Abstract: Remittances play a large and important role in certain economies, where they may exceed even 10% of GDP. Indeed, remittance flows in certain nations exceed FDI in magnitude. We study the impact of remittances on a small open economy. Our model is a stochastic limited participation model with cash in advance constraints and costly adjustment of cash holdings. We examine the impact of remittances on the steady state, as well as on the dynamic response of variables to shocks, including monetary shocks, technology shocks, and remittances shocks. We examine alternative specifications regarding the initial impact of a remittances shock on the economy, in one case allowing the monetary injection to be a helicopter drop on households, thereby loosening the cash in advance constraint, and alternatively allowing the monetary injection to be a helicopter drop on the financial intermediaries, providing an increase in the supply of loanable funds on impact. In a similar way remittances may either flow to households as increased cash for purchases, or flow to banks as additional deposits and increased lending potential. We find that a positive remittances shock forces the exchange rate to depreciate and lowers both output and the interest rate in the period of the shock, irrespective of adjustment costs on money balances, but increase output in the subsequent periods, while consumption rises on impact. We also show that a positive shock expands the dynamic responses of the nominal interest rate, output and nominal exchange rate, but reduces the magnitude of the consumption response, as we allow for a larger proportion of remittances to go through the financial system for investment.

Keywords: Migration; Remittances; Limited participation model; Overshooting; Liquidity Effect; Uncovered interest rate parity.
JEL Classification: E40; F22; J61; O15

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1 Introduction

Remittances have been on the rise for the last several decades. International estimates of official remittances flows suggest that the total amount of remittances received by developing countries has reached 167 billion U.S. dollars in 2005, up by 73 percent from 2001 (World Bank’s Global Economic Prospects). Moreover, remittances constitute a significant share of some countries’ gross domestic product (Neyapti (2004) and Heilman (2006)). The apparent increase in remittances may in part be attributed to the rapid growth of money transfer institutions, making the money flows more visible, and decreases in the average transaction cost of making remittances. However, the increase in measured remittances is also indicative of an actual increase in these monetary flows, and remittance flows have grown from only satisfying basic needs to providing durable goods for the recipient households.

Remittances gain their significance not just from their size but from the potential and actual effects of these money flows on both the society and the individual. Remittances affect labor market decisions, school retention levels, export sector competitiveness, and create moral hazard problems (Funkhouser (1992), Glytsos (2002), Edwards and Ureta (2003), Amuedo-Dorantes and Pozo (2004) and Chami et. al. (2005)).

The increasing volume of monetary remittances has led to an interest in studying the effects of remittances. Several studies have documented that for several developing countries total remittances already exceed foreign aid and compete in size with foreign direct investment (Connell and Brown (2004), De Haas (2006), Heilmann (2006) and Chami et. al. (2006)). While foreign direct investment (FDI) flows are assumed to be profit driven and therefore considered as a source of development, the increase in remittances also has the potential to promote economic growth through increased domestic demand.

Remittances may be motivated by many factors, such as altruism or self interest (Lucas and Stark (1985)). Consequently, the principal motivation behind remittances may have important implications for the effect of remittances on output in the recipient country. Some researchers believe that altruistically motivated remittances are countercyclical with domestic output; others consider remittances as procyclical with domestic output when they are mainly motivated by self-interest plans.
Figure 1 indicates the increasing importance of remittances in selected Latin American countries, comparing remittances and FDI as shares of GDP. Remittances have surpassed FDI in magnitude starting in about 1999, and remittances have been growing while FDI is shrinking. Also, while FDI has been volatile and dependent on the economic performance of the receiving countries and region, remittances have been more stable and increasing at a fairly steady pace.

![Remittances and FDI (% of GDP) for Selected Latin American Countries 1990-2004](image)

*Figure 1: Trends of FDI and Remittances for a sample of Latin American Countries*

Most of the remittance literature focuses on the microeconomic implication of such flows. The literature on the macroeconomic impact of remittances on the recipient country is sparse. This paper explores the impact of remittance flows on output, consumption, interest and exchange rates in the recipient country. In particular, we model remittances in a small open economy and analyze the impact of shocks to money, to technology and to remittances. We expand a limited participation model that requires that money balances be held to finance certain types of purchases, and that specifies agents incur an adjustment costs on money holdings. These two requirements generate a large and persistent liquidity effect consistent with the stylized facts (Hairault et. al. (2004)). The impact of the adjustment costs on the predetermined allocation of money cash available for consumption is then analyzed to see how the main real variables of the economy respond to a remittances shock.
The main contribution of this paper is to provide a model to examine the impact of a remittances shock on the main economic variables of a small open economy. We also examine the importance of how remittances enter the economy, whether as cash for use directly in consumption, or as bank deposits. This could provide useful information to domestic governments that are currently trying to develop policy tools to direct a portion of remittances towards investment. We distinguish between the direct effect of remittances on output through investment and the indirect effect through consumption and its multiplier effects. Being able to distinguish the end use of remittances is crucial in looking at the final effect on output in the economy (Burgess and Haksar (2005), Heilmann (2006) and Sayan (2006)).

The remainder of this paper is organized as follows. Section 2 presents a brief summary of the literature review. Section 3 formulates a theoretical model. Section 4 discusses the results and Section 5 provides a robustness check. Section 6 summarizes and concludes.

2 Literature Review

Residents of labor exporting countries receive substantial annual flows of remittances. Countries like India and Mexico received documented remittances of more than 9 billion U.S. dollars in 2001⁴ (IMF Balance of Payments Yearbook). Figure A.1 – in the Appendix – shows that remittances were 40% of GDP in Guatemala by 2004, approaching 15% in Honduras, above 8% in Ecuador, and over 30% in El Salvador. Even in larger economies such as Mexico remittances approached 1% of GDP by 2004.

Durand et. al. (1996) argue that remittance can stimulate economic activity both directly through investment and indirectly through consumption. Even if the large percentage of remittances is used for private consumption, some smaller portion is used in productive investment. When applied to large sums of remittances this investment portion may play a significant role in the economy. Furthermore, Durand et. al. argue that large use of remittances for consumption stimulates the demand for goods and services in the receiving country, leading to increases in production, employment and disposable income.

⁴ Several researchers believe that undocumented remittances are twice the recorded amounts. Refer to Freeman (2006) for more details.
Widgren and Martin (2002) include remittances with FDI and foreign aid as possible sources of accelerating economic growth, although they warn about the nature of remittances. Remittances are not profit driven and are often thought to be intended to mitigate the burden of poor economic performance on the local recipients. Chami et al. (2005) also suggest that remittances are compensatory in nature, and document a negative correlation between remittances and GDP growth.

Heilmann (2006) argues that remittances differ from other capital flows. Remittances consist of a transfer of ownership between two individuals, and one objective is to increase the recipients’ disposable income. Further, remittances are not evenly distributed. Heilmann outlines the case for remittances promoting a sustainable level of development but also warns of potential inflation due to stimulation of internal demand for imports due to remittances.

Chami et al. (2006) develop a stochastic dynamic general equilibrium model that includes government policies to study the implication of remittances for monetary and fiscal policy in the recipient country. They explore the behavior of a subset of real and nominal variables in remittance-dependent economies and in economies where remittances are not significant. The authors demonstrate that optimal monetary policy will differ between the remittance-dependent economy and an economy with no significant remittances.

The literature seems to present two opposing positions concerning the effects of remittances on the economy of the receiving country (Keely and Tran (1989), León-Ledesma and Piracha (2004) and De Haas (2006)). On the one hand, remittances do increase the standard of living of receiving households. These flows of funds are spent on consumption, health and education, even finding their way into productive investment. On the other hand, remittances are mainly spent on consumption and rarely directly invested in productive projects. Remittances increase dependency and may increase economic instability.

In the rest of this paper we develop and analyze a theoretical model of remittances in a small open economy. Remittances transfer resources from the rest of the world to

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5 Djajić (1998) show that remittances can also increase the welfare of all residents in the labor exporting countries not just those receiving positive amount of remittances.
households in a small open economy. Households react as optimizing agents, increasing consumption, leisure, and bond holdings in the steady state. We will model remittances as occurring in the currency of the small open economy, although the exact form of remittances is not crucial since goods are readily convertible into currency, local or foreign, and vice versa. Our small open economy focus also allows us to rationalize our assumption that remittances do not impact the remitting economy. Our model generates the expected effects of remittances on optimizing agents, and our goal is to study the quantitative and qualitative dynamic responses that lead to the steady state results or that occur in response to shocks to remittances, to the money supply, or to technology.

3 Theoretical Model

This section presents a limited participation model that requires money balances be held to finance certain types of purchases, and agents incur an adjustment cost when altering their money holdings. This model has been used to rationalize a large and persistent liquidity effect. We assume that any monetary shock occurs after households have decided on their deposit balances, and therefore these will generate a liquidity effect. To make this liquidity effect persistent we also introduce an adjustment cost on cash money holdings, $M_t^c$.

We model the cost of changing money holdings similarly to Hairault et al. (2004), who take into account the time spent on reorganizing the flow of funds. The adjustment cost is a time cost – a reduction in leisure in order to spend time adjusting money balances. The adjustment cost equation is:

$$
\Omega_t = \frac{\varepsilon}{2} \left( \frac{M_{t+1}^c}{M_t^c} - \theta_t \right)^2
$$

---

6 This skirts issues related to the transfer problem discussed in the trade literature, which we ignore. Samuelson provided a classic analysis of the transfer problem, showing that in perfectly competitive two-country two-good world, the donor always has reduced welfare and the recipient increased welfare. The donor country’s terms of trade may deteriorate if and only if the donor country’s marginal propensity to consume its own exports is lower than the recipient country’s marginal propensity to consume the donor country’s exports.
The long run value of $\frac{M_{t+1}^c}{M_t^c}$ is equal to the growth rate of money, represented by the parameter $\theta$, so both the level of $\Omega_\xi$ and its derivative with respect to $\frac{M_{t+1}^c}{M_t^c}$ is zero in the steady state. The cost of changing $M_t^c$ is an increasing function of the parameter $\xi$, and this parameter allows us to calibrate the size and persistence of the liquidity effect.

The cost of adjusting money holdings implies that bank deposits would not change significantly following a monetary shock, and consequently, the firm will have more funds to absorb as the decrease in the interest rate is stronger and more persistent. In addition, given uncovered interest rate parity (UIP), this large and persistent fall in the interest rate differential generates an overshooting in the exchange rate in accord with the stylized facts. The model is described in the following subsections.

3.1. Timing of decisions

We model a small open economy that includes a representative consumer-household, a goods-producing firm, a central bank, and a financial intermediary. We have a market for goods, labor, loanable funds, foreign assets, and a money market. Within each period the timing of decisions follows these five stages:

- At the end of period $t-1$ the representative household decides the amount of deposits ($M_t^h$) and cash ($M_t^c$) to carry into the next period.
- At the beginning of period $t$, migrants living abroad remit funds to agents in the small country. Remittances are nominal, in the currency of the small economy. After observing the remittances flow, the Central Bank makes its monetary policy decision, choosing the level of monetary injection.
- The credit market then opens. Bank deposits are available in quantity $M_t^h$ and the firm determines its demand for capital and labor to produce an internationally identical good. The firm borrows from the financial intermediary to finance the needed investment for production.
- The perfectly competitive goods market then opens. Production occurs, and purchasing decisions are made.
At the end of period \( t \), the foreign asset market opens. The representative household makes its decision to purchase or sell foreign assets, with returns given by the exogenous world interest rate. Labor is paid at this stage, and firms pay off their intra-period loans to the financial intermediary. Households decide on deposits and cash to carry into the following time period. As household owns both the bank and the firm, household receive dividend payments from the bank and firm as part of household income.

We assume that the evolution of money follows the time line presented below. The flow of nominal remittances \( (\mathcal{R}) \) occurs prior to the Central Bank decision on the monetary injection \( (X) \) necessary to achieve its desired monetary growth. Thus the monetary injection is net of the remittance flows.

\[
\begin{align*}
\begin{array}{c|c|c}
& t-1 & t \\
\hline
\mathcal{R}_{t-1} & X_{t-1} & \mathcal{R}_t \\
M_{t-1} & M_t & M_{t+1} = M_t + X_t
\end{array}
\end{align*}
\]

3.2. Structure of the model

The goods market is characterized by perfect competition, with domestic firms and the rest of the world producing an identical good whose price in domestic currency is given by \( P_t \). The law of one price holds. Letting \( e_t \) denote the price of foreign currency in terms of domestic currency, and keeping in mind that the small open economy assumption implies that the price of the good in foreign currency \( (P^*) \) and the foreign interest rate \( (i^*) \) are exogenous, purchasing power parity is given by:

\[
P_t = e_t P^* \tag{2}
\]

3.2.1. The household

The representative agent’s objective is to choose a path for consumption and asset holdings to maximize

\[
\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{3}
\]
where $C$ is real consumption and $L$ is leisure hours. We normalize the time endowment to unity, so leisure is given by

$$L_t = 1 - H_t - \Omega_t,$$

where $H$ is worked hours and $\Omega$ is time spent adjusting money balances.

We specify a parametric constant elasticity of substitution (CES) per-period utility function to facilitate calibration of our model:

$$U(C_t, L_t) = \left[ \frac{C_t^{1-\gamma} L_t^{\gamma}}{1-\sigma} \right]^{-\frac{1}{\gamma}}$$  \hspace{1cm} (4)

Here $\gamma$ is the relative weight of leisure in the above utility function and $\sigma$ define the inverse of the intertemporal elasticity of substitution with $0 < \sigma < 1$ and $0 < \gamma < 1$.

When the goods market opens – in the fourth stage – the cash-in-advance (CIA) constraint takes the form:

$$P_t C_t \leq M_t^c + \phi R_t + \varphi X_t$$  \hspace{1cm} (5)

where $M_t^c$ denotes cash brought forward from period $t-1$, $R_t$ is nominal remittances received by the household, and $X_t$ is the amount of money injected by the central bank.

The parameters $\phi$ and $\varphi$ take values between 0 and 1. The parameter $\phi$ indicates the percentage of remittances immediately available for consumption (as opposed to being held as bank deposits and only available for consumption in future periods) and the parameter $\varphi$ indicates the percentage of the monetary injection available for immediate consumption as opposed to being first channeled through the financial intermediary.\footnote{We introduce $\phi$ to allow us to study policies that induce (force) agents to keep a certain amount of remittances as deposits (increasing funds available for investment) and $\varphi$ to allow for different channels through which money is injected by the central bank, either helicopter drops on households or helicopter drops on banks.}

These parameters allow us to change the channel in which remittances and monetary injections affect the economy, and to see how the end use of remittances and monetary injections matter.

Household can hold foreign assets that yield a risk-free exogenous nominal interest rate $i^*$. In each period the household buys foreign assets $B_{t+1}$ denominated in the
foreign currency, so the nominal exchange rate becomes a key variable in the portfolio decision of the household.

The household budget constraint is given by:

\[ M^c_{t+1} + M^b_{t+1} + e_i B_{t+1} + P_t C_t \leq M^c_t + \phi R_t + \varphi X_t + P_t w_t H_t + (1 + i_r^*) M^b_t + e_i (1 + i_r^*) B_t + D^f_t + D^b_t \]  \hspace{1cm} (6)

At time \( t \) the household determines consumption \( C_t \) and labor supply \( H_t \), as well as the amount of money deposited in banks, \( M^b_{t+1} \), the amount of money kept as cash, \( M^c_{t+1} \), and the foreign asset position \( B_{t+1} \). Household income is determined by the real wage \( w_t \) and the profits (or dividends) received at the end of the period from the firm and the bank, \( D^f_t \) and \( D^b_t \), as well as interest on deposits and on foreign bonds.

The household’s maximization problem can be represented by the value function

\[ V(M^c_t, M^b_t, B_t) = \max_{\{C_t, H_t, M^c_{t+1}, M^b_{t+1}, B_{t+1}\}} \left\{ U(C_t, 1 - H_t - \Omega_t) + \beta E \left[ V(M^c_{t+1}, M^b_{t+1}, B_{t+1}) \right] \right\} \]

subject to the cash-in-advance constraint (5) and the budget constraint (6). Letting \( \lambda_t \) denote the Lagrangian multiplier associated with the budget constraint, the first order necessary conditions for the household’s choice of consumption, labor, money deposits, money-cash holdings, and foreign assets provide the following relationships:

\[ \lambda_t = \beta E_t \left[ (1 + i_r^*) \lambda_{t+1} \right] \]  \hspace{1cm} (7)

\[ -U_{H_t} = w_t P_t \lambda_t \]  \hspace{1cm} (8)

\[ e_i \lambda_t = \beta E_t \left[ e_i (1 + i_r^*) \lambda_{t+1} \right] \]  \hspace{1cm} (9)

\[ P_t w_t \lambda_t \frac{\xi}{M^c_t} \left( \frac{M^c_{t+1}}{M^c_t} - \theta \right) + \lambda_t = \beta E_t \left[ \frac{U_{C_{t+1}}}{P_{t+1}} \right] \]

\[ + \beta E_t \left[ P_{t+1} w_{t+1} \lambda_{t+1} \frac{\xi M^c_{t+2}}{(M^c_{t+1})^2} \left( \frac{M^c_{t+2}}{M^c_{t+1}} - \theta \right) \right] \]  \hspace{1cm} (10)

Equation (7) requires equality between the costs and benefits of bank deposits, while equation (8) requires equality between the marginal disutility of working and the marginal benefit – the real wage multiplied by the Lagrange multiplier. Equation (9) requires equality of the current marginal cost of buying foreign assets (in terms of wealth)
with the gains in the following period from holding such assets today, and equation (10) equates the costs and benefits related to the choice made at time $t$ of money holdings available for consumption in the following period. It is clear that if the adjustment cost is zero ($\xi = 0$) then equation (10) will just equate the household’s cost of holding money in the current period to the marginal utility of consumption in the following period, properly discounted. However, when adjustment costs exist ($\xi \neq 0$), the household will compare the cost of changing money holdings (cash) today to the benefits accrued in the next period with respect to the purchasing power of money holdings and the in-advance time saved rearranging the household portfolio.

3.2.2. The Firm

We specify the firm’s production technology using a parametric, Cobb-Douglas functional form:

$$Y_t = e^{\alpha H_t^{1-\alpha}}$$  \hspace{1cm} (11)

Here $\alpha \in [0,1]$ and $K$ is physical capital. The firm’s objective is to maximize the discounted stream of dividend payments, where we consider the value of this discounted dividend stream to the owners, households. The firm receives its profits at the end of the period, so the firm borrows funds from the bank to invest in physical capital at the beginning of the period, with the cost of borrowing given by the nominal interest rate $i_t$.

Consequently, the nominal profits of the firm are given by

$$D_t^f = P_t Y_t - P_t w_t H_t - P_t (1 + i_t) I_t$$  \hspace{1cm} (12)

with investment evolving according to the law of motion of the stock of physical capital,

$$I_t = K_{t+1} - (1 - \delta) K_t$$  \hspace{1cm} (13)

where $\delta$ is the (constant) depreciation rate. The decision about the use of dividends, either payments to households or reinvestment in the firm, is captured by the ratio of the multipliers associated with the budget constraint of the household in the value function (see equation (7)), as it reflects the consumer’s variation in wealth. The value function of the firm is then

Note that we assume that firms can only borrow for incremental investments, which need to be paid off completely by the end of the period.
\[ V(K_t) = \max \{ D_t^f + E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} V(K_{t+1}) \right] \} \]  

(14)

Note that the discount factor \( \beta \frac{\lambda_{t+1}}{\lambda_t} \) can be written as \( [E_t(1 + i_{t+1})]^{-1} \), reflecting the fact that the appropriate discount rate is time varying and reflects the expected value of the market-determined interest rate.

The first order necessary conditions for the household’s choice of labor and capital take the form:

\[ w_t = (1 - \alpha) \frac{Y_t}{H_t}, \]  

(15)

\[ 1 + i_t = \beta E_t \left[ \frac{P_{t+1} \lambda_{t+1}}{P_t \lambda_t} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)(1 + i_{t+1}) \right) \right], \]  

(16)

Equation (15) indicates that the cost of hiring an additional worker should equal that worker’s marginal productivity, and equation (16) requires equality between the cost and benefit of the marginal investment.

3.2.3. The Central Bank

The money stock evolves according to

\[ M_{t+1} = M_t + X_t, \]  

(17)

where the Central Bank’s money injection is defined as

\[ X_t = (\theta_t - 1)M_t, \]  

(18)

and where \( \theta_t \) represents the monetary growth factor, itself possibly a function of the size of the remittances flow.\(^9\) Equation (17) indicates that money growth in the economy depends on the existing stock of money \( M_t \) and the monetary injection implemented by the central bank \( X_t \). The timing here is that \( M_t \) is the beginning-of-period \( t \) money stock. After remittances occur in period \( t \), the central bank decides on the monetary injection, \( X_t \), and this injection determines the money stock carried forward into period \( t+1 \).

\(^9\) This monetary growth rate will depend on remittances in a fixed exchange rate regime, where the monetary authority will need to sterilize such flows.
The monetary growth factor $\theta_t$ is specified as:

$$\log(\theta_{t+1}) = (1 - \rho_{\theta}) \log(\bar{\theta}) + \rho_{\theta} \log(\theta_t) + \epsilon_{\theta,t+1}$$

We also define $g_t$ as the growth factor for remittances, which evolves according to the first order autoregressive process:

$$\log(g_{t+1}) = (1 - \rho_g) \log(\bar{g}) + \rho_g \log(g_t) + \epsilon_{g,t+1}$$

We specify the technology shock to the production function in the usual way,

$$\log(z_{t+1}) = (1 - \rho_z) \log(\bar{z}) + \rho_z \log(z_t) + \epsilon_{z,t+1}$$

Here $\epsilon_{g,t+1}, \epsilon_{\theta,t+1},$ and $\epsilon_{z,t+1}$ are white noise innovations with variance $\sigma_g^2, \sigma_{\theta}^2,$ and $\sigma_z^2,$ respectively.

3.2.4. The financial intermediary

At the beginning of the period, the financial intermediary or ‘bank’ receives deposits from the household, $M_t^b$, receives a portion of remittances as deposits, $(1 - \phi)R_t$, and receives a portion of the monetary injection as deposits, $(1 - \phi)X_t$. These funds are then available for lending to the firm to pay for the firm’s investment in physical capital. At the end of the period, the firm repays its loans, and the bank returns deposits to the household along with the appropriate interest payment.

To make this clearer, the bank’s nominal asset balance is given by

$$P_tI_t = M_t^b + (1 - \phi)R_t + (1 - \phi)X_t$$

Here $P_tI_t$ are the loans made to firms and the right hand side lists sources of funds including deposits, a portion of remittances, and a portion of the monetary injection.

Bank profits per period are equal to the interest on loans minus interest paid on deposits and on remittances deposited in banks. Note that the monetary injection directly into banks is a subsidy to the bank in that there is no interest expense incurred by the bank.

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10 The deposit amount from remittances could be zero if the total amount of remittances received is immediately disbursed to the agent such that it will just add to money-cash available for consumption. The monetary injection $X_t$ is a helicopter drop that can be split between households and banks. When dropped on banks, it can lend out in the current period $t$, earning interest that is then distributed back to the households at the end of the period. When dropped on households, it is directly available for consumption in period $t$. 

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11 The deposit amount from remittances could be zero if the total amount of remittances received is immediately disbursed to the agent such that it will just add to money-cash available for consumption. The monetary injection $X_t$ is a helicopter drop that can be split between households and banks. When dropped on banks, it can lend out in the current period $t$, earning interest that is then distributed back to the households at the end of the period. When dropped on households, it is directly available for consumption in period $t$. 

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13
bank on those funds. Note too that we have equality between the loan rate and the deposit rate. Absent monetary injections, the bank earns zero economic profits.

\[ D^b_i = (1 + i_i)P_i I_i - (1 + i_i)M^b_i - (1 + i_i)(1 - \phi)\Re_i \]  

(23)

Putting both expressions together results in profits of the intermediary depending only on the money injection provided by the monetary authority

\[ D^b_i = (1 + i_i)(1 - \phi)X_i \]  

(24)

3.2.5. Closing the model

To complete the model specification it is worth to note that there is an uncovered interest rate parity condition (UIP) from combining equations (7) and (9):

\[ E \left[ \frac{P_{t+1}}{1 + \pi_{t+1}} \right] = E \left[ \frac{\lambda_{t+1} e_{t+1}}{1 + \pi_{t+1}} \right] \]  

(25)

Here \( \pi \) is the net inflation rate at time \( t+1 \). Since we are modeling a small open economy with international assets freely traded, the no-arbitrage condition leads to UIP.

We assume remittances are based on the income of the receiving economy, and we further assume that remittances are negatively correlated with income deviations from the steady state – perhaps in accord with the altruistic view. Thus remittances increase when the receiving country experiences an economic downturn. Our specification follows Chami et. al. (2006), and is written as:

\[ \Re_i = E_i \left[ \frac{1}{Y_i} \right] \left[ \frac{\rho P_{t+1}}{Y_i} \right] \]  

(26)

A special cases of interest would be \( \tau = 0 \), so that remittances respond only to the domestic price level and to the growth rate \( g \). For other values of \( \tau > 0 \) remittances react to the state of the recipient economy, rising when the state of the economy worsens.

3.3. Equilibrium

The system’s equilibrium is characterized by the set of prices and quantities

\[ \Omega^p = \{w, i, P_t, e_t\}^{\infty}_{t=0} \]

\[ \Omega^C = \{C_t, H_t, B_{t+1}, M_{t+1}, M^b_t, \Re_t\}^{\infty}_{t=0} \]

\[ \Omega^Q = \{Y_t, H_t, K_{t+1}\}^{\infty}_{t=0} \]
and the vector of exogenous foreign variables \( \{P^*, i^*\} \). Given these prices and quantities, the set of quantities \( \Omega^C \) maximizes the household’s expected intertemporal utility subject to (5) and (6), the set of quantities \( \Omega^Q \) maximizes the profits of the firm subject to (12) and (13), and the set of prices \( \Omega^P \) ensures that the labor market, the loanable funds market, and the money market all clear, all while satisfying purchasing power parity.

Note that the household can, in principle, hold any quantity of foreign assets that it finds optimal, subject only to its budget constraint. From equation (6) and market equilibrium we can infer that foreign asset holdings evolve according to

\[
e_t B_{t+1} - e_t (1 + i^*) B_t = P_t (Y_t - C_t - I_t) + (1 - (1 + i_t)(1 - \phi)) \mathcal{R}_t
\]  

Equation (27) relates domestic production and absorption to an economy’s foreign asset position, giving the balance of payments equilibrium. If a country’s production is greater than its absorption, that country has a balance of trade surplus and a negative capital account, so its foreign asset holdings will increase when there are no remittances flowing into the country. Of course, the actual equilibrium impact of remittances on future bond holdings depends on its level and on its impact on output, consumption, and investment.

The set of equations given by the first order conditions, the market equilibriums, and the laws of motion for physical capital, domestic money supply, foreign assets, and the monetary growth factor constitute a non-linear dynamic stochastic system. The system of equations is presented in the appendix (A.1) together with the log-linearized system following Uhlig’s (1997) methodology. To solve this system we calibrate certain basic parameters and find the steady state values of the relevant variables to characterize the long-run equilibrium of the economy.

3.4. Calibration and steady state equilibrium

Our calibration of the standard parameters is based in part on Hairault et al. (2004), supplemented with specific parameters we derive from a sample of countries used in this study: Bolivia, Brazil, Colombia, Ecuador, El Salvador, Guatemala, Honduras, Mexico, Panama, and Peru. The periods in the model are given by quarters.

Table 1 lists the values we assign to the basic parameters. The first three parameters have a standard calibration. The capital share, \( \alpha \), is set to 0.36. The
subjective discount factor $\beta$ is set at 0.988, implying a real interest rate equal to 1.2% per quarter. The depreciation rate on capital is set to roughly 2.5% per quarter. We set the parameter $\gamma$ to 0.75, which implies that the representative household devotes 80% of its time endowment to non-working activities, roughly a 34-hour work week. The remaining parameters are derived from data from our sample of Latin American countries covering the period 1990 to 2004, and then converted into quarterly measures. The data come from the World Bank’s World Development Indicators database. The parameter $v$ represents the average of the trade balance to GDP, and is used to determine the long-run real debt-to-GDP ratio in our steady state calculation. The long run gross inflation factor is given by $\Pi$, and is based on the average inflation factor of the countries in our sample. We set the average money growth rate parameter, $\theta$, to 1.038, or 3.8% per quarter. Remittances are calibrated to be 5 percent of GDP, with a steady-state growth rate of 5.5% per quarter. The persistence coefficient of the remittance’s shock, $\rho_g$, and the standard deviation of the remittance’s innovation, $\sigma_g$, are obtained from regressions on the remittance’s base of the countries in the sample. Similarly, the persistence coefficient of the monetary shock, $\rho_{\theta}$, and the standard deviation of the monetary innovation, $\sigma_{\theta}$, are obtained from regressions on the monetary base of the countries in the sample. Finally, we calibrated the technology shock, persistence and variance, to match the parameters of Chami et. al. (2006).

We explicitly consider three values for the adjustment cost parameter, $\xi$. We examine the benchmark case of no adjustment cost, $\xi = 0$, and also the cases of small but positive adjustment costs to allow for the liquidity effect. These positive adjustment costs represent lost time rearranging money cash balances of almost three minutes per week (when $\xi = 5$) and almost 6 minutes per week (when $\xi = 10$).

Table 1: Model Calibration Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.75</td>
</tr>
<tr>
<td>$g$</td>
<td>1.055</td>
</tr>
<tr>
<td>$\rho_g$</td>
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</tr>
<tr>
<td>$\sigma_g$</td>
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</tr>
<tr>
<td>$\beta$</td>
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</tr>
<tr>
<td>$\theta$</td>
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</tr>
<tr>
<td>$\theta$</td>
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</tr>
<tr>
<td>$\rho_{\theta}$</td>
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</tr>
<tr>
<td>$\sigma_{\theta}$</td>
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</tr>
<tr>
<td>$\delta$</td>
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</tr>
<tr>
<td>$\phi$</td>
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</tr>
<tr>
<td>$\nu$</td>
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</tr>
<tr>
<td>$\rho_{\theta}$</td>
<td>0.63</td>
</tr>
<tr>
<td>$\sigma_{\theta}$</td>
<td>0.00336</td>
</tr>
</tbody>
</table>
The equations are written to describe a stationary system and are the ones presented in the beginning of A.1 in the appendix. Nominal variables are made stationary by dividing them by the lagged domestic price level. The main variables are:

\[ m_t = M_t / P_{t-1} ; m_t^b = M_t^b / P_{t-1} ; \pi_t = \pi_t / P_{t-1} ; b_t = e_{t-1} B_t / P_{t-1} ; \Gamma_t = \Re_t / P_{t-1} \]

In order to evaluate the implications of the positive exogenous shock in the limited participation model, different adjustment costs are introduced to observe the behavior of the nominal interest rate, output, nominal exchange rate, and consumption following such shocks, both in terms of impulse response functions as well as in quantitative terms.

3.4.1 Steady state equilibrium

We outline the calculation of steady state equilibrium values for the remaining variables in this section. Obviously adjustment costs disappear in the steady state, and steady state values do not need time subscripts. In the long-run equilibrium we assume the domestic gross inflation rate is given by the gross money growth rate, see equation (62), so that \( \Pi = \theta \).

We look at a steady state in which the domestic and foreign inflation levels are the same, so equation (57) implies that the change in the nominal exchange rate, \( \Delta e = \frac{e_t}{e_{t-1}} \), is constant and equal to unity\(^1\). Consequently the uncovered interest rate parity condition implies that the domestic and the foreign interest rates are equal (\( i = i^* \)). Finally, combining equations (52) and (54) and, after some manipulation, we have that the domestic nominal interest rate in steady state is

\[ i = \frac{\Pi}{\beta} - 1 \]

We can derive the steady state level of remittances from equation (26) as

\[ \Gamma = \Re \Pi \]

To find the steady state capital/output ratio (denoted \( \kappa \))\(^1\) we get, from the stationarity of equation (61):

\(^{11}\) Note that this assumption just sets the steady-state nominal exchange rate to be constant, while allowing a different steady-state foreign inflation rate will make the steady-state exchange rate to be growing at a constant rate.

\(^{12}\) This term is divided by 4 to account for the quarterly estimation.
\[ 1 + i = \beta \left[ \frac{Y}{K} + (1 - \delta)(1 + i) \right] \]

\[ \frac{1 + i}{\beta} - (1 - \delta)(1 + i) = \alpha \frac{Y}{K} \]

\[ \kappa = \frac{K}{Y} = \alpha \left[ \beta \frac{1 + i - (1 - \delta)(1 + i) \beta}{1 + i + \frac{1}{(1 - \delta)(1 + i) \beta}} \right] \]

Then from the production function we can solve for the output/labor ratio

\[ \frac{Y}{H} = \kappa^{\alpha \gamma} \]

which can be used in equation (60) to solve for the real wage

\[ w = (1 - \alpha) \frac{Y}{H} \]

Solving for \( H \) in equation (51), and substituting \( \Lambda \) from equation (54), we can solve for the consumption/output ratio

\[ \frac{C}{Y} = \frac{w \beta}{\Pi \gamma} \left[ 1 - \kappa^{\alpha \gamma} \right] \]

Letting \( TB = Y - C - I + (1 - (1 + i)(1 - \phi))(1 - \delta \kappa) \) to be the adjusted trade balance, and using the calibration for \( v = TB/Y \), we obtain the long-run real debt-to-GDP ratio that is equal to the domestic trade balance as a share of GDP

\[ b \left( 1 - \frac{1}{1 + \pi} \right) = \frac{TB}{Y} = v \]

This and equation (64), together with the capital/output ratio, allows us to write the steady state output as

\[ Y = \frac{\left[ 1 - (1 + i)(1 - \phi) \right] \beta - \frac{w \beta(1 - \gamma)}{\Pi \gamma}}{\left( v - 1 - \frac{w \beta(1 - \gamma)}{\Pi \gamma} \right) \kappa^{\alpha \gamma} + \delta \kappa} \]

then the steady state physical capital stock will be given by \( K = \kappa Y \), and the steady state investment rate will be given by \( I = \delta K \).
The steady state stock of foreign assets in real terms is derived from the balance of payments equilibrium (64), so the household’s stock of foreign assets in real terms is

\[ b = \left( \frac{1 + \pi}{\pi - i^*} \right) vY \]

Consequently, the steady state consumption level is given by:

\[ C = Y + (1 - (1 + i)(1 - \phi)) \frac{\Gamma}{\Pi} - I - \left( \frac{1 + i^*}{1 + \pi} \right) b \]

Given that real money balances is defined by equation (55), its steady state level is:

\[ m = m^b + m^c \]

Combining equations (56) and (63), the steady state for real money balances is:

\[ m = C - \frac{1}{\theta} \Gamma + I \]

Then using (56), the household’s steady state deposit balances are

\[ m^b = [\Pi - (1 - \phi)(\theta - 1)]I - (1 - \phi)(\theta - 1)C + \left[ \frac{(1 - \phi)(\theta - 1)}{\theta} - (1 - \phi) \right] \Gamma \]

From the definition of preferences, the marginal utility of wealth in the steady state is given by

\[ \Lambda = \beta(1 - \gamma)C^{\gamma - \sigma(1 - \gamma)}(1 - H)^{(1 - \sigma)}\Pi \]

The steady state values of these variables are presented in Table A.1 in the appendix, gives the steady state values under two alternative calibrations of remittances, one with remittances equal to 5% of GDP, and the other with remittances equal to 10% of GDP. The nominal interest rate is 5.06% per quarter in either instance, and the capital output ratio is unaffected by the level of remittances. We have the same inflation rate for either level of remittances – which is only dependent on the steady state money growth rate, and thus independent of the level of remittances. Output is affected somewhat by remittances, and falls by 4.3% when remittances rise from 5% to 10%. This occurs because the capital stock and labor hours worked are also about 4.3% lower. Meanwhile consumption is higher by about 1%. Thus a steady flow of outside purchasing power
results in households choosing more leisure while also having more consumption. Remittances are good for households but do not necessarily lead to an increase in steady-state domestic production.

4 Results

Given the steady states values from the previous section, we analyze the aggregate dynamics of the nominal interest rate, output, the nominal exchange rate, and consumption following expansionary monetary, technological, and remittances shocks. We present results for the case of no adjustment costs, for a small but positive adjustment cost of about 3 minutes per week \( \xi = 5 \), and a larger adjustment cost equivalent to 6 minutes per week \( \xi = 10 \).

Our model allows a variety of specifications for the percentage of remittances going to consumption and investment, and similarly for the monetary injection. It turns out that the main dynamics can be observed in our baseline specification, with remittances going almost entirely for consumption \( \phi = 0.99 \) and the monetary injection going first through the financial intermediary for investment \( \varphi = 0 \). Therefore for the sake of brevity we present impulse responses only for this case. We provide a brief discussion of how different assumptions on the distribution of remittances and monetary injections – between consumption and investment – will affect the impulse response functions.

The results presented below hold for an elasticity of substitution of 1.01. This is our baseline; we investigate the impact on these results of larger and smaller values \( \sigma = 1.5 \) and \( \sigma = 0.5 \).

4.1 Monetary Shock

The impulse response functions presented in this section are those following a 3.8% increase in the home money growth factor in period 0, a magnitude large enough to bring the monetary growth to a halt in the case of a negative shock. The case with no adjustment costs is illustrated with a solid line, the case with the smaller adjustment cost with dashed lines, and the case with the larger adjustment cost with dotted lines.

4.1.1 Nominal Interest Rate Response
The monetary injection leads to a rise in the nominal interest rate, increasing on impact by about 1% of the steady state value, equivalent to about 4.5 basis points, and peaking on the third quarter with 5.5 basis points higher than steady state when there is no adjustment cost ($\xi = 0$). This coincides with the positive response typical in CIA models. Introducing adjustment costs enables us to generate the observed liquidity effect, with the monetary shock leading to a drop in the interest rate, falling by over 4% of the steady state value, equivalent to about 21 basis points, when the adjustment cost is small ($\xi = 5$), and by 32 basis points when there is a larger adjustment cost ($\xi = 10$). At the time the shock occurs, the increased monetary injection increases the money supply, increasing inflation and putting downward pressure on the nominal interest rate because households cannot withdraw their deposits within the period. This is the liquidity effect, and its persistent effect on the interest rate can be observed below in Figure 2.

Response to a 3.8 percent deviation in monetary shock

![Graph showing the response of the nominal interest rate to a 3.8 percent monetary shock with different adjustment costs.](image)

**Figure 2: Nominal interest rate dynamics following a monetary shock**

The monetary shock raises inflation momentarily, which reduces the value of real money balances and induces households to increase their holdings of money cash the following period to satisfy their consumption level, thus reducing its money deposits ($M^h_{t+1}$). The magnitude of the drop in the interest rate is determined by the cost of money adjustments. However, even if the household reduces its money deposits the following period, the liquidity effect is persistent because firms raise their investment the period of the shock to take advantage of the lower interest rate and in anticipation of the relatively lower money supply that would result from the expected deposit withdrawals in the future. This increased investment results in a larger capital stock, which lowers the marginal product of capital and leads firms to reduce their demand for loans more than...
the household’s reduction of money deposits the following period, thus keeping the nominal interest rate below its steady state level and producing a persistent liquidity effect.

4.1.2 Output Response

The output response to a monetary shock depends on the assumed adjustment costs for cash holdings. With no adjustment costs, the monetary shock causes a temporary increase in output in the second period, followed by a long period of output below steady state. With adjustment costs, this pattern is inverted, with output first temporarily declining, followed by a long and persistent period of output above the steady state. Output peaks after 7 quarters in the case of the smaller adjustment cost and after 9 quarters in the case of the larger adjustment cost.

![Response to a 3.8 percent deviation in monetary shock](image)

Figure 3: Output dynamics following a monetary shock

An expansionary monetary shock generates a positive wealth effect, which is allocated to increase leisure in the first period because of the cash-in-advance constraint and adjustment cost of money holdings. However, from the second period onwards, when there is no adjustment cost, the increase in real wages induce agents to increase labor above the initial steady state level, which combined with the surge in capital from the second period onwards due to the lower cost of investment explains the sporadic increase in output in the short run. When adjustment costs are positive, while the fall in working hours is stronger its recovery is persistent and long-lived. Together with the surge in investment, this leads to an increase capital beginning the following period thus producing higher levels of output that continue to increase for about 8 periods.

4.1.3 Nominal Exchange Rate Response
In the baseline case of no adjustment cost, we observe that the monetary injection causes a continuous depreciation of the nominal exchange rate. When we introduce adjustment costs, the monetary injection leads to the instantaneous fall in the nominal interest rate, reducing the return on domestic savings, and inducing households to hold more foreign assets. This leads to an instantaneous depreciation of the nominal exchange rate on impact, depreciating by 13 percent on impact with the smaller adjustment cost \( \xi = 5 \), and by 15 percent with the larger adjustment cost \( \xi = 10 \). The overshooting of the nominal exchange rate shown in Figure 4 is due to the uncovered interest rate parity (equation (25)), which requires the interest rate differential to be equal to the expected rate of appreciation, leading to the subsequent appreciation until it reaches its new steady state, as the liquidity effect is expected to be persistent.

![Response to a 3.8 percent deviation in monetary shock](image)

**Figure 4: Nominal Exchange Rate dynamics following a monetary shock**

The overshooting of the nominal exchange rate is accentuated by the size of the adjustment costs, as it creates a larger and persistent liquidity effect that requires a more accentuated appreciation. In fact, the higher \( \xi \) the more limited the withdrawal of private deposits, the farther the fall in the interest rate, and the larger the initial depreciation of the exchange rate. Even if agents respond to the below-steady-state domestic interest rate with a continuously increase in their holdings of foreign bonds, the initial overshooting of the exchange rate is strong enough to allow for the subsequent appreciation, even if the demand for the foreign asset is still rising.

### 4.1.4 Consumption Response

The consumption dynamics following a monetary injection is primarily generated from the inflationary pressure during the period of the shock. Given that the consumption
level is determined by the cash-in-advance constraint, and since the amount on money-cash can not be changed during the period of the shock, inflation generated by the larger money supply reduces consumption instantaneously, mimicking the inverse dynamics of inflation when there is no adjustment costs, but returning to steady state more gradually when there is a positive adjustment cost. The consumption dynamics from the second period onwards arises from the rearrangement between money-cash and money-deposits. Since agents anticipate inflation, and in order to preserve their consumption in the future, households increase their future amount of nominal money cash the period of the shock ($M_{t+1}^c$). While it is relatively inexpensive to change the ratio $M_{t+1}^c/M_t^c$ when there are no adjustment costs, thus adjusting consumption quickly, this ratio would be adjusted smoothly when there are adjustment costs, thus inducing persistence in the adjustment of consumption.

![Response to a 3.8 percent deviation in monetary shock](image)

**Figure 5: Consumption dynamics following a monetary shock**

Our model allows us to consider the influence, if any, of how we specify the channel by which remittances first impacts the economy. We can specify that remittances first end up in the hands of households as cash, loosening the cash in advance constraint. We can also specify that some portion of remittances end up in banks as deposits, which in the period of impact will mean additional funding available for bank loans to fund firm investment. However, we note that the impact of a monetary shock is not significant in this modeling choice. The method by which remittances first enter the economy has almost nothing to do with the responses of the economy to a monetary shock.
Our model also allows us to consider the influence, if any, of how we specify the channel by which a monetary injection first impacts the economy. We consider monetary injections that are basically helicopter drops on households, loosening the cash-in-advance constraint, and helicopter drops on banks. As the fraction of a monetary injection that is initially channeled through the financial intermediary is reduced, so that monetary injections directly fall to households and hence impact household consumption, the impulse response functions show very similar patterns that vary only slightly in magnitude but not in qualitative impact or in timing. For example, as we increase the fraction of the monetary injection that goes to the household for consumption, the initial decline in the nominal interest rate and the exchange rate overshooting are reduced in magnitude, while the output and consumption responses are also reduced in magnitude, with the ‘hump’ in the output response being slightly delayed.

These results we find here are similar to those obtained in related papers (e.g. Hairault et. al. (2004), Chari et al. (2001), and Christiano and Eichenbaum (1992)).

4.2 Technology Shock

We analyze the behavioral response of the main macroeconomic variables to a positive 1 percent technology shock. We maintain our baseline assumptions, that the elasticity of substitution parameter is 1.01, that remittances go almost completely into consumption ($\phi = .99$), and that monetary injections go completely into investment via the financial intermediary ($\varphi = 0$).

As in the previous section, the cases with no adjustment costs are illustrated with solid lines, the cases with smaller adjustment costs with dashed lines, and the case with larger adjustment cost with dotted lines.

4.2.1 Nominal Interest Rate Response

A technology shock has a direct effect on output, which outweighs the fall in inflation to put upward pressure on the nominal interest rate. On impact, the nominal interest rate increases by 45 basis points with no adjustment costs ($\xi = 0$), by 55 basis points with smaller adjustment costs ($\xi = 5$), and by 60 basis points with larger adjustment costs ($\xi = 10$). The increase in output brought about by the technology shock

13 Results are available upon request.
lowers inflation initially and raises consumption the period of the shock, which fuels an important increase in investment to raise physical capital. This higher demand for loans exerts pressure to raise the nominal interest rate above its initial steady state level as shown in Figure 6.

![Response to a 1 percent deviation in technology shock](image)

**Figure 6: Nominal Interest Rate dynamics following a technology shock**

The dynamics of the nominal interest rate after the shock is determined by the adjustment of money cash balances. When there is no adjustment cost, the period following the shock is still dominated by the further increase in investment to satisfy the above-steady-state consumption level, and while the rise in inflation contributes to the continuous upward pressure on the interest rate, the larger increase in money deposits exerts a stronger pressure on the opposite direction, forcing the nominal interest rate down. The fall of the interest rate towards its steady state continues thereafter as investment, inflation, and money deposits returns to their initial steady state levels. These dynamics are also observed for the case of positive adjustment cost, but the nominal interest rate returns to the initial level at a lower pace, which is mainly due to the much smaller increase in money deposits, whose continuous increase for couple more periods is enough to outweigh the much lower decline in investment.

### 4.2.2 Output Response

The technology shock increases output by almost 1.9 percent on impact, irrespective of the adjustment costs. The positive impact on physical capital is reinforced by the increase in hours worked fueled by the rise in real wages. Since these two factors are the main determinants of the production function, their rise results in an increase in output that continues for another 7 quarters, peaking at almost 2.3 percent above the
initial steady state level when there are no adjustment costs and at almost 2 percent above the initial steady state level when there are positive adjustment costs. These subsequent dynamics arise from the continuous increase in both physical capital and hours worked during these quarters, with the increase in physical capital being fueled by the above-steady-state levels of investment and the increase in labor supply being brought about by the direct effect on the real wage.

![Response to a 1 percent deviation in technology shock](image)

*Figure 7: Output dynamics following a technology shock*

The positive effect on output is in accord with existing analyses of technological shocks, with its long lasting effect being determined by the continuous investment brought about by the large increase in money deposits that outweighs the higher than steady state interest rate.

### 4.2.3 Nominal Exchange Rate Response

The initial nominal exchange rate response to the positive technology shock is determined by the rise of the nominal interest rate, which is only partially neutralized by the fall in inflation. The nominal exchange rate appreciates by 3.2 percent on impact when there are no adjustment costs ($\xi = 0$), by 4 percent when there are smaller adjustment costs ($\xi = 5$) and by 4.3 percent when there are larger adjustment costs ($\xi = 10$), as shown in Figure 8 below. The overshooting of the exchange rate is governed by the uncovered interest rate parity condition that requires that the interest rate differential is equal to the expected rate of depreciation, which is accentuated when there is a positive adjustment cost. The expected persistent increase in the nominal interest rate ($E_t \hat{r}_{t+1} > 0$) generates a positive interest rate differential and thereby causes the persistent expected depreciation of the exchange rate ($E_t \hat{d}_{t+1} - \hat{d}_t > 0$).
From a balance of payments perspective, the above-steady-state domestic interest rate induces agents to reduce their holdings of foreign bonds, which also forces the initial appreciation. As the domestic interest rate returns to its initial level, the rearrangement of foreign bonds gets reversed, with the resulting higher demand for foreign bonds pressuring the nominal exchange rate upwards and producing its continuous depreciation. The higher the adjustment cost, the slower the return of the nominal interest rate to its initial level, causing a larger and longer fall in the demand for foreign bonds.

4.2.4 Consumption Response

The effect of the positive shock to technology on consumption is again primarily determined by the cash-in-advance constraint, which is mainly influenced by the inflation dynamics and the flexibility to adjust the money balances. In the period of the shock, the predetermined amount of cash and the fall in inflation leads to an increase in consumption, rising by almost 2.7 percent with no adjustment costs, by almost 3.5 percent with the smaller adjustment costs, and by almost 4 percent with larger adjustment costs. However, the consumption dynamics following the period of the shock are also affected by other factors. With no adjustment costs, consumption drops immediately following the initial positive response, to a level below the initial steady state. This if followed by a long and very gradual increase in consumption.
The consumption dynamics following the period of the shock are much more persistent when there are positive adjustment costs. In these cases, the fact that cash is brought back to its initial steady state level only slowly allows for above-steady-state levels of consumption to persist, returning to the steady state at the same rate as money cash.

The effect of the positive technology shock on our model is in accord with the existing literature, with the representative agent being able to increase output and consumption, which raises the domestic nominal interest rate and allows agents to reduce their holdings of foreign bonds at least in the short run, producing the initial nominal exchange rate appreciation described above. These results are robust to the alternative distributions of remittances and monetary injection, as described in the monetary shock section, and the dynamics are only affected by small changes in magnitude.

4.3 Remittances Shock

We first analyze the behavior of the economy to a 5.5% positive remittances shock in our baseline calibration through its impact on the nominal interest rate, output, nominal exchange rate, and consumption, to then examine the overall effect on the welfare of the receiving economy, measured by the utility, adjusted trade balance, and real exchange rate. Note that the magnitude of the shock is large enough to bring the growth of remittances to a halt in the negative case. Our baseline case assumes the elasticity of substitution parameter is 1.01, that remittances go almost completely into consumption ($\phi = .99$), and monetary injections go completely into investment via the financial intermediary ($\phi = 0$).
4.3.1 Nominal Interest Rate Response

The introduction of a positive remittances shock lowers the interest rate slightly on impact, irrespective of the existence of adjustment costs. Although the remittances shock increases inflation slightly on the period of the shock, the decrease in investment is relatively larger such that its downward pressure outweighs the upward pressure from inflation. This lower demand for loans exerts the pressure to lower the nominal interest rate below its initial steady state level as shown below in Figure 10. The initial impact on the nominal interest rate is larger when there are no adjustment costs ($\xi = 0$), declining by 0.45 basis points.

![Response to a 5.5 percent deviation in remittances shock](image)

*Figure 10: Nominal Interest Rate dynamics following a remittances shock*

The dynamics of the nominal interest rate after the period of the shock are governed by the dynamics of investment and money deposits. When there are no adjustment costs ($\xi = 0$), the fall in inflation below the steady state, combined with the smaller recovery in investment relative to the increase in money deposits in the subsequent period, further reduces the interest rate for an additional period before it starts to rise, which itself is due mainly to the slow but continuous rise in inflation. These same dynamics are in play for the cases of positive adjustment costs, but the further decrease in the interest rate is extended for an additional period, mainly due to the slower adjustment of money-cash balances, and money-deposits, brought about by the adjustment cost. Since both investment and money-deposits peak at levels above their initial steady state four quarters after the remittances shock, with similar proportional increases, it is only when inflation starts to rise slowly back to its steady state level that the interest rate

30
begins to rise monotonically back to its original level, creating a persistent liquidity effect.

4.3.2 Output Response

The remittances shock decreases output on impact irrespective of the existence of adjustment costs, but its long term dynamics are affected by the magnitude of the adjustment cost. When there is no adjustment costs ($\xi = 0$) the remittances shock slightly lowers the amount of hours worked on impact even if the real wage increases - wealth effect. Since the capital stock is fixed, this reduction in labor causes output to fall slightly. However, since labor further declines the following period, outweighing the increase in the capital stock, output decreases for an additional period. This decline in labor is reversed only after two periods, giving rise to an increase in labor that combines with above steady-state capital to produce an increase in output that peaks 6 periods after the shock. It is only then that the decrease in investment begins to outweigh the above steady-state labor, forcing output to fall monotonically.

![Figure 11: Output dynamics following a remittances shock](image)

When adjustment costs are introduced into the model we observe a slight decrease in output during the first couple periods, due to dynamics similar to the ones described for the case of no adjustment costs. However, since investment remains above steady state levels for additional periods, and since labor also increases in response to higher real wages, it is only in the 10th period when output peaks in this case. Since the fall in the nominal interest rate is much smaller when there are positive adjustment costs, the increase in investment is also much smaller, resulting in a weaker recovery of output. It is worth noting that the initial downward pressure on output gets relieved as adjustment
costs increase, due to the smaller increase in investment arising from the higher adjustment costs, and that the recovery of output diminishes as adjustment costs increase.

4.3.3 Nominal Exchange Rate Response

The initial exchange rate response to a positive remittances shock is mainly determined by the inflationary pressure, which leads to a proportional depreciation of the exchange rate on impact. In particular, the positive 0.058 percent deviation from steady-state in inflation is directly translated in a 0.058 percent depreciation from steady-state in the nominal exchange rate when there are no adjustment costs, while when we have a positive adjustment cost, a 0.032 percent deviation from steady-state in inflation is directly translated in a 0.032 percent depreciation from steady-state in the nominal exchange rate for the smaller adjustment cost, and a 0.028 percent deviation from steady-state in inflation is directly translated in a 0.028 percent depreciation from steady-state in the nominal exchange rate for the higher adjustment cost. This is shown in Figure 12 below.

Figure 12: Nominal Exchange Rate dynamics following a remittances shock

Note that while subsequent dynamics are determined by the uncovered interest rate parity condition, the rate of appreciation is dependent on the existence of adjustment costs. The persistent negative interest rate differential \( E_t R_{r,t+1} < 0 \) arising from the liquidity effect is counterbalanced by the expected appreciation of the nominal exchange rate in this case \( E_t \hat{e}_{t+1} - \hat{e}_t < 0 \), given rise to an overshooting of the exchange rate.

The remittances shock induces agents to hold more foreign bonds in both cases, following interest rate parity condition. With no or small adjustment costs, the initial fall in the domestic interest rate forces the exchange rate to depreciate as agents look for a
better return and increase their holdings of foreign bonds the first few periods after the shock. The increase in foreign bond’s holdings then decelerates as the domestic interest rate begins to rise and the nominal exchange rate to appreciate, improving the return on domestic deposits.

4.3.4 Consumption Response

The consumption dynamics following a remittances shock are primarily generated by the increase in purchasing power brought about by such inflows, outweighing the inflationary pressure during the period of the shock. Since remittances are assumed to go almost completely for consumption ($\phi = 0.99$), the increase in inflation by 0.058 percent the period of the shock and the fall in real money-cash are not strong enough to depress the purchasing power brought about by the remittances shock when there is no adjustment cost. Consumption rises on impact, but then it quickly falls the following period due to the below steady state money-cash balances and output, to then level-off slightly above the initial steady state level for the remaining periods as the subsequent remittances flows are large enough to outweigh the below steady-state money-cash balances.

![Response to a 5.5 percent deviation in remittances shock](image)

*Figure 13: Consumption dynamics following a remittances shock*

When we introduce adjustment costs, the initial increase in consumption is slightly larger, mainly due to the smaller increase in inflation and smaller fall in money-cash balances (a quarter of the no adjustment cost case). However, the subsequent dynamics show a slow but persistent monotonic decrease in consumption due to the sequential adjustment of money-cash and money-deposits, with the magnitude of the adjustment cost becomes relatively unimportant.
4.3.5 Utility Response

While the impact of a remittances shock on the main macroeconomic aggregates of our small open economy provides an adequate understanding of its effect at the macro level, its overall impact on the welfare of the representative agent is still somewhat elusive. In order to obtain the agent’s welfare gain from a remittances shock, we analyze the utility of the representative agent under our previous cases. Note that, in our benchmark case with remittances set at 5 percent of GDP, steady-state per-period utility is -100.3016. In the case when remittances are 10 percent of GDP, steady state per-period utility increases to -100.29124.

When we introduce the positive 5.5 percent remittances shock to the benchmark economy with zero adjustment costs, per-period utility increases on impact due to the increase of both consumption and leisure (decrease in worked hours). When we introduce the smaller adjustment costs, per-period utility continues to increase the period of the shock but it does so by almost one-quarter less than in the previous case. Although the increase in consumption is slightly larger relative to the case of no adjustment costs, the increase in leisure is much smaller, leading to a somewhat smaller improvement in utility. However, in addition to this effect, the negative impact of the adjustment cost on utility exerts a stronger effect, pulling the improvement in utility to almost one-third less than in the case of no adjustment costs. This influence of the adjustment cost on the behavior of utility can be seeing clearer as we examine the case of the larger adjustment cost, noting that the improvement in utility is almost two-thirds less than in the case of no adjustment cost even if the consumption and leisure are almost unaltered from the previous case (small adjustment cost).

The utility dynamics following the period of the shock are influenced by the effect of the adjustment cost directly and indirectly. It returns quickly to levels around its steady-state level when there are no adjustment costs, but it does so only monotonically when there are positive adjustment costs, both because consumption also returns to its steady-state monotonically but also because the adjustment cost dissipates slowly through

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14 In terms of time, the remittances shock allows the agent to use 0.1 more minutes on leisure when there are no adjustment costs and approximately 0.04 more minutes when there are positive adjustment costs, but this cost implies that she spends 3 minutes more rearranging her portfolio when she incurs in the smaller adjustment cost and 6 minutes when she incurs in the larger adjustment cost.
time. Per period utility returns smoothly to its original level only after 8 quarters in this case as shown below in Figure 14.

These utility dynamics are similar for the case of remittances being 10 percent of GDP, with the main difference being on the impact on the macroeconomic aggregates. Here the 5.5 percent remittances shock results in almost 0.5 percent increase in consumption and around 0.01 percent decrease in worked hours (increase in leisure), magnitudes that diminish the detrimental effect of the adjustment cost on per-period utility, as shown above.

4.3.6 Adjusted Trade Balance Response

We also examine the impact of the positive remittance shock on the adjusted trade balance (since we are including remittances to domestic production to then subtract domestic absorption). Figure 15 illustrate these trade balance dynamics, with the first row showing the case when we assume that steady-state remittances are 5 percent of GDP.
GDP, and the second row showing the case when we assume that steady-state remittances are 10 percent of GDP. It is clear that a remittances shock has a positive impact on the trade balance in the short run, having a larger effect when there are no adjustment costs. In the calibration with remittances being 5 percent of GDP we observe that the trade deficit declines during the period of the shock by 0.06 percent of GDP when there are no adjustment costs and by around 0.03 percent of GDP when there are adjustment costs, while in the calibration with remittances being 10 percent of GDP this effect gets magnified but remains proportional. In this case the trade deficit falls in the period of the shock by 0.1 percent of GDP when there are no adjustment costs and by 0.05 percent when there are adjustment costs.

These dynamics are determined by the behavior of output and remittances relative to the behavior of consumption and investment, which allows the trade deficit to remain lower for a few periods before starting to deteriorate again.

<table>
<thead>
<tr>
<th>Remittances</th>
<th>5% of GDP</th>
<th>10% of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>![Adj. Trade Balance Dynamics following a remittances shock](Figure 15: Trade Balance dynamics following a remittances shock)</td>
<td>![Adj. Trade Balance Dynamics following a remittances shock](Figure 15: Trade Balance dynamics following a remittances shock)</td>
</tr>
</tbody>
</table>
In terms of its effect of the behavior of foreign bonds, the representative agent increases its holding of next-period foreign bonds in all cases in response to the fall in the domestic nominal interest rate on the period of the shock, leveling off at a slightly higher level than in the beginning since the interest rate doesn’t fully recover.

4.3.7 Real Exchange Rate Response

Our model provides a good approximation of the real exchange rate appreciation following a remittances shock. One difference is that other studies find this appreciation starting the period of the shock, whereas we find that the real exchange rate is not affected in the period of the shock. In fact, the depreciation of the nominal exchange rate mimics the increase in inflation in the period of the shock, and thus neutralizes any effect on the real exchange rate. In the period after the shock, we obtain real exchange rate depreciation and overshooting, due to the larger fall in inflation relative to the small appreciation of the nominal exchange rate. From this period on, we find the real exchange rate appreciation usually found in the literature.

![Real Exchange Rate Behavior](image)

*Figure 16: Real Exchange Rate dynamics following a remittances shock*

This overshooting adjustment is a consequence of the modeling of limited participation models, and reflects in part the fact that our small open economy has only one good, instead of the usual tradable and non-tradable goods used in the study of real exchange rate fluctuations.

5 Robustness of the Remittances Shock

The qualitative dynamic response to a remittances shock in our model is robust to alternative specifications regarding the amount of remittances or money injection used for consumption and investment. Here we discuss the differences in magnitude of the
main dynamics in response to different assumptions regarding the percent of remittances that go to consumption, the magnitude of the intertemporal elasticity of substitution, and the amount of time spent working. We use our benchmark calibration, with remittances at 5 percent of GDP and the adjustment cost being almost 3 minutes per week ($\xi = 5$).

Since many governments in remittance-receiving countries are currently exploring policy tools that could direct a portion of remittances towards investment, we begin this section by increasing the proportion of remittances channeled to the financial intermediaries (and thus reducing the proportion available for consumption) and examine the impact of a remittances shock as we allow for its effect to work its way through investment. As we lower the amount of remittances available for consumption from the initial $\phi = .99$ to $\phi = .85$ and then to $\phi = .70$, the fall in the nominal interest rate becomes more accentuated, and thus generating a stronger and longer-lasting liquidity effect, as shown below in Figure 17. This stronger liquidity effect also increases investment and thus capital, generating a stronger and faster recovery of output as one allows for a greater fraction of remittances to initially go to the bank and thus be available for lending and investment. Following the nominal interest rate dynamics, as we lower the amount of remittances available for consumption from $\phi = .99$ to $\phi = .85$ and to $\phi = .70$, the initial depreciation of the nominal exchange rate also becomes accentuated, depreciating by an additional 33 percent every time the fraction available for consumption falls to a lower level. As expected, consumption’s dynamic response becomes smaller as we reduce the percentage of remittances used to finance consumption, falling by almost 40 percent as we allow for the fraction available for consumption to fall to 70 percent.

These dynamics show that as we increase the percentage of remittances devoted for investment will put a downward pressure on the nominal interest rate, as more funds are available for lending, and will consequently decrease consumption too. This lower interest rate will induce agents to adjust their investment portfolio towards foreign bonds, depreciating the domestic currency on its way, and to increase their capital accumulation, thus allowing for a significant improvement in production.
Figure 17: Main dynamics for different end uses of remittances
Since the output response to a remittances shock depends on the wealth effect and on its implied effect on worked hours, we also examine the role played by the intertemporal elasticity of substitution. The instantaneous utility function used in this study was originally calibrated such that the relative risk aversion degree parameter was equal to 1.01, implying an intertemporal elasticity of substitution equal to 0.99. We now focus attention on the effect of the remittance’s shock on the main macroeconomic aggregates as we vary $\sigma$, the inverse of the intertemporal elasticity of substitution. It is easy to show that when $\sigma > 1$ consumption and leisure are complements and that when $\sigma < 1$ consumption and leisure are substitutes. Thus as $\sigma$ increases the household’s willingness to smooth her consumption across time also increases. We allow $\sigma$ to take the values 0.5, 1.01, 1.5, and 2.5.

Rising $\sigma$ tends to reduce the demand for loans directed to investment, as firms try to take advantage of the temporary fall in the nominal interest rate, since each household is a shareholder of the firm and is trying to smooth consumption. This reduces the fall of the nominal interest rate on impact, and even raises it for the larger $\sigma$, and tends to amplify the overall liquidity effect in the long run. This adjustment in the credit market also affects the nominal exchange rate, with a higher $\sigma$ leading to a greater overshooting of the nominal exchange rate.

The output response to a larger $\sigma$ reduces and even reverts the fall in output under the smaller $\sigma$. As shown below in Figure 18, for the cases of $\sigma$ being greater than 1.01 we actually have an increase in output following the remittances shock. These dynamics follow the behavior of worked hours, since the capital stock is predetermined in the period of the shock. The parameter $\sigma$ determines the degree of substitutability between consumption and leisure, the increase in consumption resulting from the remittances shock – also shown below – leads to a decrease in the marginal utility of leisure and consequently to a decrease in leisure time (increase in worked hours) for $\sigma > 1$, and to an increase in the marginal utility of leisure and consequently to an increase in leisure time (decrease in worked hours) for $\sigma < 1$. 
Figure 18: Main dynamics for different elasticities of substitution
Increases in the relative risk aversion parameter therefore reduce, and even overturn, the fall in the nominal interest rate and in output, magnifies the overshooting depreciation of the nominal exchange rate, and does not alter the consumption response.

To conclude this robustness check, we also allow the amount of worked hours to vary to allow for conflicting views about the right calibration of $H$ for Latin America. We allow the parameter $\gamma$ – the relative weight of leisure in the utility function – to vary to examine the cases when the representative agent spends 20 percent of total time working (33.6 hours per week), 26 percent of total time working (43 hours per week), and 34 percent of total time working (57 hours per week).

Increasing $H$ reduces the fall of the nominal interest rate on impact, and therefore dampens the liquidity effect in the long run. As the household spends more time working, the positive remittances shock increases the demand for goods but reduces the fall in investment, thus alleviating the downward pressure on the interest rate. This smaller reduction in the domestic interest rate reduces the portfolio adjustment towards foreign bonds, and thus alleviating the initial depreciation of the nominal exchange rate.

The output response to a larger $H$ is determined by the relation between the predetermined capital and labor input, such that the drop in output is reduced. The household earning more for the additional time spent working makes the additional purchasing power brought about the remittances shock less influential in the increase in real wages, and thus in the initial fall of working hours. This causes the fall in output to dampen as $H$ increase, but also to reduce the subsequent increase in output as capital and worked hours recover. In fact, the greater earnings brought about from the larger amount of time spent working also reduces the impact on consumption, since the one percent remittances shock is now relatively smaller in the household’s budget constraint. The remittances shock increases consumption, but its effect on consumption is smaller, as shown below in Figure 19. Increases in the work hours parameter therefore reduces the fall in the nominal interest rate, the overshooting depreciation of the nominal exchange rate, and the increase in consumption, and smooths the output response.
Figure 19: Main dynamics for different levels of hours worked
To further investigate the differential effect of a remittances shock on a small open economy with twice the level of remittances, we also allowed for remittances to be 10 percent of GDP, a magnitude that reflects a doubling in importance of remittances in the receiving economy and is more in line with some of the economies of our sample. The results for the main macroeconomic variables are presented in the appendix. Here we find that a doubling in remittances results in the effects being twice as strong, but it does not have any behavioral effect on the main macroeconomic aggregates of our model.

6 Conclusions

Our limited participation model for a small open economy with remittances explicitly incorporated is able to capture important features from observed empirical responses of economic variables to monetary shocks. In particular, we capture important aspects in the dynamic response of the nominal interest rate, output, the exchange rate, and consumption. The introduction of adjustment costs on money holdings accentuates the persistence of the liquidity effect, and consequently expands the overshooting dynamics of the nominal exchange rate, both in accord with existing empirical evidence on the result of monetary innovations. The technology shock results are also in accord with existing findings, in particular with those regarding the overshooting exchange rate appreciation in response to a positive shock.

A novel contribution of this paper comes from our ability to examine the dynamic response of major macroeconomic aggregates – namely the nominal interest rate, output, the nominal exchange rate, and consumption – to remittances shocks. We find that a remittances shock in our model without adjustment costs will lower the nominal interest rate and create a liquidity effect, reduce output for couple periods before increasing to levels above steady state that peak a year-and-half after the shock, depreciates the nominal exchange rate on impact to then continuously appreciate, and increases consumption for one period before returning to levels marginally above steady state. When there are positive adjustment costs the dynamic responses of the nominal interest rate and the nominal exchange rate are decreased in magnitude but maintain their shape, while the dynamic response of output is significantly reduced. Not surprising, the initial increase in consumption is somewhat larger due to the smaller reduction in money cash.
Here consumption smoothly returns to its initial steady state due to the adjustment cost on money balances.

We also examined the impact of different modeling assumptions with respect to the end use of remittances on the economy, whether to loosen the cash in advance constraint facing households or to increase the amount of loanable funds available to financial intermediaries. We find that these alternative specifications have scant impact on the dynamic responses of the variables we examine to a monetary shock, but these alternatives do affect the dynamic responses of macroeconomic variables to a remittances shock. We find that the decrease in the nominal interest rate and the initial depreciation of the nominal exchange rate are accentuated as we reduce the amount of remittances that are available for consumption, and thus being available to the financial intermediaries. In addition, while this reduction in the percentage of remittances used for consumption increases significantly output, such higher level of output – and consequently remittances – is not strong enough to avoid the fall in consumption brought about by the smaller percentage allowed for consumption.

We also examine the impact of a change in remittances on the steady state of the economy. As remittances change from 5 percent of output to 10 percent of output, we find that both output and work hours fall by almost 4 percent while consumption increases by slightly more than 1 percent. Physical capital also falls by almost 4 percent. The distribution of real money balances also becomes affected by the doubling of the share of remittances, with real money cash decreasing by almost 4 percent while real money deposits decrease by almost 4 percent. It can be observed that the representative household increases its leisure as the share of remittances increases, which together with the fall in physical capital reduces output in steady state. This negative effect is counterbalanced by the increase in consumption brought about by the doubling of remittances.

While we do not include some frictions, like sluggish capital adjustment or adjustment costs in foreign assets, the current model provides sufficient insight into the effects of a remittances shock in the main macroeconomic aggregates of small open economies. We leave such extension for future research.
References


Figure 2a: Remittances as percentages of GDP in 10 Latin American Countries
Figure A.1

Remittances (% of GDP)
Bolivia 1990-2004

Source: World Bank World Development Indicators Database

Remittances (% of GDP)
Guatemala 1990-2004

Source: World Bank World Development Indicators Database

Remittances (% of GDP)
Brazil 1990-2004

Source: World Bank World Development Indicators Database

Remittances (% of GDP)
Honduras 1990-2004

Source: World Bank World Development Indicators Database

Remittances (% of GDP)
Colombia 1990-2004

Source: World Bank World Development Indicators Database

Remittances (% of GDP)
Mexico 1990-2004

Source: World Bank World Development Indicators Database

Remittances (% of GDP)
Ecuador 1990-2004

Source: World Bank World Development Indicators Database

Remittances (% of GDP)
Panama 1990-2004

Source: World Bank World Development Indicators Database

Remittances (% of GDP)
El Salvador 1990-2004

Source: World Bank World Development Indicators Database

Remittances (% of GDP)
Peru 1990-2004

Source: World Bank World Development Indicators Database
Figure A.2

Response to a 5.5 percent deviation in remittances shock

Quarters

Response to a 5.5 percent deviation in remittances shock

Quarters

Response to a 5.5 percent deviation in remittances shock

Quarters

Response to a 5.5 percent deviation in remittances shock

Quarters
Table A.1: Steady State Values

<table>
<thead>
<tr>
<th></th>
<th>Remittances 5% GDP</th>
<th>Remittances 10% GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Interest Rate</td>
<td>0.0506</td>
<td>0.0506</td>
</tr>
<tr>
<td>Capital/output ratio</td>
<td>9.2247</td>
<td>9.2247</td>
</tr>
<tr>
<td>Output</td>
<td>0.6753</td>
<td>0.6460</td>
</tr>
<tr>
<td>Labor (hours worked)</td>
<td>0.1935</td>
<td>0.1851</td>
</tr>
<tr>
<td>Remittances</td>
<td>0.0337</td>
<td>0.0646</td>
</tr>
<tr>
<td>Capital</td>
<td>6.2298</td>
<td>5.9592</td>
</tr>
<tr>
<td>Investment</td>
<td>0.1557</td>
<td>0.1490</td>
</tr>
<tr>
<td>Bonds</td>
<td>1.6236</td>
<td>1.5531</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.5715</td>
<td>0.5774</td>
</tr>
<tr>
<td>Real Money Balances</td>
<td>0.6947</td>
<td>0.6642</td>
</tr>
<tr>
<td>Real Money deposits</td>
<td>0.1349</td>
<td>0.1288</td>
</tr>
<tr>
<td>Real Money Cash</td>
<td>0.5598</td>
<td>0.5355</td>
</tr>
<tr>
<td>Inflation</td>
<td>1.0380</td>
<td>1.0380</td>
</tr>
<tr>
<td>Real Wages</td>
<td>2.2334</td>
<td>2.2334</td>
</tr>
<tr>
<td>Lambda</td>
<td>0.4176</td>
<td>0.4133</td>
</tr>
<tr>
<td>Utility</td>
<td>-100.302</td>
<td>-100.291</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>-0.01977</td>
<td>-0.01882</td>
</tr>
</tbody>
</table>
A.1. LPM Model

We denote the shadow price associated with the household real wealth by $\Lambda_t = P_t \lambda_t$. The relevant equations in the LPM model are defined the following way:

\[
\Gamma_t = E_t \left[ \theta (Y^{\pi x})^\pi Y_t - e^\pi \right]
\]

\[
\Lambda_t = \beta E_t \left[ (1 + i_{t+1}) \frac{\Lambda_{t+1}}{\pi_{t+1}} \right]
\]

\[
-U_{t,t} = w_t \Lambda_t
\]

\[
\Lambda_t = \beta E_t \left[ \frac{\Lambda_{t+1}}{\pi_{t+1}} (1 + R_{t+1}) \right]
\]

The following variable is introduced for technical convenience

\[
\Delta M_t^c = \frac{M_{t+1}^c}{M_t^c}
\]

which can be stationarized (to be used in the next equation) as

\[
\Delta M_t^c = \frac{m_{t+1}^c \pi_{t+1}}{m_t^c}
\]

\[
w_t \Lambda_t \xi \frac{\pi_t}{m_t} (\Delta M_t^c - \theta) + \Lambda_t = \beta E_t \left[ \frac{U'_{t+1}}{\pi_{t+1}} \right] + \beta E_t \left[ w_{t+1} \Lambda_{t+1} \xi \frac{\Delta M_{t+1}^c}{m_{t+1}^c} (\Delta M_{t+1}^c - \theta) \right]
\]

\[
m_t = m_t^h + m_t^e
\]

\[
\pi_t C_t = m_t^e + \phi \Gamma_t + \phi(\theta_t - 1)m_t
\]

\[
\pi_t = \frac{e_t}{e_{t+1}} \pi_t^*
\]

\[
Y_t = e^{\gamma_t} K_t^{\alpha} H_t^{1-\alpha}
\]

\[
I_t = K_{t+1} - (1 - \delta) K_t
\]

\[
w_t = (1 - \alpha) \frac{Y_t}{K_t}
\]

\[
1 + R_t = \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)(1 + R_{t+1}) \right\} \right]
\]

\[
m_{t+1} = \theta_t \frac{m_t}{\pi_t}
\]

\[
\pi_t I_t = m_t^h + (1 - \phi)(\theta_t - 1)m_t + (1 - \phi) \Gamma_t
\]
\begin{align*}
\tag{64} \frac{b_{t+1}}{e_{t-1}} - \frac{e_t}{b_t} (1+i^*_t) \frac{b_t}{\pi_t} &= Y_t - C_t - I_t + [1-(1+R_t)(1-\phi)] \frac{\Gamma_t}{\pi_t} \\
\tag{65} \log(\theta_{t+1}) &= (1-\rho_\theta) \log(\theta_t) + \rho_\theta \log(\theta_{t+1}) + \rho_g \log(g_{t+1}) + \varepsilon_{\theta t+1} \\
\tag{66} \log(g_{t+1}) &= (1-\rho_g) \log(g_t) + \rho_g \log(g_{t+1}) + \varepsilon_{g t+1} \\
\tag{67} \log(z_{t+1}) &= (1-\rho_z) \log(z_t) + \rho_z \log(z_{t+1}) + \varepsilon_{z t+1} \\
\end{align*}

Consequently, the log-linearized system of equations, following Uhlig’s methodology, is given by

\begin{align*}
\tag{27'} 0 &= E_t \left[ \hat{\pi}_t - \hat{\Gamma}_t - r\hat{Y}_t + \hat{g}_t \right] \\
\tag{50'} 0 &= E_t \left[ -\hat{\Lambda}_t + \hat{\Lambda}_{t+1} + \hat{e}_t - \hat{\pi}_{t+1} \right] \\
\tag{51'} 0 &= \hat{w}_t + \hat{\Lambda}_t - (1-\gamma)(1-\sigma)) \frac{H}{1-H} \hat{H}_t - (1-\gamma)(1-\sigma) \hat{C}_t \\
\tag{52'} 0 &= E_t \left[ -\hat{\Lambda}_t + \frac{R}{1+R} \hat{R}_{t+1} + \hat{\Lambda}_{t+1} - \hat{\pi}_{t+1} \right] \\
\tag{53'} 0 &= -\Delta M_t + \hat{m}_{t+1} + \hat{\pi}_t - \hat{m}_t \\
\tag{54'} 0 &= E_t \left[ -\hat{\Lambda}_t - S\beta\hat{\pi}_{t+1} - S\beta\gamma(1-\sigma) \frac{H}{1-H} \hat{H}_{t+1} - S\beta(\gamma + \sigma(1-\gamma)) \hat{C}_{t+1} + \beta\pi^2 \Lambda w \frac{1}{m^c} \Delta M_{t+1} - \pi^2 \Lambda w \frac{1}{m^c} \Delta M_t \right] \\
\end{align*}

where \( S = (1-\gamma)(1-H)^{\gamma(1-\sigma)}C^{-\gamma-\sigma(1-\gamma)} \)

\begin{align*}
\tag{55'} 0 &= -(m^c)\hat{m}_t + (m^b)\hat{m}_t + (m^e)\hat{m}_t \\
\tag{56'} 0 &= \hat{\pi}_t + \hat{C}_t - \frac{m^c}{C\pi} \hat{m}_t - \frac{\Gamma \hat{\pi}_t}{C\pi} - \frac{m^c \hat{\phi}_t}{C\pi} - \frac{m^c \hat{\phi}_t}{C\pi} - \frac{m^c \hat{\phi}_t}{C\pi} (\theta - 1) \hat{m}_t \\
\tag{57'} 0 &= -\hat{\pi}_t + \hat{e}_t - \hat{e}_{t+1} \\
\tag{58'} 0 &= -\hat{Y}_t + \alpha\hat{K}_t + (1-\alpha)\hat{H}_t + \hat{z}_t \\
\tag{59'} 0 &= \frac{I}{K} \hat{I}_t - \hat{K}_{t+1} + (1-\delta)\hat{K}_t \\
\tag{60'} 0 &= -\hat{w}_t + \hat{Y}_t - \hat{H}_t \\
\tag{61'} 0 &= E_t \left[ \left( \alpha\beta \frac{Y}{K} + \beta(1-\delta)(1+R) \right) \hat{\Lambda}_{t+1} + \left( \beta(1-\delta)R \right) \hat{R}_{t+1} + \alpha\beta \frac{Y}{K} \hat{Y}_{t+1} - \alpha\beta \frac{Y}{K} \hat{K}_{t+1} \\
\tag{62'} 0 &= -\hat{m}_{t+1} + \hat{m}_t - \hat{\pi}_t + \hat{\theta}_t \\
\end{align*}

52
(63') \[ 0 = -\hat{\pi}_t - \hat{I}_t + \frac{m^b}{l\pi} \hat{m}^b_t + \frac{m}{l\pi} (\theta - 1)(1 - \varphi) \hat{m}_t + \frac{(1 - \varphi)m}{l} \hat{\theta}_t + \frac{\Gamma}{l\pi} (1 - \phi) \hat{\Gamma}_t \]

(64') \[ 0 = -\hat{b}_{t+1} + \frac{(1 + i^*)}{\pi} \hat{e}_t - \frac{(1 + i^*)}{\pi} \hat{e}_{t-1} + \frac{(1 + i^* \hat{b}_t + \left( \frac{Y - C - I - b}{b} \right) \hat{\pi}_t + \frac{Y}{b} \hat{Y}_t - \frac{C}{b} \hat{C}_t}{b\pi} \hat{I}_t + \left( \frac{(1 - \phi)(1 + R)}{b\pi} \right) \hat{\Gamma}_t - \frac{(1 - \phi)R \Gamma}{b\pi} \hat{R}_t \]

(65') \[ \hat{\theta}_{t+1} = \rho_\theta \hat{\theta}_t + \rho_g \hat{g}_t + \epsilon_{\theta_{t+1}} \]

(66') \[ \hat{g}_{t+1} = \rho_g \hat{g}_t + \epsilon_{g_{t+1}} \]

(67') \[ \hat{z}_{t+1} = \rho_z \hat{z}_t + \epsilon_{z_{t+1}} \]

A.2. Solving

The system is given by 19 equations with 19 variables. The endogenous state variables \( \{ \hat{m}_t, \hat{b}_t, \hat{K}_t, \hat{m}^c_t, \hat{\epsilon}_t, \hat{\Lambda}_t \} \) include lambda and the nominal exchange rate in addition to the standard four variables, as Uhlig’s toolkit suggests that variables dated \( t-1 \) or earlier should be considered state variables (in the case of \( \hat{e}_t \)) while the matrix of other endogenous variables should be non-singular in order for its pseudo-inverse to exists, allowing to redeclare \( \hat{\Lambda}_t \) as other endogenous state variable instead. The other endogenous variables of the system are \( \{ \hat{\pi}_t, \hat{m}^b_t, \hat{C}_t, \hat{R}_t, \hat{w}_t, \hat{H}_t, \hat{Y}_t, \hat{I}_t, \Delta M_t, \hat{\Gamma}_t \} \), and the exogenous state variable are \( \{ \hat{\theta}_t, \hat{g}_t, \hat{z}_t \} \).