FEDERAL RESERVE BANK *of* ATLANTA

Inflation, Trade Frictions, and Productive Activity in a Multiple-Matching Model of Money

Derek Laing, Victor E. Li, and Ping Wang

Working Paper 2000-28
November 2000

Working Paper Series

Inflation, Trade Frictions, and Productive Activity in a Multiple-Matching Model of Money

Derek Laing, Victor E. Li, and Ping Wang

Federal Reserve Bank of Atlanta
Working Paper 2000-28
November 2000

Abstract: This paper investigates the relationship between money growth, inflation, and productive activity in a general equilibrium model of search. The use of a multiple-matching technique, where trade frictions are captured by limited consumption variety, allows us to study price determination in a search-theoretic environment with divisible money and goods. In our basic framework, productive activity and matching in the goods market are endogenized by a time allocation decision of work and shopping effort. We find that in such an environment, a positive feedback between shopping and work effort decisions creates a channel by which inflation can positively influence productive activity. This feature also creates the possibility of multiple steady state equilibria when household preferences for variety is sufficiently great. We also consider an alternative means of endogenizing the matching technology through endogenous firm entry. Consistent with the findings of our basic framework, the importance of search frictions continues to be essential for the non-uniqueness of equilibria and an additional channel which links money growth to real activity.

JEL classification: D83, E40, E31

Key words: multiple matching, endogenous search effort, money, inflation and productivity activity

---

The authors thank Daron Acemoglu, Ben Bernanke, Dean Corbae, Yi-ting Li, Peter Morgan, Shoyoung Shi, Neil Wallace, Randy Wright, Mike Woodford, and Ruelin Zhou, as well as participants at the Summer Meeting of the Econometric Society and workshops at the University of Pittsburgh, Princeton University, and Texas A&M University. The views expressed here are the authors' and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System. Any remaining errors are the authors' responsibility.

Please address questions regarding content to Victor Li, Research Department, Federal Reserve Bank of Atlanta, 104 Marietta Street, N.W., Atlanta, GA 30303, 404-521-8938, Victor.Li@atl.frb.org; Derek Laing, Penn State University; or Ping Wang, Vanderbilt University.

I. Introduction

The relationship between money growth, inflation, and real activity is a classic and much debated issue in monetary economics. Contrary to the Mundell-Tobin view established about three decades ago, optimal growth models of money tend to favor the conclusion that steady inflation is disruptive to real economic activity. For example, monetary growth in cash-in-advance models with production [Stockman (1981) and Cooley and Hansen (1989)] generates a pure inflation tax effect which discourages market activities requiring cash. As a result, consumption, work effort, output, and the capital stock all decline with the inflation rate. Under proper conditions, shopping-time and money-in-the-utility-function models also have a similar prediction [e.g., see a discussion in Wang and Yip (1992)]. However, these approaches assume an active role of money without much microfoundation. Thus, one may wonder if their findings are robust in micro-oriented models which consider the explicit role of money in reducing market frictions of goods trading.

Why would the consideration of trade frictions matter concerning the issues of the interactions between the real and the monetary activities? From a theoretical point of view, trade frictions are necessary to generate a transactions role for money. The existing literature consists largely of general equilibrium models, such as cash-in-advance, which simply approximate these trade frictions in Walrasian environments where money is otherwise valueless. Such theoretic short-cuts will most certainly overlook the potential effect of money growth and anticipated inflation on the very frictions which give rise to money as a medium of exchange. Moreover, from an empirical point of view, evidence pertaining to a consistently negative relationship between inflation and economic activity is far from conclusive. While some cross-country studies and evidence from hyperinflationary episodes [e.g., Fischer(1983), Cooley and Hansen (1989), and Aiyagari and Eckstein (1994)] detect a negative correlation between inflation and output growth, these findings may be influenced by the observation that countries with sustained high inflation also experience highly variable inflation.[1] A recent study by Bullard and Keating (1995) finds that a negative money-output growth correlation is absent from stable price industrialized countries. These results suggest that additional study is needed toward understanding the factors underlying the interplays between the real and monetary activities.

This paper evaluates the consequences of money growth and inflation on economic activity in the context of a search-equilibrium model of money that highlights the decentralized and costly nature of the exchange process. Search theoretic approaches to monetary theory emphasize that the use of a medium of exchange minimizes the time or resource costs associated with searching for exchange opportunities, hence alleviating the "double coincidence of wants" problem with barter. The seminal work of Kiyotaki and Wright (1989,1991,1993) formalizes this aspect of monetary exchange in the search-equilibrium paradigm of Diamond (1982,1984), and provides insights toward understanding the transactive role of money in micro-oriented models with market frictions. In our previous work Laing, Li and Wang (1997), we generalize the Kiyotaki-Wright structure by developing a “multiple-matching” model of money. Such an approach, while embodying the “double coincidence of wants” frictions, utilizes an environment which allows us to relax restrictions on the divisibility and storability of goods and money often imposed in search-theoretic models of money.[2] This multiple-matching approach to money can be regarded as the basis of the present paper.

The key features of the multiple-matching model that allows us to accomplish our objectives in a tractable manner are (i) abandoning a sequential search structure and having buyers contact *multiple* numbers

[1] As argued by Jones and Manuelli (1995), it may be this variability of high inflation, rather than the level itself, which generates distortions and disrupts economic activity.

[2] In the prototypical search model of money, exchange is characterized by one-for-one swaps of goods and money, implying fixed prices. Extensions of the Kiyotaki-Wright model with divisible goods but indivisible money to include pricing include Trejos and Wright (1993,1995) and Shi (1994). Among the first to consider the implications of inflation in search-theoretic models of money is Li(1994,1995). However, because of these restrictions, inflation was modeled as a tax on money balances given fixed nominal prices.

of sellers in a given period and (ii) assuming households to possess a preference for consumption variety and to consume a basket of goods. This ensures that there is always a subset of goods - with *given* measure - among those contacted which the household finds desirable. This keeps the steady-state distribution of cash/goods trivial.[3] Search frictions and market incompleteness are captured by limitations in the number of sellers that buyers can contact in a given period and hence limitations on the variety of goods ultimately consumed. An analogy of this process is a consumer who shops in a marketplace and encounters many different products but not all desired products in the economy. The model is closed by specifying that prices are set by monopolistically competitive firms, with each of them selling differentiated products, and by considering a circular flow of income between households and firms. As illustrated by Laing, Li, and Wang (1997), the double coincidence problem causes monetary exchange to improve trading opportunities relative to barter by increasing consumption variety.

Since the emphasis of this study is on inflation and monetary (rather than barter) exchange, the model simplifies elements of Laing, Li, and Wang (1997) and extends the approach to focus on a pure-currency search economy. In our basic set-up there is a competitive labor market and a product market with random matching. In contrast with Laing, Li and Wang (1997) where labor supply is inelastic, this paper allows households to allocate their time over work effort, shopping time, and leisure.[4] Households supply labor to firms to receive a cash wage payment and then proceed to the goods market where they are randomly

[3] The main (technical) difficulty behind direct extensions of the Kiyotaki-Wright framework to include prices and divisible inventories is that it leads to an endogenous distribution of cash and goods which must be determined jointly with prices. Recent work attempting to characterize pricing behavior and the distribution of cash include Green and Zhou (1995), Corbae and Camera (1996), Zhou (1996), and Molico (1996). Shi (1997) circumvents the distributional issues with a structure where large households consist of a continuum of traders.

[4] It should also be noted that our notion of “shopping time” is very different from shopping time models of money. In these models money is valued because it directly increases the value of leisure. However, while possessing fiat currency in a world where it is generally accepted reduces exchange costs, it is not immediate why the quantity of money itself saves on these costs. Our model captures the notion that shopping time is a costly activity required for exchange.

matched with a subset of monopolistically competitive, price-setting firms. It is the choice of shopping time which endogenizes the matching technology and influences the extent of trade frictions. Once cash is exchanged for desired goods, consumption occurs and firms use receipts to finance wage payments.

Upon building-up the multiple-matching model of money with endogenous labor allocation, we procced with a complete examination of the effects of trade frictions, money growth and steady inflation on exchange activity, labor allocation, and production decisions. With a given time allocated to shopping, an exogenous reduction in trade frictions increases labor supply, overall work effort, and economic activity. On the other hand, money growth creates an inflation tax inducing a reallocation away from work effort to leisure. Similar to conventional models, inflation discourages market activity and real output in this context.

However allowing shopping time *and* the matching technology to vary in response to the money growth rate leads to very different conclusions. In particular, when trade frictions are severe, not only can money growth and steady inflation encourage both work and shopping effort, but there also exists the possibility of multiple steady states. Intuitively, a greater matching rate enlarges consumption variety and encourages work effort when consumption and leisure are highly substitutable; on the other hand, the higher labor income generated from work effort encourages the demand for variety and induces shopping time to increase. It is precisely the complementarity between work and shopping effort which leads to the possibility of multiple equilibria.

Finally, we consider an extension of our basic set-up where, while fixing shopping time allocation, we endogenize the matching technology by introducing firm entry. We find that by allowing the mass of firms to affect the matching technology in a way analoguous to Diamond (1982), there generally exists a positive relationship between household incentives to work and firm entry. Similarly, firm profits and their entry decisions are also positively influenced by the household choice of work effort. It is precisely this positive feedback which again generates the possibility of multiple steady states. Furthermore, in our set-up an increase in the money growth rate encourages firm entry by raising the monopolistic mark-up. This provides an additional channel by which inflation can expand consumption variety and increase household participation in market activity.

The paper proceeds as follows. Section II will outline the basic model and develop equilibrium conditions. It will then characterize and analyze the properties the steady state of the model with both a fixed matching rate and endogenous shopping time. Section III extends the basic framework to include an endogenous determination of firm entry. Finally, Section IV will conclude with a summary.

II. A Multiple Matching Model of Money

Goods, Preferences, and Production

Time is discrete and the economy is populated by a continuum of infinitely lived households (indexed by $h \in H$) and firms, with each of their masses normalized to unity. There is a large number of differentiated commodities of mass one, indexed by $\omega \in \Omega$. Each firm can only produce a particular good using labor as the sole input so that firms can also be indexed by ω. A household of type h desires a variety of goods over a subset $\Omega(h) \subset \Omega$. The commodity space is ordered in such a way that a worker of type h, employed by a particular firm, produces a good outside of his/her preference domain, $\Omega(h)$, and thus there is no double coincidence of wants between them.[5] In this way, we rule out the uninteresting case of autarky as well as any possible matches/exchanges between a worker and his/her employer. As in Diamond-Yellen (1990), we assume that associated with each firm ω is an infinitely lived owner who desires good ω and acts as the residual claimant of the firm's output.[6] All exchanges occur between households and firms as only

---

[5] This model can support the possibility of a double coincidence of wants and barter between households and firms by specifying carefully households' and owners' preferences over a *random subset* of goods. Laing, Li and Wang (1997) does precisely this, proves the existence of both barter and pure monetary equilibria, and shows that under some conditions, the pure monetary equilibrium is welfare-enhancing compared to barter. Since the present study focuses strictly on the pure monetary equilibrium, the detailed structure to support fiat currency will not be elaborated.

[6] This is without loss of generality since it is, as we shall see below, consistent with profit maximization. Alternatively, we can also consider a more complex environment where households are themselves the

workers/shoppers are mobile. Both goods and money are perfectly divisible and agents can store money and their own production goods in any amount without cost.[7]

We make the following assumptions regarding household and firm owner preferences and the production technology.

Assumption 1: (*Household Preferences*). The lifetime utility for household h $\in$ H is given by,

$$V = \sum_{t=0}^{\infty} \beta^t U(D_t, L_t) \tag{1}$$

where U[•] is strictly increasing and quasi-concave in its arguments, and D_t is a composite consumption good given by

$$D_t = \left\{ \int_{\omega \subset \Omega(h)} c_t(\omega)^{\frac{\gamma-1}{\gamma}} d\omega \right\}^{\frac{\gamma}{\gamma-1}} \tag{2}$$

where $\beta \in (0,1)$ is the subjective time-discount factor, L_t is leisure at time t, $c_t(\omega)$ is household consumption of good ω, and the composite consumption good captures the preference for consumption variety and has the constant elasticity form with $\gamma > 1$ denoting the elasticity of substitution across varieties.[8]

Assumption 2: (*Firm Owner Preferences*). The owner of firm $\omega \in \Omega$ has a lifetime utility given by,

$$\hat{V} = \sum_{t=0}^{\infty} \beta^t \hat{c}_t(\omega) \tag{3}$$

where $\hat{c}$ is ownership consumption of his own production good.

---

owners of firms and receive dividend payments via a stock market.

[7] This feature of our model contrasts sharply with the traditional search-theoretic framework of Kiyotaki and Wright.

[8] For large values of γ, varieties are closer substitutes. This type of preferences is standard in the monopolistic competition literature, e.g., Dixit and Stiglitz (1977).

Assumption 3: (*Production Technology*). The production technology of firm ω is given by

$$y_t(\omega) \ = \ f[l_t(\omega)] \tag{4}$$

where $l(\omega)$ is the employment (density) and f satisfies $f' > 0$, $f'' < 0$, $f(0) = 0$ and the Inada conditions, $lim_{l \to 0}$ $f'(l) = \infty$ and $lim_{l \to \infty} f'(l) = 0$.

Labor and Product Markets

At the beginning of each period households allocate their time to work effort, l_t, shopping effort, s_t, and leisure, $L_t = 1 - l_t - s_t$. Household's possess the ability to produce many types of goods but are only productive at a single firm per period. Firm $\omega \in \Omega$ offers a competitive labor contract to households $h \in H$ which pays a nominal cash wage $W_t(\omega)$ in exchange for the household's labor services l_t.[9] Thus, the firm produces output $y_t(\omega)$ according to the production technology given by (4).

Once household $h \in H$ receive wages from the competitive labor market, they travel to the goods market in which they are randomly matched with a set of $\chi_t \subset \Omega(h)$ firms with measure α_t. We make the following assumption regarding this matching technology:

Assumption 4: (*Matching Technology*). The measure of firms contacted by a particular household h is given by $\alpha(s_t)$, where $\alpha'(s_t) \geq 0$ and $\alpha(0) \geq 0$.

Thus, α_t can be thought of as a "matching" rate which measures the severity of search frictions in the goods market. It is endogenized by the household investment decision in shopping time.

After matching, trades occur at monetary prices $P_t(\omega)$ set by the relevant monopolistically competitive firms. Households consume $c_t(\omega)$ for each $\omega \in \chi$, and firms' owners consume their residual output $\hat{c}(\omega)$.

---

[9] In the generalized version of the model with barter and monetary exchange, this contract can also consist of wage payments in the firm's output. The composition of this optimal contract between goods and cash then determines equilibrium trading regimes. A pure monetary economy is one where this contract pays only cash wages.

The Money Supply Process

Lump-sum transfers of cash from the monetary authority occur to both households and firms after the labor market closes but before the goods market opens. Thus, firms must finance wage payments with cash receipts accumulated from the previous period's sales.[10] Let X_t denote this cash transfer, where a portion $T_t = \theta X_t$ is given to households and $\hat{T}_t = (1-\theta)X_t$ is given to firms, with $\theta \in [0,1]$.[11] Thus, we can write the money supply process as $M^s_{t+1} = M^s_t + X_t = (1+\mu)M^s_t$ where μ is the money growth rate, and $X_t = T_t + \hat{T}_t$.

Optimization and Equilibrium

In each period, each household of type h is matched with a set of χ products with measure α in their desirable consumption set $\Omega(h)$. Included in this set are firms setting a common monetary price P and a set of positive measure of deviating firms (denoted by Ω'), with the representative deviating firm (indexed by ω') setting a monetary price of P'.[12]

The representative household's problem is given by choosing $\{c_t(\omega), c_t(\omega'), l_t, s_t, M_{t+1}\}$ to maximize (1), subject to

$$M_t + W_t l_t - \int_{\chi(h)} P_t(\omega) c_t(\omega) d\omega + T_t - M_{t+1} \geq 0 \tag{5}$$

---

[10] This timing of events should not be thought of as a cash-in-advance constraint on firms. It is the ex-post outcome of the richer environment of Laing, Li, and Wang (1997) where firms have the option of accumulating goods for the payment of wages.

[11] The liquidity effect literature [Lucas(1990), Fuerst(1992)] motivates a special case of this cash transfer process where $\theta \to 0$ and firms use the additional transfers to finance their wage bill.

[12] Since each firm is negligible in the continuum, it does not make sense to evaluate the demand for a single product. In order to obtain meaningful demand functions, we in essence posit a large "representative firm" which controls for a positive measure of the commodity space over Ω' and sets prices P'. Consumer behavior is then defined as the limit case for the measure of Ω' to approach zero such that the deviating firm's price setting has a negligible impact on each consumer's wealth.

where

$$D_t = \left\{ \int_{\chi(h)} c_t(\omega)^{\frac{\gamma-1}{\gamma}} d\omega \right\}^{\frac{\gamma}{\gamma-1}}, \tag{6}$$

M_t is the *beginning-of-period* household money holdings. With λ_t denoting the multiplier associated with (6), the first-order conditions, evaluated at the limiting case where the measure of Ω' vanishes, are given by

$$U_D(D,L)\{\int_{\chi} c_t(\omega)^{\frac{\gamma-1}{\gamma}} d\omega\}^{\frac{1}{\gamma-1}} c_t(\omega)^{-\frac{1}{\gamma}}\} = \lambda_t P_t \tag{7}$$

$$U_D(D,L)\{\int_{\chi} c_t(\omega)^{\frac{\gamma-1}{\gamma}} d\omega\}^{\frac{1}{\gamma-1}} c_t(\omega')^{-\frac{1}{\gamma}}\} = \lambda_t P_t' \tag{8}$$

$$U_L(D,L) = \lambda_t W_t \tag{9}$$

$$U_L(D,L) = U_D(D,L)\frac{\partial D}{\partial s_t} \tag{10}$$

$$M_{t+1}\{\lambda_t - \beta\lambda_{t+1}\} = 0 \tag{11}$$

Equations (7) and (8) imply a relationship between $c(\omega)$ and $c(\omega')$ given by

$$\left[\frac{c_t(\omega)}{c_t(\omega')}\right]^{-1/\gamma} = \frac{P_t}{P_t'}$$

Substituting this into (6) yields the household's consumption demands:

$$c_t(\omega) = \frac{W_t l_t + T_t}{\alpha(s_t)P_t} = c_t \; ; \quad c_t(\omega') = \frac{W_t l_t + T_t}{\alpha(s_t)P_t^{1-\gamma}(P_t')^{\gamma}} = c_t' \tag{12}$$

Equation (12) implies each consumer's demand, $c(\omega')$, decreases with its price P' and at a rate that depends upon the elasticity of substitution γ. An increase in total cash receipts, given by $W_t l_t + T_t$ raises the demand for all goods proportionately. The consumer's preference for variety implies that the share of their income apportioned to each good declines with the number of trading partners α contacted.

Noting that as the set of deviating firms are arbitrarily small, $\partial D/\partial s_t = \gamma/(\gamma\text{-}1)\alpha^{1/(\gamma-1)}\alpha'(s_t)c_t$. Using this, (7), (8), and (9), the efficiency conditions for work effort and shopping time are

$$U_L(D,L) \;=\; U_D(D,L)\frac{W_t}{P_t}\alpha(s_t)^{\frac{1}{\gamma-1}} \tag{13}$$

$$\frac{W_t}{P_t} \;=\; \frac{\gamma}{\gamma-1}\alpha'(s_t)c_t \tag{14}$$

Equation (13) simply equates the marginal disutility of work effort with the marginal utility of consumption that can be supported by the additional wage income. Equation (14) says that while work effort raises the overall level of consumption by the additional real wage, the marginal benefits of shopping time is the additional variety which can be purchased with a given level of income. The latter is strictly increasing in the preference for variety (i.e. decreasing in γ).

Finally, from (11) note that a necessary condition for $M_{t+1} > 0$ is given by $\lambda_t = \beta\lambda_{t+1}$ or $\{U_{dt}D_{ct}/\beta U_{Dt+1}D_{ct+1}\}(P_{t+1}/P_t) = 1$. This condition implies that the opportunity cost of holding cash (or implicit nominal interest rate) is zero. We will impose that this cost be strictly positive, which in the steady state corresponds to the restriction that $\mu > \beta$-1. Consequently, since the cash transfer occurs before the goods market opens, the household ends each period with zero money holdings or $M_{t+1} = 0 \;\forall\, t$. Thus, the household chooses an optimal sequence $\{c_t, c_t', l_t, s_t\}$ solving (12), (13), and (14) given prices and wage $\{P_t, P_t', W_t\}$.

We now consider the optimal price setting behavior of a deviating firm ω' which takes the price set by all other firms as given and sets the best response Nash equilibrium price P'. In a pure monetary equilibrium each owner consumes the residual output of it's firm so that the firm's problem is consistent with profit maximization. The representative deviating firm takes the consumption demands of each household contacted in (11) as given and chooses $\{\hat{c}_t(\omega'),\hat{l}_t(\omega'),P_t(\omega'),\hat{M}_{t+1}\}$ which solves,

$$\max_{\hat{c}(\omega'),l(\omega'),P'} \sum_{t=0}^{\infty} \beta^t \hat{c}_t(\omega')$$

subject to

$$f[\hat{l}_t(\omega')] - \alpha_t c_t(\omega') - \hat{c}_t(\omega') \geq 0 \tag{15}$$

$$\hat{M}_t + \alpha_t P_t(\omega') c_t(\omega') + \hat{T}_t - W_t \hat{l}_t(\omega') - \hat{M}_{t+1} \geq 0 \tag{16}$$

$$\hat{M}_t \geq W_t \hat{l}_t(\omega') \tag{17}$$

Inequality (15) is the firm's resource constraint, and says that output is either consumed or else sold to other households. Inequality (16) is the firm's flow budget constraint requiring that total cash balances at the beginning of next period cannot exceed the sum of current period money balances, receipts from sales, and the monetary transfer less cash wage payments. Finally, (17) is due to the absence of capital markets, and indicates that the firm cannot hire more labor than is warranted by its current cash balances.

It is convenient to characterize a stationary equilibrium by scaling all nominal variables by the beginning-of-period money stock: $\hat{m}_t \equiv \hat{M}_t/M_t^s$, $w_t = W_t/M_t^s$, and $p_t = P_t/M_t^s$. We can write (16) and (17) as:

$$\hat{m}_{t+1} = \frac{\hat{m}_t + \alpha p_t(\omega') c_t(\omega') - w_t \hat{l}_t(\omega') + (1-\theta)\mu}{1+\mu} \tag{16'}$$

$$\hat{m}_t \geq w_t \hat{l}_t(\omega') \tag{17'}$$

and express the firm's value function as

$$V(\hat{m}_t) = \max_{l(\omega'),p'} f[\hat{l}_t(\omega')] - \alpha c_t(\omega') + \beta V(\hat{m}_{t+1})$$

With (16') and (17') strictly binding, the first order conditions, given in the Appendix, yields a Nash equilibrium in the price-setting game where

$$p_t^{\prime} = \left(\frac{\gamma}{\gamma - 1}\right)\frac{w_{t+1}(1+\mu)}{\beta f^{\prime}(\hat{l}_{t+1})} \tag{18}$$

Intuitively, the monopolistic markup of price over next period wages depends negatively on γ and next period's marginal productivity. Since firms must finance wage payments with cash receipts carried over from last period, the marginal cost of hiring labor, and hence the markup, is increasing with the inflation rate μ. As $\mu \rightarrow \beta$-1 and $\gamma \rightarrow \infty$, the conventional result emerges in that the inverse markup approaches the marginal productivity of labor.

The firm chooses an optimal sequence $\{p_t^{\prime}, \hat{c}_t, \hat{l}_t\}$ solving (15), (17'), and (18) given prices and wages $\{p_t, w_t\}$. Labor and money market clearing implies that $l_t = \hat{l}_t$ and that $\hat{m}_t = \hat{m}_{t+1} = 1$. We now characterize the steady state of the economy's equilibrium in which no firm would deviate.

Definition 1: A symmetric steady-state monetary equilibrium is given by quantities $\{c^*, l^*, s^*\}$ and prices $\{p^*, w^*\}$ satisfying

$$p^* = \left(\frac{\gamma}{\gamma - 1}\right)\frac{w^*(1+\mu)}{\beta f^{\prime}(l^*)} \tag{19}$$

$$c^* = \frac{1+\theta\mu}{\alpha(s^*)p^*} \tag{20}$$

$$\frac{w^*}{p^*}\alpha(s^*)^{\frac{1}{\gamma-1}} = \frac{U_L(D^*,L^*)}{U_D(D^*,L^*)} \tag{21}$$

$$\frac{w^*}{p^*} = (\frac{\gamma}{\gamma-1})\alpha'(s^*)c^* \tag{22}$$

where $w^* = 1/l^*$, $L^* = 1 - l^* - s^*$, and

$$D^* = \alpha(s^*)^{\frac{\gamma}{\gamma-1}}c^* = \alpha^{\frac{1}{\gamma-1}}(\frac{1+\theta\mu}{1+\mu})(\frac{\gamma-1}{\gamma})\beta f'(l^*)l^* \tag{23}$$

Notice that a convenient way of expressing condition (21) is by substituting in (19), (20) and (21) and writing it in terms of the ratio of the elasticity of substitution of leisure to composite consumption

$$\Gamma(D^*,L^*)\frac{l^*(1+\theta\mu)}{1 - l^* - s^*} = 1 \tag{24}$$

where $\Gamma \equiv \xi_L/\xi_D$ and $\xi_L = U_L L$, $\xi_D = D_D D$.

Properties Steady State Equilibria

This section analyzes the existence of steady state equilibria and investigates the model's steady state implications for money growth, inflation, and real activity. First, we will consider equilibria with a fixed shopping effort and matching rate. Then we consider the general model which allows shopping effort to vary optimally.

For convenience, and to make our analysis more concrete, we will adopt some specific functional forms for preferences and technology. In particular, let $f(l) = l^{\phi}$, consider a linear matching technology $\alpha(s) = \alpha_0 + \alpha_1 s$, $\alpha_0, \alpha_1 \geq 0$, and let preferences be given by $U(D,L) = [\eta D^{\rho} + (1-\eta)L^{\rho}]^{1/\rho}$, where $\eta \in (0,1)$ and $\rho \in [0,1]$. This CES specification embodies both the linear case where $\rho \to 1$ and the Cobb Douglas case where

$\rho \to 0$. It implies that the elasticity of substitution ratio is given by $\Gamma = \{(1-\eta)/\eta\}(L/D)^{\rho}$. With this, condition (24) is given by

$$\frac{1-\eta}{\eta}\frac{l^*(1+\theta\mu)}{(D^*)^{\rho}(1 - l^* - s^*)^{1-\rho}} = 1 \tag{25}$$

Equilibria with a Fixed Matching Technology

In order to highlight the essential role of endogenous shopping effort and matching, we begin by considering the case where the matching rate is fixed at $\alpha = \alpha_0 > 0$ and hence $s^* = 0$. Then we have:

Proposition 1. Given $\alpha = \alpha_0$, there exists a unique steady state equilibrium $\{c^*, p^*, l^*, D^*\}$ solving (19), (20), (21) and (23).

Proof: From (25) a sufficient condition for this is that (l/D^{ρ}) is strictly increasing in l. Substituting in (23) gives

$$(l/D^{\rho}) = \frac{l^{1-\rho}(1+\mu)}{\alpha_0^{\gamma/(\gamma-1)}(1+\theta\mu)\beta f'(l)}$$

Thus, $\partial(l/D^{\rho})/\partial l > 0$ and there exists a unique l^* satisfying (24). With this, (22), gives D* and (19) and (20) gives $p^* = [\gamma/(\gamma-1)]\,[(1+\mu)/\beta f'(l^*)]$ and $c^* = [(\gamma-1)/\gamma]\,[(1+\theta\mu)/(1+\mu)]\,[\beta f'(l^*)l^*/\alpha_0]$. ■

Consider now the impact of an exogenous increase in the matching rate α_0 and money growth rate μ:

Proposition 2. (*Effect of Trade Frictions and Money Growth)* The pure monetary equilibrium with $\alpha = \alpha_0$ possesses the following properties:

(i) $\partial l^*/\partial\alpha_0 > 0$, $\partial D^*/\partial\alpha_0 > 0$, $\partial(w/p)^*/\partial\alpha_0 < 0$, and $\partial p^*/\partial\alpha_0 < 0$;

(ii) $\partial l^*/\partial\mu < 0$, $\partial D^*/\partial\mu < 0$, $\partial(w/p)^*/\partial\mu < 0$, and $\partial p^*/\partial\mu > 0$.

Proof: To prove part (i), we substitute (23) into (25) to yield the equilibrium locus determining l^*:

$$\frac{1-\eta}{\eta}(1+\theta\mu)^{1-\rho} = \left\{\alpha_0^{1/(\gamma-1)}\left(\frac{\gamma-1}{\gamma}\right)\frac{\beta f'(l)}{1+\mu}\right\}^{\rho}\left\{\frac{1-l}{l}\right\}^{1-\rho} \tag{26}$$

Since the right hand side of (26) is strictly decreasing in l and increasing in α_0, $\partial l^*/\alpha_0 > 0$. From (23), D is increasing in both α_0 and l so that $\partial D^*/\alpha_0 > 0$. From (19) $w^*/p^* = [\gamma/(\gamma-1)]\,\beta f'(l^*)/(1+\mu)$ and thus $\partial(w/p)^*/\alpha_0 < 0$. Finally, since $f'(l)l$ is increasing in l, $\partial p^*/\alpha_0 < 0$.

For part (ii), it is immediate from (26) that a higher money growth rate increases the left hand side while reducing the right hand side. As the right hand side of (26) is strictly decreasing in l, it must be that $\partial l^*/\partial\mu < 0$. Since D is increasing in l from (22), $\partial D^*/\mu < 0$. From (19) a decreasing l^* implies a higher nominal wage and lower marginal product and this gives $\partial p^*/\mu > 0$. From (21) and CES preferences, note that $(w/p)^* = [(1-\eta)/\eta][D^*/(1-l)]^{1-\rho}$; as μ discourages D* and work effort, $\partial(w/p)^*/\mu < 0$. ■

Intuitively, an increase in the matching rate increases the marginal benefit of wage income, as it is able to purchase more consumption variety. This shifts labor supply out and lowers the equilibrium real wage. The resultant increase in equilibrium work effort and matching rate increases real incomes and composite consumption.

Moreover, money growth creates an inflation tax effect which, for a given matching rate, decreases both labor demand and supply and equilibrium work effort. Real money balances used to finance labor declines and lower real incomes reduces composite consumption. For $\theta > 0$, beginning-of-period cash transfers to households create a positive wealth effect which reinforces the decline in work effort. The inflation tax effect on work effort is consistent with many standard general equilibrium models which predict a negative relationship between inflation and market activity. However, as we shall see below, the ability of traders in the economy to affect the "frequency" of exchange opportunities and the extent of search frictions can drastically change the equilibrium characterization in the steady state and even the real effects of inflation.

Equilibria with Endogenous Shopping Effort

Having endowed with the findings in an economy with exogenous shopping effort and a fixed matching rate, we are now ready to characterize pure monetary equilibrium in the general model outlined in Section II, where $\alpha = \alpha(s) = \alpha_0 + \alpha_1 s$. For a given shopping time allocation s, equation (25) corresponds to an efficiency condition for optimal work effort. Substituting (23) into (25) gives the LL locus:

$$\frac{1-\eta}{\eta}(1+\theta\mu)^{1-\rho}(1+\mu)^{\rho} = \left\{\alpha(s)^{1/(\gamma-1)}\left(\frac{\gamma-1}{\gamma}\right)\beta f'(l)\right\}^{\rho}\left\{\frac{1-l-s}{l}\right\}^{1-\rho} \tag{27}$$

For a given work effort allocation l, equation (22) corresponds to an efficiency condition for optimal shopping effort. Substituting (20) into (222) gives the SS locus:

$$l = \frac{\alpha(s)}{\alpha'(s)}\frac{\gamma-1}{\gamma(1+\theta\mu)} \tag{28}$$

A steady state can be characterized by $\{l^*, s^*\}$ satisfying (27) and (28). These conditions lead to the following:

Lemma 1. (*Characterization of LL and SS Loci)*

(i) For $\rho \geq 0$ sufficiently small, $\mathrm{d}l/\mathrm{d}s\big|_{LL} < 0$, for $\rho = 1$, $\mathrm{d}l/\mathrm{d}s\big|_{LL} > 0$, and there exists values of $0 < \rho < 1$ such that $\mathrm{d}l/\mathrm{d}s\big|_{LL} > 0$ for $s < \bar{s} < 1-l$ and $\mathrm{d}l/\mathrm{d}s\big|_{LL} < 0$ for $\bar{s} < s < 1$.

(ii) The SS locus is strictly increasing in the (s,l) space: $\mathrm{d}l/\mathrm{d}s\big|_{SS} > 0$.

Proof: See the Appendix. ■

The LL locus denotes the optimal response of work effort to a change in shopping effort. For ρ sufficiently large, a greater substitutability between composite consumption and leisure implies that an exogenous increase in s raises the marginal benefits of work effort and causes a substitution towards composite consumption. For ρ sufficiently small, less substitutability between composite consumption and leisure implies that an exogenous increase in s will actually reduce incentives for work effort as households substitutes towards leisure. The SS locus denotes the optimal response of shopping effort to a change in work effort. An exogenous increase in work effort lowers the marginal benefit of labor supply and, at the optimum, this must be equated with the marginal benefits of shopping effort. Since consumption per type, c^*, is strictly decreasing in s, an increase in shopping effort is necessary.

In light of these properties, we can divide the characterization of equilibria into several cases and analyze the effects of search frictions and inflation for each.

Proposition 3. Given $\rho > 0$ sufficiently small, there exists a unique steady state equilibrium $\{l^*, s^*\}$ satisfying:

(i) $\partial l^*/\partial \alpha_0 > 0$ and $\partial s^*/\partial \alpha_0 < 0$;

(ii) $\partial l^*/\partial \mu < 0$ for any value of θ and $\partial s^*/\partial \mu < 0$ for θ sufficiently small.

Proof: See the Appendix. ■

The uniqueness of steady state equilibria comes from Lemma 1, which verified that the SS-locus is strictly upward sloping while for $\rho \geq 0$ sufficiently small, the LL locus is strictly downward sloping. Intuitively, a reduction in trade frictions, as captured by an increase in α_0, generates a positive wealth effect that causes households to lower shopping effort and a substitution effect towards work effort. The SS locus shifts upwards in the (s,l) plane as shown in Figure 1A. For $\rho > 0$, the LL locus also shifts upwards, re-inforcing the positive impact on work effort but mitigating somewhat the negative impact on shopping time. Consequently, there is an increase in composite consumption and real balances, and real wages decline from the increase in labor supply.

Figure 1B illustrates the unique steady state and shows that an increase in μ shifts both the LL and SS loci downward. Both the inflation tax effect on work effort and the substitution away from work towards shopping time lowers equilibrium work effort, but the impact on steady state shopping effort becomes ambiguous. Notice that if $\theta = 0$ and $\rho = 0$, both l^* and s^* are invariant to the money growth rate, and if θ

is sufficiently small while ρ is small but positive, then steady state shopping time also falls as households substitute away from market activity.[13]

Proposition 4. For $\rho = 1$, there exists a unique steady state $\{l^*, s^*\}$ satisfying:

(i) $\partial s^*/\partial\alpha_0 < 0$ and $\partial l^*/\partial\alpha_0 = 0$;

(ii) $\partial s^*/\partial\mu < 0$ and $\partial l^*/\partial\mu < 0$ for $\gamma\text{-}1 > 1/(1\text{-}\phi)$;

(iii) $\partial s^*/\partial\mu > 0$ and $\partial l^*/\partial\mu > 0$ for $1 < \gamma\text{-}1 < 1/(1\text{-}\phi)$.

Proof: See Appendix. ■

Recall that with ρ sufficiently large, both the SS and LL locus are upward sloping. An increase in α_0 tends to reduce the optimal choice of *s* for a given *l*, shifting the SS locus upward in the (*s*,*l*) plane. This is the pure wealth effect of the improved matching technology . However, it also increases the optimal choice of *l* given *s*, shifting the LL locus upward. While both effects lead to a reduction in shopping time, the impact on work effort depends upon whether or not the substitution effect of α_0 outweighs the wealth effect. In the linear example where $\rho = 1$, these effects exactly cancel and there is no overall change in either the overall matching rate, $\alpha(s)$, or equilibrium work effort (see Figure 2).

A greater money growth rate lowers work effort for a given shopping effort, shifting the LL locus downward in the (*s*,*l*) plane. This is the negative wealth effect of the inflation tax. For γ sufficiently large, equilibria occrs where the SS locus is steeper LL and the decline in work effort lowers the marginal incentives to invest in shopping effort (see Figure 3B illustrates the case where $\theta = 0$ for convenience). Notice that from (23) composite consumption D* falls, and, since U_L/U_D is a constant, (21) implies an increase in real wages as the marginal product of labor rises. Intuitively, if the preference for variety is small, and hence search frictions are not important, inflation decreases investment in shopping time, employment,

[13] Higher money growth with $\theta > 0$ will create a positive wealth effects to households; however, this is an artifact of the feature that households are not required to hold cash across periods. Since such wealth effects are likely to be small in practice, we interpret θ sufficiently small as a more “natural” specification of the model.

and economic activity.

However, for γ sufficiently small, steady state occurs where the LL locus will be steeper than SS and the decline in work effort creates a substitution towards shopping effort. Consequently, the resulting increase in the matching rate increases the marginal benefits of wage income and the incentive to increase labor supply. It is precisely this positive feedback which can lead to an overall increase in work effort and employment (see Figure 3A for the case where $\theta = 0$). Intuitively, if the preference for variety is large, and hence search frictions are important, then inflation can increase shopping time, employment, and economic activity. In this case an increase in the money growth rate increases composite consumption, D*, and lowers the real wage rate. This result is in stark contrast to the model which simply assumes a fixed matching rate.

Proposition 5. *(Multiple Equilibria)* For $0 < \rho < 1$, multiple (non-degenerate) steady states may emerge, under which

(i) equilibria can be ranked by a monotone increasing relationship between l^* and s^*;

(ii) for θ sufficiently small, sign$\{\partial l^*/\partial\mu\}$ = sign$\{\partial s^*/\partial\mu\}$, and there will be at least one equilibria with $\partial l^*/\partial\mu > 0$ and $\partial s^*/\partial\mu > 0$.

Proof: This proposition can be verified graphically. Consider the case where the LL locus is upward sloping for s small and downward sloping for s large. Since the SS locus is upward sloping and all equilibria must occur along it, (i) is immediate. Since SS is linear, if LL is steeper than SS at the origin, then there is either a unique equilibria [described by Proposition 3 or Proposition 4(ii)] or an odd number of steady states (Figure 4A). Since a higher money growth rate shifts the entire LL locus downward, movement of equilibria along the SS locus implies l^* and s^* must move in the same direction and for $2n$-1, $n \geq 2$, steady states implies that n - 1 of those equilibria will be characterize by $\partial l^*/\partial\mu > 0$ and $\partial s^*/\partial\mu > 0$. If SS is steeper than LL close to the origin, there is at least two steady states or, in general, an even number (Figure 4B). Again, a higher money growth rate shifts SS downward, implying that l^* and s^* must move together. Furthermore, for every equilibria where $\partial l^*/\partial\mu < 0$ and $\partial s^*/\partial\mu < 0$, there exists one where $\partial l^*/\partial\mu > 0$ and $\partial s^*/\partial\mu > 0$. ■

The possibility of multiple equilibria arises from the complementarity between the optimal choices of work and shopping effort. Intuitively, a greater matching rate enlarges consumption variety and encourages work effort when goods and leisure are highly substitutable. On the other hand, the higher labor income generated from work effort encourages demand for variety and induces shopping time to increase. When the elasticity of substitution between goods and leisure is moderately high and the preference for consumption variety is sufficiently great, there exists nonlinear positive interactions between employment and shopping time which generate a multiplicity of non-degenerate steady states.

As an illustration of the existence of multiple equilibria, consider $\gamma = 2$, $\eta = 0.4$, $\rho = 0.8$, $\phi = 0.8$, $\alpha_0 = 0$, $\alpha_1 = 8$, $\beta = 0.99$. Figure 6 plots the roots of (27) as a function of l, where s has been substituted out from (28). For $\mu = 0$, it indicates that there is a low output equilibria, where $l = 0.104$ and $s = 0.208$ and a high output equilibria, where $l = 0.313$ and $s = 0.626$. Raising the inflation rate to $\mu = 0.10$ increases work and shopping effort in the low output equilibria to 0.125 and 0.256 and reduces work and shopping effort in the high output equilibria to 0.298 and 0.609.

One may now wonder in cases of multiple equilibria if some are stable and some are not. Obviously, by the nature of the LL and SS loci, stability cannot be analyzed graphically since the evolution of quantities (consumption and labor allocation) and prices (contemporaneous goods prices, wages and the intertemporal price) cannot be simply summarized in (s, l)-space. Moreover, even one may analyze those quantity and price dynamics, the complete analysis cannot be done with including the change of values in the Bellman equations. If one does so, then, by the nature of self-fulfilling expectations, there is always a bubble-type hyperinflationary transition path leading to the barter equilibrium. Thus, the only alternative remaining is to apply the Samuelson Correspondence Principle to examine the *ad hoc* dynamics by Liaponov functions. In this latter sense any (and indeed all) equilibria described in Proposition 5 may be stable. Thus, we have proceeded with discussion of the comparative-static properties of each type of equilibria.

Notice that these results differ dramatically from those obtained in the case of exogenous shopping effort and matching. In particular, the introduction of endogenous shopping and matching (i) permits the possibility of multiple equilibria and (ii) generates ambiguity concerning the effect of money growth on work effort. We also want to call the reader's attention that preference over variety is essential for the emergence of multiple non-degenerate steady states. When differentiated goods are perfect substitutes (i.e., $\gamma \to \infty$), it is clear from (26) that the positive feedback effect between work and shopping effort via matching (α) vanishes. In this case, the LL locus is always downward-sloping, regardless of the degree of substitution between goods and leisure. This, together with the positively sloped SS locus, results in a unique non-degenerate steady-state equilibrium.

III. Firm Entry and Trade Frictions

In this extension, we consider an alternative method of endogenizing the matching technology. Previously, we have always normalized the measure of firms to be unity; this section considers the issue of optimal firm entry and how it's interaction with the matching technology affects the properties of steady state equilibria. To isolate this effect, we fix the shopping time allocation decision exogenously so that s = 0. We normalize the measure of households to unity and let $N_t \in \mathbb{R}_+$ denote the measure of firms, where each firm possesses production technology (4). The following assumption modifies the matching technology by relating the measure of firms contacted by each household to the total measure of firms participating in the market.

Assumption 4': (*Matching Technology*) The measure of firms contacted by a particular household h is given by $\alpha(N_t)$, where $\alpha'(N_t) > 0$ and $\alpha(0) = 0$.

The money supply process is identical to the previous section, with monetary transfer $T_t = \theta X_t$ given to households and $\hat{T}_t = (1-\theta)X_t$ is given to firms, implying $\hat{T}_t/N_t$ is the transfer per firm. We will focus on symmetric steady states where cash balances will be identical across all firms in each period. Without loss of generality, a money supply process consistent with such a symmetric steady state is one where each entering firm recieves a cash transfer from the central bank in an amount which replicates the money balances of the existing fims in the market. This will allow entering firms to finance their wage payment. The transfer can then be rebated from these entrants in the following period. The transfer and rebate process works analogously for exiting firms. Since households take the matching technology and money supply process as given, the choice of $\{c_t(\omega), c_t(\omega'), l_t, M_{t+1}\}$ maximizing (1) subject to (5) must continue to satisfy the analoguous versions of (12) and (13):

$$c_t(\omega) = \frac{w_t l_t + \mu\theta}{\alpha(N_t)p_t} = c_t \; ; \quad c_t(\omega') = \frac{w_t l_t + \mu\theta}{\alpha(N_t)p_t^{1-\gamma}(p_t')^{\gamma}} = c_t' \tag{29}$$

$$U_L(D,L) = U_D(D,L)\frac{w_t}{p_t}\alpha(N_t)^{\frac{1}{\gamma-1}} \tag{30}$$

The deviating firm chooses $\{\hat{c}_t(\omega'), \hat{l}_t(\omega'), p_t(\omega'), \hat{m}_{t+1}\}$ to maximize (3) subject to (17'),

$$f[\hat{l}_t(\omega')] - \frac{\alpha(N_t)}{N_t}c_t(\omega') - \hat{c}_t(\omega') \geq 0$$

$$\hat{m}_{t+1} = \frac{\hat{m}_t + \alpha(N_t)p_t(\omega')c_t(\omega')/N_t - w_t\hat{l}_t(\omega') + (1-\theta)\mu/N_t}{1+\mu}$$

where nominal variables have once again been scaled by the aggregate beginning-of-period money stock M_t^s. Notice that this alteration in the matching technology scales both the marginal benefits and costs of setting a particular deviating price by equal proportions so that the symmetric Nash optimal pricing rule continues to be given by (18).

To close the model, firm entry is introduced by imposing a fixed per-period firm entry cost of $\kappa > 0$ and allowing the measure of firms to vary subject to an ex-post zero profit condition given by $\hat{c}(\omega') = \kappa$ or

$$f[\hat{l}_t(\omega')] - \frac{\alpha(N_t)}{N_t}c_t(\omega') = \kappa \tag{31}$$

Labor and money market clearing implies that $l_t/N_t = \hat{l}_t$ and $\hat{m}_t = 1/N_t$. Thus, a *symmetric steady state equilibrium with firm entry*, with p = p'= p* and c = c' = c* is given by {c*,l*,N*,m*} and prices {p*,w*} satisfying (29), (30), (18), (31), m* = 1/N*, and w* = 1/l*. Substituting (29) and (18) into (30) gives us an LL-locus indicating the optimal choice of l for a given measure of firms N:

$$\frac{1-\eta}{\eta}(1+\theta\mu)^{1-\rho}(1+\mu)^{\rho} = \left\{\alpha(N)^{1/(\gamma-1)}(\frac{\gamma-1}{\gamma})\beta f'(\frac{l}{N})\right\}^{\rho}\left\{\frac{1-l}{l}\right\}^{1-\rho} \tag{32}$$

Substituting (29) and (18) into (31) gives an EE-locus indicating the optimal firm entry condition for a given household choice of l:

$$f(\frac{l}{N}) - \frac{l}{N}f'(\frac{l}{N})\beta\frac{1+\theta\mu}{1+\mu}\frac{\gamma-1}{\gamma} \tag{33}$$

Thus, a steady state can be characterized as a pair {N*,l*} satisfying (32) and (33). We again adopt the Cobb-Douglas form for the production technology and a linear matching technology where $\alpha(N) = \sigma N$, $\sigma < 1$. With this, it is immediate that the EE-locus is increasing and linear in the (N,l) space as (33) can be written as:

$$N = \left\{\frac{1}{\kappa}[1 - \phi\beta(\frac{1+\theta\mu}{1+\mu})(\frac{\gamma-1}{\gamma})]\right\}^{\frac{1}{\phi}} l \tag{34}$$

The following proposition characterizes the LL-locus:

Lemma 2. *(Characterization of LL Locus)* The LL-locus possesses the following properties:

(i) for $\rho = 0$, $dl/dN|_{LL} = 0$ and $l^* = [1+(1-\eta)(1+\theta\mu)/\eta]^{-1}$;

(ii) for $\rho = 1$, $dl/dN|_{LL} > 0$, $l(N) \rightarrow 0$ as $N \rightarrow 0$, and the LL-locus is strictly convex in the (N,l) space;

(iii) for $0 < \rho < 1$, $dl/dN|_{LL} > 0$, $l(N) \rightarrow 0$ as $N \rightarrow 0$, and $l(N) \rightarrow 1$ as N becomes arbitrarily large.

Proof: See Appendix. ■

Lemma 2 implies that for the Cobb-Douglas case, where the ratio of the elasticity of substitution between consumption and leisure is independent of the matching technology, steady state work effort l^* will be constant and independent of N. For all $0 < \rho \leq 1$, an increase in the measure of firms increases the set of firms households are matched with and hence enhances product variety. This increases the opportunity cost of leisure to consumption and households substitute towards work effort. The following propositions now characterize the properties of steady state equilibria.

Proposition 5. For $\rho = 0$, there exists a unique steady state equilibrium satisfying:

(i) $\partial l^*/\partial\mu < 0$;

(ii) $\partial N^*/\partial\mu > 0$ for θ sufficiently small and $\partial N^*/\partial\mu < 0$ for θ sufficiently large.

Proof: See Appendix. ■

The uniqueness of the steady state is immediate since, from Lemma 2, the LL locus is horizontal while the EE locus is upward sloping. Again, for $\theta > 0$, it is the positive wealth effect on households from cash transfers which lowers steady state work effort. The inflation tax effect on work effort is absent in the Cobb-Douglas case where the ratio of the elasticity of substitution of leisure to consumption is constant. This is immediate seen by noting that as $\theta \rightarrow 0$, work effort becomes invariant to the money growthrate. However, in the case where θ is sufficiently small, a higher money growth rate raises the firm's markup of prices over marginal costs, encourages firm entry, and expands the steady state measure of firms.

Proposition 7. For $\rho = 1$, there exists a unique steady state where $\partial l^*/\partial\mu > 0$ and $\partial N^*/\partial\mu > 0$.

Proof: See Appendix. ■

Existence of a unique steady state arises from the increasing and convex LL locus. The impact of an increase in the money growth rate is illustrated in Figure 6. Just as in the Cobb-Douglas case, a higher money growth rate increases the monopolist mark-up and leads to firm entry, for a given choice of work effort per household. This is represented by a shift of the EE locus to the right. Furthermore, money growth now generates an inflation tax effect which discourages work effort and causes households to substitute towards leisure. This shifts the LL locus to the right. Consequently, if leisure and composite consumption are very close substitutes, the firm entry effect and it's positive impact on the matching technology dominates this inflation tax effect. Thus, as trade frictions are reduced, the result is greater steady state work effort and firm entry. While labor input per firm (l^*/N^*) diminishes, aggregate real activity increases as measured by $N^*f(l^*/N^*)$ increases.

Proposition 8. Suppose $0 < \rho < 1$. Then

(i) for $\gamma > 1/(1-\rho)$ there exists a unique non-degenerate steady state where $\partial l^*/\partial\mu < 0$, and for θ sufficiently large $\partial N^*/\partial\mu < 0$;

(ii) for $\gamma < 1/(1-\rho)$ there exists two steady states $\{l_1^*, N_1^*\}$ and $\{l_2^*, N_2^*\}$, with $l_1^* < l_2^*$ and $N_1^* < N_1^*$, such that $\partial l_1^*/\partial\mu > 0$, $\partial l_2^*/\partial\mu < 0$, $\partial N_1^*/\partial\mu > 0$, and for θ sufficiently large $\partial N_2^*/\partial\mu < 0$.

Proof: See Appendix. ■

The uniqueness of steady state requires that the preference for variety is small (γ large), or consumption and leisure are not very close substitutes (ρ small). In this case, the LL locus is strictly concave in (N,l) space and the unique steady state is illustrated in Figure 8. An increase in the money growth rate leads households to substitute away from work effort and shifts the LL locus downward. The higher monopolist mark-up leads firms to enter the market for a given per household choice of labor supply, and shifts the EE-locus downward. The overall impact is a decline in steady state work effort, but the impact on steady state firm entry depends upon the share of the monetary transfer going to firms. If this share is sufficiently small (θ large), then the wealth effect on households reinforces the negative impact on work effort and leads to an overall decline in the measure of firms.

However, if there is a sufficient preference for variety ($\gamma > 1$ small) or consumption and leisure are highly substitutable in household utility ($\rho < 1$ sufficiently large), then there will be multiple (two) equilibria. As shown in Figure 7, these equilibria can be ranked by real activity, since along the EE locus, labor per firm is constant. Intuitively, preferences for variety and the importance of trade frictions are necessary in generating multiple equilibria. All else being equal, higher inflation again causes households to substitute towards leisure, shifting the LL locus downward, and encourages firm entry, shifting EE downward. The overall impact on the steady state will thus differ across equilibria. For the low activity equilibria, both work effort and the measure of firms increases while for the high activity equilibria, work effort declines and if there is a sufficiently large wealth effect to households from the cash transfer, firm entry will decline as well.

We can also analyze the impact of a direct reduction in search frictions, captured by increasing the fraction of firms contacted per household, σ, on real activity. Notice from (32) that an increase in σ increases the work effort choice and shifts the LL locus upwards. In the unique steady state case where $\gamma > 1/(1-\rho)$, we have $\partial l^*/\partial\sigma > 0$ and $\partial N^*/\partial\sigma > 0$. For $\gamma < 1/(1-\rho)$, $\partial l_1^*/\partial\sigma < 0$, $\partial l_2^*/\partial\sigma > 0$, $\partial N_1^*/\partial\sigma < 0$, and $\partial N_2^*/\partial\sigma > 0$. This suggests that for the multiple equilibrium case the impact of an increase in the productivity of the matching technology depends upon the level of real activity. If the number of firms matched to each household is sufficiently large, as in the low activity equilibria, then a substitution effect dominates and households shift towards work effort and market activity. Notice when $\gamma < 1/(1-\rho)$ and $0 < \rho < 1$, the properties of each of the two steady states correspond to the unique steady state cases analyzes previously. In particular, the properties of the low activity equilibria are analogous to the unique steady state case when $\rho = 1$. Also, the properties of the high activity equilibria are analogous to the unique steady which arises with $\gamma > 1/(1-\rho)$ and $0 < \rho < 1$.

An overall examination of the cases of endogenous shopping effort and endogenous firm entry indicates that the main findings are qualitatively similar. In either case, multiple equilibria may emerge, while monetary growth may have positive effects on work effort. However, the endogenous entry model has an advantage in terms of welfare analysis. Specifically, by free entry, firms always reach zero profit in equilibrium and as a concequence, welfare evaluation only depends on consumer's lifetime utility. Staighforward manipulations show that consumer's welfare may be hump-shaped. When the money growth rate is low, the positive endogenous entry effect dominates the standard negative purchasing power effect via consumption reduction. Thus, an increase in the money growth rate (or anticipated inflation rate) leads to an improvement in welfare. When the money growth rate is high, the negative purchasing power effect becomes the dominant factor and a negative relationship between money growth and welfare is therefore anticipated. This Laffer-curve shaped welfare schedule with respect to changes in money growth suggests that in the presence of trade frictions and endogenous firm entry, there may be a positive rate of optimal inflation and the Friedman rule need not hold up in general.

IV. Conclusion

This paper has investigated implications of a multiple matching model of money for the effects of monetary growth and inflation on economic activity. The use of a multiple matching technique, where search frictions are captured by limited consumption variety, allows the model to generalizes various aspects of the traditional money-search literature, including price determination and the divisibility and storability of goods and money. We find that in both the basic framework with endogenous shopping effort and in an alternative specification where frim entry is endogenized, trade frictions are an important element in generating multiple steady states and providing a channel by which monetary growth positively influences real activity. These features exist in the absence of increasing returns of the matching technology or search externalities.

This finding is also complementary to some earlier work by Li (1994, 1995) evaluating the consequences of inflation in search-theoretic models of money. In a fixed price indivisible search model of money of Kiyotaki and Wright (1993), these papers concluded that a tax on money balances can indeed positively influence search activity, stimulate the accumulation of inventories, and increase welfare. Our paper suggests that these conclusions may not have been just an artifact of the indivisible nature of fiat money and inventory restrictions assumed by these models and are robust to generalizations to the search environment.

The possibility of multiple steady states suggests that the lack of strong empirical evidence supporting a positive or negative impact of steady inflation in industrialized countries may be the result of an economy in transition across multiple equilibria. However, our steady state analysis cannot address the stability of these steady states and the transitional dynamic response to changes in the money growth rate. Given our results, this undertaking appears to be a fruitful avenue for future work.

This paper demonstrates that search theoretic models of money with multiple matching can be extended and applied to a wide variety of issues in monetary economics. For example, a saving and investment decision can be incorporated into the model by the inclusion of productive capital. Such a framework can then begin to address the issue of how monetary growth impacts the capital stock and whether there exists a Mundell-Tobin effect. Also, the model can be useful in analyzing the extent by which inflation distorts relative prices across markets with varying degrees of search frictions. Finally, an incorporation of a credit market will allow the model to study the transmission of monetary policy, whether such a model can capture the liquidity effects of monetary shocks, and their implications for the cyclical behavior of real variables.

REFERENCES

Aiyagari, S. Rao and Zvi Eckstein (1994). "Interpreting Monetary Stabilization in a Growth Model with Credit Goods Production." Working Paper 525, Federal Reserve Bank of Minneapolis

Brock, W. (1975). "A Simple Perfect Foresight Monetary Model." *Journal of Monetary Economics* 1: 133-150.

Bullard, J. and J. Keating (1995) "The Long-run Relationship Between Inflation and Output in Postwar Economies." *Journal of Monetary Economics*, 36: 477-96.

Corbae, D. and G. Camera (1996). "On the Distribution of Money and Prices with Search," mimeo, University of Iowa.

Cooley, Thomas F. and Gary D. Hansen (1989). "The Inflation Tax in a Real Business Cycle Model." *American Economic Review* 79: 733-48.

Fischer, S. (1983). “Inflation and Growth” NBER Working Paper 1235.

Diamond, P. (1982). "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 89, 881-94.

———, (1984): "Money in Search Equilibrium," *Econometrica*, 52, 1-20.

Diamond, P. and J. Yellen (1990). "Inventories and Money Holdings in a Search Economy," *Econometrica*, 58(4), 929-950.

Dixit, A. and Stiglitz, J., (1977). "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 297-308.

Drazen, Allen (1981). "Inflation and Capital Accumulation under a Finite Horizon." *Journal of Monetary Economics* 8: 247-260.

Fuerst, T. S. (1992). "Liquidity, Loanable Funds, and Real Activity." *Journal of Monetary Economics* 29: 3-24.

Green, E. and R. Zhou (1995). "A Rudimentary Model of Search with Divisible Money and Prices," Working Paper, Federal reserve Bank of Minneapolis.

Jones, L. and R. Manuelli (1995). “Policy Uncertainty and Informational Monopolies: The Case of Monetary Policy.” Working Paper, University of Wisconsin.

Kiyotaki, N.and R. Wright (1989). "On Money as a Medium of Exchange," *Journal of Political Economy*, 97, 927-954.

———, and ——— (1991). "A Contribution to a Pure Theory of Money," *Journal of Eonomic Theory*, 53, 215-235.

———, and ——— (1993). "A Search-Theoretic Approach to Monetary Economics," *American Economic Review*, 83, 63-77.

Laing D., Li, V.E., and P. Wang (1997). "Money and Prices in a Model of Search and Monopolistic Competition." Manuscript, Penn State University.

Li, V.E. (1994). "Inventory Accumulation in a Search Based Monetary Economy," *Journal of Monetary Economics*, 34, 511-36.

———,(1995). "The Optimal Taxation of Fiat Money in Search Equilibrium," *International Economic Review*, 36(4), 927-942.

Lucas, R. E. Jr. (1982). "Interest Rates and Currency Prices in a Two Country World." *Journal of Monetary Economics* 10: 335-359.

Lucas, R.E. Jr., (1990). "Liquidity and Interest Rates," *Journal of Economic Theory*, 50, 237-64.

McCallum, B. and M. Goodfriend (1988). "Theoretical Analysis of the Demand for Money." *Economic Review*, Federal Reserve Bank of Richmond (Jan/Feb): 16-24.

Molico, M., (1996). "The Distribution of Money and Prices in Search Equilibrium," mimeo, University of Pennsylvania.

Mundell, Robert A. (1963). "Inflation and Real Interest." *Journal of Political Economy* 71: 280-83.

Rogers, J., and P. Wang (1993). "Sources of Fluctuations in Relative Prices: Evidence from High Inflation Countries. *Review of Economics and Statistics* 70: pp. 589-605.

Shi, S. (1994). "Money and Prices: A Model of Search and Bargaining," *Journal of Economic Theory*, 67, 467-496.

———. (1997). "A Divisible Search Model of Fiat Money" *Econometrica*, 65, 75-102.

Sidrauski, M. (1967). "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review* 57: 534-544.

Stockman, Alan C. (1981). "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy." *Journal of Monetary Economics* 8 (November): 387-93.

Trejos, A. and R. Wright (1993). "Search, Bargaining, Money and Prices: Recent Results and Policy Implications," *Journal of Money, Credit, and Banking*, 25(3), 558-576.

———, and ——— (1995). "Search, Bargaining, Money and Prices," *Journal of Political Economy*, 103(1), 118-141.

Tobin, James (1956). "The Interest-Elasticity of the Transactions Demand for Cash." *Review of Economics and Statistics* 38: 241-247.

Wang, P. and C. Yip (1992). "Alternative Approaches to Money and Growth," *Journal of Money, Credit and Banking*, 24: 553-562.

Zhou, R. (1996). "Inventory Holdings of Money and Indeterminacy of Search Equilibrium," mimeo, Federal Reserve Bank of Minneapolis.

APPENDIX

Firm's First Order Conditions

Letting $\hat{\lambda}$ be the multiplier associated with (17') and assuming the constraints are strictly binding, the firm's first order conditions for $\hat{l}_t$ and p_t' can be written as:

$$\frac{\beta V_m(\hat{m}_{t+1})}{1+\mu}[c_t(\omega') + \frac{\partial c_t(\omega')}{\partial p'}p'] = \frac{\partial c_t(\omega')}{\partial p'}$$

$$f'[\hat{l}_t(\omega')] = [\frac{\beta V_m(\hat{m}_{t+1})}{1+\mu} + \hat{\lambda}_t]w_t$$

and where, from (12), $\partial c_t'/\partial p_t' = -\gamma c_t'/p_t'$. The envelope condition is given by

$$V_m(\hat{m}_t) = \frac{\beta V_m(\hat{m}_{t+1})}{1+\mu} + \hat{\lambda}_t .$$

Proofs to Propositions

Lemma 1.

(i) Notice that the left hand side of (27) is independent of l and s. The right hand side of (27) can be written as

$$RHS_{LL} = \left\{\frac{\gamma-1}{\gamma}\beta\phi\right\}^{\rho}(\alpha_0+\alpha_1 s)^{\frac{\rho}{\gamma-1}}\frac{(1 - l - s)^{1-\rho}}{l^{1-\phi\rho}}s^{\frac{\rho}{\gamma-1}}$$

Thus, the right hand side is strictly decreasing in l [$d(RHS_{LL})/dl<0$]. Differentiating this expression with respect to s gives

$$\frac{dRHS_{LL}}{ds} = \left\{\frac{\gamma-1}{\gamma}\beta\phi\right\}^{\rho} l^{\phi\rho-1}(1-l-s)^{1-\rho}(\alpha_0+\alpha_1 s)^{\frac{\rho}{\gamma-1}}[\frac{\rho}{\gamma-1}\frac{\alpha_1}{\alpha_0+\alpha_1 s} - \frac{1-\rho}{1-l-s}] \qquad \text{(A1)}$$

For $\rho = 0$, this expression is strictly negative, implying $dl/ds|_{LL} < 0$, and for $\rho = 1$, this expression is strictly positive, implying $dl/ds|_{LL} > 0$. We also see that (i) if (A1) is positive at $s = 0$, then there exists a unique $\bar{s} < 1-l$, such that (A1) is positive for $s < \bar{s}$, implying $dl/ds|_{LL} > 0$ and negative for $s > \bar{s}$, implying $dl/ds|_{LL} < 0$, and (ii) if (A1)is negative at $s = 0$, which is true for $\rho > 0$ sufficiently small, then (A1) is negative for all $s > 0$, implying $dl/ds|_{LL} < 0$.

(ii) From (28), a sufficient condition for $\mathrm{d}l/\mathrm{d}s\big|_{SS} > 0$ is given by $\alpha''(s) \leq 0$. This is certainly satisfied with our linear matching technology, which implies an increasing and linear SS locus. ■

Proposition 3.

Consider the limiting case where $\rho = 0$, the Cobb-Douglas case. Equations (27) and (28) can be written as

$$\frac{1-\eta}{\eta}(1+\theta\mu) = \frac{1-l-s}{l} \tag{A2}$$

$$l = \frac{(\alpha_0+\alpha_1 s)}{\alpha_1}\frac{\gamma-1}{\gamma(1+\theta\mu)} \tag{A3}$$

Since the (LL) locus in (A2) is downward sloping while the (SS) locus in (A3) in upward sloping in the (s,l) plane, there exists a unique steady state $\{l^*,s^*\}$. From Proposition 4, LL is also strictly downward sloping for $\rho > 0$ sufficiently small which also guarantees uniqueness.

For $\rho = 0$, (A3) indicates that an increase in α_0 increases l and shifts SS upwards. As a result, $\partial s^*/\partial\alpha_0 < 0$, $\partial l^*/\partial\alpha_0 > 0$. For ρ sufficiently small, the LL locus will shift right, re-inforcing the positive impact on l^* while mitigating the negative impact on s^*. From (28), the overall matching rate α increases.

To analyze the effect of an increase in money growth μ, when $\rho = 0$, substituting (A3) into (A2) gives

$$\frac{1-\eta}{\eta} = \frac{(1-s)\alpha_1}{\alpha_0+\alpha_1 s}\left(\frac{\gamma}{\gamma-1}\right) - \frac{1}{1+\theta\mu} \tag{A4}$$

For $\theta > 0$, since the right hand side of (A4) is increasing in μ and decreasing is s, $\partial s^*/\partial\mu > 0$ and from (A2), $\partial l^*/\partial\mu < 0$. For $\theta = 0$, l^* and s^* are invariant with respect to μ. Now consider the case where $\rho > 0$ but small. $\partial s^*/\partial\mu$ will generally be ambiguous but negative for θ sufficiently small.

Proposition 4. Consider the limiting case where $\rho = 1$, the linear specification. Equations (27) can be written as

$$\frac{1-\eta}{\eta}(1+\mu) = \alpha(s)^{1/(\gamma-1)}\left(\frac{\gamma-1}{\gamma}\right)\beta f'(l) \tag{A5}$$

and (28) is given by (A3). Since the right hand side of (A5) is strictly increasing in s and decreasing in l, both the SS and LL locus are upward sloping in the (s,l) plane. By substituting (A3) into (A5) we can verify a unique steady state given by

$$s^* = \{E(1+\mu)(1+\theta\mu)^{\phi-1}\}^{\psi}/\alpha_1 - \alpha_0/\alpha_1 \tag{A6}$$

$$l^* = \{E(1+\mu)(1+\theta\mu)^{\phi-1}\}^{\psi}\left(\frac{\gamma-1}{\gamma}\right)\frac{1}{\alpha_1(1+\theta\mu)} \tag{A7}$$

where $E \equiv [(1-\eta)/\eta][\gamma/(\gamma-1)]^{\phi}/[\beta\phi\alpha_1^{1-\phi}]$ and $\psi \equiv (\gamma-1)/[1-(\gamma-1)(1-\phi)]$.

(i) It is immediate that $\partial s^*/\partial\alpha_0 = -1/\alpha_1 < 0$ and $\partial l^*/\partial\alpha_0 = 0$.

(ii) Consider the case where $\psi < 0$, which is guaranteed for a sufficiently large $(\gamma-1) > 1/(1-\phi)$. From (A6) it is clear that $\partial s^*/\partial\mu < 0$. From (28) it is immediate that $\partial l^*/\partial\mu < 0$.

(iii) Next, consider the case where $\psi > 0$, which is guaranteed for a sufficiently small $(\gamma-1) < 1/(1-\phi)$. From (A6) it is clear that $\partial s^*/\partial\mu > 0$. To analyze the impact on work effort, notice from (A7) that a sufficient condition for $\partial l^*/\partial\mu > 0$ for all θ is given by $\partial l^*/\partial\mu > 0$ for $\theta = 1$. Take the limiting case where $\theta = 1$, the exponential on $(1+\mu)$ becomes $\psi\phi - 1$, which is positive for $(\gamma-1) > 1$, implying $\partial l^*/\partial\mu > 0$, and negative for $(\gamma-1) < 1$, implying $\partial l^*/\partial\mu < 0$.■

Lemma 2.

(i) For $\rho = 0$, (32) can be written as $(1-\eta)(1+\mu)/\eta = (1-l)/l$ and the LL-locus can be written as $l = [1+(1-\eta)(1+\theta\mu)/\eta]^{-1}$.

(ii) For $\rho = 1$, (32) can be written as

$$\frac{1-\eta}{\eta}(1+\mu) = \alpha(N)^{\frac{1}{\gamma-1}}\frac{\gamma-1}{\gamma}\beta f'(\frac{l}{N})$$

$$\Rightarrow \quad l = \left\{\frac{1-\gamma}{\gamma}(1+\mu)\sigma^{-\frac{1}{\gamma-1}}\frac{\gamma}{\gamma-1}\frac{1}{\phi\beta}\right\}^{-1} N^{\frac{\gamma/(\gamma-1)-\phi}{1-\phi}}$$

Thus, $l(0) = 0$, $dN/dl > 0$ and since $[\gamma/(\gamma-1)-\phi]/(1-\phi) > 1$, $d^2N/dl > 0$ and the LL-locus is increasing and strictly convex in the (N,l) space.

(iii) For $0 < \rho < 1$, (32) implies that for $f''<0$ and $\alpha' > 0$, $dl/dN > 0$. Also, since $f' \to 0$ as $l/N \to \infty$ and $f' \to \infty$ as $l/N \to 0$, it is clear that the LL locus passes through the origin in the (N,l) space and $l \to 1$ as $N \to \infty$. ■

Proposition 6. From Proposition 8, $l^* = [1+(1-\eta)(1+\theta\mu)/\eta]^{-1}$, and it is immediate that $\partial l^*/\partial\mu < 0$ for $\theta > 0$. Substituting this into (34) gives the unique steady state value for N^*,

$$N^* = \left\{\frac{1}{\kappa}[1-\phi\beta\frac{\gamma-1}{\gamma}\frac{1+\theta\mu}{1+\mu}]\right\}^{\frac{1}{\phi}}\left[1+\frac{1-\eta}{\eta}(1+\theta\mu)\right]^{-1}$$

For $\theta \to 1$ we have $\partial N^*/\partial\mu < 0$ and for $\theta \to 0$ we have $\partial N^*/\partial\mu > 0$. ■

Proposition 7. From Proposition 8, uniqueness of the steady state is gauranteed by convexity of the LL-locus in (N,l) space. For $\rho = 1$, (32) becomes

$$\frac{1-\eta}{\eta} = N^{\frac{\gamma}{\gamma-1}-\phi} l^{\phi-1} \sigma^{\frac{1}{\gamma-1}} \left(\frac{\gamma-1}{\gamma}\right) \frac{\phi\beta}{1+\mu}$$

Substituting in (34) and simplifying gives

$$\left(\frac{1-\eta}{\eta}\right)\left(\frac{\gamma}{\gamma-1}\right)\frac{k^{\psi}}{\beta\phi} = \left[1-\phi\beta\frac{\gamma-1}{\gamma}\frac{1+\theta\mu}{1+\mu}\right]^{\psi-1}(1+\mu)^{-1}(l\sigma)^{\frac{1}{\gamma-1}} \tag{A8}$$

where $\psi = \gamma/\phi(\gamma-1) - 1$. Notice that for $\theta = 1$, the right-hand side of (A8) is strictly decreasing in μ while increasing in l. Hence $\partial l^*/\partial\mu > 0$. A sufficient condition for this to be true for all $\theta \in [0,1]$ is for the right-hand side of (A8) to be decreasing in μ at $\theta = 0$. To verify this, notice that the RHS of (A8) is proportional to $[1-z/x]^{\psi}/x$, where $x \equiv 1+\mu$ and $z \equiv \phi\beta(\gamma-1)/\gamma$. Differentiating this w.r.t. x gives that $d(RHS)/d\mu < 0$ iff $\psi z/x(1-z.x)^{-1} > 1$ or $\beta < 1+\mu$. This condition is true by supposition that $\mu > \beta - 1$. Therefore $\partial l^*/\partial\mu > 0$ for all θ. From (34) it is immediate that $\partial N^*/\partial\mu > 0$. ■

Proposition 8. The LL locus in (32) can be written as

$$\left(\frac{1-\eta}{\eta}\right)(1+\mu)^{\rho}(1+\theta\mu)^{1-\rho} = \left\{\frac{1-l}{l}\right\}^{1-\rho}\left\{N^{\frac{\gamma}{\gamma-1}-\phi} l^{\phi-1}\sigma^{\frac{1}{\gamma-1}}\left(\frac{\gamma-1}{\gamma}\right)\beta\phi\right\}^{\rho}$$

Substituting in (32) gives

$$\left\{\frac{1-l}{l}\right\}^{1-\rho} l^{\frac{\rho}{\gamma-1}} = (1+\mu)^{\rho}(1+\theta\mu)^{1-\rho}\left\{\frac{1}{\kappa}\left[1-\phi\beta\left(\frac{\gamma-1}{\gamma}\right)\frac{1+\theta\mu}{1+\mu}\right)\right]\right\}^{-\rho\psi}\left\{\sigma^{\frac{1}{\gamma-1}}\beta\phi\frac{\gamma-1}{\gamma}\right\}^{-\rho}\left(\frac{1-\eta}{\eta}\right) \tag{A9}$$

where $\psi = \gamma/\phi(\gamma-1) - 1$. Notice that if $\rho/(\gamma-1) - (1-\rho) < 0$, or $\gamma > 1/(1-\rho)$, the LHS of (A9) is strictly decreasing in l while the RHS is a constant. This verifies the uniqueness of the steady state for γ sufficiently small or ρ sufficiently large. If $\gamma < 1/(1-\rho)$, then it can be verified that the LHS of (A9) is parabolic, zero when $l = 0$, and attains a unique maximum at $[\rho/(\gamma-1) - (1-\rho)]\,[\rho/(\gamma-1)]^{-1} < 1$. Thus, for $\gamma < 1/(1-\rho)$, there exists two steady states $\{l_1^*, l_2^*\}$ where $l_1^* < l_2^*$.

To evaluate the impact of money growth, notice that a sufficient condition for the RHS of (A9) to be increasing in μ for all $\theta \in [0,1]$ is $d(RHS)/d\mu > 0$ for $\theta = 0$. Setting $\theta = 0$, notice that RHS is proportional to $x(1-z/x)^{-\psi}$, where $x \equiv 1+\mu$ and $z \equiv \phi\beta(\gamma-1)/\gamma$. From the proof to Proposition 10, this expression is increasing in μ for $\beta < 1+\mu$, and this condition is true by supposition. Therefore, for $\gamma > 1/(1-\rho)$ where there is a unique steady state, $\partial l^*/\partial\mu < 0$ and, form (34), $\partial l^*/\partial N < 0$ for θ sufficiently large. For $\gamma < 1/(1-\rho)$ where there are two steady states, $\partial l_1^*/\partial\mu > 0$, $\partial l_2^*/\partial\mu < 0$, and from (32) $\partial N_1^*/\partial\mu > 0$, and $\partial N_2^*/\partial\mu < 0$ for θ sufficiently large. ■

FIGURE 1A - Steady State for $\rho > 0$ sufficiently small and Effect of an Increase in α_0

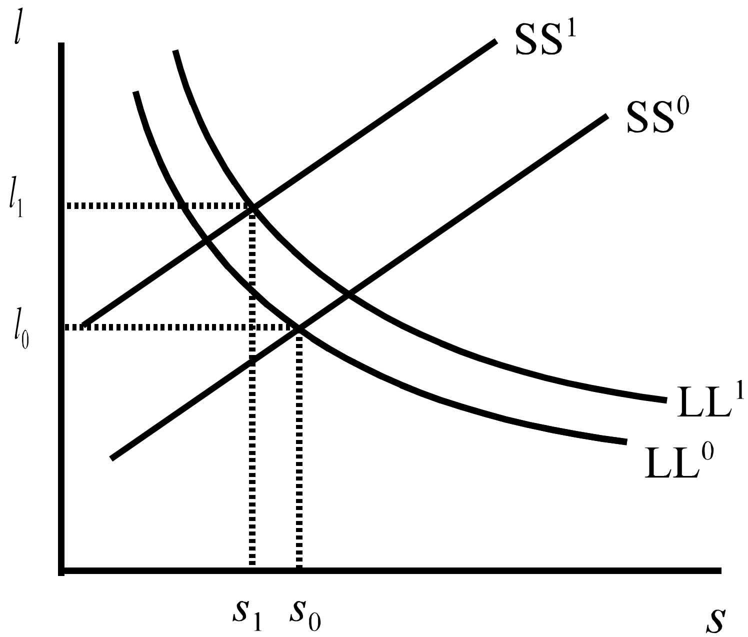

FIGURE 1B - Steady State for $\rho > 0$ sufficiently small and Effect of an Increase in μ

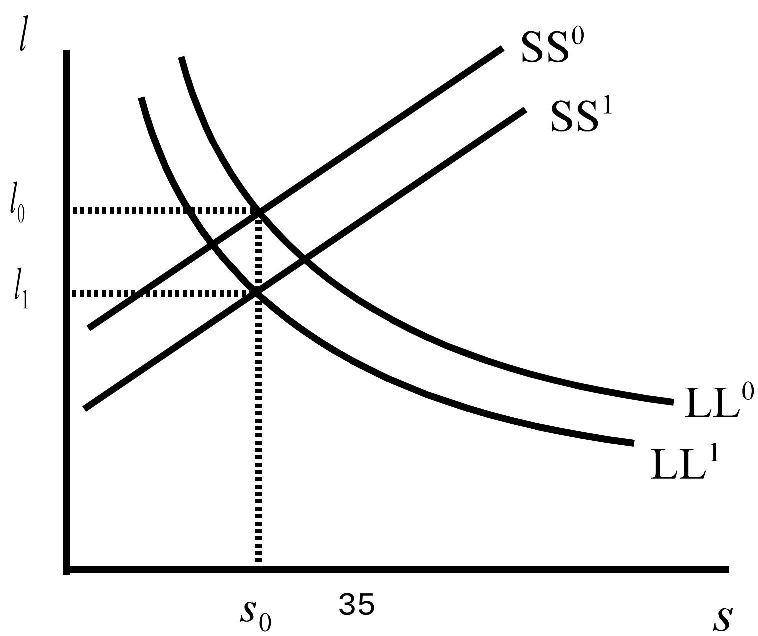

FIGURE 2 -

Steady State for $\rho = 1$

and Effect of an Increase in α_0

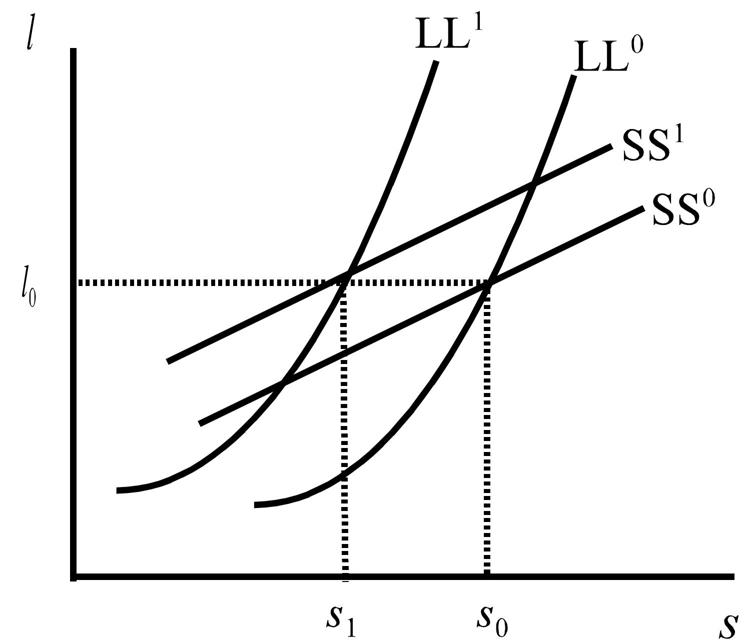

FIGURE 3A - Steady State for ρ = 1, γ-1 < 1/(1-ϕ) and Effect of an Increase in μ

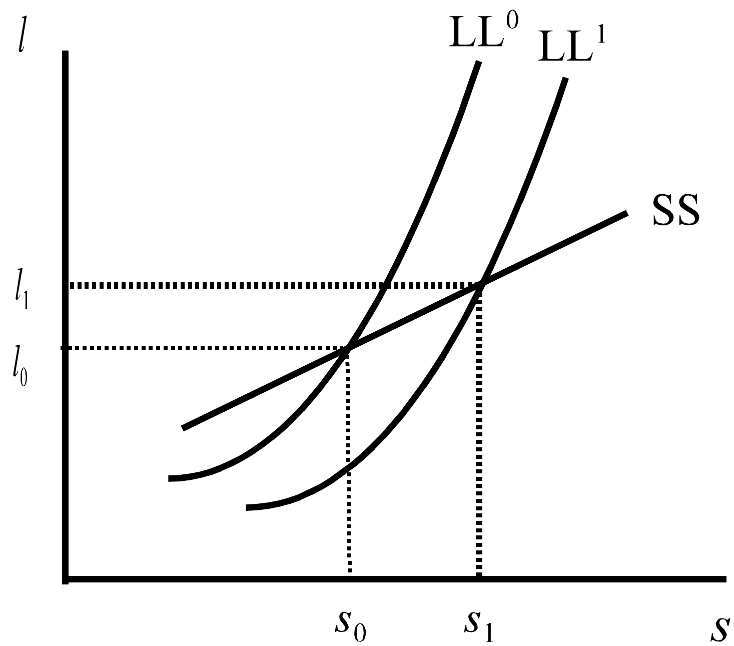

FIGURE 3B - Increase in μ for ρ = 1, γ-1 > 1/(1-ϕ)

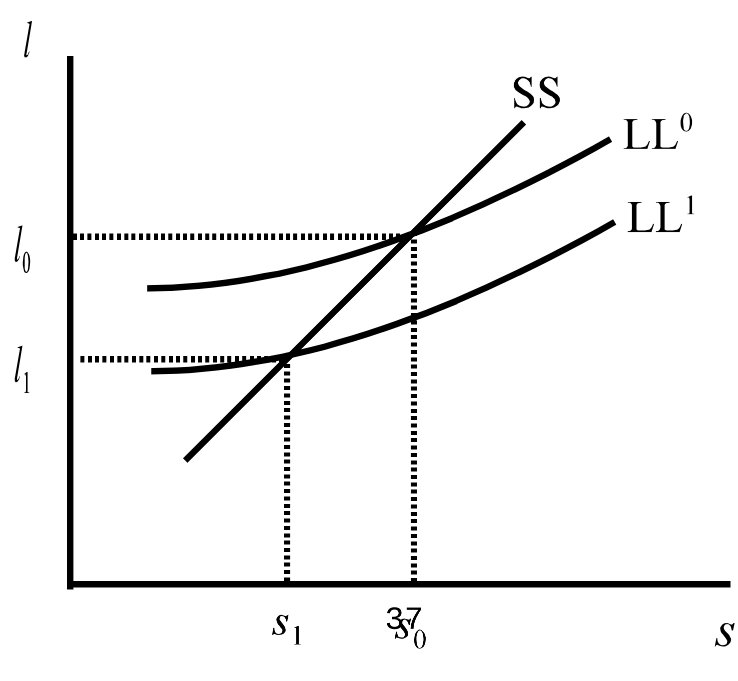

FIGURE 4A - Multiple Equilibria (Odd Number)
and Effect of an Increase in μ

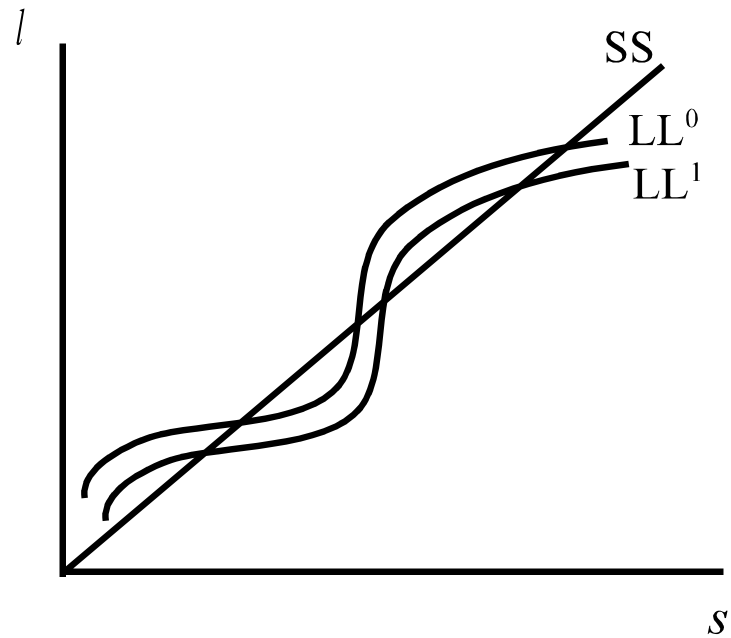

FIGURE 4B - Multiple Equilibria (Even Number)
and Effect of an Increase in μ

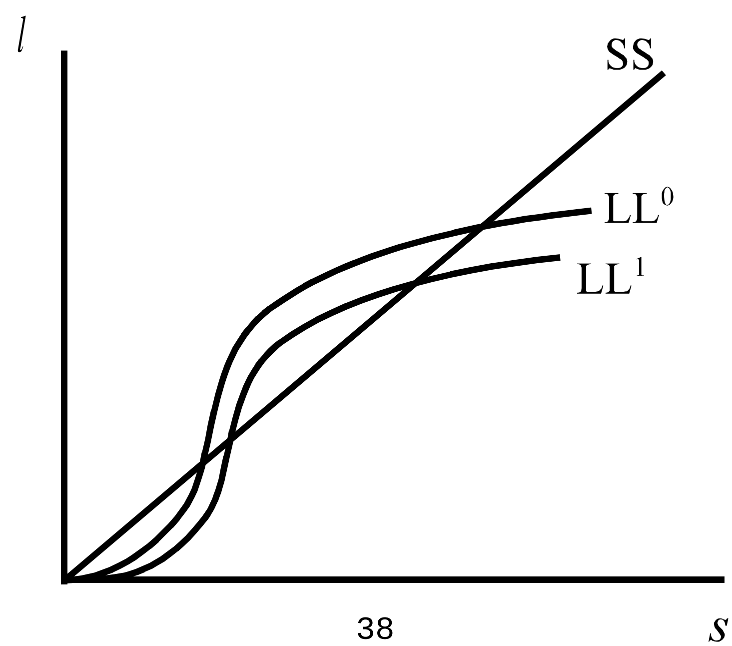

FIGURE 5 - Existence of Multiple Steady States with γ=2, η=0.4, ρ=0.8, α_0=0, α_1=8 and β=0.99.

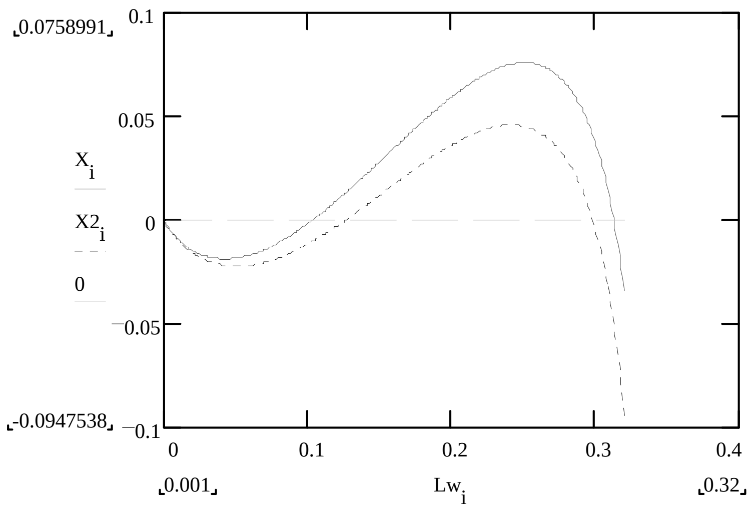

FIGURE 6 - Steady State with Firm Entry for ρ=1 and Effect of Increase in μ.

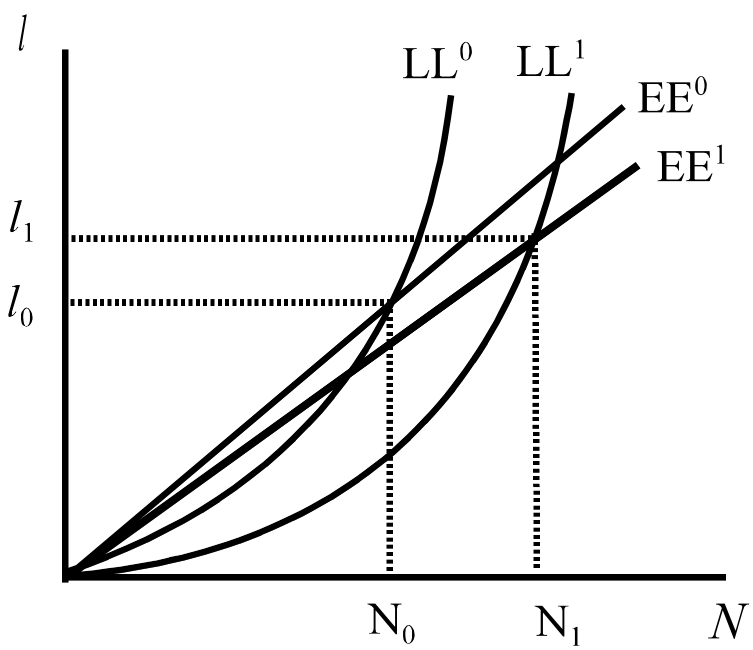

FIGURE 7 - Steady State with Frim Entry for $0 < \rho < 1$, and $\gamma > 1/(1-\rho)$. Effect of Increase in μ.

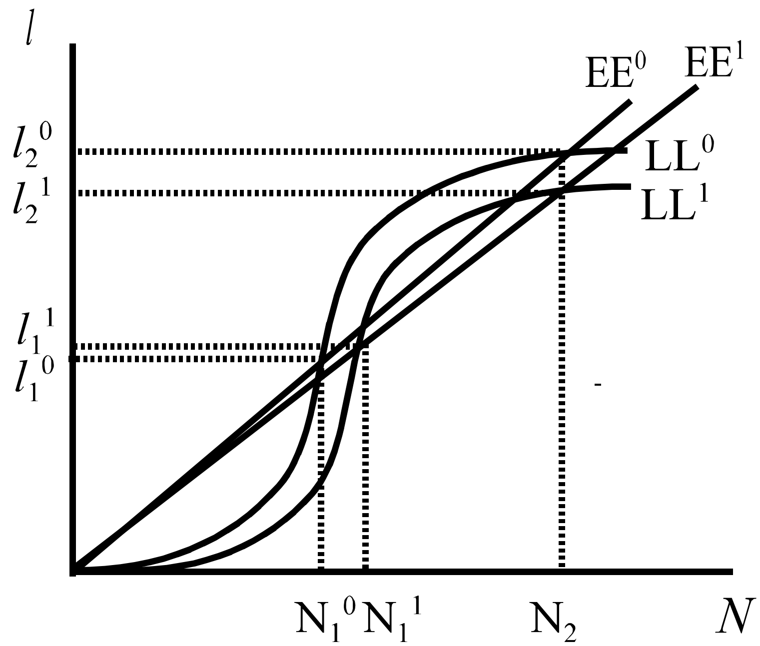

FIGURE 8 - Multiple Steady States with Firm Entry for $0 < \rho < 1$ and $\gamma < 1/(1-\rho)$. Effect of Increase in μ.