpha q_t = (1-\alpha)(1-\lambda) l_t$ in Switzerland and $\Lambda L_t = \Xi_{t+1} Q_t$ and $AQ_t = (1-A)(1-A) L_t$ in Germany. One can solve equations (1) through (3), (5), (6) through (9), and the equilibrium conditions in the currency market, loan market, and second-hand debt market simultaneously. In particular, consider a stationary equilibrium at which $h = h_t = l_t = l$, $p_x = p_{x+1} = p_s$, $P_X = P_{X+1} = P_X$, $P_Y = P_Y$, and $\Xi_{t+1} = \Xi$ are satisfied.

Assume that $(1-\alpha-\lambda) > 0$ and $(1-A-A) > 0$ throughout the following analysis, which makes the equilibrium discount rates less than one. Then, the equilibrium conditions of the second-hand debt market lead to $\rho = (\alpha(1-\alpha)(1-\lambda) < 1$ and $\Xi = (A/A)(1-A) < 1$. Those results simplify the budget constraints for old Swiss and German creditors without a taste shock to $\overline{c}_{st+1} = [\overline{l}/(1-\alpha)](l/p_s), \overline{c}_{st+1} = [(1-\lambda)/\alpha][l/p_s], \overline{c}_{st+1} = [A(1-A)](L/P_X)$ and $\overline{c}_{st+1} = [(1-A)/A](L/P_X)$. Let $l_s = l/p_s$, $h_s = h/p_s$, $p_s/p_x = p_s$, $L_s = L/P_x$, $H_s = H/P_x$, and $P_s/P_x = P_s$.

Solving equations (10) through (13):

$$u_s(\overline{x} - h_s) = \frac{1}{p_s} u_s(\frac{h_s}{p_s}),$$

$$v'(\overline{y} - \frac{l_s}{p_s}) = \gamma \lambda v'(\frac{\lambda}{1-\alpha} l_s) p_s + \gamma (1-\lambda) v'(\frac{1-\lambda}{\alpha} l_s) p_s + (1-\gamma)(1-\lambda) v'(\frac{1-\lambda}{\alpha} l_s) p_s, \tag{11}$$

$$U_X(\overline{X} - H_s) = \frac{1}{P_s} U_Y(H_s/P_s), \tag{12}$$

$$V'(\overline{Y} - \frac{L_s}{P_s}) = \Gamma \Lambda V'(\frac{A}{1-A} L_s) P_s + \Gamma (1-\Lambda) V'(\frac{1-\Lambda}{A} L_s) P_s + (1-\Gamma)(1-\Lambda) V'(\frac{1-\Lambda}{A} L_s) P_s, \tag{13}$$

with $h_s = l_s$ and $H_s = L_s$ yields six equilibrium values: $p_s$, $P_s$, $h_s$, $l_s$, $H_s$, and $L_s$.

Using the budget constraints for old creditors, equations (2) and (3) become:
\[
\frac{p_s}{P_s \bar{e}} = \frac{\Lambda(1-\Gamma)L_s}{\lambda(1-\gamma)l_s}, \quad (14)
\]
\[
\frac{p_x}{P_x \bar{e}} = \frac{(1-\Lambda)(1-\Gamma)L_x}{(1-\lambda)(1-\gamma)l_x}, \quad (15)
\]

Substituting equations (14) and (15) into equations (11) and (13), one can solve equations (10) through (13) for \(h_s^*, H_s^*, p_s^*, p_x^*, P_s^*, P_x^*\), with the superscript stars indicating the equilibrium values under a liquidity constraint. Fujiki (2003) shows that a numerical verification of the existence of solutions for equations (10) through (13) is possible if utility functions display constant relative risk aversion.

Using the budget constraints for creditors at time \(t+1\) yields equation (16):
\[
\frac{\bar{c}_x}{c_x} = \frac{\alpha \lambda}{(1-\alpha)(1-\lambda)} = \rho^* < 1, \quad \frac{\bar{C}_x}{C_x} = \frac{\Lambda \Lambda}{(1-A)(1-A)} = \Xi^* < 1. \quad (16)
\]

Substituting the results \(\bar{c}_x^* = \bar{c}_x^* (p_x^*/P_x \bar{e}^*)\), \(\bar{c}_x^* = \bar{c}_x^* (p_x^*/P_x \bar{e}^*)\), \(\bar{C}_x^* = \bar{C}_x^* (P_x \bar{e}^*/p_x^*)\), and \(\bar{C}_x^* = \bar{C}_x^* (P_x \bar{e}^*/p_x^*)\) into equations (14) and (15) leads to equations (17) and (18):
\[
\frac{\bar{c}_x^*}{c_x} = \frac{\bar{C}_x^*}{C_x} = \frac{\alpha}{(1-\alpha)(1-\lambda)} = \Xi^* < 1. \quad (17)
\]
\[
\frac{\bar{C}_x^*}{C_x} = \frac{\alpha}{1-A} = \frac{\Lambda}{1-A} = \frac{\alpha}{1-\alpha}, \quad (18)
\]

Equations (17) and (18) indicate that the ratio of the consumption of foreign goods by early-departing creditors to that of late-departing creditors is proportional to the ratio of the foreign discount rate, adjusted by the relative size of the domestic banking sector \(\alpha/(1-\alpha)\) to the foreign banking sector \(A/(1-A)\).\(^8\) Thus, under the assumptions that \(\alpha < A\) and \(\lambda < \Lambda\), \{\(\alpha(1-\alpha)\}/\{A(1-A)\}\} < 1, and hence \(\bar{c}_x^* < \bar{c}_x^*\). With \(\lambda < A\), \(\lambda A/(1-A)(1-\lambda) < AA/(1-A)(1-A) < 1\), and therefore \(\bar{C}_x^* < \bar{C}_x^*\).

### 3.2 Benchmark efficient allocation and the role of central bank discounting and intervention

Consider a benchmark stationary state allocation, with which a social planner maximizes the weighted average utilities of two citizens given the world resource constraint. This social planner

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\(^8\) Note that a high value of \(\alpha\) means that large fractions of creditors do not need to be repaid immediately and thus are ready to purchase second-hand debt. Hence, \(\alpha\) represents the size of the banking sector.
maximizes equation (19), in which the $\theta$s are weights for Swiss and German citizens subject to equations (6) through (9):

$$
\begin{align*}
\theta_1 \{ u(d_x) + u(d_y) \} \\
+ \theta_2 \{ v(c_x) + \gamma(1-\alpha)v(\tilde{c}_x) + \gamma\alpha v(\tilde{c}_x) + (1-\gamma)(1-\alpha)v(\tilde{c}_x) + (1-\gamma)\alpha v(\tilde{c}_x) \} \\
+ \theta_3 \{ U(D_x) + U(D_y) \} \\
+ \theta_4 \{ V(C_x) + \Gamma(1-\Lambda)V(\tilde{C}_x) + \Gamma\Lambda V(\tilde{C}_x) + (1-\Gamma)(1-\Lambda)V(\tilde{C}_x) + (1-\Gamma)\Lambda V(\tilde{C}_x) \}.
\end{align*}
$$

The optimal solution satisfies three properties. First, consumption by creditors leaving early and those leaving late is equal. Second, creditors’ marginal rates of substitution between creditors’ goods and debtors’ goods, and those of debtors, are equal. Third, Swiss and German creditors’ marginal rates of substitution between Swiss goods and German goods are equal.

Fujiki (2003) considers the effects of the following policy. First, central banks intervene in each domestic credit market for the early-departing creditors, where the Swiss (or German) central bank can supply $(1-\alpha-\lambda)m$ (or $(1-\alpha-\lambda)M$ in Germany) units of fiat money by purchasing IOUs at par value. Then, these central banks sell the IOUs to late-arriving debtors in exchange for fiat money, and discard money equal to $(1-\alpha-\lambda)m$ (or $(1-\alpha-\lambda)M$ in Germany).

Fujiki (2003) demonstrates that those interventions could equalize the consumption levels of creditors with and without a taste shock. However, the third condition for optimality, that Swiss and German creditors’ marginal rates of substitution between Swiss goods and German goods be equal, could not be satisfied. Equations (17) and (18) imply that the ratio of the consumption of foreign goods by early-departing creditors to that by late-departing creditors is proportional to the relative size of the domestic banking sector multiplied by foreign discount rate. The central banks’ interventions into the second-hand debt market only achieve $\rho = \Xi = 1$.

Fujiki (2003) proposes a way to achieve the third condition for optimality. First, the German central bank supplies $T = (1-\Gamma)(\Lambda-\alpha)\overline{M}$ marks in the foreign exchange market for early-departing creditors at an exchange rate of $e^\xi = (1-\gamma)m / (1-\Gamma)\overline{M}$, which is the fair exchange rate based on gold standard parity. This operation leaves some francs for their reserve. Then, in the foreign exchange market for late-departing old creditors, that reserve is sold by the German central bank, and the German central bank takes $T$ marks out of circulation once the foreign exchange markets are closed. In this equilibrium, given the price levels in each country, German intervention equates the nominal exchange rate for early-departing creditors and late-departing creditors. Since the
consumption of agents without a taste shock remains constant, this operation allocates the foreign
goods consumption of agents with a taste shock equally. Given the strictly concave utility function,
this equilibrium dominates the equilibrium with domestic intervention alone. The next section
examines other institutional arrangements that also achieve this welfare improvement.

4. Institutions that could improve efficiency: PVP with central bank credit

Despite domestic market intervention by two central banks to settle second-hand debt at par value,
the segmentation of the foreign exchange market among early-departing creditors and late-
departing creditors leads to country-specific cash-in-advance constraints that cause the market
exchange rate to deviate from gold standard parity. A central bank’s monetary policy, conducted in
its domestic credit market, and its foreign exchange interventions have different welfare effects.

With regard to a cure for this problem, observe that overall supplies of fiat monies in the foreign
exchange market are known \( \text{ex ante} \). Therefore, if early-departing creditors whose currencies are
in excess supply could wait until their counterparties were ready to make payment, all creditors
could exchange their fiat money at the gold standard parity. Thus, intraday credit provided by the
central banks in the foreign exchange market should be beneficial. It prevents fluctuations in the
spot foreign exchange rate, it is safe because there is no aggregate risk, and this is one way to
achieve PVP. In this section, I first discuss an institutional arrangement based on the idea of PVP
that replicates the equilibrium allocation proposed by Fujiki (2003). Later, I will touch on the issue
of default.

4.1 Naïve PVP proposal

Suppose that guardians work at the each tunnel entrance (the points labeled (4) in Figure 2), rather
than at the central point of the tunnel. They accept all of the money from old creditors with a taste
shock who are going through the tunnel. However, the guardians at the German end of the tunnel
(the point labeled (6) in the lower panel of Figure 2) should not distribute the marks acquired from
early-departing old German creditors with a taste shock equally among early-departing old Swiss
creditors with a taste shock. Instead, the guardians at the German end of the tunnel should
exchange francs for marks at the fair exchange rate, \( e^F \).
Suppose momentarily that the guardians also ask some of the early-departing old Swiss creditors with a taste shock to wait until the late-departing old German creditors with a taste shock enter the tunnel to highlight the idea of PVP, although, in Section 4.2, I show that there is an institutional arrangement in which the guardians do not have to ask some early-departing creditors to wait. Note that the guardians at the Swiss end of the tunnel are also asked to exchange marks for francs at the fair exchange rate. Then, at the Swiss end of the tunnel, the guardians have \((1–\gamma)(A–\alpha)\overline{m}\) francs even after the departure of the early-departing old German creditors with a taste shock. Once the late-departing old German creditors with a taste shock come to the tunnel (the point labeled (4) in the lower panel of Figure 2), the guardians are ready to exchange francs for marks with the early-departing old Swiss creditors with a taste shock at the fair exchange rate. Then, the guardians at the German end of the tunnel meet late-departing old Swiss creditors and exchange francs for marks at the fair exchange rate. At the Swiss end, the guardians have enough francs to exchange for marks obtained from late-departing old German creditors with a taste shock because they have accumulated \((1–\gamma)(A–\alpha)\overline{m}\) francs in addition to the francs obtained from late-departing old Swiss creditors with a taste shock.

The mechanism discussed in the previous paragraph could be achieved by an institution that specializes in foreign exchange settlement at the fair exchange rate on a PVP basis. This institution assigns a trading partner in the foreign exchange market to early-departing old creditors with a taste shock upon their initial arrival at the tunnel. The institution randomly selects \((1–\Gamma)(A–\alpha)\) early-departing old Swiss creditors who are not allowed to go through the tunnel until the arrival of late-departing old German creditors at the other end of the tunnel. This selection ensures PVP at the fair exchange rate \(e^F\).

### 4.2 PVP with central bank credit

The proposal in Section 4.1 seems to be problematic if it not only alters the timing of transactions made in the foreign exchange market but also affects the timing of consumption by early-departing creditors and late-departing creditors with a taste shock. However, note that the use of a daylight overdraft within each economy is consistent with domestic credit market intervention by the central banks when we regard late-departing old creditors as banks, as has been suggested by Green (1997). Moreover, it is reasonable to assume that an institution like CLS Bank is allowed to access...
the daylight overdraft from the central banks. Let us suppose that this institution is allowed to have overdrafts from the German central bank up to the amount \( (1-\Gamma)(A-\alpha)\bar{M} \). Then, this institution could sell marks at the fair exchange rate to all early-departing old Swiss creditors with a taste shock. Such a mechanism would be possible if \( (1-\gamma)(A-\alpha)\bar{m} \) francs obtained from early-departing old creditors could serve as collateral for this overdraft. Indeed, once the late-departing old German creditors arrived at the tunnel, this institution could return marks to the German central bank. Moreover, the German central bank could limit the daylight overdraft amount to \( (1-\Gamma)(A-\alpha)\bar{M} \) and force this institution to keep the overdraft amount zero after settlement in the foreign exchange market for late-departing creditors. In this way, a financial institution like the CLS Bank, which can access a daylight overdraft from the central banks, achieves the same equilibrium allocation proposed by Fujiki (2003). This financial institution changes the amount of fiat money in the foreign exchange market using daylight overdraft to keep market exchange rates at the gold standard parity. This financial institution does not affect the timing of consumption or transactions in the foreign exchange market made by early-departing creditors and late-departing creditors with a taste shock.\(^9\)

### 4.3 Herstatt risk

Let us touch on the issue of Herstatt risk, namely, the risk that one party fails after the other party has sent his/her payment into this foreign exchange market.

In this model, such risk could mean a situation in which some old creditors with a taste shock (call them “dishonest creditors”) do not submit their fiat money to the guardians at the entrance to the tunnel, but throw it away. Moreover, they successfully receive the fiat money of the other country at the other end of the tunnel.\(^10\) Dishonest creditors are able to take such actions because

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\(^9\) This result is similar to the proposal made by Kahn and Roberds (2001b). They show the optimality of having creditors who are subjected to liquidity constraints because they could not postpone the timing of their consumption with intraday free credit under a real-time gross settlement system. Kahn and Roberds stipulate a closed economy model in which the central bank knows about the flow of transfers and there is no risk associated with the intraday credit.

\(^10\) Freeman (1999) analyzes such risks in the sense that some proportion of late-departing old debtors, randomly chosen, do not go to the central clearing area. Instead, they visit young debtors’ where they are free to consume using fiat money based on a closed economy model similar to Freeman (1996). He shows that liquidity provided by the central bank improves risk sharing even though the central bank suffers losses and the price level changes to distribute risks. Zhou (2000) modifies Freeman’s (1996, 1999) model to discuss the remedy for liquidity shortage in a real-time gross
the guardians at the other end of the tunnel have no way to distinguish dishonest creditors from other creditors. However, in this model, the guardians could issue a certificate to show that the agent has submitted fiat money at the entrance of the tunnel. The payment of fiat money at the other end of the tunnel could be contingent on the submission of this certificate. Such a mechanism is easily incorporated into institutions like the CLS Bank, since the bank requires member banks to open an account for each central bank, which could facilitate the receipt of fiat money at one end of the tunnel conditioned on a pay-in at the other end of the tunnel.

5. Institutions that could improve efficiency: Eurodollar

Section 5 considers another institutional set up to replicate the equilibrium allocation proposed by Fujiki (2003). Consider a financial institution that provides a negotiable certificate of deposit for old creditors with taste shocks in exchange for their domestic fiat money, like the Eurodollar. This deposit can be withdrawn at the end of the period as foreign fiat money. Old creditors with taste shocks transfer their deposits to foreign young debtors and obtain foreign young debtors’ goods. Note that young debtors only need fiat money at the end of the first period of their lifetime, since they repay their debts in the next period. Young debtors make withdrawals and obtain domestic fiat money at the end of the first period of their lifetime. They repay their debts in the next period. This financial institution has enough fiat money to distribute to the young debtors in exchange for their negotiable certificates of deposit at the fair exchange rate, because withdrawals of deposits occur only at the end of a period. The provision of the negotiable certificate of deposit allows foreign old creditors with taste shocks to obtain domestic young debtors’ goods in exchange for their negotiable certificate of deposit, rather than domestic fiat money. This financial institution changes the timing of settlement in the foreign exchange market and achieves the same efficiency gain as Fujiki (2003) proposes. I discuss the details of this institutional arrangement.

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settlement system, the provision of intraday liquidity by the central bank, and policies to reduce the central bank’s exposure to credit risk.

11 Dishonest creditors cannot consume in the other economy without fiat money in that economy, whereas dishonest creditors who do not go to the central area in Freeman (1999) are free to use cash already in hand.
5.1 *A financial institution that provides a negotiable certificate of deposit*

Suppose that guardians work at each entrance to the tunnel (the points labeled (4) in Figure 2), rather than at the central point of the tunnel. They accept all fiat money from old creditors with a taste shock who go into the tunnel. The guardians at the German end of the tunnel (the point labeled (4) in the lower panel of Figure 2) provide a negotiable certificate of deposit of German marks whose value is equal to $m$ francs, which can be withdrawn from the guardians at the Swiss end of the tunnel at the end of time period $t$, for all old German creditors with a taste shock. The old German creditor with a taste shock presents his negotiable certificate of deposit to the guardians at the Swiss end of the tunnel (the point labeled (4) in the upper panel of Figure 2) and reports which debtor’s village he intends to visit. A young Swiss debtor is willing to sell $x - dxt$ of his endowment at the market price $pxt$ to an old German creditor born at time $t-1$ who visits his village, in exchange for this negotiable certificate of deposit, rather than fiat money $mt$ francs, since the young Swiss debtor knows that he can withdraw this negotiable certificate of deposit as Swiss francs. The young Swiss debtor, who obtains this negotiable certificate of deposit, mails this certificate to the guardians at the Swiss end of the tunnel, and the guardians deliver $mt$ francs to the young Swiss debtor by the end of period $t$.

The guardians at the Swiss end of the tunnel (the point labeled (4) in the upper panel of Figure 2) provide a negotiable certificate of deposit of Swiss francs whose value is equal to $M$ marks, which can be withdrawn from the guardians at the German end of the tunnel at the end of period $t$, for all old Swiss creditors with a taste shock. The old Swiss creditor with a taste shock presents his negotiable certificate of deposit to the guardians at the German end of the tunnel (the point labeled (4) in the lower panel of Figure 2) and reports which debtor’s village he plans to visit. The young German debtor, who obtains the negotiable certificate of deposit, gets $M$ marks from the guardians at the German end of the tunnel.

The guardians at both entrances to the tunnel have enough fiat money to distribute to the young debtors in exchange for their negotiable certificates of deposit at the fair exchange rate, $e^F = (1 - \gamma)m/(1 - \Gamma)M$, because withdrawals of deposits occur only at the end of the period. Moreover, this financial institution, operated by the guardians, does not change the timing of consumption between time $t$ and time $t+1$. In this way, this institutional arrangement achieves the same efficiency gains proposed by Fujiki (2003).
5.2 Other assumptions on the timing of transactions

The results presented in Section 5.1 depend on two important assumptions made throughout this paper, “the guardians are honest and can live without any consumption”, and “all contracts are honored”. Below, I show that at least two other assumptions on the timing of withdrawals of negotiable certificates of deposit are consistent with the results in 5.1.

First, I can assume that young debtors travel to the central hill at the end of their youth and obtain fiat money from the guardians in exchange for their negotiable certificates of deposit, instead of assuming that the guardians deliver fiat money to them. This alternative assumption is consistent with Freeman (1996), who assumes that domestic private debt incurred between two parties can only be redeemed with fiat currencies at the central clearing area.12

Second, I can suppose that the withdrawals of negotiable certificates of deposit made by debtors occur at time \( t+1 \), after old debtors climb up the central hill, and immediately before they repay their debts in fiat money. This assumption is consistent with the assumption that only transactions in the foreign exchange market involve the guardians at the tunnel because the two domestic credit markets and the foreign exchange market are spatially separated.

6. Institutions that could improve efficiency: Currency Union

Section 6 considers whether a currency union could achieve the third condition for optimality: that Swiss and German creditors’ marginal rates of substitution between Swiss goods and German goods be equal. This question is motivated by the notion that an institution that removes country-specific cash-in-advance constraints could improve the efficiency of equilibrium. It is difficult to prove that this idea is true as long as the sizes of creditors with taste shocks are not identical in two countries. However, we can present an example in which this idea works.

Suppose that at time 1, the central banks allow agents to settle their debts using a new currency called the euro. At time 1, each central bank supplies \( \bar{m} = m_0 = h_0 \) fiat money, called euros, in

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12 Mills (2004) considers a trading pattern that young debtors accept old debtors’ IOUs, rather than fiat money. The young creditors accept IOUs from young debtors not because it will be repaid in the form of fiat money when the young creditors become old, rather the young creditors anticipate the IOUs’ operation as a means of payment when they become old. Mills (2004) shows that the payment mechanism, which does not involve the use of money to repay debt, also achieves the same equilibrium proposed by Freeman (1996).
Switzerland and $\overline{M} = M_0 = H_0$ in Germany, and the two central banks still operate in their domestic credit markets. Let us consider the equilibrium conditions under liquidity constraints.

First, equation (1) and its German counterparts, (11) and (13), will be changed as follows:

\[ u_{st}(\bar{X} - h_{st}) = u_{yt}(\frac{h_{st}}{p_{st}})(\frac{1}{p_{st}}), \]  

\[ v^\prime(\bar{y} - \frac{l_{st}}{p_{st}}) = \gamma \lambda v^\prime(\frac{\lambda}{1-\alpha}, \frac{l_{st}}{p_{st}}) + \alpha(1-\lambda)v^\prime(\frac{1-\lambda}{\alpha}, \frac{l_{st}}{p_{st}}), \]  

\[ + (1-\gamma)\lambda v^\prime(\frac{\lambda}{1-\alpha}, \frac{l_{st}}{p_{st}})(\frac{p_{st}}{\Pi_{t+1}}) \]  

\[ + (1-\gamma)(1-\lambda)v^\prime(\frac{1-\lambda}{\alpha}, \frac{l_{st}}{p_{st}})(\frac{p_{st}}{\Pi_{t+1}}), \]  

\[ U_X(\bar{X} - H_{st}) = U_Y(\frac{H_{st}}{P_{st}})(\frac{1}{P_{st}}), \]  

\[ V^\prime(\bar{Y} - \frac{L_{st}}{P_{st}}) = \Lambda \alpha V^\prime(\frac{\Lambda L_{st}}{1-\Lambda \Pi_{t}}) \]  

\[ + (1-\Lambda)\alpha V^\prime(\frac{1-\Lambda L_{st}}{\Pi_{t}}) \frac{P_{st}}{P_{st}}, \]  

where $h_{st} = h_t/p_{st}$, $p_{st} = p_{yt}/p_{st}$, $\Pi_{t+1} = p_{st}/p_{st} H_{st} = H_{st}/P_{st}$, $P_{st} = P_{yt}/P_{st}$ and $\Pi_{t+1} = P_{yt}/P_{yt}$.

Equilibrium conditions in the loan markets are $l_t = h_t$ in Switzerland and $L_t = H_t$ in Germany. Those constraints and the equilibria in the secondary debt markets yield $\rho = (\alpha(1-\alpha)(\lambda(1-\lambda)) < 1$ and $\varpi = (A/1-A)(A/1-A) < 1$. Equations (6) and (8) can be used to define the price level.

It is tempting to suppose that there is a stationary equilibrium in which the demand for real balances in each economy is constant over time if the two central banks supply fiat money in their domestic credit markets to bring the prices of second-hand debts to par value. Let us see how this conjecture works. Under the proposed equilibrium, equations (6) and (8) should be read as the following equations:

\[ \bar{x} - d_{st} = m_{st} \frac{m_{t+1}}{p_{st}} + (1-\gamma)\frac{M_{t+1}}{p_{st}}, \]  

\[ \bar{X} - D_{st} = M_{st} \frac{M_{t+1}}{P_{st}} + (1-\gamma)\frac{M_{t+1}}{P_{st}}. \]
Suppose that old creditors born at time zero were supplied fiat money $m = m_0 = h_0$ in Switzerland and $M = M_0 = H_0$ in Germany. Then, equations (24) and (25) give the price levels at time 1, given that the constant value of real money balances in each economy as follows:

$$p_{x1} = \frac{\bar{x} - d_{x1}}{\gamma m + (1 - \Gamma)M},$$

$$P_{x1} = \frac{\bar{X} - D_{x1}}{(1 - \gamma)m + \Gamma M}. \tag{27}$$

Under the proposed equilibrium, fiat money delivered by old creditors (or, demand for fiat money by young debtors born at time $t$) should evolve by the following equation:

$$\left(\frac{m_t}{M_t}\right) = \left(\frac{\gamma}{1 - \gamma} \frac{1 - \Gamma}{\Gamma}\right)\left(\frac{\bar{m}}{\bar{M}}\right), t = 1, 2, 3, \ldots \tag{28}$$

It is easy to see that inflation rates, $\pi_{x+1} = p_{x+1}/p_x$ and $\Pi_{x+1} = P_{x+1}/P_x$, will be the same as growth rates of money in each economy under the hypothesis that $\bar{x} - d_{x1} = h_x = h/p_x$ and $\bar{X} - D_{x1} = H_x = H/P_x$ remain constant. Equations (21) through (24) could be solved for $p_x$ and $P_x$ given the ratio of $p_{x1}/P_{x1}$ specified by equations (26) through (28). However, since rates of inflation change in the two economies over time, relative prices in the two economies change over time, even given constant real money balances and a constant world money supply.

Note that this currency union achieves the three efficiency conditions within a generation. First, consumption by early-leaving creditors and that by late-departing creditors is equal, since the central banks’ intervention in the domestic credit market ensures that the amount of fiat money brought by each type of creditor is the same. Second, creditors’ marginal rates of substitution between creditors’ goods and debtors’ goods, and those of debtors’, are equal since we can uniquely determine relative prices $p_x$ and $P_x$. Third, Swiss and German creditors’ marginal rates of substitution between Swiss goods and German goods are equal because they both face terms of trade specified by equations (26) through (28).

The welfare improvement over time due to this currency union relative to the two-currency equilibrium studied in Section 4 is difficult to assess in general, because currency union member countries might experience either deflation or inflation. Remember that Fujiki (2003) presumes country-specific cash-in-advance constraints. The assumptions specified in that research, indeed,
help to show that there is a stationary equilibrium in which the real money balances of young debtors remain constant in each economy, as do the amounts of fiat money circulated in each economy, and thus the terms of trade remain constant over time.

Only if $\Gamma = \gamma$ and the initial money supply levels in both economies were the same could we reproduce the same stationary equilibrium studied in section 3.2. For example, if $\Gamma = \gamma = 0.5$, we obtain a stationary equilibrium that satisfies $m_t = M_t = (1/2)(\bar{m} + \bar{M})$ for all $t$ from equation (28). Table 2 presents numerical examples to show the case for currency union if $\Gamma = \gamma = 0.5$.

The stationary equilibrium under the currency union, with domestic credit market interventions by both central banks (row 4, the lower panel of Table 2), achieves the same welfare level as does foreign exchange interventions by the German central bank in the multiple currencies equilibrium (row 3, the upper panel of Table 2). The level of welfare under the multiple currencies equilibrium with central bank domestic credit market intervention (row 2, the upper panel of Table 2) is lower than that of the currency union equilibrium with domestic credit market interventions (rows 2 and 3, the lower panel of Table 2). Thus, if $\Gamma = \gamma = 0.5$, we find the case for currency union, which replicates the equilibrium proposed by Fujiki (2003).

Note that the model summarized in Figures 1 and 2 makes plausible the assumption that it is difficult to arrange risk sharing among agents living in the other villages. This feature of the model suggests that only integrated national financial markets are inter-bank market supported by the central bank, and that there are some specialists helping transactions in the foreign exchange market. Thus, the analysis in this section might illuminate the pros and cons of the currency union among economies with relatively decentralized goods and financial markets. In such economies, unless the degree of goods-market integration is high, in the sense that old creditors want to consume goods in the other economy and goods in the domestic economy with equal probability (namely, $\Gamma = \gamma = 0.5$), benefits from the currency union may be limited.

13 It is easy to show that $\begin{bmatrix} \gamma & 1-\gamma \\ 1-\gamma & \gamma \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \frac{(2\gamma-1)^t}{2(1-\gamma)} \begin{bmatrix} 1-\gamma & \gamma-1 \\ \gamma-1 & 1-\gamma \end{bmatrix}$.

14 These solutions were obtained using the “solve” command in the Mathcad 2000 Professional version by specifying parameter values listed in Table 2.

15 Hernández-Verme (2002) studies a model where two countries are identical in every respect except the size of their financial sector, and only the agents in one of the economies face stochastic needs for liquidity in a similar environment to that considered in this paper. She examines the effects of various policies, both single-currency cases and multiple-country specific-currency cases under a gold standard.
7. Conclusion and Reservations

Fujiki (2003) extends the Freeman (1996) model to show that an elastic money supply in the foreign exchange market and an elastic money supply in the domestic credit market, when combined, yield efficiency gains in monetary equilibrium. This paper discusses three other institutional designs to achieve this improvement in efficiency. First, a financial institution like the CLS Bank, together with daylight overdrafts from the central bank, will achieve the same equilibrium allocation. Second, consider a financial institution that provides negotiable certificates of deposit for old creditors with a taste shock, which can be withdrawn at the end of the period in terms of foreign fiat money. Foreign old creditors with taste shocks can purchase domestic young debtors’ goods by transferring their negotiable certificates of deposit to the domestic young debtors, rather than using domestic fiat money. This financial institution also achieves the same equilibrium allocation as Fujiki (2003) does. Third, the benefits of a currency union in terms of efficient resource allocation are clear; however, its effects on overall welfare are unclear. Unless the degree of goods market integration is high in the sense that old creditors want to consume goods in the other economy and the goods in the domestic economy with equal probability, the benefits from the currency union may be limited.

There are many reservations to consider before taking the conclusions in this paper literally. First, this paper is silent about the reason that we have country-specific cash-in-advance constraints. Kocherlakota and Krueger (1998) use a random-matching model to justify country-specific currencies when two currencies are essential in order to achieve efficient allocation of resources. They find that if the agents’ preferences are sufficiently heterogeneous over the different nationalities of goods, the presence of national currencies is socially beneficial. National currencies allow the buyer credibly to signal private preferences to the seller. However, the model in Section 4 avoids the issue of credible revelation of preferences owing to spatial separation and the assumption of travel by the old creditors.

Second, the model in Section 6 is not explicit about why a currency union is created. Matsuyama et al. (1993) use a version of a two-country random-matching model to discuss the evolution of world currency. Since an agent’s incentive to accept a nation’s currency depends on how often one would meet a member of that country, the currency of a larger country emerges as
the world currency because it gives people better chances of acquiring their consumption goods. On the other hand, Kiyotaki and Moore (2003) provide an example in which the introduction of a common currency can be costly if it leads to too little specialization and limits the variety of consumption goods available. These results suggest that there are other benefits and costs from currency unions that are not studied in this paper.

Third, the model in this paper assumes that all agents honor their contracts and that the guardians at the tunnel are honest. Kahn and Roberds (2001a) discuss strategic issues that arise when one of the parties refuses to settle—for example, because of fear that the counterparty will collapse and that all contracts may not be honored. Baxter and Heller (2000) suggest that a payment system is most efficient and safe when it operates within a clear legal framework. Those important concerns are not included in the analysis of this paper.

References


Table 1. Steps of trading under a liquidity constraint in Switzerland

<table>
<thead>
<tr>
<th>Step</th>
<th>Young</th>
<th>Old</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Young Swiss debtors visit domestic neighboring creditors’ village.</td>
<td>All old Swiss creditors and $\lambda$ early-bird Swiss debtors visit the central hill.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>$(1-\alpha)$ old Swiss creditors identify that they are early-departing creditors.</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>Debt settlement at the central hill begins.</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>$(1-\alpha)$ old Swiss early-departing creditors leave the central hill.</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>$(1-\alpha)(1-\gamma)$ old Swiss early-departing creditors identify their taste shock.</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>$(1-\alpha)\gamma$ old Swiss early-departing creditors travel to Swiss debtors’ village with francs.</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>$(1-\alpha)(1-\gamma)$ old Swiss early-departing creditors arrive at the tunnel and meet German early-departing old creditors, and foreign exchange market opens.</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>$(1-\alpha)(1-\gamma)$ old Swiss early-departing creditors obtain marks, and continue their travel to German debtors’ village via the German central hill.</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>$(1-\lambda)$ old Swiss debtors arrive at the Swiss central hill.</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>Debt settlement at the central hill begins.</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>The remaining ($\alpha$) old Swiss creditors leave the central hill with francs.</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>$\alpha(1-\gamma)$ old Swiss late-departing creditors identify their taste shock.</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>$\alpha\gamma$ old Swiss late-departing creditors travel to Swiss debtors village with francs.</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>$\alpha(1-\gamma)$ old Swiss late-departing creditors arrive at the tunnel and meet German old debtors, and foreign exchange market opens.</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>$\alpha(1-\gamma)$ old Swiss late-departing creditors obtain marks, and continue their travel to German debtors’ village via the German central hill.</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>Young Swiss debtors come back from the neighboring young creditors’ village.</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>Young Swiss debtors obtain francs from old Swiss creditors and old German creditors in exchange for their endowment.</td>
</tr>
</tbody>
</table>
Table 2. Numerical Examples

<table>
<thead>
<tr>
<th>Multiple Currencies</th>
<th>German creditor's utility</th>
<th>Swiss creditor's Utility</th>
<th>German Domestic Credit Market</th>
<th>Swiss Domestic Credit Market</th>
<th>Foreign Exchange Intervention by Germany</th>
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Notes:
The parameters and functional forms of utility functions for benchmark stationary solutions are:
\( v(z) = u(z) = z^{1−\sigma} / (1−\sigma), \quad V(Z) = U(Z) = Z^{1−\Sigma} / (1−\Sigma), \quad \sigma = \Sigma = 0.9, \quad \Gamma = \gamma = 0.5, \quad \bar{x} = 1, \quad \bar{y} = 10, \quad \bar{\lambda} = 0.2, \quad \alpha = 0.2, \quad \bar{m} = 10, \quad \bar{X} = 10, \quad \bar{Y} = 100, \quad \Lambda = 0.4, \quad \Lambda = 0.4, \) and \( \bar{M} = 100. \)

The figures shown in the column “German (Swiss) Domestic Credit Market” show the amount of fiat money issued by the German (Swiss) central bank to purchase the IOUs of early-departing creditors. I assume that the fiat money provided in this way will be taken out of circulation once all debtors arrive at the central clearing area.

The figures shown in the column “Foreign Exchange Intervention by Germany” show the amount of fiat money issued by the German (Swiss) central bank in the foreign exchange market for early-departing creditors. In the foreign exchange market for late-departing old creditors, the reserve of Swiss francs obtained in the foreign exchange market for early-departing creditors is sold by the German central bank, and the German central bank takes marks out of circulation once the foreign exchange markets are closed.

The first row in each panel shows the level of utility achieved in Germany and Switzerland without domestic credit market interventions of the two central banks, which corresponds to the liquidity-constrained equilibrium.
Figure 1. Pattern of Travels when Young

(1) Young debtor visits creditors’ village.
(2) Young debtor meets a young creditor and obtains creditor’s goods in exchange for IOU.
(3) Young debtor goes back to his village.
(4) Young debtor meets an old creditor and obtains fiat money in exchange for goods. (The travel of old creditors is explained in Figure 2.)

Note: Thick solid lines show travel by young Swiss debtors, and thick dotted lines show travel by German debtors.
Figure 2. Pattern of Travels when Old

(1) Debtors and creditors visit the central hill; however, the arrival rate of debtors is less than one. Some old creditors identify themselves as early departing.

(2) Debts are repaid with domestic fiat money, and old debtors die after debt settlement. After debt settlement, some old creditors identify their taste shock.

(3) Old creditors without a taste shock scatter to the debtors’ village and obtain young debtors’ goods in exchange for fiat money.

(4) Old creditors subject to a taste shock visit the entrance to the tunnel.

(5) In the middle of the tunnel, old creditors with a taste shock exchange their fiat money.

(6) Old creditors with a taste shock travel to the central hill of the other country with the fiat money issued in the other country.

(7) Old creditors with a taste shock scatter to the debtors’ village in the other country.

Note: The thick solid line shows travel by old Swiss debtors; the thin solid lines show travel by Swiss creditors. The thick dotted line shows travel by old German debtors, and the thin dotted lines show travel by old German creditors.