Simple Analytics of the Government Expenditure Multiplier

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Introduction

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- Recent years have seen development of a theory of stabilization policy that integrates consequences of price/wage stickiness for output determination with intertemporal optimization
  — implications for fiscal stimulus?
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- taxes guarantee intertemporal solvency
- monetary policy independent of public debt
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Hence path of public debt irrelevant, focus on implications of alternative paths for government purchases
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- the degree of price or wage stickiness?
- the monetary policy reaction?
- the degree of economic slack?
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Also: does countercyclical government spending increase welfare?
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- Preferences of representative household:
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  \sum_{t=0}^{\infty} \beta^t [u(C_t) - v(H_t)], \quad u', v' > 0, \quad u'' < 0, \quad v'' > 0
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- Production technology (capital stock fixed):

\[
Y_t = f(H_t), \quad f' > 0, \quad f'' < 0
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A Neoclassical Benchmark

Competitive equilibrium requires:

\[
\frac{v'(H_t)}{u'(C_t)} = \frac{W_t}{P_t} = f'(H_t)
\]

Hence equilibrium output \( Y_t \) must satisfy

\[
\tilde{v}(Y_t) - G_t = \tilde{v}(Y_t)
\]

where \( \tilde{v}(Y_t) \equiv v(f(Y_t) - 1) \) is the disutility of supplying output \( Y_t \) note this is also FOC for welfare-maximizing output can solve for \( Y_t \) as function of current \( G_t \) only
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Multiplier is seen to be:

\[
\frac{dY}{dG} = \Gamma \equiv \frac{\eta_u}{\eta_u + \eta_v} < 1
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where \(\eta_u, \eta_v > 0\) are the elasticities of \(u', \tilde{v}'\) respectively.
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Necessarily less than 1 (government purchases crowd out private spending)

— substantially less than 1, unless \( \eta_u >> \eta_v \)

— e.g., Eggertsson (2009) parameters: \( \Gamma = 0.4 \)
This result depends on flexibility of both wages and prices.
Beyond the Neoclassical Benchmark

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- How much the “labor wedge” changes, under any given hypothesis about sticky prices, sticky wages, or sticky information, depends on degree of monetary accommodation
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  - a useful benchmark because the answer is independent of the details of price or wage adjustment (within that broad family)
  - corresponds to the textbook “multiplier” calculation, that determines the size of the rightward shift of “IS curve”
Consider a deterministic path \( \{ G_t \} \) for government purchases, such that \( G_t \to \bar{G} \) (consider only temporary increases in \( G \))
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- can be achieved, for example, by Taylor rule with suitably time-varying intercept (to be determined)
A New Keynesian Benchmark

FOC for optimal intertemporal expenditure:

\[
\frac{u'(C_t)}{\beta u'(C_{t+1})} = 1 + r_t
\]

Then if \( r_t = \bar{r} \) for all \( t \), the \( \{C_t\} \) must be constant over time.

Hence \( C_t = \bar{C} \equiv \bar{Y} - \bar{G} \) for all \( t \).

Hence \( Y_t = \bar{C} + G_t \) for all \( t \).

Thus equilibrium \( Y_t \) again depends only on \( G_t \), and \( \frac{dY}{dG} = 1 \).

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Analytics of Multiplier
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- In fact, we can obtain a multiplier of 1 regardless of wage-price block of model
  — can easily specify to be consistent with the procyclical markups found by Nekarda and Ramey: sticky wages and prices, procyclical labor productivity due to overhead labor
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Example: flexible wages, Calvo model of price adjustment: equilibrium inflation rate given by

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where coefficient $\kappa > 0$ depends on frequency of price adjustment.
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  - labor/supply demand factors that determine \( \Gamma \) still matter, but only to determine how inflationary the hypothesized policy is.
A New Keynesian Benchmark

- Doesn’t the multiplier depend on the degree of price/wage flexibility?

Not if real interest rate is held constant! But degree of stickiness may affect plausibility of assuming that central bank will take actions required to hold it constant.

If prices, wages and information all adjust rapidly, this will require extremely inflationary policy—Calvo example: $\kappa \rightarrow \infty$ as prices adjust more frequently.

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But while the NK model implies that the multiplier can be higher than the neoclassical prediction, it need not be.

— low multipliers also possible, under other assumptions about monetary policy
Suppose, instead, that CB enforces a \textit{strict inflation target}: \( \pi_t = 0 \) for all \( t \) (not just in long run), regardless of \( \{G_t\} \).
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$$\frac{dY}{dG} = \Gamma < 1$$

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In any of these models: larger multiplier requires inflation
A common monetary policy specification: interest rate determined by a Taylor rule

Simple case (again consistent with zero-inflation steady state):

\[ i_t = \bar{r} + \phi_\pi \pi_t + \phi_y (\hat{Y}_t - \Gamma \hat{G}_t) \]

where \( \phi_\pi > 1, \phi_y > 0 \) as proposed by Taylor (1993)

— here “output gap” is interpreted as output in excess of flex-price equilibrium output
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Consider path for government purchases of form \( G_t = G_0 \rho^t \), for some \( 0 \leq \rho < 1 \).

— then forward path is same function of current \( G_t \) at all times
Monetary Policy Follows a Taylor Rule

- In the Calvo model (purely forward-looking), this implies that equilibrium $Y_t$, $\pi_t$, $i_t$ are all time-invariant functions of $G_t$

- Can again define a static “multiplier” $dY/dG$:

\[
dY / dG = 1 - \rho + \psi \Gamma 1 - \rho + \psi, \quad \psi \equiv \sigma \phi_y + \kappa (1 - \beta \rho) (\phi \pi - \rho) > 0.
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where

$$\psi \equiv \sigma \left[ \phi_y + \frac{\kappa}{1 - \beta \rho} (\phi_\pi - \rho) \right] > 0.$$

Note this implies that

$$\Gamma < \frac{dY}{dG} < 1.$$
Monetary Policy Follows a Taylor Rule

- Note that multiplier is **smaller** if
  - prices more flexible ($\kappa$ larger)
  - marginal cost more sharply increasing ($\kappa$ larger)
  - response coefficient $\phi_\pi$ or $\phi_y$ greater
  - persistence $\rho$ of fiscal stimulus greater
Note that multiplier is smaller if

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In each of these limiting cases ($\kappa \to \infty$, $\phi_\pi \to \infty$, $\phi_y \to \infty$, or $\rho \to 1$), neoclassical multiplier is recovered
Arguably more realistic specification:

\[ i_t = \bar{r} + \phi_\pi \pi_t + \phi_y \hat{Y}_t \]

— note that central banks’ measures of “potential output” aren’t typically adjusted in response to government spending
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- Multiplier in this case

\[
\frac{dY}{dG} = \frac{1 - \rho + (\psi - \sigma \phi_y) \Gamma}{1 - \rho + \psi}
\]

is necessarily smaller; for large enough \( \phi_y \), can even be smaller than the neoclassical multiplier!

— e.g. Eggertsson (2009) parameters: \( \Gamma = 0.4 \), but multiplier for Taylor rule with \( \phi_\pi = 1.5 \), \( \phi_y = 0.25 \) is only 0.3
A case of particular interest: effects of increased government purchases, when central bank’s policy rate is at zero lower bound:
Fiscal Stimulus at the Zero Lower Bound

- A case of particular interest: effects of increased government purchases, when central bank’s policy rate is at zero lower bound:

  - currently relevant case in many countries

  - interest in fiscal stimulus especially great, because further interest-rate cuts not possible

  - monetary accommodation especially plausible: even if central bank wishes to implement strict inflation target, or follow Taylor rule, it may be constrained by lower bound on interest rate, and this should not change due to modest increase in government purchases
How ZLB may sometimes be binding constraint: extend model to allow for a credit spread $\Delta_t$ between the CB policy rate $i_t$ and the interest rate that is relevant to aggregate demand determination.

Log-linearized Euler equation then becomes

$$\hat{Y}_t - \hat{G}_t = E_t[\hat{Y}_{t+1} - \hat{G}_{t+1}] - \sigma(i_t - E_t\pi_{t+1} - r_{t}^{\text{net}})$$

where

$$r_{t}^{\text{net}} \equiv -\log \beta - \Delta_t$$

decreases if a disruption of credit markets increases $\Delta_t$ (here, exogenously)

— Cúrdia and Woodford (2009) provide more detailed microfoundations
Fiscal Stimulus at the Zero Lower Bound

- ZLB more likely to bind when $r_t^\text{net}$ (real policy rate required to maintain expenditure at steady-state level $\bar{C}$) is temporarily low, due to elevated credit spreads
Fiscal Stimulus at the Zero Lower Bound

- ZLB more likely to bind when $r_t^{net}$ (real policy rate required to maintain expenditure at steady-state level $\bar{C}$) is temporarily low, due to elevated credit spreads.

- Simple example (Eggertsson, 2009):
  - In normal state (low credit spreads), $r_t^{net} = \bar{r} > 0$
  - Shock at date zero lowers $r_t^{net}$ to $r_L < 0$
  - Each period, probability $\mu$ that credit spread remains high ($r_t^{net} = r_L$) another period, if still high in last period; with probability $1 - \mu$, reversion to normal level
  - Once $r_t^{net}$ reverts to normal level $\bar{r}$, remains there forever after
Assume CB follows Taylor rule when consistent with ZLB:

\[ i_t = \max \left\{ \bar{r} + \phi_{\pi} \pi_t + \phi_y \hat{Y}_t, 0 \right\} \]
Fiscal Stimulus at the Zero Lower Bound

- Assume CB follows Taylor rule when consistent with ZLB:

\[ i_t = \max \{ \bar{r} + \phi_\pi \pi_t + \phi_y \hat{Y}_t, 0 \} \]

- Policies to consider: \( G_t = G_L \) for all \( t < T \) (random date at which credit spreads revert to normal), \( G_t = \bar{G} \) for all \( t \geq T \)

  — consider effects of varying \( G_L \) (fiscal stimulus during crisis)
Assume CB follows Taylor rule when consistent with ZLB:

\[ i_t = \max \{ \bar{r} + \phi_\pi \pi_t + \phi_y \hat{Y}_t, 0 \} \]

Policies to consider: \( G_t = G_L \) for all \( t < T \) (random date at which credit spreads revert to normal), \( G_t = \tilde{G} \) for all \( t \geq T \)

— consider effects of varying \( G_L \) (fiscal stimulus during crisis)

Markovian structure implies equilibrium in which

\[ \pi_t = \pi_L, Y_t = Y_L, i_t = i_L \] for all \( t < T \); and

\[ \pi_t = 0, Y_t = \bar{Y}, i_t = \bar{r} \] for all \( t \geq T \).
Solution: \( i_L = 0 \) (ZLB continues to bind) for all \( G_L \leq G^{\text{crit}} \), while \( i_L > 0 \) (Taylor rule applies) for all \( G_L > G^{\text{crit}} \).
Fiscal Stimulus at the Zero Lower Bound

Solution: $i_L = 0$ (ZLB continues to bind) for all $G_L \leq G^{crit}$, while $i_L > 0$ (Taylor rule applies) for all $G_L > G^{crit}$

For $G_L \leq G^{crit}$,

$$\hat{Y}_L = \vartheta_r r_L + \vartheta_G \hat{G}_L$$

where

$$\vartheta_r \equiv \frac{\sigma(1 - \beta \mu)}{(1 - \mu)(1 - \beta \mu) - \kappa \sigma \mu} > 0$$

$$\vartheta_G \equiv \frac{(1 - \mu)(1 - \beta \mu) - \kappa \sigma \mu \Gamma}{(1 - \mu)(1 - \beta \mu) - \kappa \sigma \mu} > 1$$
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- For \( G_L > G^{\text{crit}} \), equilibrium same as above for Taylor rule: \( dY / dG < 1 \), possibly less than \( \Gamma \).
Eggertsson (2009) parameter values:

\[ \beta = 0.997 \]
\[ \kappa = 0.00859 \]
\[ \sigma = 0.862 \]
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“Great Depression” shock:

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- “Great Depression” shock:
  \[r_L = -0.0104\]
  \[\mu = 0.903\]

Implications: multiplier = 2.29 for \( G < G^{crit} \), 0.32 for \( G > G^{crit} \)
Fiscal Stimulus at the Zero Lower Bound

- Effect of $G_L$ on $Y_L$, in case of a “Great Depression” shock:

\[ \hat{G}_{\text{crit}} = 13.6 \text{ percent of steady-state GDP} \]
Fiscal Stimulus at the Zero Lower Bound

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— fiscal stimulus increases inflation (reduces deflation); if $\mu > 0$, this means higher expected inflation, so lower real interest rate
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- This is precisely the case in which risk of output collapse is greatest in absence of fiscal stimulus: for $dY/dr$ becomes very large as well

  — so fiscal stimulus highly effective exactly in case where most badly needed ("Great Depression" case)
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The main difference is not their use of more complex models: Christiano *et al.* (2009) find multiplier can be 2 or more, using closely related empirical NK model
Why do Cogan et al. (2009), Erceg and Lindé (2009) find much smaller multipliers, in simulations using empirical NK models, despite assuming a situation in which ZLB initially binds?

The main difference is not their use of more complex models: Christiano et al. (2009) find multiplier can be 2 or more, using closely related empirical NK model.

Important difference: Cogan et al., Erceg and Lindé assume increase in government purchases that extends beyond the time when ZLB ceases to bind, interest rates set by Taylor rule.

— Expectation of higher government purchases after period for which ZLB binds can reduce output when it does!
Why expectation that high government spending will continue after ZLB ceases to bind can reduce output during the crisis:

- If Taylor Rule determines monetary policy post-crisis (or inflation target), higher $G$ then will crowd out private spending
  $\Rightarrow$ higher expected marginal utility of income $\Rightarrow$ less desired spending during crisis
Fiscal Stimulus: The Importance of Duration

- Why expectation that high government spending will continue after ZLB ceases to bind can reduce output during the crisis:
  - if Taylor Rule determines monetary policy post-crisis (or inflation target), higher $G$ then will crowd out private spending $\Rightarrow$ higher expected marginal utility of income $\Rightarrow$ less desired spending during crisis
  - higher $G$ then can also reduce inflation then $\Rightarrow$ lower expected inflation $\Rightarrow$ zero nominal rate implies higher real interest rate $\Rightarrow$ less desired spending during crisis
Multiplier for alternative persistence $\lambda$ of stimulus policy after ZLB no longer binds:
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- Multiplier below 1 for $\lambda > 0.8$, negative for $\lambda > 0.91$
Government Purchases and Welfare

Have shown that government purchases can increase output and employment: but does that mean they increase welfare?
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Let preferences of rep. household be

$$\sum_{t=0}^{\infty} \beta^t [u(C_t) + g(G_t) - v(H_t)], \quad g' > 0, \quad g'' < 0$$
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- additive separability implicit in previous calculations
- \( \eta_g \equiv -g'' \tilde{G} / g' \geq 0 \) a measure of degree of diminishing returns to government expenditure
Neoclassical model: FOC for optimal path $\{G_t\}$: 

$$g'(G_t) = u'(Y_t - G_t)$$
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Simple principle: choose government purchases to ensure efficient composition of aggregate expenditure: maximize \( u(Y_t - G_t) + g(G_t) \), for given aggregate expenditure \( Y_t \)

— Note this principle requires no consideration of effects of government purchases on economic activity
Sticky prices or wages: if increasing $G_t$ increases $Y_t$, welfare is increased iff

$$(u' - \tilde{v}') \frac{dY}{dG} + (g' - u') > 0$$
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But: effective monetary policy should minimize the importance of this additional consideration!
Example: flexible wages but sticky prices; and assume a subsidy so that flex-price equilibrium is efficient
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Example: flexible wages but sticky prices; and assume a subsidy so that flex-price equilibrium is efficient

Then optimal monetary policy maintains \textit{zero inflation} at all times (assuming ZLB not a problem)

— this achieves the flex-price equilibrium allocation, which is efficient, regardless of path \{\(G_t\}\}

So optimal choice of \{\(G_t\}\} is \textit{same as in neoclassical model}!

— determined purely by \textit{principle of efficient composition}
But result is different if financial disturbance causes ZLB to bind, preventing complete stabilization through monetary policy.
Fiscal Stabilization at the Zero Lower Bound

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- 2-state Markov example: assume that $\bar{G}$ is optimal steady-state level, and that central bank targets zero inflation except when constrained by ZLB.
Fiscal Stabilization at the Zero Lower Bound

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- 2-state Markov example: assume that $\bar{G}$ is optimal steady-state level, and that central bank targets zero inflation except when constrained by ZLB.

- Quadratic approximation to expected utility varies inversely with

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda_y (\hat{Y}_t - \Gamma \hat{G}_t)^2 + \lambda_g \hat{G}_t^2 \right]$$

$$= \frac{1}{1 - \beta \mu} \left[ \pi_L^2 + \lambda_y (\hat{Y}_L - \Gamma \hat{G}_L)^2 + \lambda_g \hat{G}_L^2 \right]$$

— choose $\hat{G}_L$ to minimize this.
Optimal level:

\[ \hat{G}_L = -\frac{\xi(v_G - \Gamma)v_r}{\xi(v_G - \Gamma)^2 + \lambda_g} \quad r_L > 0 \]

where

\[ \xi \equiv \left( \frac{\kappa}{1 - \beta \mu} \right)^2 + \lambda_y > 0 \]
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\[ \hat{G}_L = -\frac{\xi(\vartheta_G - \Gamma)\vartheta_r}{\xi(\vartheta_G - \Gamma)^2 + \lambda_g} \quad r_L > 0 \]

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\[ \xi \equiv \left( \frac{\kappa}{1 - \beta \mu} \right)^2 + \lambda_y > 0 \]

Optimal to choose \( \hat{G}_L > 0 \), even though principle of efficient composition would require \( \hat{G}_L < 0 \) (since \( \hat{C}_L < 0 \))

— but optimal \( \hat{G}_L \) is less than the level required to “fill the output gap” (ensure that \( \hat{Y}_L - \Gamma \hat{G}_L = 0 \))
Fiscal Stabilization at the Zero Lower Bound

Optimal $\hat{G}_L/|r_L|$ for alternative $\mu$:
Optimal $\hat{G}_L / |r_L|$ for alternative $\mu$:

Case (A): $\eta_g = 0$; Case (B): same diminishing returns as for private expenditure.
Fiscal Stabilization at the Zero Lower Bound

- Here the case for fiscal stabilization policy again depends on assuming a suboptimal monetary policy

— optimal policy would instead involve commitment to subsequent reflation (Eggertsson and Woodford, 2003)
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— optimal policy would instead involve commitment to subsequent reflation (Eggertsson and Woodford, 2003)

But the sub-optimality is of a plausible kind: inability to commit to history-dependent policy

— becomes much more problematic when ZLB binds
Conclusions

- Under “Great Depression” circumstances (ZLB reached, $\mu$ large), multiplier should be large, and it is optimal to increase government purchases aggressively, nearly to extent required to “fill the output gap”.

- If ZLB reached, but $\mu$ is small, multiplier should still be greater than 1, and it is optimal to increase $G$ beyond point consistent with efficient composition, though probably only a small fraction of what would “fill the gap”.

- When ZLB is not a constraint, output-gap stabilization should largely be left to monetary policy; decisions about government purchases governed by the principle of efficient composition of aggregate expenditure.
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Conclusions

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Conclusions

When ZLB binds, effective fiscal stimulus (and welfare-maximizing policy) require that government purchases be increased for as long as ZLB still binds, but not longer.