

Money for Nothing: The Consequences of Repo Rehypothecation *

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Abstract

This paper presents a model of repo rehypothecation where dealers intermediate funds and collateral between cash lenders (e.g., money market funds) and prime brokerage clients (e.g., hedge funds). Dealers take advantage of their position as intermediaries, setting different repo terms with each counterparty. In particular, the difference in haircuts represents a positive cash balance for the dealer which can be an important source of liquidity. The model shows that dealers with higher default risk are more exposed to runs by collateral providers rather than runs by cash lenders, who are completely insulated from a dealer's default. In addition, collateral providers' repo terms are sensitive to changes in a dealer's default probability and its correlation with the collateral's outcome. In contrast, cash lenders' repo terms are unaffected by these changes. This paper rationalizes the difference in haircuts observed in bilateral and tri-party repo markets, and reconciles the partial evidence of a run on repo during the recent financial crisis.

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1 Introduction

The 2007–09 financial crisis raised awareness of the potential destabilizing effects of financial intermediaries’ reliance on wholesale funding. The growth of financial innovation and the interconnectedness of the financial system have changed the way in which intermediaries finance themselves, and the manner in which their liquidity can come into question. In particular, many financial firms finance their positions through repurchase agreements (repos), which are in effect secured loans backed by financial assets. In the context of repos, bank runs can manifest themselves through an increase in haircuts (i.e., margin), where the cash borrower receives less money for the same amount of collateral, inducing the borrower to place more of its own cash to finance its position. Alternatively, bank runs can be seen as lender’s refusal to rollover a loans altogether, implying an abrupt withdrawal of funds.

Despite the large literature that has emerged on how repo markets developed during the crisis, the empirical evidence is not conclusive as to how bank runs evolved. On the one hand, Gorton and Metrick (2012) show a dramatic increase in haircuts during the crisis, effectively forcing borrowers to use more of their own cash holdings to finance their existing positions; or to sell off parts of their portfolios, potentially at fire sale prices. On the other hand, Krishnamurthy et al. (2014) show evidence of only a mild variation in margins. Moreover, they document that collateral classes (private label ABS) which did see a reduction in financing seemed to have lost it precipitously. This paper reconciles these two empirical facts by presenting a stylized model of repo seen from the perspective of a financial intermediary bringing together initial cash lenders with ultimate cash borrowers. The difference in margins between these two repo contracts imply a cash surplus for the intermediary, effectively giving them *money for nothing* through repo rehypothecation.

This paper develops a theoretical model where dealers act as intermediaries between a competitive cash lending market — money market funds (MMF) — and their prime brokerage clients — hedge funds (HF) — through repos and reverse repos respectively. Hedge funds use the loan to finance the purchase of the repo’s underlying collateral, and dealers use the same collateral to finance the hedge fund’s repo. The terms of trade on these two lending contracts are determined endogenously, which depend on a dealer’s need for liquidity and each contracts’ resolution in bankruptcy. Importantly, I assume dealers have a preference for cash in the initial leg of the loan. That is, a positive cash inflow is more valuable in the initial leg than on the final debt payment. The notion is that an influx of liquidity can allow dealers to finance other activities or give them flexibility to supplant any liquidity shortfalls. In the model, dealers default for reasons unrelated to their role as cash intermediaries, which counterparties price accordingly. The resolution of the repo, and reverse repo, in default plays an important role in deciding the equilibrium terms of trade. Specifically, counterparties’ reduced ability to access unsecured claims on the dealer pins down the interest rate/haircut

trade off.¹

The resulting equilibrium predicts a higher haircut for the cash borrower than for the cash lender, implying an windfall of funds for the dealer in the initial leg of the loan. The dealer has free reign to use this surplus of cash, effectively receiving money for nothing. Given cash lenders' limited recourse to the dealer's assets upon default, they set the repurchase price (final debt payment) to equal to the asset's worst case outcome, in effect insulating them from the dealer's counterparty risk. This suggests relatively stable contracting conditions between dealers and cash lenders. On the other hand, the hedge fund suffers a loss in case of a dealer's default since it loses the asset altogether, effectively forfeiting the initial margin used to purchase it. Given that in equilibrium the cash lender is insulated from the dealer through the collateral, and the cash borrower has an unsecured claim on the dealer in case of default, the hedge fund has incentives to withdraw its asset from the dealer in case its solvency comes into question. Thus a run does not ensue because of the cash provider's unwillingness to lend, but because of the collateral provider's unwillingness to borrow (i.e., deliver the initial collateral). The model also shows that whenever the correlation between the dealer's default and the collateral's outcome increases, so does the dealer-hedge fund margin. Moreover, assuming that a dealer's preference for liquidity increases with its probability of default, the model prescribes that riskier dealers charge higher margins on their dealer-hedge fund contracts. These predictions reconcile the aforementioned evidence of a run on repo: A sharp reduction in repo volumes between dealers and cash lenders, and an increase in repo haircuts between dealers and their prime brokerage clients.

The model assumes that money market funds are competitive, and given an initial loan request, they price the final debt repayment accordingly. The interaction between dealers and hedge funds is modeled as a Nash bargaining game, where the surplus of the asset's payoff and lending terms are split between players. This modeling choice makes it possible to consider different dealer/hedge fund relationships: Hedge funds may only interact with one prime broker, in effect being captured; or they may have multiple dealers to obtain funding. Hedge funds are also assumed to be optimistic to generate surplus and induce trade.

This paper captures key aspects of repo markets in the United States, which can be separated into two distinct markets. The tri-party repo market is the venue where money funds and securities lenders invest large amounts of their cash holdings with primary dealers. These funds help dealers finance their inventories, but an important fraction is distributed to other financial agents (i.e., prime brokerage clients) through the bilateral market.²³ There have been several studies documenting the size and dynamics of the tri-party

¹The relevance of these two assumptions are detailed in subsection 2.3.

²Adrian et al. (2013) have a good summary on the interactions between these markets. For earlier work on dealers' role as cash intermediaries see Mitchell and Pulvino (2012).

³It is important to note that a large fraction of the bilateral market is also used to intermediate collateral. Using data from 10-K SEC filings and estimating the amount of securities pledged to dealers through repos, securities lending, and margin loans, Singh (2011) estimates the total amount of collateral circulation in the global financial system. Although the role of rehypothecation for collateral distribution may be sizable, the focus of this paper repo rehypothecation as a means to

market, but relatively little is known about the bilateral market. Martin et al. (2014) estimate of the total size of U.S. repo markets is in the order of \$ 3 trillion dollars as of May 2012 and \$ 6.1 trillion in July 2008. Although estimates of the relative size of these markets are hard to gauge, Martin et al. (2014) report that in July 2008, the bilateral market was approximately 60% of the total repo market. The sheer size of these markets, and dealers' role as intermediaries of funds, suggest that any difference in contracting terms can have important implications on dealers' access to liquidity. Moreover, these differences can shed light on which counterparty bears the risk in case of a dealer default. Copeland et al. (2014) report differences in haircuts obtained from primary dealer survey results and tri-party clearing banks, and find considerable differences for some collateral classes.

The above implies that the difference in haircut dynamics reported by Krishnamurthy et al. (2014) and Gorton and Metrick (2012) arise because they study two different markets: tri-party vs. bilateral repo. Krishnamurthy et al. (2014) recognize there may be important differences in the dynamics between these markets two markets, which depend on the relevant frictions counterparties face in each. In particular, they mention that dealers' may be altering conditions to their clients in order to improve their cash position, "...a rise in interdealer haircuts could indicated a credit crunch in which dealers act defensively given their own capital and liquidity problems, raising credit terms to their borrowers." Copeland et al. (2014) mention that the different margin dynamics between these two markets is somewhat of a puzzle. Martin et al. (2014) argue that margin differences stem from institutional arrangements: tri-party terms are "sticky" (fixed custodial agreements), whereas bilateral terms are negotiated trade by trade. This paper offers an alternative explanation, which directly comes from primary dealers' role as intermediaries between these two markets and the relevant frictions at play.

This paper is related to the theoretical literature on the use of collateral and its impact on asset prices studied by Geanakoplos (2010), Fostel and Geanakoplos (2011), and Simsek (2013). These models consider a general equilibrium framework, where agents differences in beliefs can have important effects on what contracts are traded in equilibrium. This paper is also related to the work by Brunnermeier and Pedersen (2009) who study margin dynamics whenever financiers (i.e., lenders) use a value-at-risk rule to determine margins. Those papers view collateral markets as a direct means funding for ultimate investors or arbitrageurs. The focus of this paper is to study the pricing of a specific type of contract (i.e., repo) though the perspective of dealers intermediating funds between borrowers and lenders, and sheds light on important risks in the current financial environment: a withdrawal from intermediaries' end borrowers.

Duffie (2013) highlights that an important source of vulnerability in the financial system is the potential run of prime brokerage clients from their dealers, which has yet to be addressed in the policy discussion. In

intermediate funds.

effect, a dealer’s ability to access their clients’ assets is an important resource for a dealer bank’s business; not only for financing, but for other activities such as collateral circulation. Duffie (2013) states that the use of client assets to attract funds is an important source of liquidity, “...any difference between haircuts (over-collateralization) applied to prime brokerage clients and haircuts applied to prime brokers represents an effective source of additional cash financing to the prime brokerage. This paper provides the microfoundations for this behavior, endogenizing the haircuts on both sides of the intermediation process, and showing that collateral runs are an important source of vulnerability. Eren (2014) also explores the role of dealers as repo intermediaries, arguing that the size of the dealer-cash borrower haircut depends on the total volume of funds provided by cash lenders and dealers’ need for replace any liquidity shortfall. In this paper, I consider varying market structures between dealers and hedge funds, and the final equilibrium endogenizes both repo margins which are pinned down by the relevant frictions in the model. Specifically, margins depend on the underlying asset’s risk, the relative market power between dealer and hedge fund, and the dealer’s preference for liquidity and probability of default.

The rest of the paper is distributed as follows. Section 2 presents the model’s environment, its main frictions, and a discusses their relevance. Section 3 presents the formal problem and the main equilibrium results. Section 4 discusses the model outcome in more detail and relates is to the existing empirical evidence. Section 5 provides some additional evidence to the importance of the intermediation channel. Finally, Section 6 discusses future work and concludes.

2 Model Set Up

The model consists of two periods $t \in \{0, 1\}$. In the first period the initial repo and reverse repo are issued to the dealer and hedge fund respectively, and in the final period the loans are repaid. I assume that the hedge fund cannot bypass the dealer and receive funds directly from money market funds.⁴ A detailed discussion of the model’s assumptions are elaborated on further below.

2.1 Assets & Contracts

There is one risky asset with an uncertain payoff in the second period \tilde{a} . The asset is distributed $G(\cdot)$ under an objective probability distribution with finite support: $[\underline{a}, \bar{a}]$. Under this probability measure, the asset’s value in period 1 is $a = \mathbb{E}(\tilde{a})$, which assumed to be its trading price.

The hedge fund will receive a repo from the dealer to purchase asset \tilde{a} which also serves as collateral on

⁴This could be due to a severe “lemons” problem money funds face when dealing with hedge funds, or alternatively regulatory restrictions.

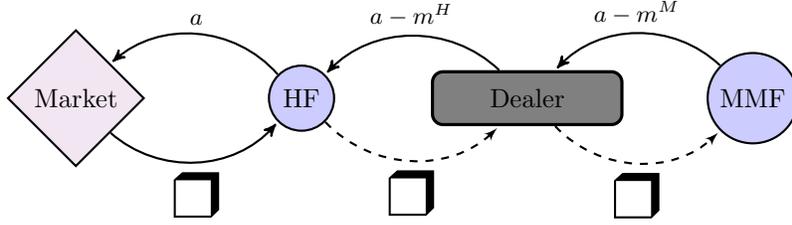


Figure 1: Initial Leg of Rehypothecation with No Default

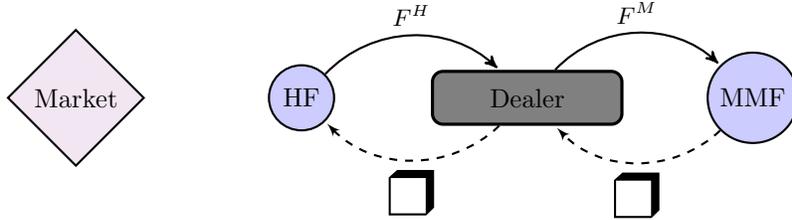


Figure 2: Closing Leg of Rehypothecation with No Default

the loan. For the intermediation between the hedge fund and dealer, the initial loan is denoted $a - m^H$ and the final payment in the following period is F^H , making the pair (m^H, F^H) the contracting terms for that repo. Simultaneously, the dealer receives the collateral and posts it with the money fund, to raise cash for the loan issued hedge fund. The money fund's initial loan is denoted $a - m^M$ and the final debt repayment is F^M . Figure 1 shows the initial leg of the rehypothecation channel, where the dealer receives $(a - m^M)$ from the money market fund and distributes $(a - m^H)$ to the hedge fund, netting $m^H - m^M$. Figure 2 shows the closing leg, where the dealer receives F^H from the hedge fund and distributes F^M to the money market fund, netting $F^H - F^M$.

If the dealer were to default, neither the hedge fund nor the money fund have recourse to the dealer's assets. In turn the dealer does have access to the hedge fund's assets above and beyond the repo collateral.

⁵ The model also takes into account the possibility that the dealer's solvency may be correlated with the asset's outcome, which will be detailed in the following subsection.

The money market fund industry is assumed to be competitive, and given the initial loan amount, prices the final debt repayment such that they break even. The outcome between the hedge fund and dealer is the result of a Nash bargaining game, where the dealer and hedge fund have market power of θ and $1 - \theta$ respectively.

⁵See subsection 2.3 for more details.

2.2 Agents

The three agents in the model is the original cash lender, i.e., the money market fund; the ultimate cash borrower, i.e., the hedge fund; and the intermediary between them, i.e., the dealer.

2.2.1 Dealer

The dealer's payoff takes the following form:

$$\begin{aligned} \mathbb{E}(U^D(m^H, F^H, m^M, F^M)) &= p [\mathbb{E}(v(\text{"payoff today"})) + \mathbb{E}(\text{"payoff tomorrow"})] \\ &= p \left[v(m^H - m^M) + \left(F^H + \int_{\underline{a}}^{m^H + F^H - W} (\eta + W - m^H - F^H) dG(\eta) - F^M \right) \right] \end{aligned}$$

where $1 - p$ is the dealers' exogenous probability of default and W is the hedge fund's initial endowment. The dealers preference for money in $t = 0$ is modeled by v with $v'(x) > 1$ for $x < a$ and $v'' < 0$. In the initial leg, the dealer nets: $m^H - m^M$, getting a payoff of $v(m^H - m^M)$. In $t = 1$ the dealer receives the full repayment F^H if the hedge fund is solvent and, since the dealer has recourse, the repo's collateral and the hedge fund's remaining portfolio if not. The dealer's payoff in $t = 1$ is the risk neutral valuation of the hedge fund's debt (with recourse) minus the repayment to the money market fund.

The dealer can only default because of exogenous reasons and in which case its payoff is zero. The dealer will face a budget constraint in $t = 0$ and $t = 1$, ensuring the dealer's ability to finance and repay both repos. Therefore, the dealer cannot default because of the intermediation process.⁶ Given that the dealer defaults independent of its rehypothecation activity, this also affects its outside option (i.e., not intermediating the repo) which is simply $pv(0) := pv_0$. Therefore, payoff's dependence on p can be omitted in the intermediation problem.

In addition, the model allows for the dealer's default and the asset's outcome to be correlated. Specifically,

$$\mathbb{P}(\text{dealer solvent} | \tilde{a} = \underline{a}) = \rho$$

with $\rho \in [0, p]$. If $\rho = p$, then the asset outcome is independent of the asset's outcome. As ρ decreases, the dealers solvency depends on asset's innovation, to the point where the dealer necessarily defaults whenever $\tilde{a} = \underline{a}$, that is, when $\rho = 0$. Using Bayes' rule, the asset's conditional distribution is denoted by $G(\cdot|s)$ or $G(\cdot|i)$ whenever the dealer is solvent or insolvent, respectively.

⁶Even with the aforementioned budget constraints, the dealer may potentially have a shortfall in case the hedge fund defaults. Though, in equilibrium, this is not a problem since the promised payment to the money market fund will be the lowest outcome of the asset, ensuring the collateral will always make the money fund whole.

2.2.2 Hedge Fund

The hedge fund is an optimistic risk neutral agent, where this optimism generates gains from trade. He is the residual claimant on of the repo, internalizing the possibility of a dealer's default. In such a case, the dealer will lose the asset. This entails a loss of the asset's upside relative to the repurchase price, i.e., $(\tilde{a} - F^H)^+$.

The hedge fund's payoff takes the following form:

$$\hat{\mathbb{E}}(U^H(m^H, F^H)) = p \int_{m^H + F^H - W}^{\tilde{a}} (\eta + W - m^H - F^H) d\hat{G}(\eta) + (1-p) \int_{m^H + F^H - W}^{\tilde{a}} (W - m^H + (\eta - F^H)^-) d\hat{G}(\eta)$$

where \hat{G} is the hedge fund's optimist distribution, with $\hat{\mathbb{E}}(\tilde{a}) = \hat{a} > a$ under the \hat{G} cdf. The hedge fund's outside option is its initial endowment W and the margin m^H must be lower than W in order to finance the asset purchase. Note that in case of a dealer's default, if the hedge fund's repurchase price is higher than the asset value, the hedge fund must pay $(\tilde{a} - F)$.⁷

2.2.3 Money Market Fund

The money market fund gives the initial loan to the dealer, internalizing the dealer's default. Because repo is exempt from automatic stay, the money market fund has immediate access to the collateral, allowing it to liquidate the collateral in order to make its claim whole.⁸ All cash flows above and beyond the face value of the original repo must be returned to the borrower. Therefore, the money fund's net payoff is given by:

$$\mathbb{E}(U^M(m^M, F^M)) = pu^M(F^M) + (1-p) \left[u^M(F^M)(1 - G(F^M)) + \int_{\underline{a}}^{F^M} u^M(\eta) dG(\eta) \right] - u^M(a - m^M)$$

with $u^{M'} > 0$ and $u^{M''} < 0$. I assume a competitive money market fund industry, thus in equilibrium they break even.

2.3 Discussion of Assumptions

In this section I briefly discuss the justification and intuition behind the main assumptions driving the equilibrium outcome between counterparties.

Dealer Preferences for Liquidity

In the model, dealer's have a preference for holding liquidity in $t = 0$. The dealer's preference for cash in $t = 0$ gives them incentives to have a higher margin when lending to the hedge fund and a lower margin

⁷See subsection 2.3 for more details.

⁸Automatic stay is a U.S. bankruptcy provision that prohibits creditors from collecting payments from a borrower who files for bankruptcy. Under current U.S. law, repos are exempt from automatic stay, allowing lenders to immediately sell the collateral in case of default.

when borrowing from the money fund. This can be motivated in several ways. Dealers may use these funds to finance other areas of their business, effectively using the cash windfall for more profitable activities. It may also reflect a dealer's need for liquidity in times of market stress. In effect, it is precisely when a dealer is perceived to be vulnerable that their need for liquidity is at its highest.

In the base model, changes in the dealer's probability of default does not alter its preference for money in $t = 0$. But in an extension, discussed in Section 4, incorporating this feature can explain an increase in margin in the bilateral market, maintaining the lending terms constant in the tri-party market. This reconciles different views on the dynamics of these markets during the 2007–09 crisis.

Recourse vs. Non-Recourse

In the model it is assumed that both the hedge fund and money fund do not have access to a dealer's balance sheet in case of default. In theory, in case either counterparty suffers a loss in their contract, they have an unsecured claim on the bankrupt dealer since repos are recourse loans. Although these firms are entitled to receive additional payments in case they are not made whole, in reality the costly and tedious task of resolving a large broker dealer may significantly hinder this process. For example, given the contracting nature between money funds and their own clients, awaiting the resolution of bankruptcy process may trigger a run on the money fund itself. Since in the U.S. repos are exempt from automatic stay, giving the money fund the ability to liquidate the collateral immediately; money funds would have a strong incentive to completely insulate themselves from the dealer. Thus, even if the dealer had some remaining assets to distribute in bankruptcy, the money market fund would still desire to make the repo risk free. In the model, this translates into having to heavily compensate the money fund for being exposed to a dealer's default, in which case the money fund receives nothing but the asset itself.

The non-recourse assumption of the hedge fund to the dealer also has important consequences. In the model, the bankruptcy resolution between the dealer and hedge fund is asymmetric. In effect, if the dealer were to default, the hedge fund loses the upside on the collateral relative to the repurchase price. But in case the asset's value is below the contracted loan repayment, the hedge fund must pay the shortfall to the bankruptcy estate. Ex ante, this cost is proportional to the hedge fund's initial margin. The hedge fund's loss in case of a dealer's defaults can be justified by the same arguments mentioned above: It may be very costly for the hedge fund wait for the resolution of a lengthy bankruptcy process to receive payments on any unsecured claim. Although this might be a strong assumption for a hedge fund, who presumably may be more patient than a money market fund, the considerable costs associated with the default of a large prime broker would make the recovery of unsecured claims more unlikely. In addition, the hedge fund's obligation to pay an asset shortfall relative to the repurchase price can be interpreted as the dealer's ability

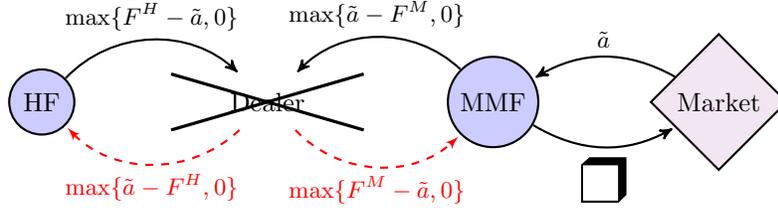


Figure 3: Cash Flows Upon Dealer Default

to seize their clients portfolio in case of any outstanding balance, irrespective of their bankruptcy status.⁹ Thus, it is reasonable to assume that conditional on the dealers default, the hedge fund would not expect any unsecured claims on the dealer to be fulfilled (at least not immediately), but would expect to pay any outstanding balances it could have.

Figure 3 highlights the relevant cash flows after a dealers default. The money market fund sells the asset to the market, and both the hedge fund and money fund owe the dealer any positive balance that may be owed (black lines). The asymmetric non-recourse assumption implies that any obligation owed by the dealer will not be fulfilled (dashed red lines).

The consequences of the recourse assumption is that both of the dealer’s counterparties stand to take a loss in case of default. This implies that the money market fund will want to insulate itself from the dealer, and that the hedge fund and dealer have to balance this cost with the dealer’s preference for liquidity. Once the equilibrium is characterized, Section 4 will revisit these issues in light of the final outcome.

3 Intermediation Problem & Equilibrium

This section formalizes the intermediation problem and presents the resulting equilibrium. Given the above setting, dealers and hedge funds enter Nash bargaining problem, setting the terms of both the dealers repo and reverse repo, to split the surplus between them. The problem can be summarized as,

$$\max_{\{m^H, m^M, F^H, F^M\}} (\mathbb{E}(U^D(m^H, F^H, m^M, F^M)) - v_0)^\theta (\mathbb{E}(U^H(m^H, F^H)) - W)^{1-\theta}$$

⁹I’d like to thank Mark Carlson for noting the dealers ability to easily seize clients assets in case there are any outstanding balances due. Eren (2014) also assumes this type of bankruptcy asymmetry between dealers and cash borrower.

subject to,

$$\begin{aligned}
m^H &\geq m^M && \text{Initial Leg Self Financing} \\
m^H + F^H &\geq m^M + F^M && \text{Overall Financing} \\
\mathbb{E}(U^M(m^M, F^M)) &= 0 && \text{MMF Break Even} \\
m^H &\leq W && \text{HF Down Payment Constraint} \\
m^H, m^M, F^H, F^M &\geq 0 &&
\end{aligned}$$

The problem's objective function is the Nash bargaining game between the dealer and hedge fund. The first restriction ensures the initial loan from the money market fund is enough to finance the hedge fund's repo. The second restriction ensures that the overall financing conditions are met. The third restriction is the money market funds break even condition. The fourth restriction makes sure the hedge fund has a large enough endowment to pay the initial loans margin to asset purchase the asset. The final restrictions are so that the contracting terms are feasible.

To simplify the analysis, I will restrict the study to analyze a binomial risky asset. Under the objective probability distribution, the asset pays \bar{a} with probability α and \underline{a} with probability $1 - \alpha$. The hedge fund's optimism is manifested through a higher probability of a good outcome: $\hat{\alpha} > \alpha$. The following Theorem characterizes an equilibrium of this model.¹⁰ The proof is relegated to the appendix.

Theorem 1 (Solution to Intermediary Problem). *In the intermediary problem with a binomial asset payoff, if the following parameter assumptions hold:*

$$i) \frac{\hat{\alpha}}{p} \left\{ \frac{p - \rho(1 - \alpha)}{p - \rho(1 - \hat{\alpha})} \right\} + \frac{\rho}{p}(1 - \alpha) < \frac{1}{1 - (1 - \alpha)(1 - \rho)}$$

$$ii) p(\hat{a} - \underline{a}) \geq W$$

$$iii) m^* := (v')^{-1} \left(\frac{\hat{\alpha}}{p} \left\{ \frac{p - \rho(1 - \alpha)}{p - \rho(1 - \hat{\alpha})} \right\} + \frac{\rho}{p}(1 - \alpha) \right) + (a - \underline{a}) \leq W$$

with $\rho \in [0, p]$, then there exists a θ^S and $\underline{\theta}$ such that:

for $\theta \in [\theta^S, 1)$ an equilibrium $(m^{H*}, F^{H*}, m^{M*}, F^{F*})$ is given by $F^{H*} = \theta F_{MonoD} + (1 - \theta) F_{MonoH}$ where,

$$\begin{aligned}
F_{MonoD} &= \bar{a} - \frac{((1 - \hat{\alpha})W + \hat{\alpha}m^H)}{p - \rho(1 - \hat{\alpha})} \\
F_{MonoH} &= \underline{a} - \frac{v(m^* - (a - \underline{a})) - v_0 + \frac{\rho(1 - \alpha)}{p}(W - m^*)}{\left(1 - \frac{\rho(1 - \alpha)}{p}\right)}
\end{aligned}$$

and $m^{H*} = m^*$, $F^{H*} = F^*$, $m^{M*} = a - \underline{a}$, $F^{M*} = \underline{a}$, and

¹⁰To simplify this version of the model I only present one possible equilibrium. More equilibrium may be characterized with similar properties.

for $\theta \in (\underline{\theta}, \theta^S]$ an equilibrium $(m^{H*}, F^{H*}, m^{M*}, F^{F*})$ is given by the solution to

$$m^{H*} + F^{H*} = W + \underline{a} \quad \text{and} \quad \frac{(1 - \theta) (\mathbb{E}(U^D) - v_0)}{\theta(\hat{\mathbb{E}}(U^H) - W)} = \frac{v'(m^{H*} - m^{M*}) - 1}{(1 - p) - (1 - \hat{\alpha})(1 - \rho)}$$

with $m^{M*} = a - \underline{a}$, $F^{M*} = \underline{a}$.

Proof. See Appendix □

The Theorem focuses on equilibria which can separate both the dealer-hedge fund problem and the dealer-money market fund problem, i.e., the financing conditions are slack. This is to focus the discussion on the frictions present in each of the intermediaries interactions, without considering the dealers hard financing constraints. The solution is divided into two sub intervals, which imply risky or safe contracting terms for hedge fund.

The dealer-money fund is relatively straight forward for the entire feasible range: charge the highest margin which results in a risky free contract for the money fund. Since the agents' relationship is not recourse, and increase in repurchase price above lowest asset outcome will entail a potential loss, for which the money market fund must be compensated. Although the dealer has a preference for money today (since $v'(\cdot) > 1$), increasing the money funds initial loan, this implies a disproportionately high repurchase price. In effect, comparing the two agents marginal rate of substitution between (m^M, F^M) in the proposed equilibrium gives,

$$v'(m^* - (a - \underline{a})) < \frac{1}{1 - (1 - \alpha)(1 - \rho)} \quad (1)$$

the (LHS) is the marginal utility increase in lowering m^M and the (RHS) is the additional dollar m^M needed to pay to compensate the money market fund.

The range of feasible θ is given by the interval $(\underline{\theta}, 1)$, which is divided into two by θ^S . For relatively high values of θ the dealer-hedge fund problem results in risky contract for the hedge fund. In this case, equalizing agents marginal rate of substitution, the condition that pins down m^{H*} is given by solution to following equation,¹¹

$$\frac{v'(m^H - m^M) - \frac{\rho(1-\alpha)}{p}}{1 - \frac{\rho(1-\alpha)}{p}} = \frac{\hat{\alpha}}{p \left(1 - \frac{\rho(1-\hat{\alpha})}{p}\right)} \quad (2)$$

Equation (2) highlights the two main frictions at play. The (LHS) is the dealer's marginal rate of substitution: Increasing m^H increases the amount of cash in the initial leg valued at v , but lowers the recovery if the hedge fund defaults whenever the dealer is solvent (with probability $\frac{\rho(1-\alpha)}{p}$); while an increase F^H increases the dealer's payoff in case the hedge fund avoids default (with probability $1 - \frac{\rho(1-\alpha)}{p}$). The (RHS) is the hedge

¹¹Which is feasible given assumption iii).

funds marginal rate of substitution: increasing m^H lowers the its initial endowment stock which lowers its payoff in case the hedge fund avoids default, irrespective of the dealer’s solvency (with probability $\hat{\alpha}$); but increasing F^H only effect its payoff if the dealer doesn’t default and the asset has a good outcome (with probability $p \left(1 - \frac{\rho(1-\hat{\alpha})}{p}\right)$), since any asset upside is forfeited otherwise.

The dealer-hedge fund’s final repurchase price is a convex combination of their respective repurchase price if each were a monopolist: F_{MonoD} and F_{MonoH} , respectively. In effect, condition ii) ensures that the dealer’s monopolist solution results in a risky repo for the hedge fund (i.e., the hedge fund may default), thus a high value of θ will give the aforementioned solution which decreases as the dealers market power is diminished. The cutoff θ^S is when the optimal dealer-hedge fund contract is risk free. Note that condition (2) and (1) hold simultaneously due to assumption i) and iii).

For $\theta \in (\underline{\theta}, \theta^S]$ the dealer hedge fund contract is risk free, resulting in the dealer forfeiting margin to the hedge fund, but maintaining the total debt burden ($m^{H*} + F^{H*}$) constant. As the hedge fund’s bargaining position becomes stronger, either the loss in margin is so sever that equation (1) is violated or $m^{H*} + F^{H*} < \underline{a} + W$, which pins down $\underline{\theta}$. In effect, in the first case the loss of liquidity from the hedge fund’s contract creates incentives for the dealer to increase m^{M*} , implying a risky contract for the money fund. The definition of $\underline{\theta}$ focuses on equilibria where the money fund’s contract is risk free and the overall debt burden is greater or equal to $W + \underline{a}$.¹²

Note that the optimal solution is in fact self financing since $m^{H*} > m^{M*}$ and $m^{H*} + F^{H*} \geq \underline{a} + W > \underline{a}$. Therefore, the dealer does reap benefits from the difference in margin, gaining liquidity today without having to suffer any financing costs; and even if the hedge fund defaults, the dealer is able to repay the money fund. In other words the dealer receives “Money for Nothing”.

3.1 Equilibrium when Default and Asset Outcome are Independent ($\rho = p$)

The main results of the paper can be appreciated in a simplified case, when \tilde{a} is independent from the dealer’s survival. Having the repo intermediation equilibrium characterized an important question is to understand how the feasibility of such an equilibrium can change with p . The following proposition shows how the bargaining power threshold is altered for different levels of dealer solvency.

Proposition 1 (Feasibility as Dealer Risk Changes). *Given the equilibrium characterized in Theorem 1 with $\rho = p$ and p sufficiently close to 1, then the lower bound threshold $\underline{\theta}$ is decreasing in p .*

Proof. See Appendix □

¹²This assumption just serves to limit the amount of equilibrium considered and is not absolutely necessary for the main result.

Proposition 1 shows that the interval where the equilibrium involves a riskless contract for the money market fund and feasible outcome between the hedge fund and dealer shrinks as the dealer's probability of default increases. Thus, it is harder for less solvent dealers to engage in repo intermediation, precisely because the collateral provider has reduced incentives to participate.

Another important issue to understand how the terms of trade change for different levels of dealer counterparty risk. The first result shows how the margin changes under the original specification.

Proposition 2 (Margins as Dealer Risk Changes). *Given the equilibrium characterized in Theorem 1 with $\rho = p$, for $\theta \in [\theta^S, 1)$,*

$$\frac{\partial m^{H*}}{\partial p} = -\frac{\alpha}{p^2 v''(m^{H*} - (a - \underline{a}))} > 0,$$

and for $\theta \in (\underline{\theta}, \theta^S]$,

$$\frac{\partial m^{H*}}{\partial p} = \frac{\theta(v'(m^* - (a - \underline{a})) - 1)\hat{\alpha}(\bar{\alpha} - F^{*H}) + (1 - \theta)(\mathbb{E}(U^D) - v_0)}{\hat{\alpha}(1 - p)(v'(m^* - (a - \underline{a})) - 1) - v''(m^{H*} - (a - \underline{a}))\theta(\mathbb{E}(U^H) - W)} > 0.$$

Proof. Both results are derived directly from taking the implicit derivative of the first order condition that pins down m^{H*} . For $\theta \in [\theta^S, 1)$ this is equation (2) and for $\theta \in (\underline{\theta}, \theta^S]$ this is equation $\frac{(1-\theta)(\mathbb{E}(U^D)-v_0)}{\theta(\mathbb{E}(U^H)-W)} = \frac{v'(m^{H*}-m^{M*})-1}{\hat{\alpha}(1-p)}$. \square

The result in Proposition 2 indicates that as the dealer's probability of remaining solvent increases so does the dealer-hedge fund margin. Although this result is rather unsatisfactory, it is very intuitive. In effect, as the dealers probability of default decreases (i.e., an increase in p), the hedge funds unsecured claims is less important, increasing the surplus in the relationship which is maximized via equation 2. That is the trade off between the hedge funds loss versus the dealers preference for liquidity is changed because the loss is smaller. Therefore, a larger margin is optimal.

But the result in Proposition 2 can be easily overturned by considering a slight modification to the model. Specifically, if one assumes that the dealer's marginal preference for cash today decreases with its solvency. That is, if $\frac{\partial v'(\cdot)}{\partial p}$ sufficiently negative; the optimal response is an increase in margin whenever the dealer is closer to default.¹³ This additional condition can be easily motivated by thinking that dealer's liquidity needs increase as its probability of default increases.

Proposition 3 (Margins as Dealer Risk Changes — Alternate Model). *Given the equilibrium*

¹³An example of this characteristic with power utility is: $v(x) = a^{\eta/p} \left(\frac{x^{1-\eta/p}}{1-\eta/p} \right)$

characterized in Theorem 1 with $\rho = p$ and assuming that $\frac{\partial v'(\cdot)}{\partial p} < -\frac{\alpha}{p^2}$, then for $\theta \in [\theta^S, 1)$,

$$\frac{\partial m^{H^*}}{\partial p} = -\frac{1}{v''(m^{H^*} - (a - \underline{a}))} \left(\frac{\partial v'(m^{H^*} - (a - \underline{a}))}{\partial p} + \frac{\alpha}{p^2} \right) < 0.$$

Proof. The result is derived directly by taking the implicit derivative of the first order condition that pins down m^{H^*} , i.e., equation (2). \square

Proposition 3 states that if the dealers preference for liquidity decreases with it's solvency, then the optimal contract will prescribe an increase in the margin offered to the hedge fund, even though its costly.¹⁴ The reduced surplus from increasing the hedge fund's cost in case of a dealer's default is outweighed by the dealer's need for liquidity in times of market stress.

3.2 Equilibrium when Default and Asset Outcome are Correlated ($\rho < p$)

In this subsection I analyze how the contracting terms change whenever the dealer's default is correlated with the repo's collateral. In particular, how the dealer-hedge fund margin changes as ρ decreases.

Proposition 4 (Margins as Correlation Changes). *Given the equilibrium characterized in Theorem 1 with $\rho < 1$ and assuming that $\frac{1-\alpha}{p} - \frac{\hat{\alpha}-\alpha}{(p-\rho(1-\hat{\alpha}))^2} > 0$, then for $\theta \in [\theta^S, 1)$,*

$$\frac{\partial m^{H^*}}{\partial \rho} = \frac{1}{v''(m^{H^*} - (a - \underline{a}))} \left\{ \frac{1-\alpha}{p} - \frac{\hat{\alpha}-\alpha}{(p-\rho(1-\hat{\alpha}))^2} \right\} < 0.$$

Proof. The result is derived directly by taking the implicit derivative of the first order condition that pins down m^{H^*} , i.e., equation (2). \square

Proposition 4 shows under what conditions an increase in the correlation implies a higher dealer-hedge fund margin. The intuition behind the result stems from the fact that the hedge fund's loss of the asset in case of a dealer default only has a negative impact if the asset has a good outcome. In effect, when the dealer default, the hedge fund's payoff stemming from the asset is $(\tilde{a} - F^H)^-$. Therefore, if it is more likely for the dealer to default whenever asset outcome is bad, then the expected hedge fund loss is smaller; implying a higher dealer-hedge fund margin.

¹⁴A similar condition can be derived for $\theta \in (\underline{\theta}, \theta^S]$.

3.3 Numerical Example

Figure 4 illustrates the properties of the equilibrium characterized in Theorem 1 when $\rho = p$ for a given parameterization. The dealer's preference for cash in $t = 0$ is modeled as power utility with $\eta < 1$, that is,

$$v(x) = a^\eta \left(\frac{x^{1-\eta}}{1-\eta} \right).$$

Given that the dealer-money fund contract is constant, the focus is on the dealer-hedge fund outcome. For θ close to 1, the solution approaches the dealer-monopolist contract, where the overall lending terms $m^{H*} + F^{H*}$ are at it's highest. As the dealer's market power decreases the overall lending terms decrease, but the margin is held constant, satisfying equation (2) which maximizes the surplus between borrower and lender. In effect, the condition in equation (2) balances the cost of hedge fund's loss in case of default with the dealer's preference for cash upfront.

A constant margin implies a reduction in the repurchase price, till $\theta = \theta^S$ when the equilibrium switches to a riskless contract between the dealer-hedge fund. In this case, the balance of power leads to a more favorable margin for the hedge fund, at the cost of a higher repurchase price, but maintain the overall debt burden constant. Here, the hedge fund is able to reduce its unsecured claim, costing the dealer the marginal benefit of more cash initially.

Figure 5 shows the resulting equilibrium in terms of the usual repo contracting terms, i.e., haircut and repo rate. The mapping of the initial margin and final repurchase price to the repo haircut and repo rate is given by,

$$\text{haircut} = \frac{a}{a - m^{H*}} - 1 \quad \& \quad \text{repo rate} = \frac{F^{H*}}{a - m^{H*}} - 1.$$

Note that the intuition of the model remains: as the hedge fund's bargaining position increases, the haircut decreases as does the corresponding repo rate.

4 Model Discussion & Relation to Existing Empirical Evidence

Theorem 1 sheds light on a number of important features of the model which are relevant for the repo intermediation process. First of all, note that the risky free dealer-money fund repo is a feature often assumed in much of the literature, but arises endogenously in this model.¹⁵ In effect, since the money fund must be heavily compensated for the dealer's default, the optimal agreement insulates the money fund from the dealer altogether, only leaving the collateral.¹⁶ In the model this occurs since the money fund has no

¹⁵This result is also arises endogenously in Infante (2013) due to borrower's ability to dilute existing creditors.

¹⁶Note that a similar version of this result would also arise under more general asset payoff distributions if one considers the money fund to be relatively more risk averse.

recourse to dealer’s other assets in case of default. Given the lengthy, and presumably costly, resolution of a large financial firm; and the immediate liquidity money market funds must provide to their clients, it is reasonable that the money fund would want to insulate itself from the dealer’s counterparty risk. Moreover, this result is in line with the common perception that repo’s in the tri-party market are relatively safe investment vehicles.

The second interesting feature, whenever $\rho = p$, is that as p decreases this class of equilibrium is harder to sustain, which is the result of Proposition 1. This is simply observed in the set up of the problem: When the dealers probability of default increases, the importance of the hedge fund’s “dead weight loss” through its unsecured claim is larger. Note that there would still be a cost to the hedge fund even if the strong no recourse assumption were relaxed. In effect, if the hedge fund had access to any cashflows of the asset sale above and beyond the money marked funds repurchase price, the hedge fund would suffer an ex ante loss proportional to $m^{H^*} - m^{M^*}$ instead of m^{H^*} . That is, the frictionless unsecured claim of the hedge fund is the additional cash the dealer reaps through intermediation. Although this version of the model complicates the analysis, the message is the same: The hedge fund holds an unsecured claim on the dealer and thus has incentives to run. Therefore, an increased probability of a dealer default, decreases the likelihood of the intermediation of repos, which is induced by a run from collateral providers rather than cash lenders who (endogenously) are not subject to the dealer’s counterparty risk.

These two points put together capture the empirical findings of Krishnamurthy et al. (2014), and partly by Copeland et al. (2014), who show that conditional on the underlying collateral’s asset class there is a high degree of homogeneity in haircuts amongst dealers in the tri-party repo market, that is, the margin is largely asset specific. Krishnamurthy et al. (2014) go on to further show relatively stable haircuts the tri-party market and a sharp contraction in the amount of repos issued with riskier collateral classes, such as non-Agency MBS/ABS. This can be interpreted as a run from collateral providers, eliminating the intermediation process altogether. In the model, margins in the tri-party market only depend on the riskiness of the collateral, thus it allows for an increase in tri-party repo haircuts, but this increase is only asset specific. The near constant margin in the U.S. Treasury market documented by Krishnamurthy et al. (2014) is in line with this model outcome.

In addition Theorem 1 can account for an increase in margin in the bilateral market, leaving the tri-party margin unchanged. In effect, the results Proposition 4 shows that an increase in the correlation between the dealer’s solvency and asset outcome actually increases the margin in the dealer-hedge fund relationship. Proposition 3 also shows the same outcome whenever the dealer’s probability of default increases by introducing a relationship between a dealer’s marginal preference for a dollar today and its probability of default. For example, if a primary dealer were to be in distress due to low asset valuations stemming from a

fire sale, or because alternative financing lines were reduced; the dealer would have a stronger desire to obtain cash from other activities. The heightened need for cash is captured by $\frac{\partial v'}{\partial p} < 0$, which in the intermediation problem implies negotiating better contracting terms in the dealer-hedge fund repo and maintaining the dealer-money fund margin stable. These model outcomes are in line with the empirical literature that shows an increase in bilateral repo margins during the financial crisis.

These results are in line with Gorton and Metrick (2012)'s findings of a sharp increase in haircuts during the financial crisis. The argument of this paper is that either the correlation between dealer's portfolio and their solvency increased, or dealer's need for liquidity increased. The effect of both of these channels is that dealers captured more funds in the initial intermediation leg. This is also in line with the survey evidence presented in Copeland et al. (2014) who show a large spread between tri-party and bilateral repo haircuts. In that paper it can be easily seen an important driver of the difference in margin is the underlying asset's riskiness.

5 Additional Empirical Evidence

To provide additional evidence to the importance of repo intermediation, I use primary dealers' financing reports collected by the New York Federal Reserve Bank: FR 2004 C. This survey collects weekly reports on primary dealers' outstanding financing, particularly on the amount of repos and reverse repos. The New York Federal Reserve releases the dealers' aggregate positions which document the amount of funds received by dealers through repos and the amount of funds issued by dealers through reverse repos.

Between July 2001 and March 2013 the repo and reverse repo transactions were reported by aggregating all underlying collateral classes. As prescribed from the model, this omits any contracting differences there may be because of different collateral riskiness. To alleviate this concern, I also show the newly revised survey which starts from April 2014 onwards, where different collateral classes are reported separately.

Given that only the initial loan amounts are reported, the ability to this data to detail the amount of liquidity reaped by primary dealers though intermediation is limited. But coupled with existing literature showing the difference in haircuts between tri-party and bilateral repo, the importance of this channel can be gauged. Observing the time series of the minimum between repos and reverse repos, the size of their intermediation (i.e., primary dealers' *matched book*) can be estimated. Though many of these repos are between dealers in the tri-party market¹⁷, evidence reported by Copeland et al. (2012) on the size of the bilateral market suggest that it makes up a sizable fraction of dealers' matched book. In all of the figures presented here, the minimum amount between repos and reverse repos is always the latter, implying that

¹⁷Specifically, in the GCF Repo market. See Fleming and Garbade (2003) for details on this interdealer market.

more money is being lent to the dealers than what is being distributed. In Figures 6 and 7, these time series are labeled as "Reverse Repo". Moreover, the difference between repos and reverse repos captures the amount of cash that stays in the dealer sector through their repo operations, which in Figures 6 and 7 is marked as "Net repo borrowing". Note that taking the difference cancels out intra dealer repos. Although what fraction of this difference is due to different contracting schemes (i.e., $m^{H^*} - m^{M^*}$) and how much is attributed to dealers funding their position is unclear, it does capture the important inflow of cash reaped through the repo market.

Information can be backed out through available estimates of haircuts and their differences, reported by Copeland et al. (2014) (See Figure 1 of that paper), to estimate the windfall of cash discussed in this paper, i.e., $m^{H^*} - m^{M^*}$. As noted in subsection 3.3, gross haircuts are reported as the ratio of the collateral value over the amount of the initial loan. For example, a haircut of ϕ percent corresponds to: $\phi + 1 = a/(a - m)$. The FR 2004 data provides us with estimates of total inflows, i.e., $loan = a - m$.¹⁸, implying $m = loan * \phi$. Therefore, a rough estimate of the windfall due to intermediation is $(\phi_H - \phi_M) * loan_H$, that is, the difference in haircuts times the amount of reverse repos.¹⁹ As documented by Copeland et al. (2014), some of these differences were in the order of 5% to 40%, especially during the financial crisis, potentially implying large inflows.

Figure 6 shows the aggregate series between July 2001 and March 2013. The measure of the matched book shows a sizable intermediation role, of the order of 2 trillion dollars the end of the sample, and a net cash inflow in the order of 500 billion. Figure 7 shows the more recent information separated by collateral class. For example, the size of the matched book for Agency MBS is between \$500 and \$300 billion dollars. This implies that a hair cut difference of 5% implies a windfall of \$25 to \$18 billion, which is about 1/8 of the total cash imputed by the net repo borrowing series. In the revised model, as with the data used by Copeland et al. (2014), the differences in margin are larger in times of market stress, implying a larger reliance on prime brokerage client's assets. This evidence is suggestive to the important channel repo intermediation can have for dealers.

6 Concluding Remarks & Future Work

This paper analyses repo markets as a cash intermediation chain. Dealers receive funds from money market funds through the tri-party market and distribute them to their prime brokerage clients in the bilateral market. This paper shows that different contracting terms in these markets can provide dealers with an

¹⁸For now I shall assume a unique haircut within collateral classes. There is an issue when adding inflows and backing out the total margin in the presence of different haircuts.

¹⁹This is an upper bound, since the reverse repo number includes interdealer trades.

important liquidity channel. Moreover, the contracting terms derived from the model imply that collateral providers have more incentives to run than cash lenders, in line with some of the risks highlighted by Duffie (2013). The model also predicts that conditional on the repo collateral class, margins are homogeneous and relatively stable in the tri-party market. In addition, if one assumes that dealers preference for liquidity increases with their probability of default, or the correlation between the dealer's solvency and the collateral increases; riskier dealers will charge higher margins to their collateral providers in the bilateral market.

One open question that remains is why does the intermediation chain exist in the first place? That is, why don't cash lenders directly fund cash borrowers? Although it is beyond the scope of this paper, one specific reason may be the relative opaque nature of cash borrowers. In effect, an important feature of the hedge fund industry is that their positions are largely unknown to outsiders. Thus cash lenders, such as money market funds, would be concerned about the borrowers counterparty risk. Arguably, prime brokers do not suffer such a severe informational problem, since they are the execute many of the hedge fund's trades. Thus, an efficient outcome may very well be cash lenders fund dealers who in turn fund their clients.

Future versions of the paper will explore the full extent of all the possible equilibria in the intermediation model. In addition, incorporating a positive correlation between a dealer's default and the collateral's payoff it an important innovation to study how margins react to asset classes which may be perceived as systemic. Sharper estimates on the fraction of repo financing stemming from dealer's relative market power in the bilateral market are also an avenue of future work.

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A Appendix

Proof of Theorem 1:

The proof of the Theorem will consist in exploring equilibria that separate each interaction of the intermediation and then puts the pieces together to characterize the final equilibrium.

Under general distribution assumptions, the intermediation problem's Lagrangean takes the following form:

$$\begin{aligned} \mathcal{L} = & \theta \log(\mathbb{E}(U^D) - v_0) + (1 - \theta) \log(\hat{\mathbb{E}}(U^H) - W) + \mu_s(m^H - m^M) + \mu_b(m^H + F^H - (m^M + F^M)) \\ & + \mu_m(W - m^H) + \mu \left(pu^M(F^M) + (1 - p) \left[u^M(F^M)(1 - G(F^M|i)) + \int_{\underline{a}}^{F^M} u^M(\eta) dG(\eta|i) \right] - u^M(a - m^M) \right) \end{aligned}$$

Taking FOC gives:

$$\frac{\partial \mathcal{L}}{\partial m^H} = \left(\frac{\theta}{\mathbb{E}(U^D) - v_0} \right) \frac{\partial \mathbb{E}(U^D)}{\partial m^H} + \left(\frac{1 - \theta}{\mathbb{E}(U^H) - W} \right) \frac{\partial \mathbb{E}(U^H)}{\partial m^H} + \mu_s + \mu_b - \mu_m \leq 0 \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial F^H} = \left(\frac{\theta}{\mathbb{E}(U^D) - v_0} \right) \frac{\partial \mathbb{E}(U^D)}{\partial F^H} + \left(\frac{1 - \theta}{\mathbb{E}(U^H) - W} \right) \frac{\partial \mathbb{E}(U^H)}{\partial F^H} + \mu_b \leq 0 \quad (4)$$

$$\frac{\partial \mathcal{L}}{\partial m^M} = \left(\frac{\theta}{\mathbb{E}(U^D) - v_0} \right) \frac{\partial \mathbb{E}(U^D)}{\partial m^M} + \mu u^{M'}(a - m^M) - \mu_s - \mu_b \leq 0 \quad (5)$$

$$\frac{\partial \mathcal{L}}{\partial F^M} = \left(\frac{\theta}{\mathbb{E}(U^D) - v_0} \right) \frac{\partial \mathbb{E}(U^D)}{\partial F^M} + \mu[p + (1 - p)(1 - G(F^M))]u^{M'}(F^M) - \mu_b \leq 0 \quad (6)$$

where

$$\begin{aligned} \frac{\partial \mathbb{E}(U^D)}{\partial m^H} &= v'(m^H - m^M) - G(m^H + F^H - W|s) \\ \frac{\partial \mathbb{E}(U^D)}{\partial F^H} &= 1 - G(m^H + F^H - W|s) \\ \frac{\partial \mathbb{E}(U^D)}{\partial m^M} &= -v'(m^H - m^M) \\ \frac{\partial \mathbb{E}(U^D)}{\partial F^M} &= -1 \\ \frac{\partial \hat{\mathbb{E}}(U^H)}{\partial m^H} &= -(1 - \hat{G}(m^H + F^H - W)) \\ \frac{\partial \hat{\mathbb{E}}(U^H)}{\partial F^H} &= -p(1 - \hat{G}(m^H + F^H - W)|s) - (1 - p)(\hat{G}(F^H|i) - \hat{G}(m^H + F^H - W|i)) \end{aligned}$$

Given that I consider the case where \bar{a} only has two outcomes $\{\bar{a}, \underline{a}\}$, I have to consider “kinks” in the payoff function. Specifically, in the dealer-HF case when $m^H + F^H \vee W + \underline{a}$ and $F^H \vee \underline{a}$, and in the dealer-MMF case when $F^M \vee \underline{a}$. Whenever the contracts are risky (i.e., either hedge fund or money fund can suffer a loss), I add a multiplier μ_{Ri} to the corresponding FOC, and whenever they are safe I subtract a multiplier μ_{Si} to the corresponding FOC; where i denotes which counterparty under consideration. This will affect equations (3) and (4) when dealing with a risky or safe dealer-HF contract, and equation (6) when dealing with a risky or safe dealer-MMF contract. The additional kink in F for dealer-HF problem will have a multiplier $\bar{\mu}_F$ or $\underline{\mu}_F$, which will only affects equation (4).

Note that if the self financing conditions aren't active, then there is no connection – other than the Dealers expected payoff – that relates the dealer-HF problem with the dealer-MMF problem. In this version of the paper, I shall focus on the separable case and drop the self financing multipliers when necessary. Also, I shall only consider optimal contracts where the upper bound on m^H is slack, i.e., $m^H < W$.

Dealer-MMF Problem (D-MMF):

In the case where the outcome of the asset is binary, and assuming $m^M, F^M > 0$, equations (5) and (6) are reduced to,

$$\begin{aligned} \left(\frac{-\theta v'(m^H - m^M)}{\mathbb{E}(UD) - v_0} \right) + \mu u^{M'}(a - m^M) - \mu_s - \mu_b &= 0 \\ \left(-\frac{\theta}{\mathbb{E}(UD) - v_0} \right) + \mu[p + (1-p)(1 - G(F^M|i))]u^{M'}(F^M) - \mu_b &= 0. \end{aligned}$$

where $1 - G(F^M) = \alpha$ if $F^M \geq \underline{a}$ and 1 if $F^M \leq \underline{a}$. Thus, there are two cases to analyze: F^M greater or less than \underline{a} .

(D-MMF) Case I – $F^M \geq \underline{a}$: Solving for μ from the first equation gives,

$$\mu = \frac{1}{u^{M'}(a - m^M)} \left[\left(\frac{\theta v'(m^H - m^M)}{\mathbb{E}(UD) - v_0} \right) + \mu_s + \mu_b \right] > 0$$

Replacing that expression for μ in equation (6) gives,

$$\begin{aligned} 0 &= \left(\frac{\theta}{\mathbb{E}(UD) - v_0} \right) \left[v'(m^H - m^M) \frac{u^{M'}(F^M)}{u^{M'}(a - m^M)} [1 - (1 - \alpha)(1 - \rho)] - 1 \right] \\ &\quad + \underbrace{\mu_b \left[\frac{u^{M'}(F^M)}{u^{M'}(a - m^M)} [1 - (1 - \alpha)(1 - \rho)] - 1 \right]}_{<0} + \mu_s \frac{u^{M'}(F^M)}{u^{M'}(a - m^M)} [1 - (1 - \alpha)(1 - \rho)] + \mu_{RM} \end{aligned}$$

Because $F^M \geq a - m^M$, only the first “large bracket term” may be zero if the Dealer’s marginal payoff for cash today is relatively large.

Assuming the self financing conditions do not bind ($\mu_s = \mu_b = 0$), there exist two possible solutions: a risk free and risky outcome.

(D-MMF) Case I.1 – Safe Outcome $F^M = \underline{a}, m^M = a - \underline{a}$, implying $u^{M'}(F^M) = u^{M'}(a - m^M)$, which gives,

$$\mu_{RM} = \left(\frac{\theta}{\mathbb{E}(UD) - v_0} \right) \underbrace{\left(1 - v'(m^H - m^M)[1 - (1 - \alpha)(1 - \rho)] \right)}_{\geq 0}$$

thus μ_{RM} is positive if and only if $1 > v'(m^H - m^M)[1 - (1 - \alpha)(1 - \rho)]$. Condition i) and iii) gives $v' < \frac{1}{1 - (1 - \alpha)(1 - \rho)}$. Intuitively the upside of additional cash today cannot outweigh the additional compensation for the MMF’s extra risk.

(D-MMF) Case I.2 – Risky Outcome $F^M > \underline{a}, m^M$ solving MMF’s IR, implying $u^{M'}(F^M) < u^{M'}(a - m^M)$ and $\mu_{RM} = 0$, which gives,

$$0 = \left(\frac{\theta}{\mathbb{E}(UD) - v_0} \right) \left(v'(m^H - m^M) \frac{u^{M'}(F^M)}{u^{M'}(a - m^M)} [1 - (1 - \alpha)(1 - \rho)] - 1 \right)$$

and only occurs if the “large bracket term is zero”. For this version of the paper, I shall only focus on safe dealer money market contracts.

(D-MMF) Case II – $F^M \leq \underline{a}$: For all of these cases, the IR constraint implies $u^M(a - m^M) = u^M(F^M)$. Solving for μ from (5) again gives,

$$\mu = \frac{1}{u^{M'}(a - m^M)} \left[\left(\frac{\theta v'(m^H - m^M)}{\mathbb{E}(UD) - v_0} \right) + \mu_s + \mu_b \right] > 0$$

replacing back in equation (6) now gives,

$$0 = \underbrace{\left(\frac{\theta}{\mathbb{E}(UD) - v_0} \right) [v'(m^H - m^M) - 1]}_{>0} + \mu_s - \mu_{SM}$$

Therefore, if we assume that the self financing conditions do not bind ($\mu_s = 0$), there exist only one solution (since $v'(\cdot) > 1$): $F^M = \underline{a}$ and $m^M = a - \underline{a}$. The intuition is that when the repo is risk free, the MMF is indifferent between m^M and F^M , making the dealer's preference for cash today dominate, and thus pushing up the final debt payment.

Dealer-HF Problem (D-HF):

The more interesting problem comes from analyzing the interaction between the Dealer and HF. Assuming that the decision variables are positive, $m^M, F^M > 0$, equations (3) and (4) are reduced to,

$$\begin{aligned} & \left(\frac{\theta(v'(m^H - m^M) - G(m^H + F^H - W|s))}{\mathbb{E}(U^D) - v_0} \right) - \left(\frac{(1-\theta)(1 - \hat{G}(m^H + F^H - W))}{\mathbb{E}(U^H) - W} \right) \\ & \quad + \mu_s + \mu_b = 0 \\ & - \left(\frac{(1-\theta)(p(1 - \hat{G}(m^H + F^H - W|s)) + (1-p)(\hat{G}(F^H|i) - \hat{G}(m^H + F^H - W|i)))}{\mathbb{E}(U^H) - W} \right) \\ & \quad + \left(\frac{\theta(1 - G(m^H + F^H - W|s))}{\mathbb{E}(U^D) - v_0} \right) + \mu_b = 0 \end{aligned} \quad (7)$$

Note that replacing the second equation in the first gives the following condition,

$$\left(\frac{\theta(v'(m^H - m^M) - 1)}{\mathbb{E}(U^D) - v_0} \right) - \left(\frac{(1-\theta)(1-p)(1 - \hat{G}(F^H|i))}{\mathbb{E}(U^H) - W} \right) + \mu_s = 0 \quad (8)$$

Under a binary asset payoff, HF's optimism is reduced to $\hat{\alpha} > \alpha$; and there are three cases to consider: $\underline{a} + W \vee m^H + F^H$ and $\underline{a} \vee F^H$.²⁰ Note that given that HF optimism is with respect to the payoff probability and not the level of payoffs, the thresholds are the same for the dealer and HF. Moreover, note that the safe and risky multiplier μ_{RH}, μ_{SH} enter the FOC for m^H and F^H , therefore they cancel out when deriving equation (8).²¹ But the same cannot be said about $\bar{\mu}_F$ and $\underline{\mu}_F$ which appears in equation (8) with the sign changed from the original FOC equation.

Case I – $m^H + F^H \geq \underline{a} + W$: In this case, the reduced FOC translate to,

$$\begin{aligned} & \left(\frac{\theta(v'(m^H - m^M) - 1)}{\mathbb{E}(U^D) - v_0} \right) - \left(\frac{(1-\theta)((1-p) - (1-\hat{\alpha})(1-\rho))}{\mathbb{E}(U^H) - W} \right) + \mu_s = 0 \\ & \left(\frac{\theta \left(1 - \frac{\rho(1-\alpha)}{p} \right)}{\mathbb{E}(U^D) - v_0} \right) - \left(\frac{(1-\theta)p \left(1 - \frac{\rho(1-\hat{\alpha})}{p} \right)}{\mathbb{E}(U^H) - W} \right) + \mu_b + \mu_{RH} = 0 \end{aligned}$$

(D-HF) Case I.1 – $m^H + F^H > \underline{a} + W$: Assume that the self financing conditions do not bind ($\mu_s = \mu_b = 0$) and that the risky threshold is slack; the above equations can two different expressions for $\frac{(1-\theta)(\mathbb{E}(U^D) - v_0)}{\theta(\mathbb{E}(U^H) - W)}$. Equalizing these expressions gives,

$$\frac{pv'(m^H - m^M) - \rho(1-\alpha)}{p - \rho(1-\alpha)} = \frac{\hat{\alpha}}{p - \rho(1-\hat{\alpha})}$$

which is equation (2). Note that this expression only depends on m^H and m^M , that is, final repayments are irrelevant. Intuitively, it trades off the cost borne by the HF due to the dealer's default and the dealer's preference for cash today, equalizing both players marginal rate of substitution between m^H and F^H . Rearranging terms gives $v' = \frac{\hat{\alpha}}{p} \left\{ \frac{p-\rho(1-\alpha)}{p-\rho(1-\hat{\alpha})} \right\} + \frac{\rho}{p}(1-\alpha)$. Having a condition for m^H , in this case the equation to solve for F^H is given by,

$$F^H = \theta F_{MonoD} + (1-\theta) F_{MonoH}$$

where F_{MonoD} is the dealer's monopolist solution (i.e., making the HF break even) and F_{MonoH} is the HF's monopolist solution

²⁰Note that $\underline{a} + W - m^H > \underline{a}$.

²¹Just like μ_b .

(i.e., making the dealer break even). In effect, with a risky contract the firms' payoffs are given by,

$$\begin{aligned}\mathbb{E}(U^D) - v_0 &= v(m^H - m^M) - v_0 + \left(\left(1 - \frac{\rho(1-\alpha)}{p}\right) F^H + \frac{\rho(1-\alpha)}{p} (\underline{a} + W - m^H) - F^M \right) \\ \mathbb{E}(U^H) - W &= p \left(1 - \frac{\rho(1-\hat{\alpha})}{p}\right) (\bar{a} + W - m^H - F^H) + (1-p) \left(1 - \frac{(1-\rho)(1-\hat{\alpha})}{1-p}\right) (W - m) - W\end{aligned}$$

note F^H appears linearly in both expressions, thus we can separate them in terms grouped by θ and $1 - \theta$. In that case, the Dealer's payoff evaluated in F_{MonoH}^H is zero and the HF's payoff evaluated in F_{MonoD}^H is zero. Finally, using expression for the ratio between payoffs which doesn't involve the Dealer's preference for cash today v , resulting in,

$$p \left(1 - \frac{\rho(1-\hat{\alpha})}{p}\right) (1-\theta) (\mathbb{E}(U^D) - v_0) = \left(1 - \frac{\rho(1-\alpha)}{p}\right) \theta (\mathbb{E}(U^H) - W)$$

where, conditional on being a risky contract, the utility functions are linear. Swapping F_{MonoD} with F_{MonoH} which are pre multiplied by $p \left(1 - \frac{\rho(1-\hat{\alpha})}{p}\right) (1-\theta)$ and $\left(1 - \frac{\rho(1-\alpha)}{p}\right) \theta$ respectively, gives the original expression of each agents payoff so that they break even, implying the solution in fact does solve the FOC's.

Finally, to ensure that this in fact is a solution, it's necessary to verify that in fact $F^H > \underline{a} + W - m^H$. Each sub expression of F^H takes the following form,

$$\begin{aligned}F_{MonoD} &= \bar{a} - \frac{((1-\hat{\alpha})W + \hat{\alpha}m^H)}{p - \rho(1-\hat{\alpha})} \\ F_{MonoH} &= \frac{v_0 - v(m^H - m^M) - \frac{\rho(1-\alpha)}{p}(\underline{a} + W - m^H) + F^M}{\left(1 - \frac{\rho(1-\alpha)}{p}\right)}\end{aligned}$$

Note that F_{MonoD} will satisfy the condition if $p(\hat{a} - \underline{a}) > W(p\hat{\alpha} + (1-\hat{\alpha})) + (1-p)\hat{\alpha}m^H$, since F_{MonoD} is decreasing in ρ and $\rho \leq p$.²² That is, the HF's optimism has to be high enough for it to assume the loss in case of default. Therefore if the Dealer has significant market power, then the weighted average can be above the threshold.

(D-HF) Case I.2- $m^H + F^H = \underline{a} + W$: Assume that the self financing conditions do not bind ($\mu_s = \mu_b = 0$) and that the risky threshold binds. Therefore we have the following equations,

$$\begin{aligned}\frac{(1-\theta) (\mathbb{E}(U^D) - v_0)}{\theta(\mathbb{E}(U^H) - W)} &= \frac{(v'(m^H - m^M) - 1)}{(1-p) - (1-\rho)(1-\hat{\alpha})} \\ \frac{(1-\theta) (\mathbb{E}(U^D) - v_0)}{\theta(\mathbb{E}(U^H) - W)} - \frac{(p - \rho(1-\alpha))}{p(p - \rho(1-\hat{\alpha}))} &= \hat{\mu}_{RH}.\end{aligned}$$

For this to be a solution it's necessary to have, $\frac{p(v'-1)}{(1-p)-(1-\rho)(1-\hat{\alpha})} > \frac{p-\rho(1-\alpha)}{p-\rho(1-\hat{\alpha})}$. This proposed equilibrium would have $m^H + F^H = \underline{a} + W$.

(D-HF) Case II - $F^H \in [\underline{a}, \underline{a} + W - m^H]$: In this case, the reduced FOC translate to,

$$\begin{aligned}\left(\frac{\theta(v'(m^H - m^M) - 1)}{\mathbb{E}(U^D) - v_0}\right) - \left(\frac{(1-\theta)((1-p) - (1-\hat{\alpha})(1-\rho))}{\mathbb{E}(U^H) - W}\right) + \mu_s - \bar{\mu}_F &= 0 \\ \left(\frac{\theta}{\mathbb{E}(U^D) - v_0}\right) - \left(\frac{(1-\theta)(p + (1-\hat{\alpha})(1-\rho))}{\mathbb{E}(U^H) - W}\right) + \mu_b - \mu_{SH} &= 0\end{aligned}$$

(D-HF) Case II.1 - $F^H \in (\underline{a}, \underline{a} + W - m^H)$: Assume that the self financing conditions do not bind ($\mu_s = \mu_b = 0$) and that repurchase price restrictions are slack. The original FOC equations give two different expressions for $\frac{(1-\theta)(\mathbb{E}(U^D) - v_0)}{\theta(\mathbb{E}(U^H) - W)}$.

²²Note that since $W > m^H$, the condition $p(\hat{a} - \underline{a}) > W$ is enough to ensure the above occurs.

Equalizing these expressions gives,

$$\frac{v'(m^H - m^M) - 1}{(1-p) - (1-\hat{\alpha})(1-\rho)} = \frac{1}{p + (1-\hat{\alpha})(1-\rho)}. \quad (9)$$

Rearranging equation (9) gives $v' = \frac{1}{p+(1-\hat{\alpha})(1-\rho)}$. Note that this expression only depends on m^H and m^M , that is, final repayments are irrelevant. Having a condition for m^H , in this case the equation to solve for F^H is given by,

$$F^H = \theta F_{MonoD} + (1-\theta)F_{MonoH}$$

where F_{MonoD} is the Dealer's monopolist solution (i.e., making the HF break even) and F_{MonoH} is the HF's monopolist solution (i.e., making the Dealer break even). In effect, with $F^H \in (\underline{a}, \underline{a} + W - m^H)$ the firms' payoffs are given by,

$$\begin{aligned} \mathbb{E}(U^D) - v_0 &= v(m^H - m^M) - v_0 + F^H - F^M \\ \mathbb{E}(U^H) - W &= (p - \rho(1 - \hat{\alpha}))\bar{a} + (1 - \hat{\alpha})\underline{a} - (p + (1 - \hat{\alpha})(1 - \rho))F^H - m^H. \end{aligned}$$

F^H appears linearly in both expressions, thus we can separate them in terms grouped by θ and $1 - \theta$. In that case, the Dealer's payoff evaluated in F_{MonoH} is zero and the HF's payoff evaluated in F_{MonoD} is zero. Evaluating F^H in the expression involving the ratio of excess payoffs verifies that the conjectured repurchase price satisfies the FOC's.

To ensure that this in fact is a solution, it's necessary to verify that in fact $F^H \in (\underline{a}, \underline{a} + W - m^H)$. Each sub expression of F^H takes the following form,

$$\begin{aligned} F_{MonoD} &= \frac{(p - \rho(1 - \hat{\alpha}))\bar{a} + (1 - \hat{\alpha})\underline{a} - m^H}{p + (1 - \hat{\alpha})(1 - \rho)} \\ F_{MonoH} &= v_0 - v(m^H - m^M) + F^M. \end{aligned}$$

(D-HF) Case II.2 – $F^H = \underline{a} + W - m^H$: Assume that the self financing conditions do not bind ($\mu_s = \mu_b = 0$) the optimal contract is marginally safe. The first equation of this case gives an expression for $\frac{(1-\theta)(\mathbb{E}(U^D)-v_0)}{\theta(\mathbb{E}(U^H)-W)}$, which included in the second FOC equation implies,

$$\frac{1}{p + (1 - \hat{\alpha})(1 - \rho)} - \frac{(v'(m^H - m^M) - 1)}{(1-p) - (1 - \hat{\alpha})(1 - \rho)} = \hat{\mu}_{SH}$$

Therefore this solution is optimal in this case when $\frac{1}{p+(1-\hat{\alpha})(1-\rho)} > \frac{(v'-1)}{(1-p)-(1-\hat{\alpha})(1-\rho)}$. The proposed equilibrium would have $F^{H*} + m^{H*} = \underline{a} + W$ and m^{H*} would be pinned down by the ratio of excess payoffs equal to $\frac{v'(m^{H*} - m^M) - 1}{(1-p) - (1-\hat{\alpha})(1-\rho)}$.

(D-HF) Case II.3 – $F^H = \underline{a}$: Assume that the self financing conditions do not bind ($\mu_s = \mu_b = 0$) and that the threshold on F^H binds. The second equation of this case gives an expression for $\frac{(1-\theta)(\mathbb{E}(U^D)-v_0)}{\theta(\mathbb{E}(U^H)-W)}$, which included in the first FOC equation implies,

$$\frac{v'(m^H - m^M) - 1}{(1-p) - (1 - \hat{\alpha})(1 - \rho)} - \frac{1}{p + (1 - \hat{\alpha})(1 - \rho)} = \hat{\mu}_F$$

Therefore this solution is optimal in this case when $\frac{v'-1}{(1-p)-(1-\hat{\alpha})(1-\rho)} > \frac{1}{p+(1-\hat{\alpha})(1-\rho)}$. This proposed equilibrium would have $F^{H*} = \underline{a}$, and m^{H*} would be pinned down by the ratio of excess payoffs equal to $\frac{1}{p+(1-\hat{\alpha})(1-\rho)}$.

(D-HF) Case III – $F^H \leq \underline{a}$: In this case, the reduced FOC translate to,

$$\begin{aligned} \left(\frac{\theta(v'(m^H - m^M) - 1)}{\mathbb{E}(U^D) - v_0} \right) - \left(\frac{(1-\theta)(1-p)}{\mathbb{E}(U^H) - W} \right) + \mu_s + \underline{\mu}_F &= 0 \\ \left(\frac{\theta}{\mathbb{E}(U^D) - v_0} \right) - \left(\frac{(1-\theta)p}{\mathbb{E}(U^H) - W} \right) + \mu_b &= 0 \end{aligned}$$

(D-HF) Case III.1 – $F^H < \underline{a}$: Assume that the self financing conditions do not bind ($\mu_s = \mu_b = 0$) and that repurchase price restriction is slack. The original FOC equations give two different expressions for $\frac{(1-\theta)(\mathbb{E}(U^D)-v_0)}{\theta(\mathbb{E}(U^H)-W)}$. Equalizing these

expressions gives,

$$v'(m^H - m^M) - 1 = \frac{(1-p)}{p}$$

Note that this expression only depends on m^H and m^M , that is, final repayments are irrelevant. Having a condition for m^H , in this case the equation to solve for F^H is given by,

$$F^H = \theta F_{MonoD} + (1-\theta)F_{MonoH}$$

where F_{MonoD} is the Dealer's monopolist solution (i.e., making the HF break even) and F_{MonoH} is the HF's monopolist solution (i.e., making the Dealer break even). In effect, with a safe contract the firms' payoffs are given by,

$$\begin{aligned} \mathbb{E}(U^D) - v_0 &= v(m^H - m^M) - v_0 + F^H - F^M \\ \mathbb{E}(U^H) - W &= p(\hat{a} - F^H) - m^H. \end{aligned}$$

Note F^H appears linearly in both expressions, thus we can separate them in terms grouped by θ and $1-\theta$. In that case, the Dealer's payoff evaluated in F_{MonoH} is zero and the HF's payoff evaluated in F_{MonoD} is zero. Evaluating F^H in the expression involving the ratio of excess payoffs verifies that this repurchase price satisfies the FOC's.

To ensure that this in fact is a solution, it's necessary to verify that in fact $F^H < \underline{a}$. Each sub expression of F^H takes the following form,

$$\begin{aligned} F_{MonoD} &= \hat{a} - \frac{m^H}{p} \\ F_{MonoH} &= v_0 - v(m^H - m^M) + F^M \end{aligned}$$

in this case it would be difficult to have the Dealer's monopolist solution satisfy the bound. In effect, one would need $\hat{a} - \frac{(1-p)m^H}{p} < \underline{a} + W$, which would be unlikely considering $m^H < W$. The case for the HF's monopolist solution with the MMF's safe solution (i.e., $F^M = \underline{a}$) then $F_{MH}^H = v_0 - v(m^H - m^M) + \underline{a} < \underline{a} + W - m^H$ always holds. In this case, the solution satisfies the condition if the HF has relatively more market power. Note that it would also need to be verified that the self financing conditions hold, but I shall leave that analysis for when I consider both solutions in conjunction.

(D-HF) Case III.2- $F^H = \underline{a}$: Assume that the self financing conditions do not bind ($\mu_s = \mu_b = 0$) and that the threshold on F^H binds. Therefore we have the following equations,

$$\begin{aligned} \left(\frac{(1-\theta)(\mathbb{E}(U^D) - v_0)}{\theta(\mathbb{E}(U^H) - W)} \right) - \left(\frac{v'(m^H - m^M) - 1}{(1-p)} \right) &= \hat{\mu}_F \\ \left(\frac{(1-\theta)(\mathbb{E}(U^D) - v_0)}{\theta(\mathbb{E}(U^H) - W)} \right) &= \frac{1}{p} \end{aligned}$$

therefore for this to be an equilibrium $\frac{1-p}{p} > v'(m^H - m^M) - 1$. This proposed equilibrium would have $F^H = \underline{a}$ and m^{H*} would be pinned down by the ratio of excess payoffs equal to $1/p$.

Bringing the Dealer-HF Problem & Dealer-MMF Problem:

The proposed safe solution in the Dealer-MMF is in fact an equilibrium iff:

$$v'(m^H - m^M) \in \left(1, \frac{1}{1 - (1-\alpha)(1-\rho)} \right) \quad (10)$$

and any proposed solution of the Dealer-HF is in fact an equilibrium iff:

$$v'(m^H - m^M) \in \left[\frac{\hat{\alpha}}{p} \left\{ \frac{p - \rho(1 - \alpha)}{p - \rho(1 - \hat{\alpha})} \right\} + \frac{\rho}{p}(1 - \alpha), \frac{1}{p} \right]$$

For a risky or marginal safe solution to in fact exist²³, I need to impose:

$$\frac{\hat{\alpha}}{p} \left\{ \frac{p - \rho(1 - \alpha)}{p - \rho(1 - \hat{\alpha})} \right\} + \frac{\rho}{p}(1 - \alpha) < \frac{1}{1 - (1 - \alpha)(1 - \rho)}$$

which is condition i).

Thus under this parameterization, for a high enough θ , given assumption ii) the optimal contract, with m^H equalizing the marginal rates of substitution characterized in equation (2):

$$\frac{pv'(m^H - (a - \underline{a}) - \rho(1 - \alpha))}{p - \rho(1 - \alpha)} = \frac{\hat{\alpha}}{p - \rho(1 - \hat{\alpha})}$$

and F^H close to the dealer's monopolist repurchase price, leading to a hedge fund default in case of a bad asset outcome. This is Case I.1 in the Dealer-HF problem and results in feasible $m \leq W$ due to assumption iii).

To keep the analysis simple, the Theorem only characterized equilibria where the MMF has a riskless contract and that at best, the hedge fund has a risky contract or marginally safe contract. The latter occurs whenever $m^H + F^H = \underline{a} + W$, which is Case II.2. For θ close to 1, the equilibrium is characterized by Case I.1. As θ decreases, the repurchase price falls till the optimal contract becomes riskless (i.e., $\underline{a} + W = m^H + F^H$), which occurs whenever $\theta = \theta^S$. Then, the equilibrium is characterized by Case II.2, where the margin is pinned down by,

$$\frac{(1 - \theta)(\mathbb{E}(U^D) - v_0)}{\theta(\mathbb{E}(U^H) - W)} = \frac{v' - 1}{(1 - p) - (1 - \hat{\alpha})(1 - \rho)} \quad (11)$$

with $F^H = \underline{a} + W - m^H$.

As θ decrease the equilibrium characterized by equation (11) is sustained till one of the following conditions are met: Either the optimal m^H is low enough for the equilibrium to be characterized by Case II.1 (that is, whenever $\frac{v' - 1}{(1 - p) - (1 - \hat{\alpha})(1 - \rho)} = \frac{1}{p + (1 - p)\alpha}$), or the optimal m^H implies a risky MMF contract (that is, $v' \geq \frac{1}{1 - (1 - \alpha)(1 - \rho)}$). The lower bound $\underline{\theta}$ is defined as the minimum θ such that both these conditions are met. Thus, the Theorem has two equilibrium characterizations: one for $\theta \in (\underline{\theta}, \theta^S]$ given by Case I.1 and another for $\theta \in [\theta^S, 1)$ given by Case II.2. ■

Proof of Proposition 1: The result stems by taking the implicit derivative of the equilibrium conditions when $\theta = \underline{\theta}$. Recall that $\underline{\theta}$ is defined as the smallest value of θ such that a riskless money market solution is possible, i.e.,

$$\frac{v'(m^H - m^M) - 1}{1 - p} = \frac{1 - \alpha}{p + (1 - p)\alpha}.$$

For this value of θ the equilibrium is characterized by $m^H + F^H = W + \underline{a}$ and

$$\frac{(1 - \underline{\theta})(\mathbb{E}(U^D) - v_0)}{\underline{\theta}(\mathbb{E}(U^H) - W)} = \frac{v'(m^{H*} - m^{M*}) - 1}{\hat{\alpha}(1 - p)}.$$

giving the following equality:

$$\hat{\alpha}(1 - \underline{\theta})(p + (1 - p)\alpha)(\mathbb{E}(U^D) - v_0) = \underline{\theta}(1 - \alpha)(\mathbb{E}(U^H) - W).$$

²³Where “marginal safe” implies that $m^H + F^H = \underline{a} + W$

First, considering the original model and deriving with respect to p gives,

$$\begin{aligned} \frac{\partial \underline{\theta}}{\partial p} \left[(p + (1-p)\alpha)(\mathbb{E}(U^D) - v_0) + (1-\alpha)(\mathbb{E}(U^H) - W) \right] &= \hat{\alpha}(1-\underline{\theta})(1-\alpha)(\mathbb{E}(U^D) - v_0) - \underline{\theta}(1-\alpha) \frac{\partial(\mathbb{E}(U^H) - W)}{\partial p} + \\ &\quad \left[\hat{\alpha}(1-\underline{\theta})(p + (1-p)\alpha)(v'(m^H - m^M) - 1) + \underline{\theta}(1-\alpha)(1-p)\hat{\alpha} \right] \frac{\partial m}{\partial p}. \end{aligned}$$

Note that due to the equilibrium condition $(p + (1-p)\alpha)(v'(m^H - m^M) - 1) = (1-\alpha)(1-p)$. Also, given that in this equilibrium, the optimal dealer hedge fund contract is safe, $\frac{\partial(\mathbb{E}(U^H) - W)}{\partial p} = \hat{\alpha}(\bar{a} - F^H)$, therefore,

$$\frac{\partial(\mathbb{E}(U^H) - W)}{\partial p} = (\mathbb{E}(U^H) - W) + \underbrace{(1-p)\hat{\alpha}(\bar{a} - F^H) + m^H - (1-\hat{\alpha})(\underline{a} - F^H)}_{:=\Gamma > 0}.$$

Using these last two observations to solve for the partial derivative of $\underline{\theta}$,

$$\frac{\partial \underline{\theta}}{\partial p} = \frac{\hat{\alpha}(1-\underline{\theta})(1-\alpha)(\mathbb{E}(U^D) - v_0) - \underline{\theta}(1-\alpha)(\mathbb{E}(U^H) - W) - \Gamma + (1-\alpha)(1-p) \frac{\partial m}{\partial p}}{[(p + (1-p)\alpha)(\mathbb{E}(U^D) - v_0) + (1-\alpha)(\mathbb{E}(U^H) - W)]}.$$

Note that $\hat{\alpha}(1-\underline{\theta})(1-\alpha)(\mathbb{E}(U^D) - v_0) - \underline{\theta}(1-\alpha)(\mathbb{E}(U^H) - W) < 0$ for p sufficiently large, given the initial equation to pin down $\underline{\theta}$. Therefore, a p sufficiently close to 1 implies $\frac{\partial \underline{\theta}}{\partial p} < 0$.

■

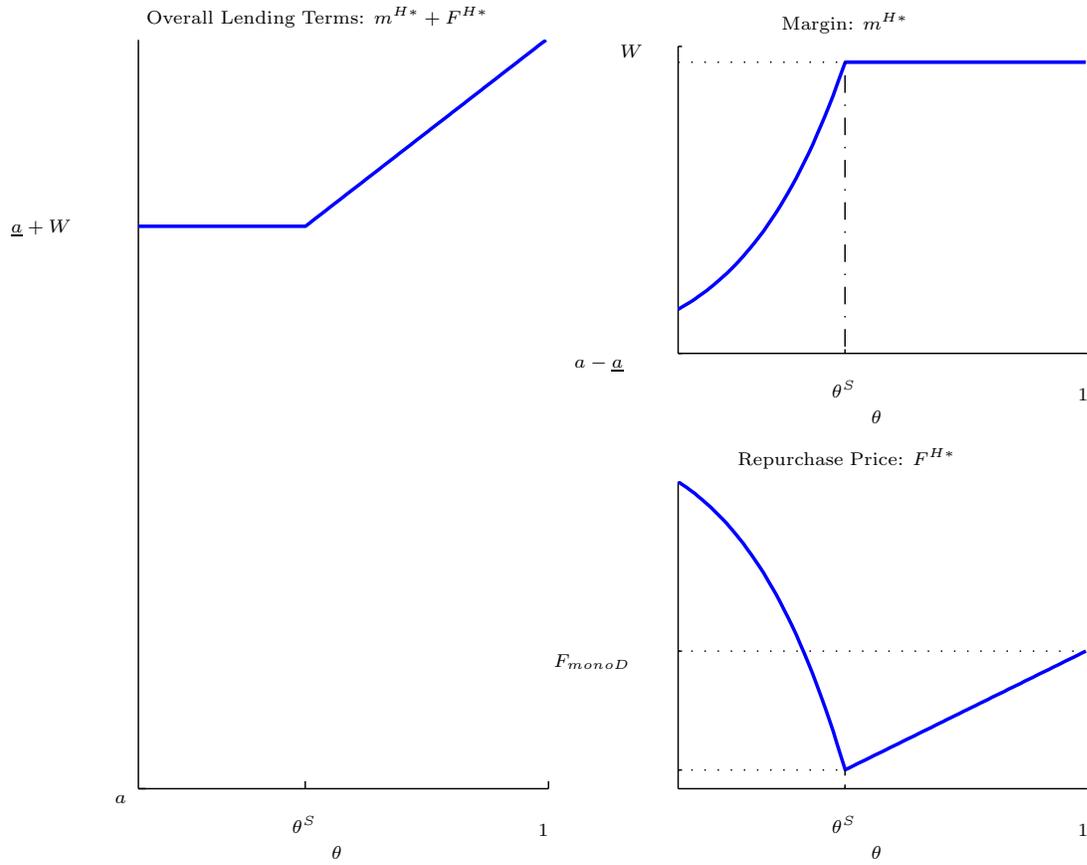


Figure 4: Dealer-Hedge Fund Nash bargaining solution for different levels of dealer market power. Left plot shows the overall lending terms on the contract. Right top plots show the contract split into the margin (top) and repurchase price (bottom). As the dealer's market power decreases, the overall lending terms are reduced. For relatively $\theta > \theta^S$ the resulting contract can induce the hedge fund to default. For $\theta \leq \theta^S$ the contract is riskless (i.e., $m^{H^*} + F^{H^*} = \underline{a} + W$). The parameter assumptions are $a = 100, \bar{a} = 120, \alpha = .35, \hat{\alpha} = .651, p = .975$ and $W = 18$; and considering $v(x) = \frac{\bar{a}^\eta (x)^{1-\eta}}{1-\eta}$ with $\eta = 1/300$.

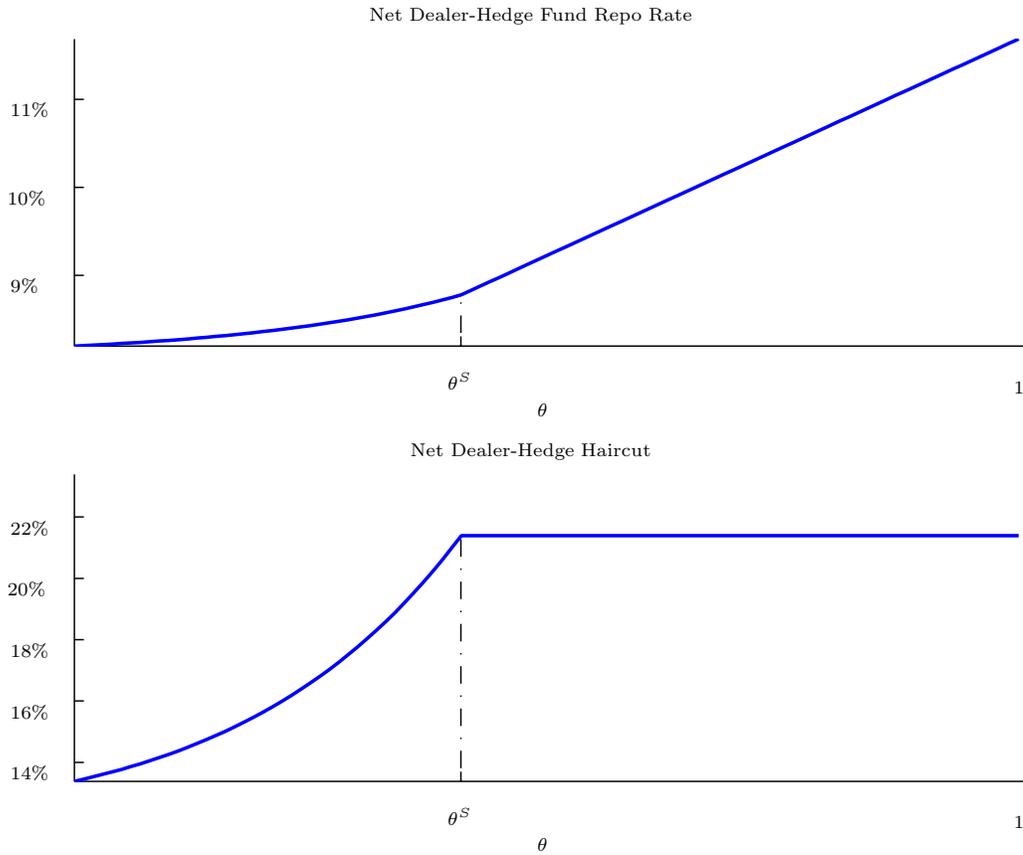
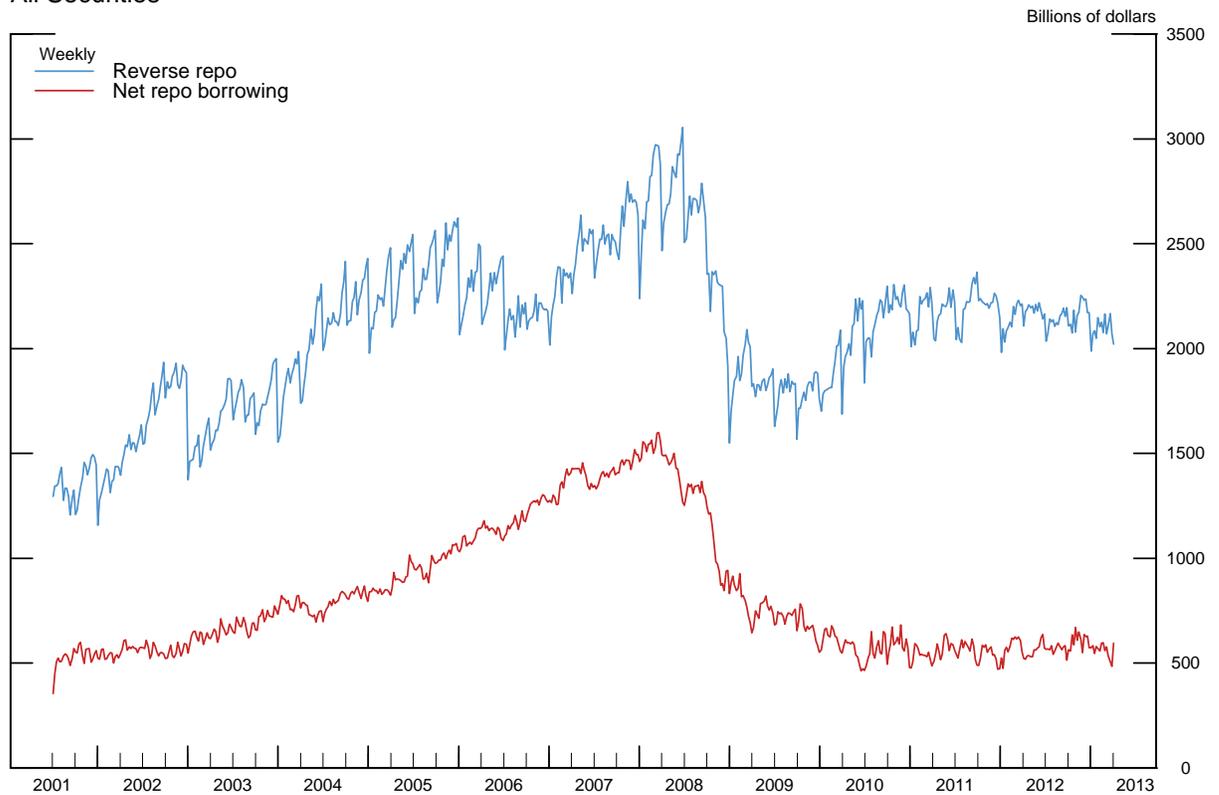


Figure 5: Dealer-Hedge Fund Nash bargaining solution for different levels of dealer market power. Top plot shows the implied repo rate. Bottom plot shows the implied haircut. The parameter assumptions are $a = 100$, $\bar{a} = 120$, $\alpha = .35$, $\hat{\alpha} = .651$, $p = .975$ and $W = 18$; and considering $v(x) = \frac{a^{\eta(x)^{1-\eta}}}{1-\eta}$ with $\eta = 1/300$.

Dealer Net Repo Borrowing by Security Type (July 2001 - March 2013)

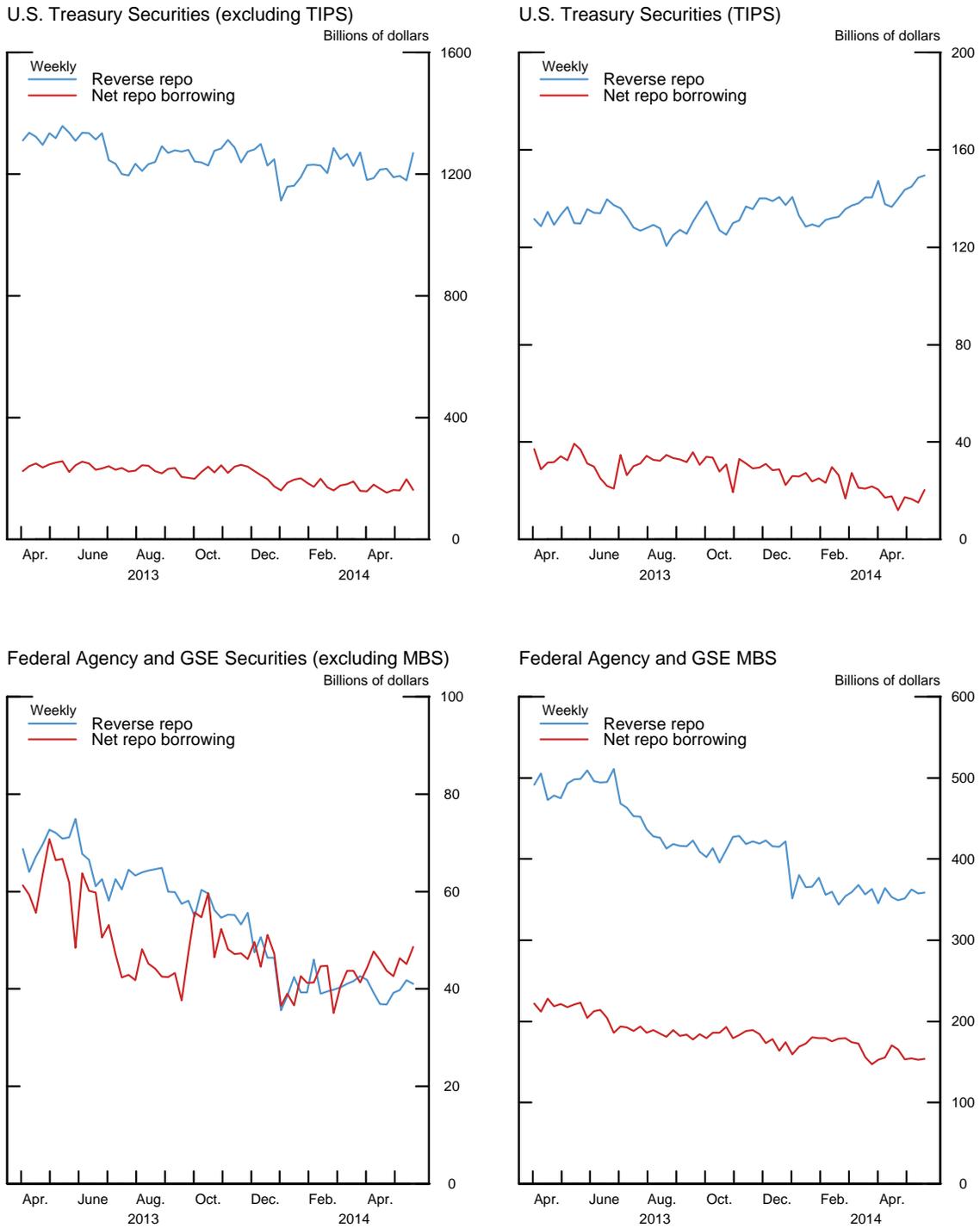
All Securities



Note: Data are through March 27, 2013. Dealer net repo borrowing includes all dealer financing through overnight and term repurchase and reverse repurchase agreements (i.e., securities out - securities in).
Source: FR 2004C.

Figure 6

Dealer Net Repo Borrowing by Security Type (April 2013 - May 2014)



Note: Data are through May 21, 2014. The FR 2004 was revised effective April 1, 2013. Dealer net repo borrowing includes all dealer financing through overnight and term repurchase and reverse repurchase agreements (i.e., securities out - securities in).
 Source: FR 2004C.

Figure 7