To sell or to borrow?
A Theory of Bank Liquidity Management*

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Abstract

This paper studies banks' decision whether to borrow from the interbank market or to sell assets in order to cover liquidity shortage in presence of credit risk. The following trade-off arises. On the one hand, tradable assets decrease the cost of liquidity management. On the other hand, uncertainty about credit risk of tradable assets might spread from the secondary market to the interbank market, lead to liquidity shortages and socially inefficient bank failures. The paper shows that liquidity injections and liquidity requirements are effective in eliminating liquidity shortages and the asset purchases are not. The paper explains how collapse of markets for securitized assets contributed to the distress of the interbank markets in August 2007. The paper argues also why the interbank markets during the 2007-2009 crisis did not freeze despite uncertainty about banks' quality.

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One of the most significant features of modern banks is their ability to turn individual illiquid loans into tradable securities and use them as source of liquidity. On the one hand, reliance on tradable securities reduces banks' cost of liquidity management. On the other hand, as the 2007-09 financial crisis showed reliance on such securities exposed the whole financial system to shocks to credit risk embedded in these securities. Banks that relied on such securities became the center of events in August 2007, when markets for securitized assets froze and stress immediately spread to interbank markets. However, as recent evidence shows despite freeze of markets for securitized assets the U.S. interbank markets were stressed but did not freeze after the initial shock in August 2007. We provide a novel theory of bank liquidity management that explains this differing performance of the interbank market and the secondary markets for the banks assets in August 2007.

In our model banks can cope with their liquidity shortfall using own cash, unsecured interbank borrowing and asset sales while they possess private information about their assets' quality. In equilibrium with endogenous price of their assets and interbank loan rate liquidity is then reallocated between illiquid banks, liquid banks and outside investors. Although asymmetric information about the banks' asset quality affects both the interbank market and the secondary market for banks' assets the banks with the good assets (the good banks) prefer to borrow because borrowing entails lower adverse selection cost than selling (as in Myers and Majluf (1984)).

If there is enough liquidity on the interbank market (either because liquidity hoarding or central bank intervention), in equilibrium the market for banks' assets freezes and the interbank market functions. Because the good illiquid banks prefer to borrow the expected quality of the sold asset is low. In turn, to profit from asymmetric information all bad illiquid banks follow the good banks to the interbank market. Although the bad banks contaminate the interbank market, the good illiquid banks have no desire to deviate and sell, because they anticipate higher adverse selection cost of selling than of borrowing. Such an equilibrium exhibits two features consistent with empirical evidence around August 2007 cited later. First, the interbank market is stressed but not frozen (Afonso, Kovner and Schoar (2011)), and (ii) the secondary market breaks down as in Akerlof (1970). All illiquid banks can find liquidity on the interbank market albeit at elevated spreads, whereas the markets for their assets freeze.

We also provide conditions under which some illiquid banks cannot cover their liquidity shortfall and go bankrupt in equilibrium. This occurs when there is little liquidity on the interbank market and the quality of the bad asset is sufficiently low. Because there is not enough loans for all illiquid banks, some illiquid banks are forced to sell. If the quality of the bad asset is sufficiently low the asset price is so depressed that the selling illiquid banks cannot generate enough cash to cover their liquidity shortfall. Since they cannot borrow and selling does not help, these banks go bankrupt. This equilibrium has an interesting feature that some illiquid banks go bankrupt although the asset price is higher than in the equilibrium with all illiquid banks borrowing.

Our model is well suited for analyzing the impact of uncertainty about quality of banks' assets on modern banks' liquidity management. First, our paper explains how this uncertainty affects liquidity redistribution via the interbank and secondary markets. This allows us to understand the conditions for effectiveness of different policy tools in addressing liquidity shortages. We show why interbank market injections are ineffective, why asset purchases might not be effective, and a trade-off the regulator faces when introducing liquidity requirements. Second, we view our model as a description of acute liquidity shocks, during which access to liquidity sources other than cash, interbank lending, asset sales and central bank is impossible. This makes our model suitable to explain the differing performance of interbank and secondary markets after the sudden breakout of the financial crisis in August 2007, triggered by an increase of uncertainty about riskiness of the mortgage-backed securities owned by banks.

Existing empirical evidence surrounding events in August 2007 is consistent with our interpretation of the model as a description of these events. First, as Acharya, Afonso and Kovner (2013) show the U.S. banks exposed to the shock at the ABCP market increased their interbank borrowing in the first two weeks of August 2007. This is consistent with our result that after an acute shock to the banks' assets' credit risk these banks look for liquidity on the interbank market rather than sell these assets. Second, Kuo, Skeie, Youle, and Vickrey (2013) point out that contrary to the conventional wisdom the term interbank markets for which the counterparty risk is an important determinant of borrowing costs did not freeze during the 2007-2009 crisis (see also

Afonso, Kovner and Schoar (2011) for similar evidence on the Fed Funds market). Specifically, the volume actually increased right after the start of the crisis in August 2007 despite the jump in the spread between the 1- and 3-month LIBOR and OIS (Figures 1 and 2). This supports our argument that the interbank market served as an important source of liquidity when the markets for banks' assets freeze. The robustness of the U.S. interbank markets might have also been supported by the Federal Reserve's liquidity injections, which occurred immediately after the start of the crisis. As we show, this is also consistent with our theory. As a central bank injects enough liquidity into the interbank market, all banks borrow and stop selling, because borrowing has a lower adverse selection cost.

Literature Review. The paper is related to an extensive literature on bank liquidity and shares many common features with other papers. However, because we model the coexistence of the secondary and interbank markets as sources of liquidity for modern banks, our contribution to the literature is two-fold. First, as argued above, we provide a theory that explains the differing performance of these two markets in August 2007. Second, our paper bridges a gap between two strands of literature on bank liquidity that are concerned with only one of these two markets at a time.

The first strand of literature concerns liquidity provision through interbank markets. Freixas, Martin and Skeie (2011) provide a model of an interbank market affected by uncertainty about distribution of liquidity in the banking system in the context of the 2007-2009 crisis. Our paper complements their work by emphasizing an effect of uncertainty about quality of banks' tradable assets on the liquidity distribution through the interbank markets. In this spirit, our model is similar to models by Freixas and Holthausen (2005) in the context of cross-border interbank markets and Heider, Hoerova and Holthausen (forthcoming) in the context of the recent financial crisis. These two papers show how asymmetric information about banks' quality can lead to the interbank market freeze. In our model the interbank market does not freeze, because we model an acute stress event, in which the highest quality banks experience the lowest adverse selection cost of purchasing liquidity at the interbank market. Our model is also related to work by Allen and Gale (2000) and Freixas, Parigi and Rochet (2000) on contagion through interbank networks.

Our paper can be viewed as a model of contagion through different markets to which banks are exposed.

The second strand of the relevant literature studies vulnerability of liquidity provision through secondary markets for banks' assets. Bolton, Santos and Scheinkman (2011), Brunnermeier and Pedersen (2009), and Malherbe (2014) analyze the effect of asymmetric information about quality of banks' assets. Bolton, Santos and Scheinkman (2011) study the timing of asset sales on secondary markets. Brunnermeier and Pedersen (2009) show how secondary markets dry up, when funding necessary to keep the secondary market liquid depends on this market's liquidity. Malherbe (2014) models self-fulfilling collapse of liquidity provision through asset sales. Our paper and Malherbe (2014) share many features, but the emphasis is different. Malherbe (2014) shows how liquidity hoarding and distribution are endogenous. We focus on liquidity provision when banks have more than one market to get liquidity from.

The remainder of the paper is organized as follows. Section 1 describes the setup. Section 2 presents benchmark results for the perfect information case. Section 3 analyzes the asymmetric information case. Section 4 discusses policy implications. Section 5 discusses model's main assumptions. The Appendix contains proofs of the results.

1 Setup

There are three dates, t=0,1,2, and one period. At t=0 each bank decides how to split one unit of its endowment between a risky asset and cash to maximize its return at t=2. The endowment belongs to the bank, i.e., we abstract from any debt except the interbank debt incurred at t=1. This simplifies algebra considerably without affecting the results and Section 5.4 discusses this assumption. At t=1 each bank receives two signals about the return structure of its asset and its liquidity need. After signals are revealed, a market for unsecured interbank loans (the interbank market) and a secondary market for banks' asset open. Each bank can use these markets to manage its liquidity. At t=2 the asset's returns are realized and payments are made.

The banks. At t=0 there is a continuum of mass one of identical banks. Each bank can invest fraction $\lambda \in [0;1]$ of endowment in cash, which returns 0 in net terms, and $1-\lambda$ in the asset.[1]

At t=1 each bank receives a private signal about its asset's return. With probability q the bank learns that the asset is good and returns $R > 1$ at t=2 with certainty (we call such a bank a "good bank"). With probability $1-q$ the bank learns that the asset is bad ("bad bank") and has the following return structure: it pays R at t=2 with probability $p > 0$ and 0 otherwise, where $pR < 1$. At t=0 it holds that $[q + (1-q)p]\, R > 1$, so the risky asset in expected terms is more valuable than cash.

At t=1 each bank receives also a private signal about its liquidity need. We model the liquidity need (shock) as a need to make a cash payment in an amount of $d < 1$.[2] We call banks that need to pay d illiquid, and the other banks liquid. With probability π the bank is liquid, and with $1-\pi$ illiquid. If an illiquid bank cannot generate enough cash to pay d it becomes bankrupt and sells all of its asset. Proceeds from such a sale accrue towards the payment of d. In the discussion section we argue why modelling the liquidity shock in such a reduced form is not relevant for our results.

We assume that shocks to the asset's return and liquidity are independent. We do so for algebraic convenience. Our results are intact as long as we assume less than perfect correlation between the realizations that a bank is illiquid and bad, which we think is a plausible assumption. If we assumed perfect correlation between these two realizations (the only illiquid banks are bad banks), there would be no asymmetric information about the quality of banks' assets at t=1, which we need for our results.

The interbank and secondary markets. In modelling the interbank and secondary market we follow the existing literature. The interbank lending is unsecured, diversified, and competitive with banks acting as price takers (e.g. Freixas and Holthausen (2005), and Freixas et al (2011)). Assumption about diversification of interbank loans at each lending bank makes default risk on interbank loans algebraically tractable, because each lender will be exposed to average risk on the credit market (Freixas and Holthausen (2005)). Each bank can sell its asset to other banks

[1] Alternatively to model a short-term liquidity management, we could assume that the bank starts out with a unit of an asset and has some spare liquidity and decides what fraction of this spare liqudity to preserve and to "consume" (see Gale and Yorulmazer (2011)).

[2] We assume $d < 1$ to reduce the amount of cases we need to consider when solving the model. Allowing for $d \geq 1$ would add cases in which bad and illiquid banks would be insolvent, which will be captured anyway for some levels of d close to 1.

and outside investors, who we think of as institutions that have a tolerance for long-term assets such as pension funds, mutual funds, sovereign wealth funds and hedge funds (Bolton, Santos and Scheinkman (2011)). The outside investors are competitive and have deep pockets, i.e., they can absorb any amount of assets that appear on the market (Malherbe (2014)).

The only crucial assumption in our setup is that the access to external funding immune to adverse selection, i.e., insured deposits, is prohibitively costly. If immediate access to such funding were cheaper than interbank borrowing or asset sales, all illiquid banks would prefer to tap it leading to a collapse of the secondary and interbank markets. Our assumption is plausible, because we model an acute liquidity shock, which is impossible to cope with by raising enough of insured deposits in a very short time.

2 Case of Perfect Information

As a benchmark we solve the model when all agents observe liquidity shocks and asset quality at individual banks. We solve the model backwards starting at t=1 after liquidity and asset quality shocks are realized and the markets open. From now on, we focus on the case in which the individual banks' level of cash is not higher than the liquidity need, $\lambda \leq d$, because holding $\lambda > d$ is not optimal in equilibrium. We also concentrate on describing behavior of illiquid banks, because this behavior is the focus of the paper. We can characterize the equilibria on the interbank and secondary markets at t=1 as follows.

Lemma 1. Assume perfect information about the individual banks' liquidity needs and asset quality. An equilibrium at t=1 has the following properties. All solvent illiquid banks are indifferent between selling their asset and borrowing from other banks. The price of the good and bad asset at t=1 is equal to their fundamental value R and pR. The interbank loan rate for the good and solvent bad banks reflects their default probabilities and is equal to 1 and $\frac{1}{p}$. Nobody lends to the insolvent illiquid banks.

Proof: In the Appendix.

Lemma 1 establishes three benchmark results, which we will contrast with our results under asymmetric information. First, under perfect information the no arbitrage condition makes the illiquid banks indifferent between selling, borrowing and using its own cash as in Modigliani and Miller (1958). Because competitive outside investors with deep pockets observe the banks' asset quality, they bid for these assets until they break even and the price of each asset reflects its fundamental value. This has two implications. Because each bank can sell its asset at its fundamental value, the cost of acquiring one unit of cash by selling its asset is 1 and is the same as the cost of using one unit of its own cash.[3] Moreover, because the sale of its asset at the fundamental value provides an opportunity cost of borrowing in the interbank market, the no arbitrage condition forces the interbank loan rates to adjust and equate the cost of acquiring a unit of cash by selling and borrowing. As a result illiquid banks are indifferent between selling and borrowing. Second, despite a fixed amount of cash held by banks at t=1, there is no cash-in-the-market effect on the interbank market. The reason is that there is always abundant liquidity on the secondary market for banks who cannot obtain interbank loans. Third, because the assets trade at their fundamental values only the bad illiquid banks can fail. They fail if they are insolvent, i.e., the expected return on their asset is lower than cash they need to cover their liquidity need ($pR\,(1-\lambda) < d-\lambda$).

Because at t=1 the cost of acquiring a unit of cash is the same when borrowing, selling and using own cash, there is no value from holding own cash at t=1. Because holding own cash is not valuable at t=1 and the asset is ex ante more valuable than cash, each bank invests its entire endowment in the asset at t=0.

Lemma 2. Assume perfect information about the individual banks' liquidity needs and asset quality. The banks' optimal choice of cash λ at t=0 is 0.

Proof: In the Appendix.

In the rest of the paper we analyze the asymmetric information case.

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[3] If the asset's price is P, then to obtain one unit of cash a bank has to sell $\frac{1}{P}$ of its asset. The bank's loss is the forgone return on the sold portion of the asset, $\frac{p_i R}{P}$, where $p_i R$ is the asset's expected return. If the asset trades at its fundamental value $P = p_i R$, the bank's loss from selling it is 1, which is the same as from using one unit of its cash. The loss from using one unit of its own cash is the foregone return from storing cash from t=1 to t=2, which is 1.

3 Banks' liquidity management under asymmetric information

We solve the model in the usual fashion. We start at t=1 and solve for the equilibrium on the secondary and interbank markets. Then at t=0 we solve for the banks' optimal choice of cash at t=0.

The crucial difference between the cases of perfect and asymmetric information is the nature of equilibrium at t=1. Under asymmetric information the asset price and the interbank loan rate are a function of agents' expectations about the quality of sold assets and the quality of borrowing banks (under asymmetric information there is only one asset price and one loan rate).[4] At the same time, the quality of sold assets and the quality of borrowing banks are functions of the illiquid banks' decisions whether to sell or borrow, where these decisions themselves depend on the anticipated asset price and loan rate as well as banks' own expectations. Hence, the asset price, the loan rate as well as agents' expectations about the quality of sold assets and borrowing banks are determined simultaneously in equilibrium. This is different from the perfect information case in which agents do not have to build any expectations, and the asset price and loan rate are functions of the observed quality of banks' assets. Moreover, our model is more complicated than usual models of asymmetric information affecting only one market, because asymmetric information affects two markets at the same time.

To better highlight the above consequences of asymmetric information we construct this section as follows. First, we derive banks' optimal liquidity management decisions for given asset price and loan rate at t=1. Second, given these optimal decisions we find an equilibrium at t=1 as a fixed point of these decisions, agents' expectations, and market clearing conditions. Finally, given an equilibrium at t=1 we derive the banks' optimal choice of cash at t=0.

[4] See Akerlof (1970), Bolton, Santos and Scheinkman (2011), Freixas and Holthausen (2005), Heider, Hoerova and Holthausen (forthcoming), and Malherbe (2014).

3.1 Banks' optimal liquidity management

After the liquidity and asset return shocks are realized at t=1, there are four types of banks: good and liquid, bad and liquid, good and illiquid, and bad and illiquid. Liquid banks, which have λ of cash, look for the best use of it on the interbank and secondary market. Illiquid banks, which have a liquidity shortfall $d - \lambda$, look for the cheapest way to cover this shortfall on the interbank and secondary market.

We can write the liquidity management decision problem of all four types of banks in a compact form after the following simplifications. First, the liquid and illiquid banks differ only in their liquidity need. We use an indicator μ that equals to 1 if the bank is illiquid and 0 if it is liquid. Second, the good and bad banks differ only in the probability of success of their asset, p_i, where $p_B = p$ for the bad bank ($i = B$) and $p_G = 1$ for the good bank ($i = G$). Finally, without loss of generality we ignore the possibility that the banks can buy other banks' assets for two reasons. First, purchasing the asset is never better than storing cash in equilibrium. The competitive outside investors bid up the asset price to its expected return resulting in a 0 net return on the purchased asset, which is also the net return on storing cash. Second, additional liquidity supplied by banks on the secondary market has no influence on the asset price because the outside investors have deep pockets.

The liquidity management decision problem of a bank reads

$$\max_{l,S} \begin{cases} p_i[(1-\lambda-S)R + (SP+\lambda-l-\mu d) + \widehat{p}R_D l] + (1-p_i)[(SP+\lambda-l-\mu d) + \widehat{p}R_D l], \text{ if } l > 0, \\ \\ p_i \max[0; (1-\lambda-S)R + (SP+\lambda-l-\mu d) + R_D l] \\ +(1-p_i)\max[0; (SP+\lambda-l-\mu d) + R_D l], \text{ if } l \leq 0. \end{cases} \tag{1}$$

s.t.

$$S \in [0; 1-\lambda], \tag{2}$$

$$SP + \lambda - \mu d \geq l. \tag{3}$$

At t=1 each bank chooses how much to borrow/lend and how much to sell to maximize its expected return at t=2 (1) under constraints (2) and (3). The first line in objective function (1) is the bank's expected return at t=2 when it lends on the interbank market ($l > 0$). $(1 - \lambda - S)R$ is the return on the asset after selling S units if the asset will pays R at t=2. $SP + \lambda - l - \mu d$ is the remaining cash after the bank receives SP from selling S of the asset at a price P, lends l on the interbank market and pays d if it is illiquid ($\mu = 1$). $\widehat{p}R_D l$ is the return on the interbank loans at t=2, where $\widehat{p}$ is the expected probability of repayment by the borrowing banks. The return on the interbank loans is deterministic because we assumed that each lending bank has a diversified interbank loan portfolio. The second line in objective function (1) is the bank's expected return at t=2 when it borrows on the interbank market ($l < 0$; we add here the case of $l = 0$). This return differs from the case when the bank lends for two reasons. First, instead of $\widehat{p}R_D l$ we have $R_D l$, which is the amount the borrowing bank has to repay. Second, we include max-operators to take into account that an illiquid bank might be insolvent, in which case its payoff is 0 due to limited liability. Constraint (2) represents the amount of the asset the bank can sell. Constraint (3) represents the maximum amount the bank can lend on the interbank market, which is equal to its own cash λ, cash raised from selling S of its asset SP net of the possible repayment of d.[5]

The following proposition presents the main result from solving the above program: the illiquid banks' selling and borrowing strategies. We present the only relevant case in equilibrium, $\widehat{p}R_D \geq 1$, $P \in [pR; R)$, and $p_G = 1 \geq \widehat{p} > p_B$.[6]

Proposition 1: Assume that $\widehat{p}R_D \geq 1$, $P \in [pR; R)$, and $p_G = 1 \geq \widehat{p} > p_B$. For a given asset price and loan rate the illiquid banks' optimal liquidity management decisions are as follows.

1. If $P < \frac{d-\lambda}{1-\lambda}$ and $R_D > \frac{1-\lambda}{d-\lambda}R$, each good and bad bank is bankrupt and indifferent between borrowing and selling.

2. If $P < \frac{d-\lambda}{1-\lambda}$ and $R_D \leq \frac{1-\lambda}{d-\lambda}R$, each good and bad illiquid bank borrows $d - \lambda$ and does not sell.

[5] An illiquid bank can also lend if by selling it generates more cash than it needs to pay.

[6] Condition $\widehat{p}R_D \geq 1$ means that lending is profitable. Condition $P \in [pR; R)$ means that the asset price cannot be lower than the fundamental value of the bad asset but always lower than the fundamental value of the good project. In equilibrium anything else cannot occur. Because the outside investors have deep pockets the asset price cannot drop below the lowest value of all projects in the economy, pR. It cannot also reach the value R of the best project in equilibrium because of adverse selection. Condition $p_G = 1 \geq \widehat{p} > p_B = p$ means that the expected quality of borrowing banks is always higher than the quality of bad banks. An equilibrium in which the good illiquid banks sell and the bad illiquid banks borrow ($\widehat{p} = p_B = p$) cannot exist, because the bad illiquid banks would then prefer to sell their asset for a price R rather than borrow.

3. If $P \geq \frac{d-\lambda}{1-\lambda}$, then we have the following solutions. Whenever a bank sells all of its asset, it lends the remaining cash $\lambda + (1-\lambda)P - d$.

For $R_D < \frac{R}{P+\left(\frac{\widehat{p}}{p_i}-1\right)\left(P-\frac{d-\lambda}{1-\lambda}\right)}$ each good and bad bank borrow $d-\lambda$ and do not sell.

For $R_D \in \left[\frac{R}{P+\left(\frac{\widehat{p}}{p_i}-1\right)\left(P-\frac{d-\lambda}{1-\lambda}\right)}; \frac{R}{P}\right)$ each bad bank sells all of its asset $1-\lambda$, but each good bank borrows $d-\lambda$ and does not sell.

For $R_D \in \left[\frac{R}{P}; \frac{R}{\widehat{p}P}\right)$ each bad bank sells all of its asset $1-\lambda$, but each good bank sells only $\frac{d-\lambda}{P}$ of its asset and does not borrow (or it is indifferent between selling and borrowing for $R_D = \frac{R}{P}$).

For $R_D \geq \frac{R}{\widehat{p}P}$ each bad and good bank sells all of its asset $1-\lambda$.

Proof: in the appendix.

Depending on the anticipated asset price and the loan rate three cases can arise. If the asset price is so low that no bank can pay d by selling all of its asset and the loan rate is so high that no bank can repay the loan $d-\lambda$ at t=1 (case 1), then each bank goes bankrupt. If the asset price is still too low for a bank to pay d by selling, but it could repay the loan $d-\lambda$ (case 2), the bank borrows all of its liquidity shortfall $d-\lambda$ and does not sell. However, once the bank can pay d by selling its asset (case 3), then each illiquid bank's choice whether to sell or borrow depends on the relative price of the asset and the loan. Generally, as the loan rate increases relative to the asset price, the illiquid banks prefer to sell rather than borrow.

The main result of Proposition 1 is that once the illiquid banks can choose between selling and borrowing (case 3), the good banks are more reluctant to sell and, therefore, more willing to borrow than the bad banks. For intermediate loan rates, $R_D \in \left[\frac{R}{P+\left(\frac{\widehat{p}}{p_i}-1\right)\left(P-\frac{d-\lambda}{1-\lambda}\right)}; \frac{R}{\widehat{p}P}\right)$, the bad banks sell all of their asset and do not borrow, but the good banks either borrow and sell nothing or do not borrow but sell only enough of their asset to cover the liquidity shortfall. The main reason is asymmetric information: the investors cannot distinguish between the quality of the banks' assets and pay a indiscriminate price $P \in [pR; R)$. In turn, the good banks are less willing to sell their valuable asset knowing that they will get a price lower than their asset's fundamental value. To the contrary, the bad banks will sell all of their asset knowing that they will get a price at least as high as their asset's fundamental value.[7]

[7] In addition, the bad banks are more willing to sell than the good banks, because the bad bank's loan portfolio would be less risky than the bank itself ($p_B < \widehat{p}$) making lending and selling more attractive for such a bank than

Proposition 1, which shows that the good banks are less willing to sell than the bad banks, is the driver of our main result that the interbank market might be active at the same time when the secondary market breaks down due to lower adverse selection discount on the interbank market. The good banks face a lower adverse selection cost on the interbank market than on the market for their assets because the share of bad borrowing banks to the share of the good borrowing banks is lower than the share of bad assets sold to the share of good assets sold. When the good and bad banks borrow at the same time, both types borrow only the amount they need to cover their liquidity shortfall, $d-\lambda$. None of them would like to borrow more because for both types borrowing is costly in the sense that for each borrowed unit the lenders demand in return more than one unit. However, when both types of banks sell at the same time they sell different amounts. As Proposition 1 shows the bad banks sell all of its asset $1-\lambda$, whereas the good banks sell maximally an amount $\frac{d-\lambda}{P}$.[8] Hence, the proportion of the bad asset on the secondary market when both types sell is higher than the proportion of the bad banks on the interbank market when both borrow. This implies that the adverse selection cost of selling is higher than of borrowing pushing the good banks towards the interbank market in equilibrium.

3.2 Equilibria on the interbank and secondary market

As described earlier the equilibrium asset price and loan rate have to reflect the banks' optimal liquidity management choices, be consistent with agents' expectations about these choices, and be such that the interbank and secondary markets clear. Because all agents build expectations about the quality of sold assets and of borrowing banks we use the concept of perfect Bayesian equilibrium as in Freixas and Holthausen (2005).

Because of the nature of the perfect Bayesian equilibrium two types of equilibria might coexist. In one equilibrium the interbank market is active. In other equilibrium all illiquid banks sell and the interbank market is inactive if we impose sufficiently pessimistic off-equilibrium beliefs about the quality of borrowing banks. In the proof of Proposition 2 we derive conditions under which the

keeping its asset.

[8] In fact, there is no equilibrium in which both good and bad illiquid banks sell all of its asset in order to lend excess cash on the interbank market. In such a case, there would be no demand for interbank loans and the interbank market would not clear.

Cho-Kreps intuitive criterion eliminates the equilibrium with the inactive interbank market. In what follows we restrict the discussion to the set of parameters fulfilling these conditions, because we think the equilibrium with the active interbank market is the most relevant scenario for our positive and normative implications.

To simplify the exposition of our results we discuss equilibrium outcomes in two sections. We start with an equilibrium in which all illiquid banks can cover their liquidity shortfall $d - \lambda$ by selling their assets ("equilibrium without liquidity shortage"), and then we discuss the opposite case ("equilibrium with liquidity shortage").

3.2.1 Equilibrium without liquidity shortage

We derive the equilibrium in a usual way. First, we assume specific banks' liquidity management choices in equilibrium. Then we write down equilibrium conditions consistent with these choices. Finally, we check whether the asset price and loan rate that solve these conditions are consistent with the assumed banks' liquidity management choices.

The process of finding an equilibrium with specific liquidity management choices is tedious because the discrete distribution of bank types implies many specific cases to be analyzed.[9] For that reason we present in the text only derivation of an equilibrium in which all good and bad illiquid banks borrow and only the bad liquid banks sell. Such an equilibrium describes best our main result that the interbank market can thrive while the secondary market can break down, and contrasts most with the Modigliani-Miller result from Lemma 1. We analyze other cases in more detail in the proof of Proposition 2.

The conditions describing the equilibrium in which all illiquid banks borrow without selling any of their asset are the following. All equilibrium values are denoted with *. Because we assume that all illiquid banks borrow the aggregate loan demand is $(1 - \pi)(d - \lambda)$, where $1 - \pi$ is the number of all illiquid banks and $d - \lambda$ is individual banks' liquidity shortfall. Good and bad liquid banks are at least indifferent between lending or not and supply their cash λ and $\lambda + P^*(1 - \lambda)$,

[9] If we work with a continuous distribution of asset quality, we are able to write down a single set of equilibrium equations at t=1. However, even for a uniform distribution the set of equilibrium eqautions has no analytical solution at t=1.

resulting in the aggregate loan supply $\pi\left[q\lambda+(1-q)\left(\lambda+P^{*}(1-\lambda)\right)\right]$.[10] Hence, in an equilibrium with all illiquid banks borrowing the interbank market clears if the aggregate supply of loans is bigger than or equal to the aggregate demand:

$$\pi\left[q\lambda+(1-q)\left(\lambda+P^{*}(1-\lambda)\right)\right]\geq(1-\pi)(d-\lambda). \tag{4}$$

If condition (4) holds with strict inequality sign, there is excess supply of loans and the interbank market clears at a loan rate $R_D^*=\frac{1}{\widehat{p}^*}$, for which the liquid banks break even on lending to banks with the expected probability of interbank loan repayment $\widehat{p}^*$ are indifferent between lending or storing cash. If condition (4) holds with equality, then the interbank market clears for any loan rate such that all liquid banks lend and all illiquid banks borrow, $R_D^*\in\left[\frac{1}{\widehat{p}^*};\frac{R}{P^*+\left(\frac{\widehat{p}^*}{p_i}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)}\right)$. Agents' equilibrium beliefs about the expected probability of interbank loan repayment $\widehat{p}^*$ have to be consistent with the fact that all illiquid banks borrow in equilibrium

$$\widehat{p}^*=q+(1-q)\,p. \tag{5}$$

Because the competitive outside investors supply abundant liquidity on the secondary market, the demand for the banks' asset is perfectly elastic at a price for which the outside investors break even. Hence, the equilibrium price P^* is equal to agents' beliefs about the quality of sold assets. Because the good banks do not sell in the assumed equilibrium, the investors anticipate this and expect only the bad asset to be sold on the market:

$$P^*=pR. \tag{6}$$

The equilibrium characterized above exists if the above equilibrium loan rate R_D^*, asset price P^*, the expected probability of interbank loan repayment $\widehat{p}^*$, and each bank's cash level λ are such that condition (4) is satisfied, all illiquid banks are ready to borrow which occurs for $R_D^*<\frac{R}{P^*+\left(\frac{\widehat{p}^*}{p_i}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)}$ and $i=G,B$, and that all liquid banks are ready to lend which occurs for $\widehat{p}^*R_D^*\geq 1$. Inserting the equilibrium values of R_D^*, P^* and $\widehat{p}^*$ into these three conditions provides

[10] The proof of Proposition 1 describes the behavior of the good and bad liquid banks.

the set of parameters for which the assumed equilibrium exists (see the proof of Proposition 1 for details).

The above equilibrium, in which all illiquid banks borrow and do not sell, contrasts sharply with the Lemma 1's Modigliani-Miller-type result, in which the illiquid banks are indifferent between selling and borrowing. To best explain this contrast let's assume that in the asymmetric information case en equilibrium exists in which the good banks are indifferent between selling and borrowing. From Proposition 1 we know the good banks are indifferent between selling and borrowing if $R_D^* = \frac{R}{P^*}$. For such a loan rate Proposition 1 also implies that the bad illiquid banks prefer to sell all of their asset. Hence, in an equilibrium with $R_D^* = \frac{R}{P^*}$ some good banks would borrow and the rest of them together with all bad illiquid banks would sell. Moreover, in such an equilibrium the expected probability of loan repayment would be $\widehat{p}^* = 1$ and the expected value of sold assets would be somewhere between pR and R. With abundant liquidity on the interbank market (condition (4) holds) and $\widehat{p}^* = 1$ the interbank market would clear for a loan rate $R_D^* = \frac{1}{\widehat{p}^*} = 1$. However, the loan rate $R_D^* = 1$ and the indifference condition for the good banks $R_D^* = \frac{R}{P^*}$ would imply that the equilibrium asset price should be $P^* = R$. But this contradicts with the fact that the equilibrium asset price must be lower than R because the bad banks sell their asset too. Hence, it is impossible that the good banks would be indifferent between selling and borrowing in equilibrium, because all good banks would prefer to borrow.

With all good banks borrowing all bad illiquid banks prefer to borrow rather than sell because they want to profit from asymmetric information on the interbank market. Although such a behavior by the bad illiquid banks implies that the good illiquid banks incur an adverse selection discount when borrowing (as reflected in the equilibrium loan rate $R_D^* = \frac{1}{\widehat{p}^*}$), the good banks do not deviate to the secondary market anticipating that the adverse selection cost there would be even higher as explained earlier.

The above equilibrium arises when there is enough liquidity on the interbank market for all the illiquid banks. As this liquidity gets scarcer, the good and bad banks have to look for liquidity on the secondary market. They find this liquidity there if the equilibrium asset price is such that cash raised by selling all of its asset $P^*(1-\lambda)$ is sufficient to cover the liquidity shortfall $d-\lambda$. We do not describe in detail the equilibria in which some illiquid banks have to sell, but we characterize them in the following Proposition and briefly explain the intuition.

Proposition 2: For sufficiently high cash level λ all good and bad illiquid banks borrow. For any other λ assume that in equilibrium it holds that $P^* (1-\lambda) + \lambda > d$. For intermediate cash level λ all good and some of the bad illiquid banks borrow, and the rest of bad illiquid banks sells. For sufficiently low cash level λ only some of the good illiquid banks borrow, and the rest of illiquid banks sells.

Proof: in the appendix.

Proposition 2 restates first the result that for abundant liquidity all illiquid banks borrow. Once the banks' cash levels are such that not all illiquid banks can borrow, the bad banks are the first ones to leave the interbank market and start selling. When the interbank loans become scarce, the illiquid banks start looking for an alternative source of liquidity. This alternative source is the secondary market. Because the bad illiquid banks are more willing to sell than the good illiquid banks, the bad banks are the first ones to sell when there are too little loans for all illiquid banks. Finally, when there is not enough interbank loans for all good illiquid banks, these banks follow the bad banks to the secondary market.

To close the discussion of equilibria without liquidity shortage we characterize the equilibrium price, loan rate and total interbank borrowing volume as functions of cash level λ.

Lemma 3: Suppose that in an equilibrium with some illiquid banks selling it holds that $P^* (1-\lambda) + \lambda > d$.

1. Equilibrium price P^* is decreasing in λ until all good illiquid banks leave the secondary market. At this point P^* reaches pR and stays there for other λ.

2. Equilibrium loan rate R_D^* is increasing in λ until all good illiquid banks leave the secondary market. Then it can be either increasing or decreasing until all bad illiquid banks leave the secondary market, and equals $\frac{1}{q+(1-q)p}$ when all illiquid banks borrow.

3. Total lending volume is decreasing in λ until all good illiquid banks leave the secondary market, can be non-monotonic (first increasing and then decreasing) until all bad illiquid banks leave the secondary market, and decreasing when all illiquid banks borrow.

Proof: Proof follows from deriving the equilibrium values with respect to λ.

The equilibrium asset price P^* decreases with cash level as long as some of the good illiquid banks sell. As banks' cash levels increase and interbank loans are more abundant, the increasing number of the good illiquid banks switches from selling to borrowing. As the higher number of the good banks stops selling the expected quality of the sold assets goes down and, with it, their price. Once there is enough interbank loans for all good illiquid banks, only the bad banks sell and the price stays at pR.

Changes in the equilibrium loan rate R_D^* reflect the interplay between the expected repayment probability and interbank loan supply due to changes in banks' cash levels. On one hand, the equilibrium loan rate increases with cash levels, because more of the bad illiquid banks borrow resulting in falling expected probability of interbank loan repayment. On the other hand, the equilibrium loan rate decreases with banks' cash levels, because supply of interbank loans becomes more abundant and the liquidity premium due to the cash-in-the-market effect decreases.[11] Total lending volume is generally decreasing in λ because each illiquid bank's loan demand $d-\lambda$ decreases. However, for intermediate cash reserves total lending volume can be non-monotonic. The reason is that increasing share of the borrowing bad illiquid banks increases the total loan demand.

3.2.2 Equilibrium with liquidity shortage

In the previous section we worked under the assumption that in equilibrium in which some illiquid banks have to sell the equilibrium asset price is such that the selling illiquid banks can raise enough cash to cover their liquidity shortfall $d-\lambda$, $P^*(1-\lambda)+\lambda > d$. Now we turn to the opposite case.

Lemma 4: Condition $P^*(1-\lambda)+\lambda > d$ from Proposition 2 does not hold for low λ if the share of good banks q is low, or for intermediate λ if the payment d and the share of good banks q are high.

Proof: In the Appendix.

The fact that the asset price might be such that selling all of the asset does not generate enough

[11] In two knife-edge cases for $\lambda = \lambda_1$ and $\lambda = \lambda_2$ as defined in the proof of Proposition 1 R_D^* is indeterminate, because the demand and supply of interbank loans are inelastic (see also Freixas, Martin and Skeie (2011) for a similar result). In addition, in a case with continuous distribution of asset quality the interpplay between asymmetric information and limited loan supply gives a clear-cut inverse-U-shape of R_D^* as a function of λ.

cash to cover the liquidity shortfall is not surprising. As in Lemma 1, in which for sufficiently low bad asset's value the bad illiquid banks cannot raise enough cash to cover their liquidity shortfall, in the asymmetric information case sufficiently low share of good banks q drags the (pooling) asset price so low that the selling illiquid banks cannot generate enough cash to cover their liquidity shortfall $d - \lambda$.

In addition, for sufficiently high good banks' share but high liquidity need d the asset price might be too low to raise enough liquidity by selling for intermediate cash levels λ. This is due to the fact that the selling bank's total cash, $\lambda + P^* (1 - \lambda)$, is first decreasing and then increasing in banks' cash. The total cash falls in banks' own cash because per Lemma 3 the equilibrium asset price falls in cash until all good banks switch to the interbank market. After that the equilibrium asset price is equal to pR and the total cash must increase with banks' own cash λ. Hence, the banks' total cash after selling, $\lambda + P^* (1 - \lambda)$, is the lowest for intermediate λ for which all good illiquid banks switch to the interbank market.

Proposition 3: Suppose condition $P^* (1 - \lambda) + \lambda > d$ from Proposition 2 does not hold. There exists an equilibrium with liquidity shortage, in which the asset price is smaller than $\frac{d-\lambda}{1-\lambda}$ and the loan rate is $\frac{1-\lambda}{d-\lambda}R$. In such an equilibrium only some of the illiquid banks become liquid by borrowing, and the rest of the illiquid banks goes bankrupt. Otherwise there is an equilibrium in which the asset price is $\frac{d-\lambda}{1-\lambda}$, the loan rate is $\frac{1-\lambda}{d-\lambda}R$, and all illiquid banks are indifferent between borrowing and selling.

Proof: In the Appendix.

We call an equilibrium in which the asset price is smaller than $\frac{d-\lambda}{1-\lambda}$ "an equilibrium with liquidity shortage". In such an equilibrium the asset price is so low that none of the illiquid banks can cover their liquidity shortfall by selling its asset. In such a case, the only option to cover the shortfall is the interbank market. Each illiquid bank would like to borrow for any loan rate R_D for which it can repay the interbank loan at t=2, $R_D < \frac{1-\lambda}{d-\lambda}R$. For any other loan rate $R_D \geq \frac{1-\lambda}{d-\lambda}R$ the illiquid banks are bankrupt because they cannot repay their interbank loans and, therefore, they are indifferent between selling and borrowing. The liquid banks supply loans only if the loan rate is so low that the illiquid banks can repay their loans at t=2, $R_D \leq \frac{1-\lambda}{d-\lambda}R$. Because there is not enough interbank loans for all illiquid banks to borrow, the interbank market clears at the highest loan rate at which the liquid banks lend and the illiquid are indifferent between selling and borrowing, $R_D = \frac{1-\lambda}{d-\lambda} R$. Then the available interbank loans are assigned randomly to illiquid banks and only some of them get the loans. The rest of the illiquid banks becomes bankrupt, because they either cannot borrow or sell to cover the liquidity shortfall.[12] The contrast with Lemma 1 is that in the asymmetric information case some good illiquid banks might become bankrupt.

There is also an equilibrium in which the asset price is equal to $\frac{d-\lambda}{1-\lambda}$. This equilibrium arises due to discrete distribution of bank types coupled with random assignment of loans. As we vary banks' cash level λ at the point where the equilibrium without liquidity shortage seizes to exist the number of good banks forced to sell increases discontinuously due to rationing of scarce interbank loans. But such a discontinuous increase in the share of the good selling banks increases the asset price discontinuously, which means that an equilibrium with liquidity shortage cannot arise. Because the banks' cash level is such that an equilibrium without liquidity shortage cannot arise either, there exists a "hybrid" equilibrium in which the equilibrium asset price is exactly $\frac{d-\lambda}{1-\lambda}$.[13]

3.2.3 Social welfare

Social welfare at t=1 is the sum of banks' expected profits at t=2 and the sum of payments d made by the illiquid banks at t=1. Although we do not model explicitly agents who receive these payments, including these payments in welfare calculation makes them welfare neutral if they are made.[14] We also assume that missing a payment by a bankrupt bank costs $\tau > 0$ units of welfare per unit of missed payment.

Lemma 5: Social welfare at t=1 in an equilibrium without liquidity shortage is equal to the expected value of the asset at t=0 and cash kept by all banks, $(1-\lambda)(q+(1-q)p)R+\lambda$. Social

[12] Instead of rationing the share of borrowing illiquid banks we could ration the size of the interbank loan. In such a case each illiquid bank would borrow less than $d-\lambda$ and sell its asset to come up with the missing cash. Given that all illiquid banks would do the same they would all default in an equilibrium with liquidity shortage. Once we assume that defaults are socially costly, rationing the loan size size is less inefficient than the assumed rationing of the number of borrowing banks which limits the number of defaulting banks. For that reason we stick to the latter form of rationing.

[13] Continuous distribution of bank types eliminates such hybrid equilibria but it makes the solution of the model analytically untractable.

[14] If the payments d were not added to welfare, their execution by banks would be socially costly, because they would reduce banks' returns. Hence, it would be socially beneficial to close all illiquid banks and sell their assets without making the payment d.

welfare at t=1 in an equilibrium with liquidity shortage is lower than $(1-\lambda)(q+(1-q)p)R+\lambda$ if $\tau>0$.

Proof: In the appendix.

In an equilibrium without liquidity shortage the highest possible social welfare at t=1 is achieved. Although the borrowing banks have to generally pay a liquidity premium, this is just a welfare-neutral transfer to the lending banks. In equilibrium with liquidity shortage the highest possible social welfare at t=1 cannot be achieved. The bankrupt illiquid banks do not have enough cash to cover their liquidity shortfall. Hence, each unit of missed payment by these banks is socially costly.

3.3 Optimal choice of cash reserves

To close the model we solve for the banks' optimal choice of cash holdings at t=0. At t=0 the banks maximize their expected profits at t=2 by choosing their cash holding λ consistent with an equilibrium that they anticipate at t=1.

Lemma 6: In general, the banks' cash holdings at t=0 increase when the risky asset becomes less profitable (R decreases). For intermediate R there are multiple equilibria: the banks hold either no cash or a positive cash level. If parameters are such that an equilibrium with liquidity shortage at t=1 can exist, for intermediate R the banks choose at t=0 such cash holdings that an equilibrium with liquidity shortage at t=1 occurs.

Proof: In the appendix.

The first statement in Lemma 6 holds in general because (i) there may be multiple equilibria, and (ii) for parameters such that an equilibrium with liquidity shortage at t=1 can exist we can use only numerical solution.

Contrary to Lemma 2 for the perfect information case, the banks can choose to hold positive cash reserves. The reason is the cash-in-the-market effect, and the resulting liquidity premium in the loan rate, on the interbank market that affects the borrowing banks at t=1. This liquidity premium means that cash is valuable at t=1 for speculative and pre-cautionary reasons.[15] If the

[15]See also Acharya and Skeie (2011), Ashcraft, McAndrews and Skeie (2011), and Gale and Yorulmazer (2013) for models or precautionary and speculative motives for liquidity hoarding against liquidity shocks.

bank is liquid, it will use its cash to earn a positive net return on interbank lending. If the bank is illiquid, its own cash reduces the need to borrow and pay the liquidity premium. Given that cash is valuable at t=1 (contrary to the perfect information case), at t=0 the bank trades off this value of cash with the expected return on the risky asset. As the risky asset becomes less profitable, the bank invests more in cash. Finally, since the banks do not internalize the social cost of missed payments, it is possible that for some intermediate values of R they choose such cash reserves for which they might go bankrupt at t=1.

We need to discuss also the multiple equilibrium result. This result occurs for the exactly same reasons as it occurs in Malherbe (2014): coordination failure among banks when choosing cash at t=0 and the falling equilibrium asset price in cash level at t=1. As shown in Lemma 3, the equilibrium asset price at t=1 is decreasing in cash λ at a certain interval. This however coincides with banks' preferences for cash at t=0 that arise due to coordination failure. If a bank anticipates a high (low) asset price at t=1, it prefers low (high) cash λ at t=0. Combining the banks' preference for cash at t=0 with the equilibrium asset price at t=1 means that two equilibria can coexist. In one of them each bank has no cash and the asset price is high in equilibrium. In the other, each bank has high cash and the asset price is low. Multiple equilibria arise for intermediate values of the risky asset, (intermediate) R. For sufficiently high return R by selling their asset the bad banks can provide so much loan supply that all illiquid banks can borrow for any cash reserves. Hence, there is a unique equilibrium in which banks choose no cash. For sufficiently low R the anticipated asset price is so low that a unique equilibrium is the one in which the banks choose positive cash levels at t=0.

Contrary to Malherbe (2014), whose goal is to show that a secondary market for bank assets can freeze due to coordination failures, we do not put much emphasis on multiple equilibria in the choice of cash at t=0. Instead our paper focuses on how the interbank market can thrive and the secondary market can be impaired at the same time at t=1 for given banks' cash holding. This result is independent of the multiple equilibrium result at t=0. In fact, we can see that there is a unique equilibrium with high cash levels at t=0 for low values of the risky asset, which leads to a collapse of secondary market and thriving interbank market at t=0. This is consistent with the events in August 2007. Moreover, existence of multiple equilibria is sensitive to assumption of abundant liquidity on the secondary market. Once we allow for fire sales on the secondary market, the equilibrium asset price at t=1 might be increasing in banks' cash holdings, because more own cash reduces need for asset sales. If this is the case, multiple equilibria seize to exist, since banks' preference to hold cash at t=0 decreases with the anticipated asset price ensuring that there is a unique equilibrium fix point at t=0.

3.4 Welfare analysis

Social welfare at t=0 is equal to social welfare from Lemma 5 for a given optimal choice of cash holdings λ at t=0. In general, banks' privately optimal choice of cash reserves is not socially efficient for two reasons.

First, the socially efficient choice of cash at t=0 is 0 if it does not lead to an equilibrium with liquidity shortage at t=1. Zero cash is socially optimal, because the investment in the risky asset is more valuable than in cash, and the transfer of interbank loan payments between banks is welfare-neutral. However, the individual banks care about the speculative and pre-cautionary motives, and do not internalize the impact of their cash holding's choice on the asset price and loan rate. Hence, they choose positive cash reserves.

Second, the banks might choose cash holdings at t=0 such that an equilibrium with liquidity shortage occurs at t=1, because they do not internalize the social cost of their bankruptcy. The socially optimal cash in such a case depends on the set of parameters for which the liquidity shortage occurs. If the liquidity shortage occurs for sufficiently low banks' cash holdings, then the socially optimal cash is higher than what the banks choose. If the liquidity shortage occurs for some intermediate cash holdings, then the socially optimal cash holdings is zero and lower than what the banks have chosen. The reason is the same as in the preceding paragraph.

The important observation is that the socially optimal amount of cash holdings might be higher or lower than the banks' optimal choice depending on the model's parameters. This is an important issue that we discuss in the next section.

4 Policy Implications

We discuss our model's policy implications from two perspectives: ex post (t=1) and ex ante (t=0).

4.1 Policy Implications Ex Post

As shown in Lemma 5 the policy intervention at t=1 is warranted only when an equilibrium with liquidity shortage occurs and the bankrupt illiquid banks miss their payments. In such an equilibrium two conditions are met at the same time: (i) there are not enough interbank loans for all illiquid banks, and (ii) the asset price is so low that the illiquid banks cannot raise enough cash by selling. A policy intervention is successful if it remedies one of the two conditions and pushes the banking system into an equilibrium without liquidity shortage.

The remedy for not enough interbank loans is an injection of sufficient amount of cash into the banks by a central bank to allow all illiquid banks to borrow. In our stylized model, such injections could be implemented in various ways: through unsecured loans to all banks at the market loan rate or through secured loans backed by the same collateral requirements for all banks.[16] It needs to be mentioned that the side effect (without social cost in this model) would be a breakdown of the secondary market because flooding the interbank market with liquidity moves the banking system to an equilibrium in which all illiquid banks borrow (the same equilibrium we described in Section 3.2.1).

If the equilibrium with liquidity shortage occurs for intermediate cash λ, draining of cash from banks would also push the banking system towards an equilibrium without liquidity shortage, in which *only* the good banks borrow. Draining of cash would further reduce the supply of interbank loans forcing more of the good illiquid banks to sell their assets. Increasing share of the good asset on the secondary market would reduce the adverse selection discount and increase the asset price. If the central bank drains enough reserves, the asset price increases to a point in which banks can cover their liquidity shortfall by selling and reduce their reliance on the interbank market. Draining of reserves could be implemented through central bank's borrowing mirroring lending proposed in

[16] Since we assume that the central bank has the same information about individual banks as the market, it has to protect itself against adverse selection by offering the same conditions to all banks.

the preceding paragraph.

Although draining of banks' cash reserves is aimed at lifting the equilibrium asset price, changing the supply of cash directly on the secondary market does not work, because there is already unlimited amount of cash provided by outside investors. Specifically, asset purchases by the central bank are ineffective, because the good illiquid banks do not sell because of adverse selection and not because of lack of liquidity on the secondary market.

4.2 Policy Implications Ex Ante

As highlighted in Section 3.4 on social welfare at t=0, when the type of equilibrium that occurs at t=0 is deterministic, the socially optimal amount of cash reserves might be higher or lower than the banks' optimal choice of cash reserves depending on the equilibrium at t=0. In such a case policy implications are simple, because the social planner must only ensure that the banks have proper amount of cash at t=0 consistent with social optimum. If the social planner anticipates an equilibrium without liquidity shortage (or with shortage but for intermediate cash holding chosen by the banks) at t=1, then it should mandate the banks to hold 0 cash. If the planner anticipates an equilibrium with liquidity shortage for low cash reserves at t=1, then it should mandate the banks to hold a positive amount of cash that is sufficient to ensure an equilibrium without shortage.

However, typically when choosing a policy the social planner faces uncertainty about the type of equilibrium at t=0. In our setup such an uncertainty could come from uncertainty about the quality of banks in the banking sector at t=1, as proxied by uncertainty about the value of parameter q. We can illustrate such a situation with a simple example. At t=0 with probability $1-\varepsilon$ the share of good banks at t=1 is such that an equilibrium without liquidity shortage occurs at t=1, and with probability ε the share of good banks is such that an equilibrium with liquidity shortage occurs at t=1 for sufficiently low cash reserves (including zero).

Such aggregate uncertainty about the banks' quality creates the following trade-off for the social planner: banks' positive cash holdings are socially valuable during crises (equilibrium with liquidity shortage), but socially costly during normal times (equilibrium without liquidity shortage). Simple tools such as liquidity requirements force the social planner to weigh the costs and benefits of liquidity requirements which is very hard to do.

However, our model offers a simple solution to the planner's problem: a combination of a maximum liquidity requirement and interbank market intervention. The social planner would require the banks to hold zero cash, which would guarantee the maximum investment in the risky asset. If the realization of parameter q is such that a liquidity shortage would occur at t=1, the social planner would inject liquidity to the interbank market eliminating the liquidity shortage. Such a solution guarantees the highest social welfare without a need to make any assumptions about bankruptcy cost or distribution of banks' quality. One caveat to the above policy prescription is that it proposes a unconditional bailout of banks with inadequate cash holdings. Without further assumptions this model cannot answer the question why such a bailout would be undesirable.

5 Discussion

In this section we discuss the importance of the main assumptions of our model.

5.1 Market segmentation

The most striking assumption that we kept for the purposes of realism was the segmentation of the secondary and interbank markets. It means that the abundant liquidity from the secondary market cannot flow to the liquidity-starved interbank market. This assumption is not important for our main result that the secondary market can break down and the interbank market can be thriving. As we showed in Section 3.2.1 such an outcome occurs when there is enough liquidity on the interbank market for all illiquid banks. In fact, a free flow of liquidity between the two markets would mean that the secondary market would collapse and the interbank market would be thriving for any cash reserves held by banks, because there would be automatically enough liquidity for all borrowing banks. At the same time, this assumption is critical for the existence of the equilibrium with liquidity shortage. A free flow liquidity from the secondary market to the interbank market would prevent this equilibrium from occurring.

5.2 Abundant liquidity on the secondary market

Another assumption that differentiates the interbank and secondary markets in our model is the abundant liquidity on the secondary market, which eliminates the cash-in-the-market effect on the secondary market that exists on the interbank market. Limited liquidity on the secondary market would add another factor to the determination of the asset price in equilibrium. As argued earlier, higher banks' cash holdings would decrease the asset supply relative to the demand for it and, therefore, lessen the fire sale effect. This would mean that the equilibrium asset price would be hump-shaped in banks' cash holdings: the price would be first increasing (where the fire-sale effect would dominate) and then decreasing (as the adverse selection effect from the paper would dominate).[17]

This assumption is also not crucial for our main result, because this result as argued earlier has nothing to do with availability of liquidity on the secondary market. Because the fire sales depress the equilibrium asset price relative to the case described in the paper, the equilibrium with liquidity shortage would exist for a wider set of parameters and cash reserves.

5.3 Secured borrowing

One important funding alternative that we do not explicitly consider is secured borrowing. In our model the banks could borrow in a secured manner using cash or the risky asset as collateral. However, we can neglect secured borrowing in our model, because it is not more attractive than the funding sources we consider explicitly. This happens for two reasons. First, secured borrowing against cash held by banks as reserves at t=1 cannot be cheaper than using cash directly, because both are equivalent and cash reserves are directly available to the bank at no additional cost. Second, secured borrowing against the risky asset is the same as unsecured borrowing in our model. In our simple model the probability of individual bank's default is exactly the same as the probability that its asset pays 0 (given that the illiquid banks will use cash as the first line of

[17] Other related papers study the impact of fire sales on intermediaries' liquidity. Fecht (2004) using the Diamond (1997)-extension of Diamond and Dybvig (1983) shows how fire sales impact banks linked by a common secondary market. Martin, Skeie and von Thadden (2012) show how reliance on securitization might increase probability of bank runs when the investors from whom banks borrow value banks collateral very low.

defense against liquidity shocks). Hence, secured borrowing does not create any advantage for the borrower or lender over unsecured borrowing.

5.4 Modelling of liquidity shock

The exact underpinnings of the liquidity need d are not crucial for the model. As the liquidity need d is currently modelled, we could interpret it as payment tied to some contingent liabilities (such as derivatives or committed lines of credit) to which the banks committed before t=0. Alternatively, we could model the banks' liquidity needs using a Diamond-Dybvig-style framework identical to the one proposed in similar papers on the commercial banks' liquidity management such as Freixas and Holthausen (2005), Freixas et al (2011), and Heider et al (forthcoming). We would have to make the following adjustment to the original Diamond-Dybvig (1983) setup to be able to apply it to our model. First, the size of impatient consumers' liquidity need at t=1 would be different across banks as in Freixas et al. (2011) to generate liquid and illiquid banks. Second, consumers as all other agents could not observe the true return on banks' assets to preserve the private information component crucial for our analysis.

6 Conclusion

The paper provides a generic model of bank liquidity management in which the illiquid banks can choose between cash, interbank borrowing and asset sales as sources of liquidity. We show that asymmetric information has quite a dramatic effect on the performance of the interbank and secondary markets. Under perfect information we obtain Modigliani-Miller-type result, in which the illiquid banks are indifferent between selling or borrowing. Under asymmetric information we show that the banks prefer to borrow rather than sell, because the adverse selection cost of selling is higher than of borrowing. We then obtain two equilibrium results. First, if there are no bank bankruptcies, liquidity redistribution is welfare neutral despite banks preferring one liquidity source than the other. Second, it is possible that banks' inability to become liquid by selling their assets forces them to borrow, resulting in liquidity shortage on the interbank market and socially inefficient bank failures.

The paper main contribution is to provide a novel explanation for a transmission of a shock from the secondary market for bank assets to the interbank markets. As such the paper can be viewed as an interpretation of events in August 2007. We explain why the interbank markets did not freeze despite collapse of markets for securitized assets. We argue that in case of an acute stress the interbank markets provide outside liquidity at the lowest adverse selection cost.

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7 Proofs

Proof of Lemma 1

Because the competitive investors with deep pockets observe the quality of the banks' assets, they pay the expected return on these assets: R for the good asset and pR for the bad asset. Now we analyze the interbank market clearing. We start with the case in which the bad banks are solvent at t=1: the expected return on their asset and cash is not lower than the payment d, $pR(1-\lambda)+\lambda \geq d$. The banks with excess cash want to lend to the good and bad banks as long as they break even on lending, i.e, when the loan rate on lonas to the good banks is at least 1 and to the bad banks at least $\frac{1}{p}$. These break even loan rates are also the highest loan rates at which the illiquid banks want to borrow. To see this observe the following. Illiquid banks' payoff from selling given the above asset prices is $R(1-\lambda)+\lambda-d>0$ for the good banks and $pR(1-\lambda)+\lambda-d \geq 0$ for the bad banks. If $R_{D,G}$ and $R_{D,B}$ are the loan rates for the good and bad banks, illiquid banks' payoff from borrowing the liquidity shortfall $d-\lambda$ is $R(1-\lambda)-R_{D,G}\,(d-\lambda)$ for the good and

$p\left[R(1-\lambda)-R_{D,B}\left(d-\lambda\right)\right]$ for the bad banks. Comparing payoffs from selling and borrowing for both banks' types shows that the illiquid banks want to borrow as long as $R_{D,G}\leq 1$ and $R_{D,B}\leq \frac{1}{p}$. Hence, it must follow that (i) the interbank market for the loans to the good banks clears at 1 for the good and at $\frac{1}{p}$ for the bad banks, and (ii) at such equilibrium loan rates and asset prices the illiquid banks are indifferent between selling and borrowing. If $pR(1-\lambda)+\lambda<d$, the equilibrium on the market for interbank loans for the good banks is the same as in the previous case. The bad illiquid banks go bankrupt, because they cannot cover their liquidity shortfall by selling all of their assets and nobody lends to them because the value of their asset $pR(1-\lambda)$ is not enough to repay the loan in the amount of $d-\lambda$ even at the break even loan rate $\frac{1}{p}$.

Proof of Lemma 2

At t=0 each bank choose λ to maximize its profit at t=2 anticipating a given equilibrium that occurs for a chosen λ and taking the asset prices P_i and loan rates $R_{D,i}$ for each $i=B,G$ at t=1 as given. Because the illiquid solvent banks are indifferent between selling and borrowing for each λ, with γ_i we describe the share of the illiquid banks of type i that borrows. Hence, at t=0 each bank chooses λ to maximize its expected return at t=2, which reads:

$$\begin{cases} \pi q\left[R\left(1-\lambda\right)+R_{D,i}p_i\lambda\right]+\left(1-\pi\right)q\left[\gamma_G\left(R\left(1-\lambda\right)-R_{D,G}\left(d-\lambda\right)\right)+\left(1-\gamma_i\right)\left(\left(R-s\right)\left(1-\lambda\right)+sP+\lambda-\right.\right. \\ +\pi\left(1-q\right)\left[pR\left(1-\lambda\right)+\lambda\right]+\left(1-\pi\right)\left(1-q\right)\left[pR\left(1-\lambda\right)+\lambda-d\right], \\ \text{for } pR\left(1-\lambda\right)+\lambda-d\geq 0 \\ \pi q\left[R\left(1-\lambda\right)+\lambda\right]+\left(1-\pi\right)q\left[R\left(1-\lambda\right)+\lambda-d\right] \\ +\pi\left(1-q\right)\left[pR\left(1-\lambda\right)+\lambda\right], \\ \text{for } pR\left(1-\lambda\right)+\lambda-d<0 \end{cases}$$

where

$$\Pi_{GL}=\left(R-s_{GL}\right)\left(1-\lambda\right)+s_{GL}P_G+R_{D,i}p_i\left(\lambda+\right)$$

$R\left(1-\lambda\right)+\lambda$ is the return of a good liquid bank, $R\left(1-\lambda\right)+\lambda-d$ of a good illiquid bank, $pR\left(1-\lambda\right)+\lambda$ of a bad liquid bank, and $pR\left(1-\lambda\right)+\lambda-d$ of a bad illiquid bank. If $pR\left(1-\lambda\right)+\lambda-d\geq 0$, the bad illiquid bank is solvent at t=1, otherwise it is not. Given that $\left[q+(1-q)p\right]R>1$

we can show that the above return is striclty decreasing in λ. This means that each bank will hold $\lambda = 0$ at t=0.

Proof of Lemma 3

We make the following observation that the liquid banks will never borrow, because the proceeds from borrowing have no use for these banks: they have no payment d to make nor there are profitable opportunities to invest them into (asset purchases have return on storage so borrowing to buy would be waste of resources). After rewriting (1) for $\mu = 0$ the liquid bank's decision problem reads:

$$\max_{l,S}(1-\lambda-S)p_iR + (SP+\lambda-l) + \widehat{p}R_Dl, \text{ s.t. } S \in [0;1-\lambda],\ l \in [0;SP+\lambda]\,.$$

The bank's expected return is linear in l and in S. Then we have that for $\widehat{p}R_D > 1$ $l_{iL} = S_{iL}P + \lambda$, and for $\widehat{p}R_D = 1$ $l_{iL} \in [0; S_{iL}P + \lambda]$ for $i = B, G$.

For $\widehat{p}R_D \geq 1$ after taking into account the optimal lending decisions the bank's expected return becomes

$$(1-\lambda)p_iR + \widehat{p}R_D\lambda + (P\widehat{p}R_D - p_iR)S.$$

The first two terms in the last expression are constant. Inspection of the last term delivers the second result concerning the optimal selling decision: $S_{iL} = 1 - \lambda$ for $P\widehat{p}R_D \geq p_iR$, and $S_{iL} = 0$ for $P\widehat{p}R_D < p_iR,$ for $i = B, G.$ The third result about the banks' willingness to sell for a given P, R_D and $\widehat{p}$ follows from the fact that $p_B < p_G = 1$.

Proof of Proposition 1

As in the body of Lemma 4 we split the discussion into two cases depending on the price P.

In the first case the anticipated price is such that $P(1-\lambda) + \lambda > d$. As noted in the text we concentrate on the case when $\widehat{p}R_D \geq 1$. If $\widehat{p}R_D = 1$, the bank lends out all excess cash as assumed in the text, implying that the constraint $SP + \lambda - l \geq d$ binds. If $\widehat{p}R_D > 1$, the bank finds optimal to exhaust the constraint $SP + \lambda - l \geq d$. If this constraint were slack, then the bank would have excess cash to store at a gross return of 1 till t=2. However, holding excess cash is not profitable, because either the bank borrows too much, which is costly ($R_D > 1$ due to $\widehat{p}R_D > 1$ and $\widehat{p} \leq 1$), or lend too little if it has spare cash after paying d (it looses $\widehat{p}R_D > 1$ on every unit of excess cash stored).

Substituting $l = SP + \lambda - d$ into (1) gives us

$$\max_S \begin{cases} p_i(1-\lambda)R - \widehat{p}R_D\,(d-\lambda) + S(P\widehat{p}R_D - p_iR), \text{ if } S > \frac{d-\lambda}{P}, \\ p_i[(1-\lambda)R - R_D\,(d-\lambda) + S(PR_D - R)], \text{ if } S \leq \frac{d-\lambda}{P}. \end{cases} \tag{7}$$

We make two observations: both parts of the last expression are linear in S, and the expected return has a kink at $S = \frac{d-\lambda}{P}$. These observations together with $S \in [0; 1-\lambda]$ imply that the illiquid bank's optimal selling decision S_{iI} is one of three possibilities: 0, $\frac{d-\lambda}{P}$ or $1-\lambda$. To find which of these possibilities is optimal we compare the values of the expected return for each of these possibilities by inserting them into the corresponding part of the expected return (7):

$$\begin{cases} p_i[(1-\lambda)R - R_D\,(d-\lambda)], \text{ if } S = 0, \\ \frac{p_iR}{P}[P\,(1-\lambda) + \lambda - d], \text{ if } S = \frac{d-\lambda}{P}, \\ \widehat{p}R_D[P\,(1-\lambda) + \lambda - d], \text{ if } S = 1-\lambda. \end{cases}$$

For each of these optimal selling decisions there is a corresponding optimal lending/borrowing decision given by the fact that $l = SP + \lambda - d$. We have to note that for our comparison between these three values makes sense only if $(1-\lambda)R - R_D\,(d-\lambda) > 0$. Otherwise borrowing would always be dominated by selling under $P\,(1-\lambda) + \lambda > d$. We skip the details of the algebra from comparing of these three values and only report the results in Lemma 2.

In order to prove the result on the willingness to sell by banks we make several observations. First, the only interesting case if when $p_G \geq \widehat{p} > p_B$. This occurs because of the binary nature of the distribution of type i. $\widehat{p}$ will be the average probability of borrowing banks' repaying their loans. Hence, in an economy with two types we cannot have that $p_G > p_B \geq \widehat{p}$. In addition, the case $p_B \geq \widehat{p} > p_G$ will not occur in our equilibria, so we choose to ignore it. Second, the good bank does not sell at all and borrows for $R_D < \frac{R}{P}$, and the bad bank sells for $R_D \geq \frac{R}{P+\left(\frac{\widehat{p}}{p_i}-1\right)\left(P-\frac{d-\lambda}{1-\lambda}\right)}$.

Solving these two inequalities for P delivers that the good illiquid bank does not sell for $P < \frac{R}{R_D}$ and the bad illiquid bank sells for $P \geq \frac{p_B R}{\widehat{p} R_D} + \frac{d-\lambda}{1-\lambda}\left(1 - \frac{p_B}{\widehat{p}}\right)$. The inequality delivering our result

$$\frac{p_B R}{\widehat{p} R_D} + \frac{d-\lambda}{1-\lambda}\left(1 - \frac{p_B}{\widehat{p}}\right) < \frac{R}{R_D}$$

is equivalent to $R\,(1-\lambda) > R_D\,(d-\lambda)$. This proves our result, because our result holds whenever borrowing delivers positive return for the bank (as explained at the end of previous paragraph).

In the second case we have that $P\,(1-\lambda) + \lambda \leq d$. We can simplify (**??**) by observing that for $P\,(1-\lambda) + \lambda \leq d$ the bank's return at t=2 when the asset pays zero is never positive. The reason is that the bank has to borrow ($l < 0$) and the lowest possible R_D is 1 if $\widehat{p} R_D \geq 1$. In such a case, when the asset pays zero the highest possible return at t=2 is not positive:

$$(SP + \lambda - l - d) + R_D l \leq (SP + \lambda - l - d) + l \leq SP + \lambda - d \leq P\,(1-\lambda) + \lambda - d \leq 0.$$

Hence, (**??**) boils down to

$$\max_{l,S} \begin{cases} p_i[(1-\lambda-S)R + (SP+\lambda-l-d) + R_D l], \text{ if } (1-\lambda-S)R + (SP+\lambda-l-d) + R_D l \geq 0, \\ 0, \text{ otherwise} \end{cases}$$

$$\text{s.t. } S \in [0; 1-\lambda], SP + \lambda - l \geq d.$$

Whether the bank gets a positive return at t=2 depends on the amount it sells and borrows. Hence, we have to find optimal lending and selling decision as well as the condition for which the bank achieves a non-negative return when the asset pays at t=2 simultaneously.

Assume the bank can achieve a positive return at t=2. If it is the case, the bank borrows the least possible amount, $l = SP + \lambda - d$, because it has to borrow and it is costly to do so (as we will show we will always have $R_D > 1$ in equilibrium). Substituting $l = SP + \lambda - d$ into the first part of the last expression for the expected return yields

$$p_i[(1-\lambda)R - R_D\,(d-\lambda) + S(PR_D - R)]. \tag{8}$$

Because (8) is linear in S, the optimal selling decision depends on the sign of $PR_D - R$. If $PR_D - R \leq 0$, the bank does not sell or is indifferent how much it sells. In either case (8) reads $p_i[(1-\lambda)R - R_D\,(d-\lambda)]$ and is not negative for $R_D \leq \frac{1-\lambda}{d-\lambda}R$. For such R_D it holds that $PR_D - R < 0$, because

$$PR_D - R \leq P\frac{1-\lambda}{d-\lambda}R - R \leq \frac{R}{d-\lambda}\left[P\,(1-\lambda) + \lambda - d\right] < 0.$$

Hence, if $R_D < \frac{1-\lambda}{d-\lambda}R$, the illiquid bank borrows $l_{iI} = -\,(d-\lambda)$ and sells nothing $S_{iI} = 0$ and can repay its interbank loans at t=2. If $R_D = \frac{1-\lambda}{d-\lambda}R$, the bank is indifferent between borrowing and selling, because both deliver return of 0.

Now we show that for $R_D > \frac{1-\lambda}{d-\lambda}R$ (8) is always negative, so that the illiquid bank always receives a payoff 0 for such loan rates. First, $R_D > \frac{1-\lambda}{d-\lambda}R$ implies that the first two terms in (8) are negative, $(1-\lambda)R - R_D\,(d-\lambda) < 0$. This also implies that the sign of the highest value of (8) depends on the value of its third term, $S(PR_D - R)$. If $PR_D - R \leq 0$, (8) is maximized for $S = 0$, but then (8) boils down to the sum of its first two terms, which is negative. If $PR_D - R > 0$, (8) is maximized for $S = 1-\lambda$. After inserting $S = 1-\lambda$ into (8) we have that

$$\begin{aligned} & p_i[(1-\lambda)R - R_D\,(d-\lambda) + (1-\lambda)\,(PR_D - R)] \\ = \quad & p_i R_D[P\,(1-\lambda) + \lambda - d] < 0. \end{aligned}$$

This concludes our proof that the illiquid bank receives always the payoff 0 for any $R_D > \frac{1-\lambda}{d-\lambda}R$. In such a case the illiquid bank is indifferent between borrowing or selling. This concludes the proof of Lemma 2.

Proof of Proposition 2

We postpone the discussion of the multiple equilibria till the end of the proof and concentrate now on equilibria, in which the banks borrow. We also present the derivation of the conditions for which $\lambda + P^*\,(1-\lambda) > d$ in the proof of Lemma 4.

We construct equilibria one at a time. This is dictated by the discrete distribution of types. We start with an equilibrium in which all illiquid banks borrow. Then we proceed to an equilibrium, in which all the GI banks borrow and some of the BI banks have to sell. Finally, we discuss an equilibrium, in which all BI banks and some of the GI banks have to sell.

We denote as P^*, R_D^* and $\widehat{p}^*$ the equilibrium values of price, loan rate and the expected fraction of borrowing banks that will repay their loans at t=2. The share of the banks that repay their interbank loans at t=2, $\widehat{p}$, equals to the expected probability of success of the asset held by the borrowing banks. The reason is that, as shown in Lemma 2, a borrowing bank does not hold any cash reserves till t=2 so the probability of bank's default is equal to the probability of its asset paying 0.

We construct the equilibria in the following way. We first assume a certain equilibrium, in which requires us to stipulate certain banks' optimal choices. Then we write down equilibrium equations that are consistent with these optimal choices, and derive P^*, R_D^* and $\widehat{p}^*$. Finally, we check if the assumed optimal choice of the banks are indeed optimal under the derived P^*, R_D^* and $\widehat{p}^*$. This requires checking two things. First, we need to make sure that the conditions from Lemmata 1 and 2 relating to the assumed banks' optimal choices are satisfied for the derived P^*, R_D^* and $\widehat{p}^*$. Second, we have to check whether there is enough cash reserves λ carried from t=0 for the interbank market to clear.

Equilibrium in which all illiquid banks borrow Let's assume that we have an equilibrium, in which all illiquid banks borrow. Using the assumption that all illiquid banks borrow, we can provide some more structure to our equilibrium. First, in such equilibrium the GL banks do not sell. This is implied by the assumption that the GI banks borrow. From Lemma 2 the GI banks borrow if $R > PR_D$, which implies that $R > PR_D \geq P\widehat{p}R_D$. $R \geq P\widehat{p}R_D$ implies per Lemma 1 that the GL do not sell. Second, only the BL banks sell and the equilibrium price P^* equals pR. This follows because none of the other types of banks sell and outside investors' deep pockets imply that they bid up the price till it hits the anticipated return on the purchased asset. Third, in such an equilibrium the liquid banks have to supply enough liquidity for all illiquid banks on the interbank market. Total demand for interbank loans is $(1-\pi)(d-\lambda)$, where each of $1-\pi$ illiquid banks demands a loan equal to its liquidity shortfall $d-\lambda$. Total supply of interbank loans is $\pi q\lambda + \pi(1-q)(\lambda + P(1-\lambda))$, where $\pi q\lambda$ is the sum of cash reserves provided by all GL banks, and $\pi(1-q)(\lambda + P^*(1-\lambda))$ is the sum of cash reserves λ and cash raised from selling the asset by all BL banks. Hence, there is enough liquidity on the interbank market for all illiquid banks, when total supply is not lower than total demand:

$$\pi\left[q\lambda + (1-q)(\lambda + P^*(1-\lambda))\right] \geq (1-\pi)(d-\lambda),$$

or (using $P^* = pR$)

$$\lambda \geq \max\left[0; \frac{(1-\pi)d - (1-q)\pi pR}{1-(1-q)\pi pR}\right] \equiv \lambda_2.$$

We split the discussion of equilibria into two cases: $\lambda > \lambda_2$ and $\lambda = \lambda_2$.

For $\lambda > \lambda_2$ there is excess supply of interbank loans. Hence, the interbank market clears only if the lending banks are indifferent between lending and storing cash, i.e., $\widehat{p}^* R_D^* = 1$. Given that in equilibrium all illiquid banks borrow, the expected fraction of borrowing banks repaying their loans at t=2 is $\widehat{p}^* = q + (1-q)p$ implying an equilibrium loan rate $R_D^* = (q + (1-q)p)^{-1}$. Using Lemma 1 we confirm that for the derived P^*, $\widehat{p}^*$ and R_D^* the liquid banks are indifferent between lending and cash storage, the GL banks do not sell (because $P^*\widehat{p}^* R_D^* = pR < R$) and the BL sell all of their asset (because $P^*\widehat{p}^* R_D^* = pR \geq pR$). Using Lemma 2 we also confirm that borrowing is optimal for the GI and BI banks. For the GI banks it holds that $R > P^* R_D^* = \frac{pR}{\widehat{p}^*}$, because $p < \widehat{p}^*$. For the BI banks it holds that $R_D^* < \frac{R}{P^* + \left(\frac{\widehat{p}^*}{p} - 1\right)\left(P^* - \frac{d-\lambda}{1-\lambda}\right)}$, which is equivalent to $p < \widehat{p}^*$ in equilibrium.

For $\lambda = \lambda_2$ the supply and demand for loans are equal. Hence, the equilibrium loan rate is indeterminate because both supply and demand are inelastic for certain ranges of R_D. The liquid banks lend for R_D^* such that lending is at least as profitable as cash storage, $(q + (1-q)p) R_D^* \geq 1$ and such that the illiquid banks can repay their loans when the asset pays, $R(1-\lambda) - (d-\lambda) R_D^* \geq 0$. From Lemma 2 we know that the first banks to withdraw from the interbank market are the BI banks. Hence, the upper bound on the loan rate for which all illiquid banks borrow is given by the BI banks' indifference condition from Lemma 2, i.e., $R_D^* \leq \frac{R}{P^* + \left(\frac{\widehat{p}^*}{p} - 1\right)\left(P^* - \frac{d-\lambda}{1-\lambda}\right)}$ for $P^* = pR$ and $\widehat{p}^* = q + (1-q)p$. The upper bound on R_D^* for which all illiquid banks borrow is lower than the upper bound for which the liquid banks lend, $\frac{R}{P^*+\left(\frac{\widehat{p}^*}{p}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)} < \frac{1-\lambda}{d-\lambda}R \Leftrightarrow \lambda + P^*(1-\lambda) > d$. Hence, the interbank market clears for any $R_D^* \in \left[\frac{1}{\widehat{p}^*}; \frac{R}{P^*+\left(\frac{\widehat{p}^*}{p}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)}\right]$. It is again easy to check that P^*, R_D^* and $\widehat{p}^*$ satisfy the conditions from Lemmata 1 and 2 for which the stipulated banks' choices are optimal.

Equilibrium in which all GI banks and only some of the BI banks borrow Again we first assume that such an equilibrium exists: all GI banks and only some of the BI banks borrow. From Lemma 2 we know that such an equilibrium can exist, because the GI banks prefer to borrow for some loan rates for which the BI banks do not. Again, when the GI banks borrow, the GL banks do not sell. Hence, as above, the equilibrium price is still $P^* = pR$. The equilibrium equations, (6)-(**??**), were provided and explained at the beginning of Section 3.1, hence we provide here only their solution.

Solving the interbank market clearing equation for σ^* delivers

$$\sigma^* = \frac{(1-\pi)\,d - \pi\,(1-q)\,P^* - \lambda\,(1-(1-q)\,\pi P^*)}{(1-\pi)\,(1-q)\,(1-\lambda)\,P^*}$$

Hence, the equilibrium exists for $\sigma^* \in (0;1)$. It can be easy verified that $\sigma^* > 0$ for $\lambda < \lambda_2$ and $\sigma^* < 1$ for $\lambda > \lambda_1 \equiv \max\left[\frac{(1-\pi)d-(1-q)pR}{1-(1-q)pR}; 0\right]$, where $\lambda_1 \leq \lambda_2$. It is again straightforward to check that all the required conditions from Lemmata 1 and 2 are satisfied.

Using the above condition we can also characterize an equilibrium for $\lambda = \lambda_1$ when $\sigma^* = 0$. The interbank market clears because the amount of loans supplied by all liquid banks and selling BI banks is the same as demand for loans from all GI banks. No BI banks borrow. The construction of the equilibrium loan rate, which is again indeterminate, is similar to the case $\lambda = \lambda_2$. The interbank market clears for such loan rates that the GI banks want to borrow and the BI banks want to sell, $R_D^* \in \left[\frac{R}{P^*+\left(\frac{1}{p}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)}; \frac{1}{p}\right]$, where $\widehat{p}^* = 1$ and $P^* = pR$.

Equilibrium with only some GI banks borrowing For $\lambda \in [0;\lambda_1)$ the supply of loans from all liquid and BI banks is lower than the demand for loans from all GI banks. Hence, the interbank market clears when only some of the GI banks sell. Because only the GI banks borrow we have $\widehat{p}^* = 1$. Moreover, for this equilibrium to arise the GI banks have to be indifferent between borrowing and selling. In fact, Lemma 2 implies for $p_G = \widehat{p}^* = 1$ that the GI banks are indifferent between all three options listed in case $p_G = \widehat{p}^*$. Hence, in equilibrium it has to be that $R = P^* R_D^*$. In addition, $R = P^* R_D^*$ together with Lemma 1 implies that the GL banks are also indifferent between keeping and selling all. Denote γ_S the fraction of the GI banks selling only $\frac{d-\lambda}{P}$ and γ_A the fraction of the GI banks selling all, and γ the fraction of the GL banks selling all. After taking into account that $\widehat{p}^* = 1$, the equilibrium conditions are

$$\begin{aligned} & \pi q \left[(1-\gamma)\lambda + \gamma\left(\lambda + P^* (1-\lambda)\right)\right] \\ & + \pi (1-q)\left(\lambda + P^*(1-\lambda)\right) \\ & + (1-\pi)(1-q)\left(\lambda + P^*(1-\lambda) - d\right) \\ & + (1-\pi) q \gamma_A \left(\lambda + P^*(1-\lambda) - d\right) \\ = \; & (1-\pi) q (1-\gamma_S - \gamma_A)(d-\lambda) \end{aligned} \tag{9}$$

$$\begin{aligned} P^* = \; & \frac{\pi q \gamma (1-\lambda) + (1-\pi) q \left[\gamma_S \frac{d-\lambda}{P} + \gamma_A (1-\lambda)\right]}{(1-q)(1-\lambda) + \pi q \gamma (1-\lambda) + (1-\pi) q \left[\gamma_S \frac{d-\lambda}{P} + \gamma_A (1-\lambda)\right]} R \\ & + \frac{(1-q)(1-\lambda)}{(1-q)(1-\lambda) + \pi q \gamma (1-\lambda) + (1-\pi) q \left[\gamma_S \frac{d-\lambda}{P} + \gamma_A (1-\lambda)\right]} pR, \end{aligned} \tag{10}$$

$$R_D^* = \frac{R}{P^*} \tag{11}$$

(9) is the market clearing condition for the interbank market. The left hand side is the supply of interbank loans, provided by a fraction $1-\gamma$ of the GL banks that lend only cash reserves, as well as by a fraction γ of the GL banks, all bad banks, and a fraction γ_A of the GI banks that sell all of their asset and lend out all of its cash (after covering paying d in case of illiquid banks). The right hand side is the demand for interbank loans by a fraction $1-\gamma_S-\gamma_A$ of the GI banks. (10) is the equilibrium price paid by investors, who anticipate that a fraction γ_S of the GI banks sell only $\frac{d-\lambda}{P}$ as well as all bad banks, a fraction γ of the GL banks, and fraction γ_A sell all of their asset. (11) is the loan rate that guarantees that the GI banks are indifferent between borrowing and selling

and the GL banks between keeping and selling all. Moreover, it is easy to check Lemma 2 to see that the BI banks prefer to sell all and lend out under (11). Despite the fact that there are more unknowns than equations, P^* is uniquely determined by (9)-(11)

$$P^* = \frac{R((1-\pi)d-\lambda)}{(1-\pi)d-\lambda+(1-p)(1-q)(1-\lambda)R}, \tag{12}$$

implying the uniqueness of R_D^* as well.

Multiple equilibria In addition to the above equilibria, there is also a perfect Bayesian equilibrium in which all GI banks sell and none of the banks borrows. First, we show that such an equilibrium can exist. Second, we show when an intuitive criterion can eliminate it.

The equilibrium price P^e is

$$P^e = \frac{(1-\pi)\,q\frac{d-\lambda}{P^e}}{(1-q)\,(1-\lambda)+(1-\pi)\,q\frac{d-\lambda}{P^e}}R + \frac{(1-q)\,(1-\lambda)}{(1-q)\,(1-\lambda)+(1-\pi)\,q\frac{d-\lambda}{P^e}}pR. \tag{13}$$

The investors buying the asset anticipate that all bad banks sell all of their assets and the GI banks sell only the amount $\frac{d-\lambda}{P^e}$ they need to cover their liquidity shortfall $d-\lambda$. Hence, the aggregate amount of the bad asset on the market is $(1-q)\,(1-\lambda)$ and of the good asset $(1-\pi)\,q\frac{d-\lambda}{P}$. The above equation has two solutions for P^e that have opposite signs. To see this, denote $a = (1-q)\,(1-\lambda)\,pR - q\,(1-\pi)\,(d-\lambda)$ and $b = 4\,(1-q)\,(1-\lambda)\,q\,(1-\pi)\,(d-\lambda)\,R > 0$. Then the solution to (13) reads $\frac{a\pm\sqrt{b+a^2}}{2(1-q)(1-\lambda)}$. Because $b>0$ we have for any a that $a-\sqrt{b+a^2}<0$ and $a+\sqrt{b+a^2}>0$. Hence, the equilibrium price P^e is the positive solution:

$$\begin{aligned} P^e \; &= \; \frac{1}{2\,(1-q)\,(1-\lambda)}\left[(1-q)\,(1-\lambda)\,pR - q\,(1-\pi)\,(d-\lambda)\right. \\ &\quad \left. + \sqrt{4\,(1-q)\,(1-\lambda)\,q\,(1-\pi)\,(d-\lambda)\,R + \left((1-q)\,(1-\lambda)\,pR - q\,(1-\pi)\,(d-\lambda)\right)^2}\,\right]. \end{aligned} \tag{14}$$

Such an equilibrium can be supported by arbitrary and sufficiently negative off-equilibrium beliefs that rule out borrowing. For example, we can impose an off-equilibrium belief such that any bank that borrows is deemed to be a bad bank. Under such beliefs the lowest loan rate for which liquid banks are willing to lend is $\frac{1}{p}$. Now we can show that under such a loan rate no GI bank borrows. Selling delivers the GI banks a payoff of $R\left(1-\lambda-\frac{d-\lambda}{P^e}\right)$. Because the GI banks sell a positive amount of the asset it has to be that the equilibrium price is higher than the expected return on the bad asset $P^e > pR$. Hence, the GI banks' payoff from selling is higher than $R\left(1-\lambda-\frac{d-\lambda}{pR}\right) = R(1-\lambda) - \frac{d-\lambda}{p}$, which is exactly the payoff when the GI banks would decide to borrow an amount $d-\lambda$ at the loan rate $\frac{1}{p}$. Hence this proves the existence of the stipulated equilibrium. Of course, it is obvious that all bad banks want to sell, given that they are subsidized by the good banks.

However, the intuitive criterion can eliminate such an equilibrium for certain parameters. This criterion puts some discipline on the off-equilibrium beliefs. It requires that in an equilibrium when a deviation is profitable for one type but not for the other, agents when building their beliefs cannot associate this deviation with the type for which it is not profitable for any of their beliefs. If an equilibrium does not satisfy this criterion it should be discarded (Bolton and Dewatripont (2005)). We will apply this criterion to show that our equilibrium with selling which is supported by any beliefs for which that a borrowing bank is not good can be discarded.

For our purposes it suffices to show that for which parameter constellations the only bank that would like to deviate is a GI bank and not the BI bank for the most favorable off-equilibrium belief about which bank borrows. The most favorable off-equilibrium belief upon seeing a borrowing bank is that it is a GI bank. Hence, a competitive loan rate R_D would be 1 for such a bank. First, we show that the GI bank would deviate. For a loan rate of 1 the deviating GI bank's payoff is $R(1-\lambda)-(d-\lambda)$. Then this must be higher than the payoff from selling in equilibrium because $P^e < R$ due to selling bad banks: $R\left(1-\lambda-\frac{d-\lambda}{P^e}\right) < R\left(1-\lambda-\frac{d-\lambda}{R}\right) = R(1-\lambda)-(d-\lambda)$. Second, we show for which parameters no BI bank wants to deviate. We have to show when the BI banks' payoff from selling for P^e, $P^e(1-\lambda)+\lambda-d$, is not lower than the payoff from deviating to borrowing for $R_D = 1$, $p(R(1-\lambda)-(d-\lambda))$. We can show that inequality

$$P^e(1-\lambda)+\lambda-d \geq p(R(1-\lambda)-(d-\lambda))$$

is equivalent to

$$D - dE \geq \lambda \left(D - E \right), \tag{15}$$

where

$$\begin{aligned} D &= R\left[q\left(1-\pi\right) - p\left(1-q\right)\right], \\ E &= q\left(1-\pi\right) + \left(1-\overline{p}\right), \\ \overline{p} &= q + \left(1-q\right)p. \end{aligned}$$

Any time (15) holds the equilibrium with no borrowing does not exist, because we can find such "reasonable" off-equilibrium beliefs, for which the GI banks want to deviate from the equilibrium and the BI banks do not. The region where it happens depends on λ. In order to narrow the set of possible equilibria, we opt out to find such parameters for which (15) holds for all $\lambda \in [0; d]$. Because $d \in (0; 1)$ we have that $D - dE > D - E$. From this follows that (15) holds for all $\lambda \in [0; d]$ when $D \geq dE$. To see this we consider four cases. First, when $D - E > 0$, then (15) becomes $\lambda \leq \frac{D-dE}{D-E}$. We require that $\frac{D-dE}{D-E} \geq d$, which is equivalent to $D > 0$, but this has to hold when $D - E > 0$, because $E > 0$. Hence, $D - E > 0$ means that (15) holds for all λ. Second, for $D - E = 0$ we can use similar arguments to show that (15) holds for all λ. Third, when $D - dE \geq 0 > D - E$, (15) holds for all λ, because its right-hand side is non-negative and the left hand side negative. Fourth, the last case is when $D - dE < 0$, then (15) becomes $\lambda \geq \frac{D-dE}{D-E} > 0$. Hence, (15) holds either only for some $\lambda \in (0; d)$ or for none if $\frac{D-dE}{D-E} > d$. Summarizing we have that (15) holds for all λ for $D - dE \geq 0$ or

$$\pi \leq 1 - \frac{pR\left(1-q\right) + d\left(1-\overline{p}\right)}{q\left(R-d\right)}.$$

This inequality holds for some $\pi \in (0; 1)$ when $1 - \frac{pR(1-q)+d(1-\overline{p})}{q(R-d)} > 0 \Leftrightarrow q\left[(1-p)R - dp\right] > pR + (1-p)\,d$. In turn, this inequality holds for some $q \in (0; 1)$ when $d < (1-2p)\,R$. To see this realize that t his inequality holds for some $q \in (0; 1)$when $(1-p)R - dp > 0$ and $\frac{pR+(1-p)d}{(1-p)R-dp} < 1$, which are equivalent to $d < \frac{1-p}{p}R$ and $d < (1-2p)\,R$ respectively. But we can show that $1 - 2p < \frac{1-p}{p}$

for any p, implying our claim. Hence, (15) holds for all $\lambda \in [0; d]$ when

$$\pi \in \left(0; 1 - \frac{pR(1-q) + d(1-\overline{p})}{q(R-d)}\right], \; q \in \left(\frac{pR + (1-p)d}{(1-p)R - dp}; 1\right] \text{ and } d < (1-2p)R.$$

It is straightforward to see that as p converges to zero, the above set of parameters expands and converges to:

$$\pi \in \left(0; 1 - \frac{d(1-q)}{q(R-d)}\right], \; q \in \left(\frac{d}{R}; 1\right] \text{ and } d < R.$$

Given that for p close to 0 we have to have $qR > 1$, hence, the last two inequalities are satisfied in such a case. Then the first inequality gives also a broad range of π for which it is satisfied. Moreover, it does so for the most interesting case of lower π, i.e., a low fraction of liquid banks.

Proof of Lemma 4

Equilibria from Proposition 2 in which at least some banks sell exist when $\lambda \in [0; \lambda_2)$ and $(1-\lambda)P^* + \lambda > d$. We derive conditions under which this last inequality does not hold. Observe that necessary condition for $(1-\lambda)P^* + \lambda \leq d$ has to be that $d \geq pR$. Because the lowest equilibrium pirce we can have is pR, if $pR > d$, then for any $\lambda \in [0; d]$ we have that $(1-\lambda)P^* + \lambda > d$. Hence, we are working under the assumption $d \geq pR$.

We define the expression $(1-\lambda)P^* + \lambda - d$ as a function f of λ and find out for which λ we have $f(\lambda) \leq 0$. Given P^* from Proposition 1 we have

$$f(\lambda) = \begin{cases} (1-\lambda)\frac{R((1-\pi)d-\lambda)}{(1-\pi)d-\lambda+(1-p)(1-q)(1-\lambda)R} + \lambda - d, \text{ for } \lambda \in [0; \lambda_1) \text{ and } \lambda_1 > 0, \\ (1-\lambda)pR + \lambda - d, \text{ for } \lambda \in [\max[0; \lambda_1]; \lambda_2). \end{cases}$$

First, we discuss the case $\lambda_1 \leq 0$, which is equivalent to $d \leq \frac{1-q}{1-\pi}pR$. If $\lambda_1 \leq 0$ we have that $f(\lambda) = (1-\lambda)pR + \lambda - d$ for $\lambda \in [0; d]$. Solving $f(\lambda) = (1-\lambda)pR + \lambda - d \leq 0$ with respect to λ delivers that there is no equilibrium from Proposition 2 for $\lambda \in [0; \lambda_{UB}]$ if $\lambda_{UB} < \lambda_2$ or $\lambda \in [0; \lambda_2)$ if $\lambda_{UB} \geq \lambda_2$, where $\lambda_{UB} = \frac{d-pR}{1-pR}$.

Second, we discuss the case $\lambda_1 > 0$. To prove our main claim we will first show the following properties of $f(\lambda)$.

Result: $f(\lambda)$ is strictly decreasing for $\lambda \in [0; \lambda_1)$, strictly increasing for $\lambda \in [\lambda_1; \lambda_2)$ and continuous at $\lambda = \lambda_1$.

Proof: We estbalish the four propoerties from the Result one at a time. 1. We show that f is strictly decreasing for $\lambda \in [0; \lambda_1)$ by showing that the first derivative of f for such λ is negative. The derivative of f for $\lambda \in [0; \lambda_1)$ is

$$\frac{A\lambda^2 + B\lambda + C}{[(1-\pi)d - \lambda + (1-p)(1-q)(1-\lambda)R]^2},$$

where

$$A = -[1 + (1-p)(1-q)R]\left((q + (1-q)p)R - 1\right),$$

$$B = 2[d(1-\pi) + (1-p)(1-q)R]\left((q + (1-q)p)R - 1\right),$$

$$C = -(1-p)(1-q)(q + (1-q)p)R^2 + 2d(1-p)(1-q)R(1-\pi) + d^2(1-\pi)^2(R-1),$$

and

$$\overline{p} = q + (1-q)p.$$

The negative sign of the above derivative obtains because we can show that $A\lambda^2 + B\lambda + C$ is always negative for all λ. To show it it is sufficient to establish the following two claims. First, we have that $A < 0$ because we assume in the paper that $(q + (1-q)p)R > 1$. Second, after some algebra we can show that the determinant of the polynomial, $B^2 - 4AC$, is negative:

$$B^2 - 4AC = -4(1-p)(1-q)R^2(\overline{p}R - 1)(1 - d(1-\pi))^2 < 0,$$

because all terms in brackets are positive under our assumptions.

2. $f(\lambda) = (1-\lambda)pR + \lambda - d = \lambda(1 - pR) + 1 - d$ for $\lambda \in [\lambda_1; \lambda_2)$ is increasing in λ because $pR < 1$.

3. $f(\lambda)$ is continuous at $\lambda = \lambda_1$ because $\frac{R((1-\pi)d-\lambda)}{(1-\pi)d-\lambda+(1-p)(1-q)(1-\lambda)R}$ equals to pR for $\lambda = \lambda_1$.$\blacksquare$

Our result implies that $f(\lambda)$ obtains a minimum on the interval $[0; \lambda_2)$ at $\lambda = \lambda_1$. If $f(\lambda_1) > 0$, the equilibria from Proposition 1 exist for all $\lambda \in [0; \lambda_2)$. If $f(\lambda_1) \leq 0$, then for some λ around λ_1

these equilibria do not exist. To be more precise (and combining it with the result for $\lambda_1 \leq 0$), these equilibria do not exist for low cash reserves ($\lambda \in [0; \lambda_{UB}]$ if $\lambda_{UB} < \lambda_2$ or $\lambda \in [0; \lambda_2)$ if $\lambda_{UB} \geq \lambda_2$) if $\lambda_1 \leq 0$ or $\lambda_1 > 0$, $f(\lambda_1) \leq 0$, and $f(0) \leq 0$, or for intermediate cash reserves ($\lambda \in [\lambda_{LB}; \lambda_{UB}]$ if $\lambda_{UB} < \lambda_2$ or $\lambda \in [\lambda_{LB}; \lambda_2)$ if $\lambda_{UB} \geq \lambda_2$) if $f(\lambda_1) \leq 0$ and $f(0) > 0$, where $\lambda_{LB} < \lambda_1$ is such that $f(\lambda_{LB}) = d$. We can solve each of the three conditions, $\lambda_1 = 0$, $f(\lambda_1) = 0$, and $f(0) = 0$, for d, which delivers $d = \frac{1-q}{1-\pi} pR$, $d = \frac{qpR}{qpR+\pi(1-pR)}$, and $d = \frac{\bar{p}-\pi}{1-\pi} R$. Then we can plot these conditions in a $(q; d)$-diagram, where all conditions together with $d = pR$ cross at $q = \pi$ (see Fig. 4).

Proof of Proposition 3

Now the parameters are such that we are in one of the four intervals for λ that we determined in the last paragraph of the previous proof.

Equilibrium with liquidity shortage We start with the equilibrium with liquidity shortage in which only some of the illiquid banks obtain enough liquidity to pay d. This means that the anticipated price P^{**} has to fulfill $P^{**}(1-\lambda) + \lambda - d < 0$. From Proposition 1 we know that once $P^{**}(1-\lambda) + \lambda - d < 0$ all illiquid banks borrow when $R_D < \frac{1-\lambda}{d-\lambda} R$ and are indifferent between selling and borrowing when $R_D \geq \frac{1-\lambda}{d-\lambda} R$. At the same time the liquid banks lend if $R_D \in \left[\frac{1}{\hat{p}}; \frac{1-\lambda}{d-\lambda} R\right]$. For any $R_D > \frac{1-\lambda}{d-\lambda} R$ the loan supply is zero because no borrower would repay its loans at t=2. For any $R_D < \frac{1}{\hat{p}}$ the loan supply is also zero, because the loan rate does not compensate for the anticipated risk. Because there is not enough liquidity for all the illiquid banks on the interbank market ($\lambda < \lambda_2$), this market clears for $R_D^{**} = \frac{1-\lambda}{d-\lambda} R$. Then all illiquid banks are indifferent between borrowing and selling, and only a fraction of the illiquid banks gets loans. Those banks that cannot obtain interbank loans cannot raise enough liquidity from the secondary market to pay d and become insolvent.

Now we determine the asset price. The share of the illiquid banks that get interbank loans is given by the total supply of loans divided by the loan size, $\frac{\pi[\lambda+(1-q)(1-\lambda)P^{**}]}{d-\lambda}$. Hence, the share of the illiquid banks that cannot pay d is the rest of the illiquid banks, $\nu = 1 - \pi - \frac{\pi[\lambda+(1-q)(1-\lambda)P^{**}]}{d-\lambda}$. Per our intial assumptions the insolvent banks sell all of their asset and proceeds from sale go directly towards the payment of d. The equilibrium price is given by

$$P^{**}=\frac{\nu q\left(1-\lambda\right)}{\nu\left(1-\lambda\right)+\pi\left(1-q\right)\left(1-\lambda\right)}R+\frac{\nu\left(1-q\right)\left(1-\lambda\right)+\pi\left(1-q\right)\left(1-\lambda\right)}{\nu\left(1-\lambda\right)+\pi\left(1-q\right)\left(1-\lambda\right)}pR. \tag{16}$$

At the same time, because the loans are allocated randomly to the pool of illiquid banks, by law of large numbers we have that the expected probability of repayment is the same as the one for the entire population of illiquid banks, $q+\left(1-q\right)p$. That means that for the equilibrium loan rate $R_D^{**}=\frac{1-\lambda}{d-\lambda}R$ the lending banks make profits and therefore are ready to lend ($\overline{p}\frac{1-\lambda}{d-\lambda}R>1\Leftrightarrow\overline{p}R>1>\frac{d-\lambda}{1-\lambda}$). Equation (16) has two solutions of the form

$$P_{1,2}^{**}=\frac{F\pm\sqrt{F^2+G}}{2\left(1-q\right)\left(1-\lambda\right)\pi},$$

where

$$\begin{aligned} F &= d(1-q\pi)+(1-q)\pi(\overline{p}R(1-\lambda)-\lambda)-\lambda, \\ G &= 4(1-q)R\pi(1-\lambda)\left[(\overline{p}+p(1-q)\pi)\lambda-d(\overline{p}-q\pi)\right]. \end{aligned}$$

For the solution to exist we have to have $F^2+G\geq 0$. Whenever $F^2+G<0$, there cannot be an equilibrium with the liquidity shortage. After some cumbersome, and therefore omitted, algebra we can show that smaller solution is to (16) is lower and the bigger is higher than the average quality of the asset in the banking system, $\left(q+\left(1-q\right)p\right)R$.[18] Hence, the equilibrium price is given by the smaller solution, because the bad liquid banks also sell resulting the expected return on the asset smaller than $\left(q+\left(1-q\right)p\right)R$:

$$P^{**}=\frac{F-\sqrt{F^2+G}}{2\left(1-q\right)\left(1-\lambda\right)\pi}. \tag{17}$$

The equilibrium with liquidity shortage exists whenever the set of λ for which this inequality $P^{**}\left(1-\lambda\right)+\lambda<d$ holds overlaps with our intervals for which the equilibria from Proposition 2

[18] To show this, we can show that the right hand side of expression (16) is a hyperbole with two arms from which one is smaller and the other is bigger than $\left(q+\left(1-q\right)p\right)R$.

do not exist. This inequality is quadratic in λ and reads

$$(1-\alpha)\lambda^2 + [\alpha + d(\alpha - \pi\overline{p}R + \pi - 2)]\lambda + d(d(1-\pi) - (\alpha - \pi\overline{p}R)) < 0,$$

where

$$\alpha = R[\overline{p} - q\pi(1-q)(1-p)]$$

and

$$\overline{p} = q + (1-q)p.$$

The exact set of parameters for which the solution set for the above inequality overlaps with the intervals for which the equilibria from Proposition 2 do not exist is impossible to determine algebraically. However, we can convince the reader that such an equilibrium exists using a numerical example. For values $\pi = 0.7$, $p = 0.5$, $R = 1.5$, $d = 0.78$, $q = 0.62$ we have that $\lambda_{LB} \approx -0.14$, $\lambda_{UB} = 0.12$, and $\lambda_2 = 0.043$. Hence, the equilibria from Proposition 2 do not exist for $\lambda \in [0; 0.043)$. Moreover, the inequality $P^{**}(1-\lambda) + \lambda < d$ holds approximately for $\lambda \in (0.02; 2.88)$. Combining this last interval with the interval in which the equilibria from Proposition 2 do not exist, we have that the equilibrium with liquidity shortage exists for $\lambda \in (0.02; 0.43)$.

Equilibrium with $P^*(1-\lambda) + \lambda - d = 0$ As argued earlier there is a possibility that for some λ there must be equilibria in which the equilibrium price is $P^* = \frac{d-\lambda}{1-\lambda}$. This can also be seen from the above numerical example. For $\lambda \in [0; 0.02]$ equilibria from Proposition 2 (in which the equilibrium asset price is bigger than $\frac{d-\lambda}{1-\lambda}$) and the equilibrium with liquidity shortage (in which the equilibrium asset price is smaller than $\frac{d-\lambda}{1-\lambda}$) do not exist. Hence, it must be the case that the equilibrium price is $\frac{d-\lambda}{1-\lambda}$.

When the illiquid banks anticipate $P^* = \frac{d-\lambda}{1-\lambda}$, they all would like to borrow for $R_D < \frac{1-\lambda}{d-\lambda}R_D$. Because we are in the case that $\lambda < \lambda_2$, then there is not enough liquidity on the interbank market for all illiquid banks. Hence, the interbank market clears for $R_D^* = \frac{1-\lambda}{d-\lambda}R_D$. Given that $R_D^* = \frac{1-\lambda}{d-\lambda}R_D$ and $P^* = \frac{d-\lambda}{1-\lambda}$ all illiquid banks are indifferent between selling or borrowing. The good illiquid banks are also indifferent between selling all $(1-\lambda)$ or only $\frac{d-\lambda}{P^*}$ of the asset, because

$\frac{d-\lambda}{P^*} = 1 - \lambda$. For notational simplicity we assume the good illiquid banks that do not get the interbank loans sell all of their asset.

Because all illiquid banks are indifferent between selling or borrowing we have an equilibrium in mixed strategies. We denote as γ_M (β_M) the share of the good (bad) illiquid banks that borrow. Hence, the equilibrium in mixed strategies is described by the following two equations:

$$\frac{d-\lambda}{1-\lambda} = \frac{q(1-\pi)(1-\gamma_M) + p[(1-q)(1-\pi)(1-\beta_M) + \pi(1-q)]}{q(1-\pi)(1-\gamma_M) + (1-q)(1-\pi)(1-\beta_M) + \pi(1-q)} R \ . \tag{18}$$

and

$$\frac{\pi\left[\lambda + (1-q)(1-\lambda)\frac{d-\lambda}{1-\lambda}\right]}{d-\lambda} = (1-\pi)(\gamma_M q + \beta_M(1-q)).$$

The first equation describes the secondary market, where the equilibrium price $\frac{d-\lambda}{1-\lambda}$ equals to the expected quality of the sold asset. The second equation describes the clearing of the interbank market, where the left (right) hand side is the total loan supply (demand). The solution to these two equations reads

$$\gamma_M = \frac{-d^2(1-\pi) + d\overline{p}R(1-\pi) - d\lambda(\overline{p}R(1-\pi) + \pi - 2) - \lambda\left[R(\overline{p} - (1-p)q\pi)(1-\lambda) + \lambda\right]}{(1-p)qR(1-\pi)(d-\lambda)(1-\lambda)}$$

and

$$\beta_M = \frac{d^2(1-\pi) - dR(\overline{p} - \pi) + d\lambda(R(\overline{p} - \pi) + \pi - 2) + \lambda(\overline{p}R(1-\lambda) + \lambda)}{(1-p)(1-q)R(1-\pi)(d-\lambda)(1-\lambda)}.$$

We can show the following properties of γ_M and β_M. First, γ_M and β_M cross at one of the solutions $P^{**}(1-\lambda) + \lambda = d$. This is to be expected since the moment there is an equilibrium with liquidity shortage the amount of bad and good illiquid lending is the same. Second, $\gamma_M > 0$ and $\beta_M = 0$ for $\lambda = \lambda_{LB} \geq 0$. This is also to be expected since for $\lambda = \lambda_{LB}$ we have that $P^*(1-\lambda) + \lambda = d$, meaning that all bad illiquid sell and some good illiquid borrow.

Proof of Lemma 5

For the case $P^*(1-\lambda) + \lambda > d$ social welfare is a sum of profits for each type of bank, $\Pi(\lambda; R_D; P; \widehat{p})$, and payments made by illiquid banks, $d(1-\pi)$, for given λ and the corresponding equilibrium values R_D^*, P^*, and $\widehat{p}^*$. Hence, social welfare at t=1 is $SW_1 = \Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right) + d\left(1-\pi\right)$ where $R_D = R_D^*$, $P = P^*$, $\widehat{p} = \widehat{p}^*$, and

$$\Pi\left(\lambda; R_D; P; \widehat{p}\right) = \begin{cases} \pi q\left[\left(1-\gamma\right)\left[R\left(1-\lambda\right)+\widehat{p}R_D\lambda\right]+\gamma\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)\right]+ \\ \pi\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)+ \\ \left(1-\pi\right)q\left[\left(1-\gamma_S-\gamma_L\right)\left(R\left(1-\lambda\right)-R_D\left(d-\lambda\right)\right)+\gamma_S R\left(1-\lambda-\frac{d-\lambda}{P}\right)\right]+ \\ +\left(1-\pi\right)q\gamma_L\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda-d\right)+\left(1-\pi\right)\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda-d\right), \\ \text{for } \lambda\in[0;\lambda_1) \\ \\ \pi\left[q\left[R\left(1-\lambda\right)+\widehat{p}R_D\lambda\right]+\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)\right]+ \\ \left(1-\pi\right)\left[q\left[R\left(1-\lambda\right)-R_D\left(d-\lambda\right)\right]+\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda-d\right)\right], \\ \text{for } \lambda=\lambda_1 \\ \\ \pi\left[q\left[R\left(1-\lambda\right)+\widehat{p}R_D\lambda\right]+\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)\right]+ \\ \left(1-\pi\right)\left[q\left[R\left(1-\lambda\right)-R_D\left(d-\lambda\right)\right]+\left(1-q\right)\left(1-\beta\right)p[(1-\lambda)R-R_D\left(d-\lambda\right)]\right]+ \\ \left(1-\pi\right)\beta\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda-d\right), \\ \text{for } \lambda\in(\lambda_1;\lambda_2) \\ \\ \pi\left[q\left[R\left(1-\lambda\right)+\widehat{p}R_D\lambda\right]+\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)\right]+ \\ \left(1-\pi\right)\left[q\left[R\left(1-\lambda\right)-R_D\left(d-\lambda\right)\right]+\left(1-q\right)p[(1-\lambda)R-R_D\left(d-\lambda\right)]\right], \\ \text{for } \lambda\in[\lambda_2;d] \end{cases} \tag{19}$$

We can show that for all $\lambda\in[0;d]$

$$\Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right) = \left(1-\lambda\right)\left(q+\left(1-q\right)p\right)R+\lambda-d\left(1-\pi\right).$$

Hence, social welfare in equilibria with $P^*\left(1-\lambda\right)+\lambda > d$ is $SW_0 = \left(1-\lambda\right)\left(q+\left(1-q\right)p\right)R+\lambda$.

For $P^*(1-\lambda)+\lambda = d$ all illiquid banks pay d. The banks' profits are given by

$$\begin{aligned}
& \Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right) && (20) \\
= \;& \pi\left[q\left[R(1-\lambda)+\widehat{p}^* R_D^* \lambda\right]+(1-q)\,\widehat{p}^* R_D^*\left((1-\lambda) P^*+\lambda\right)\right] \\
& +(1-\pi)\, q\left[\gamma_M\left(R(1-\lambda)-R_D(d-\lambda)\right)+\left(1-\gamma_M\right)\left((1-\lambda) P^*+\lambda-d\right)\right] \\
& +(1-\pi)(1-q)\left[\beta_M p\left(R(1-\lambda)-R_D(d-\lambda)\right)+\left(1-\beta_M\right)\left((1-\lambda) P^*+\lambda-d\right)\right],
\end{aligned}$$

where $R_D^* = \frac{1-\lambda}{d-\lambda} R$, $P^* = \frac{d-\lambda}{1-\lambda}$ and $\widehat{p}^*$ is given by (**??**). We can show that (21) equals to

$$(1-\lambda)(q+(1-q)\,p)\,R+\lambda-d(1-\pi).$$

Hence, again we have that social welfare is $SW_0 = (1-\lambda)(q+(1-q)\,p)\,R+\lambda$.

For $P^*(1-\lambda)+\lambda < d$ only a fraction $(1-\pi-v)$ of illiquid banks pay d, a fraction of v pays $P^*(1-\lambda)+\lambda < d$ and only the liquid banks have positive profits. We can show that

$$\begin{aligned}
& \Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right) && (21) \\
= \;& \pi\left[q\left[R(1-\lambda)+\widehat{p}^* R_D^* \lambda\right]+(1-q)\,\widehat{p}^* R_D^*\left((1-\lambda) P^*+\lambda\right)\right]
\end{aligned}$$

where $R_D^* = \frac{1-\lambda}{d-\lambda} R$, P^* as given by (17), $\widehat{p}^* = q+(1-q)\,p$ and $\nu = 1-\pi-\frac{\pi[\lambda+(1-q)(1-\lambda)P^*]}{d-\lambda}$. Using the expressions for the equilibrium price (16) and for the amount of illiquid banks that cannot become liquid v we can show that (21) equals to

$$\begin{aligned}
& \Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right) \\
= \;& (1-\lambda)(q+(1-q)\,p)\,R+\lambda-\left[(1-\pi)-\nu\right] d-v\left((1-\lambda) P^*+\lambda\right).
\end{aligned}$$

This implies the following social welfare

$$\begin{aligned} SW_{LS} &= \Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right) \\ &\quad + \left[(1-\pi) - \nu\right] d + v\left((1-\lambda) P^* + \lambda\right) \quad (22) \\ &\quad - v\left[d - (1-\lambda) P^* + \lambda\right] \tau \quad (23) \\ &= SW_0 - v\left[d - (1-\lambda) P^* + \lambda\right] \tau \leq SW_0, \quad (24) \end{aligned}$$

because $\tau \leq 0$.

Proof of Lemma 6

At t=0 each bank chooses optimal λ anticipating a certain equilibrium at t=1 as derived in Proposition 1 and 2 for this given λ or an interval of λ. In looking for the optimal choice of λ we will assume again assume that the bank anticipates an equilibrium at t=1 for a given choice of λ or its region, and then we will maximize the bank's profits at t=0 and see whether this choice of λ for the anticipated equilibrium at t=1 is consistent with this anticipated equilibrium.

Using $\Pi\left(\lambda; R_D; P; \widehat{p}\right)$ to denote the banks' expected profit at t=0 as a function of λ, P, R_D, the bank solves the following problem at t=0 taking as given P, R_D and $\widehat{p}$:

$$\max_{\lambda \in [0;d]} \Pi\left(\lambda; R_D; P; \widehat{p}\right),$$

where the functional form of $\Pi\left(\lambda; R_D; P; \widehat{p}\right)$ depends on the anticipated equilibrium as in (19)-(21).

We tackle first the most general case in which $\lambda_2 > \lambda_1 > 0$, which occurs for some $R < \frac{(1-\pi)d}{(1-q)p}$.

At the beginning observe that at t=0 any choice of $\lambda \geq \lambda_2$ guarantees that an illiquid bank will pay d at t=1 regardless of whether $P^*(1-\lambda) + \lambda$ is higher or lower than d, because then there is enough liquidity on the interbank market for all banks, so that no illiquid bank needs to sell. Now, it is clear that none of the banks will choose $\lambda > \lambda_2$, because taking more than λ_2 would be waste of resources. For any $\lambda > \lambda_2$ there would be excess supply of liquidity on the interbank market, implying that the banks keep too much of cash reserves.

The anticipated values of equilibrium variables at t=1 for $\lambda = \lambda_2$ have to be such that the first order condition with respect to λ holds with equality at t=0. Otherwise, the bank would either prefer to set λ smaller or bigger than λ_2. The first order condition is given by deriving the fourth expression in (19) with respect to λ:

$$\pi \left[q\left(-R+\widehat{p}R_D\right)+(1-q)\,\widehat{p}R_D\left(-P+1\right)\right]+(1-\pi)\left[q\left(-R+R_D\right)+(1-q)\,p\left(-R+R_D\right)\right].$$

In equilibrium for $\lambda = \lambda_2$ at t=1 the banks expect that $\widehat{p}^* = q + (1-q)\,p$ and $P^* = pR$, but R_D^* is indeterminate. Hence, it is given by the binding first order constraint as in Freixas et al. (2011) after inserting $\widehat{p}^*$ and P^* into it: $R_D^* = \frac{qR+(1-\pi)(1-q)pR}{(q+(1-q)p)(1-\pi(1-q)pR)}$.

This choice of λ and equilibrium values of R_D^*, $\widehat{p}^*$ and P^* constitute an equilibrium at t=0, if $R_D^* = \frac{qR_G+(1-\pi)(1-q)pR}{(q+(1-q)p)(1-\pi(1-q)pR)}$ is in the interval $\left[\frac{1}{q+(1-q)p}; \frac{1}{[q+(1-q)p]-\frac{q(1-p)}{pR}\frac{d-\lambda_2}{1-\lambda_2}}\right]$ as determined in the proof of the Proposition 1. We can see that is always holds that $\frac{qR+(1-\pi)(1-q)pR}{(q+(1-q)p)(1-\pi(1-q)pR)} > \frac{1}{q+(1-q)p}$ (which is equivalent to $(q+(1-q)\,p)\,R > 1$). $\frac{[q+(1-\pi)(1-q)p]R}{(q+(1-q)p)(1-\pi(1-q)pR)} \leq \frac{1}{[q+(1-q)p]-\frac{q(1-p)}{pR}\frac{d-\lambda_2}{1-\lambda_2}}$ holds for $R \leq \underline{R}$, where $\underline{R}$ is given by solving the last inequality for R with equality sign

$$\underline{R} = \frac{dq^2\pi - p^2\left(1-q\right)\left(1+d\left(1-\pi\right)\left(1+q\pi\right)\right)+pq(1-d(1+2\left(1-q\right)\pi - \cdots}{p(q^2(1-\pi+q\pi-d(1-(2-q)\pi))-p\left(1-q\right)q(2-(1-2q)\pi+\left(1-q\right)\pi^2 d(2-\pi)(1+\left(1-q\right)\pi))+p^2 \cdots}$$

Hence, the bank chooses at t=0 $\lambda = \lambda_2$ for $R \leq \underline{R}$.

Now we split the discussion into cases in which $P^*\left(1-\lambda\right)+\lambda > d$ and $P^*\left(1-\lambda\right)+\lambda \leq d$.

The case $P^*\left(1-\lambda\right)+\lambda > d$ **for all** $\lambda \in [0;d)$ For $\lambda \in (\lambda_1;\lambda_2)$ we use the similar procedure as for $\lambda = \lambda_2$. The derivative of (19) with respect to λ is

$$\begin{aligned}
&\pi\left[q\left(-R+\widehat{p}R_D\right)+(1-q)\,\widehat{p}R_D\left(-P+1\right)\right]+\\
&(1-\pi)\left[q\left(-R+R_D\right)+(1-q)\left(1-\beta\right)p\left(-R+R_D\right)\right]+\\
&(1-\pi)\,\beta\left(1-q\right)\widehat{p}R_D\left(1-P\right)
\end{aligned}$$

The difference to the case with $\lambda = \lambda_2$ is that this time equilibrium values of P, R_D, σ and $\widehat{p}$ at t=1 are functions of λ. Hence, the equilibrium choice of λ, λ^*, and corresponding equilibrium variables at t=1 is an interior solution to a system of five equations: the binding above first order condition, and equations (6)-(**??**). Because equilibrium values of λ, R_D, σ and $\widehat{p}$ are complicated objects, we do not providing them (the reader is more than welcome to ask the author for the Mathematica code that offers the solutions). The solution to this system of equations is an equilibrium if the equilibrium value of λ, λ^*, is between $(\lambda_1; \lambda_2)$. This occurs for $R \in \left(\underline{R}; \overline{R}\right)$, where $\overline{R}$ is the solution of an equation in which λ_1 is equal to the chosen λ^*. Again we refrain from providing the exact form of $\overline{R}$. Hence, the bank chooses optimally some $\lambda^* \in (\lambda_1; \lambda_2)$ for $R \in \left(\underline{R}; \overline{R}\right)$.

For $\lambda = \lambda_1$ again we apply the same procedure as for $\lambda = \lambda_2$. The first order condition with respect to λ (obtained for the second expression in (19)) equals zero for $R_D^* = \frac{qR}{1-(1-q)pR}$, which has to be in the interval $\left[\frac{1}{1-\frac{1-p}{pR}\frac{d-\lambda_1}{1-\lambda_1}}; \frac{1}{p}\right]$. It always holds that $\frac{qR}{1-(1-q)pR} \leq \frac{1}{p}$, and $\frac{qR}{1-(1-q)pR} \geq \frac{1}{1-\frac{1-p}{pR}\frac{d-\lambda_1}{1-\lambda_1}}$ is equivalent to $R \geq \overline{R}$. Hence, the bank chooses optimal $\lambda = \lambda_1$ for any $R \geq \overline{R}$.

For $\lambda \in [0; \lambda_1)$ (this interval is not empty for $d > \frac{p(1-q)R}{1-\pi}$) the derivative of the first expression in (19) with respect to λ reads

$$\begin{aligned}
&(R_D - R)\, q\, (1 - \pi\gamma - (1-\pi)(\gamma_S + \gamma_A)) + \\
&(1 - P)\left[R_D\, (1 - q + \pi q\gamma + (1-\pi)\, q\gamma_S) + \frac{R}{P}(1-\pi)\, q\gamma_S\right].
\end{aligned}$$

We have to evaluate the sign of this derivative at the equilibrium arising at t=1. Using the equilibrium loan rate (11) the above derivative boils down to

$$\frac{R}{P^*}(1 - P^*)\,. \tag{25}$$

The expression (25) implies the following. If the anticipated equilibrium price P^* at t=1 is below 1, then the bank will choose any $\lambda \geq \lambda_1$, because (25) is negative, and, therefore, the bank's profits at t=0 are increasing in λ. If the anticipated price P^* at t=1 is higher than 1, then the bank will choose $\lambda = 0$ at t=0. If the anticipated price is 1, then the bank is indifferent between any $\lambda \in [0; \lambda_1)$. At the same time from Proposition 1 we know that the equilibrium price at t=1 given by (12) is decreasing in λ. Hence, as depicted in Fig. 5 we might have either a unique or multiple equilibria depending on the parameters.

First, a unique equilibrium exists when the equilibrium price (12) is lower than 1 for all $\lambda \in [0;\lambda_1)$ (it can never be above 1 for all $\lambda \in [0;\lambda_1)$, because it converges to $pR < 1$ for λ converging to λ_1 from the left). This occurs when the highest possible price at t=1, which obtains for $\lambda = 0$, is not higher than 1. Then the bank will not choose any $\lambda \in [0;\lambda_1)$. Hence, then we have a unique equilibrium given by one of the optimal λ derived for the cases in which $\lambda \geq \lambda_1$. Formally, this occurs when the expression (12) for $\lambda = 0$ is not higher than 1:

$$\frac{Rd(1-\pi)}{d(1-\pi)+(1-p)(1-q)R} \leq 1.$$

This holds for any R if $d \in \left(\frac{p(1-q)R}{1-\pi}; \frac{(1-p)(1-q)R}{1-\pi}\right]$ and $p < \frac{1}{2}$ or for $R \in \left(\frac{1}{\overline{p}}; \widetilde{R}\right)$ if $d > \max\left\{\frac{p(1-q)R}{1-\pi}; \frac{(1-p)(1-q)R}{1-\pi}\right\}$, where $\widetilde{R} \equiv \frac{d(1-\pi)}{d(1-\pi)-(1-q)(1-p)}$ (under our assumptions on d and π it holds that $\widetilde{R} > \frac{1}{\overline{p}}$).

Second, when $R \geq \widetilde{R}$ and $d > \max\left\{\frac{p(1-q)R}{1-\pi}; \frac{(1-p)(1-q)R}{1-\pi}\right\}$, then we have multiple equilibria as seen in Fig. 5. One equilibrium is $\lambda^* = 0$, because then the equilibrium price (12) is higher than 1 and choice of $\lambda = 0$ is consistent with that price because (25) is negative for that price. Another equilibrium is a choice of $\lambda^* \geq \lambda_1$ as in the case of the above unique equilibrium. In such a case the equilibrium price is $pR < 1$, which is consistent with the choice of $\lambda \geq \lambda_1$ and negative sign of (25) for such a price. This equilibrium leads to lower profits and is less efficient than the one with $\lambda^* = 0$. Observe that the expected profit of the banks boils down to $(1-\lambda)\overline{p}R + \lambda - d(1-\pi)$ (and social welfare is $(1-\lambda)\overline{p}R + \lambda$) as shown previously. Hence, lower choice of λ implies higher profits and welfare.

It has to be noted that although $P = 1$ nullifies (25) it is an unstable equilibrium. The reason is that if we take any arbitrarily small perturbation around $P = 1$ the bank would prefer to set either $\lambda^* = 0$ or $\lambda^* \geq \lambda_1$, given that the equilibrium loan rate would adjust and the interbank market would always clear (see also Malherbe (2014)).

There is also a possibility of two additional cases: $\lambda_2 \leq 0$, which occurs for $R \geq \frac{(1-\pi)d}{\pi(1-q)p}$, and $\lambda_2 > 0 \geq \lambda_1$, which occurs for $R \in \left[\frac{(1-\pi)d}{(1-q)p}; \frac{(1-\pi)d}{\pi(1-q)p}\right)$. In case $\lambda_2 \leq 0$, there is enough liquidity for any illiquid bank on the interbank market for any $\lambda > 0$. Hence, the optimal choice of λ at t=0 is 0 and we always have an equilibrium in which all illiquid banks lend. In case $\lambda_2 > 0 \geq \lambda_1$, we need to use the above results for the case when $\lambda^* \in (\lambda_1;\lambda_2)$. The banks choose λ_2 optimally for

$R \in \left[\frac{(1-\pi)d}{(1-q)p}; \min\left[\underline{R}; \frac{(1-\pi)d}{\pi(1-q)p}\right]\right)$ and $\lambda^* \in [0; \lambda_2)$ for $R \in \left[\max\left[\underline{R}; \frac{(1-\pi)d}{(1-q)p}\right]; \frac{(1-\pi)d}{\pi(1-q)p}\right)$. IN addition, here we can have multiple equilibria if $\widetilde{R} < \frac{(1-\pi)d}{\pi(1-q)p}$.

Given the algebraic complexity of the solution in the body of the Lemma we just report the qualitative results.

The case $P^*(1-\lambda)+\lambda \leq d$ Finally, we analyze the case when there is a potential for an equilibrium with liquidity shortage for some λ. Here, it is impossible to obtain such clear cut conditions for the optimal choice of λ as in the case when $P^*(1-\lambda)+\lambda > d$ for all $\lambda \in [0; d)$. The reason is that it is very hard to obtain analytically clear cut conditions for the existence of equilibria at t=1. However, as highlighted in the proof of Proposition 2, once we have parameters such that a solution λ_{FR} exists, we can obtain such conditions. In what follows we use two numerical examples to provide some insight on the existance of equilibria in which $P^*(1-\lambda)+\lambda \leq d$ and the optimal choice of λ in such cases.

Example 1 We use values $p = 0.1$, $q = 0.7$, $\pi = 0.4$ and $d = 0.5$, and we vary R. We know that for $pR(1-\lambda_1)+\lambda_1 > d$ or $R > \widehat{R} \approx 3.636$ we always have $P^*(1-\lambda)+\lambda > d$ for any $\lambda \in [0; d)$. Hence, we are interested in the cases in which $R \in \left(\frac{1}{q+(1-q)p}; \widehat{R}\right]$, where $\frac{1}{q+(1-q)p} \approx 1.37$. We proceed in such a way that we first pin down which equilibria exist at t=1 for given λ and R, and then we look for optimal λ at t=0 given these equilibria at t=1.

As shown in the proof of Proposition 2, for $R \in \left[\widehat{\widehat{R}}; \widehat{R}\right]$ there is no equilibrium with liquidity shortage. However, because for such R we can have $pR(1-\lambda_1)+\lambda_1 < d$ for some λ it must mean that there exists an equilibrium with $P^*(1-\lambda)+\lambda = d$ at t=1. In fact, at t=1 for a given λ we have an equilibrium with only some good banks borrowing for $\lambda \in [0; \lambda_{LB})$, where P^* is given by (12), an equilibrium with illiquid banks being indifferent between borrowing and selling and $P^* = \frac{d-\lambda}{1-\lambda}$ for $\lambda \in [\lambda_{LB}; \lambda_{UB}]$, as well as an equilibria in which all GI banks borrow and some or all BI banks borrow for $\lambda \in (\lambda_{UB}; d)$. We have that $\lambda_{LB} > 0$, for two reasons. First, we know from Lemma 4 that λ_{LB} exists once $R \leq \widehat{R}$. Second, solving $\lambda_{LB} > 0$ for R reveals that it holds for $R > \frac{d(1-\pi)}{p(1-q)+\pi-q} \approx 0.9$.

Now we can look for the optimal choice of λ anticipating a certain equilibrium at t=1. First, the bank does not find optimal to choose any $\lambda \in [0; \lambda_{LB})$. From the previous analysis we know that it occurs for $R < \widetilde{R} = 10$. Because $\widetilde{R} = 10 > \widehat{R}$, we obtain our claim. Second, the bank find optimal to choose $\lambda = \lambda_2$ when anticipating equilibria in which all the GI banks borrow at t=1. From the previous analysis this happens for $R < \underline{R} \approx 4.1$. Because $\underline{R} \approx 4.1 > \widehat{R} \approx 3.636$, we obtain our claim. Third, we are left with the optimal choice of λ at t=1 when the banks anticipate an equilibrium in which $P^* = \frac{d-\lambda}{1-\lambda}$ for $\lambda \in [\lambda_{LB}; \lambda_{UB}]$. Deriving (20) with respect to λ we obtain

$$\begin{aligned}
&\pi \left[q \left[\widehat{p} R_D - R \right] + (1-q)\, \widehat{p} R_D \,(1-P) \right] \\
&+ (1-\pi)\, q \left[\gamma_M \left(R_D - R \right) + \left(1 - \gamma_M \right)(1-P) \right] \\
&+ (1-\pi)(1-q) \left[\beta_M p \left(R_D - R \right) + \left(1 - \beta_M \right)(1-P) \right].
\end{aligned}$$

Inserting all the equilibrium values for R_D, P, $\widehat{p}$, γ_M and β_M into the last expression yields a following expression

$$\begin{aligned}
&1 - \overline{p} R + \frac{(1-d)\left(\overline{p} - \pi \right) R}{d - \lambda} + \frac{(1-d)^2 (1-q)(1-q\pi)}{(1-\lambda)\left(d(1-q) + q \right)} \\
&+ \frac{qR(\overline{p} - q\pi) - d(1 - p(1-q)^2 R - q((1-q)(1-\pi)R + \pi))}{(d(1-q)+q)(d(1-q)+q\lambda)},
\end{aligned} \tag{26}$$

which becomes a polynomial of the third degree in λ, once we take out a common denominator. Therefore, an analytical solution for λ and providing conditions when such a solution might not be in $[\lambda_{LB}; \lambda_{UB}]$ is not possible. In our numerical example, we can show that the only real root of the above equation is above λ_{UB} for all $R \in \left[\widehat{\widehat{R}}; \widehat{R} \right]$, implying that the bank's profits are increasing in λ for all $\lambda \in [\lambda_{LB}; \lambda_{UB}]$. However, we know that once $\lambda > \lambda_{UB}$ the bank would like to choose λ_2. Hence, in our example for all $R \in \left[\widehat{\widehat{R}}; \widehat{R} \right]$ we have that the bank would always choose $\lambda = \lambda_2$.

Now we take up the case when $R \in \left(\frac{1}{\overline{p}}; \widehat{\widehat{R}} \right)$. We know that this time an equilibrium with liquidity shortage exists. At t=1 for a given λ we have an equilibrium with only some good banks borrowing for $\lambda \in [0; \lambda_{LB})$ if $\lambda_{LB} > 0$, where P^* is given by (12), an equilibrium with illiquid banks being indifferent between borrowing and selling and $P^* = \frac{d-\lambda}{1-\lambda}$ for $\lambda \in [\max[0; \lambda_{LB}]; \lambda_{FR}]$ if $\lambda_{FR} > 0$, an equilibrium in which some illiquid bank become insolvent for $\lambda \in (\max[\lambda_{UB}; \lambda_{FR}]; \lambda_2)$

as well as an equilibrium in which all illiquid banks can borrow on the interbank market for $\lambda \in [\lambda_2; d)$.

From the previous analysis we know that in all equilibria in which all illiquid banks are solvent the bank chooses the optimal λ as λ_2. In our numerical example again the expression (26) is always positive. Now, we are looking into optimal choice of λ when the bank anticipates an equilibrium with liquidity shortage at t=1. Deriving (21) with respect to λ we obtain

$$\pi \left[q \left[\widehat{p} R_D - R \right] + (1-q) \widehat{p} R_D (1-P) \right]. \tag{27}$$

Again after inserting equilibrium values for R_D, P and $\widehat{p}$, we obtain a derivative whose sign determines the optimal choice of λ. Due to complexity of (17) this derivative becomes also very complex. In our numerical example however, its value for all $R \in \left(\frac{1}{p}; \widehat{\widehat{R}} \right)$ is positive, implying again that the bank would like to choose λ_2.

Example 2 We use values $p = 0.1$, $q = 0.7$, $\pi = 0.85$ and $d = 0.9$, and we vary R again. Observe that there are so many liquid banks (high π) that $\lambda_2 \leq 0$ is for all $R \geq \frac{1-\pi}{1-q} \frac{d}{p\pi} \approx 5.29$. For such R we always have that the banks can take $\lambda = 0$ and there is enough interbank loans at t=1 after the bad illiquid bank sell there loans. Hence, we are interested in the case $R < \frac{1-\pi}{1-q} \frac{d}{p\pi} \approx 5.29$. Then for $\lambda \in [0; \lambda_2)$ the banks have to sell at t=1 to become liquid.

Next, we can show that for $R < \frac{1-\pi}{1-q} \frac{d}{p\pi} \approx 5.29$ and all $\lambda \in [0; \lambda_2)$ we have that $P^* (1-\lambda) + \lambda \leq d$. To see this, we make the following observations based on previously established claims. First, $\lambda_2 < \lambda_{UB}$ holds for $R < \widehat{\widehat{R}} \approx 9.11$. Hence, it holds for all the interesting cases for $R < \frac{1-\pi}{1-q} \frac{d}{p\pi} \approx 5.29$. Second, we can use expressions (**??**) and (**??**). Expression (**??**) applies when $\lambda_1 < 0$ or $R > \frac{1-\pi}{1-q} \frac{d}{p} = 4.5$. Moreover, we have just seen that $\lambda_2 < \lambda_{UB}$ in our interval of interest. Hence, from (**??**) our claim follows for $R \in \left(\frac{1-\pi}{1-q} \frac{d}{p}; \frac{1-\pi}{1-q} \frac{d}{p\pi} \right)$. Once we have that $R \leq \frac{1-\pi}{1-q} \frac{d}{p}$, we can use expression (**??**). Here we can show that the highest value $f(0) = -0.45 < 0$, which obtains for the highest R in this interval, $R = \frac{1-\pi}{1-q} \frac{d}{p}$, given that $f(0)$ is increasing in R. Given the last observation and $\lambda_2 < \lambda_{UB}$ expression (**??**) also gives us that we have $P^* (1-\lambda) + \lambda \leq d$ for all $R \leq \frac{1-\pi}{1-q} \frac{d}{p}$.

In fact, in our numerical example we have only an equilibrium with liquidity shortage for all

$R < \frac{1-\pi}{1-q}\frac{d}{p\pi} \approx 5.29$. In our numerical example, $\lambda_{FR} < 0$ or does not exist for all $R < \frac{1-\pi}{1-q}\frac{d}{p\pi} \approx 5.29$. Hence, we could only find examples in which we had to take a look only at the derivative (27) evaluated at the equilibrium values at t=1. In all of our examples this derivative evaluated at the equilibrium values at t=1 is increasing in λ. This means that we have a possibility of multiple equilibria that occur in a similar way to the ones in the case without liquidity shortage when $\lambda \in [0; \lambda_1)$. Multiple equilibria occurs, because given that each bank has to look for the highest profits at the corners of the interval $[0; \lambda_2)$, it has to also take into account what other banks do. As in Malherbe (2014) this might lead to coordination problems and multiple equilibria in the choice of λ at t=0. We could find examples in which the derivative is always negative for R close to the upper bound on the interval $\frac{1-\pi}{1-q}\frac{d}{p\pi}$, implying that the bank would like to choose optimally $\lambda = 0$. This is sensible given that for $R > \frac{1-\pi}{1-q}\frac{d}{p\pi}$ this is also the optimal choice of λ. As R decreases, the derivative can become negative for low λ and positive for high λ, indicating multiple equilibria. As R decreases further the derivative becomes positive for all λ, indicating optimal choice of λ_2.

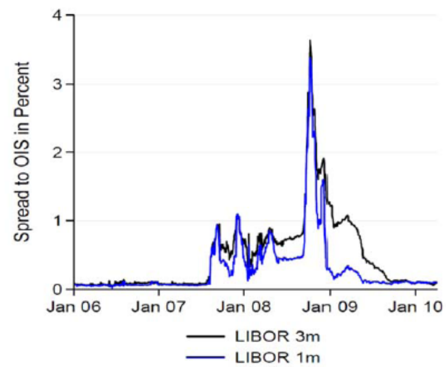

Figure 1: Source: Kuo, Skeie, Youle, and Vickrey (2013). This figure (Figure 1 in Kuo, Skeie, Youle, and Vickrey (2013)) depicts the spread between the 1- and 3-month Libor and OIS.

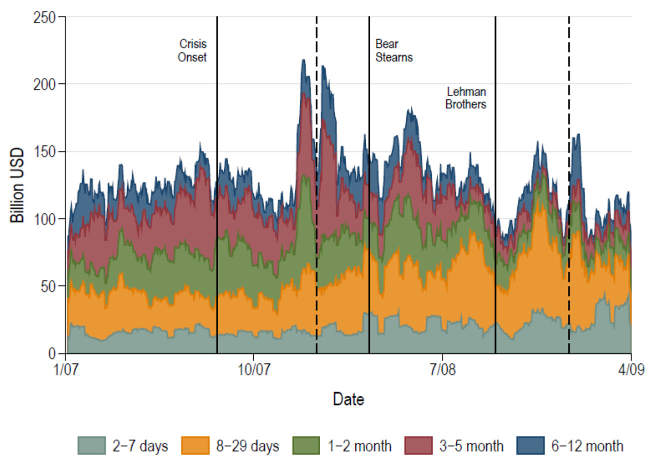

Figure 2: Source: Kuo, Skeie, Youle, and Vickrey (2013). This figure (Figure 5 in Kuo, Skeie, Youle, and Vickrey (2013)) depicts maturity-weighted volume of term interbank loans orginiated between January 2007 and March 2009.

Figures

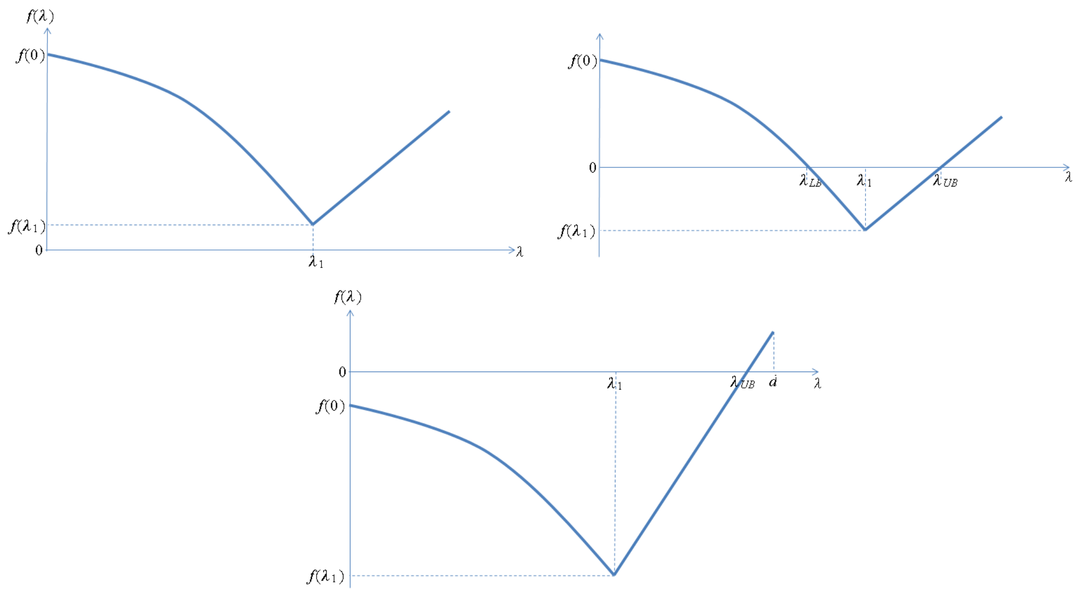

Figure 3: Function $f(\lambda)$. The first panel show the case in which $f(\lambda) > 0$ for any λ. The second shows the case when $f(\lambda) < 0$ occurs for intermediate λ, and the third for low λ.

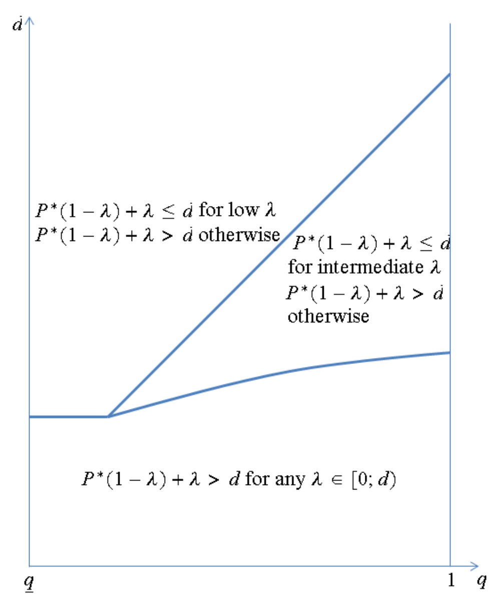

Figure 4: Results of Lemma 4

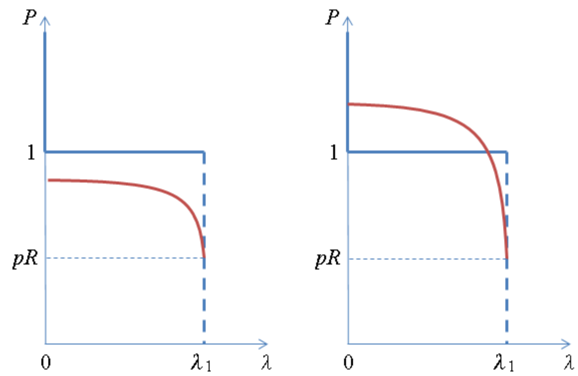

Figure 5: The case of $\lambda \in [0; \lambda_1)$. The blue step-wise curve is the optimal choice of λ at t=0 based on the aniticipated price at t=1. The dashed part represents that $\lambda = \lambda_1$ does not belong to the interval $[0; \lambda_1)$. The red solid curve is the equilibrium price at t=1 as a function of λ. The crossing points of these two curves pin down the equilibrium values of λ and P. The left panel represents the case in which the bank does not choose any $\lambda \in [0; \lambda_1)$ as optimal. The right panel show the case with multiple equilibria in which the optimal $\lambda = 0$ or the optimal $\lambda \geq \lambda_1$ as derived earlier.

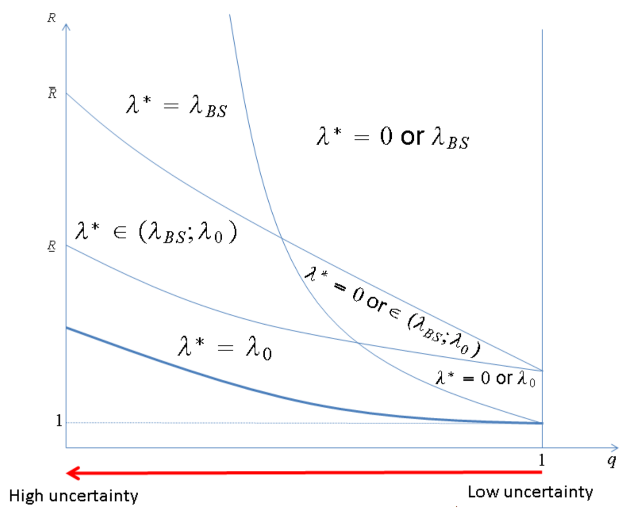

Figure 6: Lemma 6 in case of equilibria without liquidity shortage for any λ.