Trade, Unemployment, and Monetary Policy*

Matteo Cacciatoré†
HEC Montréal

Fabio Ghironi‡
Boston College,
Federal Reserve Bank of Boston,
EABCN, and NBER

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Abstract

We study the effects of trade integration for the conduct of monetary policy in a two-country model with heterogeneous firms, endogenous producer entry, and labor market frictions. The model reproduces important empirical regularities related to international trade, namely synchronization of business cycles across trading partners and reallocation of market shares across producers. Three key results emerge. First, when trade linkages are weak, the optimal policy is inward-looking but requires significant departures from price stability both in the long run and over the business cycle. Second, as trade integration reallocates market share toward more productive firms, the need of positive inflation to correct long-run distortions is reduced. Third, increased business cycle synchronization implies that country-specific shocks have more global consequences. Welfare gains from cooperation are small relative to optimal non-cooperative policy, but sizable relative to historical Federal Reserve behavior. The constrained efficient allocation generated by optimal cooperative policy can still be achieved by appropriately designed inward-looking policy rules. However, sub-optimal (historical) policy implies inefficient fluctuations in cross-country demands that result in large welfare costs when trade linkages are strong.

JEL Codes: E24; E32; F16; E52, F41, J64.

Keywords: Exchange rate regime; Optimal monetary policy; Producer entry; Trade integration; Unemployment.

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†Institute of Applied Economics, HEC Montréal, 3000, chemin de la Côte-Sainte-Catherine, Montréal (Québec), Canada. E-mail: matteo.cacciatore@hec.ca. URL: http://www.hec.ca/en/profs/matteo.cacciatore.html

‡Department of Economics, Boston College, 140 Commonwealth Avenue, Chestnut Hill, MA 02467-3859, U.S.A. E-mail: Fabio.Ghironi@bc.edu. URL: http://www2.bc.edu/fabio-ghironi.
“I would like to know how the macroeconomic model that I more or less believe can be reconciled with the trade models that I also more or less believe. [...] What we need to know is how to evaluate the microeconomics of international monetary systems. Until we can do that, we are making policy advice by the seat of our pants” (Krugman, 1995).

1 Introduction

The optimal conduct of monetary policy is a traditional subject of research in macroeconomics and international macroeconomics. Vast literatures have been written in both fields. In the open economy context, both policy debates and academic literature have often tied the analysis of monetary policy to the openness characteristics of the countries involved and their degree of trade integration. The consequences of increased trade for incentives to cooperate across countries in monetary matters and for the desirability of alternative exchange rate arrangements are classic topics of discussion and research.

In the policy arena, the implementation of the European Single Market after 1985 was viewed as a crucial step toward adoption of the euro. The argument was that the mere possibility of exchange rate movements may eventually destabilize the Single Market, thus making monetary union desirable for the viability of a broader integration agenda (Eichengreen and Ghironi, 1996). The view that increased trade integration makes monetary cooperation—and, in this case, adoption of a shared currency—more desirable is fully embraced in official European Union documents. Influential articles by Frankel and Rose (1998) and Clark and van Wincoop (2001) provided highbrow backing for this argument by finding evidence that trade integration results in stronger business cycle comovement, thus potentially resulting in countries endogenously satisfying one of Mundell’s (1961) optimum currency area criteria. At the other end of the spectrum, the limited weight of international trade in U.S. GDP was often invoked among the reasons for small international spillovers to the United States, and therefore small incentives for the Federal Reserve to engage in international monetary coordination in the post-Bretton Woods era. The recent financial crisis brought global monetary cooperation to the forefront as it had not been since perhaps the Plaza Accord of 1985, when policymakers of France, West Germany, Japan, the United Kingdom, and the

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1 For recent surveys see Corsetti, Dedola, and Leduc (2010) and Schmitt-Grohé and Uribe (2010).
United States agreed to implement concerted intervention to depreciate the dollar. While the extent of the recent crisis was such that financial sector matters are likely to remain under the spotlight of international policy for the foreseeable future, increasing trade in the modern era of globalization is also likely to keep trade flows among the key determinants of international discussions on monetary matters.\(^4\)

In the more academic realm, the recent New Keynesian literature on optimal monetary policy in open economies has made an effort to incorporate trade integration among the determinants of policy incentives. This literature, however, tends to characterize trade integration in terms of the degree of home bias in consumer preferences and/or the weight of imported inputs in production (for instance, Coenen, Lombardo, Smets, and Straub, 2007, Faia and Monacelli, 2008, Pappa, 2004, Lombardo and Ravenna, 2010). While there is undisputed merit in this exercise, proxying a policy outcome (the extent of trade integration) with structural parameters of preferences and technology risks confounding the consequences of a policy change (the removal—or lowering—of trade barriers) with those of features of agents’ behavior that may have little to do with policy.

In this paper, we re-examine the classic question of trade integration and optimal monetary policy in a two-country model that incorporates the standard ingredients of the current workhorse frameworks in international trade and macroeconomics: heterogeneous firms and endogenous producer entry in domestic and export markets (Melitz, 2003); nominal rigidity; and dynamic, stochastic, general equilibrium. Reflecting the attention of policymakers to labor market dynamics and unemployment, we introduce search-and-matching frictions in labor markets, following Diamond (1982a,b) and Mortensen and Pissarides (1994). By combining these ingredients, we answer Krugman’s (1995) “call for research” that opens the paper.

We show that the model reproduces empirical regularities for the U.S. and international business cycle, including increased comovement following trade integration (captured by a reduction in “iceberg” trade costs, including tariffs). The combination of endogenous producer entry and labor market frictions is central to this result—a traditional challenge for international business cycle models, as shown by Kose and Yi (2006).\(^5\) In the long run, trade integration results in reallocation of market shares toward the relatively more efficient producers, consistent with the evidence that has contributed to the success of the Melitz (2003) model of trade.

\(^4\)Frequent references by U.S. officials to Chinese “exchange rate manipulation” in the context of the trade imbalance between China and the United States provide a clear example. From the U.S. perspective, a substantial appreciation of the renminbi would constitute cooperative monetary policy by the Chinese.

\(^5\)The positive relation between trade and comovement is not captured by standard New Keynesian models that proxy trade integration with reduction in home bias, such as Faia and Monacelli (2008) or Pappa (2004).
With respect to monetary policy, three key results emerge. First, when trade linkages are weak, the optimal, cooperative policy is inward-looking but requires significant departures from price stability both in the long run and over the business cycle. Optimal policy uses inflation to narrow inefficiency wedges relative to the efficient allocation. Second, as trade integration reallocates market share toward more productive firms, the need of positive inflation to correct long-run distortions is reduced. The reallocation of market share results in an endogenous increase in average firm productivity. This makes job matches more valuable and pushes employment toward the efficient level, reducing the need for average inflation to accomplish that by eroding markups. Third, increased business cycle synchronization implies that country-specific shocks have more global consequences, and welfare gains from cooperation are small relative to optimal non-cooperative policy. This echoes Benigno and Benigno’s (2003) finding that there are no gains from cooperation when shocks (and, therefore, business cycles) are perfectly correlated across countries. Our model provides a structural microfoundation for their finding, by making increased business cycle correlation an endogenous consequence of trade integration. (Increased comovement per se makes an exchange rate peg more desirable for the pegger. However, if the center country follows historical Federal Reserve behavior, this generates inefficient spillovers with strong trade linkages, offsetting the gain from increased comovement.) Importantly, we show that gains from cooperation are sizable relative to historical Federal Reserve behavior. The constrained efficient allocation generated by optimal cooperative policy can still be achieved by appropriately designed inward-looking policy rules, but sub-optimal (historical) policy implies inefficient fluctuations in cross-country demands that result in large welfare costs when trade linkages are strong.

Besides the literature on trade and monetary policy, our paper contributes to the recent literature that studies how endogenous entry and product variety affect business cycles and optimal policy in closed and open economies. In this literature, our work is most closely related to Cacciatore (2010), Contessi (2010), Ghironi and Melitz (2005), Rodriguez-Lopez (2011), and Zlate (2010), among others. On optimal policy with endogenous producer entry, see Bergin and Corsetti (2008), Bilbiie, Fujiwara, and Ghironi (2011), Cacciatore, Fiori, and Ghironi (2011), Chugh and Ghironi (2011), Faia (2010), and Lewis (2010), among others. Auray and Eyquem (2011) and Cavallari (2011) study the role of monetary policy for shock transmission in two-country versions of Bilbiie, Ghironi, and Melitz’s (2012) model, but they do not analyze optimal monetary policy. Auray and Eyquem find that policies of price stability in each country imply welfare costs relative to interest

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6See Pissarides and Vallanti (2007) for evidence that higher productivity lowers unemployment in the long run.
7Put differently, as long as each central bank influences domestic distortions appropriately, increased synchronization dampens the effect of international distortions (lack of risk sharing, incentives to manipulate the terms of trade, lack of exchange rate pass-through). With weak trade linkages, these international distortions have second-order welfare effects; when trade linkages are strong, they are not more costly (if inward-looking policies are designed optimally) precisely because of synchronization.
ciatore (2010), who studies the consequences of trade integration in a flexible-price model that merges Ghironi and Melitz (2005) with the Diamond-Mortensen-Pissarides framework. We extend Cacciatoro’s model to a framework with sticky prices and wages to study the interaction of trade integration and monetary policy. Our results on optimal monetary policy extend those in Bilbiie, Fujiwara, and Ghironi (2011—BFG) and Cacciatoro, Fiori, and Ghironi (2011—CFG): As in BFG, an inefficiency wedge in product creation is among the reasons for the Ramsey central bank of our to use positive long-run inflation, but our model features a wider menu of sources of inefficiency, with the labor margin affected by a larger number of distortions. Differently from Bilbiie, Fujiwara, and Ghironi, we find that the interaction of distortions in our model can result in sizable, optimal departures from price stability over the business cycle. In this respect, our approach and results are closer to the analysis of market deregulation and optimal monetary policy in a monetary union in Cacciatoro, Fiori, and Ghironi (2012), whose model, however, does not incorporate the firm heterogeneity and reallocation effects that are central to the recent trade literature.

The paper is also related to the vast literature on monetary transmission and optimal monetary policy in New Keynesian macroeconomic models. In particular, we contribute to the strand of this literature that incorporates labor market frictions, such as Arseneau and Chugh (2008), Faia (2009), and Thomas (2008), and to the literature on price stability in open economies (Benigno and Benigno, 2003 and 2006, Catão and Chang, 2012, Galí and Monacelli, 2005, Dmitriev and Hoddenbagh, 2012, and many others) by studying hitherto unexplored mechanisms that affect monetary policy incentives.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes monetary policy: In our benchmark exercise, we compare the Ramsey-optimal, cooperative monetary policy to the consequences of historical behavior by the Federal Reserve and its symmetric counterpart. We follow Sims (2007) in considering historical behavior a more realistic benchmark for comparison than optimal, non-cooperative policies. To build intuition for the tradeoffs for monetary policymaking, Section 4 discusses the inefficiency wedges and sources of distortions that characterize the market economy. Section 5 presents the calibration of the model. Section 6 studies optimal monetary with weak trade linkages. Section 7 addresses the consequences of trade integration for monetary policy and performs a series of robustness exercises, including the comparison of optimal, cooperative policy to optimal, non-cooperative behavior and a fixed exchange rate.

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Section 8 concludes.

2 The Model

We model a world economy that consists of two countries, Home and Foreign. Foreign variables are denoted with a superscript star. We focus on the Home economy in presenting our model, with the understanding that analogous equations hold for Foreign. We abstract from monetary frictions that would motivate a demand for cash currency in each country, and we resort to a cashless economy following Woodford (2003).

Each country is populated by a unit mass of atomistic households, where each household is thought of as an extended family with a continuum of members along the unit interval. In equilibrium, some family members are unemployed, while some others are employed. As common in the literature, we assume that family members perfectly insure each other against variation in labor income due to changes in employment status, so that there is no \textit{ex post} heterogeneity across individuals in the household (see Andolfatto, 1996, and Merz, 1995).

Household Preferences

The representative household in the Home economy maximizes the expected intertemporal utility function $E_0 \sum_{t=0}^{\infty} \beta^t[u(C_t) - l_t v(h_t)]$, where $\beta \in (0, 1)$ is the discount factor, $C_t$ is a consumption basket that aggregates domestic and imported goods as described below, $l_t$ is the number of employed workers, and $h_t$ denotes hours worked by each employed worker. Period utility from consumption, $u(\cdot)$, and disutility of effort, $v(\cdot)$, satisfy the standard assumptions.

The consumption basket $C_t$ aggregates Home and Foreign sectoral consumption outputs $C_t(i)$ in Dixit-Stiglitz form:

$$C_t = \left[ \int_0^1 C_t(i) \frac{\phi}{\phi-1} \, di \right]^{\frac{\phi}{\phi-1}}.$$

where $\phi > 1$ is the symmetric elasticity of substitution across goods. A similar basket describes consumption in the Foreign country. The corresponding consumption-based price index is given by:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\phi} \, di \right]^{\frac{1}{1-\phi}},$$

where $P_t(i)$ is the price index for sector $i$, expressed in Home currency.
Production

In each country, there are two vertically integrated production sectors. In the upstream sector, perfectly competitive firms use labor to produce a non-tradable intermediate input. In the downstream sector, each consumption-producing sector \(i\) is populated by a representative monopolistically competitive multi-product firm that purchases intermediate input and produces differentiated varieties of its sectoral output. In equilibrium, some of these varieties are exported while the others are sold only domestically.\(^{10}\)

**Intermediate Goods Production**

There is a unit mass of intermediate producers. Each of them employs a continuum of workers. Labor markets are characterized by search and matching frictions as in the DMP framework. To hire new workers, firms need to post vacancies, incurring a cost of \(\kappa\) units of consumption per vacancy posted. The probability of finding a worker depends on a constant-return-to-scale matching technology, which converts aggregate unemployed workers, \(U_t\), and aggregate vacancies, \(V_t\), into aggregate matches, \(M_t = \chi U_t^{1-\varepsilon} V_t^\varepsilon\), where \(\chi > 0\) and \(0 < \varepsilon < 1\). Each firm meets unemployed workers at a rate \(q_t \equiv M_t / V_t\). As in Krause and Lubik (2007) and other studies, we assume that newly created matches become productive only in the next period. For an individual firm, the inflow of new hires in \(t + 1\) is therefore \(q_t v_t\), where \(v_t\) is the number of vacancies posted by the firm in period \(t\).\(^{11}\)

Firms and workers can separate exogenously with probability \(\lambda \in (0, 1)\). Separation happens only between firms and workers who were active in production in the previous period. As a result the law of motion of employment, \(l_t\) (those who are working at time \(t\)), in a given firm is given by \(l_t = (1 - \lambda) l_{t-1} + q_{t-1} v_{t-1}\).

As in Arsenau and Chugh (2008), firms faces a quadratic cost of adjusting the hourly nominal wage rate, \(w_t\). For each worker, the real cost of changing the nominal wage between period \(t - 1\) and \(t\) is \(\vartheta \pi_{w,t}^2 / 2\), where \(\vartheta \geq 0\) is in units of consumption, and \(\pi_{w,t} \equiv (w_t / w_{t-1}) - 1\) is the net wage inflation rate. If \(\vartheta = 0\), there is no cost of wage adjustment.

The representative intermediate firm produces output \(y_t^I = Z_t l_t h_t\), where \(Z_t\) is exogenous aggre-
gate productivity.\textsuperscript{12} We assume the following bivariate process for Home and Foreign productivity:

\[
\begin{bmatrix}
\log Z_t \\
\log Z_t^*
\end{bmatrix} = \begin{bmatrix}
\phi_{11} & \phi_{12} \\
\phi_{21} & \phi_{22}
\end{bmatrix} \begin{bmatrix}
\log Z_{t-1} \\
\log Z_{t-1}^*
\end{bmatrix} + \begin{bmatrix}
\epsilon_t \\
\epsilon_t^*
\end{bmatrix},
\]

where \(\phi_{11}\) and \(\phi_{22}\) are strictly between 0 and 1, and \(\epsilon_t\) and \(\epsilon_t^*\) are normally distributed innovations with variance-covariance matrix \(\Sigma_{\epsilon_t\epsilon_t^*}\).

Intermediate goods producers sell their output to final producers at a real price \(\varphi_t\) in units of consumption. Intermediate producers choose the number of vacancies, \(v_t\), and employment, \(l_t\), to maximize the expected present discounted value of their profit stream:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \frac{u_{C,t}}{u_{C,0}} \left( \varphi_t Z_t l_t h_t - \frac{w_t}{P_t} l_t h_t - \kappa v_t - \frac{\vartheta}{2} \pi_{w,t}^2 l_t \right),
\]

where \(u_{C,t}\) denotes the marginal utility of consumption in period \(t\), subject to the law of motion of employment. Future profits are discounted with the stochastic discount factor of domestic households, who are assumed to own Home firms.

Combining the first-order conditions for vacancies and employment yields the following job creation equation:

\[
\frac{\kappa}{q_t} = E_t \left\{ \beta_{t,t+1} \left( 1 - \lambda \right) \frac{\kappa}{q_{t+1}} + \varphi_{t+1} Z_{t+1} h_{t+1} - \frac{w_{t+1}}{P_{t+1}} h_{t+1} - \frac{\vartheta}{2} \pi_{w,t+1}^2 \right\},
\]

where \(\beta_{t,t+1} \equiv \beta u_{C,t+1}/u_{C,t}\) is the one-period-ahead stochastic discount factor. The job creation condition states that, at the optimum, the vacancy creation cost incurred by the firm per current match is equal to the expected discounted value of the vacancy creation cost per future match, further discounted by the probability of current match survival \(1 - \lambda\), plus the profits from the time-\(t\) match. Profits from the match take into account the future marginal revenue product from the match and its wage cost, including future nominal wage adjustment costs.

**Wage and Hours** The nominal wage is the solution of an individual Nash bargaining process, and the wage payment divides the match surplus between workers and firms. Due to the presence of nominal rigidities, we depart from the standard Nash bargaining convention by assuming that

\textsuperscript{12}Note that the assumption of a unit mass of intermediate producers ensures that \(y_t^*\) is also the total output of the intermediate sector.
bargaining occurs over the nominal wage payment rather than the real wage payment. With zero costs of nominal wage adjustment (\( \vartheta = 0 \)), the real wage that emerges would be identical to the one obtained from bargaining directly over the real wage. This is no longer the case in the presence of adjustment costs.

We relegate the details of wage determination to the Appendix. We show there that the equilibrium sharing rule can be written as 
\[ \eta_t H_t = (1 - \eta_t) J_t, \]
where \( \eta_t \) is the bargaining share of firms, \( H_t \) is worker surplus, and \( J_t \) is firm surplus (see the Appendix for the expressions). As in Gertler and Trigari (2009), the equilibrium bargaining share is time-varying due to the presence of wage adjustment costs. Absent these costs, we would have a time-invariant bargaining share \( \eta_t = \eta \), where \( \eta \) is the weight of firm surplus in the Nash bargaining problem.

The bargained wage satisfies:
\[
\frac{w_t}{P_t} h_t = \eta_t \left( \frac{v(h_t)}{u_{C,t}} + b \right) + (1 - \eta_t) \left( \varphi_t Z_t h_t - \frac{\vartheta}{2} \mu_{w,t} \right) \\
+ E_t \left\{ \beta_{t,t+1} J_{t+1} \left[ (1 - \lambda) (1 - \eta_t) - (1 - \lambda - \iota_t) (1 - \eta_{t+1}) \right] \right\},
\]
where \( v(h_t)/u_{C,t} + b \) is the worker’s outside option (the utility value of leisure plus an unemployment benefit \( b \)), and \( \iota_t \) is the probability of becoming employed at time \( t \), defined by \( \iota_t \equiv M_t / U_t \). With flexible wages, the third term in the right-hand side of this equation reduces to 
\[ (1 - \eta) \iota_t E_t \left( \beta_{t,t+1} J_{t+1} \right), \]
or, in equilibrium, \( \kappa (1 - \eta) \iota_t / q_t \). In this case, the real wage bill per worker is a linear combination—determined by the constant bargaining parameter \( \eta \)—of worker’s outside option and the marginal revenue product generated by the worker (net of wage adjustment costs) plus the expected discounted continuation value of the match to the firm (adjusted for the probability of worker’s employment). The stronger the bargaining power of firms (the higher \( \eta \)), the smaller the portion of the net marginal revenue product and continuation value to the firm appropriated by workers as wage payments, while the outside option becomes more relevant. When wages are sticky, bargaining shares are endogenous, and so is the distribution of surplus between workers and firms. Moreover, the current wage bill reflects also expected changes in bargaining shares.

As common practice in the literature we assume that hours per worker are determined by firms and workers in a privately efficient way, i.e., so as to maximize the joint surplus of their employment.

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\(^{13}\)The same assumption is made by Arseneau and Chugh (2008), Gertler, Trigari, and Sala (2008), and Thomas (2008).
The joint surplus is the sum of the firm’s surplus and the worker’s surplus, i.e., $J_t + H_t$. Maximization yields a standard intratemporal optimality condition for hours worked that equates the marginal revenue product of hours per worker to the marginal rate of substitution between consumption and leisure: $v_{h,t}/u_{C,t} = \varphi_t Z_t$, where $v_{h,t}$ is the marginal disutility of effort.

**Final Goods Production**

In each consumption sector $i$, the representative, monopolistically competitive producer $i$ produces the sectoral output bundle $Y_t(i)$, sold to consumers in Home and Foreign. Producer $i$ is a multi-product firm that produces a set of differentiated product varieties, indexed by $\omega$ and defined over a continuum $\Omega$:

$$Y_t(i) = \left( \int_{\omega \in \Omega} y_t(\omega, i)^{\theta+1} \frac{d\omega}{\omega} \right)^{\frac{1}{\theta+1}},$$

where $\theta > 1$ is the symmetric elasticity of substitution across product varieties. Each product variety $y_t(\omega, i)$ is created and developed by the representative final producer $i$. Since consumption-producing sectors are symmetric in the economy, from now on we omit the index $i$ to simplify notation. The cost of the product bundle $Y_t$, denoted with $P_t^y$, is:

$$P_t^y = \left( \int_{\omega \in \Omega} p_t^y(\omega)^{1-\theta} \frac{d\omega}{\omega} \right)^{\frac{1}{1-\theta}},$$

where $p_t^y(\omega)$ is the nominal marginal cost of producing variety $\omega$.

The number of products (or features) created and commercialized by each final producer is endogenous. At each point in time, only a subset of varieties $\Omega_t \subset \Omega$ is actually available to consumers. To create a new product, the final producer needs to undertake a sunk investment, $f_{e,t}$, in units of intermediate input. Product creation requires each final producer to create a new plant that will be producing the new variety. Plants produce with different technologies indexed by relative productivity $z$. To save notation, we identify a variety with the corresponding plant productivity $z$, omitting $\omega$. Upon product creation, the productivity level of the new plant $z$ is drawn from a common distribution $G(z)$ with support on $[z_{\min}, \infty)$. Foreign plants draw

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15 Alternatively, $Y_t(i)$ can be thought as a bundle of product features that characterize the final product $i$.
16 Sectors (and sector-representative firms) are of measure zero relative to the aggregate size of the economy. Notice that $Y_t(i)$ can also be interpreted as a bundle of product features characterizing the product $i$.
17 Alternatively, we could decentralize product creation by assuming that monopolistically competitive firms produce product varieties (or features) that are sold to final producers, in this case interpreted as retailers. The two models are isomorphic. Details are available upon request.
productivity levels from an identical distribution. This relative productivity level remains fixed thereafter. Each plant uses intermediate input to produce its differentiated product variety, with real marginal cost:

$$\varphi_{z,t} = \frac{p(z)}{P_t} = \frac{\varphi_t}{z}. \quad (7)$$

At time \(t\), each final Home producer commercializes \(N_{d,t}\) varieties and creates \(N_{e,t}\) new products that will be available for sale at time \(t + 1\). New and incumbent plants can be hit by a “death” shock with probability \(\delta \in (0, 1)\) at the end of each period. The law of motion for the stock of producing plants is:

$$N_{d,t+1} = (1 - \delta)(N_{d,t} + N_{e,t}).$$

When serving the Foreign market, each final producer faces per-unit iceberg trade costs, \(\tau_t > 1\), and fixed export costs, \(f_{z,t}\). Fixed export costs are denominated in units of intermediate input and paid for each exported product. Thus, the total fixed cost is \(f_{x,t} = N_{x,t}f_{z,t}\), where \(N_{x,t}\) denotes the number of product varieties (or features) exported to Foreign. Absent fixed export costs, each producer would find it optimal to sell all its product varieties in Home and Foreign. Fixed export costs imply that only varieties produced by plants with sufficiently high productivity (above a cutoff level \(z_{x,t}\), determined below) are exported.\(^1\)

Define two special “average” productivity levels (weighted by relative output shares): an average \(\tilde{z}_d\) for all producing plants and an average \(\tilde{z}_{x,t}\) for all plants that export:

$$\tilde{z}_d = \left[ \frac{\int_{z_{\text{min}}}^{\infty} z^{\theta - 1} dG(z)}{\int_{z_{\text{min}}}^{\infty} dG(z)} \right]^{1/\theta}, \quad \tilde{z}_{x,t} = \left[ \frac{1}{1 - G(z_{x,t})} \right] \left[ \int_{z_{x,t}}^{\infty} z^{\theta - 1} dG(z) \right]^{1/\theta}. \quad (8)$$

Assume that \(G(\cdot)\) is Pareto with shape parameter \(k_p > \theta - 1\). As a result, \(\tilde{z}_d = \alpha \frac{1}{\theta - 1} z_{\text{min}}\) and \(\tilde{z}_{x,t} = \alpha \frac{1}{\theta - 1} z_{x,t}\), where \(\alpha = k_p / [k_p - (\theta - 1)]\). The share of exporting plants is given by:

$$N_{x,t} \equiv [1 - G(z_{x,t})] N_{d,t} = \left( \frac{z_{\text{min}}}{z_{x,t}} \right)^{-k_p} \alpha^{k_p} N_{d,t}. \quad (8)$$

The output bundles for domestic and export sale, and associated unit costs, are defined as
follows:

\[
Y_{d,t} = \left[ \int_{z_{\text{min}}}^{\infty} y_{d,t}(z)^{\frac{\theta-1}{\theta}} \, dG(z) \right]^{\frac{\theta}{\theta-1}}, \quad Y_{x,t} = \left[ \int_{z_{x,t}}^{\infty} y_{x,t}(z)^{\frac{\theta-1}{\theta}} \, dG(z) \right]^{\frac{\theta}{\theta-1}},
\]

(9)

\[
P_{d,t}^y = \left[ \int_{z_{\text{min}}}^{\infty} p_{d}^y(z)^{1-\theta} \, dG(z) \right]^{\frac{1}{1-\theta}}, \quad P_{x,t}^y = \left[ \int_{z_{x,t}}^{\infty} p_{x}^y(z)^{1-\theta} \, dG(z) \right]^{\frac{1}{1-\theta}}.
\]

(10)

Using equations (7) and (10), the real costs of producing the bundles \( Y_{d,t} \) and \( Y_{x,t} \) can then be expressed as:

\[
\frac{P_{d,t}^y}{P_t} = N_{d,t}^{\frac{1}{1-\theta}} \frac{\varphi_t}{z_d}, \quad \frac{P_{x,t}^y}{P_t} = N_{x,t}^{\frac{1}{1-\theta}} \frac{\varphi_t}{z_{x,t}}.
\]

(11)

The total present discounted cost facing the final producer is thus:

\[
Et \left\{ \sum_{s=t}^{\infty} \beta_{t,s} \left[ \frac{P_{d,s}^y}{P_s} Y_{d,s} + \tau_s \frac{P_{x,s}^y}{P_s} Y_{x,s} + \left( \frac{N_{s+1}}{1-\delta} - N_s \right) f_c,s \varphi_s + N_{x,s} f_{x,s} \varphi_s \right] \right\}.
\]

The producer determines \( N_{d,t+1} \) and the productivity cutoff \( z_{x,t} \) to minimize this expression subject to (8), (11), and \( z_{x,t} = \alpha z_{x,t} \).

The first-order condition with respect to \( z_{x,t} \) yields:

\[
\frac{P_{x,t}^y}{P_t} Y_{x,t} \tau_t = \frac{(\theta - 1) k_p}{[k_p - (\theta - 1)]} f_{x,t} N_{x,t} \varphi_t.
\]

The above conditions states that, at the optimum, marginal revenue from adding a variety with productivity \( z_{x,t} \) to the export bundle has to be equal to the fixed cost. Thus, varieties produced by plants with productivity below \( z_{x,t} \) are distributed only in the domestic market. The composition of the traded bundle is endogenous and the set of exported products fluctuates over time with changes in the profitability of export.

The first-order condition with respect to \( N_{d,t+1} \) determines product creation:

\[
\varphi_t f_{e,t} = Et \left\{ (1-\delta) \beta_{t+1,t+1} \left[ \varphi_{t+1} \left( \frac{f_{e,t+1}}{N_{d,t+1}} - \frac{N_{x,t+1} f_{x,t+1}}{N_{d,t+1}} \right) + \frac{1}{\theta-1} \left( \frac{\varphi_{t+1} P_{d,t+1} Y_{d,t+1}}{P_{t+1} N_{d,t+1}} + \frac{\varphi_{t+1} Y_{x,t+1} N_{x,t+1} f_{x,t+1} N_{d,t+1}}{P_{t+1} N_{d,t+1}} \right) \right] \right\}.
\]

In equilibrium, the cost of producing an additional variety, \( \varphi_t f_{e,t} \), must be equal to its expected benefit (expected savings on future sunk investment costs augmented by the marginal revenue from commercializing the variety, net of fixed export costs, if it is exported).

\footnote{Equation (8) implies that by choosing \( z_{x,t} \) the producer also determines \( N_{x,t} \).}
We are now left with the determination of domestic and export prices. Denote with $P_{d,t}$ the price (in Home currency) of the product bundle $Y_{d,t}$ and let $P_{x,t}$ be the price (in Foreign currency) of the exported bundle $Y_{x,t}$. Each final producer faces the following domestic and foreign demand for its product bundles:

$$Y_{d,t} = \left( \frac{P_{d,t}}{P_t} \right)^{-\phi} Y_t^C, \quad Y_{x,t} = \left( \frac{P_{x,t}}{P_t} \right)^{-\phi} Y_t^{C*},$$

where $Y_t^C$ and $Y_t^{C*}$ are aggregate demands of the consumption basket in Home and Foreign. Aggregate demand in each country includes sources other than household consumption, but it takes the same form as the consumption basket, with the same elasticity of substitution $\phi > 1$ across sectoral bundles. This ensures that the consumption price index for the consumption aggregator is also the price index for aggregate demand of the basket.

Prices in the final sector are sticky. We follow Rotemberg (1982) and assume that final producers must pay quadratic price adjustment costs when changing domestic and export prices. In the benchmark version of the model we assume producer currency pricing (PCP): Each final producer sets $P_{d,t}$ and the domestic currency price of the export bundle, $P_{x,t}^h$, letting the price in the foreign market be $P_{x,t} = \tau_t P_{x,t}^h / S_t$, where $S_t$ is the nominal exchange rate. The nominal costs of adjusting domestic and export price are, respectively, $\Gamma_{d,t} \equiv \nu \pi_{d,t}^2 P_{d,t} Y_{d,t}/2$, and $\Gamma_{x,t}^h \equiv \nu \pi_{x,t}^h P_{x,t}^h Y_{x,t}/2$, where $\nu \geq 0$ determines the size of the adjustment costs (domestic and export prices are flexible if $\nu = 0$), $\pi_{d,t} = (P_{d,t}/P_{d,t-1} - 1)$ and $\pi_{x,t}^h = (P_{x,t}^h/P_{x,t-1}^h - 1)$.

Absent fixed export costs, the producer would set a single price $P_{d,t}$ and the law of one price (adjusted for the presence of trade costs) would determine the export price as $P_{x,t} = \tau_t P_{x,t} = \tau_t P_{d,t}/S_t$. With fixed export costs, however, the composition of domestic and export bundles is different, and the marginal costs of producing these bundles are not equal. Therefore, final producers choose two different prices for the Home and Foreign markets even under PCP.

We relegate the details of optimal price setting to the Appendix. We show there that the (real) price of Home output for domestic sales is given by:

$$\left( \frac{P_{d,t}}{P_t} \right) = \frac{\phi}{(\phi - 1) \Xi_{d,t}} \left( \frac{P_{d,t}^h}{P_t} \right),$$

where $\Xi_{d,t}$ is defined by:

$$\Xi_{d,t} \equiv \left( 1 - \frac{\nu}{2} \pi_{d,t} \right) \nu (\pi_{d,t} + 1) \pi_{d,t} - \frac{\nu}{(\phi - 1)} E_t \left[ \beta_{t,t+1} (\pi_{d,t+1} + 1) \pi_{d,t+1} \frac{Y_{d,t+1}}{Y_{d,t}} \right].$$

12
As expected, price stickiness introduces endogenous markup variations: The cost of adjusting prices gives firms an incentive to change their markups over time in order to smooth price changes across periods. When prices are flexible ($\nu = 0$), the markup is constant and equal to $\phi / (\phi - 1)$.

The (real) price of Home output for export sales is equal to:

$$
\left( \frac{P_{x,t}}{P_t^*} \right) = \frac{\phi}{(\phi - 1)} \Xi_{x,t}^h \left( \frac{\tau_t P_{x,t}^y}{Q_t P_t} \right),
$$

(14)

where $Q_t \equiv SP_t^*/P_t$ is the consumption-based real exchange rate (units of Home consumption per units of Foreign), and:

$$
\Xi_{x,t}^h \equiv \left( 1 - \frac{\nu}{2} \pi_{x,t}^h \right) \nu \left( \pi_{x,t}^h + 1 \right) \pi_{x,t}^h - \frac{\nu}{(\phi - 1)} E_t \left[ \beta_{t,t+1} \left( \pi_{x,t+1}^h + 1 \right) \pi_{x,t+1}^h \frac{Y_{x,t+1}}{Y_{t}} \right].
$$

(15)

Absent fixed export costs $z_{x,t} = z_{\text{min}}$ and $\Xi_{x,t} = \Xi_{x,t}^d$. Plant heterogeneity and fixed export costs, instead, imply that the law of one price does not hold for the exported bundles.

For future purposes, define the average price of a domestic variety, $\bar{\rho}_{d,t} \equiv N_{d,t}^b (P_{d,t}/P_t)$ and the average price of an exported variety, $\bar{\rho}_{x,t} \equiv N_{x,t}^b (P_{x,t}/P_t^*)$. Combining the equations (11), (12), and (14), we have:

$$
\bar{\rho}_{d,t} = \mu_{d,t} \frac{\phi}{z_d}, \quad \bar{\rho}_{x,t} = \mu_{x,t} \frac{\tau_t \phi}{Q_t \Xi_{x,t}},
$$

(16)

where $\mu_{d,t} \equiv \phi / [(\phi - 1) \Xi_{x,t}]$ and $\mu_{x,t} \equiv \phi / [(\phi - 1) \Xi_{x,t}^h]$. Finally, denote with $\tilde{y}_{d,t}$ and $\tilde{y}_{x,t}$ the average output of, respectively, a domestic and exported variety:

$$
\tilde{y}_{d,t} \equiv \rho_{d,t} \frac{\theta - \phi}{N_{d,t}^b Y_t^C}, \quad \tilde{y}_{x,t} \equiv \rho_{x,t} \frac{\theta - \phi}{N_{x,t}^b Y_t^C}.
$$

(17)

### Household Budget Constraint and Intertemporal Decisions

The representative household can invest in non-contingent bonds that are traded domestically and internationally. International assets markets are incomplete as only risk-free bonds are traded across countries. Home bonds, issued by Home households, are denominated in Home currency. Foreign bonds, issued by Foreign households, are denominated in Foreign currency. Let $A_{t+1}$ and $A_{x,t+1}$ denote, respectively, nominal holdings of Home and Foreign bonds at Home.\footnote{Foreign nominal holdings of Home bonds are denoted with $A_t^*$, while Foreign nominal holdings of Foreign bonds are denoted by $A_{x,t}^*$.} To induce steady-state determinacy and stationary responses to temporary shocks in the model, we follow Turnovsky (1985), and, more recently, Benigno (2009), and we assume a quadratic cost of
adjusting bond holdings. The cost of adjusting Home bond holdings is \( \psi (A_{t+1}/P_t)^2 / 2 \), while the cost of adjusting Foreign bond holdings is \( \psi (A_{*,t+1}/P_t^*)^2 / 2 \). These costs are paid to financial intermediaries whose only function is to collect these transaction fees and rebate the revenue to households in lump-sum fashion in equilibrium.

The Home household’s period budget constraint is:

\[
A_{t+1} + S_t A_{*,t+1} + \frac{\psi}{2} P_t \left( \frac{A_{t+1}}{P_t} \right)^2 + \frac{\psi}{2} S_t P_t^* \left( \frac{A_{*,t+1}}{P_t^*} \right)^2 + P_tC_t = (1 + i_t)A_t + (1 + i_t^*)A_{*,t}S_t + w_tL_t + P_t b(1 - l_t) + T_t^A + T_t^i + T_t^f,
\]

where \( i_{t+1} \) and \( i_{t+1}^* \) are, respectively, the nominal interest rates on Home and Foreign bond holdings between \( t \) and \( t+1 \), known with certainty as of \( t - 1 \). Moreover, \( T_t^g \) is a lump-sum transfer (or tax) from the government, \( T_t^A \) is a lump-sum rebate of the cost of adjusting bond holdings from the intermediaries to which it is paid and \( T_t^i \) and \( T_t^f \) are a lump-sum rebate of profits from intermediate and final goods producers.\(^{23}\)

Let \( a_{t+1} \equiv A_{t+1}/P_t \) denote real holdings of Home bonds (in units of Home consumption) and let \( a_{*,t+1} \equiv A_{*,t+1}/P_t^* \) denote real holdings of Foreign bonds (in units of Foreign consumption). The Euler equations for bond holdings are:

\[
(1 + \psi a_{t+1}) = (1 + i_{t+1}) E_t \left\{ \frac{\beta_{t,t+1}}{1 + \pi_{C,t+1}} \right\},
\]

\[
(1 + \psi a_{*,t+1}) = (1 + i_{*,t+1}) E_t \left\{ \frac{Q_{t+1}}{Q_t} \frac{\beta_{t,t+1}}{1 + \pi_{C,t+1}^*} \right\},
\]

where \( \pi_{C,t} \equiv (P_t/P_{t-1}) - 1 \) and \( \pi_{C,t+1}^* \equiv (P_t^*/P_{t-1}^*) - 1 \). We present the details of the equilibrium of our model economy in the Appendix, and we limit ourselves to presenting the law of motion for net foreign assets below.

\(^{23}\)In equilibrium,

\[
T_t^g = -P_t b(1 - l_t),
\]

\[
T_t^A = P_t \frac{\psi}{2} \left( \frac{A_{t+1}}{P_t} \right)^2 + S_t P_t^* \frac{\psi}{2} \left( \frac{A_{*,t+1}}{P_t^*} \right)^2,
\]

\[
T_t^i = P_t \left( \varphi_l Z_{sl} - \frac{w_t l_t}{P_t^*} - \kappa V_t - \frac{\eta}{2} \pi_{w,t} \right),
\]

\[
T_t^f = \left( \frac{\mu_{d,t} - 1}{\mu_{d,t}} - \frac{\nu}{2} (\pi_{d,t})^2 \right) \tilde{p}_{d,t} N_{d,t} \tilde{y}_{d,t} + Q_t \left( \frac{\mu_{x,t} - 1}{\mu_{x,t}} - \frac{\nu}{2} (\pi_{x,t})^2 \right) \tilde{p}_{x,t} N_{x,t} \tilde{y}_{x,t} - \varphi_t (N_{x,t} f_{x,t} + N_{c,t} f_{c,t}).
\]
Net Foreign Assets and the Trade Balance

Bonds are in zero net supply, which implies the equilibrium conditions $a_{t+1} + a_{t+1}^* = 0$ and $a_{t+1}^* + a_{t+1} = 0$ in all periods. We show in the Appendix that Home net foreign assets are determined by:

$$a_{t+1} + Q_t a_{t+1} = \frac{1 + i_t}{1 + \pi_{C,t}} a_t + Q_t \frac{1 + i_t^s}{1 + \pi_{C,t}^s} a_{t,t} + Q_t N_{x,t} \tilde{p}_{x,t} \tilde{y}_{x,t} - N_{x,t}^s \tilde{p}_{x,t}^s \tilde{y}_{x,t}^s.$$ 

Defining $1 + r_t \equiv (1 + i_t) / (1 + \pi_{C,t})$, the change in net foreign assets between $t$ and $t+1$ is determined by the current account:

$$(a_{t+1} - a_t) + Q_t (a_{t+1} - a_{t,t}) = CA_t \equiv r_t a_t + Q_t r_t^s a_{t,t} + TB_t,$$

where $TB_t$ is the trade balance:

$$TB_t \equiv Q_t N_{x,t} \tilde{p}_{x,t} \tilde{y}_{x,t} - N_{x,t}^s \tilde{p}_{x,t}^s \tilde{y}_{x,t}^s.$$ 

3 Monetary Policy

To close the model described in the previous section, we must specify the behavior of monetary policy. In our benchmark exercises, we compare the Ramsey-optimal conduct of monetary policy to a representation of historical central bank behavior under a flexible exchange rate. Historical policy is captured by a standard rule for interest rate setting in the spirit of Taylor (1993) and Woodford (2003) for both central banks.\textsuperscript{24}

Data-Consistent Variables and Historical Monetary Policy

Before describing the interest-rate setting rule that characterizes historical policy, we must address an issue that concerns the data that are actually available to the central bank, i.e., we need to determine the empirically-relevant variables that should enter the theoretical representation of historical policy. As pointed out by Ghironi and Melitz (2005), in the presence of endogenous product creation and “love for variety” in the production of final consumption-varieties, variables measured in units of consumption do not have a direct counterpart in the data, i.e., they are not data-consistent. As the economy experiences entry of Home and Foreign firms, the welfare-consistent aggregate price index $P_t$ can fluctuate even if product prices remain constant. In the data,\textsuperscript{24} Later on, we also consider non-cooperative optimal policy and a fixed exchange rate regime (sections 6 and 7).

\textsuperscript{24}
however, aggregate price indexes do not take these variety effects into account. To resolve this issue, we follow Ghironi and Melitz (2005), and we construct an average price index \( \tilde{P}_t = \Omega_t^{1/2} P_t, \) where \( \Omega_t = N_{d,t} + N_{x,t}^*. \) The average price index \( \tilde{P}_t \) is closer to the actual CPI data constructed by statistical agencies than the welfare-based index \( P_t, \) and, therefore, it is the data-consistent CPI implied by the model. In turn, given any variable \( X_t \) in units of consumption, its data-consistent counterpart is \( X_{R,t} = X_t P_t / \tilde{P}_t = X_t \Omega_t^{1/2}. \)

With a flexible exchange rate regime, each country’s central bank sets its policy instrument following an historical interest rule. Since we calibrate the model to match features of the U.S. post-Bretton Woods, we assume that each country’s central bank sets its interest rate to respond to data-consistent CPI inflation and GDP gap relative to the equilibrium with flexible prices and wages:

\[
1 + i_{t+1} = (1 + i_t) \left( 1 + \tilde{\pi}_{C,t} \right)^{\theta} \left( \tilde{Y}_{g,t} \right)^{\theta_y} \]  

(18)

where \( \tilde{\pi}_{C,t} \) is the data-consistent CPI inflation and \( \tilde{Y}_{g,t} \) is the data-consistent output gap. An analogous rule for interest rate setting applies to Foreign.

Table 1 summarizes the key equilibrium conditions of the model. We rearranged some equations appropriately for transparency of comparison to the planner’s optimum, which we will use to build intuition for the tradeoffs facing the Ramsey policymaker. The table contains 25 equations that determine 25 endogenous variables of interest: \( C_t, \tilde{p}_{d,t}, i_t, h_t, V_t, N_{d,t}, w_t / P_t, \tilde{z}_{x,t}, \pi_{w,t}, \pi_{C,t}, i_{t+1}, a_{t+1}, \) their foreign counterparts, and \( Q_t. \) (Other variables that appear in the table are determined as described above.)

**Ramsey-Optimal, Cooperative Monetary Policy**

The Ramsey authority maximizes aggregate welfare under the constraints of the competitive economy. Let \( \{A_{1,t}, ..., A_{23,t}\}_{t=0}^{\infty} \) be the Lagrange multiplier associated to the equilibrium conditions in

\[25\] There is much empirical evidence that gains from variety are mostly unmeasured in CPI data, as documented most recently by Broda and Weinstein (2010).

\[26\] The Federal Reserve’s has a mandate of price stability, defined in terms of a (harmonized) index of consumer price inflation, and maximum employment.

\[27\] We define GDP, denoted with \( Y_t, \) as total income: the sum of labor income, dividend income from final producers, and profit income from intermediate producers. Formally: \( Y_t \equiv (w_t / P_t) l_t + (T_{x,t}^f - \varphi_t N_{x,t} f_{x,t}) + T_{x,t}^i. \) We define output gap \( \tilde{Y}_{g,t} = Y_{R,t} / Y_{R,t}^f \) where \( Y_{R,t}^f \) is GDP under flexible prices and wages.
Table 1 (excluding the two interest-rate setting rules). 28 The Ramsey problem consists in choosing:

\[
\{C_t, C^*_t, \bar{\rho}_{d,t}, \bar{\rho}^*_{d,t}, l_t, l^*_t, h_t, h^*_t, V_t, V^*_t, N_{d,t}, N^*_d, J_t, J^*_t, \bar{z}_{x,t}, \bar{z}^*_{x,t}, \pi_{w,t}, \pi^*_{w,t}, \pi_C, \pi^*_C, i_{t+1}, i^*_t, a_{t+1}, a^*_{t+1}, Q_t, \Lambda_{1,t}, ..., \Lambda_{23,t}\}_{t=0}^{\infty}
\]

to maximize:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} [u(C_t) - l_t v(h_t)] + \frac{1}{2} [u(C^*_t) - l^*_t v(h^*_t)] \right\}, \tag{19}
\]

subject to the constraints in Table 1 (excluding the interest rate rules). 29

As common practice in the literature, we write the original non-stationary Ramsey problem in a recursive stationary form by enlarging the planner’s state space with additional (pseudo) co-state variables. Such co-state variables track the value to the planner of committing to the pre-announced policy plan along the dynamics.

4 Inefficiency Wedges

The Ramsey planner uses its policy instruments (the Home and Foreign interest rates) to address the consequences of a set of distortions that exist in the market economy. To understand these distortions and the tradeoffs they create for optimal policy, it is instructive to compare the equilibrium conditions of the market economy to those implied by the solution to a first-best, optimal planning problem. This allows us to define inefficiency wedges for the market economy (relative to the planner’s optimum) and describe Ramsey policy in terms of its implications for these wedges.

In the Appendix we derive the first-best allocation chosen by a benevolent social planner for the world economy, summarized in Table 2. We define the inefficiency wedges that characterize the market economy by comparing the equilibrium allocation in the decentralized economy (Table 1) to the one chosen by the social planner (Table 2).

The presence of price and wage stickiness, firm monopoly power, positive unemployment benefits and incomplete markets induces ten sources of distortion (summarized in Table 3) in the market economy. These distortions affect three margins of adjustment and the resource constraint for

\[28\text{We assume that the other variables that appear in the table have been substituted out by using the appropriate equations and definitions above.}\]

\[29\text{In the primal approach to Ramsey policy problems described by Lucas and Stokey (1983), the competitive equilibrium is expressed in terms of a minimal set of relations involving only real allocations. In the presence of sticky prices and wages, it is impossible to reduce the Ramsey planner’s problem to a maximization problem with a single implementability constraint.}\]
consumption output in the decentralized economy:

- **Product Creation Margin:** Comparing the term in square brackets in equation (9) in Table 1 to the term in square brackets in equation (9) in Table 2 implicitly defines the inefficiency wedge along the market economy’s product creation margin (see the Appendix for details). The wedge $\Sigma_{PC,t}$ is induced by the presence of sticky prices which result in inefficient time-variation and lack of synchronization of domestic and export markups: $\Upsilon_{\mu_d,t} \equiv \mu_{d,t-1}/\mu_{d,t} - 1$ and $\Upsilon_{\mu_x,t} \equiv \mu_{d,t}/\mu_{x,t} - 1$. Absent sticky prices ($\Upsilon_{\mu_d,t} = \Upsilon_{\mu_x,t} = 0$), the product creation wedge $\Sigma_{PC,t}$ is zero.

- **Job creation margin:** Comparing the term in square brackets in equation (11) in Table 1 to the term in square brackets in equation (11) in Table 2 implicitly defines the inefficiency wedge along the market economy’s job creation margin (see the Appendix for details; equation (17) in Table 1 determines the real wage in the market economy). The wedge $\Sigma_{JC,t}$ is a combination of various distortions. Monopoly power in the final sector distorts the job creation decision by inducing a suboptimally low return from vacancy posting, captured by $\Upsilon_{\phi,t} \equiv 1/\mu_{d,t}$. Failure of the Hosios condition (for which equality of the firm’s bargaining share and the vacancy elasticity of the matching function is necessary for efficiency) is an additional distortion in this margin, measured by $\Upsilon_{\eta,t} \equiv \eta_t - \varepsilon$. This is affected both by the flexible-wage value of the bargaining share ($\eta_t$, which can be different from $\varepsilon$) and the presence of wage stickiness, which induces time variation of $\eta_t$. Sticky wages are sufficient to generate a wedge between private and social returns to vacancy posting. Moreover, they distort job creation also by affecting the outside option of firms through an additional term $\Upsilon_{w,t} \equiv \vartheta \pi^2_{w,t}/2$. Finally, unemployment benefits increase the workers’ outside option above its efficient level: $\Upsilon_{b,t} \equiv b$. When $\Upsilon_{\phi,t} = \Upsilon_{\eta,t} = \Upsilon_{b,t} = \Upsilon_{w,t} = 0$, the real wage is determined by

$$\frac{u_t}{P_t} h_t = \varepsilon \frac{v(h_t)}{u_C,t} + (1 - \varepsilon) \rho_{d,t} Z_t h_t + \kappa (1 - \varepsilon) \tau_t/q_t,$$

and $\Sigma_{JC,t} = 0$.

- **Labor supply margin:** With endogenous labor supply, monopoly power in product markets, $\Upsilon_{\phi,t} \equiv (1/\mu_{d,t}) - 1$, induces a misalignment of relative prices between consumption goods and leisure. This is the distortion that characterizes standard New Keynesian models without labor market frictions. The associated wedge $\Sigma_{h,t} \equiv \Upsilon_{\phi,t}$, which is time-varying for the
presence of sticky prices.

- **Cross-country risk sharing margin:** Incomplete markets imply inefficient risk sharing between Home and Foreign households, resulting in the distortion $\Upsilon_{Q,t} \equiv (u_{C^*,t}/u_{C,t})/Q_t$. The departure of relative consumption from the perfect risk sharing outcome is also affected by the costs of adjusting bond holdings (the distortion $\Upsilon_{a,t} \equiv \psi a_{t+1} + \psi a_{s,t+1}$ and its Foreign mirror image in the Euler equations for Home and Foreign holdings of bonds).

  We summarize the combined effect of these distortions with the financial inefficiency wedge $\Sigma_{RS,t} \equiv (u_{C^*,t}/u_{C,t})/Q_t = \Upsilon_{Q,t}$. Efficiency along this margin requires $\Sigma_{RS,t} = 1$.

- **Consumption resource constraint:** Sticky prices and wages imply diversion of resources from consumption and creation of new product lines and vacancies, with the distortions $\Upsilon_{\pi,w,t} \equiv \vartheta \pi_{w,t}^2/2$, $\Upsilon_{\pi,d,t} \equiv \nu \pi_{d,t}^2/2$ and $\Upsilon_{\pi,s,t} \equiv \nu \pi_{s,t}^2/2$. The associated wedge (defined by $\Sigma_{\upsilon^C,t} \equiv \Upsilon_{\pi,w,t} + \Upsilon_{\pi,d,t} + \Upsilon_{\pi,s,t}$) is zero under flexible wages and prices.

The market allocation is efficient only if all the distortions and associated inefficiency wedges are zero at all points in time. Since we abstract from optimal fiscal policy and focus on asymmetric shocks, it follows that we work in a second-best environment in which the efficient allocation cannot be achieved. In this second-best environment, the Ramsey central bank optimally uses its leverage on the economies via the sticky-price and sticky-wage distortions, trading off its costs (including the resource costs) against the possibility of addressing the distortions that characterize the market economy under flexible wages and prices.

5 **Calibration**

We interpret periods as quarters and calibrate the model to match U.S. macroeconomic data from 1954:Q1 to 1980:Q1.\textsuperscript{30} Table 4 summarizes the calibration, which is assumed symmetric across countries. (Variables without time indexes denote steady-state levels.) We set the discount factor $\beta$ to 0.99, implying an annual real interest rate of 4 percent. The period utility function is given by $u_t = C_t^{1-\gamma_C}/(1-\gamma_C) - \frac{1}{2} h_t^{1+\gamma_h} (1+\gamma_h)$. The risk aversion coefficient $\gamma_C$ is equal to 2, while the Frisch elasticity of labor supply $1/\gamma_h$ is set to 0.4, a mid-point between empirical micro and

\textsuperscript{30} This time period featured relatively weak trade linkages between the U.S. economy and the rest of the world. The growth in U.S. trade began at the beginning of the 80’s, experiencing a five-fold growth in nominal terms over the next twenty five years— in 1980 US two-way merchandise trade was 467 billion U.S. dollars, reaching 2,942 billion U.S. dollars in 2006 (UNComtrade via WITS 2008).
The elasticity of substitution across product varieties, $\theta$, is set to 3.8 following Bernard, Eaton, Jensen, and Kortum (2003), who find that this value fits U.S. plant and macro trade data. Following Ghironi and Melitz (2005), we set the elasticity of substitution across Home and Foreign goods, $\phi$, equal to $\theta$. As in Ghironi and Melitz (2005), we also set $k_p = 3.4$, normalize $z_{\text{min}}$ to 1 and calibrate the fixed export cost $f_x$ so that the share of exporting plants is equal to 21 percent. We choose iceberg trade costs, $\tau$, so that total trade (imports plus exports) over GDP is equal to 10 percent, the average value for the U.S. in the sample period. This requires setting $\tau = 1.75$.\(^{32}\)

To ensure steady-state determinacy and stationarity of net foreign assets, we set the bond adjustment cost $\psi$ to 0.0025 as in Ghironi and Melitz (2005). The scale parameter for the cost of adjusting prices, $\nu$, is equal to 80, as in Bilbiie, Ghironi, and Melitz (2008). We choose $\vartheta$, the scale parameter of nominal wage adjustment costs, so that the model reproduces the volatility of unemployment relative to GDP observed in the data. This implies $\vartheta = 60$. To calibrate the entry costs, we follow Ebell and Haefke (2009) and set $f_e$ so that regulation costs amount to 5.2 months of per capita output.

We set unemployment benefits, $b$, so that the model reproduces the average replacement rate, $b / (wh)$, for the U.S. reported by OECD (2004). The steady-state bargaining share of firms, $\eta$, is equal to 0.4, as estimated by Flinn (2006) for the U.S. The elasticity of the matching function, $\varepsilon$, is also equal to 0.4, within the range of estimates reported by Petrongolo and Pissarides (2006) and such that the Hosios condition holds in steady state. The exogenous separation rate between firms and workers, $\lambda$, is 10 percent, as reported Shimer (2005). To pin down exogenous producer exit, $\delta$, we target the portion of worker separation due to plant exit equal to 40 percent (see Haltiwanger, Scarpetta, and Schweiger, 2008).

Two labor market parameters are left for calibration: the scale parameter for the cost of vacancy posting, $\kappa$, and the matching efficiency parameter, $\chi$. We calibrate these parameters to match the steady-state probability of finding a job and the probability of filling a vacancy. The former is 60 percent, while the latter is 70 percent, in line with Shimer (2005).

For the bivariate productivity process, we set persistence and spillover parameters consistent

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31 The value of this elasticity has been a source of controversy in the literature. Students of the business cycle tend to work with elasticities that are higher than microeconomic estimates, typically unity and above. Most microeconomic studies, however, estimate this elasticity to be much smaller, between 0.1 and 0.6. For a survey of the literature, see Card (1994). Our results are not affected significantly if we hold hours constant at the optimally determined steady-state level.

32 This value is remarkably close to the estimates of trade costs reported by Anderson and van Wincoop (2003).
with evidence in Baxter (1995) and Baxter and Farr (2005), implying zero spillovers across countries and persistence equal to 0.999. Moreover, we set the standard deviation of productivity innovations at 0.73 percent and the covariance of innovations at 0.19 percent, as in Baxter (1995) and Backus, Kehoe, and Kydland (1992, 1994).

Finally, the parameter values in the historical rule for the Fed’s interest rate setting are those estimated by Clarida, Galí, and Gertler (2000). The inflation and GDP gap weights are 1.62 and 0.34, respectively, while the smoothing parameter is 0.71.

In the Appendix, we provide a detailed discussion of the impulse responses to a Home productivity shock and the second-moment properties of the model under the historical policy and a flexible exchange rate. We show that the model successfully replicates several features of the U.S. and international business cycle. In particular, the model captures reasonably well the cyclical behavior of imports and exports and it reproduces (at least qualitatively) a ranking of cross-country correlations that represent a traditional challenge for international business cycle models.33

6 Optimal Monetary Policy with Weak Trade Linkages

We begin our discussion of optimal policy by characterizing the Ramsey-optimal monetary policy in the presence of weak trade linkages. First, we study optimal monetary policy in the long run, then we turn to the Ramsey allocation over the business cycle.

Optimal Monetary Policy in the Long Run

Our interest in this section is in how the two Ramsey central banks determine the optimal inflation rates \( \pi_C \) and \( \pi_C^* \) to address the distortions discussed in Section 4. To begin, it is immediate to verify that long-run inflation is always symmetric across countries regardless of symmetry or asymmetry of the calibration. This result follows from the steady-state Euler equations of households once it is observed that Home and Foreign assets holdings are always zero in steady state:

\[
1 + \pi_C = \beta(1 + i) = 1 + \pi_C^*.
\]

Moreover, wage inflation and domestic and export producer price inflation are always equal to consumer price inflation: \( \pi_C = \pi_d = \pi_x = \pi_w \).

33 The model correctly predicts that imports and exports are more volatile than GDP. Moreover, net exports are countercyclical and the volatility of the trade balance relative to GDP is in line with the data. These stylized facts are not reproduced by standard international business cycle models (see Engel and Wang, 2009).
Table 5 shows that the optimal long-run target for net inflation before trade integration is positive and equal to 1.4 percent. To understand why the Ramsey allocation prescribes positive long-run inflation, observe first that a symmetric long-run equilibrium with constant endogenous variables eliminates some of these distortions: A constant markup removes the markup variation distortion from the product creation margin ($\gamma_{\mu_k} = \gamma_{\mu_s} = \Sigma_{PC} = 0$); Symmetry across countries removes the risk-sharing distortion of incomplete markets ($\gamma_{Q} = \Sigma_{RS} = 0$), and constant, zero net foreign assets eliminate the effect of asset adjustment costs ($\gamma_{a} = \gamma_{Q} = 0$). All the the remaining steady-state distortions but the costs of wage and price adjustment require a reduction of markups. Firms’ monopoly power in the downstream sector and positive unemployment benefits imply a suboptimally low job-creation. Since $\pi_C = \pi_w$, positive inflation raises the firm bargaining power $\eta$, favoring vacancy posting by firms. However, the Ramsey authority in each country must trade the beneficial welfare effects of reducing these distortions against the costs of non-zero inflation implied by allocating resources to wage and price changes and by the departure from the Hosios condition (since $\eta > \varepsilon$). Compared to the zero inflation outcome, the Ramsey authority reduces the inefficiency wedge in job creation ($\Sigma_{JC}$).

The finding of an optimal positive long-run inflation is in contrast with the prescription of near zero inflation delivered by the vast majority of New Keynesian models in closed and open economy. While the costs of inflation outweigh the benefits of reducing other distortions in those models, this is no longer the case with a richer microfoundation of labor markets. In particular, the prescription of an optimal positive long-run inflation stems from the presence of wage stickiness and search and matching frictions in the labor market. Wage stickiness, in fact, allows the Ramsey authority to optimally manipulate the firm’s bargaining power to reduce inefficiencies in job creation. Absent sticky wages, a policy of zero inflation would be optimal also in our model.

Table 5 also presents the welfare gain from implementing the long-run optimal policy relative to the Fed’s historical behavior. To compute this welfare gain avoiding spurious welfare reversals, we assume identical initial conditions across different monetary policy regimes and include transition dynamics in the computation. Specifically, we assume that all the state variables are set at their steady-state levels under the historical policy at time $t = -1$, regardless of the monetary regime from $t = 0$ on. We compare welfare under the continuation of historical policy from $t = 0$ on (which implies continuation of the historical steady state) to welfare under the optimal long-run policy from $t = 0$ on (which implies a transition between the initial implementation at $t = 0$ and the Ramsey steady state). We measure the long-run welfare gains of the Ramsey policy in the two
countries (which are equal by symmetry) by computing the percentage increase $\Delta$ in consumption that would leave the household indifferent between policy regimes. In other words, $\Delta$ solves:

$$\sum_{t=0}^{\infty} \beta^t u \left( \epsilon^{\text{Ramsey}, t}, h^{\text{Ramsey}, t} \right) = u \left[ \frac{\left( 1 + \frac{\Delta}{100} \right) C^{\text{Hist}, t} h^{\text{Hist}, t} h^{\text{Hist}, t}}{1 - \beta} \right].$$

Table 5 shows that the welfare gains from the Ramsey-optimal policy amount to 0.34 percent of annualized steady-state consumption.$^{34}$

**Optimal Monetary Policy over the Business Cycle**

Stochastic fluctuations in aggregate productivity modify the policy tradeoffs facing the Ramsey authorities by reintroducing the distortions eliminated by symmetry and absence of time variation in steady state. Moreover, Ramsey-optimal long-run policy does not close the steady-state inefficiency wedges. Thus, the Home and Foreign economies fluctuate around a steady state where unemployment is inefficiently high and the number of producers serving domestic and export markets is inefficiently low. As a result, shocks trigger larger fluctuations in product and labor markets (in both economies) than in the efficient allocation: Both producer entry and unemployment are suboptimally volatile.

Figure 1 (dashed lines) shows impulse responses to a Home productivity increase under the Ramsey-optimal policy. Relative to the historical rule (i.e., a policy of near producer price stability, defined as zero deviation of average domestic producer inflation from trend), the Ramsey authority generates a much smaller increase in wage inflation and a larger departure from price stability (disinflation).

To understand these results, it is instructive to characterize the policy tradeoffs facing the Ramsey central bank over the business cycle. First, as in steady state, there is a tension between the beneficial effects of manipulating inflation and its costs. Second, there is a tradeoff between stabilizing consumer price inflation (which contributes to stabilizing domestic markups) and wage inflation (which stabilizes unemployment). Finally, there is a tension between stabilizing domestic markups, $\mu_{d,t}$, and export markups, $\mu_{x,t}$.

Policy tradeoffs explain why a policy of price stability is suboptimal. First, wage inflation is too volatile, and markup stabilization correspondingly too strong, under this policy. Following fluctuations in aggregate productivity, sticky wages and positive unemployment benefits generate

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$^{34}$Our results are not sensitive to the choice of (identical) initial conditions for the state variables.
real wage rigidities, i.e., a positive (negative) productivity shock is not fully absorbed by the rise (fall) of the real wage, affecting job creation over the cycle. Higher Home productivity pushes the real wage above its steady-state level, as the real value of existing matches has increased. Under a policy of price stability, the effect of wage stickiness is magnified, since the real wage becomes even more rigid. Firms post too many vacancies and, in equilibrium, nominal wage adjustment costs are too large.\textsuperscript{35} Domestic price stability can also be suboptimal due to the asymmetric dynamics of domestic and export markups. Endogenous fluctuations in the export productivity cutoff $z_{x,t}$ open a wedge between domestic and export inflation in each country. Since the law of one price does not hold, the central bank cannot stabilize export markups by setting domestic producer price inflation equal to zero.\textsuperscript{36}

Historical Fed behavior results in positive employment comovement across countries. In contrast, the Ramsey authority pushes unemployment rates in opposite directions by engineering wage disinflation rather than inflation in the Foreign country. This results in higher unemployment in the relatively less productive economy. To understand the result, notice that in the Home country, producers have a weaker incentive to post vacancies as more stable wage inflation implies that their effective bargaining power rises by less than under the historical policy. Lower job creation translates into smaller employment gains, which reduces domestic aggregate demand for Home and Foreign goods. Moreover, Foreign households find more profitable to invest in the more productive economy in response to the shock, shifting resources towards Home.

As for the long-run optimal policy, we compare policy regimes by computing the welfare gains for the two countries from optimal policy over the cycle. Specifically, we compute the percentage $\Delta$ of steady-state consumption that would make households indifferent between living in a world with uncertainty under monetary policy $m$, where $m = \text{Ramsey}$ or $\text{Hist}$, and living in a deterministic

\textsuperscript{35}Notice, however, that a policy that completely stabilizes wage inflation is also suboptimal. In this case, there would be too much inflation and markup volatility, and the response of unemployment would be too small. Moreover, while the standard New Keynesian prescription of price stability amounts to a prescription of procyclical monetary policy, with expansion in response to favorable productivity shocks to mimic the flexible-price equilibrium, optimal policy in our monetary union with multiple distortions is more countercyclical than historical behavior. The Ramsey central bank induces a larger drop in inflation (and markups) in both countries following an expansionary shock at Home, but it expands more aggressively in the opposite case of a contractionary shock.

\textsuperscript{36}The Ramsey authority would face a tradeoff between stabilizing domestic and export markups even if price stickiness was the only distortion in the market economy. Figure 1 shows that this tradeoff is quantitatively not important when trade linkages are weak (the dynamics of $\mu_{d,t}$ and $\mu_{x,t}$ are very similar under the Ramsey-optimal policy).
Ramsey world:

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^m, h_t^m) = \frac{u \left( (1 + \frac{\Delta}{100}) C^{Ramsey}, h^{Ramsey} \right)}{1 - \beta}.
\]

First-order approximation methods are not appropriate to compute the welfare associated with each monetary policy arrangement. The solution of the model implies that the expected value of each variable coincides with its non-stochastic steady state. However, in an economy with a distorted steady state, volatility affects both first and second moments of the variables that determine welfare. Hence, we compute welfare by resorting to a second-order approximation of the policy functions. As shown in Table 6, by implementing the Ramsey-optimal policy the welfare cost of business cycle falls by approximately 20 percent: Optimal departures from price stability lower the cost of business cycles from 1.02 percent of steady-state consumption under the historical policy to 0.82 percent.

From a policy perspective, it is important to know whether the Ramsey-optimal policy can be implemented by mean of simple interest rate rules, and whether such optimal rules can be purely inward looking. To address this question, we consider a constrained Ramsey problem in which the Ramsey authority maximizes aggregate welfare in (19) by optimally choosing the response coefficients in a general class of inward-looking interest rate rules.\(^{37}\) For simplicity, we allow current-period interest rates in Home and Foreign to respond to four domestic variables: previous-period interest rate, producer price inflation, wage inflation and output gap. For the Home economy, the interest rule has the following functional form (a similar expression holds for the Foreign country):

\[
1 + i_{t+1} = (1 + i_t) ^{\varrho_i} \left[ (1 + \tilde{\pi}_{d,t}) ^{\varrho_{\pi_d}} (1 + \tilde{\pi}_{w,t}) ^{\varrho_{\pi_w}} \left( \tilde{Y}_{g,t} \right) ^{\varrho_Y} \right] ^{-\varrho_i}.
\]

The welfare maximizing rule implies: \(\varrho_i = .60, \varrho_Y = 0, \varrho_{\pi_d} = 1.45 \) and \(\varrho_{\pi_w} = 3.75\). As shown in Table 6, the welfare loss implied by the (constrained) optimal interest rate rule relative to the (unconstrained) Ramsey allocation is very small (less than 1 percent, corresponding to 0.008 percent of steady state consumption). As a result, when trade linkages are weak, the Ramsey-optimal policy is well approximated by an inward-looking interest rate rule, i.e., each central bank can achieve the constraint, efficient allocation by appropriately responding to domestic targets.

\(^{37}\)We only consider combinations of policy parameters that deliver a unique rational expectations equilibrium.
7 Optimal Monetary Policy and Trade Integration

How does trade integration affect optimal monetary policy? Stronger trade linkages pose different challenges for the central banks of integrating countries. First, trade integration has permanent effects that may alter the optimal long-run inflation target. Second, stronger trade linkages affect the way economies respond to aggregate shocks, with consequences for the optimal conduct of monetary policy over the business cycle.

In our exercises, we interpret trade integration as a symmetric reduction of iceberg trade costs, \( \tau \) and \( \tau^* \), capturing a decrease in several impediments to international trade such as tariffs and transportation costs.\(^{38}\) We consider two scenarios. First, we re-calibrate \( \tau \) and \( \tau^* \) so that in the new steady state the ratio of trade to GDP is 22 percent, the average value observed in the U.S. during the period 1980 – 2011. Second, we consider a further reduction in trade costs that implies a trade-to-GDP ratio equal to 35 percent.

Optimal Monetary Policy in the Long Run

The starting point of our analysis is a robust conclusion reached by empirical work using micro-level data: When the exposure to trade changes, the probability of exporting among non-exporters increase.\(^{39}\) Given the productivity advantage of exporters, this induces reallocations in favor of the more productive exporting plants, increasing average industry productivity (see Bernard, Jensen, and Schott, 2006).

Our model, as in Melitz (2003), is consistent with these stylized facts. Define a weighted productivity average \( \tilde{z}_t \) that reflects the combined market shares of all Home firms and the output shrinkage linked to exporting:

\[
\tilde{z} = \left\{ \left[ \tilde{z}_d^{\theta-1} + \left( \frac{\tilde{z}_x}{\tau} \right)^{\theta-1} \frac{N_x}{N_d} \right] \frac{1}{\theta} \right\}^{1/\theta}.
\]

In response to trade integration, the relative more productive non-exporting plants begin to export and the market shares of the domestic plants shrink due to increased foreign competition. Even if the average productivity of the exporters (\( \tilde{z}_x \)) falls, the gain in market shares of existing

\(^{38}\) Trade integration can also be interpreted as a permanent decrease in fixed export costs. Qualitatively, none of our results is affected by the specific nature of the “integration shock”.

\(^{39}\) There is well-documented evidence about trade-induced self-selection: Firms are relatively more productive prior to their entry into export markets. Several studies further reject the hypothesis of firm-level productivity growth following export market entry, although some studies, especially for developing countries, do report such a link.
and new exporting plants is strong enough to guarantee that the average productivity \( \bar{z} \) increases.

This result has implications for the conduct of monetary policy. Consider steady-state inefficiency wedges under a long-run zero net inflation, \( \pi_C = 0 = \Upsilon_{\pi_d} = \Upsilon_{\pi_x} = \Upsilon_{\pi_w} \). Markups are constant and equal to one, and so \( \Upsilon_{\mu_d} = \Upsilon_{\mu_x} = 0 \). Moreover, the Hosios condition implied by our calibration ensures that \( \eta = \varepsilon \) and \( \Upsilon_{\eta} = \Upsilon_{\pi_w} = 0 \). Finally, full symmetry across countries implies that \( Q = 1 \) (as a result of symmetry \( \Upsilon_{Q} = \Sigma_{RS} = 1 \)). Thus, two distortions remain: the monopoly power distortion on job creation, \( \Upsilon_{\varphi} = (1/\mu_d) - 1 \), and non-zero unemployment benefits, leaving \( \Upsilon_b \) unaffected. The effects of trade integration on welfare operates indirectly by reducing the welfare losses induced by \( \Upsilon_{\varphi} \) and \( \Upsilon_b \). More precisely, trade integration raises average productivity and dampens the negative consequences of firms’ monopoly power and distortionary unemployment benefits. To see this, let \( \varkappa \equiv q/\iota \) be the labor market tightness. Since \( U = \lambda/(\lambda + \varkappa^2) \), the effect of trade integration on job creation is summarized by the response of \( \varkappa \) to changes in trade costs. In the Appendix, we show that labor market tightness is an increasing function of the marginal revenue from a match, \( \varphi \), i.e. \( d\varkappa/d\varphi > 0 \). Moreover, we also show that \( \varphi = (1/\mu_d) N_d^{\frac{1}{3-\iota}} \bar{z} \). Thus, the marginal revenue of a match and labor market tightness depend positively on the number of domestic varieties available to consumers, \( N_d \), and the average productivity of firms \( \bar{z} \). Trade openness always decreases \( N_d \) but increases \( \bar{z} \). For any realistic parametrization of the model, the productivity effects dominate, implying that \( \partial\varkappa/\partial\varphi > 0 \). Thus, our model features a negative link between trade and unemployment, given that \( \partial U/\partial\varkappa = -\partial\varkappa/\partial\varphi < 0 \). As in Cacciatore (2010) and Felbermayr, Prat, and Schmerer (2011), the increase in \( \bar{z} \) makes workers on average more productive, increasing the average marginal revenue of a match, and pushing employment toward its efficient level.\(^{40}\)

We can now discuss the implications of stronger trade linkages for optimal monetary policy. Table 5 shows that trade lowers steady-state optimal inflation, which becomes 1 percent when trade integration reaches its maximum. The intuition is straightforward: trade-induced productivity gains make price stability relatively more desirable since they reduce the need to resort to positive inflation to correct for steady-state distortions. Table 5 also reveals that the welfare gains from implementing the optimal policy response to trade integration are positive but smaller that in the pre-integration scenario (welfare gains reduce from 0.34 percent of steady state consumption to 0.16 percent).\(^{41}\)

\(^{40}\)Dutt, Mitra and Ranjan (2009) and Felbermayr, Prat and Schmerer (2011) document empirically the negative long-run relationship between trade openness unemployment.

\(^{41}\)Welfare calculations include the adjustment to trade integration. Impulse responses are available upon request.
Optimal Monetary Policy over the Business Cycle

A second robust conclusion of empirical work is that, among industrialized economies, business cycles become more synchronized when trade linkages are stronger. In particular, by running cross-country regressions, the slope coefficient estimates in Frankel and Rose (1998) and Clark and van Wincoop (2001) imply that countries with 3.5 times larger trade intensity have a correlation that is 0.089 higher and 0.125 higher, respectively.42

Table 7 shows that the model correctly predicts business cycle synchronization in response to trade integration. In particular, under the historical monetary policy making, the model predicts that cross-country GDP correlation increase from 0.36 to 0.49 when trade volumes are 3.5 larger.43

The ability of the model to account for the business cycle synchronization observed in the data has often eluded standard international business cycle models. Kose and Yi (2001) show that the Backus, Kehoe and Kydland (1992) model augmented with transportation costs yields the counterfactual prediction of a smaller cross-country GDP correlation following reductions in trade costs, the so called trade-comovement puzzle. The reason is that in the benchmark model, demand complementarities generated by reductions in trade barriers are too weak and the reallocation of production towards more productive locations over the cycle dominates.

As discussed in Cacciatore (2010), endogenous product dynamics and labor market frictions explain why the model can successfully address the trade-comovement puzzle. First, they introduce a strong internal propagation mechanism in the model, which translates in long-lasting effects of domestic shocks abroad in the presence of strong trade linkages. Specifically, aggregate disturbances trigger spikes in job creation, generating persistent employment dynamics on account of matching frictions. The sluggish adjustment in the number of plants serving domestic and export markets feeds back into employment dynamics, magnifying the future output effects of the shock. The domestic amplification of aggregate shocks results into persistent effects on Foreign output dynamics through cross-country demand linkages.

42 The numbers are the average of the coefficient estimates in Frankel and Rose (1998) and Clark and van Wincoop (2001). Using aggregate data, Kose and Yi (2006), Calderon, Chong and Stein (2007), and Baxter and Kouparitsas (2005) also find that country-pairs that trade more with each other exhibit a higher degree of business cycle comovement. di Giovanni and Levchenko confirm this finding using sector-level data.

43 These results are not directly comparable with empirical estimates since the latter refer to an increase in the average bilateral trade intensity across countries while in the model we consider an increase in total trade volumes. To make the comparison between the model predictions and empirical evidence more transparent, we have also considered an alternative calibration of trade costs, setting the initial value of \( \tau \) and \( \tau^* \) to generate a 0.5 percent bilateral trade intensity, the average value for the U.S. in the period 1954 – 1980. Then, we reduced trade costs to increase the bilateral trade intensity by a factor of 3.5. In this case the predicted increase in GDP comovement is 0.085, in line with empirical estimates.
Second, firm heterogeneity mitigates the terms of trade ($\text{TOT}_t \equiv S_t p_{x,t}/p_{x,t}^*$) effects of aggregate shocks, reducing the incentives to shift resources across countries over the cycle. For example, following an increase in Home productivity, Home’s terms of trade depreciate, i.e., Home goods become relatively cheaper. Compared to standard international business cycle models, however, terms of trade depreciation is milder: The endogenous selection of relatively low-productive firms into the export market, summarized by a lower export productivity cutoff $z_{x,t}$, partially offsets the reduction in marginal costs and export prices generated by higher aggregate productivity.

We can now discuss the consequences of trade integration for the conduct of monetary policy over the business cycle. Figure 2 shows that the optimal monetary policy does not change after trade integration. The Ramsey authority continues to strike a balance between stabilizing price and wage inflation in both countries.\textsuperscript{44} Moreover, the optimized inward-looking interest rate rules derived in the previous section can still replicate the constrained efficient allocation. Even when the trade-to-GDP ratio is 35 percent, there are virtually no differences between the welfare costs of business cycle under the Ramsey-optimal policy and the optimized rules (see Table 6).

Our results echo the finding in Benigno and Benigno (2003), who show that when aggregate shocks are perfectly correlated across countries there are no gains from coordinating policies.\textsuperscript{45} In our model, increased trade integration results (endogenously) in stronger business cycle comovement. Thus, inward-looking interest rate rules can still replicate the constrained efficient allocation and the need of cooperation remains muted. Put different, when stronger trade linkages result into plausible business cycle synchronization, there is no shift in the focus of monetary stabilization to redressing domestic as well as external distortions, i.e., trade integration does not require targeting rules involving misalignments in the terms of trade or cross-country demand imbalances.

A question remains open: what are the consequences of trade integration when monetary policy is not optimally designed? Table 6 shows that historical (Fed) policy behavior results in more sizable welfare costs relative to the pre-integration scenario: The welfare gains from implementing the Ramsey-optimal policy relative to historical policy making increase from 18 percent (with high trade costs) up to 30 percent. To understand this result, recall that historical policy results in suboptimal unemployment dynamics in each country, inducing inefficient fluctuations in terms

\textsuperscript{44} Notice that the Ramsey-optimal policy generates a positive comovement of employment and investment across countries when trade linkages are strong (it was negative in the pre-integration scenario). Intuitively, when cross-country demand linkages are strong, shifting resources toward the relatively more productive economy is more costly for the Ramsey authority.

\textsuperscript{45} When productivity shocks are perfectly correlated across countries, the optimal cooperative policy in Benigno and Benigno (2003) dictates a flexible exchange rate and domestic price stability. Notice that their model features Walrasian labor markets and flexible wages.
of trade and cross-country demand. For example, Figure 2 shows that following an increase in Home productivity, terms of trade depreciation is too weak relative to the constrained efficient allocation since the Home economy expands production beyond its efficient level. When trade linkages are strong, sub-optimal terms-of-trade fluctuations combine with incomplete risk sharing across countries, resulting in inefficient international spillovers and larger welfare costs of historical policy.

To summarize, our analysis has two main implications for the conduct of monetary policy following trade integration. First, stronger trade linkages do not make monetary policy cooperation necessarily more desirable. Provided that central banks appropriately use inflation to smooth domestic unemployment fluctuations, inward-looking interest rate rules (and a flexible exchange rate) are optimal. However, sub-optimal inward-looking policies (such as a policy of price stability), become more costly when trade linkages are stronger: The increase in comovement is not sufficient to offset the negative consequences of (inefficient) international spillovers.

**Robustness**

Our analysis assumed complete exchange rate pass-through and abstracted from strategic considerations in monetary policy setting. The sole international distortions we have considered so far is the lack of efficient risk sharing between Home and Foreign households. In the data, however, exchange rate pass-through is far from complete and monetary policy can involve strategic currency devaluations. As a result, our benchmark model could underestimate the importance of external distortions for the optimal conduct of monetary policy. We turn to these issues next, investigating the robustness of our findings to the presence of local currency pricing (LCP) and non-cooperative monetary policy setting.

**Local Currency Pricing**

Under LCP, firms set prices in domestic currency for the domestic market, and in foreign currency for the market of destination. As a result, nominal exchange rate movements cannot be expected to have expenditure switching effects: Nominal depreciation does not make goods produced in the country cheaper worldwide, thus re-allocating demand in favor of them.\(^{46}\) In the Appendix, we show that the only difference between producer and local currency pricing involves the determination of

\(^{46}\)With LCP, the exchange rate pass-through is zero and nominal depreciation raises the local-currency revenue from selling goods abroad at an unchanged price.
the export markup. Under LCP, the optimal export markup, $\mu_{x,t} \equiv \frac{\phi}{(\phi - 1)\Xi_{x,t}}$, is now defined by:

$$\Xi_{x,t} \equiv \left(1 - \frac{\nu^2}{2} \pi_{x,t}\right)\nu (\pi_{x,t} + 1) \pi_{x,t} - \frac{\nu}{(\phi - 1)} E_t \left[\beta_{t+1} (\pi_{x,t+1} + 1) \pi_{x,t+1} \frac{Y_{x,t+1}}{Y_{x,t}}\right],$$

where:

$$\frac{1 + \pi_{x,t}}{1 + \pi_{x,t}^*)} = \frac{\tilde{\rho}_{x,t}}{\tilde{\rho}_{x,t-1}} \left(\frac{N_{x,t}}{N_{x,t-1}}\right)^\frac{1}{1-\pi^*}.$$

How does LCP affect the policy tradeoffs faced by the Ramsey authority? A well-known theoretical result in the literature is that incomplete pass-through makes it is impossible to simultaneously stabilize domestic and export markups since the law of one price does not hold. The optimal-policy prescription is that policymakers should pay attention to international relative price misalignments, as the exchange rate cannot be expected to correct them. In our model, however, the law of one price does not hold regardless of the currency denomination of exported goods. As a result, LCP does not introduce new policy tradeoffs for the Ramsey authority, but it modifies their nature with respect to PCP.

As shown by Figure 3, when trade linkages are weak, the optimal policy continues to stabilize unemployment fluctuations, generating higher domestic markups volatility in the relatively more productive economy. The international transmission of aggregate shocks differ under LCP and PCP. Under LCP there is no expenditure switching towards Home good and the increase of Home demand for Foreign goods is strong enough to increase the present discounted value of entry in Foreign. As a result, differently from PCP, the optimal policy induce a positive comovement between employment and investment across countries. Nevertheless, Table 6 shows that the welfare costs of historical policy under PCP and LCP remain very close, and the optimized inward-looking interest rate rule obtained under PCP continue to approximate well the Ramsey allocation (see Table 7). These results are not surprising since differences in the international transmission of aggregate shocks are expected to have second-order welfare implications when trade linkages are weak.

The key finding, instead, is that the optimized interest rate rules that we derived under PCP and weak trade linkages continue to be optimal after trade integration. Intuitively, provided that

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47 For simplicity we assume that all the producers set export prices in Foreign currency. The model could be easily extended to allow for an exogenous partition of firms operating under PCP and LCP.

48 Devereux and Engle (2003) have shown that a fixed exchange is part of the optimal policy when price stickyness is the only distortion in the economy, shocks are efficient and PPP holds. In general, however, the presence of local currency pricing does not motivate a complete stabilization of the nominal exchange rate under the optimal policy, and volatility can remain quite high (even if lower than under PCP).

49 Even under PCP, stabilization of marginal cost of domestic producers does not coincides with markups stabilization in all markets.
each central bank responds appropriately to movements in price and wage inflation, business cycle synchronization offsets international distortions: When shocks are more global, asymmetries in the dynamics of domestic and export markups are reduced and the need to correct for real exchange rate misalignment and cross-country misallocation in consumption correspondingly mitigated.

Table 6 also shows that sub-optimal domestic stabilization continues to be costly in terms of welfare. Importantly, the welfare costs are larger under LCP compared to what observed in the presence of PCP as the relative welfare loss under historical policy is now doubled compared to the pre-integration scenario. As shown by Figure 4, historical policy implies that Home terms of trade do not depreciate enough in response to an increase in Home productivity due the lack of unemployment stabilization. Since the optimal depreciation engineered by the Ramsey authority is larger under LCP relative to PCP, historical policy becomes more costly.

Non-Cooperative Monetary Policy

We now investigate how strategic considerations affect the conduct of monetary policy in the presence of trade integration. As common practice in the literature, we consider two self-oriented central banks that set monetary policy to maximize the welfare of domestic consumers. The Home central bank maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(C_t) - l_t v(h_t)].$$ (20)

(The Foreign central bank maximizes an analogous welfare criterion.)

To characterize the non-cooperative allocation, we need to specify the strategic game. Following Benigno and Benigno (2006), each policymaker’s strategy is specified in terms of each country’s consumer price in inflation rate, $\pi_{C,t}$, as a function of the sequence of shocks, taking as given the sequence of the other country’s consumer price inflation rates (two-country, open-loop Nash equilibrium). Formally, let $\{\Lambda_{1,t}, ..., \Lambda_{23,t}\}_{t=0}^{\infty}$ be the Lagrange multiplier associated to the equilibrium conditions in Table 1 (once again excluding the two interest-rate setting rules). The Home central bank chooses:

$${\{C_t, C^*_t, \bar{\rho}_{d,t}, \bar{\rho}^*_{d,t}, l_t^*, h_t^*, V_t^*, V^*_t, N_{d,t}^*, N_{d,t}^*, w_t / P_t, w^*_t / P^*_t, z_{x,t}, \bar{z}_{x,t}, \bar{z}^*_{x,t}, \pi_{w,t}, \pi_{C,t}, i_{t+1}, a_{t+1}, a^*_{t+1}, Q_t, \Lambda_{1,t}, ..., \Lambda_{23,t}\}_{t=0}^{\infty}}$$

$$\pi^*_{w,t}, \pi_{C,t}, i_{t+1}, a^*_{t+1}, a^*_{t+1}, Q_t, \Lambda_{1,t}, ..., \Lambda_{23,t}$$

to maximize equation (20), taking as given $\left\{\pi^*_{C,t}\right\}_{t=0}^{\infty}$. The central in Bank in Foreign solves an
analogous maximization problem, taking as given \( \{ \pi_{C,t} \}_{t=0}^{\infty} \).

In a Nash equilibrium, domestic policymakers have an incentive to manipulate their country’s terms of trade, resulting into inefficient exchange rate volatility relative to the constrained efficient benchmark of policy cooperation. Table 6 shows that when trade linkages are weak, the welfare loss associated to the non-cooperative outcome is very modest (almost 0 percent, regardless of the assumptions about the currency denomination of export). Intuitively, weak trade linkages imply that each policymaker has no incentives to manipulate terms of trade.

Stronger trade linkages do not significantly change this conclusion. Table 6 shows that the welfare costs of non-cooperative monetary policy relative to the Ramsey-optimal allocation reach at most 2 percent. Once again, this result is explained by the large increase in comovement induced by trade integration (see Table 7): business cycle synchronization reduces the incentives to manipulate terms of trade since shocks become more global.

**Trade Integration and Fixed Exchange Rates**

Our analysis has shown that optimized inward-looking interest rate rules together with a flexible exchange rate can mimic the constraint efficient allocation regardless of the intensity of trade linkages. To conclude our analysis, we investigate what are the consequences of an exchange rate peg in the presence of trade integration.

We model a fixed exchange rate regime by assuming that Home (assumed to be the “leader” or “center” country) sets its policy instrument following the historical rule described in (18). Foreign (the “follower” or “periphery”) commits instead to the following rule:

\[
i_t^* = i_t + S_t^\phi_S,
\]

with \( \phi_S < 0 \). As shown by Benigno, Benigno, and Ghironi (2007), the credible threat to increase (decrease) the interest rate if the nominal exchange rate depreciates (appreciates) implies that the exchange rate does not move and results endogenously in interest rate equalization across countries.

A fixed exchange rate introduces an additional distortion in the market economy, since now the adjustment of international relative prices in the model is summarized by the condition that ties real exchange rate dynamics to relative inflation in consumer price indexes: 

\[
Q_t/Q_{t-1} = \left(1 + \pi_{C,t}^*\right)/(1 + \pi_{C,t}).
\]

A policy of fixed exchange rate distorts this margin of adjustment by
removing adjustment through the nominal exchange rate. Unfortunately, this distortion cannot be summarized by an analytically defined wedge relative to the planner’s optimum, because the planned economy does not feature nominal rigidity.

Table 6 shows that when trade linkages are weak, an exchange rate peg is significantly more costly for the follower country. Intuitively, absent strong trade linkages, a fixed exchange rate regime is costly for the follower since business cycle synchronization between center and periphery is relatively low, resulting in poor stabilization of aggregate fluctuations in the country that pegs the exchange rate.

Stronger trade linkages, however, do not make a fixed exchange rate more desirable for the follower. As shown in Table 6, the welfare loss from a peg is either unaffected or it slightly increases, depending on the alternative model specifications that we considered. This result is explained by the offsetting effects of stronger business cycle synchronization and lack of domestic stabilization in the center. Increased comovement reduces the cost of the peg for the follower country since the leader’s monetary policy is less destabilizing when shocks have more global consequences. However, historical policy does not stabilize aggregate fluctuations efficiently, and stronger trade linkages also result in inefficient terms-of-trade fluctuations which are costly for both center and periphery.

8 Conclusions

We re-examined classic questions on trade integration and international monetary policy using a dynamic, stochastic, general equilibrium model with micro-level trade dynamics and labor market frictions. We have shown that departures from price stability are optimal in the long run and over the business cycle in an environment of low trade integration, but trade-induced productivity gains reduce the need of positive inflation to correct long-run distortions. Over the business cycle, trade integration results in larger benefits from cooperation relative to historical policy behavior, but optimized inward-looking policy rules can still approximate the cooperative outcome. The increase in business cycle synchronization across countries that is generated by trade integration is a key reason why gains from cooperation are small relative to optimal non-cooperative behavior. Increased comovement per se makes an exchange rate peg more desirable for the pegger. However, if the center country follows historical Federal Reserve behavior, this generates inefficient spillovers with strong trade linkages, offsetting the gain from increased comovement.

50 With flexible exchange rates, it would be $\frac{Q_t}{Q_{t-1}} = (1 + \pi_{C,t}^*) S_t / [(1 + \pi_{C,t}) S_{t-1}]$. As we already know, a fixed nominal exchange rate is never optimal in the model.
Much remains to be done in this area of research. We modeled trade integration as an exogenous reduction in tariffs (iceberg trade costs), but trade integration may also take the form of lower fixed costs of trade. Moreover, we did not analyze optimal trade policy nor its potentially strategic interdependence with monetary policymaking.\footnote{An interesting contribution in this vein, using non-microfounded tools, is Basevi, Delbono, and Denicolo’ (1990).} We view these as important, promising areas where to take this research next.

References


\[1 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[2 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[3 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[4 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[5 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[6 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[7 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[8 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[9 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[10 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[11 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[12 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[13 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[14 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[15 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[16 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[17 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

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\[20 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[21 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[22 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[23 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[24 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]

\[25 = \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \tilde{\beta}_{t-1}^* \tilde{\beta}_{t+1}^* \]
### TABLE 2: SOCIAL PLANNER

\[ 1 = \tilde{\rho}_{d,t}^{1-\theta} N_{d,t} + \tilde{\rho}_{x,t}^{1-\theta} N_{x,t} \]  
(1)

\[ 1 = \tilde{\rho}_{d,t}^{1-\theta} N_{d,t} + \tilde{\rho}_{x,t}^{1-\theta} N_{x,t} \]  
(2)

\[ \rho_{x,t} N_{x,t} Y_{x,t}^{s,C} = k_{p} \rho_{x}^{(\theta-1)} f_{x,t} \]  
(3)

\[ \rho_{x,t} N_{x,t} Y_{x,t}^{s,C} = k_{p} \rho_{x}^{(\theta-1)} f_{x,t} \]  
(4)

\[ l_{t} = N_{d,t} \frac{\hat{y}_{x,t}}{Z_{d,t}} + N_{x,t} \frac{\hat{y}_{x,t}}{Z_{x,t}} r_{t} + N_{e,t} f_{x,t} Z_{t} + N_{x,t} f_{x,t} Z_{t} \]  
(5)

\[ l_{t}^{*} = N_{d,t} \frac{\hat{y}_{x,t}}{Z_{d,t}} + N_{x,t} \frac{\hat{y}_{x,t}}{Z_{x,t}} r_{t} + N_{e,t} f_{x,t} Z_{t} + N_{x,t} f_{x,t} Z_{t} \]  
(6)

\[ l_{t} = (1 - \lambda) l_{t-1} + q_{t-1} V_{t-1} \]  
(7)

\[ l_{t}^{*} = (1 - \lambda) l_{t-1}^{*} + q_{t-1}^{*} V_{t-1} \]  
(8)

### TABLE 3: DISTORTIONS

\[ Y_{\mu_{d,t}} \equiv \frac{\mu_{d,t}}{\mu_{d,t-1}} - 1 \]  
time varying domestic markups, product creation

\[ Y_{\mu_{x,t}} \equiv \frac{\mu_{x,t}}{\mu_{d,t}} - 1 \]  
time varying export markups, product creation

\[ Y_{\varphi_{t}} \equiv 1 - 1 \]  
monopoly power, job creation and labor supply

\[ Y_{\eta_{t}} \equiv \eta_{t} - \varepsilon \]  
failure of the Hosios condition*, job creation

\[ Y_{b_{t}} \equiv b \]  
unemployment benefits, job creation

\[ Y_{Q_{t}} \equiv \frac{u_{z_{t}}^{*}}{u_{G_{t}}} - Q_{t} \]  
incomplete markets, risk sharing

\[ Y_{a_{t}} \equiv \psi a_{t+1} + \psi a_{s,t+1} \]  
cost of adjusting bond holdings, risk sharing

\[ Y_{\pi_{w,t}} \equiv \rho_{w,t}^{2} \]  
wage adjustment costs, resource constraint and job creation

\[ Y_{\pi_{d,t}} \equiv \rho_{d,t}^{2} \]  
domestic price adjustment costs

\[ Y_{\pi_{x,t}} \equiv \rho_{x,t}^{2} \]  
export price adjustment costs

* From sticky wages and/or \( \eta \neq \varepsilon \).
### TABLE 4: CALIBRATION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Aversion</td>
<td>$\gamma_C = 1$</td>
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<tr>
<td>Frisch elasticity</td>
<td>$1/\gamma_h = 0.4$</td>
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<tr>
<td>Discount Factor</td>
<td>$\beta = 0.99$</td>
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<tr>
<td>Elasticity Matching Function</td>
<td>$\varepsilon = 0.4$</td>
</tr>
<tr>
<td>Firm Bargaining Power</td>
<td>$\eta = 0.4$</td>
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<tr>
<td>Home Production</td>
<td>$b = 0.54$</td>
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<tr>
<td>Exogenous separation</td>
<td>$\lambda = 0.10$</td>
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<tr>
<td>Vacancy Cost</td>
<td>$\kappa = 0.16$</td>
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<tr>
<td>Matching Efficiency</td>
<td>$\chi = 0.68$</td>
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<tr>
<td>Elasticity of Substitution</td>
<td>$\theta = 3.8$</td>
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<td>Plant Exit</td>
<td>$\delta = 0.026$</td>
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<tr>
<td>Pareto Shape</td>
<td>$k_p = 3.4$</td>
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<td>Pareto Support</td>
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<td>Sunk Entry Cost</td>
<td>$f_e = 0.69$</td>
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<tr>
<td>Fixed Export Costs</td>
<td>$f_x = 0.005$</td>
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<tr>
<td>Iceberg Trade Costs</td>
<td>$\tau = 1.75$</td>
</tr>
<tr>
<td>Rotemberg Wage Adj. Cost</td>
<td>$\vartheta = 60$</td>
</tr>
<tr>
<td>Rotemberg Price Adj. Cost</td>
<td>$\nu = 80$</td>
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<tr>
<td>Taylor - Interest Rate Smoothing</td>
<td>$\varphi_i = 0.71$</td>
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<tr>
<td>Taylor - Inflation Parameter</td>
<td>$\varphi_i = 1.62$</td>
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<tr>
<td>Taylor - Output Gap Parameter</td>
<td>$\varphi_Y = 0.34$</td>
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<tr>
<td>Bond Adjustment Cost</td>
<td>$\psi = 0.0025$</td>
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</table>

### TABLE 5: TRADE INTEGRATION – NON STOCHASTIC STEADY STATE

<table>
<thead>
<tr>
<th>$\frac{\text{Trade}}{\text{GDP}}$</th>
<th>Ramsey Gain</th>
<th>Ramsey Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.34%</td>
<td>1.40%</td>
</tr>
<tr>
<td>0.2</td>
<td>0.22%</td>
<td>1.20%</td>
</tr>
<tr>
<td>0.35</td>
<td>0.16%</td>
<td>1.05%</td>
</tr>
</tbody>
</table>
### TABLE 6: TRADE INTEGRATION – NON STOCHASTIC STEADY STATE

#### Relative Gain from Coordination — PCP

<table>
<thead>
<tr>
<th>Trade GDP</th>
<th>Optimal Rule</th>
<th>Historical Rule</th>
<th>Peg</th>
<th>Nash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade GDP = 0.1</td>
<td>0.88%</td>
<td>18.62%</td>
<td>18.81%</td>
<td>43.45%</td>
</tr>
<tr>
<td>Trade GDP = 0.2</td>
<td>3.13%</td>
<td>25.36%</td>
<td>26.90%</td>
<td>45.40%</td>
</tr>
<tr>
<td>Trade GDP = 0.35</td>
<td>3.15%</td>
<td>29.69%</td>
<td>32.31%</td>
<td>48.39%</td>
</tr>
</tbody>
</table>

#### Relative Gain from Coordination — LCP

<table>
<thead>
<tr>
<th>Trade GDP</th>
<th>Optimal Rule</th>
<th>Historical Rule</th>
<th>Peg</th>
<th>Nash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade GDP = 0.1</td>
<td>2.17%</td>
<td>20.91%</td>
<td>20.89%</td>
<td>44.90%</td>
</tr>
<tr>
<td>Trade GDP = 0.2</td>
<td>2.66%</td>
<td>29.09%</td>
<td>29.49%</td>
<td>47.34%</td>
</tr>
<tr>
<td>Trade GDP = 0.35</td>
<td>3.16%</td>
<td>36.16%</td>
<td>37.00%</td>
<td>51.97%</td>
</tr>
</tbody>
</table>

Note: gains are the percentage reduction in welfare costs of business cycle under the Ramsey-optimal policy.

### TABLE 7: TRADE INTEGRATION AND GDP COMOVEMENT

#### $\Delta corr(Y_{R,t}, Y_{R,t}^{s})$—Producer Currency Price

<table>
<thead>
<tr>
<th>Trade GDP</th>
<th>Historical Rule</th>
<th>Peg</th>
<th>Ramsey</th>
<th>Nash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade GDP = 0.1</td>
<td>0.36</td>
<td>0.05</td>
<td>0.07</td>
<td>0.28</td>
</tr>
<tr>
<td>Trade GDP = 0.2</td>
<td>0.45</td>
<td>0.19</td>
<td>0.29</td>
<td>0.35</td>
</tr>
<tr>
<td>Trade GDP = 0.35</td>
<td>0.49</td>
<td>0.27</td>
<td>0.43</td>
<td>0.48</td>
</tr>
</tbody>
</table>

#### $corr(Y_{R,t}, Y_{R,t}^{s})$—Local Currency Price

<table>
<thead>
<tr>
<th>Trade GDP</th>
<th>Historical Rule</th>
<th>Peg</th>
<th>Ramsey</th>
<th>Nash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade GDP = 0.1</td>
<td>0.33</td>
<td>0.05</td>
<td>0.36</td>
<td>0.28</td>
</tr>
<tr>
<td>Trade GDP = 0.2</td>
<td>0.42</td>
<td>0.20</td>
<td>0.53</td>
<td>0.36</td>
</tr>
<tr>
<td>Trade GDP = 0.35</td>
<td>0.47</td>
<td>0.27</td>
<td>0.62</td>
<td>0.42</td>
</tr>
</tbody>
</table>
Figure 1: Home Productivity Shock, no trade linkages and producer currency pricing.

Variables are in percentage deviations from the steady state. Unemployment and inflation are in deviations from the steady state.
Figure 2: Home Productivity Shock, trade integration and producer currency pricing.

Variables are in percentage deviations from the steady state. Unemployment and inflation are in deviations from the steady state.
Figure 3: Home Productivity Shock, no trade linkages and local currency pricing.

Variables are in percentage deviations from the steady state. Unemployment and inflation are in deviations from the steady state.
Figure 4: Home Productivity Shock, trade integration and local currency pricing.

Variables are in percentage deviations from the steady state. Unemployment and inflation are in deviations from the steady state.
Appendix

A Wage Determination

The nominal wage is the solution of an individual Nash bargaining process, and the wage payment divides the match surplus between workers and firms. Due to the presence of nominal rigidities, we depart from the standard Nash bargaining convention by assuming that bargaining occurs over the nominal wage payment rather than the real wage payment.\textsuperscript{52} With zero costs of nominal wage adjustment ($\vartheta = 0$), the real wage that emerges would be identical to the one obtained from bargaining directly over the real wage. This is no longer the case in the presence of adjustment costs.

Let $J_t$ be the real value of an existing, productive match for a producer, determined by:

$$J_t = \varphi_t Z_t h_t - \frac{w_t}{P_t} h_t - \vartheta \frac{2}{\pi} \pi_{w,t} + E_t \beta_{t,t+1}(1 - \lambda) J_{t+1}. \tag{21}$$

Intuitively, $J_t$ is the per period marginal value product of the match, $\varphi_t Z_t h_t$, net of the wage bill and costs incurred to adjust wages, plus the expected discounted continuation value of the match in the future.\textsuperscript{53}

Next, denote with $W_t$ the worker’s asset value of being matched, and with $U_{u,t}$ the value of being unemployed. The value of being employed at time $t$ is given by the real wage bill the worker receives plus the expected future value of being matched to the firm. With probability $1 - \lambda$ the match will survive, while with probability $\lambda$ the worker will be unemployed. As a result:

$$W_t = \frac{w_t}{P_t} h_t + E_t \left\{ \beta_{t,t+1} \left[ (1 - \lambda) W_{t+1} + \lambda U_{u,t+1} \right] \right\}. \tag{22}$$

The value of unemployment is given by:

$$U_{u,t} = \frac{v(h_t)}{u_{C,t}} + b + E_t \left\{ \beta_{t,t+1} \left[ t_t W_{t+1} + (1 - t_t) U_{u,t+1} \right] \right\}. \tag{23}$$

In this expression, $v(h_t)/u_{C,t}$ is the utility gain from leisure in terms of consumption, $b$ is an un-
employment benefit from the government (financed with lump sum taxes), and \( \iota_t \) is the probability of becoming employed at time \( t \), equal to the ratio between the total number of matches and the total number of workers searching for jobs at time \( t \): \( \iota_t \equiv M_t / U_t \).

Equations (22) and (23) imply that the worker’s surplus \( H_t \equiv W_t - U_{w,t} \) is determined by:

\[
H_t = \frac{w_t}{P_t} h_t - \left( \frac{v(h_t)}{u_{C,t}} + b \right) + (1 - \lambda - \iota_t) E_t \left( \beta_{t,t+1} H_{t+1} \right).
\]

(24)

Nash bargaining maximizes the joint surplus \( J_t^0 H_t^{1-\eta} \) with respect to \( w_t \), where \( \eta \in (0, 1) \) is the firm’s bargaining power. The first-order condition implies:

\[
\eta H_t \frac{\partial J_t}{\partial w_t} + (1 - \eta) J_t \frac{\partial H_t}{\partial w_t} = 0,
\]

(25)

where:

\[
\frac{\partial J_t}{\partial w_t} = -\frac{h_t}{P_t} - v \frac{\pi_{w,t}}{w_{t-1}} + (1 - \lambda) \partial E_t \left[ \beta_{t,t+1} (1 + \pi_{w,t+1}) \frac{\pi_{w,t+1}}{w_t} \right],
\]

(26)

and:

\[
\frac{\partial H_t}{\partial w_t} = \frac{h_t}{P_t}.
\]

(27)

The sharing rule can then be rewritten as:

\[
\eta_t H_t = (1 - \eta_t) J_t,
\]

(28)

where:

\[
\eta_t = \frac{\eta}{\eta - (1 - \eta) \left( \frac{\partial H_t}{\partial w_t} / \frac{\partial J_t}{\partial w_t} \right)}.
\]

(29)

Equation (28) shows that, as in Gertler and Trigari (2009), bargaining shares are time-varying due to the presence of wage adjustment costs. Absent wage adjustment costs, we would have \( \partial J_t / \partial w_t = -\partial H_t / \partial w_t \) and a time-invariant bargaining share \( \eta_t = \eta \).

Equation (4) in the main text for the bargained wage implies that the value of a match to a producer can be rewritten as:

\[
J_t = \eta_t \left[ \varphi_t Z_t h_t - \frac{v(h_t)}{u_{C,t}} + b \right] + E_t \left\{ \beta_{t,t+1} J_{t+1} \left[ (1 - \lambda) \eta_t + (1 - \lambda - \iota_t) (1 - \eta_{t+1}) \frac{\eta_t}{\eta_{t+1}} \right] \right\}.
\]

(30)

The second term in the right-hand side of this equation reduces to \([1 - \lambda - (1 - \eta) \iota_t] E_t (\beta_{t,t+1} J_{t+1})\)
when wages are flexible. The firm’s equilibrium surplus is the share $\eta$ of the marginal revenue product generated by the worker, net of wage adjustment costs and the worker’s outside option, plus the expected discounted future surplus, adjusted for the probability of continuation, $1 - \lambda$, and the portion appropriated by the worker, $(1 - \eta) \epsilon_t$. Sticky wages again introduce an effect of expected changes in the endogenous bargaining shares.

B Pricing Decisions

Producer Currency Pricing

Each final producer sets $P_{d,t}$ and the domestic currency price of the export bundle, $P_{x,t}^h$, letting the price in the foreign market be $P_{x,t} = \tau_t P_{x,t}^h / S_t$, where $S_t$ is the nominal exchange rate. When selling at Home, the producer chooses $P_{d,t}$ to maximize the present discounted value of the stream of domestic profits $d_{d,t}$:

$$d_{d,t} = E_t \sum_{s=t}^{\infty} (1 - \delta)^{s-t} \beta_{t,t+s} \left[ \left( \frac{P_{d,s}}{P_s} \right)^{1-\phi} \left( 1 - \frac{\nu}{2} \left( \frac{\pi_h}{\pi_{d,s}} \right)^2 \right) Y_s^C - \frac{P_{d,s}^y}{P_s} \left( \frac{P_{d,s}}{P_s} \right)^{-\phi} Y_s^C \right].$$

The first order condition for $P_{d,t}$ yields:

$$\frac{P_{d,t}}{P_t} = \frac{\phi}{(\phi - 1) \Xi_{d,t}} \frac{P_{d,t}^y}{P_t},$$

where $\Xi_{d,t}$ is given by:

$$\Xi_{d,t} = \left( 1 - \frac{\nu}{2} \beta_{d,t} \right) \nu (\pi_{d,t} + 1) \pi_{d,t} - \frac{\nu}{(\phi - 1)} E_t \left[ \beta_{t,t+1} (\pi_{d,t+1} + 1) \pi_{d,t+1} \frac{Y_{d,t+1}}{Y_{d,t}} \right].$$

To determine $P_{x,t}^h$, first notice that the export demand, defined by equation (??), can be written as:

$$Y_{x,t} = \left( \frac{P_{x,t}}{Q_t P_t} \right)^{-\phi} Y_t^{C*},$$

where we have used the fact that $P_{x,t} / P_t^* = P_{x,t}^h / (Q_t P_t)$. The expected present discounted value of the stream of real export profits, $d_{x,t}$, is given by:

$$d_{x,t} = E_t \sum_{s=t}^{\infty} (1 - \delta)^{s-t} \beta_{t,t+s} \left[ \left( \frac{P_{x,s}^h}{P_s} \right) \left( \frac{P_{x,s}^h}{Q_s P_s} \right)^{-\phi} \left( 1 - \frac{\nu}{2} \left( \frac{\pi_h}{\pi_{x,s}} \right)^2 \right) Y_s^{C*} - \frac{P_{x,s}^y}{P_s} \left( \frac{P_{x,s}^h}{Q_s P_s} \right)^{-\phi} Y_s^{C*} - N_{x,t} \right].$$

A-3
where

\[ 1 + \pi_{x,t}^h = \frac{P_{x,t}^h}{P_{x,t-1}^h} = \frac{S_t}{S_{t-1}} \frac{P_{x,t}}{P_{x,t-1}}. \]

The first order condition with respect to \( P_{x,t}^h \) implies

\[
\frac{P_{x,t}^h}{P_t} = \frac{\phi}{(\phi - 1)} \frac{P_{y,t}^y}{P_t} = \xi_{x,t}^h \quad (34)
\]

where \( \xi_{x,t}^h \) is defined by:

\[
\xi_{x,t}^h = \left( 1 - \frac{\nu}{2} \pi_{x,t}^2 \right) \nu \left( \pi_{x,t}^h + 1 \right) \pi_{x,t}^h - \frac{\nu}{(\phi - 1)} E_t \left[ \beta_{t,t+1} \left( \pi_{x,t+1}^h + 1 \right) \pi_{x,t+1}^h \frac{Y_{x,t+1}^h}{Y_{x,t}} \right]. \quad (35)
\]

Equation (34) can be rearranged to obtain the expression in the text:

\[
\frac{P_{x,t}}{P_t^*} = \frac{\phi}{(\phi - 1)} \frac{P_{y,t}^y}{P_t} Q_t P_t.
\]

**Local Currency Pricing**

Equation (31) still determines the domestic price \( P_{x,t} \). However, when the export price is set in foreign currency, each producer chooses \( P_{x,t} \) to maximize:

\[
d_{x,t} = E_t \sum_{s=t}^\infty \beta_{t,t+s} \left[ Q_t \left( \frac{P_{x,t}^h}{P_s} \right)^{1-\phi} \left[ 1 - \frac{\nu}{2} (\pi_{x,t})^2 \right] \pi_{x,t}^h - \frac{\nu}{(\phi - 1)} E_t \left[ \beta_{t,t+1} \left( \pi_{x,t+1}^h + 1 \right) \pi_{x,t+1}^h \frac{Y_{x,t+1}^h}{Y_{x,t}} \right] \right]. \quad (36)
\]

where \( \pi_{x,t} = (P_{x,t}/P_{x,t-1}) - 1 \). \(^{54}\) The first order condition with respect to \( P_{x,t} \) implies:

\[
\frac{P_{x,t}}{P_t^*} = \frac{\phi}{(\phi - 1)} \frac{P_{y,t}^y}{P_t} Q_t P_t,
\]

where \( \xi_{x,t} \) is now given by:

\[
\xi_{x,t} = \left( 1 - \frac{\nu}{2} \pi_{x,t}^2 \right) \nu (\pi_{x,t} + 1) \pi_{x,t} - \frac{\nu}{(\phi - 1)} E_t \left[ \beta_{t,t+1} \left( \pi_{x,t+1} + 1 \right) \pi_{x,t+1} \frac{Y_{x,t+1}}{Y_{x,t}} \right].
\]

\(^{54}\) Notice that under LCP the costs of adjusting the export price, expressed in units of Home currency, is given by \( \Gamma_{x,t} \equiv \nu \pi_{x,t}^2 S_t P_{x,t} Y_{x,t}/2 \).
C Equilibrium

The aggregate stock of employed labor in the Home economy is determined by \( l_t = (1 - \lambda)l_{t-1} + q_{t-1}V_{t-1} \). Wage inflation and consumer price inflation are tied by \( 1 + \pi_{w,t} = (w_t^r/w_{t-1}^r)(1 + \pi_{C,t}) \), where \( w_t^r \) denotes the real wage, \( w_t/P_t \), at time \( t \). Moreover, domestic and export price inflation are tied to consumer price inflation by:

\[
(1 + \pi_{w,t}) = \pi_{C,t},
\]

Aggregate demand of the consumption basket must be equal to the sum of market consumption, the costs of posting vacancies, and the costs of adjusting prices and wages:

\[
Y_t^C = C_t - h_p(1 - L_t) + \kappa V_t + \frac{\vartheta}{2}\pi_{w,t}^2 L_t + \Gamma_{d,t} + \Gamma_{x,t}^h.
\]

Labor market clearing requires:

\[
l_t h_t = \frac{N_{d,t}y_{d,t}}{Z_t \bar{z}_d} + \tau_t \frac{N_{x,t}y_{x,t}}{Z_t \bar{z}_{x,t}} + \frac{N_{e,t}f_{e,t}}{Z_t} + \frac{N_{x,t}f_{x,t}}{Z_t}.
\]

D The Law of Motion for Net Foreign Assets

Recall the representative household’s budget constraint:

\[
A_{t+1} + S_t A_{s,t+1} + \frac{\psi}{2} P_t \left( \frac{A_{t+1}}{P_t} \right)^2 + \frac{\psi}{2} S_t P_t^* \left( \frac{A_{s,t+1}}{P_t^*} \right)^2 + P_t C_t =
\]

\[
= (1 + i_t)A_t + (1 + i_t^*)S_t A_{s,t} + w_t L_t + P_t b(1 - l_t) + T_t^g + T_t^A + T_t^f + T_t^f.
\]

The budget constraint of the government implies:

\[
T_t^g = -P_t b(1 - l_t).
\]
Moreover,

\[ T_t^A = P_t \psi \left( \frac{A_t}{P_t} \right)^2 + S_t P_t \psi \left( \frac{A_{t+1}}{P_t^*} \right)^2, \]

\[ T_t^I = P_t \left( \varphi_t Z_t l_t h_t - \frac{w_t}{P_t} l_t h_t - \kappa V_t - \frac{i}{2} \pi_{w,t}^2 l_t \right), \]

and:

\[ T_t^f = \left( \frac{\mu_{d,t} - 1}{\mu_{d,t}} - \frac{\nu}{2} (\pi_{d,t})^2 \right) \tilde{p}_{d,t} N_{d,t} \bar{y}_{d,t} + Q_t \left( \frac{\mu_{x,t} - 1}{\mu_{x,t}} - \frac{\nu}{2} (\pi_{x,t})^2 \right) \tilde{p}_{x,t} N_{x,t} \bar{y}_{x,t} - \varphi_t (N_{x,t} f_{x,t} + N_{e,t} f_{e,t}). \]

Therefore:

\[ A_{t+1} + S_t A_{s,t+1} + P_t C_t = (1 + i_t) A_t + (1 + i_{t}^*) S_t A_{s,t} + P_t \varphi_t Z_t l_t h_t - P_t \kappa V_t - P_t \frac{\theta}{2} \pi_{w,t}^2 l_t + \left( \frac{\mu_{d,t} - 1}{\mu_{d,t}} - \frac{\nu}{2} (\pi_{d,t})^2 \right) \tilde{p}_{d,t} N_{d,t} \bar{y}_{d,t} + Q_t \left( \frac{\mu_{x,t} - 1}{\mu_{x,t}} - \frac{\nu}{2} (\pi_{x,t})^2 \right) \tilde{p}_{x,t} N_{x,t} \bar{y}_{x,t} - \varphi_t (N_{x,t} f_{x,t} + N_{e,t} f_{e,t}). \] (38)

(39)

It is possible to simplify the consolidated budget constraint of the economy further. Recall the expression for Home’s aggregate demand of the consumption basket:

\[ Y_t^C = C_t + \kappa V_t + \frac{\vartheta}{2} \pi_{w,t}^2 l_t + \frac{\nu}{2} \pi_{d,t}^2 \tilde{p}_{d,t} N_{d,t} \bar{y}_{d,t} + \frac{\nu}{2} \pi_{x,t}^2 \tilde{p}_{x,t} N_{x,t} \bar{y}_{x,t}. \]

After rearranging, equation (38) can be rewritten in real terms as:

\[ a_{t+1} + Q_t a_{s,t+1} = \frac{1 + i_t}{1 + \pi_{C,t}} a_t + Q_t \frac{1 + i_{t}^*}{1 + \pi_{C,t}^*} a_{s,t} + N_{d,t} \tilde{p}_{d,t} \bar{y}_{d,t} + Q_t N_{x,t} \tilde{p}_{x,t} \bar{y}_{x,t} - Y_t^C + \frac{\varphi_t Z_t l_t h_t}{\mu_{d,t}} N_{d,t} \bar{y}_{d,t} + \frac{Q_t \tilde{p}_{x,t}}{\mu_{x,t}} N_{x,t} \bar{y}_{x,t} - \varphi_t N_{x,t} f_{x,t} - \varphi_t N_{e,t} f_{e,t}. \] (40)

(41)

Recall that pricing equations imply:

\[ \frac{\tilde{p}_{d,t}}{\mu_{d,t}} = \frac{\varphi_t}{\bar{z}_d}, \quad \frac{Q_t \tilde{p}_{x,t}}{\mu_{x,t}} = \frac{\tau_t \varphi_t}{\bar{z}_{x,t}}, \]

and labor market clearing requires:

\[ l_t h_t = N_{d,t} \frac{\bar{y}_{d,t}}{Z_t \bar{z}_d} + N_{x,t} \frac{\bar{y}_{x,t}}{Z_t \bar{z}_{x,t}} \tau_t + N_{e,t} \frac{f_{e,t}}{Z_t} + N_{x,t} \frac{f_{x,t}}{Z_t}. \]

It follows that home’s net foreign assets entering period \( t + 1 \) are determined by the gross interest
income on the assets position entering period $t$ plus the difference between home’s total production and total demand (or absorption) of consumption:

$$a_{t+1} + Q_t a_{t+1} = \frac{1 + i_t}{1 + \pi_{C,t}} a_t + Q_t \frac{1 + i^*_t}{1 + \pi_{C,t}^*} a_{t+1} + N_{d,t} \tilde{y}_{d,t} + Q_t N_{x,t} \tilde{y}_{x,t} - Y_t^C. \quad (42)$$

A similar equation holds in Foreign:

$$a_{*,t+1} + \frac{1}{Q_t} a_{*,t+1} = \frac{1 + i^*_t}{1 + \pi_{C,t}^*} a_{*,t} + \frac{1 + i_t}{1 + \pi_{C,t}} a_{*,t+1} + N_{d,t}^* \tilde{y}_{d,t}^* + \frac{1}{Q_t} N_{x,t}^* \tilde{y}_{x,t}^* - Y_t^{*C}. \quad (43)$$

Now, multiply equation (43) by $\frac{1}{Q_t}$, subtract the resulting equation from (42) and use the bond market clearing conditions $a_{t+1} + Q_t a_{t+1}^* = 0 = a_{*,t+1} + Q_t a_{*,t+1}$ in all periods. It follows that:

$$a_{t+1} + Q_t a_{t+1}^* = \frac{1 + i_t}{1 + \pi_{C,t}} a_t + Q_t \frac{1 + i^*_t}{1 + \pi_{C,t}^*} a_{*,t+1} + \frac{1}{2} \left[ N_{d,t} \tilde{y}_{d,t} + Q_t N_{x,t} \tilde{y}_{x,t} - Q_t N_{d,t}^* \tilde{y}_{d,t}^* - N_{x,t}^* \tilde{y}_{x,t} - Y_t^C - Q_t Y_t^{*C} \right]. \quad (44)$$

This is the familiar result that net foreign assets depend positively on the cross-country differential in production of final consumption output and negatively on relative absorption.

Notice next that home absorption of consumption must equal absorption of consumption output from home firms and output from foreign firms:

$$Y_t^C = N_{d,t} \tilde{y}_{d,t} + N_{x,t} \tilde{y}_{x,t},$$

where we used the fact that $\rho_{x,t}^* = Q_t \rho_{d,t}^*$. Similarly,

$$Y_t^{*C} = N_{d,t}^* \tilde{y}_{d,t}^* + N_{x,t} \tilde{y}_{x,t}.$$

Substituting these results into equation (44) yields net foreign assets as a function of interest income on the initial asset position and the trade balance:

$$a_{t+1} + Q_t a_{t+1}^* = \frac{1 + i_t}{1 + \pi_{C,t}} a_t + Q_t \frac{1 + i^*_t}{1 + \pi_{C,t}^*} a_{*,t+1} + Q_t N_{x,t}^* \tilde{y}_{x,t} - N_{x,t} \tilde{y}_{x,t}.$$
E  Social Planner Allocation and Inefficiency Wedges

Planner Economy

The benevolent social planner chooses:

\[
\{C_t, C_t, l_t, l^*_t, h_t, h^*_t, V_t, V_t, Y_{d,t}, Y_{d,t}, Y_{x,t}, Y_{x,t}, Y_{x,t}, Y_{x,t}, N_{d,t+1}, N_{d,t+1}\}_{t=0}^{\infty}
\]

to maximize the welfare criterion (19) subject to six constraints (three for each economy). We assume that the productivity distribution \(G(z)\), sunk costs of product creation \(N_{e,t} f_{e,t}\), fixed export costs \(N_{x,t} f_{x,t}\), per-unit iceberg trade costs \(\tau_t\) and the cost of vacancy posting \(\kappa V_t\) are all features of technology—the technology for product and job creation—that characterizes also the planner’s environment.

The first constraint in the social planner’s problem is that intermediate inputs are used to produce final goods, create new product lines and pay for fixed export costs:

\[
Z_t l_t = \frac{1}{\phi} Y_{d,t} + \frac{1}{\phi} \tau_t Y_{x,t} + \left( \frac{N_{d,t+1}}{1 - \delta} - N_{d,t} \right) f_{e,t} + N_{x,t} f_{x,t},
\]

where \(N_{x,t}\) is defined by (8) in the text. We denote the Lagrange multiplier associated to the constraint (46) with \(\varpi_t\), which corresponds to the social marginal cost of producing an extra unit of intermediate output.

The second constraint is that total output can be used for consumption and vacancy creation:

\[
C_t + \kappa V_t = \left[ Y_{d,t}^{\frac{1}{\phi} - 1} + Y_{x,t}^{\frac{1}{\phi} - 1} \right]^{\frac{1}{\delta}}.
\]

The Lagrange multiplier associated to this constraint, \(\zeta_t\), represents the social marginal utility of consumption resources. In the social planner’s environment, \(Y_t^C = C_t + \kappa V_t\).

Finally, the third constraint is that the stock of labor in the current period is equal to the number of workers that were not exogenously separated plus previous period matches that become productive in the current period:

\[
l_t = (1 - \lambda) l_{t-1} + \chi(1 - l_{t-1})^{1-\varepsilon} V_{t-1}^c.
\]

The Lagrange multiplier associated to this constraint, \(\zeta_t\), denotes the real marginal value of a match to society.
The first-order condition for consumption implies that $\xi_t = u_{C,t}$. Defining the social real exchange rate as $Q_t \equiv \frac{\xi_t}{\xi_t}$, the planner’s outcome is characterized by optimal risk sharing: $Q_t = \frac{u_{C,t}}{u_{C,t}}$.

The demand schedules for Home output are obtained by combining the first-order conditions with respect to $\varphi_d$, $\varphi_x$, $\varphi_d^*$ and $\varphi_x^*$:

$$Y_{d,t} = \left[\left(\frac{\omega_t}{z_d} N_{d,t} \frac{1}{1-\varphi} \right)^{-\varphi} Y_t^C \right], \quad Y_{x,t} = \left[\left(\frac{\omega_t \tau_t}{z_x\xi_t^*} N_{x,t} \frac{1}{1-\varphi} \right)^{-\varphi} Y_t^C \right].$$

(49)

To facilitate the comparison between planned and market economy, we define the following relative prices for the planner’s equilibrium: $\tilde{\rho}_{d,t} \equiv \omega_t / (\bar{z}_d \xi_t)$ and $\tilde{\rho}_{x,t} \equiv (\tau_t \omega_t) / (\bar{z}_x \xi_t^*)$. Analogous definitions hold for Foreign. Using the results in (49) and the analogs for Foreign output, it is possible to re-write equation (47) as:

$$1 = \tilde{\rho}_{d,t}^{1-\varphi} N_{d,t}^{1-\varphi} + \tilde{\rho}_{x,t}^{1-\varphi} N_{x,t}^{1-\varphi}.$$

The first-order condition for the average export-productivity, $\bar{z}_{x,t}$, implies:

$$\tau_t Y_{x,t} N_{x,t}^{1-\varphi} \left[-\frac{1}{\bar{z}_{x,t}} + \frac{k_p}{(\theta - 1)\bar{z}_{x,t}^2} \right] - \frac{k_p}{\bar{z}_{x,t}} f_{x,t} = 0.$$

Using $Y_{x,t} = \tilde{\rho}_{x,t}^{1-\varphi} N_{x,t}^{1-\varphi} Y_t^C$ we can rearrange the above expression, obtaining:

$$\tilde{\rho}_{x,t}^{1-\varphi} N_{x,t}^{1-\varphi} Y_t^C = \frac{(\theta - 1)k_p}{[k_p - (\theta - 1)]} \bar{z}_{x,t} f_{x,t}.$$

The optimality condition for $N_{d,t+1}$ equates the cost of creating a new product to its expected discounted benefit:

$$f_{c,t} = \beta(1 - \delta)E_t \left\{ \frac{\omega_{t+1}}{\omega_t} \left[ f_{e,t+1} - \frac{N_{d,t+1}}{N_{d,t+1}} f_{x,t+1} + \frac{1}{1-\theta} \left( \frac{N_{d,t+1}^{\theta} Y_{d,t+1}}{\bar{z}_d} + \frac{\tau_t N_{x,t+1}^{\theta} Y_{x,t+1}}{\bar{z}_{x,t+1}} \right) \right] \right\}.$$

(50)

The average output produced by the representative of Home firm for the domestic market is $\tilde{y}_{d,t} \equiv N_{d,t}^{\theta} Y_{d,t}$. Analogously, the amount of output produced by the representative Home firm for the export market is $\tilde{y}_{x,t} \equiv N_{x,t}^{\theta} Y_{x,t}$. Finally, recall that $\omega_t \equiv \tilde{\rho}_{d,t} \bar{z}_d \xi_t = \tilde{\rho}_{x,t} \bar{z}_x \xi_t^* / \tau_t$, and $\xi_t = u_{C,t}$.
Therefore, equation (50) can be written as:

\[
f_{e,t} = E_t \left\{ \beta_{t,t+1} \frac{\tilde{\rho}_{d,t+1}}{\tilde{\rho}_{d,t}} \left[ f_{e,t+1} - \frac{N_{x,t+1}}{N_{d,t+1}} f_{x,t+1} + \frac{1}{1 - \theta} \left( \frac{\tilde{y}_{d,t+1}}{\tilde{\rho}_{d,t+1}} + Q, \frac{N_{x,t+1}}{N_{d,t+1}} \frac{\tilde{\rho}_{x,t+1}}{\tilde{\rho}_{d,t+1}} \frac{\tilde{y}_{x,t+1}}{\tilde{\rho}_{d,t+1}} \right) \right] \right\}. \tag{51}
\]

The first-order conditions for vacancies and employment yield:

\[
\frac{\kappa}{q_t} = \beta E_t \left\{ \left( \frac{\xi_{t+1}}{\xi_t} \right) \left[ \varepsilon \left( \frac{\omega_{t+1}}{\xi_{t+1}} Z_{t+1} h_t - h_p \right) + \left[ 1 - \lambda - (1 - \varepsilon) \mu_{t+1} \right] \frac{\kappa}{q_{t+1}} \right] \right\}, \tag{52}
\]

where \( q_t \equiv M_t/V_t = \chi [(1 - l_t)/V_t]^{1 - \varepsilon} \) is the probability of filling a vacancy implied by the matching function \( M_t = \chi (1 - l_t)^{1 - \varepsilon} V_t^\varepsilon \), and \( \mu_t \equiv M_t/(1 - l_t) = \chi [V_t/(1 - l_t)]^{\varepsilon} \) is the probability for a worker to find a job. By applying the usual transformations, equation (52) can be written as:

\[
\frac{\kappa}{q_t} = \beta E_t \left\{ \frac{u_{C,t+1}}{u_{C,t}} \left[ \varepsilon \left( \frac{\tilde{\rho}_{d,t+1}}{\tilde{\rho}_{d,t}} \xi_{t+1} Z_{t+1} h_t - h_p \right) + \left[ 1 - \lambda - (1 - \varepsilon) \mu_{t+1} \right] \frac{\kappa}{q_{t+1}} \right] \right\}, \tag{53}
\]

The expected cost of filling a vacancy \( \kappa/q_t \) must be equal to its (social) expected benefit. The latter is given by the average value of output produced by one worker net of the disutility of labor, augmented by the continuation value of the match.

Finally, the first-order condition for hours implies \( v_{h,t} = \omega_t Z_t \). Table 2 summarizes the equilibrium conditions for the planned economy.

**Inefficiency Wedges**

Comparing the term in square brackets in equation (9) in Table 1 to the term in square brackets in equation (9) in Table 2 implicitly defines the inefficiency wedge along the market economy’s product creation margin. Specifically, subtracting the term for the planned outcome from that for the market economy and scrolling time indexes backward by one period allows us to define:

\[
\Sigma_{PC,t} = E_t \left\{ \tilde{\beta}_{t,t+1} \frac{\tilde{\rho}_{d,t+1}}{\tilde{\rho}_{d,t}} \left[ \left( \frac{\mu_{x,t+1}}{\mu_{x,t}} \right) \left( \frac{f_{x,t+1}}{f_{x,t}} - \frac{N_{x,t+1} f_{x,t+1}}{N_{d,t+1} f_{e,t}} \right) + \frac{1}{\theta} \left( \frac{\tilde{y}_{d,t+1}}{\tilde{\rho}_{d,t+1}} + Q \frac{N_{x,t+1} \tilde{\rho}_{x,t+1} \tilde{y}_{x,t+1}}{N_{d,t+1} \tilde{\rho}_{d,t+1} \tilde{y}_{d,t+1}} \right) \right] \right\}. \tag{A-10}
\]

In analogous fashion, comparing the term in square brackets in equation (11) in Table 1 to the term in square brackets in equation (11) in Table 2 implicitly defines the inefficiency wedge along the market economy’s job creation margin. As for the product creation wedge, subtracting the term for the planned outcome from that for the market economy and scrolling time indexes backward...
by one period yields:

$$\Sigma_{JC,t} \equiv \frac{q_{t-1}}{\kappa} \left[ \left( \varphi h_t + \frac{w_t}{P_t} h_t - \frac{\vartheta}{2} \pi_{w,t} \right) - \varepsilon \left( \rho_{d,t} Z_t h_t - \frac{v(h_t)}{w_{C,t}} \right) \right] + \frac{q_{t-1}}{q_t} (1 - \varepsilon) u_t. \quad (54)$$

F Model Properties

Impulse Responses

Figure 1 (solid lines) shows impulse responses to a one-percent innovation to Home productivity under the historical rule for the Fed interest rate setting. Focus on the Home country first. Unemployment ($U_t$) does not respond on impact, but it falls in the periods after the shock. The higher expected return of a match induces domestic intermediate input producers to post more vacancies on impact, which results in higher employment in the following period. Firms and workers (costly) renegotiate nominal wages because of the higher surplus generated by existing matches, and wage inflation ($\pi_{w,t}$) increases. Wage adjustment costs make the effective firm’s bargaining power procyclical, i.e., $\eta_t$ rises.55 Other things equal, the increase in $\eta_t$ dampens the response of the renegotiated equilibrium wage, amplifying the response of job creation to the shock.

Employment and labor income rise in the more productive economy, boosting aggregate demand for final goods and household consumption ($C_t$). The larger present discounted value of future profits generates higher expected return to product creation, stimulating product creation ($N_{e,t}$) and investment ($I_t \equiv N_{e,t} \varphi f_{e,t}$) at Home. The number of domestic plants that produce for the export market also increases, since higher aggregate productivity reduces the export productivity cutoff $z_{x,t}$.

Foreign households shift resources to Home to finance product creation in the more productive economy. Accordingly, Home runs a current account deficit in response to the shock ($CA_t$ falls on impact), and Foreign households share the benefit of higher Home productivity by shifting resources to Home via lending. Mirroring current account dynamics, trade balance moves countercyclically as in the data.

Home’s terms of trade depreciate, i.e., Home goods become relatively cheaper. Compared to

---

55To understand why this happens, recall equations (26), (27), and (29). Notice that $\partial J_t/\partial w_t$ is the change in firm surplus due to a change in nominal wages. The first term in the expression (26) for $\partial J_t/\partial w_t$ reflects the fact that, when the nominal wage increases by one dollar, the nominal surplus is reduced by the same amount (times the number of worked hours); the second term is the wage adjustment cost paid by the firm; and the last term represents the expected savings on future wage adjustments if wages are renegotiated today. When the first two effects are larger than the third one, the firm’s bargaining share rises. Intuitively, $\eta_t$ shifts upward to ensure optimal sharing of the cost of adjusting wages between firms and workers.
standard international business cycle models (IRBC), terms of trade depreciation is mild: The countercyclical response of $z_{x,t}$ counteracts, other things equal, the effects of higher productivity on marginal costs, and domestic export prices follow by less compared to a model that abstract from plant heterogeneity. Finally, in contrast to the results of standard IRBC models, our model predicts a positive comovement of GDP ($Y_t$), employment and investment across countries. The increase in aggregate demand at Home (which falls on both domestic and imported goods) and the moderate size of expenditure switching effects induced by terms of trade dynamics ensure that trade linkages, even if weak, generate positive comovement.

Second Moments

Table 1A presents model-implied, HP-filtered second moments (normal fonts). Bold fonts denote data moments, where cross-country correlations are averages of bilateral GDP and consumption correlations between the U.S. and its four largest trading partners during the period considered for the model calibration (Canada, Japan, Germany and UK).

The model correctly reproduces the volatility of U.S. consumption investment, and real wages relative to GDP. Moreover, it generates a negative Beveridge curve, and all the first-order autocorrelations are in line with the data. This successful performance is a result of the model’s strong propagation mechanism. Investment volatility is lowered relative to the excessive volatility generated by a standard IRBC framework because product creation requires hiring new workers. This process is time consuming due to search and matching frictions in the labor market, dampening investment dynamics. In contrast, consumption is more volatile than in traditional models as shocks induce larger and longer-lasting income effects.

With respect to the international dimension of the business cycle, the model is quite successful in matching the cyclical properties of trade data: imports and exports are more volatile than GDP, net exports are countercyclical and the volatility of the trade balance relative to GDP is in line with the data. These stylized facts not reproduced by standard IRBC models (see Engel and Wang, 2001). The model can also reproduce a ranking of cross-country correlations that is a challenge for standard IRBC models: GDP correlation is larger than consumption correlation. As shown in Figure 1, an increase in Home productivity generates Foreign expansion through trade linkages, as demand-side complementarities more than offset the effect of resource shifting to the

56 The close match between data and model-implied real wage moments provides indirect support for our calibration of the nominal wage adjustment cost.
more productive economy. Moreover, absent technology spillovers, Foreign consumers have weaker incentives to increase consumption on impact, which reduces cross-country consumption correlation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma_{X_R^U}$</th>
<th>$\sigma_{Y_R^U}$</th>
<th>$\sigma_{Y_R^U}/\sigma_{Y_R^U}$</th>
<th>1st Autocorr</th>
<th>corr($X_{R,t}^U$, $Y_{R,t}^U$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_R$</td>
<td>1.71</td>
<td>1.50</td>
<td>1</td>
<td>0.83</td>
<td>0.79 1</td>
</tr>
<tr>
<td>$C_R$</td>
<td>1.11</td>
<td>0.94</td>
<td>0.64</td>
<td>0.63</td>
<td>0.70 0.67 0.87</td>
</tr>
<tr>
<td>$I_R$</td>
<td>5.48</td>
<td>5.50</td>
<td>3.20</td>
<td>3.68</td>
<td>0.89 0.80 0.87</td>
</tr>
<tr>
<td>$l$</td>
<td>0.97</td>
<td>0.82</td>
<td>0.56</td>
<td>0.56</td>
<td>0.88 0.79 0.81</td>
</tr>
<tr>
<td>$w_R$</td>
<td>0.91</td>
<td>0.79</td>
<td>0.52</td>
<td>0.53</td>
<td>0.91 0.92 0.56 0.76</td>
</tr>
<tr>
<td>$X_R$</td>
<td>5.46</td>
<td>2.40</td>
<td>3.18</td>
<td>1.66</td>
<td>0.67 0.70 0.18 0.17</td>
</tr>
<tr>
<td>$I_R$</td>
<td>4.35</td>
<td>2.08</td>
<td>2.54</td>
<td>1.39</td>
<td>0.32 0.69 0.70 0.77</td>
</tr>
<tr>
<td>$TB_R/Y_R$</td>
<td>0.25</td>
<td>0.39</td>
<td>0.14</td>
<td>0.26</td>
<td>0.43 0.71 -0.47 -0.48</td>
</tr>
<tr>
<td>corr($C_{R,t}, C_{R,t}^*$)</td>
<td>0.44</td>
<td>0.16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr($Y_{R,t}, Y_{R,t}^*$)</td>
<td>0.51</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Bold fonts denote data moments, normal fonts denote model generated moments.

G  Steady-State Analysis

Job Creation

First notice that in a steady state with zero wage inflation the real wage is given by:

$$\frac{w}{\bar{p}} = \eta (h_p + b) + (1 - \eta) (\varphi_t Z_t + \kappa \kappa).$$

By substituting the wage equation into the job creation equation (3), and using $q = \chi^{1-\varepsilon}$, we obtain:

$$\kappa \varphi^{1-\varepsilon} \left[ \frac{1}{\chi} - \beta (1 - \lambda) \right] + \beta \eta (h_p + b) + (1 - \eta) \kappa \kappa = \eta \varphi Z.$$  \hspace{1cm} (55)

Taking the total differential of equation (55) we obtain:

$$\frac{d\kappa}{d\varphi} = \frac{\eta}{(1 - \varepsilon) \left( \frac{1}{\chi} - \beta (1 - \lambda) \right) + (1 - \eta)} > 0.$$
Since our calibration implies that $\chi < 1$, then $(1/\chi) > \beta (1 - \lambda)$ and $d\varepsilon/\partial \varphi > 0$.

**Marginal Revenue**

In the symmetric steady state $Q = 1$, $\widetilde{\rho}_x = \widetilde{\rho}_x^*$. and $N_x = N_x^*$. Moreover using $\phi = \theta$ (implied by our calibration), we have:

\[
1 = \frac{1}{\mu_d} N_d + \frac{1}{\mu_x} N_x,
\]

\[
1 = \left( \frac{\varphi}{\mu_d} \right)^{1-\theta} N_d \left[ \frac{\varphi}{\mu_x} \right]^{\theta-1} N_x,
\]

\[
1 = \left( \frac{\varphi}{\mu_d} \right)^{1-\theta} N_d \frac{\varphi}{\mu_x} \theta-1 N_x.
\]

It follows that $\varphi = (1/\mu_d) N_d^{\theta-1} \varphi$.