Trickle-Down Consumption, Monetary Policy, and Inequality*

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-PRELIMINARY-

Abstract

We propose a model that extends the limited asset market participation (LAMP) framework of Gali et al. (2004) and Bilbiie (2008) by introducing trickle-down externalities (TCE), whereby households preferences are assumed to depend on consumption by a reference group, or “the Joneses” (namely the higher income households) in the spirit of Bertrand and Morse (2013). We pose two questions: first, does monetary policy exacerbate consumption inequality? Second, does the presence of LAMP, TCE, and consumption inequality affect the design of optimal monetary policy? Our answer is affirmative to both questions. Hence, the model provides novel considerations for monetary policy intervention.

Keywords: Trickle-Down Consumption; Rule-of-Thumb Consumers; Limited Asset Market Participation; Interest Rate Rules; Consumption Inequality.

JEL Classifications: E4, E5

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1 Introduction

One fact about consumption inequality in the US is indisputable: it is increasing. The revamped interest in inequality from an academic point of view, was probably sparked with the pioneering work of Piketty and Saez (2003) on income inequality, and, more recently, was amplified in the public debate by the great recession of 2008-09 and the 99% movement.\textsuperscript{1}

The main aim of this paper is to embed macroeconomic considerations about consumption inequality into a DSGE model with heterogeneous agents, and a central bank that follows a Taylor rule. We try to assess whether it is possible for monetary policy to have a re-distributive role. In particular, we are interested in investigating the interplay between limited market asset participation and consumption externalities in an otherwise standard Neo-Keynesian model and study, under those assumptions, whether and the extent to which consumption inequality is amplified by monetary policy shocks.

In our model wealth inequality is exogenous and we do not aim to explain it. However, consumption and labor are endogenous choices for two kind of agents. The first group of households has their consumption choices directly affected by the monetary authority and its interest rate rule because they can smooth their consumption path inter-temporally. The other group of households cannot dynamically optimize their consumption bundles instead, and thus it is affected only indirectly by this rule through its effect on real wages and as far as it reaches the output stabilization objective. However, the presence of consumption externality established another channel that can influence the second set of agents through its effect on the behavior of the first group of agents.

Using the Gini coefficient and data from the Consumer Expenditure Survey, Fisher et al. (2013) find that consumption inequality in the US has increased over the 1985-2010 period by 6.4 percent. Aguiar and Bils (2011) dispute to some extent early findings by Krueger and Perri (2006) that consumption inequality does not mirror income inequality in the US: they show that both relative before and after-tax income inequality increased by about 33 percent between 1980 and 2010, where their measure of income inequality is the ratio of those in the 80-95th percentiles to those in the 5-20th percentiles. Using a measure of consumption inequality based on how high- versus low-income households allocate spending toward luxuries versus necessities, they also find an increase in consumption inequality of almost 43 percent over their sample period. An analysis for the 2000-2011 period by Meyer and Sullivan (2013) finds that the pattern followed by consumption inequality before and after the great recession of 2008-09 heavily depends on the percentiles of the US income distribution: there is an asymmetry in the way consumption changed over time depending on where within the income distribution individuals and households are located.

If it did not carry any negative externality or impair growth, why would consumption inequality increase over the past three decades?\textsuperscript{1}

\textsuperscript{1}Using individual tax returns data, Piketty and Saez (2003) show that top income and wages display an inverted Kuznets curve over the twentieth century in the US and that income inequality has been steadily increasing since the 70s.

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matter, per se, as long as real aggregate income and consumption keep rising (or at least not decreasing) over time like we have observed in the US since at least WWII? The answer probably lies in the fact that people evaluate their well-being not in absolute terms, but relative to others. This conjecture has its roots in the idea of Veblen (2005) and the work of Duesenberry (1949).

As first observed by Bertrand and Morse (2013), the increase in income inequality documented in the studies cited above has as a by-product outcome the fact that, over time, the median household in the US has been increasingly exposed to rich (or very rich) households. They empirically document at the micro level that a consequence of this phenomenon gives rise to a peculiar behavior: middle and lower income households end up consuming a larger share of their current income when exposed to individuals belonging to the upper part of the income and consumption distributions. They label this empirical regularity as \textit{trickle-down consumption}. This behavior, according to their computations, could for example explain approximately 25 percent of the decrease in US saving since the 80s.

In this paper we pose and try to answer two simple, but yet crucial questions. The first one is: can monetary policy exacerbate consumption inequality in a stochastic dynamic general equilibrium model? To answer this question, we propose a model that extends the Neo-Keynesian framework of Bilbiie (2008) with exogenous limited asset market participation (LAMP) coming from the agents by introducing trickle-down consumption externalities (TCE), whereby households preferences are assumed to depend on consumption by a reference group, or “the Joneses” (namely the higher income households) in the spirit of Bertrand and Morse (2013). We obtain the following results: we derive an analytical expression for the Gini coefficient for consumption inequality, showing that it is counter-cyclical (a qualitative feature also present in the data). The index counter-cyclicality is reinforced for higher degrees of LAMP and TCE. Quantitatively we find that, under the benchmark calibration, a 1% (yearly) increase in the policy rate (a contractionary monetary policy shock) raises the Gini coefficient by about, 0.8 percent. This result is consistent with the recent empirical evidence provided by Coibion et al. (2012). The adverse effect of monetary policy on consumption inequality is increasing in both LAMP and TCE, but decreasing in the aggregate Frisch elasticity of labor supply.

The second question we are interested in is whether the presence of trickle-down consumption and consumption inequality affect the design of optimal monetary policy under discretion. We find that the TCE parameter affects both the relative cost of inflation versus output gap stabilization, and the policy-marker’s preference for inflation stabilization in its policy objective. In particular, by flattening the Phillips curve, a stronger externality makes inflation stabilization more costly. In this sense, our model provides an explanation for the decrease in the slope of the Phillips curve recently documented by Coibion and Gorodnichenko (2013). However, the presence of TCE also makes inflation stabilization more desirable, i.e., it raises the relative weight of inflation volatility in the welfare-based loss function. We evaluate quantitatively this marginal cost- marginal benefit
trade-off, showing that the optimal relative volatility of inflation to output is strictly decreasing in the extent of the TCE, for any degree of LAMP. In other words, the presence of TCE requires the policy-maker to focus more on price stability.

Several studies formulate an explanation for the rise in consumption inequality, many of those studies focus on fiscal policy as the usual culprit capable of redistributing resources and thus of explaining the staggering patterns outlined above; other explanations are skill biased technological change (Bound and Johnson (1992)), trade "globalization" (Fenstra and Hanson (2001)), and the presence of unions (Card (2000)). However, to the best of our knowledge, there are only very few studies on the topic of monetary policy and inequality. The first one is the one from Coibion et al. (2012). This study has at least two fundamental merits: first, it is an important empirical paper that investigates the dynamic responses of measures of consumption and income inequality to monetary policy shocks. Second, it fleshes out a list of four potential channels through which monetary policy could affect inequality in theory beside the classical savings redistribution channel (between lenders and borrowers); those are:

1. The income composition channel: in the presence of heterogeneity in the income composition across households, those with more financial/business income as a share of their total income could be affected differently by monetary policy from those who have labor income as their primary/only source of income.

2. The financial segmentation channel whereby there is an asymmetry across agents in the degree to which they are “connected” to the central bank handling of the money supply. The idea here is to think of the extent of the frequency at which some agents trade in the financial markets with respect to other non trading agents. The former are clearly potentially more affected by changes in the monetary policy with respect to the latter.

3. The portfolio channel whereby low income agents hold on to more currency rather than alternative assets when compared to higher income agents.

4. The earnings heterogeneity channel whereby high and low labor earnings may be impacted differently by monetary policy shocks.

Our paper complements Coibion et al. (2012) in one important dimension: we provide a complete and parsimonious DSGE model that can be used, to some extent, to identify and quantitatively assess in a more disciplined way the importance of some of the mechanisms mentioned above. The model we propose is capable of capturing the first two (and to some extent also the third) channels of monetary policy transmission on consumption inequality outlined above and to gauge numerically their importance. We show that the presence of trickle-down consumption could have large and pervasive effects in the way consumption inequality respond to monetary policy shocks.
Another relevant paper on the effect of monetary policy and inequality is Romer and Romer (1998). This is probably the first paper that empirically investigates the effects of monetary policy on poverty in the long run and in the short run in the US and across countries. The effect of monetary policy on inequality in the short run is downplayed in the paper. For the long run, Romer and Romer (1998) regressions find that among industrialized countries there is a significant relationship between average inflation and the well-being of the poor: the long-run performance of monetary policy can be powerful to fight poverty. An important caveat of the message in that paper is that their results establish mostly correlations, not causation, and that they are obtained from cross-country regression with all the standard limitations of that type of analysis.

An impressive recent paper on the redistributive channel of monetary policy is Gornemann et al. (2012). They calibrate an incomplete markets general equilibrium New Keynesian sticky price model with agents that display heterogeneity in employment status, labor income, and savings. In their framework, monetary policy can effect the income and consumption distributions asymmetrically. For example, they find that a monetary tightening of 1 percent point induces a gain to the top 5% of the wealth distribution whereas it induces a loss of income/consumption to those at the bottom 5%.

One last paper related to ours is Motta and Tirelli (2013). They build a LAMP model with physical capital accumulation and catching up with the Joneses to study the dynamic stability properties of the steady state and they analyze the redistributive outcome of fiscal policies on income/consumption inequality. Their analysis is not concerned with the power of monetary policy in redistributing income/consumption.

The rest of the paper is organized as follows. Section 2 presents the building blocks of the model. Section 3 defines the aggregate equilibrium and the log-linearization around the steady state with its determinacy analysis. Section 4 provides an analysis of the relationship between monetary policy and consumption inequality. In particular, we formalize a quantitative evaluation of a contractionary monetary policy shock on key aggregate economic variables and we assess the role of LAMP and consumption externalities for inequality. In this section we also investigate the implications of TCE and LAMP for the optimal monetary policy stance. Section 5 concludes. All the proofs are in the Appendix.

2The only result they reach in a framework comparable to one of our results goes in the polar opposite direction. In the only numerical analysis they conduct on the issue of monetary policy shocks, they obtain that in the presence of LAMP and without fiscal policy, an interest rate contractionary shock induces an increase in output, and an increase in consumption for Ricardian agents. We arrive at a totally different conclusion. Our result is in line with the empirical evidence put forth by Coibion et al. (2012).
2 The Model

In this paper we build upon Airaudo and Bossi (2013) who modify Bilbiie (2008) New Keynesian setting without physical capital. This framework allow us, among other things, to prove analytically that a steady state exists, that it is locally determinate under some conditions, and to identify the monetary policy transmission channels inducing inequality. In this section we outline the building blocks of our environment.

2.1 Households

Consider an infinite horizon economy populated by a continuum of households. A fraction $\gamma \in [0, 1)$ of the population does not hold any type of assets and simply consumes its labor income every period. We refer to these household as rule-of-thumb or non-Ricardian consumers. The remaining share $1 - \gamma$ behaves like standard forward-looking Ricardian consumers: they optimally choose their infinite horizon consumption plan taking advantage of all intertemporal trading opportunities available. Our economy is “cashless” à la Woodford (2011), in the sense that money does not provide any liquidity service to households. It is simply a numeraire in which all prices are denominated.

2.1.1 Ricardian Households

Ricardian households choose their optimal consumption-leisure plans to maximize their expected discounted lifetime utility. The latter is given by:

$$U^o \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c_t^o)^{1-\sigma} - 1}{1 - \sigma} - \frac{(h_t^o)^{1+\chi}}{1+\chi} \right], \quad (1)$$

where the superscript $o$ stands for “optimizer”, $\beta \in (0, 1)$ is the discount factor, $\chi > 0$ is the inverse of the Frisch elasticity of labor supply, and $\sigma > 0$.

A Ricardian household has access to two financial assets: state-contingent bonds, and risky equity. His period by period budget constraint reads as follows:

$$P_t c_t^o + P_t \int_{0}^{1} Q_{j,t} A_{j,t}^o dj + E_t F_{t,t+1} B_{t+1}^o = B_t^o + P_t \int_{0}^{1} (Q_{j,t} + D_{j,t}) A_{j,t-1}^o dj + W_t h_t^o. \quad (2)$$

At the end of period $t$, the households holds a portfolio of contingent claims with one-period ahead stochastic nominal payoff $B_{t+1}^o$ (where $F_{t,t+1}$ is the appropriate stochastic discount factor), and a continuum of risky equity shares issued by monopolistically competitive firms operating in the productive sector, i.e. $A_{j,t}^o$ for $j \in [0, 1]$. The real price of a share issued by the $j$--th firm is denoted by $Q_{j,t}$. The first two terms on the right hand side of (2) correspond to the nominal financial wealth carried over from the previous period. This includes the nominal payoffs on the contingent claims,
and the “price plus dividend” on each share of the equity portfolio, \( Q_{j,t} + D_{j,t} \) for \( j \in [0,1] \). The household also accrues labor income \( W_t h^o_t \). The wage is equal across all agents in the economy due to the assumption of perfect substitutability between Ricardian and non-Ricardian hours worked in production.

Taking first order conditions with respect to the household’s choice variables, after simple manipulation, we obtain the following relationships:

\[
\frac{W_t}{P_t} = (h^o_t)^{\chi} (c^o_t)^{\sigma},
\]

\[(c^o_t)^{-\sigma} = \beta R_t E_t \left[ \frac{(c^o_{t+1})^{-\sigma}}{\Pi_{t+1}} \right],\]

\[Q_{j,t} = \beta E_t \left[ \frac{(c^o_{t+1})^{-\sigma}}{(c^o_t)^{-\sigma}} (Q_{j,t+1} + D_{j,t+1}) \right] \text{ for any } j \in [0,1],
\]

where \( \Pi_{t+1} \equiv \frac{P_{t+1}}{P_t} \) is the gross inflation rate and \( R_t \) is the riskless rate, defined as \( R_t = (E_t F_{t,t+1})^{-1} \). Expression (3) equates the real wage to the marginal rate of substitution between consumption and leisure, and corresponds to the labor supply schedule of Ricardian agents. Expression (4) is the Ricardian household’s Euler equation which defines a relationship between individual consumption and the real interest rate. Expression (5) is the pricing equation for the \( j \)-th firm’s equity share, for \( j \in [0,1] \).

### 2.1.2 Non-Ricardian Households

Non-Ricardian households behave like hand-to-mouth consumers: they do not save, and consume all their disposable income in every period. Non-Ricardian households define their consumption-leisure choice by solving a simple static optimization problem of maximizing the following utility:

\[
U^r \equiv \left[ \frac{(c^r_t)^{1-\sigma} - 1}{1 - \sigma} (X_t)^{\sigma \theta} - \frac{(h^r_t)^{1+\chi}}{1 + \chi} \right],
\]

Subject to the budget constraint:

\[P_t c^r_t = W_t h^r_t,\]

where the superscript \( r \) stands for “rule-of-thumb”. The functional specification of the consumption part of utility follows Gali (1994) and Alonso-Carrera et al. (2008). The term \( X_t \) represents consumption by the household’s reference group (to be defined below), and captures the existence of consumption externalities in our economy.\(^3\) The parameter \( \theta > 0 \) indexes the importance of such

\(^3\)Note that, since our analysis is conducted by log-linearizing around the steady state, our results are qualitatively robust to alternative functional forms for the externality term.
Manipulating the first order conditions with respect to consumption and labor yield the following labor supply schedule for the non-Ricardian agents:

\[
\frac{W_t}{P_t} = (h_t^r)^\chi (c_t^r)^\sigma (X_t)^{-\sigma \theta}.
\]

(8)

Since real wages are homogeneous, equations (3) and (8) imply the equalization of the marginal rate of substitution between consumption and leisure across all agents in the economy:

\[
\frac{W_t}{P_t} = (h_t^o)^\chi (c_t^o)^\sigma = (h_t^r)^\chi (c_t^r)^\sigma (X_t)^{-\sigma \theta}. \tag{9}
\]

Moreover, by combining (8) with the budget constraint (7), we obtain an expression for the optimal labor supply of non-Ricardian households:

\[
(h_t^r)^{\chi+\sigma} = (\frac{W_t}{P_t})^{1-\sigma} (X_t)^{\sigma \theta}. \tag{10}
\]

In the model studied by Bilbiie (2008) where consumption externalities are absent ($\theta = 0$), under a log-utility specification ($\sigma = 1$), non-Ricardian households supply a constant amount of hours. This is not the case in our framework: as long as $\theta > 0$, even if $\sigma = 1$, non-Ricardian hours may vary due to the externality term $(X_t)^{\sigma \theta}$ in their utility function.

### 2.2 Consumption Externalities

We consider the following specification for consumption externalities.

**Definition 1 (Trickle-Down Consumption Externality (TCE))** Let $C^o_t$ and $C^r_t$ be total consumption by, respectively, Ricardian and non-Ricardian households, and define aggregate consumption as $C_t = \gamma C^r_t + (1 - \gamma) C^o_t$. The consumption externality term $X_t$ entering (6), takes the following functional form: $X_t = C_t^o$. Moreover, since $\theta > 0$,

\[
\frac{\partial^2 U^r}{\partial c^r \partial X} > 0 \tag{11}
\]

The assumption above says that, under the TCE set-up, the “Joneses” are the representative Ricardian households. The marginal utility of private consumption is increasing with respect to consumption by the reference group so that non Ricardian households exhibit *keeping up with the Joneses* (KUJ) preferences: as consumption by the Joneses increases, non Ricardian households...
wish to consume more as their marginal utility is higher.\textsuperscript{4}

As in Bilbiie (2008), in our economy the monetary policy transmission channel can have a Standard Aggregate Demand Logic (SADL), or an Inverted Aggregate Demand Logic (IADL) depending on the degree of limited market asset participation (LAMP).\textsuperscript{5} However, the interaction between the size of LAMP and consumption externalities in this economy shrinks the possible range of the parameter space in which an IADL emerges.

### 2.3 Production

The supply-side of the economy is standard. It consists of two sub-sectors: a retail sector and a wholesale sector. The retail sector is perfectly competitive and produces the final consumption good $Y_t$ out of a continuum of intermediate goods through the following CRS technology:

$$Y_t = \left[ \int_0^1 Y_{j,t} \epsilon_t^{-1} d_j \right]^{\epsilon_t t},$$  \hspace{1cm} (12)

where $\epsilon_t > 1$ is the intratemporal elasticity of substitution between any two varieties of intermediate goods. We assume that this elasticity is stochastic, and more specifically that $\hat{\epsilon}_t \equiv \ln \frac{\epsilon_t}{\epsilon}$ (where $\epsilon > 1$ is its unconditional mean) follows a stationary AR(1) process: $\hat{\epsilon}_t = \rho_\epsilon \ln \hat{\epsilon}_{t-1} + \hat{\epsilon}_{t,t}$, where $\rho_\epsilon \in (0,1)$ and $\hat{\epsilon}_{t,t}$ is a mean-zero iid disturbance.\textsuperscript{6} Prices in the retail sector are perfectly flexible.

From (12), the optimal demand for the intermediate good $Y_{j,t}$ is given by $Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{\epsilon_t t} Y_t$, while $P_t = \left[ \int_0^1 P_{j,t}^{1-\epsilon_t} d_j \right]^{1/(1-\epsilon_t)}$ is the price of the final consumption good.

The wholesale sector is made of a continuum of firms indexed by $j$, for $j \in [0,1]$. They act under monopolistic competition and are subject to nominal rigidities in price setting. The $j$--th firm in the wholesale sector hires labor through a competitive labor market to produce the $j$--th variety of a continuum of differentiated intermediate goods which are sold to retailers. Production follows a simple linear technology:

$$Y_{j,t} = Z_t H_{j,t},$$  \hspace{1cm} (13)

Aggregate total factor productivity $Z_t$ is deterministic and without loss of generality we set it constant, $Z_t = Z$. We make this assumption to focus on the stochastic disturbances that may come from monetary policy rather than the real side of the economy and to clearly insulate how the former affect the economy. Adding and extra layer of uncertainty on TFP would not change

\textsuperscript{4}Our definition of KUJ preferences is based on Dupor and Liu (2003), adapted to account for the strict separability between consumption and labor in utility (see Chen and Hsu, 2007).

\textsuperscript{5}A IADL obtains when an increase in the short-term interest rate induces an expansion rather than a contraction in aggregate demand. In the presence of SADL, increasing the interest rate is always contractionary.

\textsuperscript{6}Due to this assumption, a cost-push shock will enter the equilibrium New-Keynesian Phillips curve. The introduction of intrinsic uncertainty will allow us to study the dynamic responses of the economy to structural shocks. However, as is well-known, it will not play any role in the equilibrium determinacy analysis.
qualitatively the results we obtain in this paper.

Firms in the wholesale sector are subject to nominal rigidities. These rigidities are modeled as in Ireland (2003): in every period \( t \), the \( j \)-th firm faces a Rotemberg-style quadratic resource cost to price changes given by

\[
\vartheta \left( \frac{P_{j,t}}{P_{j,t-1}} - \Pi \right)^2 Y_t,
\]

where \( \vartheta > 0 \) and \( \Pi \) is steady state gross inflation. The firm’s objective is given by:

\[
E_0 \sum_{t=0}^{\infty} \mathcal{F}_{0,t} \left\{ (P_{j,t} - MC_t) Y_{j,t} - \frac{\vartheta}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - \Pi \right)^2 P_t Y_t - P_t T_t \right\},
\]

(14)

where \( \mathcal{F}_{0,t} \) is the Ricardian agents’ stochastic discount factor. Nominal marginal costs are equal across firms and defined as

\[
MC_t \equiv (1 - \tau) \frac{W_t}{Z_t}.
\]

The term \( \tau \in (0, 1) \) is a constant rate at which the government subsidizes labor costs, whose determination is specified in Appendix (A.1). Firms are assumed to pay lump-sum taxes \( P_t T_t \), which the government will use to finance the labor subsidy.\(^7\)

The maximization of (14) subject to market demand

\[
Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon_t} Y_t\]

implies the following optimal price setting condition:

\[
\left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon_t} Y_t \left( 1 - \epsilon_t + \epsilon_t \frac{MC_t}{P_{j,t}} \right) - \vartheta \left( \frac{P_{j,t}}{P_{j,t-1}} - \Pi \right) \frac{P_t}{P_{j,t}} Y_t = \vartheta \left[ \mathcal{F}_{t,t+1} \left( \frac{P_{j,t+1}}{P_{j,t}} - \Pi \right) \frac{P_{t+1}}{P_{j,t}} \right] Y_t - T_t.
\]

(15)

Profits generated by each monopolistically competitive firm are distributed to Ricardian households as dividends, in proportion to their equity shares. For the \( j \)-th firm, real dividends are equal to:

\[
D_{j,t} \equiv \left( \frac{P_{j,t}}{P_t} - MC_t \right) Y_{j,t} - \frac{\vartheta}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - \Pi \right)^2 Y_t - T_t.
\]

(16)

### 2.4 Fiscal and Monetary Policy

The fiscal authority levies lump-sum taxes on wholesale firms to finance their labor subsidy. Its budget constraint reads:

\[
P_t T_t = \tau W_t H_t,
\]

(17)

where \( H_t \equiv \int_0^1 H_{j,t} \, dj \) is total labor employed in the wholesale sector. The subsidy rate \( \tau \) is set to eliminate the real wage distortion coming from monopolistic competition, at the steady state.\(^8\)

Monetary policy takes the form of a Taylor-type interest rate rule whereby the short term nominal interest rate is set in response to deviations of current (gross) inflation and output from

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\(^7\)This assumption is not crucial for the analysis. Our analysis would be effectively unchanged if we instead assumed that lump-sum taxes were levied on households.

\(^8\)See Appendix A.1.
their (target) steady state level $\Pi^*$, and $Y^*$:

$$R_t = V_t R^* \left[ \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left( \frac{Y_t}{Y^*} \right)^{\phi_y} \right],$$

where the policy coefficient $\phi_\pi$ is assumed to be non-negative.\(^9\) The factor $V_t$ represents a shock to the policy rate with mean equal to unity. Letting $\hat{\nu}_t \equiv \ln(V_t)$, we assume that $\hat{\nu}_t = \rho \nu \hat{\nu}_t + \hat{\epsilon}_{\nu,t}$, where $\rho \in (0, 1)$ and $\hat{\epsilon}_{\nu,t}$ is a mean-zero iid disturbance. The term $R^*$ is the interest rate target, which is attained if $\Pi_t = \Pi^*$, $Y_t = Y^*$ and $V_t = 1$ (no monetary policy shocks).

### 3 Equilibrium and Aggregation

We focus on a symmetric equilibrium, whereby: a) all price setting firms set the same price and produce the same quantities: $P_{j,t} = P_t$ and $Y_{j,t} = Y_t$ for $j \in [0, 1]$; b) for both Ricardian and non-Ricardian households, individual and own-group consumption coincide: $c^o_t = C^o_t$ and $c^r_t = C^r_t$. Two consequences of this symmetry are that all price setting firms make the same profits, and distribute the same amount of real dividends: $D_{j,t} = D_t$ for $j \in [0, 1]$.

Price and output equalization together with (15), imply a non-linear Phillips curve:

$$\vartheta \Pi_t (\Pi_t - 1) = \vartheta E_t \left[ \Pi_{t+1} (\Pi_{t+1} - 1) F_{t,t+1} \frac{P_{t+1} Y_{t+1}}{Y_t} \right] + \epsilon_t \left( \frac{MC_t}{P_t} - \frac{\epsilon_t - 1}{\epsilon_t} \right).$$

Letting $C^i_t$ and $H^i_t$ be, respectively, average consumption and average hours worked by type $i$ households, for $i = o, r$, the economy-wide aggregate consumption and hours worked are defined, respectively, as:

$$C_t = \gamma C^r_t + (1 - \gamma) C^o_t,$$

$$H_t = \gamma H^r_t + (1 - \gamma) H^o_t.$$

Market clearing in the goods market accounts for the real resource costs of price changes:

$$Y_t = C_t + \frac{\vartheta}{2} (\Pi_t - \Pi)^2 P_t Y_t,$$

where $Y_t = Z H_t$. The fact that only a fraction $1 - \gamma$ of economic agents holds financial assets and owns firms implies that

$$A^o_t = \frac{1}{1 - \gamma} \quad \text{and} \quad D^o_t = \frac{D_t}{1 - \gamma},$$

where, without loss of generality, we have assumed that each firm’s supply of equity shares is

\(^9\)To better highlight the impact of consumption externalities on the sufficiency of the Taylor principle for determinacy, we do not consider policy rules that also respond to output.
constant and equal to one.

In Appendix A.1 we detail a Proposition that characterizes the deterministic steady state equilibrium of this economy and shows its uniqueness. In steady state consumption and labor hours are equal across all agents.

3.1 Linearized Model

We now proceed to linearize the economy around the steady state equilibrium. We start by imposing the following assumption:

**Assumption 1** $\theta \in [0, 1)$.

From the steady state consumption and labor equalization across all agents in the economy, the log-linearization of (20) and (21) gives, respectively:

\[
\hat{c}_t = \gamma \hat{c}_t^o + (1 - \gamma) \hat{c}_o^t \tag{24}
\]
\[
\hat{h}_t = \gamma \hat{h}_t^o + (1 - \gamma) \hat{h}_o^t \tag{25}
\]

From the non-Ricardian household’s optimization problem, some algebra allows us to derive expressions for labor supply and consumption by non-Ricardian agents:

\[
\hat{h}_t^r = \frac{1 - \sigma}{\sigma + \chi} (\hat{w}_t - \hat{p}_t) + \frac{\sigma \theta}{\sigma + \chi} \hat{c}_o^t \tag{26}
\]
\[
\hat{c}_t^r = \frac{1 + \chi}{\sigma + \chi} (\hat{w}_t - \hat{p}_t) + \frac{\sigma \theta}{\sigma + \chi} \hat{c}_o^t \tag{27}
\]

where the real wage, $\hat{w}_t - \hat{p}_t$, is derived from equation (3):

\[
\hat{w}_t - \hat{p}_t = \chi \hat{h}_t^o + \sigma \hat{c}_t^o \tag{28}
\]

The description of the demand-side of the economy is completed by the log-linear version of the Ricardian household’s Euler equation (4):

\[
\sigma \hat{c}_t^o = \sigma E_t \hat{c}_{t+1}^o - (\hat{r}_t - E_t \hat{\pi}_{t+1}) \tag{29}
\]

The supply side consists of a linearized New-Keynesian Phillips curve coming from (19) together with the definition of real marginal costs, $\hat{m} \hat{c}_t - \hat{p}_t = \hat{w}_t - \hat{p}_t$:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\hat{w}_t - \hat{p}_t) + \hat{u}_t \tag{30}
\]

---

10From now on, a “hat” on top of a variable will denote its log-deviation from the respective steady state value.
where \( \kappa \equiv \frac{\epsilon - 1}{\theta \chi} \). The term \( \hat{u}_t \) appearing in (30) is the cost-push shock implied by the assumption of a stochastic elasticity of substitution across the variety of goods: more specifically, \( \hat{u}_t \equiv -\frac{1}{\theta} \epsilon_t \), such that \( \hat{u}_t = \rho \ln \hat{u}_{t-1} - \theta \hat{x}_{t,t} \). The model is closed by assuming the TCE specification, and by letting the interest rate \( \hat{r}_t \) be determined according to a linearized version of policy rule (37):

\[
\hat{r}_t = \phi \hat{\pi}_t + \phi_y \hat{y}_t + \hat{u}_t \tag{31}
\]

For analytical tractability, we introduce the following assumption:

**Assumption 2** \( \sigma = 1 \).

This assumption, of course, implies log-preferences with respect to individual consumption, and, more importantly, makes hours worked by non-Ricardian households not a function of the real wage as in Bilbiie (2008).

As shown in Appendix A.2, the linearized equilibrium conditions can be reduced to the following linear system:

\[
\begin{align*}
\hat{y}_t &= E_t \hat{y}_{t+1} - \delta (\hat{r}_t - E_t \hat{\pi}_{t+1}) \tag{32} \\
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa y \hat{y}_t + \hat{u}_t \tag{33}
\end{align*}
\]

where

\[
\delta \equiv \frac{1 - \gamma}{[1 - \gamma (1 + \chi)]}, \quad \kappa_y \equiv \kappa \left(1 - \gamma \frac{1 + \theta}{1 + \chi} - \gamma^2 \theta \right) \tag{34}
\]

Expression (32) is a dynamic IS curve that captures the demand-side block of our economy.

It is important to remark the following two results: the first one is related to the impact of real interest rate changes on current activity. For any given \( E_t \hat{y}_{t+1} \), a higher real interest rate, \( \hat{r}_t - E_t \hat{\pi}_{t+1} \), determines a decrease in current real activity \( \hat{y}_t \) (i.e., \( \delta > 0 \)) -and the economy display a standard aggregate demand logic (SADL) of policy transmission- when the share of non-Ricardian consumers, \( \gamma \), is either below \( \gamma^* \equiv \frac{1}{1 + \chi} \) or above a threshold value \( \hat{\gamma} > \gamma^* \). If instead \( \gamma^* < \gamma < \hat{\gamma} \), the opposite holds: in this case \( \delta < 0 \), and a higher real interest rate increases current activity (the inverted aggregate demand logic, IADL, takes over).

The second result is about the relationship between the real wage and aggregate output. The real wage \( \hat{w}_t - \hat{p}_t \) is positively (respectively, negatively) related to \( \hat{y}_t \) for \( \gamma \) larger (respectively, smaller) than \( \hat{\gamma} \). These two results combined imply that if the central bank sets the policy rate following a Taylor-type forward-looking interest rate rule - whereby, \( \hat{r}_t \) responds to movements in expected inflation - then the equilibrium determinacy conditions are isomorphic to those obtained by Bilbiie (2008) in a no-externality-LAMP economy. These results are summarized in the following proposition.

**Proposition 1** Let \( \tilde{\phi}_\pi \equiv 1 + \frac{2(1+\beta)}{\delta \kappa \eta} \). If the central bank follows the simple forward-looking Taylor
rule $\hat{\tau}_t = \phi_\pi E_t \hat{\pi}_{t+1}$ then the equilibrium is locally determinate if and only if a) $1 < \phi_\pi < \bar{\phi}_\pi$ when $\gamma \in [0, \gamma^*)$; and b) $\max \{\bar{\phi}_\pi, 0\} < \phi_\pi < 1$ when $\gamma \in (\gamma^*, 1)$.

The proof of this proposition is provided in Appendix (A.3). Since there is well-documented evidence about the FED having adhered to the Taylor principle with an active monetary policy in the post-Volcker era, and because of this proposition, throughout the rest of the analysis we will impose the following assumptions that also guarantee local determinacy.

**Assumption 3** $\gamma < \gamma^*$ and $1 < \phi_\pi < \bar{\phi}_\pi$.

Equation (33) is the New Keynesian Phillips Curve (PC) that always emerges in these kind of models to describe the supply side. As it will be clearer in the section below in which we analyze the optimal monetary policy, which allocation the policy-maker implements under the optimal policy depends also on the relative cost of inflation versus output stabilization as indexed by the slope of the New-Keynesian Phillips curve $\kappa_y$ (also the temporary elasticity of inflation to current marginal cost). While in a LAMP model without externalities such slope is simply $\kappa (1 + \chi)$ and hence it does not depend on the fraction of Non-Ricardians (see Bilbiie, 2008), in our framework, $\kappa_y$ is a function of both the degree of LAMP and the extent of TCE. The following lemma provides a complete characterization of the slope of the PC with respect to trickle down consumption and the share of Non-Ricardians.

**Lemma 1** The slope of the PC $\kappa_y$ has the following properties:
1) $\kappa_y < \kappa (1 + \chi)$;
2) $\frac{\partial \kappa_y}{\partial \rho} < 0$ and
3) $\frac{\partial \kappa_y}{\partial \gamma} < 0$.

The introduction of TCE has two important implications for the PC. First, by lowering $\kappa_y$, it makes the PC flatter, and, as a result, implies a worse trade-off for the central bank between output vs. inflation stabilization. This feature of our model paired with the rising relevance of TCE as suggested in Bertrand and Morse (2013), is in line with very recent empirical evidence that seem to indicate that the PC has indeed flattened in the data in recent years (Coibion and Gorodnichenko (2013)). Second, it makes the slope also a function of the amount of people with no access to savings. Now a larger degree of LAMP flattens the curve too. Therefore, lower participation to asset markets (higher $\gamma$) and stronger TCE mutually reinforce each other, making inflation stabilization more costly in terms of output and affecting volatility: the lower $\kappa_y$, the larger the decline in the output gap the central bank has to accept for a given reduction in inflation.
4 Monetary Policy and Consumption Inequality

This section focuses on the relationship between monetary policy and consumption inequality. In particular, we analyze how shocks to the policy rate (i.e., monetary policy shocks) and to goods’ mark-up (i.e., cost push shocks) affect consumption inequality, and how the transmission mechanism depends on both structural (e.g., the degree of LAMP and the strength of TCE) and policy-related (e.g., the policy’s anti-inflationary stance) features of the economy. Moreover, by showing how LAMP and TCE affect both the slope of New Keynesian Phillips curve and the policy-maker’s welfare-based relative concern for output gap stabilization, we derive some novel implications for the design of an optimal monetary policy plan under discretion.

4.1 Dynamic Responses under Instrumental Taylor Rules

The analysis in this section tries to answer the following questions: Does the model imply that a contractionary monetary policy shock generates consumption inequality, as documented by Coibion et al. (2012) with U.S. data? Does the presence of TCE amplify the impact of this shock?

We measure consumption inequality using the Gini coefficient, which, in our 2-type agents economy, is simply equal to $G_t = \gamma \left(1 - \frac{\hat{c}_t}{\bar{c}_t}\right)$. Given the assumption of equitable steady state consumption, we have that $G_{ss} = 0$. Around such steady state, $G_t$ can be expressed with the following first order approximation:

$$G_t \simeq -\gamma (\hat{c}_t - \bar{c}_t) = -\gamma \left[\frac{\hat{c}_t - (1 - \gamma) \bar{c}_t}{\gamma} - \hat{c}_t\right]$$

$$= -\gamma \left[1 - \frac{(1 - \gamma) \xi}{\gamma} - 1\right] \hat{c}_t = -\gamma (1 - \gamma) \frac{\chi + \theta (\gamma^* - \gamma)}{1 - \gamma + \gamma \frac{\theta}{1 + \chi} - \gamma^2 \theta} \hat{y}_t$$

(35)

Where the coefficient $\xi$ is defined in appendix (A.2), and the parameter $\Gamma$ in the expression above is always negative for any $\gamma \in (0, \gamma^*)$, it is zero at $\gamma = 0$ (full asset market participation, hence homogeneous agents). It is simple to show that $\Gamma$ is also strictly decreasing in the externality parameter $\theta$ and the in the inverse Frisch labor elasticity $\chi$:

$$\frac{\partial \Gamma}{\partial \theta} < 0, \quad \frac{\partial \Gamma}{\partial \chi} < 0$$

(36)

From the characterization above, it immediately follows that consumption inequality is counter-cyclical in our model: as long as the degree of LAMP does not induce an IADC ($\gamma < \gamma^*$), the Gini index of consumption inequality increases during recessions (where $\hat{y}_t$ is negative) and decreases during booms (where $\hat{y}_t$ is positive). The size of $\Gamma$ measures the degree of counter-cyclical of the Gini index. Moreover, the adverse impact of economic recessions on consumption inequality is
larger the stronger TCE and the lower labor demand elasticity are.\footnote{11}

To assess the quantitative implications of a monetary policy shock for consumption inequality, we parametrize the model as follows. We set the discount factor $\beta$ equal to 0.99, so that the economy features an annual real interest rate of about 4%. We let the unconditional mean of the intratemporal elasticity of substitution across intermediate goods, $\epsilon$, be equal to 6, as in the benchmark New-Keynesian model presented in Gali (2008).\footnote{12} This value implies a 20% steady state net mark-up in the wholesale sector. The labor disutility parameter $\chi$ is set equal to 1, which corresponds to a unit Frisch elasticity of labor supply, $\chi^{-1}$, at the steady state.\footnote{13} This value implies that, absent the externality, the threshold value across which the economy goes from a SADL to a IADL - that is, $\gamma^{*}$ - is equal to 0.5. The price adjustment cost parameter $\vartheta$ is chosen by taking advantage of the reduced-form equivalence between a Rotemberg-style and a Calvo-style approach to price stickiness. More specifically, we consider an economy with an average duration of price contracts equal to three quarters, which, under Calvo pricing, corresponds to a probability of no price change equal to 0.66. This choice is based on the recent micro-evidence provided by Nakamura and Steisson (2008). We then compute the temporary elasticity of inflation to real marginal costs in the (linearized) Phillips curve: that is, $\kappa_{\text{Calvo}} \equiv \frac{(1-\beta)(1-p)}{\beta} = 0.17$ for $p = 0.66$. For given $\beta$ and $\epsilon$, we then set the adjustment cost parameter $\vartheta$ such that $\frac{\epsilon^{-1}}{\vartheta} = 0.085$. This procedure gives $\vartheta = 28$. For the parameterization of the share of non Ricardian households, we use the evidence on financial holdings by U.S. households provided by Wolff (2012). According to his analysis, between 1962 and 2010 in the U.S., the average share of household with zero or negative net wealth has been about 20%. We therefore set $\gamma = 0.2$.

We consider the following interest rate rule specification:

$$\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \hat{\nu}_t$$ (37)

where $\hat{\nu}_t$ is a stationary monetary policy shock following an AR(1) process. For what concerns the policy parameters $\phi_\pi$ and $\phi_y$, available estimates vary considerably. For the response to inflation $\phi_\pi$, they go from slightly above unity (Boivin and Giannoni, 2006) to about 3 (Carillo et al, 2008). For the response to output (or the output gap) they vary between zero to about unity. Our benchmark parameterization features $\phi_\pi = 2$ and $\phi_y = 0.5$, which are close to the original estimates proposed

\footnote{11}{This result does not hinge on the assumption of full consumption equality at the steady state. The latter serves the sole purpose of allowing us to attain some clear analytical results. Without such assumption (e.g., if we allowed for positive inequality at the steady state) we would be forced to resort to numerical methods only to obtain qualitatively the same results.}

\footnote{12}{Available estimates for $\epsilon$ vary from slightly above 2 (from the micro-estimates of the industrial organization and international trade literature) to about 10 (from the macroeconomic literature).}

\footnote{13}{Empirical estimates of the Frisch elasticity of labor supply vary considerably in the literature, depending on 1) whether they are based on micro or macro data, and 2) whether they refer to the intensive or the extensive margin. Domeij and Floden (2006) briefly review the empirical literature and provide some new estimates. Chetty et al. (2011) try to reconcile the existing estimates from the micro and the macro literature.}
by Taylor (1993).\textsuperscript{14} We set the AR(1) coefficients for both the monetary shock and the cost push shock both equal to 0.9. Table 1 summarizes the benchmark values for the parameters in the model.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\theta$</th>
<th>$\epsilon$</th>
<th>$\chi$</th>
<th>$p$</th>
<th>$\theta$</th>
<th>$\phi_\pi$</th>
<th>$\phi_\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.2</td>
<td>0.5</td>
<td>6</td>
<td>1</td>
<td>0.66</td>
<td>28</td>
<td>2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 1: Benchmark Parameterization

Figure 1 and 2 display the impulse responses of key variables to a 1% (annual, hence 0.25% quarterly) contractionary monetary policy shock. For each variable we present the impulse response for the case of no TCE ($\theta = 0$) as well as for the case of TCE ($\theta = 0.5$). Interestingly, the contractionary policy shock does not drive down the real interest rate. This is because inflation drops substantially and the policy-maker aggressively responds to it with a coefficient $\phi_\pi = 2$. However, since the drop in the nominal interest rate is smaller than that in inflation, the real interest rate increases. This brings down output, and, as a direct consequence the real wage. Ricardian households cut their labor supply, while non-Ricardians - for whom the labor wage is the only source of income - do not respond to the policy shock. Notice that, due to the real interest rate hike, Ricardian household decrease their consumption. Non-Ricardians end up consuming less too, as a result of a lower wage per hour worked.

While interest rates, inflation, the real wage, as well as Ricardian hours and consumption are not affected by the introduction of a positive consumption externality, the introduction of trickle down consumption amplifies the impacts of a monetary contraction on output (more negative), total hours (more negative) and consumption inequality (more positive). The larger contraction in real activity is driven by a reduction in hours worked by non-Ricardian households: following the contractionary policy, aggregate demand decreases as Ricardian households prefer to save; non-Ricardian households decide then to work less to “keep-up” with the lower demand by the Joneses, leading to a larger reduction in their labor income and hence consumption: consumption inequality increases even more.

Figure 2 shows that the drop in consumption by the non-Ricardians - who do not have any way to smooth the negative shock across periods - outweighs the corresponding drop in consumption by the Ricardians. This creates a positive spread between Ricardian and non-Ricardian consumption, and therefore it generates consumption inequality. The Gini index of consumption inequality moves from the normalized value of 0 to 0.008 (a 0.8 percentage increase). For the sake of comparison, and to give an idea of the magnitude of the change we obtain our model, Romer and Romer (1998)

\textsuperscript{14}This policy parameterization guarantees equilibrium determinacy.
Figure 1: IRF of key economic variables to a 1% contractionary monetary policy shock.

Figure 2: IRF of consumption and inequality to a 1% monetary policy shock.
empirically estimate for the US that a one percentage point of unanticipated inflation corresponds
to a decline of the Gini coefficient of 0.1 (when the Gini coefficient is on a scale of 0 to 100 though).

Since the mechanism through which TCE affects the impulse responses is related to the labor
market, a key parameter affecting the dynamic response of inequality is the inverse Frisch elasticity
of labor, \( \chi \). Figure 3 therefore plots the on-impact response (i.e., at the time when the monetary
policy shock hits) of the Gini index of consumption for different values of \( \chi \) (from 0 to 2) and \( \theta \)
(from 0 to 1). As expected from the analysis of expression (36) and displayed below in the left
panel, such impact is largest in an economy featuring a high \( \chi \) (low labor elasticity) and a high \( \theta \)
(strong TCE). For any given \( \chi \), the Gini’s response is strictly increasing in \( \theta \), a pattern that appears
more significant at low values of \( \chi \) where households’ labor supply is more elastic (right panel). The
Gini’s response to \( \chi \) is strictly increasing in \( \chi \) when the strength of TCE is mild-to-intermediate
(\( \theta = 0, 0.25, \) and \( 0.5 \)) but becomes non-monotonic when the externality is quantitatively sizable
(\( \theta = 0.75 \) and 0.99).

![Figure 3: Behavior of the Gini coefficient as \( \theta \) and \( \chi \) vary.](image)

To explain this pattern, consider first the cases of no externality (\( \theta = 0 \)), and assume that
\( \chi = 1 \), our benchmark parameterization. An increase in labor elasticity (lower \( \chi \)) makes Ricardian
households’ labor supply flatter. Hence, following the leftward shift in labor demand generated by
the monetary contraction (sticky price firms hire less due to lower aggregate demand), the drop in
the real wage is smaller with respect to the benchmark case, while the decrease in hours worked by
the Ricardians is larger (non Ricardians do not change their labor supply decision). As a result,
Ricardian consumption decreases by more, while non-Ricardian consumption decreases by less (its decrease matches one-to-one the cut in wages). While consumption inequality still increases, it does so by less with respect to the benchmark case of $\chi = 1$.

Next, consider the case of a large TCE, e.g., $\theta = 0.99$. Similar to the case we have just discussed, a higher labor elasticity generates a smaller drop in the real wage, a larger decrease in Ricardian hours, and, as a result, a larger cut in Ricardian consumption. In order to keep up with the decreasing consumption level of the Joneses, the non-Ricardians end up cutting their consumption by more with respect to the case of a higher labor elasticity. Because of that, a higher labor elasticity might lead to a larger (not smaller) positive response of the Gini index of consumption inequality to the contractionary monetary policy shock.

Can a more aggressive monetary policy dampen the adverse effects of policy shocks on consumption inequality? As displayed in Figure 4, the answer is yes. A more aggressive response to endogenous changes in either inflation or output diminishes the increase in consumption inequality driven by the unexpected interest rate hike. The reason is that a more aggressive policy does a better job at shielding real output from the adverse consequences of a policy shock (i.e., the output response is less negative). Since, as shown above, consumption inequality is counter-cyclical, such policy is also effective at containing the impact on inequality.

![Figure 4: Behavior of the Gini coefficient as $\phi_\pi$ and $\phi_y$ vary.](image-url)
4.2 Implications for Optimal Monetary Policy

In this section, we focus on how the interaction between LAMP and TCE affects the design of optimal monetary policy under discretion. We assume that the central bank maximizes a weighted average of individual utilities. That is:

$$\max W \equiv \max_{\beta} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \gamma \left[ \ln c_t^{o} - \frac{(h_t^{o})^{1+\chi}}{1+\chi} \right] + (1-\gamma) \left[ (c_t^{r})^{\theta} \ln c_t^{r} - \frac{(h_t^{r})^{1+\chi}}{1+\chi} \right] \right\}$$  (38)

Using standard techniques, we derive in appendix A.4 a second order approximation to the welfare criterion in (38). In the same appendix we show that the maximization of (38) is equivalent to the minimization of the following quadratic objective function:

$$\max W \Leftrightarrow \min L \equiv \min_{\beta} E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left( \hat{\pi}_t^2 + \alpha \hat{x}_t^2 \right)$$  (39)

where $\hat{x}_t \equiv \hat{y}_t - \hat{y}_t^e$ is the output gap, expressed as the (log) deviation of output from its efficient level, and $\alpha$ (see the Appendix A.4 for its definition) represents the policy-maker’s relative concern for output gap versus inflation stabilization in the objective. $\alpha$ is a function of structural parameters and since it is key for the derivation of optimal monetary policy, it is useful to compare it to the output gap weight arising in the benchmark representative agent (RA) New Keynesian model (where $\gamma = \theta = 0$) and in the heterogeneous agents LAMP model with no externality (NE) (where $\gamma > 0$ but $\theta = 0$). These are, respectively:

$$\alpha^{RA} \equiv \frac{1+\chi}{\vartheta} \quad \text{and} \quad \alpha^{NE} \equiv \frac{1+\chi}{\vartheta (1-\gamma)}$$  (40)

**Lemma 2** The weight $\alpha$ entering (39) satisfies the following properties:
1) $\alpha < \alpha^{NE}$;  
2) $\alpha$ is strictly decreasing in $\theta$.

The introduction of consumption externalities lowers the policy-maker’s concern for output stabilization in a LAMP economy. In particular, it makes it smaller with respect to the case of an economy with limited participation, but no externalities, like the one studied by Bilbiie (2008). Given that LAMP implies a higher weight on output than in the benchmark full participation NK model ($\alpha^{NE} > \alpha^{RA}$), for a sufficiently large $\theta$, the introduction of trickle down consumption might neutralize the effect of LAMP and induce a weight $\alpha$ smaller than $\alpha^{RA}$. In other words, despite the presence of LAMP, the policy-maker might have a stronger welfare-based incentive to stabilize inflation in the presence of TCE.

We analyze optimal monetary policy under discretion. Because of the lack of commitment, current central bank’s actions and announcements cannot affect private expectations. The optimal policy problem therefore reduces to the period-by-period minimization of the temporary loss
\( \frac{1}{2} (\bar{\pi}_t^2 + \alpha \hat{x}_t^2) \), subject to the Phillips curve \( \bar{\pi}_t = \kappa_y \hat{x}_t + \hat{u}_t + \mathcal{F}_t \), where \( \mathcal{F}_t \equiv \beta E_t \hat{\pi}_{t+1} \) and the shock \( \hat{u}_t \) are both taken as given by the policy-maker. The first order condition to this problem gives a standard targeting rule:

\[
\bar{\pi}_t = -\frac{\alpha}{\kappa_y} \hat{x}_t \tag{41}
\]

This rule defines the target inflation-output gap trade-off that the central bank wants to implement, period-by-period, by moving the policy instrument accordingly. The targeting rule (41) also implies an optimal relative volatility (ORV) of inflation to output:

\[
ORV \equiv \frac{SD(\bar{\pi}_t)}{SD(\hat{x}_t)} = \frac{\alpha}{\kappa_y} \tag{42}
\]

The ORV is the ratio of the cost \( \frac{1}{\kappa_y} \) to benefit \( \frac{1}{\alpha} \) of inflation stabilization. From the PC, a 1% decline in inflation requires a \( \frac{1}{\kappa_y} \) reduction in output. From the policy objective, the relative benefit of inflation stabilization versus output gap stabilization is \( \frac{1}{\alpha} \).

Since, by Lemma 1 and 2, both \( \alpha \) and \( \kappa_y \) are decreasing in \( \theta \), an increase in the extent of TCE makes the central bank more inflation averse (by lowering \( \alpha \)) but, at the same time, makes the cost of inflation stabilization larger (by lowering \( \kappa_y \)). Thus it is not \emph{a priori} clear whether stronger TCE lead to lower or higher relative inflation volatility under the optimal plan. Given the rather complicated parametric structure of \( \alpha \), we analyze this issue numerically. To do so, we use the benchmark parameterization of the model as discussed above.

The three panels in Figure 5 display, from left to right, the slope of the PC \( \kappa_y \), the policy-maker’s relative concern for inflation stabilization \( \alpha^{-1} \), and the optimal relative volatility \( \frac{\alpha}{\kappa_y} \) as functions of the externality parameter \( \theta \). We display the results for both the case of full participation in asset markets \( \gamma = 0 \) as well as for three alternative degrees of LAMP. Note that with full asset market participation, all households are Ricardian, and, by definition, trickle-down consumption does not exists. Hence, the slope of the PC, the policy-maker’s preference for inflation stabilization, and ORV are flat lines. In the presence of limited asset market participation, a stronger TCE makes the PC flatter (left panel). This effect is more predominant for larger values of \( \gamma \). A larger \( \theta \) also increases the policy-maker’s preference for stabilizing inflation (middle panel). Interestingly, if \( \theta \) passes a certain threshold, the relative weight on inflation becomes larger than what required in a benchmark RA model. In our numerical example, this occurs for \( \theta \) larger than 0.22 (when \( \gamma = 0.1 \)), 0.33 (when \( \gamma = 0.2 \)), and 0.62 (when \( \gamma = 0.3 \)). A stronger TCE leads to a decline of optimal inflation volatility relative to that of output (right panel). This result implies that, for a given degree of LAMP, the policy-maker should be relatively more concerns about inflation stabilization if TCE becomes a more pervasive phenomenon. Notice also that, as the externality parameter \( \theta \) becomes sufficiently large, the ORV drops below the value implied by the benchmark RA model.
Figure 5: Optimal Monetary Policy trade-offs as $\theta$ vary.

Figure 6: Optimal Monetary Policy trade-offs as $\gamma$ vary.
Figure 6 looks at the same objects as functions of $\gamma$ for both the case of no externality ($\theta = 0$) as well as for three alternative parametrizations of $\theta$. Without externalities, the slope of the PC is unaffected by the presence of LAMP (left panel). A positive TCE has two effects. First, it generates a negative impact of the degree of LAMP on the slope of the PC. Hence, differently from Bilbiie (2008), LAMP affects the relative cost of inflation versus output stabilization: it makes it larger. Second, the effect of $\gamma$ on $\kappa_y$ is non-monotonic. For given level of TCE, the PC is flattest for intermediate values of $\gamma$, while it reverts back to the benchmark value as $\gamma$ approaches the threshold $\gamma^*$ (e.g. 0.5 in the specific example). The relative weight on inflation in the policy objective displays a similar pattern, but reversed (middle panel). Without TCE, a decline in participation lowers the importance of the inflation stabilization objective. With positive externalities, inflation stabilization is initially increasing in $\gamma$. This effect is stronger for larger values of $\theta$. Optimal relative volatility displays an interesting pattern. First, positive TCE lower the ORV, for any $\gamma \in (0, \gamma^*)$. Second, ORV is no longer a strictly increasing function of the degree of LAMP. With $\theta > 0$, an increase in limited participation leads to more stable inflation under the optimal plan.

5 Conclusions

We introduce non-Ricardian households with limited asset market participation into an otherwise standard New-Keynesian economy. We consider the empirically relevant case (Bertrand and Morse (2013)) in which households preferences depend on consumption by the high-income Ricardian households (trickle-down consumption externalities) and study the impact of trickle-down consumption externalities and limited asset market participation on the conduct of monetary policy. In particular, since the re-distributive content of monetary policy has been neglected in modern macroeconomic literature so far, we are interested in studying whether those factors can induce monetary policy to have a redistributive impact.

Our analysis provides a quantitative assessment of how shocks to the policy rate (i.e., monetary policy shocks) and to goods’ markup (i.e. cost push shocks) affect consumption inequality, and how this channel depends on both structural (e.g. the degree of LAMP, the strength of TCE, the Frisch elasticity of labor) and policy-related (e.g. the policy’s anti-inflationary stance) features of the economy. We show that a 1% (yearly) contractionary monetary policy shock raises the Gini coefficient by about, 0.08 percentage points. The model thus provides a novel consideration for monetary policy intervention and, to a less extent, a complementary stylized explanation for the rise of consumption inequality in the US as witnessed in the data. Moreover, by showing how LAMP and TCE affect both the slope of New Keynesian Phillips curve and the policy-maker’s welfare-based relative concern for output gap stabilization, our paper also derives some novel implications for the design of an optimal monetary policy plan (under discretion). Our result is in line with very recent empirical evidence that seem to indicate that the PC has indeed flattened in the data.
in recent years (Coibion and Gorodnichenko (2013)).
A Appendix

A.1 Steady State Equilibrium

We consider a zero inflation steady state equilibrium, where $\Pi^* = \Pi = 1$. All shocks are fixed at their mean values, and, without loss of generality, we let $Z = V = 1$. From the Euler equation of Ricardian agents (4) - and for the specification of the externality - we obtain the steady state (nominal and real) interest rate: $R = \beta^{-1}$.

As standard, the Phillips curve (19) implies that real marginal costs are equal to the inverse of the gross-markup in the retail sector: $rac{MC}{P} = \frac{\epsilon - 1}{\epsilon} \equiv \frac{1}{\mu}$. This implies that at the steady state the real wage is equal to $W^P = \frac{1}{(1-\gamma)\mu}$. We assume that the subsidy rate $\tau$ is set by the fiscal authority to offset the distortion coming from monopolistic competition, that is, $\tau$ is chosen such that the steady state real wage is equal to one (the marginal productivity of labor), as it would occur in a frictionless economy. This gives $\tau = \frac{\mu - 1}{\mu}$.

With $\frac{W^P}{\mu} = 1$, the steady state Ricardian and the non-Ricardian households budget constraints become, respectively:

$$C^o = H^o + D^o \quad \text{and} \quad C^r = H^r$$

(43)

where from (23) and the steady state version of (16) $D^o = \frac{D}{(1-\gamma)} = \frac{(1-MC)}{(1-\gamma)}Y - T$. From the fact that $\frac{MC}{P} = \frac{1}{\mu}$, the government balanced-budget (17) and the definition of the labor subsidy $\tau = \frac{\mu - 1}{\mu}$, it follows that steady state aggregate dividends are zero: $D = 0$. From (43), we then have that $C^o = H^o$ and $C^r = H^r$.

By these last two equalities, at the steady state, the labor supply conditions (3) and (8) are then equivalent to:

$$(C^o)^{\chi + \sigma} = 1$$

(44)

$$(C^r)^{\chi + \sigma} = (X)^{\sigma \theta}$$

(45)

Proposition 2 (Steady State) Let $\mu = \frac{\epsilon}{\epsilon - 1}$ be the gross mark-up in the wholesale sector. If the government subsidizes firms’ labor costs at rate $\tau = \frac{\mu - 1}{\mu}$ via lump-sum taxes on monopolistically competitive firms, there always exists an equitable steady state equilibrium whereby consumption and hours are equal across all agents in the economy: $C^o = C^r = C$ and $H^o = H^r = H$. Furthermore, this is the unique steady state for any $\theta$.

Proof. Given that $X = C^o$, from equations (44)-(45) simple algebra implies that $C^o = C^r = 1$ is the unique steady state solution.

A.2 Reduced-Form System under TCE

From Definition 1, we obtain:

$$\dot{x}_t = c_t^0$$

(46)
Using assumption 2, the Ricardian household’s Euler equation (29) becomes:
\[ \sigma \hat{c}_t^o = \sigma E_t \hat{c}_{t+1}^o - (\hat{r}_t - E_t \hat{\pi}_{t+1}) \]  (47)

From equations (26) and (27), we obtain expressions for non-Ricardian households consumption and labor supply when \( \sigma = 1 \):
\[ \hat{c}_t^r = \hat{w}_t - \hat{p}_t + \frac{\theta}{1 + \chi} \hat{c}_t^o \]  and  \[ \hat{h}_t^r = \frac{\theta}{1 + \chi} \hat{c}_t^o \]  (48)

The expression for \( \hat{h}_t^r \) can then be combined with (25) and the market clearing condition, \( \hat{c}_t = \hat{h}_t \), to give hours worked by Ricardian agents:
\[ \hat{h}_t^o = \frac{1}{1 - \gamma} \left[ \hat{c}_t - \gamma \frac{\theta}{1 + \chi} \hat{c}_t^o \right] \]  (49)

The latter together with (28) and (46) allows us to obtain an expression for the real wage as a fraction of aggregate consumption:
\[ \hat{w}_t - \hat{p}_t = \frac{(1 - \gamma)(1 + \chi)}{1 - \gamma + \gamma \frac{\theta}{1 + \chi} - \gamma^2 \theta} \hat{c}_t \]  (50)

After some rearrangement, the combination of equations (24), (48) and (50) gives the following relationship between Ricardian consumption, \( \hat{c}_t^o \), and aggregate consumption, \( \hat{c}_t \):
\[ \hat{c}_t^o = \frac{1 - \gamma (1 + \chi)}{1 - \gamma + \gamma \frac{\theta}{1 + \chi} - \gamma^2 \theta} \hat{c}_t \]  (51)

Plugging (51) into (47), and using the market clearing condition \( \hat{c}_t = \hat{y}_t \), we obtain the aggregate IS curve (32) in the main text. Plugging (50) into (30) we obtain the New-Keynesian Phillips curve (33) in the main text.

A.3 Proof of proposition 1

**Proof.** The local equilibrium dynamics are described by a linear system made of equations (32) and (33), together with the forward-looking interest rate rule (37). The equilibrium system can be rewritten in matrix form as:
\[
\begin{bmatrix}
\dot{y}_t \\
\dot{\pi}_t
\end{bmatrix} = \begin{bmatrix}
1 & \delta (1 - \phi) \\
\kappa_y & \beta + \kappa_y \delta (1 - \phi)
\end{bmatrix}
E_t \begin{bmatrix}
\dot{y}_{t+1} \\
\dot{\pi}_{t+1}
\end{bmatrix} + \begin{bmatrix}
-\delta & 0 \\
-\delta \kappa_y & 1
\end{bmatrix}
\begin{bmatrix}
\dot{\nu}_t \\
\dot{\bar{u}}_t
\end{bmatrix}
\]

\(^{15}\)The assumption \( \theta < 1 \) guarantees a negative relationship between the real interest rate and Ricardian consumption.
Since both variables are non-predetermined, the equilibrium is locally determinate if and only if both eigenvalues of the Jacobian matrix $J$ are inside the unit circle in the complex plane. Let $P(e) = e^2 - Tr(J)e + Det(J)e$ be the characteristic polynomial of $J$, where $Tr(J)$ and $Det(J)$ denote, respectively, its trace and its determinant. Necessary and sufficient conditions for both roots of $P(e) = 0$ (i.e. both eigenvalues of $J$) to be inside the unit circle are

1) $|Det(J)| < 1$;
2) $P(+1) > 0$ and $P(-1) < 0$.

Simple algebra shows that $Det(J) = \beta \in (0, 1)$, and

$$P(+1) = \kappa_y \delta (\phi_\pi - 1)$$

Let $\bar{\phi}_\pi \equiv 1 + \frac{2(1+\beta)}{\kappa_y \delta}$. Given the definitions of $\delta$ and $\kappa_y$, and their related properties, it immediately follows that

$$\kappa_y \delta \gtrless \text{ for } \gamma \lesssim \gamma^*$$

We can then conclude that both $P(+1) > 0$ and $P(-1) < 0$ hold (hence, the equilibrium is locally determinate) if and only if

$$1 < \phi_\pi < \bar{\phi}_\pi \text{ when } \gamma \in [0, \gamma^*)$$

$$\max\{\phi_\pi, 0\} < \phi_\pi < 1 \text{ when } \gamma \in (\gamma^*, 1)$$

### A.4 Second Order Approximation of Aggregate Welfare and Lemma 1

The policy-maker seeks to maximize aggregate welfare, which is defined as the weighted average of Ricardian and non-Ricardian households lifetime utilities. Namely:

$$W \equiv E_0 \sum_{t=0}^{\infty} \beta^t \{ \gamma (U^o_t - V^o_t) + (1 - \gamma) (U^r_t - V^r_t) \}$$

where

$$U^o_t \equiv \ln c^o_t, \quad V^o_t \equiv \frac{(h^o_t)^{1+\chi}}{1 + \chi}, \quad U^r_t \equiv (c^o_t)^{\theta} \ln c^r_t, \quad V^r_t \equiv \frac{(h^r_t)^{1+\chi}}{1 + \chi} \quad (52)$$

A second-order Taylor approximation of each term in (52) around their respective steady state gives the following results:

$$U^o_t \approx U^o(ss) + \hat{c}^o_t$$

$$V^o_t \approx V^o(ss) + \hat{h}^o_t + \frac{1+\chi}{2} \left( \hat{h}^o_t \right)^2$$

$$U^r_t \approx U^r(ss) + \hat{c}^r_t + 2\theta \hat{c}^o_t \hat{c}^r_t$$

$$V^r_t \approx V^r(ss) + \hat{h}^r_t + \frac{1+\chi}{2} \left( \hat{h}^r_t \right)^2$$
Let $W_t \equiv \gamma (U_t^r - V_t^r) + (1 - \gamma) (U_t^o - V_t^o)$. Then, using the fact that in:

$$
W_t = \gamma \left\{ U^r (ss) - V^r (ss) + \bar{c}_t^r + 2 \theta \bar{c}_t^o \right\} + \left\{ U^o (ss) - V^o (ss) + \bar{c}_t^o \right\} - \left[ \hat{h}_t^r + \frac{1 + \chi}{2} \left( \hat{h}_t^r \right)^2 \right] + (1 - \gamma) \left[ \hat{h}_t^o + \frac{1 + \chi}{2} \left( \hat{h}_t^o \right)^2 \right]
$$

$$
= W^{ss} + \bar{c}_t - \hat{h}_t - \frac{1 + \chi}{2} \left[ \gamma \left( \hat{h}_t^r \right)^2 + (1 - \gamma) \left( \hat{h}_t^o \right)^2 \right] + 2 \gamma \theta \bar{c}_t^o \bar{c}_t^o
$$

where the last line follows from a second order approximation to the market clearing condition.

Consider the term within squared brackets. From the linearized equilibrium condition we have that:

$$
\hat{h}_t^r = \frac{\theta \left( \gamma^* - \gamma \right)}{1 - \gamma + \gamma \theta (\gamma^* - \gamma)} \hat{y}_t \quad \text{and} \quad \hat{h}_t^o = \frac{1}{1 - \gamma + \frac{\theta}{1 + \chi} - \gamma^2 \theta} \hat{y}_t
$$

Then:

$$
\gamma \left( \hat{h}_t^r \right)^2 + (1 - \gamma) \left( \hat{h}_t^o \right)^2 = \frac{\gamma \theta^2 (\gamma^* - \gamma)^2}{\left( 1 - \gamma + \frac{\theta}{1 + \chi} - \gamma^2 \theta \right)^2} \hat{y}_t^2
$$

Consider now the term $2 \gamma \theta \bar{c}_t^o \bar{c}_t^o$. From the fact that $\bar{c}_t^o = \xi \hat{y}_t$ and $\bar{c}_t^r = \frac{\hat{c}_t^r - (1 - \gamma) \bar{c}_t^o}{\gamma} = \frac{1 - (1 - \gamma) \xi}{\gamma} \hat{y}_t$ (where simple algebra shows that, for $\gamma < \gamma^*$, we have $\xi$ and $1 - (1 - \gamma) \xi$ both positive), we have that:

$$
2 \gamma \theta \bar{c}_t^o \bar{c}_t^o = \theta [1 - (1 - \gamma) \xi] \xi \hat{y}_t^2
$$

Aggregate welfare can then be written as follows:

$$
W = E_0 \sum_{t=0}^{\infty} \beta^t W_t
$$

$$
= \frac{W^{ss}}{1 - \beta} - E_0 \frac{\theta}{2} \sum_{t=0}^{\infty} \beta^t \left\{ \hat{h}_t^r \right\}^2 + \left[ \frac{1 + \chi}{\theta} - \frac{4 \theta [1 - (1 - \gamma) \xi] \xi}{\hat{y}_t^2} \right] \xi \hat{y}_t^2
$$

Aggregate welfare maximization is therefore equivalent to the minimization of the following intertemporal loss function:

$$
\min L \equiv E_0 \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left\{ \hat{h}_t^r + \alpha \hat{y}_t^2 \right\}
$$

where

$$
\alpha \equiv \left[ \frac{1 + \chi}{\theta} - \frac{4 \theta [1 - (1 - \gamma) \xi] \xi}{\hat{y}_t^2} \right]
$$

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is the relative weight on output stabilization.

Consider the parameter $\alpha$. First it immediately follows that $\alpha < \frac{1+\chi}{\vartheta}$. Consider $\varXi$. Simple algebra shows that $\varXi < \frac{1}{1-\gamma}$ if and only if

$$\frac{\theta}{1+\chi} \xi < 2$$

where $\xi = \frac{1-\gamma(1+\chi)}{1-\gamma(1+\chi) - \chi^2 \theta}$. Let $f(\theta) \equiv \frac{\theta}{1+\chi} \xi$. Simple calculus shows that $f(0) = 0$, $f'(\theta) > 0$, $f''(\theta) < 0$ and $f(1) < 1$, which imply that the inequality always holds. We then have

$$\alpha < \frac{1+\chi}{\vartheta} \Xi < \frac{1+\chi}{\vartheta (1-\gamma)} \equiv \alpha^{NE}$$

where the latter is the weight on output stabilization in the loss function that would occur in a LAMP model without consumption externalities (like Bilbiie).

To show that $\alpha$ is strictly decreasing in $\theta$ proceed as follows. Consider the term $\frac{4\theta[1-(1-\gamma)\xi]\xi}{\partial\theta}$. We have that:

$$\frac{\partial}{\partial \theta} \left[ \frac{4\theta[1-(1-\gamma)\xi]\xi}{\partial\theta} \right] = \frac{4}{\partial} \left[ -\theta \xi (1-\gamma) \frac{\partial \xi}{\partial \theta} + [1-(1-\gamma)\xi] \frac{\partial (\xi \theta)}{\partial \theta} \right] > 0$$

Now consider $\frac{1+\chi}{\vartheta} \Xi$. Simple calculus shows that

$$\text{sign} \left( \frac{\partial \Xi}{\partial \theta} \right) = \text{sign} \left\{ [1-\gamma + \gamma \theta (\gamma^* - \gamma)] [\theta (\gamma^* - \gamma) - 1] \right\}$$

$$= \text{sign} \left\{ \theta (\gamma^* - \gamma) - 1 \right\}$$

where the second equality follows from the assumption $\gamma < \gamma^*$. Then

$$\text{sign} \left( \frac{\partial \Xi}{\partial \theta} \right) = \text{sign} \left\{ \theta (\gamma^* - \gamma) - 1 \right\}$$

$$= \text{sign} \left\{ \frac{\theta}{1+\chi} - (\theta \gamma + 1) \right\}$$

Since $\theta < 1$, the last expression is always negative. Hence $\frac{\partial \Xi}{\partial \theta} < 0$. It then follows that $\frac{\partial \alpha}{\partial \theta} < 0$. 

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References


Airaudo, Marco and Luca Bossi (2013) “Monetary Policy and the Well-Being of the Poor,” mimeo.


