A Note on Extracting Inflation Expectations from Market Prices of TIPS and Inflation Derivatives*

Nikolay Gospodinov† and Bin Wei‡

November 2015

Abstract

This note discusses the extraction of inflation expectations using an affine term structure model that incorporates information from markets for U.S. Treasuries, inflation-protected securities (TIPS), inflation options, and swaps. The empirical analysis suggests that despite the observed volatility in market-based inflation compensation, the U.S. inflation expectations appear to be anchored and stable over time. The recent variations in the observed breakeven inflation is largely driven by a significant liquidity premium. As a by-product, the model provides a valuable platform for conducting policy experiments such as estimation of probability of a particular inflation scenario.

JEL Classification: G12, E43, E44, C32.

Keywords: Bond prices, TIPS, inflation caps and floors, affine models, inflation expectations, liquidity and risk premium.

---

*This note is based on “Forecasts of Inflation and Interest Rates in No-Arbitrage Affine Models” by the same authors, which can serve as a reference for various details and technical aspects of the model. We would like to thank Paula Tkac for very helpful comments and suggestions, and Hongwei Wu for expert research assistance. The views expressed here are the authors’ and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System.

†Research Department, Federal Reserve Bank of Atlanta, 1000 Peachtree Street, N.E., Atlanta, GA 30309-4470, Email: nikolay.gospodinov@atl.frb.org

‡Research Department, Federal Reserve Bank of Atlanta, 1000 Peachtree Street, N.E., Atlanta, GA 30309-4470, Email: bin.wei@atl.frb.org
1 Introduction

Price stability is a key objective of any central bank. Anchoring inflation expectations is a necessary condition for achieving the desired price stability. Furthermore, inflation expectations play a key role in determining the long-term interest rates and the shape of the yield curve which in turn affect the state of macroeconomic activity and long-run economic growth. As a consequence, measuring inflation expectations and inflation risk premium is of utmost importance for policy makers, investors and market participants. Some potentially useful information about inflation expectations can be inferred from market prices of Treasury inflation protected securities (TIPS), inflation swaps and derivatives that have been introduced in the U.S. over the last 15-20 years.¹ For example, the spread between nominal yields and TIPS of the same maturity provides a closely followed measure of inflation compensation ("breakeven inflation"). Decomposing this inflation compensation into inflation expectations and risk premium has spurred a large recent literature that includes Abrahams, Adrian, Crump and Moench (2015), Ang, Bekaert and Wei (2008), D’Amico, Kim and Wei (2014), Chen, Liu and Cheng (2010), Christensen, Lopez and Rudebusch (2010), Grishchenko and Huang (2013), Haubrich, Pennacchi and Ritchken (2012), Hördahl and Tristani (2012), Joyce, Lildholdt and Sorensen (2010), among others.²

Recently, the short-term movements in breakeven inflation (BEI) have been increasingly scrutinized by commentators and the popular press. For example, the 60 basis points decline in the 5-year BEI from June 2015 to September 2015 (with a value of 1.02% on September 28, 2015) was largely viewed as suppressed inflation expectations. This low level of market-based inflation compensation, however, is at odds with the level and dynamics of survey inflation expectations over the same horizon. To reconcile these seemingly puzzling observations, we start by noting that the TIPS-based BEI contains several latent components and attributing the observed variations to the individual components requires a more careful analysis. The challenge in identifying these unobserved components is two-fold. First, the observed BEI differs from the “true” BEI by a liquidity premium component. Furthermore, the “true” BEI

¹Other countries that issue inflation-indexed securities include Canada, France, Japan and U.K. For example, U.K. started issuing these securities as early as 1981. Campbell, Shiller and Viceira (2009) provide a comprehensive review of the inflation-indexed bond markets.

²For some specific aspects of the U.S. TIPS market, see Fleckenstein, Longstaff and Lustig (2014), Fleming and Krishnan (2012), Gürkaynak, Sack and Wright (2010), and Sack and Elsasser (2004).
contains an inflation expectation and a risk premium component. The potential identification of these latent components requires additional independent information from other asset markets. In particular, in addition to nominal bond and TIPS prices, we employ information from inflation options (caps and floors) and inflation swaps.

In order to disentangle the components of the observed market-base inflation compensation, we develop a general affine no-arbitrage model that incorporates information from several asset markets. Modeling different asset prices within an affine no-arbitrage framework, however, presents some challenges. For instance, option prices are typically highly nonlinear functions of the state variables that cannot be accurately approximated by affine functions. Fortunately, the option- and swap-implied inflation expectations can be expressed as affine functions of the state variables. Under the put-call parity, the values of an inflation cap and an inflation floor with the same tenor and strike price imply expected inflation under the forward measure. Furthermore, under no-arbitrage, the forward, risk-neutral, and physical measures are all connected through the prices of risk, and this relation also has an affine structure that can be readily incorporated in an affine term structure model. Thus, introducing inflation options enhances the information about the risk premiums. A similar role is played by inflation swaps in identifying the liquidity premium. From an estimation perspective, adding information from inflation derivatives is expected to mitigate the identification problem (flatness of the likelihood) that plagues the estimation of traditional term structure models (Wright, 2014).

Our modeling approach is most closely related to D’Amico, Kim and Wei (2014) and Kitsul and Wright (2013). D’Amico, Kim and Wei (2014) augment the traditional term structure model with a liquidity factor but they maintain the constant volatility assumption and use only information in nominal yields, real yields and inflation (as well as survey inflation expectations and forecasts of interest rates). On the other hand, Kitsul and Wright (2013) are primarily interested in the option-implied distribution of inflation and do not build explicitly a term structure model as a link to nominal and real yields. We derive a closed-form solution of the inflation caps (floors) under the forward measure, which is a function of the structural parameters, the traditional state variables (interpreted as level, slope and curvature of the term structure) and an inflation factor. Importantly, the information in inflation options is incorporated through option-implied inflation expectations which
preserves the affine structure of the model.\(^3\)

To preview the main results, we find that the inflation expectations have remained anchored despite the recent oil price decline, weak global demand and heightened financial market volatility. The inflation risk premium is estimated to be small and fairly stable over time. By contrast, the recent 60 basis points decrease in the observed BEI is almost entirely driven by movements in the liquidity premium.

The rest of the note is structured as follows. Section 2 provides a brief introduction to our affine model of nominal yields, TIPS and inflation option prices. Section 3 describes the data and discusses the main empirical results. Section 4 concludes.

2 A Brief Description of the Model

Let \( y_{t,\tau}^N \), \( y_{t,\tau}^R \) and \( y_{t,\tau}^{TIPS} \) be the yields on \( \tau \)-year nominal bond, real bonds and TIPS, respectively, and \( Q_t \) denote the price (CPI) level at time \( t \). Under no-arbitrage, there exist nominal and real stochastic discount factors, labeled as \( M_t^N \) and \( M_t^R \), satisfying the restriction \( M_t^N = M_t^R Q_t \). It immediately follows that the breakeven inflation (BEI) rate, defined as the difference between nominal and TIPS yields of bonds with the same maturity, can be decomposed into inflation expectation (IE), inflation risk premium (IRP) and liquidity premium (LP) as follows:

\[
BEI_{t,\tau} = y_{t,\tau}^N - y_{t,\tau}^{TIPS} = y_{t,\tau}^N - (y_{t,\tau}^R + LP) = \\
\frac{1}{\tau} \ln E_t \left[ \frac{Q_{t+\tau}}{Q_t} \right]_{\text{IE}} + \frac{1}{\tau} \ln E_t \left[ \frac{(M_{t+\tau}^R/M_t^R) (Q_t/Q_{t+\tau})}{E_t [M_{t+\tau}^R/M_t^R] E_t [Q_t/Q_{t+\tau}]} \right]_{\text{IRP}} - LP.
\]

It is worth noting that the above decomposition is model-independent.

In order to identify these latent components, we use a no-arbitrage framework that extends the model of D’Amico, Kim, and Wei (2014) by introducing stochastic volatility in inflation and incorporating inflation swap and options data. In particular, we assume the

\(^3\)Incorporating additional independent information from surveys or other asset markets also help the potential identification of hidden or unspanned factors (Fisher and Gilles, 2000; Duffee, 2011; Chernov and Mueller, 2012; Joslin, Priebsch and Singleton, 2014) that pass undetected through the term structure of interest rates. For example, as Chernov and Mueller (2012) point out, a factor can remain hidden from the nominal and real term structure of interest rates if it has an equal but opposite effect on inflation expectations and inflation risk premium.
following price level process:

\[ dq_t = \pi_t dt + \sigma_q' dW_{x,t} + \sqrt{\nu_t} dW_{\perp,t}, \]

where \( q_t \equiv \log Q_t \) is the logarithm of the price level \( Q_t \), \( \pi_t \) is the instantaneous expected inflation rate, and \( W_{x,t} = (W_{1,t}, W_{2,t}, W_{3,t})' \) and \( W_{\perp,t} \) are independent standard Brownian motions. The instantaneous expected inflation rate is assumed to be affine in the latent factors \( x_t = (x_{1t}, x_{2t}, x_{3t})' \) and \( \nu_t \):

\[ \pi_t = \rho_q^\pi + \rho_{x}^\pi x_t + \rho_v^\pi \nu_t. \]

The state variables \( x_t \) capture the drivers in the yield curve movements (level, slope and curvature) that are orthogonal to the volatility process \( \nu_t \). The shock \( dW_{\perp,t} \) is introduced to capture inflation-specific dynamics that is not reflected in \( x_t \). By shutting off the stochastic volatility, we obtain the specification in D’Amico, Kim, and Wei (2014). The model is also augmented with a liquidity factor \( l_t \) which is modeled as in D’Amico, Kim, and Wei (2014).

The inflation option prices are introduced through the put-call parity. Let \( IE_{t,\tau} \) and \( IU_{t,\tau} \) denote inflation expectations and inflation uncertainty under the forward measure \( \tilde{P} \). Under the assumption that the change in log price levels follows a normal distribution, i.e., \( (q_t + \pi_t) \sim N(\pi_t, IE_{t,\tau}, IU_{t,\tau}) \), and that the put-call parity holds, then the option-implied inflation expectations have the following (approximate) exponential-affine form:

\[
y_{t,\tau,K}^{O} = \frac{1}{\tau} E_{t}^{\tilde{P}} \left[ \frac{Q_{t+\tau}}{Q_{t}} \right] = \frac{1}{\tau} \left[ \exp \left( \tau y_{t,\tau}^{N} \right) \left( P_{t,\tau,K}^{CAP} - P_{t,\tau,K}^{FLO} \right) + (1 + K)^{\tau} \right]
\approx \exp \left[ IE_{t,\tau} + \frac{1}{2} IU_{t,\tau} \right],
\]

where \( P_{t,\tau,K}^{CAP} \) (\( P_{t,\tau,K}^{FLO} \)) denotes the price of a \( \tau \)-year inflation cap (floor) with a strike price \( K \). The affine term structure model allows us to link the inflation expectations under the forward and physical measures. Then, \( y_{t,\tau}^{N}, y_{t,\tau}^{R} \) and \( y_{t,\tau,K}^{O} \) are expressed as affine functions of the state variables, the underlying model parameters and the prices of risk.

Let \( Y_t = (q_t, y_{t,\tau}^{N}, y_{t,\tau}^{TIPS}, y_{t,\tau,K}^{O})' \) denote the \( m \times 1 \) vector of observables and \( X_t = (q_t, x_{1t}, \nu_t, l_t)' \) be the augmented state vector. Note that the nominal yields, TIPS yields and inflation option prices are obtained at weekly frequency while the CPI inflation is available only at monthly frequency. The estimation is performed on the discretized state-space.
system of the model in which the bond yields and option-implied inflation expectations are assumed to be observed with a measurement error:

\[
X_t = \mathcal{A} + \mathcal{B} X_{t-1} + \eta_t \\
Y_t = a + b X_t + e_t,
\]

where \( \mathcal{A}, \mathcal{B}, a, \) and \( b \) are functions of the underlying model and prices-of-risk parameters and \( e_t \) is a vector of i.i.d. measurement errors (with 0 as a first element). The latent state variables and the parameters are then estimated by the extended Kalman filter.

3 Data and Empirical Results

3.1 Data

All data variables are converted to weekly frequency and end in the last week of September 2015 (although they may have different start dates). Continuously-compounded, zero-coupon yields on U.S. Treasury notes with 1-, 2-, 4-, 7- and 10-year maturities are obtained from the U.S. Treasury yield curve of Gürkaynak, Sack and Wright (2007), maintained by the Federal Reserve Board (available at http://www.federalreserve.gov/Pubs/feds/2006/200628/200628abs.html). The 3- and 6-month rates are obtained from the 3- and 6-month T-bill rates with constant maturity from the Federal Reserve Board’s H.15 statistical release by converting them from discount basis to continuously-compounded rates. The sample period for the nominal yields starts in the first week of January 1990. For the TIPS yields, we use data for 5-, 7- and 10-year continuously-compounded, zero-coupon yields from the TIPS yield curve of Gürkaynak, Sack and Wright (2010), maintained by the Federal Reserve Board (http://www.federalreserve.gov/pubs/feds/2008/200805/200805abs.html). The sample period for TIPS yields starts in the first week of January 1999. As of the end of August 2015, there is $1.1 trillion of TIPS outstanding versus $11.4 trillion of nominal Treasuries outstanding. The principal of the TIPS is linked to the non-seasonally adjusted CPI for all urban consumers, and is accredited monthly. TIPS offer a deflation protection (floor) as the greater of the inflation-adjusted principal and the original principal is paid at maturity.

Data for inflation cap and floor prices, starting in the first week of October 2009, with strike prices from 1% to 6% in increments of 0.5% (for caps) and -2% to 3% in increments of 0.5% (for floors) with 1-, 3-, 5- 7- and 10-year maturities, were provided by BGC Partners.
We focus on in-the-money options by selecting strike prices of 1% and 2% for caps and 2% and 3% for floors. Weekly series for nominal yields, TIPS yields and inflation option prices are constructed by using the Wednesday observation of each week (if the market is closed on Wednesday, we take the Tuesday observation or Thursday’s observation if the Tuesday’s is not available). Inflation swap data, obtained from Bloomberg and starting in 2004, is constructed similarly.

We use the CPI for all urban consumers (all items, seasonally adjusted) from the U.S. Bureau of Labor Statistics, covering the period January 1990 – September 2015. Following D’Amico, Kim and Wei (2014), the monthly CPI is assumed to be observed on the last Wednesday of each month. The remaining weeks are treated as missing observations which are filled in via the Kalman filter. Similarly, the missing weekly observations up to January 1999 for TIPS and October 2009 for inflation options are also estimated using the Kalman filter.

Figure 1 plots the 5-year Treasury and TIPS yields along with the 5-year breakeven inflation rate. For most of the period, the 5-year breakeven rate varies between 1% and 3% except for a sharp decrease in the wake of the recent financial crisis. There are some regularities in the breakeven rate that have become more pronounced after the financial crisis and may have been caused by a seasonal carry that characterizes the TIPS market. In historical context, the recent decline in the breakeven inflation, noted in the introduction, is not unusual. The 5-year/5-year forward breakeven inflation provides a better measure of long-run inflation expectations and its dynamics is plotted in Figure 2. The graph reveals that it has a higher level and smoother dynamics than the 5-year breakeven inflation but it also exhibits a notable, albeit smaller, decline over the recent months.

Figure 3 superimposes on the variables in Figure 1 the 5-year (zero-coupon) inflation swap-implied real rate (top graph) and the swap-based breakeven inflation rate. The spread between the swap-based and TIPS-based breakeven rates is a proxy measure of the liquidity premium of the TIPS market relative to the inflation swap market. Finally, Figure 4 presents the 1- and 3-year option-implied inflation expectations that are computed as described in Section 2. These different information sources will be used for extracting the latent inflation expectation, risk and liquidity premium components embodied in the observed market-based inflation compensation.
3.2 Results

We present results for the version of the model with constant volatility. Figure 5 plots the three components of 5-year breakeven inflation – inflation expectations, liquidity premium and risk premium – implied by the model. The results suggest that most of the variability in BEI over the last few years is driven by the liquidity premium. Inflation expectations are fairly stable since 2010 at a level around 2%. We should point out that the 1-year inflation expectations exhibit more variability (see Figure 7) but increasing the horizon averages out some of the short-term movements and inflation expectations load primarily on the most persistent state variable: the level of the yield curve.

Interestingly, despite the downward movement in the observed market-based inflation compensation, inflation expectations actually increased slightly above 2% in recent months. To highlight the recent movements in BEI and its components, Figure 6 presents the dynamics of these variables over the period October 2014 - September 2015. The graph illustrates that the drop in BEI since July 2015 was not accompanied by a decrease in inflation expectations. This suggests that despite some global shocks (sharp decreases in oil and other commodity prices, slowdown in economic activity in China and heightened volatility in the world financial markets), the inflation expectations in the U.S. have remained stable and well-anchored at around 2%. Note, however, that TIPS are based on the CPI-U index which is running historically about 0.5% (0.2% since 2010) higher, on annualized basis, than the PCE inflation, which is more closely monitored by policy makers.

The model-implied inflation expectations help us to reconcile another puzzling observation: the discrepancy between the market-based inflation compensation and survey-based inflation expectations. A direct comparison of these measures reveals that survey-based inflation expectations consistently exceed market-based expectations. It turns out that after modeling and separating the different components of the observed breakeven inflation, the difference between the two measures largely disappears. Figure 7 reports the 1-year inflation expectations extracted from our model and the 1-year inflation expectations from the Survey of Professional Forecasters (SPF). Apart from some divergence at the beginning of the period when TIPS yields were not available, the two measures agree both in terms of level and dynamics over time. This is reassuring for the usefulness of the market inflation expectations which are available at a higher frequency and are characterized by a more detailed...
term structure.

The inflation risk premium appears to be small and stable over time. In fact, our estimate of the inflation risk premium is negative (although statistically insignificant) which differs from other estimates reported in the literature. Campbell, Shiller and Viceira (2009) provide theoretical arguments for low and even negative risk premium of TIPS relative to short-term safe assets.

As argued earlier, the largest contributor to the recent variability of the TIPS-base breakeven inflation proves to be the liquidity premium. Our liquidity factor aggregates a number of specificities and institutional features of the TIPS market that may cause the observed BEI to deviate from the “true” BEI. In addition to the relative illiquidity of TIPS to the Treasury and swaps markets, these factors include indexation lag, deflation floor, tenor-specific liquidity concentration, carry generated by seasonality of CPI-U, limits to arbitrage, institutional investors’ re-allocations and redemptions etc. For example, a semi-annual carry is generated by the accredited TIPS principal tied to the not-seasonally adjusted CPI-U. Moreover, negative shocks (for example, a sharp oil price drop or a liquidity squeeze in other asset markets) can exacerbate the expected negative carry or dampen the expected positive carry. This may explain why the seasonal carry has become more pronounced over the last few years as illustrated in Figure 8.

Finally, Figure 9 plots the estimated liquidity factor and the spread between the inflation swaps and TIPS breakeven inflation. As pointed out above, the latter serves as a proxy for the relative liquidity between the two markets. Despite the different level, the dynamics of the two series is very similar which provides some validation of the relevance of the information captured by the liquidity factor in the model.

While not discussed in this note, our model also delivers the model-implied distribution, under different probability measures, of inflation. This allows us to perform various policy experiments such as computing the implied probability of deflation or above-target inflation at a pre-specified horizon.

4 Conclusions

This note attempts to elicit some difficulties in interpreting and analyzing the dynamics of market-based inflation compensation. We argue that the proper identification and extraction
of the latent components of breakeven inflation requires a no-arbitrage framework that integrates information from bond and inflation derivative markets. This allows us to construct a term structure of market-based inflation expectations and perform a variance decomposition of the contributions of the different components. Our results suggest that most of the variation in the observed breakeven inflation rate is due to a large and variable liquidity premium. On the other hand, market-based inflation expectations exhibit strong similarities with survey-based inflation expectations and appear to be stable and well-anchored in recent years.
References


Fisher, M., and C. Gilles, 2000, Modeling the state-price deflator and the term structure of interest rates, Unpublished manuscript.


Figure 1: 5-year Treasury and TIPS yields (top graph) and 5-year breakeven inflation (bottom graph).
Figure 2: 5-year, 5-year forward breakeven inflation for the period January 1999 - September 2015 (left graph) and October 2014 - September 2015 (right graph).
Figure 3: 5-year Treasury, TIPS and inflation swap yields (top graph), and TIPS-based and inflation swap-based breakeven inflation rates (bottom graph).
Figure 4: 1-year and 3-year option-implied inflation expectations (top graph), and 3-year option-implied inflation expectations and TIPS-based breakeven inflation rate (bottom graph).
Figure 5: Decomposition of the 5-year breakeven inflation into inflation expectations, liquidity premium and risk premium components for the period January 1999 - September 2015.
Figure 6: Decomposition of the 5-year breakeven inflation into inflation expectations, liquidity premium and risk premium components for the period October 2014 - September 2015.
Figure 7: 1-year model-implied and survey-based (Survey of Professional Forecasters, SPF) inflation expectations.
Figure 8: 5-year breakeven inflation (left axis) and CPI seasonality computed as the (accumulated over the year) difference between the not-seasonally-adjusted and seasonally-adjusted CPI inflation (right axis).
Figure 9: 5-year model-implied liquidity premium and the spread between TIPS-based breakeven inflation and inflation swaps.