On the Importance of the Participation Margin for Labor Market Fluctuations*

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Abstract

Conventional analyses of the cyclical behavior of labor market stocks ascribe a minor role to the labor force participation margin in driving unemployment fluctuations. In contrast, using a novel decomposition of the variation in labor market stocks into components accounted for by underlying worker flows, we find that transitions between unemployment and nonparticipation account for a substantial fraction—around one-third—of the cyclical variation in the unemployment rate. This result continues to hold after adjustments of data for spurious transitions, and for time aggregation. We show that inferences from conventional, stocks-based analyses of labor force participation are subject to a stock-flow fallacy, neglecting the offsetting forces of worker flows that underlie the modest cyclicality of the participation rate. Further empirical investigation of heterogeneity in worker flows across labor market histories reveals that part of the contribution of the participation margin can be traced to cyclical fluctuations in the composition of the unemployed by labor market attachment.

Keywords: Worker flows; unemployment; business cycles; labor force participation.

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1 Introduction

What is the role of the labor force participation margin in shaping fluctuations in the unemployment rate? The majority of modern theoretical and empirical research on labor market fluctuations has operated under the assumption that movements of individuals in and out of the labor force play little or no role in driving fluctuations in the unemployment rate. Recent models of labor market fluctuations, such as those informed by models in the search and matching tradition of Mortensen and Pissarides (1994), typically proceed under a two-state abstraction, focusing on the margin between employment and unemployment.\(^1\) Mirroring this focus, recent empirical research on labor market flows has emphasized the relative contributions of the processes of job loss and job finding in driving cyclical unemployment, neglecting the participation margin.\(^2\)

In this paper we take a closer look at the role of the participation margin in shaping the evolution of unemployment over the business cycle. Based on the cyclical behavior of labor market stocks it is tempting to conclude that movements of individuals in and out of the labor force have a small impact on unemployment variation, seemingly reinforcing the two-state abstraction of recent literature. In particular, while there are clear, opposite cyclical patterns in rates of employment and unemployment, the labor force participation rate displays only a modest cyclicality in the United States (see, for example, Figure 1).

We show that such an inference is an example of a stock-flow fallacy. Using standard estimates of worker flows among three labor market states—employment, unemployment, and nonparticipation—we find that the moderate cyclicality of the stock of labor force participants masks substantial cyclicality in worker flows between unemployment and nonparticipation. In addition, we show that the cyclicality of these flows accounts for a substantial fraction—around one-third—of the cyclical variation in the unemployment rate. Our results therefore present a challenge to the practice of abstracting from labor force participation in theoretical and empirical work.

The starting point for our analysis is the standard data source for the study of worker flows in the United States: the longitudinally-linked monthly Current Population Survey (CPS) microdata, known as the “gross flows”. These data have been analyzed extensively

\(^1\)Theoretical papers that adopt a two-state abstraction are too numerous to cite. Exceptions to this tendency include: Alvarez and Veracierto (1999); Andolfatto and Gomme (1996); Andolfatto, Gomme, and Storer (1998); Kim (2001); Krusell, Mukoyama, Rogerson and Şahin (2010a, 2010b, 2012); Garibaldi and Wasmer (2005); Pries and Rogerson (2009); Shimer (2011); Veracierto (2008).

\(^2\)Examples include: Braun, De Bock, and DiCecio (2006); Elsby, Michaels, and Solon (2008); Fujita and Ramey (2008); Hall (2005a,b); and Shimer (2012).
in prior empirical work on labor market dynamics.\footnote{Early examples include Kaitz (1970), Perry (1972), and Marston (1976). More recent analyses include Blanchard and Diamond (1990), Fujita and Ramey (2006), and Shimer (2012).} In Section 2 we update these estimates and review their basic cyclical properties. There we confirm the countercyclicality of employment-to-unemployment transition probabilities, and the corresponding procyclicality of unemployment-to-employment transition probabilities that have been widely documented in previous literature. But, we also highlight an often-neglected feature of the gross flows estimates: During recessions, unemployed workers are less likely to flow out of the labor force, and nonparticipants are more likely to flow into unemployment. These two forces both are likely to contribute to the rise in the level of unemployment that accompanies recessions. The remainder of this paper investigates the robustness of this observation, and provides an accounting framework that allows one to quantify the magnitude of this channel.

A particular issue that arises when one uses the gross flows data is that they are thought to be particularly susceptible to classification errors in recorded labor market status (National Commission on Employment and Unemployment Statistics, 1979). While such errors may largely cancel in measured labor market stocks, they can accumulate in estimates of worker flows, leading to spurious measured transitions. Previous research has found these errors to be substantial, especially for transitions between unemployment and nonparticipation (see, for example, Abowd and Zellner, 1985; Poterba and Summers, 1986; and Chua and Fuller, 1987).

In section 3, we take this possibility seriously and examine whether adjustments for misclassification errors have an impact on the cyclicity of worker flows at the participation margin. We consider two approaches. First, following Blanchard and Diamond (1990), we apply Abowd and Zellner’s (1985) estimates of misclassification probabilities inferred from CPS reinterview surveys to adjust the gross flows estimates for spurious transitions. As noted in prior literature, this adjustment substantially reduces the estimated flows, especially those that involve transitions in and out of the labor force. We further show that, even though these estimates of classification errors are assumed to be time-invariant, they may nevertheless impart a countercyclical bias in gross flows estimates of the number of workers transitioning between unemployment and nonparticipation. Intuitively, times of recession are accompanied by a rise in the number of nonemployed individuals at risk of being misclassified. Consistent with this intuition, the countercyclicality of the inflow rate into unemployment from nonparticipation is shaded down in the adjusted data, and the rate of outflow of unemployed workers into nonparticipation remains prominently procyclical.
The estimates of classification errors reported in Abowd and Zellner (1985) are inferred under a particular assumption about the nature of misclassification errors—specifically that they are independently and identically distributed across time. For this reason, we also examine a second adjustment of the data. This exploits the fact that the rotation structure of the CPS makes it possible to match individual responses across four consecutive months, albeit for a more restricted sample of respondents. Using this smaller sample, we assess the effect of recoding sequences of recorded labor market states to eliminate high-frequency reversals of transitions between unemployment and nonparticipation. A prominent example of the latter are consecutive monthly transitions from nonparticipation to unemployment and then back to nonparticipation again. Since our method involves removing these NUN sequences, we sometimes will refer to these adjusted flows as “deNUNified” flows. Although not a definitive adjustment for classification error, such sequences of worker flows are more likely to reflect measurement errors, and provide a sense of whether the cyclicality of flows between unemployment and nonparticipation is particularly driven by improbable sequences of flows in and out of the labor force.

A striking feature of the results of this more practical recoding approach is that the adjusted flows line up closely with those implied by the Abowd and Zellner (1985) correction. This is the case despite the fact that the two adjustments are based on very different motivations: The Abowd and Zellner correction is inferred from data on resolved labor force status from CPS reinterview surveys; the deNUNified flows simply iron out reversals of a set of worker transitions. What emerges from this analysis is that, while the countercyclicality of the nonparticipation-to-unemployment rate is diminished by both conventional and practical adjustments for classification error, the procyclicality of the rate of outflow of unemployed workers into nonparticipation appears to be a robust feature of the dynamics of the labor market in the United States.

In addition to classification errors that tend to inflate estimates of worker transition rates, the gross flows data also are subject to an offsetting bias that leads to an underestimation of flows. In particular, due to the discrete monthly nature of the CPS, measured transition probabilities are subject to a time aggregation problem—they may miss multiple transitions that occur between consecutive monthly surveys—and thus may not accurately reflect the underlying flows (Darby, Haltiwanger and Plant, 1985; Shimer, 2012). In section 4, we provide an analytical correction for time aggregation that can be applied to estimates of worker flows among arbitrarily many labor force states, such as the usual three. A virtue
of this correction is that it is very simple to implement. We apply this methodology to infer a set of estimates of the underlying continuous-time flow hazards among employment, unemployment and nonparticipation, both for the raw gross flows estimates, as well as those adjusted for classification error. These estimates suggest that time aggregation bias has a substantial effect on the measured levels of worker flows—standard gross flows measures of transition probabilities are estimated to miss 15 to 30 percent of the underlying flows. However, we find that the cyclical behavior of the estimated flow hazards, including those between unemployment and nonparticipation, is nonetheless preserved.

Taking these estimates of flow hazards as our data, in Section 5 we devise a novel decomposition of the time-series variation in each of the labor market states into components accounted for by each of the associated worker flow hazards. The decomposition exploits a partial-adjustment representation of the dynamics of labor market stocks that allows one to express the change in each of the stocks as a distributed lag of current and past changes in the flow hazards among labor market states. This in turn motivates a simple decomposition of variance that allows one to compute the fraction of variance in each labor force state accounted for by variation in each flow hazard.

We apply this decomposition to our estimates of worker flows, both with and without adjustments for classification error. The results of this exercise inform a key result of the paper: that the participation margin accounts for a substantial fraction, around one-third, of the rise in unemployment during recessions in the United States. Moreover, we find that the contribution of the participation margin remains substantial even after adjustments for classification error. Thus, the cyclicity of flows between unemployment and nonparticipation that we highlight has an important quantitative impact on the evolution of the

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4This time aggregation correction already has been applied in recent work on worker flows—see, for example, Barnichon and Figura (2012). Since our work on early versions of this paper, we learned of concurrent work by Shimer (2011) that develops the same time aggregation correction.

5While other recent work has analyzed the role of flows between the three labor force states, our approach makes important progress on a number of dimensions. First, following Shimer (2012), many papers have examined the consequences for flow-steady-state unemployment of holding constant one or more of the worker flow hazards (Gomes, 2012; King, 2011; Kudlyak and Schwartzman, 2012). Our approach instead provides an explicit analytical decomposition that accounts both for the nonlinear relationship between flows and stocks, as well as the out-of-steady-state transmission of past movements in worker flows. Second, since the earlier work of Petrongolo and Pissarides (2008), many papers have lumped together one or more of the contributions of flows in and out of the labor force (Barnichon and Figura, 2012; Elsby, Smith and Wadsworth, 2011; Smith, 2011). In constrast, our approach infers individual contributions for all of the underlying worker flows. Third, due to the generality of our approach, it can be used to investigate the flow origins of variation in any combination of labor market stocks, such as the participation rate. The latter provides an important perspective on why decompositions based respectively on stocks and flows yield divergent results on the role of the participation margin.
unemployment rate.

As discussed in the opening paragraphs of this Introduction, our finding of a significant role for the participation margin in driving unemployment fluctuations strikes at the heart of conventional wisdom on the topic. The latter instead holds that the modest reductions in labor force participation that accompany recessions in fact serve to reduce slightly the associated rise in unemployment. In section 6, we explain why such reasoning is an example of a stock-flow fallacy. We show that it is important to realize that the cyclical behavior of the labor force participation rate is itself the outcome of subtle interactions of movements in worker flow rates, just like the unemployment rate.

We illustrate this point by exploiting a virtue of our flows-based variance decomposition, namely that it can be applied to any combination of labor market stocks, in particular the participation rate. Using a set of examples, we show that the much of the variation in labor force participation can in fact be traced to movements in flows between employment and nonparticipation. Such flows have only a very indirect effect on the unemployment rate, yet an analysis of labor market stocks would incorrectly ascribe to this variation an unemployment-reducing effect in times of recession.

The message of this paper, then, is that a complete understanding of fluctuations in unemployment in the United States requires an understanding of the apparent cyclical movements in worker flows at the participation margin. In the final section of the paper, we identify one fruitful avenue of research toward this end. We focus in particular on the procyclicality of the rate of outflow of unemployed workers to nonparticipation.

We propose and quantify a novel hypothesis for why this occurs, based on cyclical shifts in the degree to which unemployed workers are attached to the labor market. Using CPS microdata matched across all available months in sample, we provide new estimates of worker flows conditional on past labor market status, defined as status one year prior to the CPS survey. Quite sensibly, we find that unemployed individuals that were employed in the previous year are much less likely to exit the labor force than their nonemployed counterparts. Importantly, during recessions, the composition of the unemployment pool shifts towards workers who are more attached to the labor market, in particular male, prime-aged individuals who were employed in the past. The latter is consistent with the wave of job loss that occurs at the onset of downturns in the United States. We find that this compositional shift along just these few dimensions accounts for between one-third and two-thirds of the recessionary decline in the rate of exit of unemployed workers from the labor force since the
late 1970s.\(^6\)

2 Data on labor market flows

The data we use are the “gross flows” data from the Current Population Survey (CPS). These measures of worker flows are obtained by exploiting a rotating-panel element in the CPS sample design. Addresses selected into the survey remain in the sample for four consecutive months, rotate out for eight months, and then rotate back in again for a further four months. A consequence is that, in any given month, the CPS is comprised of eight “rotation groups,” six of which will be surveyed in the subsequent month. In principle, then, a maximum of three-quarters of the sample in a given month can be linked longitudinally to their responses one month later. In practice, however, it is possible to match approximately two-thirds of the CPS sample across consecutive months due to non-response, changes of residence and so on.

Using these longitudinally-linked microdata, it is straightforward to estimate worker flows and their associated transition probabilities. For example, the probability that an unemployed worker finds a job and is employed one month later can be computed simply as the fraction of the unemployed in a given month who subsequently report that they are employed in the next month’s survey. Using this method, one can compute monthly flow transition probabilities among employment, unemployment and nonparticipation for each month of available data.

Measures of worker flows based on this approach have been made available from a number of sources. Data for February 1990 onwards are posted on the Bureau of Labor Statistics website. Shimer (2012) has computed analogous measures using CPS microdata from January 1976. Data from June 1967 to December 1975 have been tabulated by Joe Ritter and made available by Hoyt Bleakley.

These measures have become the standard source for estimating worker flows among

\(^6\)This finding contrasts with those of Baker (1992) and Shimer (2012), who investigate the role of compositional shifts on the total rate of outflow from unemployment, and find only small effects. Our own analysis suggests that this difference can be traced to two factors: First, a novel aspect of our analysis is that it adjusts additionally for composition across past labor market status, a dimension that we find to be especially important. Second, we emphasize composition effects on the outflow rate to nonparticipation, which is just one part of the total outflow rate analyzed by Baker and Shimer. Interestingly, we find offsetting effects of composition on the other component, outflows to employment. This, of course, is consistent with the notion that the composition of the unemployment pool shifts in recessions towards those who are relatively more attached to the labor market.
labor force states. They are the basis of a long line of research on unemployment flows, and have informed much of what we know about labor market dynamics (see, among many others, Kaitz, 1970; Perry, 1972; Marston, 1976; Blanchard and Diamond, 1990; Fujita and Ramey, 2006; and Shimer, 2012). While these data are known to be subject to a number of drawbacks that are the subject of the ensuing sections, it is instructive first to summarize the basic cyclical properties of worker flows in the gross flows data. The “unadjusted” series in Figure 2 plot the raw gross flows transition probabilities between employment, unemployment and nonparticipation. There are clear, systematic empirical regularities in the behavior of these measures over the business cycle. Among these, a particularly well-emphasized observation is the notable countercyclicality of the employment-to-unemployment probability, and the prominent procyclicality of the unemployment-to-employment probability, a feature confirmed in panels (a) and (b) of Figure 2. Clearly, both of these contribute to the cyclicality of the unemployment rate.

Considerably less emphasis has been given to fluctuations in flow probabilities between unemployment and nonparticipation over the business cycle, however. Panels (c) and (d) of Figure 2 reveal that rates of inflow to unemployment from nonparticipation rise substantially in recessions, while rates of outflow to nonparticipation decline substantially. By the same token, these flows in and out of the labor force also must contribute to the rise in unemployment that accompanies recessions in the United States. The magnitude of this contribution and its robustness are the focus of the remainder of the paper.

3 Adjustments for classification error

A drawback of the gross flows estimates of worker flows is that they are sensitive to classification errors in recorded labor market states, which may lead to spurious measured transitions. For example, imagine a respondent who is in fact unemployed for three consecutive surveys, but who is misclassified as out of the labor force in the second survey. In this example, we would observe two spurious measured transitions—from unemployment to nonparticipation and vice versa. Estimates of such classification errors suggest that spurious transitions are particularly important for such transitions between unemployment and nonparticipation (Abowd and Zellner, 1985; Poterba and Summers, 1986).

Because these transitions between unemployment and nonparticipation are the particular focus of our study, we take the potential effects of such classification errors seriously. In order to consider whether our results are affected by these errors, we examine the effect of two
specific adjustments for classification error. In the remainder of this section we introduce these two adjustment methods and document their effects on the time series behavior of labor market stocks and flows.

3.1 Abowd and Zellner (1985) correction

The first adjustment we consider is based on a literature that has sought to estimate the magnitude of classification errors in recorded labor market status using data from a subsample of the CPS (around one-thirtieth of the overall sample) that is reinterviewed each month (see, for example, Abowd and Zellner, 1985; Poterba and Summers, 1986; and Chua and Fuller, 1987). Denoting the measured stocks of employed, unemployed and nonparticipants respectively as \( \hat{E} \), \( \hat{U} \), and \( \hat{N} \), these studies assume the following relation between measured stocks and their “true” counterparts \( E \), \( U \), and \( N \):

\[
\begin{bmatrix}
\hat{E} \\
\hat{U} \\
\hat{N}
\end{bmatrix}
= \begin{bmatrix}
1 - \varepsilon_{EU} - \varepsilon_{EN} & \varepsilon_{UE} & \varepsilon_{NE} \\
\varepsilon_{EU} & 1 - \varepsilon_{UE} - \varepsilon_{UN} & \varepsilon_{NU} \\
\varepsilon_{EN} & \varepsilon_{UN} & 1 - \varepsilon_{NE} - \varepsilon_{NE}
\end{bmatrix}
\begin{bmatrix}
E \\
U \\
N
\end{bmatrix},
\]

(1)

where, \( \varepsilon_{ij} \) is the probability that an individual with true labor market state \( i \) is recorded as measured state \( j \).

Estimates of the elements of the matrix of classification error probabilities \( E \) are based on a series of CPS reinterview surveys in which CPS respondents were contacted for a follow-up interview to check the validity of their original responses. Table 1 reproduces the estimate of \( E \) from Abowd and Zellner (1985, Table 6). It can be seen that the most common classification error relates to individuals counted as nonparticipants who in fact should be classified as unemployed. This is true for approximately 10 percent of persons who were determined to be unemployed upon reinterview.

These estimates of \( E \) allow one to infer estimates of the underlying corrected worker flows from the raw measured gross flows. Specifically, if we denote the number (as opposed to the transition probabilities) of individuals flowing from state \( i \) in month \( t - 1 \) to state \( j \) in month
t by $ij_t$, and the associated matrix of these flows by

$$N_t = \begin{bmatrix} EE & UE & NE \\ EU & UU & NU \\ EN & UN & NN \end{bmatrix}_t$$

(2)

then Poterba and Summers (1986) show that measured flows, $\hat{N}_t$, can be related to their true counterparts $N_t$ according to the relation $\hat{N}_t = EN_tE'$. One may then infer the matrix of corrected flows simply by inverting this relation to obtain

$$N_t = E^{-1}\hat{N}_t (E^{-1})'.$$

(3)

An implicit assumption that underlies this adjustment is that classification errors are time-invariant. A priori, then, it would seem unlikely that such misclassification could explain the cyclical fluctuations in these flows we document above. We argue that such a conclusion would be premature. To see why, it is helpful to consider a simple special case in which classification errors exist only between unemployment and nonparticipation—that is, $\varepsilon_{ij} = 0$ for all $ij \notin \{UN, NU\}$. For small $\varepsilon_{UN}$ and $\varepsilon_{NU}$, we show in the Appendix that measured flows between unemployment and nonparticipation can be related to error-free flows according to the simple approximations:

$$\hat{UN}_t \approx (1 - \varepsilon_{UN} - \varepsilon_{NU}) UN_t + \varepsilon_{UN}UU_t + \varepsilon_{NU}NN_t,$$

and

$$\hat{NU}_t \approx (1 - \varepsilon_{UN} - \varepsilon_{NU}) NU_t + \varepsilon_{UN}UU_t + \varepsilon_{NU}NN_t.$$  

(4)

The first terms in these expressions respectively capture the fraction of true flows that show up in measured transitions. The subsequent terms capture spurious transitions driven by classification errors.

Equation (4) highlights why even time-invariant classification errors can imply a bias in measured flows that varies over the cycle. The key is that the number of individuals who remain unemployed $UU_t$ rises substantially in recessions as the stock of unemployed workers itself rises. As a result, this imparts a countercyclical bias in measured transitions between unemployment and nonparticipation, $UN_t$ and $NU_t$. The intuition is simple: During a recession, there are more nonemployed individuals at risk of being misclassified.
3.2 Recoding of unemployment-nonparticipation cyclers

The Abowd-Zellner correction for classification errors summarized has two potential shortcomings. First of all, it is based on data from past reinterview surveys. Second, it relies on a maintained assumption that measurement errors are time-invariant. We therefore examine an alternative adjustment of measured transitions which, for reasons that will become clear, we sometimes will refer to as *deNUNified* flows. This adjustment takes a more practical approach: It identifies individuals whose measured labor market state cycles between unemployment and nonparticipation from month to month, and assesses the effect of omitting such transitions on the cyclical properties of the associated flows.

In order to identify such transitions, it is necessary to match an individual’s labor market status across more than just two months. The rotation structure of the CPS is such that each household is surveyed for two sets of four consecutive months, with an intervening eight-month hiatus. Thus, the CPS allows one to identify an individual’s labor market status for a maximum of four successive months. These are the data that we use for our recoding procedure.

Our approach is first to isolate sequences of transitions that involve the reversal of a transition from unemployment to nonparticipation and vice versa. We denote a sequence of transitions from unemployment to nonparticipation to unemployment as *UNU*s, and analogously *N*-to-*U*-to-*N* sequences as *NUN*s. We then examine the effects of recoding the data to eliminate these transition reversals—hence “*deNU*Nified” flows. Table 2 summarizes the combinations of flow sequences that are recoded in this way.

The goal of this exercise is not to provide a definitive correction of labor market flows for classification errors: The approach inevitably will miss some spurious transitions between unemployment and nonparticipation, and will purge some genuine transitions. The goal is rather to investigate whether the recoding of transitions that are more likely to reflect measurement error has a significant impact on the cyclicality of flow transitions between unemployment and nonparticipation. This approach complements the correction in the previous subsection in the sense that it relies neither on the use of reinterview data from the past nor on an assumption of time-invariant classification errors.

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7 Unfortunately, CPS reinterview surveys are no longer being implemented. It is therefore not possible to update the estimates of $E$ in Table 1.
3.3 Stocks and flows adjusted for classification error

Figure 1 plots the published unemployment and participation rates together with those implied by the AZ correction and the deNUNified flows. The left and right panels respectively depict the time series for the associated unemployment rates and labor force participation rates.

We find that both adjustments for classification errors imply quite small adjustments of labor market stocks. The reason relates to the intuition that classification errors will tend to cancel out in the cross section (see, for example, National Commission on Employment and Unemployment Statistics, 1979). In accordance with this intuition, we find that the number of NU$N$s and UN$U$s tend almost to offset one another, so that our recoding procedure leaves the implied stocks almost unchanged. The Abowd and Zellner (1985, AZ) correction induces a modest adjustment to the levels of the unemployment and participation rates. This arises because the most common error is the misclassification of someone who is unemployed as being out of the labor force (see Table 1). As a result, the correction reclassifies a number of people from nonparticipation into unemployment, thus raising slightly both the unemployment rate and the participation rate. In addition, Figure 1 suggests that both adjustments have a very small effect on the cyclicality of labor market stocks.8

In contrast, we find that estimated worker flows are more sensitive to the presence of classification errors, consistent with the intuition above. The effects of each adjustment for classification error on estimated worker flows are illustrated in Figure 2. This plots the estimated transition probabilities $p_{ij_t} \equiv i_{jt}/i_{t-1}$ for $i, j \in \{E,U,N\}$, that have been adjusted for classification errors, together with their unadjusted counterparts for reference. The AZ-adjusted flows are obtained by applying the adjustment in equation (3) to the gross flows data described above in Section 2. The deNUNified flows instead are based on CPS microdata in which individuals’ outcomes have been matched over all months in sample.

In keeping with prior literature, for all plotted series we implement a correction for margin error that restricts the estimates of worker flows to be consistent with the evolution of the corresponding labor market stocks depicted in Figure 1.9 Our approach is similar to

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8Recent work by Feng and Hu (2012) applies a different classification error adjustment that implies larger increases in the unemployment rate and a smaller rise in the participation rate. The directions of the adjustments are similar though and their method does not yield adjusted gross flows estimates that we use for our analysis.

9Margin error can arise for a number of reasons. First, we ignore movements in and out of the working-age population, such as those who turn 16, die, emigrate, immigrate and so on, that are classified as “other” in the BLS gross flows data. In addition, it is possible that attrition of households from our matched CPS samples is not random with respect to labor force status. For both these reasons, implied changes in labor
that employed by Poterba and Summers (1986), and solves for the set of stock-consistent transition probabilities that minimizes the weighted sum of squares of the margin-error adjustments, and is described in detail in the Appendix. In practice, however, we find that the margin-error adjustment has a very small effect on the estimated transition probabilities.

Consistent with the notion that classification errors can accumulate in estimated flows leading to spurious estimated transitions, Figure 2 reveals that the adjusted flows lie systematically below their unadjusted counterparts. As noted in prior literature, flows in and out of the labor force particularly are affected. Transition rates between employment and nonparticipation are approximately halved, while those between unemployment and nonparticipation are adjusted down by around one third.

Interestingly, the cyclicality of rates of transition between $U$ and $N$ also appears to be affected in a manner consistent with the intuition of equation (4). While the nonparticipation-to-unemployment transition rate remains countercyclical, its fluctuations are seen to be less volatile than in the raw gross flows data. In contrast, the adjusted unemployment-to-nonparticipation rate retains its procyclicality. Both of these observations dovetail with the logic above that classification errors can lead to a countercyclical biases in flows between unemployment and nonparticipation.

Figure 2 also illustrates the impact of the adjustment for classification error based on the recoding of unemployment-nonparticipation cyclers. Unsurprisingly, the adjustment has little effect on flow transition rates between employment and unemployment, and employment and nonparticipation. The time series for these flow hazards differ slightly from those implied by the raw gross flows because the adjusted flows are based on the smaller sample of households that can be matched across four consecutive months (rather than just two).

A striking aspect of Figure 2, however, is that the de$NUN$ified transition rates between unemployment and nonparticipation correspond very closely to the adjusted flows based on the Abowd and Zellner (1985) estimates of time-invariant classification errors. Note that there is no mechanical reason to expect this: The AZ adjustment is based on error probabilities implied by resolved labor force status from reinterview data; the recoding approach simply unwinds reversals of transitions between unemployment and nonparticipation. The correspondence between the two adjustments holds both in terms of the levels of these flow hazards, as well as their cyclicality. Both the rates of inflow to and outflow from unemployment on the participation margin are reduced by around one-third. As in the AZ-adjusted market stocks in our matched samples may not necessarily replicate changes in the published stocks. Our finding, however, is that there is only a small discrepancy between implied and published changes in stocks.
data, inflows into unemployment from out of the labor force are weakly countercyclical. Importantly, the rate at which the unemployed flow out of the labor force continues to fall substantially in times of recession.

4 Adjustments for temporal aggregation

Due to the monthly frequency of the CPS data, the gross flows provide us only with a series of snapshots of an individual’s labor force status observed at discrete points in time. In practice, however, a person may make multiple transitions between consecutive surveys. For this reason, the gross flows estimates will not provide an accurate picture of the underlying flows—they will miss some transitions and incorrectly include others.

To see this, imagine an individual who is recorded as a nonparticipant in one month and as employed in the next month. In principle, there is an infinity of possible (though not equally-probable) paths that would yield this observation in discrete-time data. For example, the person could have flowed from nonparticipation to unemployment, and then from unemployment to employment. In that case, the discrete-time data would miss the latter two transitions, and would incorrectly ascribe them to a single employment to nonparticipation flow.

This temporal aggregation problem was noted by Darby, Haltiwanger and Plant (1985), and Shimer (2012) provided a correction for this bias in the context of a model with two labor force states—employment and unemployment. Here we provide a simple analytical correction for time aggregation bias in worker flows that can be applied to arbitrarily many labor market states, such as the usual three—employment, unemployment and nonparticipation.\(^\text{10}\)

The task is to back out from estimates of the discrete-time transition probabilities \(p_{ij}\) corresponding estimates of the underlying instantaneous flow hazard rates, which we shall denote \(f_{ij}\). To understand how this correction works, note first that the estimates of transition probabilities \(p_{ij}\), satisfy the discrete-time Markov chain

\[
\begin{pmatrix}
 E \\
 U \\
 N \\
 I_t
\end{pmatrix}_t = \begin{pmatrix}
 1 - p_{EU} - p_{EN} & p_{UE} & p_{NE} \\
 p_{EU} & 1 - p_{UE} - p_{UN} & p_{NU} \\
 p_{EN} & p_{UN} & 1 - p_{NE} - p_{NU}
\end{pmatrix}
\begin{pmatrix}
 E \\
 U \\
 N \\
 I_{t-1}
\end{pmatrix}_{t-1}
\]

\[ (5) \]

\(^{10}\)As noted in footnote 4, this correction is the same as that developed concurrently in Shimer (2011).
Following Shimer (2012), we assume that the flow hazards are constant between monthly surveys, but may shift across months. Thus, we can express the underlying evolution of labor market stocks in a given month $t$ in the form of the following continuous-time analogue to equation (5),\footnote{A drawback of this expression of the underlying worker flows is that it assumes that there is a contemporaneous mapping between an individual’s labor market activities—working, searching, not searching—and their recorded labor market states—employment, unemployment and nonparticipation. In practice, there is a dynamic mapping between activities and recorded states. For example, to be recorded as unemployed, a respondent must have looked for work during the last month under the CPS definition. It is an important topic for future research to disentangle these more subtle time aggregation issues.}

$$
\dot{\mathbf{l}}_t = \begin{bmatrix}
-f_{EU} - f_{EN} & f_{UE} & f_{NE} \\
-f_{EU} & -f_{UE} - f_{UN} & f_{NU} \\
f_{EN} & f_{UN} & -f_{NE} - f_{NU}
\end{bmatrix}
\mathbf{F}_t
\mathbf{l}_t.
$$

(6)

Each month, the CPS provides measures of the initial and terminal conditions for this system, $\mathbf{l}_t(t) = \mathbf{l}_{t-1}$, as well as the matrix of discrete-time transition probabilities, $\mathbf{P}_t$. To see how one can use this information to infer the associated continuous-time hazards, note that one may solve forward the differential system (6) from time $t - 1$ to time $t$ to obtain

$$
\mathbf{l}_t = \mathbf{V}_t \mathbf{D}_t \mathbf{V}_t^{-1} \mathbf{l}_{t-1},
$$

(7)
the raw gross flows miss around 30 percent of inflows into unemployment, and 15 percent of outflows from unemployment to both employment and nonparticipation. In contrast, temporal aggregation in the raw gross flows leads to a slight overstatement of transitions between employment and nonparticipation.

The intuition for these results can be traced in large part to the magnitude of the probability of exiting unemployment in the United States. Figure 3 shows that unemployed individuals flow into both employment and nonparticipation with an average probability of around 25 percent over the course of a month. As a result, the likelihood that an individual who flows into unemployment between CPS surveys exits unemployment prior to the next survey is nontrivial. Consequently, the raw gross flows will understate transitions in and out of unemployment. For the same reason, the overstatement of transitions between employment and nonparticipation in the gross flows data arises because an individual is more likely to experience an intervening unemployment spell when transitioning between these two states.

Aside from the effect of temporal aggregation on the estimated levels of worker flows, a notable feature of the adjusted flows in Figure 3 is that the cyclical properties of the corrected series are qualitatively unchanged. Importantly for the focus of this paper, the rate of outflow from unemployment to nonparticipation continues to fall during recessionary episodes after adjusting for temporal aggregation.

5 Measuring the role of the participation margin

With measures of the instantaneous transition rates $f_{ij}$ in hand, we can use them to inform a decomposition of the time-series variance of each of the labor market stocks into parts accounted for by each of the respective flow hazards. In this section, we devise such a decomposition using analytical approximations to a partial-adjustment representation of labor market dynamics. We then apply this decomposition to the estimates of the flow hazards described above.

5.1 A three-state decomposition of unemployment fluctuations

In order to motivate our decomposition of variance, it is helpful first to simplify the system of equations in (5). Specifically, we normalize labor market stocks by the civilian non-institutional working-age population, so that $E_t$, $U_t$ and $N_t$ are to be interpreted as shares
of the population. It follows that $E_t + U_t + N_t \equiv 1$ for all $t$, and the three-equation system (5) can thus be rewritten as a two-dimensional system of the form

$$
\begin{bmatrix}
E \\
U
\end{bmatrix}_t =
\begin{bmatrix}
1 - p_{EU} - p_{EN} - p_{NE} & p_{UE} - p_{NE} \\
p_{EU} - p_{NU} & 1 - p_{UE} - p_{UN} - p_{NU}
\end{bmatrix}
\begin{bmatrix}
E \\
U
\end{bmatrix}_{t-1} +
\begin{bmatrix}
p_{NE} \\
p_{NU}
\end{bmatrix}_t
$$

We denote the flow steady state of this Markov chain by $\bar{s}_t = \left(I - \tilde{P}_t\right)^{-1} q_t$.

As in the two-state case described in Elsby, Hobijn, and Şahin (forthcoming), changes over time in the flow hazards $f_{ij}$ shift the discrete-time transition probabilities $p_{ij}$, as well as the steady state that the labor market is converging to, $\bar{s}_t$. It is through this chain of events that changes in the underlying flows affect the path of employment and unemployment over time. We show in the Appendix that this intuition can be formalized as

$$
\Delta s_t = A_t \Delta \bar{s}_t + B_t \Delta s_{t-1},
$$

where $A_t = \left(I - \tilde{P}_t\right)$ and $B_t = \left(I - \tilde{P}_t\right) \tilde{P}_{t-1} \left(I - \tilde{P}_{t-1}\right)^{-1}$. The first term in (9) captures the changes in labor market stocks that are driven by contemporaneous changes in the flow transition rates which shift the flow steady state, $\bar{s}_t$. The second term in equation (9) summarizes the transmission of past changes in transition rates onto the current labor market state.

This partial adjustment representation can be used to motivate a decomposition of variance for the change in labor market stocks over time, $\Delta s_t$. To see how, note first that one can iterate backward on equation (9) to express $\Delta s_t$ as a distributed lag of past changes in the steady-state labor market stocks $\Delta \bar{s}_t$,

$$
\Delta s_t = \sum_{k=0}^{t-1} C_{k,t} \Delta \bar{s}_{t-k} + D_t \Delta s_0,
$$

where $C_{k,t} = \left(\prod_{n=0}^{t-k-1} B_{t-n}\right) A_{t-k}$ and $D_t = \prod_{k=0}^{t-1} B_{t-k}$, and $\Delta s_0$ is the change in labor market stocks in the first period of available data.

As we noted above, changes in the flow hazards $f_{ij}$ shape the present and future evolution of $\Delta s_t$ by shifting its flow-steady-state counterpart, $\Delta \bar{s}_t$. Thus, to link changes in labor
market stocks to changes in the flow hazards, we take a first-order approximation to the
change in the steady-state labor market stocks,

\[
\Delta \bar{s}_t \approx \sum_{i \neq j} \frac{\partial \bar{s}_t}{\partial f_{ij}} \Delta f_{ij},
\]

(11)

where the approximation has been taken around the lagged flow hazard rates, \( f_{ij, t-1} \). To compute the derivatives in equation (11), note that we can write the continuous-time analogue to the reduced-state Markov chain in (8) as

\[
\dot{s}_t = \begin{bmatrix}
-f_{EU} - f_{EN} - f_{NE} & f_{UE} - f_{NE} \\
 f_{EU} - f_{NU} & -f_{UE} - f_{UN} - f_{NU}
\end{bmatrix} s_t + \begin{bmatrix}
f_{NE} \\
f_{NU}
\end{bmatrix} g_t.
\]

(12)

It follows that we can write the flow steady state of the system as

\[
\bar{s}_t = \bar{F}^{-1} g_t.
\]

Using this, the associated derivatives in equation (11) are straightforward to compute analytically.

Piecing these components together yields the following decomposition of variance:

\[
\text{var}(\Delta s_t) \approx \sum_{i \neq j} \text{cov}(\Delta s_t, \sum_{k=0}^{t-1} C_{k,t} \frac{\partial s_{t-k}}{\partial f_{ijt-k}} \Delta f_{ijt-k}).
\]

(13)

A direct implication of (13) is that one can compute the fraction of the variance in any given labor market stock variable accounted for by variation in any given flow transition hazard. For example, if one were interested in computing the contribution of changes in the employment-to-unemployment flow hazard, \( f_{EU} \), to changes in the unemployment stock, then one could compute:

\[
\beta_{EU}^U = \frac{\text{cov} \left( \Delta U_t, \left[ \sum_{k=0}^{t-1} C_{k,t} \frac{\partial s_{t-k}}{\partial f_{EUt-k}} \Delta f_{EUt-k} \right] \right)}{\text{var}(\Delta U_t)}.
\]

(14)

Of course, the latter decomposition of variance applies to the stock of unemployed workers as a fraction of the working-age population, and therefore not directly to the unemployment rate, \( u_t \equiv U_t / L_t \), where \( L_t \equiv E_t + U_t \) is the labor force participation rate. However, it is straightforward to derive a decomposition of changes in \( u_t \) using the approximate transform,

\[
\Delta u_t \approx (1 - u_{t-1}) \frac{\Delta U_t}{L_{t-1}} - u_{t-1} \frac{\Delta E_t}{L_{t-1}}.
\]

(15)
Since the labor force participation rate is the sum of $E_t$ and $U_t$, a decomposition of the labor force participation rate in terms of the contribution of changes in the flow hazards can be derived in a similar way to that of the unemployment rate.

### 5.2 Results

Table 3 summarizes the results of applying this decomposition to the estimates of the flow hazards $f_{ij}$ derived above. It reports the shares of the variance of the unemployment rate accounted for by each $f_{ij}$ based on both the unadjusted flows, as well as those adjusted for classification errors. Overall, the approach appears to provide an accurate decomposition of unemployment variance, in the sense that the contributions of each flow sum approximately to one—the residual variance is generally less than 6 percent.

Consider first the results for the unadjusted gross flows estimates in the first row of Table 3. These confirm the well-known result that both countercyclical rates of job loss and procyclical rates of job finding account for a substantial fraction of the fluctuations in the aggregate unemployment rate. Over the whole sample period, around one-quarter of the cyclicality of the unemployment rate can be traced to the employment-to-unemployment hazard, and one-third to the unemployment-to-employment hazard, with a total contribution of approximately 60 percent. Thus, it is clear that an explanation of the processes of job loss and job finding is crucial to an understanding of the cyclical behavior of the labor market.

The next two columns of Table 3, however, reaffirm the visual impression of Figure 3 that the participation margin also accounts for a substantial fraction of the rise in unemployment during recessions. The combined contribution of flows between unemployment and nonparticipation accounts for around one-third of unemployment variation. Consistent with the countercyclicality of inflows into unemployment from nonparticipation, and the procyclicality of the $U$-to-$N$ flow hazard, both flows matter. However, the $U$-to-$N$ flow hazard contributes more than the $N$-to-$U$ flow hazard.

Together, flows between unemployment and employment and flows between unemployment and nonparticipation explain the vast majority of unemployment movements; the indirect effect of flows between employment and nonparticipation is negligible.

The message of this analysis, then, is that the standard gross flows estimates of labor market transitions imply an economically-significant role for the participation margin. In what follows, we examine whether this baseline result is robust to the adjustments for classification error discussed earlier.
The remaining rows of Table 3 provide a quantitative sense of this. They implement the variance decomposition using instead estimates of flows hazards based on the Abowd and Zellner (1985) correction and the deNUNified flows. The contributions of flows between unemployment and employment are adjusted upward somewhat by both corrections, accounting for approximately two-thirds of unemployment fluctuations over the whole sample period. In addition, the variance contribution of flows from $U$ to $N$ remains in the neighborhood of 20 percent in the adjusted data. Consistent with the visual impression of Figure 3, and the message of equation (4), the estimated contribution of $N$-to-$U$ flows is shaded down relative to the unadjusted gross flows data, especially for the AZ correction. Despite this, the joint contribution of the participation margin in the adjusted flows remains at around 30 percent of the variation in the unemployment rate. Thus, even after implementing adjustments for classification error, the participation margin is estimated to play a prominent role in driving cyclical unemployment dynamics.

It is instructive to compare these findings to prior literature that has focused on the respective roles of unemployment inflows and outflows in accounting for unemployment fluctuations in the context of a two-state framework. The results in Table 3 imply a joint variance contribution of unemployment outflows (the sum of the contributions of $U$-to-$E$ and $U$-to-$N$ flows) of approximately 60 percent for the unadjusted data, and 68 percent for the Abowd and Zellner (1985) correction. This is broadly consistent with the findings of earlier literature that has suggested something like a two-thirds outflows to one-third inflows decomposition of unemployment fluctuations (see for example Elsby, Michaels, and Solon, 2009; and Fujita and Ramey, 2009).\footnote{A drawback of the earlier two-state literature is that the estimated “inflow rate” into unemployment unavoidably conflates inflows from employment and nonparticipation respectively in a non-additive way. An advantage of the three-state decomposition provided in the present paper is that it disentangles these separate effects.}

6 Stock vs. flow decompositions: a stock-flow fallacy

The message of the above flows-based decomposition that worker transitions between unemployment and nonparticipation contribute substantially to cyclical fluctuations in the unemployment rate is a provocative one in the light of conventional wisdom. A prominent heuristic used to quantify the role of the participation margin in accounting for cyclical unemployment fluctuations is implicit in Figure 1. Specifically, a simple stocks-based decomposition of the variation in the unemployment rate can be derived from the following
approximate relation,
\[ \Delta u_t \approx (1 - u_{t-1}) (\Delta \log L_t - \Delta \log E_t). \]  
(16)

Thus, a close approximation to the change in the unemployment rate \( \Delta u_t \) is the difference in the logarithmic changes in the labor force participation rate \( \Delta \log L_t \), and the employment-to-population ratio \( \Delta \log E_t \).

Application of this stocks-based decomposition to quarterly-averages of published labor market stocks from the Bureau of Labor Statistics for the period 1967 to 2012 implies a contribution of variance in the labor force participation rate to variance in the unemployment rate of

\[ \beta_{u,lfpr}^u = \frac{\text{cov}(\Delta u_t, (1 - u_{t-1}) \Delta \log L_t)}{\text{var}(\Delta u_t)} \approx -7 \text{ percent}. \]  
(17)

This result stands in stark contrast to the implications of the flows-based decomposition summarized in Table 3. According to (17), the role of the participation margin is both quantitatively small, and of opposite sign, relative to that implied by the flows. The reason, of course, is that the labor force participation rate is mildly procyclical in the data. It follows that a simple stocks-based decomposition will suggest that the small declines in participation that accompany recessions in fact offset slightly the rise in unemployment. Comparisons of the relative cyclicality of labor market stocks, such as this, have informed a conventional wisdom that participation decisions are not of first-order importance for an understanding of unemployment fluctuations (see, for example, Hall, 2008, 2009). In the remainder of this section, we explain why this conclusion is an example of a stock-flow fallacy.

The key to understanding the seeming tension between these two approaches is to note that, in a dynamic labor market, the labor force participation rate is itself shaped by the underlying behavior of worker flows, just like the unemployment rate.\(^\text{14}\) Thus, the observed mild procyclicality of the participation rate is in fact the outcome of a subtle interaction of offsetting cyclical movements in worker flow hazards.

To illustrate this point, we exploit a virtue of the flows-based decomposition in equation

\(^\text{14}\)It is worth noting that the stocks-based and flows-based decompositions would deliver the same conclusion if the labor market were relatively static, which is the assumption implicit in a stocks-based decomposition. For example, if recessionary declines in labor force participation were brought about by the movement of a small group of individuals from unemployment to nonparticipation that subsequently were reversed during times of recovery, increases in unemployment during recessions would be mitigated by an upward spike in the \( U \)-to-\( N \) hazard, and the two approaches would concur. Notwithstanding the fact that the \( U \)-to-\( N \) hazard in fact appears to fall during recessions, this view of the labor market also implies low levels of worker flows. Several decades of research on worker flows supports the exact opposite view, namely that worker flows are large, and that consequently the identities of individuals in each of the labor market states are shifting continually.
(13), namely that it can be applied to any combination of labor market stocks, including the labor force participation rate, \( L \equiv E+U \). Table 4 reports the results of such a decomposition. Interestingly, flows between unemployment and nonparticipation contribute only modestly to the variation in \( L \). Instead, a large fraction of the variation in the participation rate can be traced to variation in worker flows between employment and nonparticipation. The next most prominent contribution is accounted for by flows between employment and unemployment.\(^{15}\)

To understand these results, it is instructive to consider the implied steady-state labor force participation rate, which can be written as

\[
L^* = 1 - N^* = 1 - \frac{f_{EN} E^* + f_{UN} U^*}{f_{NE} + f_{NU}}. \tag{18}
\]

Since labor market flows in the United States are particularly abundant, the evolution of realized labor force participation \( L \) is well-approximated by the latter flow-steady-state counterpart \( L^* \). Equation (18) makes clear that movements in \( f_{EN} \) and \( f_{NE} \) will have an important bearing on \( L^* \). Specifically, the secular decline in \( f_{EN} \), and the procyclicality of \( f_{NE} \), in Figure 3 respectively contribute to the secular rise, and mild procyclicality, in the participation rate. In addition, equation (18) reveals why the countercyclical of \( f_{EU} \), and procyclical of \( f_{UE} \), contribute significantly to labor force participation rate variation. Although the primary effect of these forces in times of recession is to reduce employment \( E^* \) and raise unemployment \( U^* \), unemployed workers are much more likely to leave the labor force compared to employed workers, that is \( f_{UN} \gg f_{EN} \).

Importantly for the purposes of understanding the stock-flow fallacy, equation (18) also provides a resolution for why movements in worker flows between unemployment and nonparticipation (that is, \( f_{NU} \) and \( f_{UN} \)), which contribute substantially to unemployment fluctuations in Table 3, play only a small role in the evolution of the participation rate in Table 4: It is because the direct effect of recessionary increases in \( f_{NU} \), and reductions in \( f_{UN} \), which tend to raise labor force participation in equation (18), is offset by a prominent indirect effect through raising the unemployment stock \( U^* \) that serves to reduce participation. Thus, the finding that flows between unemployment and nonparticipation play an important role in unemployment fluctuations does not stand in contradiction to the observed mild procyclical of labor force participation.

\(^{15}\)One drawback of the decompositions of labor force participation in Table 4 is that in some cases the implied residuals can be large. The reason for this is that the variation in the participation rate is quite limited over the cycle. Thus, while the residuals are a larger fraction of the variance in comparison say to unemployment decompositions in Table 3, they are larger fractions of much smaller numbers.
As a further illustration of the pitfalls of inferring the role of the participation margin from a stocks-based decomposition, in the remainder of this section we present a case study that contrasts the twin recessions of the early 1980s with the Great Recession of the late 2000s. Both episodes were associated with a rise in the unemployment rate in excess of 5 percentage points. This is confirmed in Table 5, which reports the cumulative changes in the unemployment rate $\Delta u$, the log labor force participation rate $\Delta \log L$ and the log employment-to-population ratio $\Delta \log E$ respectively for the periods May 1979 to December 1982, and March 2007 to October 2009.

Viewed through the lens of the stocks-based decomposition in (17), Table 5 suggests that the contribution of the participation margin to unemployment fluctuations changed signs across the two episodes, seemingly reinforcing the rise in unemployment in the 1980s recessions, but moderating the rise during the Great Recession. The reason, of course, is that the labor force participation rate was rising as a trend phenomenon in the earlier episode, and now appears to be on a trend decline, as shown in Figure 1.

Should one conclude from this that the role of the participation margin in accounting for cyclical unemployment has shifted fundamentally as a result of these differing secular trends? The message of the worker flows is a resounding no. Figure 4 presents the estimated contribution of each labor market flow to the changes in the unemployment rate during these two episodes. The role of flows between unemployment and nonparticipation is both quantitatively significant, and of similar magnitude, across the two recessionary periods, accounting for approximately one-third of the rise in the unemployment rate in each case.

The flows-based decomposition also reconciles the divergent behavior of the participation rate across the two recessionary periods. The final panels of Figure 4 present the analogous contributions of worker flows to the evolution of labor force participation. In both downturns, flows between unemployment and nonparticipation placed upward pressure on participation, consistent with the cyclical behavior of these flows discussed earlier. Offsetting this tendency is the effect of flows between unemployment and employment, for the reason that unemployed workers are much more likely to leave the labor force than employed workers, as in the discussion surrounding Table 4.

The key to the different trajectories in participation between the 1980s recessions and the Great Recession, however, is the comparative effects of flows between employment and nonparticipation. In particular, these flows imparted a substantial negative effect on participation during the most recent downturn, while their effect was more muted in the early 1980s. This difference, which can be attributed to changing secular trends in the employment-to-
nonparticipation flow rate in Figure 3, is what drives the opposite paths of the labor force participation rate across the two episodes. Since flows between employment and nonparticipation are largely neutral with respect to the unemployment rate, it would be fallacious to infer the contribution of the participation margin to recessionary increases in unemployment from the behavior of the stock of labor force participants, which is itself shaped by (different) worker flows.

7 Understanding the behavior of flows between unemployment and nonparticipation

The preceding sections have highlighted that the flow transition rates between unemployment and nonparticipation are prominently cyclical; that adjustments for classification errors and time aggregation do not eliminate this cyclicality; and that this variation contributes substantially to cyclical unemployment fluctuations. An important question, then, is what might explain the observed cyclicality of these flows.\textsuperscript{16}

In this section, we advance and quantify one hypothesis for why the participation margin appears to be so important. Our motivation revives an insight made by Akerlof and Main (1981) that, in practice, the structure of worker flow transitions may depart considerably from the descriptive first-order Markov structure in equation (5) that has informed the majority of research on labor market flows.\textsuperscript{17} In particular, worker flows may exhibit history dependence, whereby individual workers’ transition rates are related to their past labor market status, and may also vary across workers. Consequently, cyclical changes in the composition of the unemployed workers across these different dimensions of heterogeneity can influence the evolution of average worker flows over the cycle.

The specific hypothesis we explore is whether the behavior of the average unemployment-to-nonparticipation rate depicted in Figure 3 can be traced to cyclical shifts in labor force attachment of the nonemployed.\textsuperscript{18} In particular, a stylized feature of recessions in the United

\textsuperscript{16}A natural candidate explanation might be the role of extensions in the duration of unemployment insurance (UI) that accompany recessions, with the Great Recession of 2008 to 2010 being a prominent example. However, estimates of the impact of such UI extensions suggest a very modest impact on unemployment (see Aaronson, Mazumder, and Schecter, 2010; Farber and Valletta, 2011; Fujita, 2010; Nakajima, 2010; Rothstein, 2011; Valletta and Kuang, 2010; and Valletta, 2010).

\textsuperscript{17}A recent exception is Gomes (2012), which highlights the existence of history dependence in worker flows in the United Kingdom.

\textsuperscript{18}We also examined the role of such compositional forces on other labor market flows, but found only modest effects on flows originating from employment and nonparticipation. The simple reason is that both
States is the burst of job loss that occurs at the onset of a downturn. If such workers are more than averagely attached to the labor market, it is plausible that they will continue searching for employment rather than transitioning out of the labor force. An important potential signal of labor market attachment would be history-dependence in worker flows: Individuals who have been attached to the labor market in the past would exhibit a lower propensity to exit the labor force, and cyclical changes in the distribution of labor market attachment will in turn drive cyclical changes in average worker flows at the participation margin.

In the remainder of this section, we study the magnitude of this channel in accounting for recessionary declines in the rate at which unemployed workers exit the labor force. We do so using Current Population Survey microdata for which individual records have been matched across all eight months in sample. Using these data, we compute the unemployment-to-nonparticipation transition probability conditional on a full interaction of past labor force status (defined as status one year prior to the survey), age, gender, and education.

Table 6 reports the relevant flow transition probabilities for different groups of workers, averaged over the period 1979 to 2010. The table reveals that female workers, both younger and older workers, less educated workers, and workers who were not employed a year ago all are more likely to flow from unemployment to nonparticipation. Quite sensibly, and consistent with the premise underlying the above hypothesis, the common thread that unites these observations is that flows between unemployment and nonparticipation are more common among workers who tend to be less attached to the labor force.

Importantly, we also find that the composition of the unemployment pool becomes skewed towards workers that are more attached to the labor force during recessions. Specifically, during recessions we observe increases in the unemployment shares of prime-aged men, as well as workers who were employed one year prior. Since unemployed members of all three of these groups are less likely to exit the labor force, these compositional shifts potentially could account for the observed decline in the average $U$-to-$N$ flow rate during recessions.

To quantify the magnitude of this effect, we compute “counterfactual” $U$-to-$N$ transition probabilities for each of the last five recessionary episodes, holding constant the unemployment shares by prior status, age, education and gender at their beginning-of-recession values. The employment and nonparticipant stocks are much larger than the unemployment stock. Consequently, the composition of these larger stocks is influenced less by cyclical fluctuations.

Note that these transition probabilities differ slightly from those reported in Figure 3. In particular, they are based on the raw transition probabilities computed from CPS microdata matched across all eight months in sample, and are not adjusted for margin error or temporal aggregation.
Table 7 reports the actual and counterfactual percentage declines in the \(U\)-to-\(N\) transition probability over the course of each recessionary trough-to-peak ramp up in the unemployment rate since 1979.

The bottom line of the results of Table 7 is that a nontrivial part of the cyclicality of \(U\)-to-\(N\) flows can be attributed to cyclical shifts in the composition of unemployed workers. In particular, depending on the recession, one-third to two-thirds of the recessionary decline in the rate at which unemployed workers exit the labor force can be traced to compositional shifts.

An alternative visualization of this result is presented in Figure 5. This illustrates the results of a shift-share analysis of the time-series variation in the unemployment-to-nonparticipation transition probability (mathematical details are provided in the Appendix). This allows one to isolate the role of changing unemployment composition to changes in the \(U\)-to-\(N\) flow rate, taken over the whole sample period. The “share” part of Figure 5 depicts the component of changes in \(p_{UN}\) that is accounted for by changes in unemployment shares by gender, age, education and prior status; the “shift” part accounts for the changes in the group-specific \(U\)-to-\(N\) flow rates. Figure 5 confirms the findings in Table 7, with the share component contributing substantially during recessions, somewhat lower than the contribution of the shift part.\(^{20}\)

This result is striking from a number of perspectives. First, it is important to remember that the compositional adjustment in Table 7 and Figure 5 is based on just a few observable factors, specifically prior labor force status, age, education and gender. Since this small set of variables are imperfect proxies for labor force attachment, it is possible that additional unobservable dimensions of attachment would imply an even more dramatic composition effect.

A second notable feature of the results of Table 7 and Figure 5 is that they contrast interestingly with the analyses of Baker (1992) and Shimer (2012). Both of the latter two analyses examined the hypothesis that compositional shifts in the unemployment pool could account for cyclical changes in the rate of outflow from unemployment—the “heterogeneity hypothesis” of Darby, Haltiwanger and Plant (1986). Seemingly in contrast to the results in Table 7, however, Baker and Shimer find little role for compositional effects on the cyclical behavior of the outflow rate from unemployment.

\(^{20}\)An anomaly highlighted in Figure 5, and also visible in Figures 2 and 3, is that the redesign of the CPS in 1994 appeared to induce a sharp upward shift in the unemployment-to-nonparticipation probability. Unfortunately, to our knowledge, there are no available corrections of the worker flows that account for this shift.
Our own analysis highlights two factors that explain the difference between the results of Table 7 and those of Baker and Shimer. The main factor is that these prior analyses did not adjust for compositional shifts in prior labor market status, which we find play an important role in the composition adjustment in Table 7. In addition, though, it is important to recall that the outflow rate analyzed by Baker and Shimer is in fact the sum of the unemployment-to-nonparticipation rate that we analyze, and the unemployment-to-employment rate. Interestingly, there are some small offsetting composition effects of the latter that serve to mute the overall effect of composition on total unemployment outflows. This makes sense: The cyclical shift toward higher labor market attachment that we have highlighted will tend to elevate the $U$-to-$E$ transition rate as well as depress the $U$-to-$N$ rate that we have focused on in this section.

8 Conclusion

An often-neglected empirical regularity in standard estimates of worker flows in the United States is that flows between unemployment and out of the labor force display prominent fluctuations over the business cycle. Moreover, these fluctuations at the participation margin contribute towards increasing unemployment in times of recession: Inflows into unemployment from nonparticipation rise in downturns; the rate at which jobseekers exit the labor force falls in times of recession.

In this paper, we have quantified the magnitude of this channel in accounting for cyclical unemployment, and considered its robustness to an array of adjustments for time aggregation in measured flows and classification errors in recorded labor market status. We have found that the contribution of the participation margin is quantitatively substantial, accounting for around one-third of cyclical unemployment movements. Moreover, this conclusion continues to hold after adjustments of the data to correct for spurious transitions. Finally, we have shown why conventional wisdom on the participation margin informed by the cyclical behavior of labor market stocks is based on a stock-flow fallacy that implicitly neglects the role of worker flows in shaping the stock of labor force participants.

An important topic for further research, then, is to identify explanations for this phenomenon. We have highlighted one particular fruitful line of research. We have shown that part of the cyclical variation in worker flows at the participation margin can be traced to shifts in the composition of labor market attachment among the nonemployed in times of recession. The wave of job losses that accompany the start of recessions skews the unem-
ployment pool towards a group of workers who are more than averagely attached to the labor market, and can be expected to continue searching for employment rather than transitioning out of the labor force. We show that individuals who have been attached to the labor market in the past exhibit a lower propensity to exit the labor force, and that there are notable cyclical changes in the distribution of labor market attachment over the business cycle. We show that this mechanism is quantitatively important and a nontrivial part of the cyclicality of unemployment-to-nonparticipation flows is due to the shift in the composition of unemployed workers towards more attached workers during the recessions.

Beyond this, our paper also has broader implications for recent research that has viewed the labor market from a three-state perspective. A feature of much of this research is that it often has focused on devising models that can account for the cyclical comovement of labor market stocks. While such research is a distinguished outlier relative to the abundance of theoretical and empirical research that has ignored the labor force participation margin, our analysis emphasizes that the latter is a necessary, but not sufficient condition for a further desideratum, namely that our models provide an account of the cyclical behavior of underlying worker flows.

21See, for example, Tripier (2004); Veracierto (2008); Christiano et al. (2010); Galí et al. (2011); Ebell (2011); Haefke and Reiter (2011); and Shimer (2011).
9 References


Table 1: Abowd and Zellner (1985) estimates of classification error probabilities.

<table>
<thead>
<tr>
<th>Original interview status</th>
<th>Status determined on reinterview</th>
<th>Employed</th>
<th>Unemployed</th>
<th>Non-participant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employed</td>
<td>98.78</td>
<td>1.91</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.18</td>
<td>88.57</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Non-participant</td>
<td>1.03</td>
<td>9.52</td>
<td>99.21</td>
<td></td>
</tr>
</tbody>
</table>

Source: Abowd and Zellner (1985, Table 6).

Table 2: Recoding of unemployment-nonparticipation cyclers.

<table>
<thead>
<tr>
<th>Measured</th>
<th>Recoded</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUN</td>
<td>NNNN</td>
</tr>
<tr>
<td>NUNN</td>
<td>NNNN</td>
</tr>
<tr>
<td>ENUN</td>
<td>ENNN</td>
</tr>
<tr>
<td>NUNE</td>
<td>NNNE</td>
</tr>
<tr>
<td>.NUN</td>
<td>.NNN</td>
</tr>
<tr>
<td>NUN.</td>
<td>NNN.</td>
</tr>
<tr>
<td>UUNU</td>
<td>UUUU</td>
</tr>
<tr>
<td>UNUU</td>
<td>UUUU</td>
</tr>
<tr>
<td>EUNU</td>
<td>EUUU</td>
</tr>
<tr>
<td>UNUE</td>
<td>UUUE</td>
</tr>
<tr>
<td>.UNU</td>
<td>.UUU</td>
</tr>
<tr>
<td>UNU.</td>
<td>UUU.</td>
</tr>
</tbody>
</table>

Note: The notation ABCD refers to a sequence of transitions associated with up to four consecutive monthly individual labor market states (that is, from A to B to C to D). A “.” is used to denote missing observations.
Table 3: 3-state decomposition of unemployment rate by classification error adjustment method.

<table>
<thead>
<tr>
<th>Class. error adjustment</th>
<th>Start of sample</th>
<th>Share of variance</th>
<th>Total between</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>EU</td>
<td>UE</td>
<td>NU</td>
<td>UN</td>
<td>EN</td>
<td>NE</td>
<td>residual</td>
<td>U and E</td>
<td>U and N</td>
<td>E and N</td>
</tr>
<tr>
<td>Unadjusted</td>
<td>1967</td>
<td>24.9</td>
<td>34.9</td>
<td>9.5</td>
<td>23.9</td>
<td>-0.3</td>
<td>1.0</td>
<td>6.0</td>
<td>59.8</td>
<td>33.4</td>
<td>0.8</td>
</tr>
<tr>
<td>DeNUNified</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Abowd-Zellner</td>
<td>1967</td>
<td>29.6</td>
<td>41.7</td>
<td>-0.7</td>
<td>26.7</td>
<td>-1.3</td>
<td>2.1</td>
<td>1.8</td>
<td>71.4</td>
<td>26.0</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>1978</td>
<td>22.3</td>
<td>35.1</td>
<td>13.2</td>
<td>22.3</td>
<td>-0.7</td>
<td>1.5</td>
<td>6.3</td>
<td>57.4</td>
<td>35.5</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>1978</td>
<td>25.2</td>
<td>42.5</td>
<td>11.6</td>
<td>17.1</td>
<td>-0.8</td>
<td>1.1</td>
<td>3.3</td>
<td>67.7</td>
<td>28.7</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>1978</td>
<td>25.6</td>
<td>44.4</td>
<td>3.9</td>
<td>26.4</td>
<td>-1.7</td>
<td>2.3</td>
<td>-0.9</td>
<td>70.0</td>
<td>30.3</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Note: Decomposition of variance of change in quarterly average of unemployment rate. All samples end in February 2012.
Table 4: 3-state decomposition of participation rate by classification error adjustment method.

<table>
<thead>
<tr>
<th>Class. error adjustment</th>
<th>Start of sample</th>
<th>Share of variance</th>
<th>Total between</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>EU</td>
<td>UE</td>
</tr>
<tr>
<td>Unadjusted</td>
<td>1967</td>
<td>16.7</td>
<td>21.3</td>
</tr>
<tr>
<td>DeNUNnified</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Abowd-Zellner</td>
<td>1967</td>
<td>8.5</td>
<td>36.6</td>
</tr>
<tr>
<td>Unadjusted</td>
<td>1978</td>
<td>16.8</td>
<td>22.7</td>
</tr>
<tr>
<td>DeNUNnified</td>
<td>1978</td>
<td>10.5</td>
<td>22.7</td>
</tr>
<tr>
<td>Abowd-Zellner</td>
<td>1978</td>
<td>10.2</td>
<td>40.6</td>
</tr>
</tbody>
</table>

Note: Decomposition of variance of change in quarterly average of participation rate. All samples end in February 2012.
Table 5: Decomposition of the rise in the unemployment rate for the May 1979-Dec 1982 and March 2007-Oct 2009 periods.

<table>
<thead>
<tr>
<th>Period</th>
<th>$\Delta u$</th>
<th>$\Delta \log(lfpr)$</th>
<th>$\Delta \log(E/P)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 1979-Dec 1982</td>
<td>0.052</td>
<td>0.013</td>
<td>-0.045</td>
</tr>
<tr>
<td>March 2007-Oct 2009</td>
<td>0.056</td>
<td>-0.018</td>
<td>-0.079</td>
</tr>
</tbody>
</table>

Table 6: $U$-to-$N$ flow probabilities for different groups.

<table>
<thead>
<tr>
<th>Gender</th>
<th>$UN$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>17.8</td>
</tr>
<tr>
<td>Women</td>
<td>26.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Age</th>
<th>$UN$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16-24</td>
<td>28.6</td>
</tr>
<tr>
<td>25-54</td>
<td>17.5</td>
</tr>
<tr>
<td>55+</td>
<td>23.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Education</th>
<th>$UN$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; High-school</td>
<td>28.5</td>
</tr>
<tr>
<td>HS Degree</td>
<td>19.1</td>
</tr>
<tr>
<td>Some College</td>
<td>20.1</td>
</tr>
<tr>
<td>College</td>
<td>15.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LFS a year ago</th>
<th>$UN$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>14.7</td>
</tr>
<tr>
<td>$U$</td>
<td>19.4</td>
</tr>
<tr>
<td>$N$</td>
<td>36.6</td>
</tr>
</tbody>
</table>

Note: The $U$ to $N$ transition probabilities are calculated using Current Population Survey micro data matched across all eight months in sample.
Table 7: Actual and counterfactual declines in U-to-N transition probabilities.

<table>
<thead>
<tr>
<th>Period</th>
<th>Actual</th>
<th>Counterfactual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979Q2-1980Q3</td>
<td>16.8</td>
<td>5.1</td>
</tr>
<tr>
<td>1981Q2-1982Q4</td>
<td>21.7</td>
<td>9.1</td>
</tr>
<tr>
<td>1989Q1-1992Q2</td>
<td>19.2</td>
<td>12.8</td>
</tr>
<tr>
<td>2000Q4-2003Q2</td>
<td>14.5</td>
<td>6.5</td>
</tr>
<tr>
<td>2006Q4-2009Q4</td>
<td>26.3</td>
<td>14.2</td>
</tr>
</tbody>
</table>

Note: Reported are actual and counterfactual percentage declines in UN transition probabilities from quarterly trough to peak in unemployment rate. Counterfactual declines are based on composition adjustment for age, gender, education, and prior labor force status using microdata from the CPS.
Figure 1: Unemployment and labor force participation rates: unadjusted and adjusted for spurious transitions.
Figure 2: Monthly flow transition probabilities corrected for margin error: unadjusted and adjusted for spurious transitions.
Figure 3: Implied monthly flow transition probabilities corrected for margin error and time aggregation: unadjusted and adjusted for spurious transitions.
Figure 4: Evolution of labor market flows and stocks in the twin recessions of 1980s and during the most current downturn.
Figure 5: Shift-share analysis with shares based on gender, age, education, and prior labor force status a year ago.
A Mathematical details

A.1 Derivation of equation (4)

Given the classification errors in equation (1), measured flows between unemployment and nonparticipation can be written as

\[ \text{UN}_t = \varepsilon_{EU} [\varepsilon_{EE} E_t^* + \varepsilon_{UN} E_t^{*} + \varepsilon_{NN} E_t^{*}] + \varepsilon_{UU} [\varepsilon_{EN} E_t^* + \varepsilon_{NU} U_t^* + \varepsilon_{NN} U_t^*] + \varepsilon_{NU} [\varepsilon_{EU} E_t^* + \varepsilon_{UN} U_t^* + \varepsilon_{NN} N_t^*], \]

and

\[ \text{NU}_t = \varepsilon_{EN} [\varepsilon_{UE} E_t^* + \varepsilon_{UU} U_t^* + \varepsilon_{NU} E_t^*] + \varepsilon_{UN} [\varepsilon_{EU} U_t^* + \varepsilon_{UU} U_t^* + \varepsilon_{NN} U_t^*] + \varepsilon_{NN} [\varepsilon_{EU} E_t^* + \varepsilon_{UU} U_t^* + \varepsilon_{NU} N_t^*]. \]

Under the assumption that \( \varepsilon_{ij} = 0 \) for all \( ij \notin \{UN, NU\} \), it is possible to rewrite the latter as

\[ \text{UN}_t = \varepsilon_{UU} [\varepsilon_{UN} U_t^* + \varepsilon_{NN} N_t^*] + \varepsilon_{NU} [\varepsilon_{UN} N_t^* + \varepsilon_{NN} N_t^*], \]

and

\[ \text{NU}_t = \varepsilon_{UN} [\varepsilon_{UU} U_t^* + \varepsilon_{NU} N_t^*] + \varepsilon_{NN} [\varepsilon_{UU} U_t^* + \varepsilon_{NU} N_t^*]. \]

Noting that \( \varepsilon_{UU} = 1 - \varepsilon_{UN} \), \( \varepsilon_{NN} = 1 - \varepsilon_{NU} \), and that any product of the errors is second order in the presence of small \( \varepsilon_{UN} \) and \( \varepsilon_{NU} \) yields the approximation in equation (4).

A.2 Margin-error adjustment

We use the following method to adjust the transition probabilities that we get from the data to make them consistent with the labor market status vector, \( s_t \). Note that

\[ \Delta s_t = s_t - s_{t-1} = \begin{bmatrix} -p_{EU} - p_{EN} & p_{UE} & p_{NE} \\ p_{EU} & -p_{UE} - p_{UN} & p_{NU} \end{bmatrix} \begin{bmatrix} E_{t-1} \\ U_{t-1} \\ N_{t-1} \end{bmatrix} \]

\[ = \begin{bmatrix} -E_{t-1} & -E_{t-1} & U_{t-1} & 0 & N_{t-1} & 0 \\ E_{t-1} & 0 & -U_{t-1} & -U_{t-1} & 0 & N_{t-1} \end{bmatrix} \begin{bmatrix} p_{EU} \\ p_{EN} \\ p_{UE} \\ p_{UN} \\ p_{NE} \\ p_{NU} \end{bmatrix} = X_{t-1}p \]
Note that the vector of transitional probabilities that we get from the data, which we denote by \( \hat{p} \), has a covariance matrix that is proportional to a matrix that is consistently estimated using

\[
W = \begin{bmatrix}
\hat{p}_{EU}(1-\hat{p}_{EU}) & -\hat{p}_{EU}\hat{p}_{EN} & 0 & 0 & 0 & 0 \\
-\hat{p}_{EU}\hat{p}_{EN} & \hat{p}_{EN}(1-\hat{p}_{EN}) & 0 & 0 & 0 & 0 \\
0 & 0 & \hat{p}_{UE}(1-\hat{p}_{UE}) & -\hat{p}_{UE}\hat{p}_{UN} & 0 & 0 \\
0 & 0 & -\hat{p}_{UE}\hat{p}_{UN} & \hat{p}_{UN}(1-\hat{p}_{UN}) & 0 & 0 \\
0 & 0 & 0 & 0 & \hat{p}_{NE}(1-\hat{p}_{NE}) & -\hat{p}_{NE}\hat{p}_{NU} \\
0 & 0 & 0 & 0 & -\hat{p}_{NE}\hat{p}_{NU} & \hat{p}_{NU}(1-\hat{p}_{NU})
\end{bmatrix}^{-1}
\]

(23)

We apply a weighted-restricted-least-squares adjustment method in the sense that we choose the vector of transition probabilities that are consistent with the labor market status vector, which we denote by \( p \), to minimize \( (p-\hat{p})'W(p-\hat{p}) \), subject to \( \Delta s_t = X_{t-1}p \)

(24)

Given the associated Lagrangian

\[
L = (p-\hat{p})'W(p-\hat{p}) - 2\mu' (\Delta s_t - X_{t-1}p)
\]

(25)

where \( \mu \) is the 2 \( \times \) 1-vector with Lagrange multipliers, it is fairly straightforward to derive that

\[
\begin{bmatrix}
p \\
\mu
\end{bmatrix} = \begin{bmatrix} W & X_{t-1}' \\ -X_{t-1} & 0 \end{bmatrix} \begin{bmatrix} W\hat{p} \\
\Delta s_t
\end{bmatrix}.
\]

(26)

Since all the terms on the right hand side are known, we can use this equation to adjust the transition probabilities to \( p \).

A.3 Temporal-aggregation correction

The continuous-time system in equation (6) can be represented as a two-state system using the adding up constraint \( E_t + U_t + N_t = 1 \):

\[
s_t = \begin{bmatrix}
-f_{EU} - f_{EN} - f_{NE} \\
f_{EU} - f_{NU}
\end{bmatrix}_t s_t + \begin{bmatrix} f_{NE} \\
f_{NU}
\end{bmatrix}_t = \tilde{F}_t s_t + g_t
\]

(27)

Similarly the discrete-time transition probabilities satisfy \( s_t = \tilde{P}_t s_{t-1} + q_t \). Both of these systems have a steady state \( \bar{s} \) that satisfies

\[
\bar{s} = -\tilde{F}^{-1}g = -\tilde{P}^{-1}q
\]

(28)

Let \( \xi_t = (s_t - \bar{s}) \). Then we can represent \( \xi_t \) both using the continuous-time and discrete-time systems as

\[
\dot{\xi}_t = F\xi_t
\]

(29)

and

\[
\xi_t = P\xi_{t-1}
\]

(30)
The solution to (29) can be represented as

$$\xi_t = V \Lambda V^{-1} \xi_{t-1}$$  \hfill (31)

where

$$V = \begin{bmatrix} s & v_1 & v_2 \end{bmatrix} \quad \text{and} \quad \Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{\lambda_1} & 0 \\ 0 & 0 & e^{\lambda_2} \end{bmatrix}$$  \hfill (32)

and \(\lambda_1, \lambda_2\), being the two non-zero eigenvalues of \(F\) and \(v_1\) and \(v_2\) being the associated eigenvectors.

Similarly for the discrete-time probability matrix, \(P\)

$$P = V \Lambda V^{-1}$$  \hfill (33)

where \(V\) is still the matrix with eigenvectors, but now of \(P\), and where \(1, \lambda_1^* = e^{\lambda_1}\), and \(\lambda_2^* = e^{\lambda_2}\) are the corresponding eigenvalues. Hence, after we calculate the eigenvalues and eigenvectors of the transition matrix \(P\), we can use the eigenvalue decomposition above to obtain the values of the flow hazard rates and thus the matrix \(F\).

A.4 Derivation of equation (9)

Note first that one can decompose the change in labor market state into parts,

$$\Delta s_t = (s_t - \bar{s}_t) - (s_{t-1} - \bar{s}_{t-1}) + \Delta \bar{s}_t.$$  \hfill (34)

Then note that the reduced Markov chain \(s_t = \bar{P}_t s_{t-1} + q_t\) can be written as:

$$(s_t - \bar{s}_t) = \bar{P}_t (s_{t-1} - \bar{s}_{t-1})$$

$$= \bar{P}_t (s_{t-1} - \bar{s}_{t-1}) - \bar{P}_t \Delta \bar{s}_t.$$  \hfill (35)

Substituting for \((s_t - \bar{s}_t)\) in (34) implies:

$$\Delta s_t = - (I - \bar{P}_t) (s_{t-1} - \bar{s}_{t-1}) + (I - \bar{P}_t) \Delta \bar{s}_t.$$  \hfill (36)

Similarly, noting from (35) that \((s_{t-1} - \bar{s}_{t-1}) - \Delta \bar{s}_t = \bar{P}_t^{-1} (s_t - \bar{s}_t)\) implies that (34) can be rewritten as

$$\Delta s_t = (\bar{P}_t - I) \bar{P}_t^{-1} (s_t - \bar{s}_t).$$

Combining the latter with (36) confirms the proposed solution.

A.5 Shift-share analysis of \(U\)-to-\(N\) transition probability

Imagine there are \(i = 1, \ldots, n\) demographic groups. At time \(t\), let each group have a share in the pool of unemployed equal to \(s^U_{it}\). Let the transition probability from \(U\) to \(N\) for that group at time \(t\) be \(P_{t,UN}\).

The aggregate \(U\)-to-\(N\) transition probability is a weighted average of the transition probabilities
by group, where the weights are each groups share of persons in the pool of unemployed, such that

\[ P_{UN_t} = \sum_{i=1}^{n} s_{it}^{U} P_{i,UN_t}. \]  (37)

The shift-share decomposition uses the following result

\[ \Delta P_{UN_t} = P_{UN_t} - P_{UN_{t-1}} \]  (38)

\[ = \sum_{i=1}^{n} s_{it}^{U} P_{i,UN_t} - s_{it}^{U} P_{i,UN_{t-1}} \]

\[ = \left\{ \begin{array}{c}
\sum_{i=1}^{n} s_{it}^{U} P_{i,UN_t} - \frac{1}{2} \sum_{i=1}^{n} s_{it}^{U} P_{i,UN_{t-1}} \end{array} \right\} - \left\{ \begin{array}{c}
\sum_{i=1}^{n} s_{it}^{U} P_{i,UN_{t-1}} - \frac{1}{2} \sum_{i=1}^{n} s_{it}^{U} P_{i,UN_{t-1}} \end{array} \right\} \]

\[ = \frac{1}{2} \sum_{i=1}^{n} s_{it}^{U} \Delta P_{i,UN_t} + \frac{1}{2} \sum_{i=1}^{n} \Delta s_{it}^{U} P_{i,UN_t} \]

\[ + \frac{1}{2} \sum_{i=1}^{n} \Delta s_{it}^{U} P_{i,UN_{t-1}} + \frac{1}{2} \sum_{i=1}^{n} s_{it}^{U} \Delta P_{i,UN_{t-1}} \]

\[ = \sum_{i=1}^{n} \left( \frac{s_{it}^{U} + s_{it}^{U-1}}{2} \right) \Delta P_{i,UN_t} + \sum_{i=1}^{n} \left( \frac{P_{i,UN_t} + P_{i,UN_{t-1}}}{2} \right) \Delta s_{it}^{U} \]

Shift part \quad Share part

\[ = \Delta P_{UN_t}^{shift} + \Delta P_{UN_t}^{share}. \]

We then reaccumulate from \( t = 1, \ldots, T \), in the sense that

\[ P_{UN_t} = P_{UN_0} + \sum_{s=1}^{t} \Delta P_{UN_s}. \]  (39)

\[ = P_{UN_0} + \sum_{s=1}^{t} \Delta P_{UN_s}^{shift} + \sum_{s=1}^{t} \Delta P_{UN_s}^{share}. \]

Here, the first term is just an initial value that should not affect the cyclicity of \( P_{UN_t} \). The second term are the movements in \( P_{UN_t} \) due to the changes in the transition probabilities. The third term is the term that captures the influence of the changes in the composition of the unemployed.