A Dynamic Network Model of the Unsecured Interbank Lending Market

Francisco Blasques\textsuperscript{a}  Falk Bräuning\textsuperscript{b}  Iman van Lelyveld\textsuperscript{a,c}

\textsuperscript{a}VU University Amsterdam  \textsuperscript{b}Federal Reserve Bank of Boston  \textsuperscript{c}De Nederlandsche Bank

The Role of Liquidity in the Financial System
Atlanta, November 19th, 2015

\textsuperscript{1}The views expressed in this presentation do not necessarily represent those of the Federal Reserve Bank of Boston, the Federal Reserve System, De Nederlandsche Bank, or the Eurosyste.
This Paper in a Nutshell

- Model of **formation of interbank lending relationships**, implications for credit availability and conditions (interest rates and volumes)
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- Role of **credit risk uncertainty** and **peer monitoring** in OTC market
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- **Parameter estimation** using Dutch interbank loan-level data 2008-2011
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- **Parameter estimation** using Dutch interbank loan-level data 2008-2011
- **Model analysis**: network structure, dynamic behavior and monetary policy
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1. Motivation

2. Model

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4. Results
Dutch Interbank Market during Crisis

Before Lehman 08/2008

Figure: Nodes: banks; links: ON loans; big green node: central bank; small green nodes: banks only relying on central bank; pink nodes: banks without use of central bank facilities, see Video 3 Heijmans et al. (2014)
Dutch Interbank Market during Crisis

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Why should central banks not resume the role of central counterparty for money market transactions also in normal times (i.e. non-crisis times)?
Relevance of Private Information

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- Efficiency of liquidity allocations, Rochet & Tirole (1996)

"Specifically, in the unsecured money markets, where loans are uncollateralised, interbank lenders are directly exposed to losses if the interbank loan is not repaid. This gives lenders incentives to collect information about borrowers and to monitor them [...]. Therefore, unsecured money markets play a key peer monitoring role."

from speech by Benoît Cœuré (ECB Executive Board), June 2012
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- **Key issue**: Role of credit risk uncertainty, peer monitoring and private information in the interbank market? In OTC market we need to consider uncertainty as bank-to-bank specific problem!
Preview of Main Results

- Network model of credit risk uncertainty and peer monitoring explains two stylized facts of decentralized interbank lending markets
  - Sparse core-periphery structure of lending network
  - Stable long-term trading relationships, relationship lending

- Estimated model generates dynamic amplification mechanism of shocks due to interrelation between directed search and peer monitoring
- Shocks to credit risk uncertainty lead to extended period of market turmoil
- Trading more concentrated towards bank pairs with strong relations

- Monetary policy implication for size of interest rate corridor
  - Wider corridor increases interbank lending (direct effect on outside options)
  - Indirect multiplier effect through changes in monitoring and search
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- Monetary policy implication for size of **interest rate corridor**
  - wider corridor increases interbank lending (direct effect on outside options)
  - indirect **multiplier effect** through changes in monitoring and search
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A Model of Bilateral Link Formation

\[ \text{Bank } i \quad \text{Search frictions} \quad \text{Bank } j \]

\[ \text{Bank 1} \quad \text{Bank 2} \quad \text{Bank 3} \quad \text{Bank 4} \quad \text{Bank 5} \quad \text{Bank } N \]
A Model of Bilateral Link Formation

\[ \text{Liq shock} \quad \zeta_{i,t} (> 0) \quad \rightarrow \quad \text{Bank } i \]

\[ \text{Search frictions} \quad \text{CR uncertainty} \quad \text{monitoring} \quad s_{i,j,t} \]

\[ \text{Liq shock} \quad \zeta_{j,t} (< 0) \quad \leftarrow \quad \text{Bank } j \]
A Model of Bilateral Link Formation

Liq shock $\zeta_{i,t} (> 0)$ -> Bank $i$

Liq shock $\zeta_{j,t} (< 0)$ <- Bank $j$

Central Bank

Standing facilities $\bar{r} > r$
A Model of Bilateral Link Formation

Bank $i$ 

Bank $j$ 

Liq shock $\zeta_{i,t} (> 0)$

Liq shock $\zeta_{j,t} (< 0)$

Loan $y_{i,j,t}$, $r_{i,j,t}$

Standing facilities $\bar{r} > r$
A Model of Bilateral Link Formation

Bank $i$, $j$, $t$, $r$

Bank 1

Bank 2

Bank 3

Bank 4

Bank 5

Bank $N$

Central Bank

Liq shock $\zeta_{i,t} (> 0)$

Liq shock $\zeta_{j,t} (< 0)$

Search frictions

CR uncertainty

Standing facilities $\bar{r} > r$
A Model of Bilateral Link Formation

Bank $i$ lends to Bank $j$ with a loan $y_{i,j,t}$, $r_{i,j,t}$.

Liquidity shock $\zeta_{i,t} (>0)$ for Bank $i$ and $\zeta_{j,t} (<0)$ for Bank $j$.

Monitoring $m_{i,j,t}$ between banks.

Standing facilities $\bar{r} > r$ provided by the Central Bank.

Network of banks including Bank 1, Bank 2, Bank 3, Bank 4, Bank 5, and Bank $N$. 

Search frictions and CR uncertainty also play roles in this model.
A Model of Bilateral Link Formation

Bank 1

Bank 2

Bank 3

Bank 4

Bank 5

Bank N

Central Bank

search \( s_{i,j,t} \)

loan \( y_{i,j,t}, r_{i,j,t} \)

monitoring \( m_{i,j,t} \)

Liq shock \( \zeta_{i,t} (> 0) \)

Liq shock \( \zeta_{j,t} (< 0) \)

standing facilities \( \bar{r} > r \)
A Model of Bilateral Link Formation

Bank 1
Bank 2
Bank 3
Bank 4
Bank 5
Bank N

Liq shock \( \zeta_{i,t} (> 0) \)

monitoring \( m_{i,j,t} \)

search \( s_{i,j,t} \)

loan \( y_{i,j,t}, r_{i,j,t} \)

Central Bank

standing facilities \( \bar{r} > r \)

Liq shock \( \zeta_{j,t} (< 0) \)
Model Perspective

- Model focuses on **formation of bilateral lending relationships** under credit risk uncertainty and search frictions
  - peer monitoring and directed search
- Model **does not** take into account for:
  - endogenous true riskiness (unrelated to uncertainty, liquidity shocks, monitoring)
  - other assets/liabilities (treasury perspective)
  - serial correlation in liquidity shocks, common factors
  - other monetary policy instruments than standing facilities (MROs, LTROs)
  - liquidity hoarding for precautionary reasons
  - bank heterogeneity other than in liquidity shocks (default risk, bargaining power)
Liquidity Shocks

- Banks are hit by exogenous liquidity shocks $\zeta_{i,t}$

$$\zeta_{i,t} \sim i.i.d. \mathcal{N}(\mu_{\zeta_i}, \sigma_{\zeta_i}^2) \quad \text{where} \quad \mu_{\zeta_i} \sim \mathcal{N}(\mu_{\mu}, \sigma_{\mu}^2) \quad \text{and} \quad \log \sigma_{\zeta_i} \sim \mathcal{N}(\mu_{\sigma}, \sigma_{\sigma}^2)$$

and correlation parameter $\rho_{\zeta} := corr(\mu_{\zeta_i}, \log \sigma_{\zeta_i})$

- Heterogeneity related to scale of bank’s business ($\sigma_{\zeta_i}$) and structural liquidity deficit or surplus ($\mu_{\zeta_i}$)
Credit Risk Uncertainty and Peer Monitoring

- Perceived financial distress: \( z_{i,j,t} = z_{j,t} + e_{i,j,t} \)
  - \( z_{j,t} \sim (0, \sigma^2) \) is true financial distress of \( j \), true PD: \( \mathbb{P}(z_{j,t} > \epsilon) \)
  - \( e_{i,j,t} \sim (0, \tilde{\sigma}^2_{i,j,t}) \) is independent perception error
Perceived financial distress: $z_{i,j,t} = z_{j,t} + e_{i,j,t}$

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Perceived probability of default

$$\mathbb{P}(z_{i,j,t} > \epsilon) \leq \frac{\sigma^2 + \tilde{\sigma}^2_{i,j,t}}{\sigma^2 + \tilde{\sigma}^2_{i,j,t} + \epsilon^2} =: P_{i,j,t}$$
Credit Risk Uncertainty and Peer Monitoring

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- **Perceived probability of default**
  \[
  \mathbb{P}(z_{i,j,t} > \epsilon) \leq \frac{\sigma^2 + \tilde{\sigma}_{i,j,t}^2}{\sigma^2 + \tilde{\sigma}_{i,j,t}^2 + \epsilon^2} =: P_{i,j,t}
  \]

- **Evolution of $\tilde{\sigma}_{i,j,t}^2$ (credit risk uncertainty)**
  \[
  \log \tilde{\sigma}_{i,j,t+1}^2 = \alpha \sigma + \gamma \sigma \log \tilde{\sigma}_{i,j,t}^2 - \beta \sigma m_{i,j,t} + u_{i,j,t}, \quad u_{i,j,t} \sim \mathcal{N}(0, \sigma_u^2)
  \]
  where $m_{i,j,t}$ is bank-to-bank monitoring expenditure
Link Formation, Interest Rates and Loan Volumes

- $B_{i,j,t} \sim \text{Bernoulli}(\lambda_{i,j,t})$ indicates link between bank $i$ and $j$ at time $t$ with

$$\lambda_{i,j,t} = \frac{1}{1 + \exp(-\beta \lambda (s_{j,i,t} - \alpha \lambda))}$$

where $s_{j,i,t}$ is bank-to-bank search expenditure.
Link Formation, Interest Rates and Loan Volumes

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- If $B_{i,j,t} = 1$, bilateral Nash bargaining about rates

$$r_{i,j,t} = \theta \bar{r} + (1 - \theta) \frac{P_{i,j,t}}{1 - P_{i,j,t}}$$

where $\theta$ is bargaining power of lender, with $\bar{r} = r > r = 0$. 

[details]
Link Formation, Interest Rates and Loan Volumes

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- If $B_{i,j,t} = 1$, bilateral Nash bargaining about rates

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r_{i,j,t} = \theta r + (1 - \theta) \frac{P_{i,j,t}}{1 - P_{i,j,t}}
$$

where $\theta$ is bargaining power of lender, with $\bar{r} = r > \underline{r} = 0$

- If $r_{i,j,t} \in [0, r]$, loan granted ($l_{i,j,t} = 1$) with exogenous volume

$$
y_{i,j,t} = \min\{\zeta_{i,t}, -\zeta_{j,t}\} I(\zeta_{i,t} \geq 0) I(\zeta_{j,t} \leq 0),
$$

where $\zeta_{i,t}$ and $\zeta_{j,t}$ are liquidity shocks specific to each transaction
Dynamic Optimization Problem

- Dynamic optimization problem of each bank $i$:

$$\max_{\{m_{i,j,t},s_{i,j,t}\}} \mathbb{E}_t \sum_{s=t}^{\infty} \left( \frac{1}{1+r^d} \right)^{s-t} \sum_{j=1}^{N} (l_{i,j,t} \tilde{R}_{i,j,t} y_{i,j,t} + l_{j,i,t} (r - r_{j,i,t}) y_{j,i,t} - m_{i,j,t} - s_{i,j,t})$$

s.t. constraints; where $\tilde{R}_{i,j,t} = (1 - P_{i,j,t}) r_{i,j,t} - P_{i,j,t}$, no default occurs!
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- Linearized policy function for optimal monitoring

$$m_{i,j,t} = a + b \bar{\sigma}_{i,j,t}^2 + c \mathbb{E}_t \bar{\sigma}_{i,j,t+1}^2 + d \mathbb{E}_t y_{i,j,t+1} + e \mathbb{E}_t B_{i,j,t+1}$$

- Non-linear policy function for optimal search

$$s_{i,j,t} = h(\mathbb{E}_t (r - r_{j,i,t}) y_{j,i,t}) \quad h(\cdot)' \geq 0$$
Dynamic Optimization Problem

- Dynamic optimization problem of each bank $i$:

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- Non-linear policy function for optimal search

$$s_{i,j,t} = h(\mathbb{E}_t (r - r_{j,i,t}) y_{j,i,t}) \quad h(\cdot)' \geq 0$$

- Adaptive expectations of $x_{i,j,t}$ using exponentially weighted moving average

$$\mathbb{E}_t x_{i,j,t+1} =: x_{i,j,t}^* = (1 - \lambda_x) x_{i,j,t-1} + \lambda_x B_{i,j,t} x_{i,j,t}$$
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Data Characterization Using Network Statistics

- Observed variables are $l_{i,j,t}$ (link/loan indicator), $y_{i,j,t}$ (volumes) and $r_{i,j,t}$ (spreads), for unsecured overnight loans between $N = 50$ Dutch banks from 01-02-2008 to 30-04-2011 ($T = 810$).

- At each $t$, we compute statistics of trading network implied by $\{l_{i,j,t}\}$, with link weights $\{y_{i,j,t}\}, \{r_{i,j,t}\}$ to characterize network topology.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>Fraction of existing trading relations (links) relative to all potential relations</td>
</tr>
<tr>
<td>Reciprocity</td>
<td>Fraction of reciprocal relationships among all existing trading relationships</td>
</tr>
<tr>
<td>Stability</td>
<td>Fraction of relationships that did not change as compared to previous network</td>
</tr>
<tr>
<td>Degree Centrality</td>
<td>In- and out-degree of node: number of different lenders/borrowers per bank $\rightarrow$ cross-sectional degree distribution</td>
</tr>
<tr>
<td>Clustering</td>
<td>How close are a node’s neighbors are to being a clique (complete network) $\rightarrow$ average distribution as global measure</td>
</tr>
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</table>

\[ Corr(l_{i,j,t}, \#l_{i,j,t}^{rw} - 1) \] Stability of bilateral trading relationship

\[ Corr(r_{i,j,t}, \#r_{i,j,t}^{rw} - 1) \] Price impact of intensity of bilateral relationship (relationship lending)

- From dynamic lending network we obtain sequences of network statistics.
Indirect Inference Estimator of Network Model

- **Idea**: characterize data $X$ by vector of auxiliary statistics $\beta$ in a way that identifies structural parameters $\theta$, then simulate $s = 1, \ldots, S$ different datasets $X_s$ and choose $\hat{\theta}$ as

$$
\hat{\theta} := \arg\min_{\theta \in \Theta} \| \hat{\beta}(X) - \frac{1}{S} \sum_{s=1}^{S} \hat{\beta}(X_s(\theta)) \|.
$$

- Indirect inference estimator $\hat{\theta}$ is consistent and asymptotically normal, see Gouriéroux et al. (1993)

- Moments of sequence of network statistics and moments of bilateral volumes and spreads as auxiliary statistics, see Blasques and Bräuning (2014)
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Observed and Simulated Lending Network

Figure: Interbank network during one week. Nodes are scaled according to total trading volume.

(a) Observed network

(b) Simulated network (under $\hat{\theta}_T$)
Comparison of Auxiliary Statistics

<table>
<thead>
<tr>
<th>Auxiliary statistic (mean)</th>
<th>Observed $\hat{\beta}_T$</th>
<th>Simulated $\tilde{\beta}_{TS}(\hat{\theta}_T)$</th>
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<tr>
<td>Density</td>
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<tr>
<td>$\text{Corr}(l_{i,j,t}, #l_{i,j,t-1}^{rw})$</td>
<td>0.644</td>
<td>0.586</td>
</tr>
<tr>
<td>$\text{Corr}(r_{i,j,t}, #l_{i,j,t-1}^{rw})$</td>
<td>-0.072</td>
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<tr>
<td>Avg log volume</td>
<td>4.117</td>
<td>4.137</td>
</tr>
<tr>
<td>Std log volume</td>
<td>1.690</td>
<td>1.136</td>
</tr>
<tr>
<td>Avg spread</td>
<td>0.286</td>
<td>1.075</td>
</tr>
<tr>
<td>Std spread</td>
<td>0.107</td>
<td>0.112</td>
</tr>
</tbody>
</table>
Simulated Degree Distributions

(a) Out-degree (\# borrowers)

(b) In-degree (\# lenders)

<table>
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<tr>
<td>Avg degree</td>
<td>1.038</td>
<td>0.991</td>
</tr>
<tr>
<td>Std outdegree</td>
<td>1.841</td>
<td>1.753</td>
</tr>
<tr>
<td>Skew outdegree</td>
<td>2.882</td>
<td>2.451</td>
</tr>
<tr>
<td>Std indegree</td>
<td>1.600</td>
<td>1.687</td>
</tr>
<tr>
<td>Skew indegree</td>
<td>2.403</td>
<td>2.076</td>
</tr>
</tbody>
</table>
Parameter Estimates: What Drives the Lending Patterns?

- 16 structural parameters estimated, 8 calibrated (not identified)
- Some key results:

<table>
<thead>
<tr>
<th>Economic hypothesis</th>
<th>$H_0$</th>
<th>$\hat{\theta}$</th>
<th>reject at 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitoring has no effect on information</td>
<td>$\beta_\sigma = 0$</td>
<td>9.662</td>
<td>Yes</td>
</tr>
<tr>
<td>Search has no effect on link probability</td>
<td>$\beta_\lambda = 0$</td>
<td>72.83</td>
<td>Yes</td>
</tr>
<tr>
<td>No liquidity shock heterogeneity in mean</td>
<td>$\sigma^*_\mu = 0$</td>
<td>1.990</td>
<td>Yes</td>
</tr>
<tr>
<td>No liquidity shock heterogeneity in variance</td>
<td>$\sigma_\sigma = 0$</td>
<td>1.981</td>
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</tbody>
</table>

- Linear policy rule for monitoring:

<table>
<thead>
<tr>
<th>Variable</th>
<th>CR Uncertainty $\tilde{\sigma}_{i,j,t}$</th>
<th>$E_t \tilde{\sigma}_{i,j,t+1}$</th>
<th>Link $E_t B_{i,j,t+1}$</th>
<th>Volume $E_t y_{i,j,t+1}$</th>
</tr>
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<tr>
<td>Coefficient</td>
<td>0.002</td>
<td>-0.0055</td>
<td>0.0383</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

- Persistent expectations about bilateral link probabilities ($\lambda_B = 0.93$) and volumes ($\lambda_Y = 0.85$), less for spreads ($\lambda_r = 0.41$)
Heterogeneous Liquidity Shock Distributions

\[ \zeta_{i,t} \sim \mathcal{N}(\mu_{\zeta_i}, \sigma^2_{\zeta_i}) \]

where

\[ \begin{pmatrix} \mu_{\zeta_i} \\ \log(\sigma_{\zeta_i}) \end{pmatrix} \sim \mathcal{N} \left( \begin{pmatrix} \mu_{\mu} \\ \mu_{\sigma} \end{pmatrix}, \begin{pmatrix} \sigma^2_{\mu} & \rho \sigma_{\sigma} \sigma_{\mu} \\ \rho \sigma_{\sigma} \sigma_{\mu} & \sigma^2_{\sigma} \end{pmatrix} \right) \]
$\zeta_{i,t} \sim \mathcal{N}(\mu_{\zeta_i}, \sigma_{\zeta_i}^2)$  where  

$\begin{pmatrix} \mu_{\zeta_i} \\ \log \sigma_{\zeta_i} \end{pmatrix} \sim \mathcal{N} \left( \begin{pmatrix} \mu_\mu \\ \mu_\sigma \end{pmatrix}, \begin{pmatrix} \sigma_\mu^2 & \rho \sigma_\sigma \sigma_\mu \\ \rho \sigma_\sigma \sigma_\mu & \sigma_\sigma^2 \end{pmatrix} \right)$
Role of Peer Monitoring on Lending Structure

- **Comparison with no monitoring calibration** $\hat{\theta}_A$, where $\beta_\sigma = 0$, and all other parameters fixed at estimated values $\hat{\theta}_U$

- **And comparison with restricted estimates** $\hat{\theta}_R$, where $\beta_\sigma = 0$, and all other parameters re-estimated

<table>
<thead>
<tr>
<th>Auxiliary statistic (mean)</th>
<th>Calibrated $\tilde{\beta}_{TS}(\hat{\theta}_A)$</th>
<th>Restricted $\tilde{\beta}_{TS}(\hat{\theta}_R)$</th>
<th>Unrestricted $\tilde{\beta}_{TS}(\hat{\theta}_U)$</th>
<th>Observed $\hat{\beta}_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr($l_{i,j,t}$, $l_{i,j,t-1}^{rw}$)</td>
<td>0.2345</td>
<td>0.4259</td>
<td>0.6001</td>
<td>0.6439</td>
</tr>
<tr>
<td>Corr($r_{i,j,t}$, $l_{i,j,t-1}^{rw}$)</td>
<td>0.0000</td>
<td>-0.1578</td>
<td>-0.1231</td>
<td>-0.0716</td>
</tr>
<tr>
<td>Skew outdegree</td>
<td>0.4512</td>
<td>1.3604</td>
<td>2.3649</td>
<td>2.8821</td>
</tr>
<tr>
<td>Skew indegree</td>
<td>0.3300</td>
<td>1.3971</td>
<td>2.2801</td>
<td>2.4030</td>
</tr>
</tbody>
</table>
Dynamic Network Responses to Credit Risk Uncertainty Shock

Figure: Simulated network responses to common shock to credit risk uncertainty
Responses of Monitoring and Search

Figure: Amplification mechanism due to feedback between monitoring and search
Monetary Policy Analysis: Changes in Interest Rate Corridor

Figure: Responses of lending network structure
Changes in lending network are driven by two effects

- **Direct effect** on interbank lending activity by altering outside options
- **Indirect multiplier effect** through changes in monitoring and search efforts
Conclusion

We introduce and estimate structural interbank network model where banks monitor and search counterparties for bilateral bargaining in OTC market.

CR uncertainty and monitoring are key driver of sparse core-periphery structure trading network and existence of relationship lending.

Dynamic analysis reveals importance of monitoring and search as driver behind prolonged market inactivity after shock to uncertainty.

Changes in discount window lead to direct effect on interbank lending and indirect multiplier effect through altered monitoring and search efforts.
A Dynamic Network Model of the Unsecured Interbank Lending Market

Francisco Blasques\textsuperscript{a}  Falk Bräuning\textsuperscript{b}  Iman van Lelyveld\textsuperscript{a,c}

\textsuperscript{a}VU University Amsterdam
\textsuperscript{b}Federal Reserve Bank of Boston
\textsuperscript{c}De Nederlandsche Bank

The Role of Liquidity in the Financial System
Atlanta, November 19th, 2015

\textsuperscript{1}The views expressed in this presentation do not necessarily represent those of the Federal Reserve Bank of Boston, the Federal Reserve System, De Nederlandsche Bank, or the Eurosystem.
Details of Bilateral Interest Rate Bargaining

- For bank $i$, lending funds to bank $j$ at time $t$ is a risky investment

$$R_{i,j,t} = \begin{cases} 
rijt & \text{w.p. } 1 - P_{i,j,t} \\
-1 & \text{w.p. } P_{i,j,t}.
\end{cases}$$

with expected return (expectation under perceived probability measure)

$$\bar{R}_{i,j,t} = \mathbb{E}_t R_{i,j,t} = (1 - P_{i,j,t})rijt - P_{i,j,t}.$$

- For borrowing bank $j$ cost of borrowing are simply $r_{i,j,t}$

- Bilateral Nash bargaining solution then satisfies

$$r_{i,j,t} \in \arg\max_r \left( (1 - P_{i,j,t})r - P_{i,j,t} - r \right)^\theta (\bar{r} - r)^{1-\theta}.$$

- Back to Bargaining
Details of Indirect Inference Estimation

- We use quadratic form with diagonal weight matrix (equal unit weights, except density and RL measures which are set to 10 and 50), \( S = 24 \) simulated networks with each \( T^* = 3000 \), burning 1000 periods.

- The reduced form can be written as a nonlinear Markov autoregressive process,

\[
X_t = G_\theta(X_{t-1}, e_t)
\]

- Restrict parameter space \( \Theta \) to ensure model identification and stability; contraction condition to ensure stability of dynamic network

\[
\mathbb{E} \log \sup_x \| \nabla G_\theta(x, e_t) \| < 0
\]

where \( \nabla G_\theta \) denotes the Jacobian of \( G_\theta \) and \( \| \cdot \| \) is a norm.

- Lyapunov stability of the dynamic stochastic network model

<table>
<thead>
<tr>
<th>Parameter vector</th>
<th>initial point: ( \theta_0 )</th>
<th>estimated point: ( \hat{\theta}_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lyapunov exponent</td>
<td>-0.6451</td>
<td>-0.2462</td>
</tr>
</tbody>
</table>

- Back to [Estimation](#)
### Details of Auxiliary Statistics

**Table** : Auxiliary network statistics. The table reports the values of the observed auxiliary statistics $\hat{\beta}_T$ used in the indirect inference estimation along with the HAC robust standard errors. The simulated average of the auxiliary statistics $\tilde{\beta}_{TS}$ for $S = 24$ paths is shown for the estimated parameter vector $\hat{\theta}_T$ and the alternative calibration $\theta_a$ (model without monitoring). The observed statistics are computed on a sample of daily frequency from 18 February 2008 to 28 April 2011 of size $T = 810$.

<table>
<thead>
<tr>
<th>Auxiliary statistic</th>
<th>Simulated</th>
<th>Estimated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (mean)$^a$</td>
<td>0.1121</td>
<td>0.0193</td>
<td>0.0212</td>
</tr>
<tr>
<td>Reciprocity (mean)</td>
<td>0.0453</td>
<td>0.0627</td>
<td>0.0819</td>
</tr>
<tr>
<td>Stability (mean)</td>
<td>0.8247</td>
<td>0.9795</td>
<td>0.9818</td>
</tr>
<tr>
<td>Avg clustering (mean)</td>
<td>0.1097</td>
<td>0.0347</td>
<td>0.0308</td>
</tr>
<tr>
<td>Avg degree (mean)</td>
<td>5.4948</td>
<td>0.9441</td>
<td>1.0380</td>
</tr>
<tr>
<td>Std outdegree (mean)</td>
<td>3.2901</td>
<td>1.6547</td>
<td>1.8406</td>
</tr>
<tr>
<td>Skew outdegree (mean)</td>
<td>0.4512</td>
<td>2.3649</td>
<td>2.8821</td>
</tr>
<tr>
<td>Std indegree (mean)</td>
<td>4.7450</td>
<td>1.6950</td>
<td>1.6001</td>
</tr>
<tr>
<td>Skew indegree (mean)</td>
<td>0.3300</td>
<td>2.2801</td>
<td>2.4030</td>
</tr>
<tr>
<td>Corr($r_{i,j,t}, l^r_{i,j,t-1}$) (mean)</td>
<td>0.0000</td>
<td>-0.1231</td>
<td>-0.0716</td>
</tr>
<tr>
<td>Corr($l_{i,j,t}, l^w_{i,j,t-1}$) (mean)</td>
<td>0.2345</td>
<td>0.6001</td>
<td>0.6439</td>
</tr>
<tr>
<td>Avg log volume (mean)</td>
<td>2.8298</td>
<td>3.9422</td>
<td>4.1173</td>
</tr>
<tr>
<td>Std log volume (mean)</td>
<td>1.0547</td>
<td>1.0865</td>
<td>1.6896</td>
</tr>
<tr>
<td>Skew log volume (mean)</td>
<td>-0.1187</td>
<td>-0.1357</td>
<td>-0.3563</td>
</tr>
<tr>
<td>Avg interest rates (mean)</td>
<td>1.0348</td>
<td>1.1353</td>
<td>0.2860</td>
</tr>
<tr>
<td>Std interest rates (mean)</td>
<td>0.0000</td>
<td>0.1004</td>
<td>0.1066</td>
</tr>
<tr>
<td>Skew interest rates (mean)</td>
<td>0.0251</td>
<td>1.6010</td>
<td>0.6978</td>
</tr>
<tr>
<td>Corr(density,stability)</td>
<td>-0.4688</td>
<td>-0.3837</td>
<td>-0.7981</td>
</tr>
<tr>
<td>Corr(density,rates)</td>
<td>0.0296</td>
<td>0.0896</td>
<td>0.7960</td>
</tr>
<tr>
<td>Autocorr(density)</td>
<td>0.0034</td>
<td>0.2455</td>
<td>0.8174</td>
</tr>
<tr>
<td>Autocorr(avg volume)</td>
<td>0.0014</td>
<td>0.0760</td>
<td>0.4926</td>
</tr>
<tr>
<td>Autocorr(avg rate)</td>
<td>0.9991</td>
<td>0.2425</td>
<td>0.9655</td>
</tr>
</tbody>
</table>

| Objective function value                        | 227.3328  | 4.2407    |          |
| Euclidean norm $\|\hat{\beta}_T - \tilde{\beta}_{TS}\|$ | 6.8563    | 2.0035    |          |
| Sup norm $\|\hat{\beta}_T - \tilde{\beta}_{TS}\|_\infty$ | 4.4568    | 0.9032    |          |

$^a$ Not included in vector of auxiliary statistics as proportional to average degree.
Table: Estimated parameter values. The table reports the estimated structural parameters ($\hat{\theta}_T$) and corresponding standard errors and 90% confidence bounds. The parameter $\theta_a$ represents an calibrated model parametrization without monitoring ($\beta_{\phi,1} = 0$). For calibrated parameters no standard errors and confidence bounds are reported. The indirect inference estimator is based on $S = 24$ simulated network sequences of length $T^* = 3000$. Note also that $\sigma^*_\mu = \log(\sigma_{\mu})$.

<table>
<thead>
<tr>
<th>Structural parameters</th>
<th>Calibrated $\theta_a$</th>
<th>Estimated $\hat{\theta}_T$</th>
<th>St.Errors $\text{ste}(\hat{\theta}_T)$</th>
<th>90% Bounds LB</th>
<th>90% Bounds UB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Added information</td>
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<tr>
<td>$\alpha_{\phi}$</td>
<td>-1.5000</td>
<td>-1.5000</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$\beta_{\phi,1}$</td>
<td>0.0000</td>
<td>9.6631</td>
<td>0.0006</td>
<td>9.6619</td>
<td>9.6643</td>
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<td>$\beta_{\phi,2}$</td>
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<td>0.0001</td>
<td>0.0445</td>
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<td>0.0873</td>
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<td>Perception error variance</td>
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<td>$\alpha_{\sigma}$</td>
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<td>1.2890</td>
<td>0.0028</td>
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<td>$\beta_{\sigma}$</td>
<td>-2.0000</td>
<td>-2.0000</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$\gamma_{\sigma}$</td>
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<td>0.6648</td>
<td>0.0183</td>
<td>0.6289</td>
<td>0.7007</td>
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<td>$\delta_{\sigma}$</td>
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<td>0.3383</td>
<td>0.0451</td>
<td>0.2499</td>
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<td>Search technology</td>
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<td>$\alpha_{\lambda}$</td>
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<td>0.0001</td>
<td>0.1159</td>
<td>-0.2271</td>
<td>0.2273</td>
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<tr>
<td>$\beta_{\lambda}$</td>
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<td>72.8331</td>
<td>0.0006</td>
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<td>Liquidity shocks</td>
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<tr>
<td>$\mu_{\mu}$</td>
<td>0.0000</td>
<td>0.0000</td>
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<td>1.9810</td>
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<td>Expectations</td>
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<tr>
<td>$\rho_{\zeta}$</td>
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<td>-0.7826</td>
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<td>$\lambda_{Y}$</td>
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<td>0.8472</td>
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<td>$\lambda_{B}$</td>
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<td>0.9278</td>
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<td>$\lambda_{I}$</td>
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<td>$\lambda_{\tilde{\sigma}}$</td>
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<td>Bargaining lender</td>
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<td>$\theta$</td>
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<td>0.6896</td>
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<td>$r$</td>
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<td>Default threshold</td>
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<tr>
<td>$r^d$</td>
<td>1.7500</td>
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<td>Scale logistic</td>
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</tr>
<tr>
<td>$\beta_{I}$</td>
<td>200.00</td>
<td>200.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</tbody>
</table>