Demandable Debt as a Means of Payment: Banknotes versus Checks

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Abstract: We examine the question of whether transactable forms of privately issued, demandable debt are better used as “banknotes” or “checks.” The distinction between the two is that a check must be redeemed by the issuing bank with each use, whereas a banknote can circulate. We find that the answer to the question depends critically on the cost of early redemption. If this cost is small, banknotes will not circulate, so the question is moot. If this cost is large, incentive problems will prevent the issue of banknotes. For intermediate values of the early redemption cost, the option of early redemption limits the bank’s risk-taking behavior, so that banknotes will be preferred over checks.

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Marketplace transactions generally take place using one of two types of payments media. The first type of consists of government-issued currency or coin, while the second type consists of privately issued debt. This paper is concerned with the design of the second type of payments medium. Examples of this type of payments medium include privately issued banknotes, checks drawn on bank deposits, credit and debit cards, and more recently, stored-value cards and “electronic cash.”

Transactable debt has historically been issued in the form of demandable debt by banks or bank-like intermediaries. Several researchers (e.g., Calomiris and Kahn (1991), Flannery (1994)) have shown that the issue of demandable debt by banks has desirable incentive effects. Specifically, demandable debt can limit the incentive of bank owners or managers to abscond with bank assets after a negative shock to its net worth (Calomiris and Kahn), or to incorporate excessively risky assets into the bank’s investment portfolio (Flannery).

Calomiris and Kahn (1994, 508-509) also argue that the demandable nature of bank debt makes it a natural choice as a “liquid” transactions medium, since demandability reduces uncertainty about the market value of the debt. The liquidity of the debt may carry with it the potential for its own destruction, however, since the use of debt as a circulating medium of exchange can degrade the benefits imparted by its demandable character. The more easily the debt can be passed to another participant in the market, the less the incentive for monitoring by the original debtholders (see Maug (1998) or Kahn and Winton (1998) for explorations of this idea in corporate finance settings).
According to White (1995), this problem was associated with circulating banknotes as early as the eighteenth century (see e.g., Smith ([1994] 1776, 351-52)). A succinct description of the problem is provided by Thornton ([1962] 1802, 179-80):

A certain degree of currency being thus given to inferior paper, even the man who doubts the ultimate solvency of the issuer is disposed to take it, for the time during which he intends to detain it is very short, and his responsibility will cease almost as soon as he shall have parted with it.

Historically, there have been two institutional/regulatory responses to this dilemma. The first response has been to prohibit circulation of bank debt by mandating “redemption,” i.e., clearing and settlement, of transactable bank debt with each transaction, which essentially forces monitoring to occur with each payment. Payments media that require clearing and settlement traditionally include checks, credit and debit cards, as well as certain stored-value cards which require clearing and settlement with every transaction (e.g., those issued by Visa). The other response has been to allow bank debt to circulate, but only under certain legal restrictions. Examples of circulating payments media include privately issued banknotes and stored-value cards that do not require clearing and settlement with every transaction (e.g., those issued by the Mondex consortium).¹

White (1995) surveys the early history of legal restrictions on the private issue of banknotes in the U.S., and finds that two restrictions were placed on banknote issuers in virtually all jurisdictions: (1) notes were required to be demandable at par value, and (2) notes below a minimum denomination were prohibited. These restrictions were eventually augmented by state laws requiring banknotes to be backed by certain types of assets. During the Civil War, the U.S. government passed legislation that essentially forced all
privately issued banknotes to be backed by U.S. government bonds (see e.g., Friedman and Schwartz (1963, 20-3) or Champ, Wallace, and Weber (1994)).

In this paper, we investigate the design of transactable bank debt using a formal model that incorporates the incentive problem described above. Specifically, we are interested in the following questions. First, when is it desirable to require clearing and settlement of the debt with each transaction—when do “checks” dominate “banknotes” or vice versa? Second, are restrictions (1) and (2) desirable for circulating payments media such as banknotes? Third, is it desirable to require that circulating payments media be backed by specific types of assets?

We investigate these questions using a variant of the banking model proposed by Calomiris and Kahn (1991). In our model, there are three agents: a consumer, a bank, and a merchant. Time is discrete and divided into three periods \((t=0,1,2)\). The consumer is endowed in the period zero with an investment good that she deposits with the bank in return for a debt claim on the bank. She uses the debt claim to purchase a consumption good from the merchant in period one. The bank allocates the consumer’s endowment between two investment projects whose outcomes are realized in period two. The first project has a positive expected net present value; the second has a negative expected NPV, but offers a higher payoff to the bank. We consider two arrangements under which the consumer’s debt claim may be transferred to the merchant as a means of payment. We argue that these arrangements correspond to the use of checkable bank deposits and the use of banknotes.

Two types of contracting environments are examined. In the first environment, the consumer and the bank can write a complete, binding contract, which specifies both the
face value of the bank’s indebtedness to the consumer and the bank’s portfolio allocation. In the second environment, binding contracts can only specify the face value of the debt and not the portfolio allocation. The consumer can (costlessly) observe the bank’s portfolio allocation after the fact, however, and has the option of redeeming her debt claim in period one, thereby forcing the bank into early liquidation. In order to exercise this redemption option, a small but positive cost must be incurred.

Our results are as follows. If complete contracting is possible, then payment by banknote always dominates payment by check. It can further be shown that in this environment the optimal face value of the banknote will be less than “par.” In the incomplete contracting environment, the desirability of checks vis-à-vis banknotes depends critically on the costs of early redemption. If the cost of early redemption is sufficiently small, then banknotes will always be redeemed early, so banknotes and checks are equivalent arrangements. If the cost of early redemption is sufficiently large, then payment by check will be the only possible arrangement, as the incentive problems associated with the banknote arrangement become too severe. Finally, for intermediate values of the early redemption costs, payment by banknote will dominate payment by check.

We are also able to show that the restrictions (1) and (2) above are supported by the banknote arrangement in the incomplete contracting environment. Regarding restriction (1), we find that the only feasible face value for banknotes is par value as defined below. Regarding restriction (2), our results support a minimum denomination restriction in the sense that banknote contracts are not feasible if the cost of early redemption is too large relative to the face value of the note.
Finally, it is straightforward to show that in cases where the banknote arrangement dominates under incomplete contracting, it is a purely second-best arrangement. The inability of the bank and the consumer to write binding contracts on the bank’s portfolio allocation induces the bank to substitute less productive assets for productive ones, and thus carries with it a positive welfare cost. Hence, when demandable debt is issued in circulating form (as banknotes) there is some potential for welfare improvement by placing a priori restrictions on the bank’s scope for asset substitution, even in cases where such restrictions would be unnecessary if the demandable debt were used to create checkable deposits. Our results for the complete contracting case show that such restrictions may be incompatible with par valuation of banknotes, however.

I. The Environment

A consumer, a merchant, and a bank each live for three periods. For simplicity, all agents are risk-neutral and there is no time discounting. The consumer and the merchant are each endowed with one unit of their respective endowment goods. The consumer wishes to consume the merchant’s endowment good in period \( t=1 \), i.e., the consumer is “impatient” in the sense of Diamond and Dybvig (1983). The merchant is “patient” and is willing to wait until period \( t=2 \) to consume. The merchant is also willing to exchange his endowment good for desirable goods, or fairly priced claims on future goods. The merchant does not value the consumer’s endowment good, so there is no “double coincidence of wants” and hence no scope for barter between the two. For a transaction to occur, the use of a payments technology is required.
We suppose that one payments technology is available by default. Using this “storage” technology, the consumer can transform her one unit of endowment into $S$ units of a good valued by the merchant. One could imagine this technology deriving from the consumer’s trading her endowment for government-issued currency or specie, but the details of this technology are outside our model.\textsuperscript{3} For algebraic convenience we set $S=1$.\textsuperscript{4}

The consumer and the merchant can also make payment arrangements with a third party known as a bank. The bank has no endowment good but owns two production technologies that use the consumer’s endowment good as input at $t=0$, and produce as output at $t=2$ a good that is valued by the merchant. In return for the use of her endowment good, the bank issues to the consumer a debt claim on its period two production that can be used to purchase the merchant’s endowment good in period one.

To model the incentive problems resulting from such arrangements, the distribution of returns from the investments must allow for at least three outcomes $T_i$, where

$$T_3 > T_2 > T_1$$

(1)

For convenience we set $T_1 = 0$. We also require

$$T_3 > 1 > T_2$$

(2)

The incentive problem studied below derives from the bank’s ability to privately allocate the consumer’s endowment between two types of investment projects. A type $H$ or “high-effort” project has positive expected net present value, but has a lower probability of success than a “low-effort” or type $L$ project, i.e.,

$$p_3^H T_3 + p_2^H T_2 > 1 > p_3^L T_3 + p_2^L T_2$$

(3)

$$p_3^L > p_3^H$$

(4)
where $p_i^J$ equals the probability of the $i$th outcome, $i=2,3$, for project $J$, $J=H,L$. It must also be possible for the low effort project to completely fail, i.e., we require that

$$p_2^L + p_3^L < 1$$  
(5)

The bank will choose a proportion $\varepsilon$ of the consumer’s endowment that will be invested in the type $H$ project, with the remainder going into the type $L$ project. Let the random variable $V(\varepsilon)$ denote the period 2 value of the bank’s portfolio, so that its expected value is given by

$$EV(\varepsilon) = p_3(\varepsilon)T_3 + p_2(\varepsilon)T_2$$  
(6)

where

$$p_3(\varepsilon) = \varepsilon p_3^H + (1-\varepsilon) p_3^L$$  
(7)

$$p_2(\varepsilon) = \varepsilon p_2^H + (1-\varepsilon) p_2^L$$  
(8)

The bank’s choice of $\varepsilon$ is made privately in period zero, and is costlessly observed by the consumer after the fact, i.e., in period one.\footnote{The bank’s choice of $\varepsilon$ is made privately in period zero, and is costlessly observed by the consumer after the fact, i.e., in period one.}

Two types of contracting environments are examined below. In the first environment, the consumer and the bank can write a complete, binding contract, which specifies the face value of the bank’s indebtedness to the consumer, the bank’s portfolio allocation, and the rules for transferring the consumer’s claim on the bank to the merchant. In the second environment, a binding contract can only specify the face value of the debt and the rules for transferring the debt, but not the bank’s portfolio allocation. The consumer has the option of redeeming her debt claim in period one, thereby forcing the bank into early liquidation. In order to exercise this redemption option, the consumer bears a small fixed cost $\Lambda>0$.  


II. Payment Arrangements

II.A. Checks

We consider two types of arrangements under which the consumer’s debt claim may be used as a means of payment. Under the first arrangement, which we call payment by *check*, the consumer assigns her debt claim to the merchant in the second period. Since a check cannot circulate, this claim is of no value unless it is immediately presented to the bank for clearing and settlement. This restriction forces early liquidation of the bank’s portfolio, and necessarily incurs the costs associated with early redemption. If the face value of the debt issued by the bank is \( F \), then the value of a checkable deposit to the consumer is (assuming asset liquidation takes place at fair value)

\[
\min\{F, EV(\varepsilon)\} - \Lambda
\]  

The profit to the bank under the check arrangement is

\[
\max\{EV(\varepsilon) - F, 0\}
\]  

In order to maximize profits under the check arrangement, the bank will choose a face value of the consumer’s debt \( F \) and a portfolio decision \( \varepsilon \) so as to maximize (10), subject to the constraint that the consumer’s utility must at least equal that of storage, i.e.

\[
\min\{F, EV(\varepsilon)\} - \Lambda \geq 1
\]  

Note that the check arrangement is only feasible when early redemption costs do not exceed the highest possible expected NPV of the bank’s portfolio, i.e., when \( \Lambda \leq EV(1) - 1 \). We describe the bank’s behavior under the check arrangement in the following proposition.
**Proposition 1.** Suppose that $\Lambda \leq EV(1) - 1$. Under the check arrangement, the bank will invest its portfolio entirely in the high effort project, i.e., it will choose $\varepsilon = 1$. The face value of the debt issued to the consumer will be given by $F = 1 + \Lambda$.

**Proof:** Note that for any value of $F$, bank profits (10) are maximized by choosing the portfolio with the highest expected NPV, i.e., choosing $\varepsilon = 1$. Evaluating (11) at equality then implies $F = 1 + \Lambda$.

Q.E.D.

In words, Proposition 1 says that under the check arrangement, the bank will not be tempted to shift its portfolio into the lower-yielding, higher-risk asset. The check arrangement commits the consumer (via the merchant) to early (period one) monitoring of the bank, which forces the bank to invest only in the higher-yielding, lower-risk asset. Under the check arrangement, it is irrelevant whether the bank and the consumer can contract on the bank’s portfolio choice, since the bank always ends up choosing the most desirable portfolio, from the consumer’s perspective.

**II.B. Banknotes under complete contracting**

If the early redemption cost $\Lambda$ is large enough, then it is easy to show that the bank and the consumer will want to consider other payment arrangements besides payment by check. Suppose that $\Lambda$ is sufficiently large that the expected value of the consumer’s check in period two would exceed its value under early redemption, i.e.,
\[
\min\{1 + \Lambda, EV(1)\} - \Lambda < E \min\{1 + \Lambda, V(1)\}
\]
\[\Leftrightarrow\]
\[1 < p'_2(1 + \Lambda) + p''_2 T_2\]
\[\Leftrightarrow\]
\[\Lambda > \Delta \equiv \frac{1 - p''_2 T_2 - p''_3}{p''_3}\]

In view of inequality (12), we now consider a payment arrangement that corresponds to the issue of banknotes. Under payment by banknote, the consumer again assigns her debt claim to the merchant in the second period. Since the banknote is allowed to circulate, the merchant has the option of either redeeming the banknote in period one (at a cost), or avoiding the costs of early redemption by holding the note until period two.8

9 We initially consider the banknote arrangement in an environment where the consumer and the bank may write a binding contract in period zero, that specifies both the face value of the bank’s indebtedness \(F\) and the bank’s portfolio allocation \(\epsilon\).10 In such an environment, the consumer and the bank will clearly wish to avoid the costs associated with early redemption of the note. If the consumer transfers the note to the merchant and the merchant holds it until the second period, the bank’s expected profit will be

\[
E \max\{V(\epsilon) - F, 0\} = p_3(\epsilon)(T_2 - F)
\]

The expected utility of the consumer will equal the value of the banknote to the merchant. This must at least equal the value of storage, which requires that

\[
E \min\{F, V(\epsilon)\} = p_3(\epsilon) F + p_2(\epsilon) T_2 \geq 1
\]

Under complete contracting, the optimal banknote arrangement can be described as follows.
Proposition 2. Suppose that complete contracting is possible. Under the check arrangement, the bank will invest its portfolio entirely in the high effort project, i.e., it will choose $\varepsilon = 1$. The face value of the debt issued to the consumer will be given by $F = 1 + \Lambda$.

Proof (sketch): Since contracts can be enforced on the bank’s choice of $\varepsilon$, it is clearly in both the bank’s and the consumer’s interest to maximize the expected NPV of the bank’s investment portfolio and to choose $\varepsilon = 1$. The only question concerns the distribution of the proceeds. Evaluating constraint (14) at equality for $\varepsilon = 1$, we obtain

$$p_3^H F + p_2^H T_2 = 1$$

(15)

Solving (15) we obtain $F = 1 + \Lambda$.

Q.E.D.

In words, Proposition 2 says the following. In a first-best setting where informational asymmetries can be contracted away, the bank will always choose the investment portfolio with the highest expected NPV, which is quite intuitive. What may be less intuitive is that the face value of the banknote in this environment need not correspond to “par,” where par is defined as the optimal face value of a debt claim on the bank under the check arrangement, i.e., $1 + \Lambda$. In particular, if $\Lambda > \Lambda$, then the first-best value of the banknote will be below par. The last result is explained by the fact that under complete contracting, early redemption need never take place, hence the bank need not compensate the consumer for the costs of early redemption, as is the case under the check arrangement.
II.C. Banknotes under incomplete contracting

As discussed by Calomiris and Kahn (1991), Williamson (1992), Flannery (1994), and many others, a characteristic feature of banking environments is the inability of bank debtholders to directly control their banks’ portfolio allocations. To make our model consistent with this stylized fact, we now drop the assumption that the bank and the consumer can write binding contracts on the bank’s portfolio decision. Instead, the consumer can monitor the bank only after the fact in period one. If the consumer is sufficiently displeased with bank’s portfolio allocation, she can demand early redemption of the debt issued by the bank.

In this case, it is also necessary to specify how much information is available to merchant. In what follows, we will analyze the benchmark case where the merchant has access to the same information as the consumer, i.e., where the merchant also finds out the bank’s portfolio decision as of period one.\textsuperscript{11} This assumption concerning the merchant’s information is a sensible one, for the following reasons. First, the classic commentaries on banknotes by Smith, Thornton and other early observers tended to focus not on informational asymmetries but instead on the circulating nature of the notes. Second, there is a fair amount of historical evidence to suggest that market participants were reasonably well informed concerning the quality of banks’ circulating liabilities. For example, Gorton (1996, 363-5) finds that even during the U.S. Free Banking Era (1838-60), which saw many banknote issues of widely varying quality, marketplace participants were able to quickly assess the probability that a bank would be able to redeem its notes. Finally, since banknotes and banknote-like payments media such as stored-value cards are considered “unconventional” by modern standards, we want to initially consider envi-
ronments that are not a priori biased against this “innovative” type of payments me-
dium.12

Even if the consumer and the merchant have access to costless, perfect informa-
tion on the bank’s portfolio decision, monitoring will only be effective for sufficiently
small values of the early redemption cost $\Lambda$. To see this, suppose that the bank makes the
worst possible investment from the consumer’s viewpoint, i.e., the bank chooses $\varepsilon = 0$. Then the consumer or will only have an incentive to demand early redemption if the face
value of the note under early redemption exceeds its value “in circulation,” i.e., if kept by
the merchant until period two, which requires

$$\min\{F, EV(0)\} - \Lambda \geq E \min\{F, V(0)\}$$

$$\Leftrightarrow p_3^L T_3 + p_2^L T_2 - \Lambda \geq p_3^L F + p_3^L T_2$$

$$\Leftrightarrow p_3^L (T_3 - F) \geq \Lambda$$

As was the case under complete contracting, we will initially assume that the bank
wishes to avoid early redemption of its notes. In this case the bank’s expected profit will
be given by (13). The bank must also choose a face value for the debt and a portfolio al-
location such that the value of the banknote is at least equal to that of storage, i.e., con-
straint (14) must hold. Incomplete contracting imposes another constraint on the bank’s
choice of $\varepsilon$ and $F$, which is that the value of the banknote in circulation must at least
equal its value if redeemed early, i.e.,

$$E \min\{F, V(\varepsilon)\} \geq \min\{F, EV(\varepsilon)\} - \Lambda$$

$$\Leftrightarrow p_3(\varepsilon) F + p_2(\varepsilon) T_2 \geq \min\{F, p_3(\varepsilon) T_3 + p_2(\varepsilon) T_2\} - \Lambda$$
Under incomplete contracting, the bank’s problem is to maximize expected profits (13), subject to the “participation” constraint (14) and the “circulation” constraint (17). As under complete contracting, the bank maximizes expected profits by choosing a face value for the banknote $F$, and a portfolio allocation $\varepsilon$. The difference is that while the bank can contract over $F$, it cannot contractually specify its portfolio allocation at the time the banknote is issued, i.e., as of period zero. The bank is of course free to announce a portfolio allocation. Applying Townsend’s (1979) revelation principle, it follows that for a given face value of $F$, a credible announcement of $\varepsilon$ must cause constraint (17) to hold with equality. To see this, suppose that (17) held as a strict inequality for $(F, \varepsilon) = (F_0, \varepsilon_0)$, for a contracted face value of $F$ and an announced portfolio allocation $\varepsilon_0$. Since the bank’s expected profit (13) is decreasing in $\varepsilon$, the bank could then increase its expected profit by privately choosing a portfolio allocation $\varepsilon'$ such that $\varepsilon' < \varepsilon_0$, but where (17) is still satisfied and early redemption does not occur. Hence (17) must hold as an equality. Using this fact, we can now describe the optimal banknote arrangement.

Proposition 3. Suppose that $\Lambda \in [\Lambda, EV(1) − 1]$, and define $\varepsilon_1 \in (0,1]$ implicitly as

$$p_3(\varepsilon_1)(1 + \Lambda) + p_2(\varepsilon_1)T_2 = 1$$

Then if

$$\Lambda \leq EV(\varepsilon_1) − 1$$

is satisfied, then the optimal banknote arrangement under incomplete contracting will specify a face value for the banknote $F = (1 + \Lambda)$ and a portfolio decision $\varepsilon = \varepsilon_1$. If condition (19) is not satisfied, then no banknote can be issued.
Proof: By the discussion above, the optimal choice of $F$ and $\varepsilon$ must satisfy (14) and (17) as equalities. To solve (14) and (17) for $F$ and $\varepsilon$, we must consider two (not necessarily mutually exclusive) candidate optima.

Case 1. Suppose that $F \leq EV(\varepsilon)$ at the optimum. In this case, subtracting (14) from (17) yields

$$F = F_1 \equiv 1 + \Lambda$$

(20)

Then substitute (20) into (14) to obtain $\varepsilon_1$ as defined in (18). Note that if $EV(1) - 1 \geq \Lambda \geq \Lambda$, a weak version of inequality (12) holds so that (18) must be satisfied for some $\varepsilon_1 \in (0, 1]$. Solving (18) for $\varepsilon_1$ we obtain

$$\varepsilon_1 = \frac{1 - \left(p_3^T (1 + \Lambda) + p_2^T T_2 \right)}{(p_3^n - p_3^L)(1 + \Lambda) + (p_2^n - p_2^L)T_2}$$

(21)

Case 2. Suppose that $F \geq EV(\varepsilon)$ at the optimum. In this case, subtracting (14) from (17) yields

$$EV(\varepsilon) = 1 + \Lambda$$

(22)

Solving (22) for $\varepsilon$ yields

$$\varepsilon = \varepsilon_2 \equiv \frac{1 - EV(0) + \Lambda}{EV(1) - EV(0)}$$

(23)

Substituting (23) into (14) and solving for $F$, we obtain

$$F = F_2 \equiv \frac{1 - p_2(T_2)}{p_2(\varepsilon_2)}$$

(24)

We now show that case 2 cannot hold unless it coincides with case 1. To show this, we first show the case 2 is not feasible unless case 1 is feasible. For suppose to the contrary that case 2 is feasible and that case 1 is not. Then it must be true that...
\[ EV(\epsilon_i) < F_i \Leftrightarrow p_3(\epsilon_i)T_3 + p_2(\epsilon_i)T_2 < 1 + \Lambda \] 

(25)

We also know from (22) that

\[ p_3(\epsilon_2)T_3 + p_2(\epsilon_2)T_2 = 1 + \Lambda \] 

(26)

Define the function \( \epsilon(k) \) as the inverse function of

\[ k(\epsilon) = p_3(\epsilon)T_3 + p_2(\epsilon)T_2 \] 

(27)

Equations (25) and (26) are equivalent to

\[ k(\epsilon_2) = 1 + \Lambda > k(\epsilon_1) \] 

(28)

It then follows that

\[ \frac{d\epsilon}{dk} = \frac{1}{(p_3^h - p_3^l)T_3 + (p_2^h - p_2^l)T_2} > 0 \] 

(29)

Hence \( \epsilon(k) \) is increasing in \( k \), which implies \( \epsilon_2 > \epsilon_1 \). We will now show that

\[ \epsilon_2 > \epsilon_1 \Rightarrow F_2 < F_1 \]. Define \( F(\epsilon) \) implicitly via the condition \( H(F, \epsilon) = 0 \), where

\[ H(F, \epsilon) = p_3(\epsilon)F + p_2(\epsilon)T_2 - 1 \] 

(30)

Evaluating the participation constraint (14) at equality implies

\[ H(F_1, \epsilon_1) = H(F_2, \epsilon_2) = 0 \] 

(31)

To apply the implicit function theorem, first note that

\[ \frac{\partial H}{\partial F} = p_3(\epsilon) > 0 \] 

(32)

and that

\[ \frac{\partial H}{\partial \epsilon} = (p_3^h - p_3^l)F + (p_2^h - p_2^l)T_2 \] 

(33)

We now claim that \( \frac{\partial H}{\partial \epsilon} > 0 \) for feasible values of \( F \). To see this, first note that for any value of \( \epsilon \),

\[ \frac{\partial H}{\partial \epsilon} \bigg|_{F=1+\Lambda} = (p_3^h - p_3^l)(1 + \Lambda) + (p_2^h - p_2^l)T_2 > 0 \] ,

(34)
since $\Lambda > \underline{\Lambda}$ implies $p_3^H (1 + \Lambda) + p_2^H T_2 > 1$ by (12), and

$$p_3^L (1 + \Lambda) + p_2^L T_2 < p_3^L T_3 + p_2^L T_2 < 1$$

(35)

from condition (3). Now suppose that $\partial H / \partial \varepsilon < 0$ for $F = F^*$. Then since $\partial H / \partial \varepsilon$ is linear and decreasing in $F$, then there exists some $F'$ such that $F' \in (1 + \Lambda, F^*)$ and

$$\partial H / \partial \varepsilon \bigg|_{F = F'} = (p_3^H - p_3^L) F' + (p_2^H - p_2^L) T_2 = 0$$

(36)

Recall that from condition (3) we have

$$p_3^H T_3 + p_2^H T_2 > p_3^L T_3 + p_2^L T_2$$

(37)

Subtracting (36) from (37) and rearranging terms yields

$$p_3^H (T_3 - F') > p_3^L (T_3 - F')$$

(38)

which implies $p_3^H > p_3^L$, in contradiction to inequality (4). Hence $\partial H / \partial \varepsilon > 0$, which in turn implies that

$$\frac{dF}{d\varepsilon} = -\frac{\partial H / \partial \varepsilon}{\partial H / \partial F} < 0$$

(39)

Hence $F(\varepsilon)$ is decreasing, which implies $F_2 < F_1$ since $\varepsilon_2 > \varepsilon_1$. But

$$F_1 = 1 + \Lambda = EV(\varepsilon_2),$$

(40)

which implies that $F_2 < EV(\varepsilon_2)$, a contradiction. Hence case 2 is only feasible if case 1 is feasible.

We now show that condition (19) must hold in order for case 1 to apply. Recall that case 1 applies only if
\[ F_1 \leq EV(\epsilon_1) \]
\[
\iff
\]
\[
1 + \Lambda \leq p_3(\epsilon_1)T_3 + p_2(\epsilon_1)T_2
\]
\[
\iff
\]
\[
\Lambda \leq p_3(\epsilon_1)T_3 + p_2(\epsilon_1)T - 1 = p_3(\epsilon_1)(T_2 + (1 + \Lambda))
\]

where the last equality follows from (18).

Now note that under case 2, the profit of the bank is given by
\[
p_3(\epsilon_2)(T_3 - F_2) = p_3(\epsilon_2)T_3 + p_3(\epsilon_2)T_2 - 1 = \Lambda
\]

where the first equality follows from (24) and the second equality follows from (22). So from (41) it follows that the profit of the bank must be at least as great in case 1 as in case 2, and with strict equality unless (19) holds with equality, which means that the two cases coincide.

Finally, we note that constraint (19) implies that the profit of the bank is greater if the note is not redeemed early, and that condition (17) is also implied by (19).

Q.E.D.

In words, Proposition 3 says that in the incomplete contracting environment, the bank will issue banknotes with par face value, just as under the check arrangement (see Proposition 1). Intuitively, the bank cannot offer notes with a face value less than par and still induce the consumer to accept the banknote. Nor does the bank have anything to gain by offering more than par. The consumer knows that since there is nothing to stop the bank undermining such an offer ex post by shifting more of its portfolio into the low-effort project. The profitability of such portfolio shifting would in turn be limited by the consumer’s ability to demand early redemption.
Proposition 3 also implies that the “market” value of the banknote—the value of the banknote to the merchant, which is constrained to equal unity—will be less than its face value, i.e., that the note will circulate below par. This result is consistent with the historical experience of the Free Banking Era, during which many banknotes circulated below par at any distance from their bank of issue. In our model, the discount on the banknote results not so much from the cost of early redemption, but from the fact that this cost offers the bank an opportunity to shift its portfolio into the low-effort project. The bank’s scope for asset substitution is limited by the note holder’s (either the consumer or the merchant) incentive to redeem the note early if the value of the bank’s portfolio is driven too low. Consequently, the bank dilutes the value of the banknote until the note holder is indifferent between holding the note or redeeming it early.

Condition (19) essentially requires that the bank cannot be a “wildcat” bank in the sense of Rockoff (1975). That is, the face value of the banknote \((1 + \Lambda)\) cannot exceed what the bank could afford to redeem in period one \((EV(\epsilon_1))\), if necessary. This restriction is also largely consistent with the Free-Banking-Era experience. Recent historical research has demonstrated that wildcat banking under Rockoff’s definition was the exception rather than the rule during this period.

Condition (19) also places an upper bound on the value of the redemption cost \(\Lambda\). If we substitute for \(\epsilon_1\) from (21) and solve for \(1 + \Lambda\), then it can be shown that (19) is equivalent to \(\Lambda \leq \overline{\Lambda}\), where \((1 + \overline{\Lambda})\) is the smaller root of the quadratic inequality

\[
(p^h - p^l)(1 + \Lambda)^2 + [1 + p^l - p^h)(p^h - p^l)T_2](1 + \Lambda) + EV(0)(1 - p^hT_2)(1 - p^hT_2) - EV(1)(1 - p^lT_2) \leq 0
\] (43)
If \( \Lambda > \bar{\Lambda} \), then the banknote arrangement is not feasible, because in this case the high cost of early redemption means that ex post monitoring cannot provide an effective check on the bank’s incentives to invest in the low-effort project. It is also straightforward to show \( \Lambda \leq EV(1) - 1 \).

### III. Implications

The first question we consider is that of when banknotes are preferable to checks, and vice versa, in environments where complete contracting is not possible. The following proposition shows that answer depends on the cost of early redemption.

**Proposition 4.** If \( \Lambda < \underline{\Lambda} \), then checks and banknotes are equivalent arrangements. If \( \underline{\Lambda} \leq \Lambda \leq \bar{\Lambda} \), then banknotes dominate checks. If \( \bar{\Lambda} \leq \Lambda \leq EV(1) - 1 \), then checks are feasible while banknotes are not. If \( \Lambda > EV(1) - 1 \), then neither checks nor banknotes are feasible.

**Proof:** If \( \Lambda < \underline{\Lambda} \), then inequality (12) is reversed, which means that the bank cannot allocate a portfolio of sufficiently high value so that a note holder will forego early redemption. Hence a banknote will always be redeemed and is therefore equivalent to a check.

If \( \underline{\Lambda} \leq \Lambda \leq \bar{\Lambda} \), then under the check arrangement, the bank’s profit will be

\[
EV(1) - (1 + \Lambda) = p^HT_3 + p^HT_2 - (1 + \Lambda)
\]  

But this is less than what the bank’s expected profit would be if the check were not redeemed until period 2, which would be

\[
p^HT_3(1 + \Lambda)
\]  

(44)
This follows since the value of the bank’s payoff to the consumer under early redemption exceeds the expected value of what the bank would pay if redemption were postponed, i.e.,

\[ 1 + \Lambda > p_3 h (1 + \Lambda) + p_2 h T_2 \]  (46)

since \( T_2 < 1 \). Condition (4) implies that (45) is less than the bank’s expected profit under the banknote arrangement, i.e., \( p_3 (1 + \Lambda) (T_3 - (1 + \Lambda)) \), which implies that the bank is more profitable under the banknote arrangement than under the check arrangement. Since the expected utility of the consumer and the merchant are equal under the two arrangements, banknotes dominate.

If \( \Lambda > \overline{\Lambda} \), then by Proposition 3 banknotes are not feasible. Conditions (18) and (19) imply that \( \overline{\Lambda} \leq EV(1) - 1 \). By contrast, the bank’s profits under the check arrangement (44) are positive as long as \( \Lambda < EV(1) - 1 \).

Q.E.D.

Proposition 4 establishes that the banknote arrangement is preferred for intermediate values of the early redemption cost. If this cost is small, banknotes will always be redeemed early and will essentially function as checks. In this case, banknotes will be valued at par. If this cost is sufficiently large, the incentive problems associated with the banknote arrangement make this arrangement infeasible. For intermediate values of this cost, the banknote arrangement will be preferred, even though the market value of the notes will be below par in this case.

The desirability of banknotes in the last case contrasts with results obtained by Williamson (1992). In an environment that allows for the circulation of both “good” notes
(i.e., those backed by higher quality assets) and “bad” notes, Williamson finds that in equilibrium bad notes tend to drive out good notes via a “lemons” effect. Hence, welfare can be improved by eliminating circulating notes in favor of checkable deposits.

Our model yields a different conclusion from Williamson’s due to differences in (1) information structure and (2) the definition of a “banknote.” In our setup, the quality of a banknote is transparent to someone accepting the note as payment (i.e., the merchant), whereas in Williamson’s setup this information is not always available. This clearly tilts our results in favor of banknotes. A more critical feature of our setup, however, is the definition of a “banknote.” In contrast to Williamson, we require that banknotes be redeemable on demand. As shown above, the redeemability feature serves to limit the bank’s incentive to back its notes with lower quality assets.

Our analysis is also consistent with the restrictions that banknotes be demandable at par and limited to denominations above a minimum amount. The par demandability feature is established by Proposition 3, and the denominational restriction is also implied by Proposition 3 if we interpret the redemption cost as a cost relative to the size of the debt issue, so that a large value of the redemption cost corresponds to a small face value of the note. It should be pointed out that our model suggests that par demandability and minimum denominations are natural features of the banknote arrangement, so that there should be little need to impose these as legal restrictions. On the other hand, our model suggests that imposition of such restrictions would not constrain equilibrium arrangements among private parties.

Proposition 2 suggests that there may be some possibility for improving on the banknote arrangement by limiting the bank’s scope for asset substitution. Specifically, if
the bank can be required to hold more of the high-effort, and less of the low-effort asset, then welfare can be improved. Such requirements are potentially incompatible with par demandability, however. In the complete contracting environment of Proposition 2, the face value of the banknote need not equal par, and for values of the early redemption cost where banknotes would be normally be preferred ($\Lambda \leq \lambda \leq \bar{\lambda}$), the equilibrium face value of the banknote under complete contracting would be less than par. In this case, a legal requirement that notes be demandable at par could provide issuing banks an incentive to circumvent restrictions on their asset holdings.

IV. Implications for Regulation of New Forms of Payment

Recent years have seen the development of a number of new forms of retail payment. These new forms of payment are designed to exploit the efficiencies offered by modern communications technology. Certain of the new forms, particularly stored-value cards and electronic cash, resemble traditional forms of payment such as checks and banknotes, in the sense that they represent technologies for transferring demandable debt claims on banks and similar institutions. As with checks and banknotes, transferred claims are almost always demandable at par.

The vast majority of these new forms are more check-like than banknote-like, in the sense that clearing and settlement is required with each use of a given payments instrument. Our analysis suggests that this feature should serve to lessen incentive problems of the type described above. It is also worth noting that to date, most stored-value or electronic cash schemes are associated with commercial banks or other regulated depository institutions. Presumably liabilities issued by regulated institutions would not be
subject to the same range of incentive problems as liabilities issued by regulated institutions. However, in most G-10 countries there are currently few legal barriers to the entry of non-regulated firms into the provision of new types of payment services.17

At least one of the new forms of payment, i.e., stored-value cards offered by Mondex, can be “circulated” in the sense that a holder of one Mondex card can transfer value to any other cardholder, without such transactions having to be cleared and settled through traditional banking channels. Systems that allow for such “peer-to-peer” transactions would be more likely subject to the banknote-type incentive problems studied above. In practice, however, such problems could be circumvented if (1) the payment liabilities are issued by a regulated financial institution, (2) the cost of redemption is sufficiently low so that the liabilities rarely circulate, or (3) the issuer of the payment liabilities can credibly commit to holding only a narrow class of backing assets. Apparently the last option will be followed by the issuer of Mondex stored-value cards within the U.S. Current plans call for backing of Mondex stored value with a portfolio of U.S. Treasury and agency securities. A Mondex stored-value card will represent a claim on a separately chartered corporation holding this portfolio.18 Japan’s “Prepaid Card Law,” in effect since 1990, likewise mandates that 50% of outstanding liabilities on all stored-value cards (irrespective of whether “peer-to-peer” transactions are allowed) be backed a portfolio of cash and eligible securities.19

In summary, there is little evidence that any currently operating or planned electronic payment system is likely to be subject to the type of incentive problem historically associated with banknotes. The situation may bear watching, however, as new entrants
come into the industry and the market for electronic payment services continues to evolve.

V. Conclusions and Future Research

We have examined the question of whether transactable forms of privately issued, demandable debt are best used as “checks” or “banknotes.” The distinction between the two is that a check must be redeemed by the issuing bank with each use whereas a banknote can circulate. Redemption matters because of the monitoring benefits it provides. We find that the answer to this question depends critically on the costs of early redemption. If this cost is relatively small, banknotes will fail to circulate so the question is moot. If this cost is large, incentive problems will prevent the issue of banknotes. For intermediate values of the early redemption cost, the possibility of early redemption provides a sufficient check on the bank’s risk-taking behavior, so that banknotes will be preferred over checks, even though the notes will circulate below par.

These results depend on the assumption that marketplace participants—whose role is played by the merchant in our model—are well informed about the issuing banks’ portfolio allocations. This assumption is consistent with historical evidence put forth in Gorton’s (1996) study of the Free-Banking-Era note issues, which suggests that after a brief “learning” period, market participants are able to sort “good” from “bad” banknote issuers. Since our model is essentially static, however, it cannot capture dynamics of the learning process.

This process could be modeled in a dynamic extension of the setup above. It is unclear whether the advantages of banknotes would be amplified or diminished in a mul-
tiperiod setting. Weighing against banknotes would be large initial information asymme-
tries, whereas the monitoring benefits provided by the option of (costly) early redemption
could be reinforced in a dynamic context.
Appendix: Numerical Example

For purposes of illustration consider an example where

\[
T_2 = 0.98 \quad p_h^2 = 0.2 \quad p_l^2 = 0.01 \\
T_3 = 1.1 \quad p_h^3 = 0.8 \quad p_l^3 = 0.9
\]

These parameter values imply that

\[
EV(0) = 0.9998 \\
EV(1) = 1.076
\]

\[
\Lambda = 0.005 \\
\bar{\Lambda} = 0.045647
\]

These values imply the following values for the bank’s profits under the banknote arrangement (incomplete contracting), the bank’s profits under the check arrangement, the proportion of the bank’s portfolio invested in the high-effort project under the banknote arrangement, and the market discount on the bank’s notes as of period 1.

Table 1

<table>
<thead>
<tr>
<th>Value of early redemption cost ( \Lambda )</th>
<th>Bank’s profit under checks = ( EV(1) - (1 + \Lambda) )</th>
<th>Bank’s profits under notes = ( p^3 (\varepsilon_1) (T_2 - (1 + \Lambda)) )</th>
<th>Fraction of portfolio in high-effort project ( \varepsilon_1 )</th>
<th>Period one discount on face value of notes = ( 1 - (1 + \Lambda)^{-1} )</th>
</tr>
</thead>
</table>
| \( \Lambda \) \(
| .01                                      | .066                                         | .0724                                         | .953                                          | .0099                                         |
| .02                                      | .056                                         | .0651                                         | .857                                          | .0196                                         |
| .03                                      | .046                                         | .0577                                         | .760                                          | .0291                                         |
| .04                                      | .036                                         | .05                                           | .659                                          | .0385                                         |
| \( \bar{\Lambda} \)                      | .0304                                        | .0456                                         | .602                                          | .0437                                         |
References


Notes

1 To receive value from a Visa stored-value card requires that the recipient of the stored value have a merchant account with Visa, and that the transaction be cleared through Visa’s clearing system. By contrast, any holder of a Mondex stored-value card may receive value from any other holder of a Mondex card. Information on the Visa and Mondex stored-value cards may be found at their respective web sites: http://www.visa.com/ and http://www.mondexusa.com/.

2 A modern analog to these laws can be found in Japan’s Prepaid Card Law, which is discussed in section IV below.

3 We note this may not be an innocuous assumption. Williamson (1992) constructs several environments in which the availability of private, circulating liabilities causes fiat money not to be valued in equilibrium. The type of circulating liabilities allowed in Williamson’s setup, however, differ from those we consider. See the discussion below.

4 Since there is no time discounting in our model, imposing $S=1$ implies that the storage in effect “pays interest” at the rate of time discount. The model could easily be modified to incorporate a negative real return on storage.

5 Costly observation of the bank’s portfolio decision would be necessary to formally motivate the demandable debt arrangement between the consumer and bank (see, e.g., Calomiris and Kahn (1991)). Here we will note that the issue of debt is universally prevalent in banking environments, and simply take such arrangements as given.

6 Since in our model there is no scope for fraud by the consumer or by the merchant, the “check” arrangement as described above can be interpreted as a cashier’s check or certified check (which are considered liabilities of the institution on which they are drawn), rather than a personal check (which is considered a liability of the check writer). This arrangement could also be interpreted as a “giro” or credit (i.e., payor-initiated) transfer rather than a debit (payee-initiated) transfer.

7 Throughout the paper we assume limited liability on the part of the bank; this assumption is incorporated into expression (9). Hence the analysis below will not apply to banking systems which require unlimited liability, as was the case in the Scottish free banking system 1800-1845 (see White (1984)).

8 The optional redemption feature of the banknote arrangement, as defined above, distinguishes it from Williamson’s (1992) “circulating notes,” which lack this feature.

9 As was the case with the check arrangement, different institutional interpretations can be put on the banknote arrangement. In particular, the transaction modeled above could be reinterpreted as a check transaction in certain cases. For example, if the consumer and the merchant both having an account with the bank, the consumer’s assignment of her debt claim would correspond to an “on-us” check transaction, i.e., a transfer of balances from one of the bank’s customers (the consumer) to another (the bank). A similar interpretation would apply if the merchant and the consumer held accounts at different banks, but where the banks were linked by a correspondent-respondent (“nostro-vostro”) rela-
tionship. However, since today the predominant mechanism for settling check and similar types of retail payments is the transfer of reserves held at a central bank, we would argue that the requirement for “redemption” is applicable to checks and other non-circulating forms of payment.

10 Since the bank has limited liability, there must be some non-pecuniary means of enforcing such a contract.

11 Once again our setup differs from that of Williamson (1992). In Williamson’s setup, the quality of a note’s backing is either unobservable, or observable only at the option of the issuing bank.

12 In this case we are also assuming that the transfer of the banknote to the merchant represents final payment, i.e., the consumer will not be liable should the banknote fail to be redeemed at face value in period two. While the de jure applicability of this assumption might vary across legal jurisdictions, de facto this is a reasonable assumption if the transaction between consumer and merchant is anonymous. See Task Force on Stored-Value Cards (1997) for a discussion of this issue in the context of stored-value cards.

13 See, e.g., Gorton (1996). As Gorton points out, another important factor behind the Free-Banking-Era note discounting was the risk of counterfeit or fraud, which we abstract from in our model.


15 See Good (1997, 14-16) for a list of stored-value and electronic cash products currently available in the G-10 countries.

16 Although in the U.S. case it is unlikely that the full protection of the governmental safety net will apply to many of the new forms of payment. See Board of Governors of the Federal Reserve System (1996) and Federal Deposit Insurance Corporation (1996).


19 See Good (1997, 23); information on the Prepaid Card Law can also be found online at http://www.halsp.hitachi.com/smartcard/em/chapter3.html.