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Dynamic Strategies, Asset Pricing Models, and the Out-of-Sample Performance of the Tangency Portfolio

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Dynamic Strategies, Asset Pricing Models, and the Out-of-Sample Performance of the Tangency Portfolio

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Abstract: In this paper, I study the behavior of an investor with unit risk aversion who maximizes a utility function defined over the mean and the variance of a portfolio's return. Conditioning information is accessible without cost and an unconditionally riskless asset is available in the market.

The proposed approach makes it possible to compare the performance of a benchmark tangency portfolio (formed from the set of unrestricted estimates of portfolio weights) to the performance of a restricted tangency portfolio which uses single-index and multi-index asset pricing models to constrain the first moments of asset returns.

The main findings of the paper are summarized as follows: i) The estimates of the constant and time-varying tangency portfolio weights are extremely volatile and imprecise. Using an asset pricing model to constrain mean asset returns eliminates extreme short positions in the underlying securities and improves the precision of the estimates of the weights. ii) Partially restricting mean asset returns according to single-index and multi-index asset pricing models improves the out-of-sample performance of the tangency portfolio. iii) Active investment strategies (i.e., strategies that incorporate the role played by conditioning information in investment decisions) strongly dominate passive investment strategies in-sample but do not provide any convincing pattern of improved out-of-sample performance.

JEL classification: G11, G12, G15

Key words: asset allocation, conditioning information, dynamic strategies, tangency portfolio

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Introduction

In recent years, several researchers and practitioners in finance have tried to develop an adequate regression framework to implement Markowitz's (1952) mean-variance analysis. Mean-variance efficient portfolios play an important role in empirical finance for two main reasons. First, the tangency portfolio is relevant in normative portfolio analysis and investment performance evaluation. Second, mean-variance tangency portfolio weights are central to several asset pricing theories (see, e.g., Fama (1996)). Although the mean-variance approach has a solid theoretical background, several problems arise when researchers and practitioners in finance try to construct sample efficient portfolios. Construction of such portfolios requires the expected values of the returns on the set of primitive assets available to the investor and the covariance matrix giving their pair-wise covariances as inputs. Given that portfolio weights are highly sensitive to changes in asset means and covariances, sampling error in the first and second sample moments of asset returns feeds through to the estimates of optimal portfolio weights (see, e.g., Best and Grauer (1991) and Britten-Jones (1999)). Hence the main difficulties in doing asset management concern the extreme estimates of the tangency portfolio weights and the presence of non-negligible sampling error in the estimates of these weights. In order to reduce the influence of sampling error in the estimates of the tangency portfolio weights, several researchers consider direct restrictions on portfolio weights,[1] while other researchers focus on Bayesian shrinkage methods.[2] Since covariances can be estimated more precisely than means (see Merton (1980)), most of the work on portfolio weights focuses on restrictions on asset means, partially disregarding the sampling error associated with the sample covariance matrix of asset returns. Exceptions to the common practice mentioned above are the papers by Jagannathan and Ma (2000), Chan, Karceski, and Lakonishok (1999), Stevens (1998), Ledoit (1997), Cohen, Hawawini, Maier, Schartz, and Whitcomb (1983), and Elton and Gruber (1973), where the focus of their analysis is on the second moments of asset returns. MacKinlay and Pástor (2000) ex-

[1]See, e.g., Haugen (1997).

[2]See, e.g., Jorion (1985, 1986, 1991), Frost and Savarino (1986, 1988), Black and Litterman (1992), and Bawa, Brown and Klein (1979).

ploit the possible mispricing embedded in linear factor-based asset pricing models to derive expected return estimates that are more precise and stable than the estimates delivered by standard methods. Nonetheless, their methodology becomes computationally cumbersome when fixed-income securities are added to the analysis.

Despite the possible damaging role of sampling error in the construction of sample efficient portfolios, few papers concentrate on the finite-sample and large-sample properties of the tangency portfolio weights. Jobson and Korkie (1980) derive an asymptotic distribution for the estimates of the optimal portfolio weights under normality of asset returns. Ledoit (1995) and Gibbons, Ross, and Shanken (1989) argue that asymptotic results can be misleading when the number of assets is large. Britten-Jones (1999) shows how to do exact statistical inference on the estimates of the tangency portfolio weights assuming multivariate normality and independence of asset returns.

In this paper, I take a different route. In fact, the primary goal of a portfolio manager is the one of maximizing the out-of-sample risk-return trade-off of a given portfolio, and not necessarily the one of finding weight estimates with tight standard errors. Moreover, there is no obvious relationship between the out-of-sample performance of the tangency portfolio and the precision of the estimates of the weights. As a consequence, in the following analysis, I model portfolio weights with the explicit goal of maximizing the out-of-sample Sharpe ratio of the tangency portfolio.

This paper adds to the existing literature in several ways. First, the generality of the proposed approach makes it possible to empirically identify constant and time-varying tangency portfolio weights using a Generalized Method of Moments (GMM) technique and to test for their statistical significance. Even though the focus of my analysis is not limited to the study of the properties of weight estimates, I am able to do statistical inference on the weights similarly to Britten-Jones (1999) and Aït-Sahalia and Brandt (2001). The time-varying estimates of the weights can be obtained by enlarging the set of assets under consideration to include dynamic strategies. Hansen and Richard (1987) and Hansen and Jagannathan (1997) provide only general characterizations of the set of mean-variance efficient strategies in the presence of conditioning information. Gallant, Hansen and Tauchen (1990) and Bansal and

Harvey (1997) characterize unconditionally efficient strategies and show how to draw the relevant mean-variance diagram, but they do not display explicit solutions for portfolio weights. Duffie and Richardson (1991) display the optimal weights for a related problem in continuous time, assuming Brownian motions. Ferson and Siegel (1997, 1998) explore explicit solutions for unconditionally efficient portfolio strategies and optimal weights by focusing on the efficient use of conditioning information in portfolio analysis. Instead of modeling tangency portfolio weights as linear functions of a vector of state variables (which is common in the asset pricing literature), I empirically identify the set of time-varying weights starting from dynamic strategies available to the investor. This approach is convenient because it does not require direct estimation of means and variances of asset returns. This intermediate estimation step is one of the main problems of portfolio choice under predictability because even if there is ample evidence that returns are predictable, the patterns of predictability are actually quite weak. My approach is similar in spirit to the method proposed by Aït-Sahalia and Brandt (2001), but the focus of their paper is different. They are interested in selecting and combining variables to best predict an investor's optimal portfolio weights in sample, building on the belief that the relationship between portfolio weights and predictors is less noisy than the relationship between individual moments and predictors. On the contrary, the focus of my paper is on the out-of-sample behavior of the tangency portfolio in presence of predictability of asset returns.

Moreover, my approach does not require assumptions of multivariate normality and independence of asset returns. Asymptotic standard errors associated with weights and Sharpe ratios can be computed without imposing distributional assumptions on the return generating process. Given that the unconditional and conditional estimates of the weights are obtained by GMM, the estimates are asymptotically normal and consistent even if not necessarily unbiased. Weights and Sharpe ratios can be jointly estimated using a one-step procedure as opposed to the two-step procedure required by an OLS approach. Moreover, when the composition of the tangency portfolio is estimated outside of the GMM algorithm, the proposed methodology delivers Sharpe ratio estimates that are preciser than the values obtained by jointly estimating the mean and the volatility of the tangency portfolio. In this paper, I also use a bootstrap experiment to investigate the small-sample properties of constant and time-varying standardized tangency portfolio weights. This exercise allows me to recover the whole distribution of the tangency portfolio weights without assuming normality and independence of asset returns. This characterization of the small-sample properties of the weights of the tangency portfolio with and without constraints complements the existing asymptotic findings as well as the small-sample findings based on normality and independence of asset returns.

The method used in this study allows me to estimate tangency portfolio weights and Sharpe ratios even when the time series dimension T is smaller than the number of assets N. In contrast, direct estimation of the covariance matrix requires the number of assets to be smaller than the number of time series observations, so that the sample estimate of the covariance-matrix is non-singular.

Second, this study runs a horse race between an unrestricted tangency portfolio and a tangency portfolio that incorporates restrictions on mean excess returns delivered by single-index and multi-index partial equilibrium models. On one side, this study investigates the potential gain, in terms of in-sample and out-of-sample performance of the tangency portfolio, from imposing restrictions on mean excess returns according to a specific asset pricing model. Pástor (1999) proposes a Bayesian framework that incorporates a prior degree of belief in an asset pricing model. His approach is similar in spirit to mine, but he does not take explicitly into account the role played by conditioning information and does not investigate the out-of-sample performance of active investment strategies with different degrees of belief in a model.[3] On the other side, this study evaluates the potential gain in the out-of-sample performance of the tangency portfolio from imposing non-negativity constraints on constant and time-varying tangency portfolio weights. The out-of-sample

[3]Black and Litterman (1992) suggest using the CAPM as a benchmark toward which the investor can shrink his subjective views about expected returns. The extent of the deviations from the CAPM depends on the investor's degree of confidence in his subjective views. Nonetheless, that study makes no direct use of sample information about mean excess returns. In contrast, my approach shrinks the sample means toward their values implied by the model. The extent of the deviations from the model depends on the strength of the violations of the model in the data as well as on the investor's degree of confidence in the model.

experiment uses 60-month and 120-month rolling windows to recover the ex-post Sharpe ratios of alternative tangency portfolios.

Finally, I analyze the impact of conditioning information made available to the investor on the in-sample and out-of-sample Sharpe ratio of the tangency portfolio. In light of the existing literature on predictability of domestic and international asset returns, I analyze the role played by prespecified sources of macroeconomic and financial uncertainty in investment portfolio evaluation. Using recent developments in Bayesian forecasting and dynamic modeling, Novomestky and Kling (2000) estimate the inputs for portfolio optimization in the presence of regression parameter and error covariance time variation, as well as conditioning information. Then, they use these inputs for evaluating in-sample and out-of-sample portfolio performance.

The subsequent empirical analysis uses four sets of assets corresponding to varying degrees of aggregation of stock and bond returns: i) individual stocks; ii) aggregate stocks; iii) aggregate stock and bond returns; and iv) international stocks. The passive strategies I consider are either unconditional strategies (i.e., strategies that disregard the role played by relevant conditioning information available in the market), or strategies that do not explicitly embed the theoretical restrictions delivered by single-index and multi-index asset pricing models, or myopic strategies that invest equal dollar amounts in the underlying securities. Among the active strategies, I consider unconditional and conditional strategies that incorporate the theoretical predictions of competing asset pricing models.

The main results of the paper are summarized as follows: First, fully constraining mean excess returns according to an asset pricing model worsens the out-of-sample performance of the tangency portfolio. The strategies of investing equal dollar amounts in each asset available for investment or of imposing no short sale constraints on the underlying securities consistently outperform the strategy of fully restricting mean excess returns according to an equilibrium model. Nonetheless, portfolios based on several linear combinations of restricted and unrestricted mean excess returns often outperform the benchmark of investing equal dollar amounts in each asset. Second, for aggregate domestic stocks, there is an out-of-sample gain from including conditioning information only when a 120-month rolling window is used. For international stocks, there is an out-of-sample gain from considering conditioning information only when a 60-month rolling window is used. Passive strategies as well as no-short sale strategies consistently dominate active strategies in the remaining cases. With regard to individual stocks, the strategy of imposing no short sale constraints dominates any other active or passive strategy. These results are somehow consistent with the low patterns of predictability of stock returns at an individual level and with the the increased number of assets included in the analysis. In summary, the ambiguous role played by conditioning information in portfolio formation is consistent with other recent finding in the asset pricing literature.[4]

Finally, when decile and bond portfolios and individual stocks are considered, portfolios based on the CAPM outperform portfolios based on the Fama-French (1993) five-factor and three-factor models. With regard to international equities, portfolios based on competing versions of the International CAPM perform equally poorly and no model clearly outperforms the others.[5]

The paper proceeds as follows: Section I derives constant and time-varying tangency portfolio weights and Sharpe ratios and discusses the related estimation issues; Section II shows how to impose restrictions from single-index and multi-index partial equilibrium models; Section III describes the data used in the empirical analysis; Section IV presents the empirical results; and the paper concludes in Section V.

[4]See, for example, Bossaerts and Hillion (1999), Pesaran and Timmermann (1995), Handa and Tiwari (2001), Ayadi and Kryzanowski (2001).

[5]See Section II for a description of the International Asset Pricing models considered in this study: the International Static CAPM (IS–CAPM), the International Intertemporal CAPM in presence of currency risk (II-CAPM (SPOT)), the International Intertemporal CAPM in presence of deviations from Purchasing Power Parity (II–CAPM (PPP)), the International CAPM in presence of currency risk (I–CAPM (SPOT)), the International CAPM in presence of inflation risk (I–CAPM (PPP)).

I. Tangency Portfolio Weights and Sharpe Ratios

A. Investor's Problem without Conditioning Information

Consider an investor who maximizes a utility function defined over the mean and the variance of a portfolio's return in presence of a riskless asset with (gross) rate of return r_f. Assume for the moment that her information set is empty.[6] The portfolio choice problem is

$$\max_{\alpha}\{E[\mathbf{r}^\top\alpha + (1-\alpha^\top\mathbf{1})r_f] - \frac{\gamma}{2}Var[(\mathbf{r}-r_f\mathbf{1})^\top\alpha]\} \ , \tag{1}$$

where $\mathbf{r}$ represents the $(N \times 1)$ vector of (gross) security returns on the N risky assets, α is the $(N \times 1)$ vector of unscaled portfolio weights, and γ is the parameter of risk aversion.[7] Note that, in an unconditional setting, each asset should be thought of as intrinsically risky and that the two-fund separation theorem in general does not hold. However, the assumption of the existence of a risk-free asset can be at least partially justified by the low variability of the one-month TB rate over time and by the negligible covariance between the TB rate and equity and bond returns. The assumption of constant rate of return on the riskless asset allows me to substitute the covariance matrix of asset excess returns with the covariance matrix of asset returns. Assuming, without loss of generality, unit risk aversion ($\gamma = 1$), the set of first order conditions with respect to α can be written as

$$E(\mathbf{r}-r_f\mathbf{1}) - E\{[\mathbf{r}-E(\mathbf{r})]\ \mathbf{r}^\top\alpha\} \ = \mathbf{0} \ . \tag{2}$$

Rearranging terms,

$$E(\{1-[\mathbf{r}-E(\mathbf{r})]^\top\alpha\}\ \mathbf{r}) = E(r_f\ \mathbf{1}) \ . \tag{3}$$

Hence, the previous maximization problem yields the following set of orthogonality conditions:

$$E(v\mathbf{r}) = E(r_f\ \mathbf{1}) \ , \tag{4}$$

[6] This assumption will be relaxed in Section B.

[7] See Ingersoll (1987), p.88–90.

where $v = 1 - [\mathbf{r} - E(\mathbf{r})]^\top \alpha$ is an admissible pricing kernel with unconditional mean equal to one.[8] Using (4), the α's are given by the following equation

$$\alpha = \mathbf{\Sigma}_{rr}^{-1}[E(\mathbf{r} - r_f\ \mathbf{1})]\ , \tag{5}$$

where $\mathbf{\Sigma}_{rr}$ is the unconditional covariance matrix of risky asset returns (which is assumed to be invertible). Using (5), it follows that

$$v = 1 - [\mathbf{r} - E(\mathbf{r})]^\top \mathbf{\Sigma}_{rr}^{-1}[E(\mathbf{r} - r_f\ \mathbf{1})]\ . \tag{6}$$

The pricing kernel v has two properties worth noting. First, the vector α is proportional to the vector of portfolio weights of the *tangency* portfolio obtained from the risky security returns $\mathbf{r}$.[9] The vector of scaled tangency portfolio weights is then simply given by $\omega^\star = \frac{\alpha}{\mathbf{1}^\top \alpha}$. Hence, v is perfectly negatively correlated with the rate of return on the tangency portfolio r_τ. Second, the unconditional variance of v equals the mean of the square conditional Sharpe ratio of the tangency portfolio, Sh_τ^2. Specifically,

$$[\mathrm{Cov}(v, r_\tau)]^2 = \mathrm{Var}(r_\tau)\mathrm{Var}(v)\ . \tag{7}$$

Since v prices correctly all the securities under consideration,[10] it also correctly prices the tangency portfolio

$$-\mathrm{Cov}(v, r_\tau) = E(r_\tau - r_f)\ . \tag{8}$$

Using this result and rearranging equation (7) above, it follows that

$$\left[\frac{E(r_\tau - r_f)}{\sqrt{\mathrm{Var}(r_\tau)}}\right]^2 \equiv Sh_\tau^2 = \mathrm{Var}(v)\ . \tag{9}$$

Hence, the unconditional variance of v equals the squared unconditional Sharpe ratio of the tangency portfolio.[11] Moreover, the squared Sharpe ratio of the tangency portfolio is a non-decreasing function of the number of assets considered.[12]

[8]Notice the analogy between the pricing function v and the Hansen and Jagannathan (1991) stochastic discount factor.

[9]For example, see Ingersoll (1987), p.89.

[10]In fact, v receives the interpretation of a normalized pricing kernel and, by the law of one price, $E[v(\mathbf{r} - r_f\mathbf{1})] = \mathbf{0}$.

[11]In the out-of-sample analysis, the Sharpe ratio of the tangency portfolio is calculated as follows: $\frac{mean[(\mathbf{r}_{t+1} - r_{ft}\mathbf{1})^\top \alpha_t]}{std(\mathbf{r}_{t+1}\alpha_t)}$.

[12]See Appendix A for a proof of this result.

The estimates of the unconditional scaled weights, $\frac{\alpha}{\mathbf{1}^\top \alpha}$, are asymptotically normally distributed. This result builds on the normality of the α's and is a direct consequence of the application of a Taylor series expansion to a non-linear transformation of the original estimates. Moreover, the estimates of the tangency portfolio weights are consistent even if not necessarily unbiased.

B. Investor's Problem with Conditioning Information

The investor's problem described in the previous section can be extended to incorporate conditioning information in the analysis. The problem faced by a mean-variance investor with unit risk aversion is now to optimize her risk-return tradeoff using all the information contained in her information set. Conditioning information is introduced by enlarging the set of original assets to include managed portfolios, which is common in the asset pricing literature. Instead of scaling asset returns with state variables, the focus of the analysis is on dynamic strategies of the following type:

$$\mathbf{r}^z \equiv \mathbf{z} \otimes \mathbf{r} + (\iota_J - \mathbf{z}) \otimes r_f \mathbf{1} \; , \tag{10}$$

where $\otimes$ denotes the Kronecker product, $\mathbf{z}$ is a $(J \times 1)$ state vector with $z_1 = 1$, and ι_J is a $(J \times 1)$ vector of ones. When $\mathbf{z} = z_1$, the investor's problem reduces to the one described in the previous section.[13] An alternative way of introducing conditioning information is to assume that the coefficients α's of the pricing kernel are linear functions of $\mathbf{z}$: $\alpha_t = \alpha(\mathbf{z})$.[14] The choice of the first approach over the second one relies on a few considerations: i) Time-varying α's would transform the unconditional maximization problem into a conditional one. This would require a model for the conditional mean of asset returns and would introduce additional structure in the analysis. ii) Dynamic strategies have a unit price and the corresponding returns can be described by alternative asset pricing models; iii) The objective of maximizing the out-of-sample average Sharpe ratio would be partially inconsistent with an in-sample conditional optimization problem.

[13] Examples of the use of these dynamic strategies are contained in Hansen and Jagannathan (1997).

[14] See Aït-Sahalia and Brandt (2001) for an application of this methodology to the problem of finding instruments that, in sample, optimally predict weights.

The portfolio choice problem becomes

$$\max_{\alpha_z}\{E[\mathbf{r}^{z\top}\alpha_z + (1-\alpha_z^\top \iota) r_f] - \frac{1}{2}Var[(\mathbf{r}^z - r_f\iota)^\top \alpha_z]\} \ , \tag{11}$$

where α_z and ι represent $(NJ \times 1)$ vectors of weights associated with each dynamic strategy and ones, respectively. The new set of orthogonality conditions can be rewritten as

$$E(v^z \mathbf{r}^z) = E(r_f \ \iota) \ , \tag{12}$$

where $v^z = 1 - [\mathbf{r}^z - E(\mathbf{r}^z)]^\top \alpha_z$. Using (12), the α_z's are given by the following equation

$$\alpha_z = \mathbf{\Sigma}_{r^z r^z}^{-1}[E(\mathbf{r}^z - r_{ft} \ \iota)] \ , \tag{13}$$

where $\mathbf{\Sigma}_{r^z r^z}$ is the unconditional covariance matrix of the augmented set of strategies (which is assumed to be invertible). Hence, the new tangency portfolio has the form

$$\begin{aligned} r_\tau \quad = \quad & (\alpha_{z,1} + \alpha_{z,2} z_2 + \ldots + \alpha_{z,J} z_J) r_1 \\ & + \ldots \\ & + (\alpha_{z,(N-1)J+1} + \alpha_{z,(N-1)J+2} z_2 + \ldots + \alpha_{z,NJ} z_J) r_N \ . \end{aligned} \tag{14}$$

Specifically, the sequence of scaled time-varying tangency portfolio weights is given by the following expression:

$$\omega_i^\star = \frac{\sum_{s=1}^J \alpha_{z,[s+(i-1)J]} z_s}{\sum_{s=1}^J \sum_{k=1}^N \alpha_{z,[s+(k-1)J]} z_s} \ , \tag{15}$$

for $i = 1, ..., N$. Note that, once the alphas (the sensitivities of the tangency portfolio to dynamic strategies, long $\mathbf{z}$ dollars in the original securities and short $(\iota_J - \mathbf{z})$ dollars in the risk-free rate) have been estimated, time variation in (15) is simply given by the set of instruments $\mathbf{z}$.[15]

Given the realizations of the state vector $\mathbf{z}$, the time-varying weights in (15) are asymptotically normally distributed. This result builds on the normality of the α_z's and is a direct consequence of the application of a Taylor series expansion to a non-linear transformation of the original estimates. Moreover, the estimates of the tangency portfolio weights are consistent even if not necessarily unbiased.

[15]Even if the weights are linear in the conditioning variables, the underlying raw returns do not need to be linear functions of the $\mathbf{z}$'s.

In this setting, the tangency portfolio weights as well as the Sharpe ratio of the tangency portfolio can be jointly estimated. An OLS approach would require two sets of regressions to deliver an estimate of the Sharpe ratio of the tangency portfolio. As a consequence, the estimation error from the first stage would carry over to the second stage.

C. GMM vs Artificial Regression

In this section, I compare the Euler equation approach described in the two previous sections to the artificial regression methodology proposed by Britten-Jones (1999). I also test whether the assumption of a constant risk-free rate significantly affects the portfolio weight estimates when I take the model to the data. The following analysis shows that the artificial regression approach delivers the same estimates of the tangency portfolio weights assigned by the GMM approach. Britten-Jones (1999) considers an OLS regression of a $(T \times 1)$ vector of 1's onto a $(N \times 1)$ vector of realized asset's excess returns $(\mathbf{r} - r_f\ \mathbf{1})$, such that

$$1 = (\mathbf{r} - r_f\ \mathbf{1})^{\top} \alpha + u \ , \tag{16}$$

where u is a $(T \times 1)$ vector of error terms. Compare the set of orthogonality conditions associated with equation (16) to get

$$E(\{1 - [\mathbf{r} - r_f\ \mathbf{1}]^{\top} \alpha\}\ \mathbf{r}) = E(r_f\ \mathbf{1}) \ , \tag{17}$$

with the set of conditions implied by equation (3). Equation (17) delivers the following estimates of the α's:

$$\begin{aligned} \alpha &= \{E(\mathbf{r} - r_f\mathbf{1})(\mathbf{r} - r_f\mathbf{1})^{\top}\}^{-1} E[(\mathbf{r} - r_f\mathbf{1})] \\ &= [\mathbf{\Sigma}_{(r-r_f)} + E(\mathbf{r} - r_f\mathbf{1})E(\mathbf{r} - r_f\mathbf{1})^{\top}]^{-1} E[(\mathbf{r} - r_f\mathbf{1})] \ , \end{aligned} \tag{18}$$

where $\mathbf{\Sigma}_{(r-r_f)}$ is the variance-covariance matrix of asset excess returns. The two expressions coincide when the riskless asset exhibits constant rate of return. In Section IV, I test whether imposing the condition $[\mathbf{\Sigma}_{(r-r_f)} + E(\mathbf{r} - r_f\mathbf{1})E(\mathbf{r} - r_f\mathbf{1})^{\top}] = \mathbf{\Sigma}_{rr}$ significantly affects portfolio weight estimates.

II. Analysis of Restrictions

In this section, I describe the techniques used to estimate the composition of the tangency portfolio under restrictions. The subsequent empirical analysis runs a horse race between a benchmark (unrestricted) tangency portfolio and a tangency portfolio which embeds restrictions from single-index and multi-index asset pricing models. The set of unrestricted moment conditions,

$$\mathrm{E}(v^z \mathbf{r}^z) = \mathrm{E}(r_f \iota) \ , \tag{19}$$

is equivalent to

$$\mathrm{E}(\mathbf{r}^z - r_f \iota) = -\mathrm{Cov}(v^z, \mathbf{r}^z) \ . \tag{20}$$

Consider, for example, the pricing relation behind the usual formulation of the unconditional CAPM with constant betas:[16]

$$\mathrm{E}(\mathbf{r}^z - r_f \iota) = \beta_m \mathrm{E}(r_m - r_f) \ , \tag{21}$$

where $\beta_m \equiv \mathrm{E}[(\mathbf{r}^z - r_f \iota)(r_m - r_f)]/\mathrm{E}(r_m - r_f)^2$ is a $(NJ \times 1)$ vector of unconditional asset-specific market betas and $\mathrm{E}(r_m - r_f)$ is the unconditional premium on the market. It follows that in the restricted case

$$\beta_m \mathrm{E}(r_m - r_f) = -\mathrm{Cov}(v^z, \mathbf{r}^z) \ . \tag{22}$$

Equation (22) can be written as

$$\mathrm{E}(v^z \mathbf{r}^z) = \mathrm{E}(\mathbf{r}^{m,z}) \ , \tag{23}$$

where

$$\mathbf{r}^{m,z} \equiv [\mathbf{r}^z - \beta_m E(r_m - r_f)] \ . \tag{24}$$

Hence, a portfolio manager who aims to invest into a subset of assets forming the market index can consistently exploit the predictions delivered by the market CAPM. Using (23), the composition of the restricted tangency portfolio can be estimated by exactly identified

[16] Ferson and Korajczyk (1995), for example, attribute less than 1% of the predictable variation in returns to changing conditional betas.

GMM. In synthesis, this way of constraining the weights of the tangency portfolio leaves the variance-covariance matrix of asset returns unchanged but reduces the variability of excess expected returns. As a consequence, the standard deviation of the pricing kernel v, which receives the interpretation of unconditional Sharpe ratio, is also smaller. While a decrease of the in-sample Sharpe ratio is to be expected, the out-of-sample Sharpe ratio of the restricted tangency portfolio is not necessarily smaller than the one of its unrestricted counterpart.

In a similar way, it is possible to estimate the composition of the restricted tangency portfolio: i) using the predictions of the Consumption Capital Asset Pricing Model (C-CAPM). The rate of return on the market portfolio would be replaced by the portfolio that does the mimicking of the rate of growth in per-capita consumption between two points in time. Mimicking portfolios[17] are maximally correlated with the factors and exact factor pricing holds with such portfolios; ii) using the pricing relationship delivered by the Fama-French (1993) three-factor and five-factor models: $\mathrm{E}(\mathbf{r}^z - r_f \iota) = \mathbf{B}\delta_k$, where $\mathbf{B}$ is a $(NJ \times K)$ matrix of factor sensitivities and δ_k is a $(K \times 1)$ vector of factor risk premia. For portfolios of equities, the model underlines that three factors – the excess return on the market, the return on a zero-investment portfolio designed to capture risk associated with size (market capitalization), and the return on a zero-investment portfolio designed to capture risk associated with the book-to-market equity – explain cross-sectional differences in average returns. For portfolios of stocks and bonds, the augmented Fama-French model implies that the previous three stock-market factors plus two bond-market factors related to maturity and default risks explain average returns.

The previous setup can be extended to account for varying degrees of investor's confidence in a specific asset pricing model. Assume that mean excess returns are described by the following process:

$$\mu = \lambda E(\mathbf{r}^z - r_f \iota) + (1 - \lambda) E[\beta_m (r_m - r_f)] \ , \tag{25}$$

[17]My mimicking (hedging) portfolios are designed to track the *contemporaneous* realizations of the economic variables and differ from Lamont's (1999) "economic tracking portfolios" that track changes in expectations of *future* realizations of the economic variables. See Robotti (2000) for a description of hedging portfolios and economic risk premia.

where λ ($0 \leq \lambda \leq 1$) represents the weight attributed to the unrestricted and restricted mean excess returns. Then, the set of orthogonality conditions (23) holds with $\mathbf{r}^{m,z} \equiv (\mathbf{r}^z - \mu)$. The subsequent empirical analysis uses alternative values of λ: $\lambda = 0, 0.25, 0.5, 0.75, 1$. For each λ and each equilibrium model, the Sharpe ratios associated with alternative tangency portfolios are computed and compared.

Consider, now, the set of international equities described above. Four international asset pricing models are used to constrain the average excess returns on international stocks: i) the International Static CAPM (IS–CAPM); ii) the International CAPM (I-CAPM (PPP)) in presence of deviations from Purchasing Power Parity (PPP); iii) the International CAPM in presence of currency risk (I-CAPM (SPOT)); and iv) the International Intertemporal CAPM in presence of currency risk (II–CAPM (SPOT)) or the International Intertemporal CAPM in presence of inflation risk (II-CAPM (PPP)). The IS–CAPM was first derived by Solnik (1974) and postulates a linear relationship between the cross-section of international expected excess equity returns and the excess return on a world market portfolio. The I-CAPM, as introduced by Adler and Dumas (1983), links nominal excess returns on international equity denominated in a reference currency to a world market portfolio and portfolios hedging against deviations from PPP. The II-CAPM is a combination of Merton's (1973) intertemporal CAPM and Adler and Dumas's (1983) international CAPM. Namely, the cross-section of nominal excess returns denominated in a reference currency is explained by three hedging funds: a nationless (or logarithmic) world market portfolio; portfolios hedging against deviations from PPP (or against currency risk); and portfolios hedging against variations in the investment opportunity set of an international investor.[18] The II–CAPM collapses to the IS–CAPM when the investment opportunity set is constant or, equivalently, when the weights associated with the hedging demands are equal to zero. Moreover, when the inflation rate of country l ($l = 1, \ldots, L+1$), expressed in its home currency, is zero or non-stochastic, the $L+1$ inflation hedging funds of Adler and Dumas (1983) collapse to the L exchange rate hedging funds,[19] i.e., the I–CAPM (PPP) collapses to the I–CAPM (SPOT).

[18] See Appendix B for a formal derivation of this result.

[19] See, for example, Solnik (1974), Sercu (1980), and Grauer, Litzenberger and Stehle (1976).

The absence of money illusion allows one to express nominal returns in a reference currency and, without loss of generality, in excess of a measurement currency risk-free rate.[20]

Denote with $\mathbf{y}$ a $(K \times 1)$ vector of risk factors. Without loss of generality, assume that $E(\mathbf{y}) = \mathbf{0}$. Consider now an admissible pricing function v_y which is linear in the risk variables:

$$v_y = 1 - \mathbf{y}^\top \delta \ . \tag{26}$$

Equation (20) above implies that

$$E(\mathbf{r}^z - r_f \iota) = \beta^z \delta \ , \tag{27}$$

where $\beta^z = E(\mathbf{r}^z \mathbf{y}^\top) \equiv \mathbf{\Sigma}_{r^z y}$. Hence, returns satisfy the linear factor model

$$\mathbf{r}^z - r_f \iota = \beta^z \delta + \beta \mathbf{y} + \epsilon \ , \tag{28}$$

where δ is a $(K \times 1)$ vector of unconditional risk premia and ϵ is an $(NJ \times 1)$ vector of disturbances orthogonal to $\mathbf{y}$. Define the vector $\mathbf{y}$ as

$$\mathbf{y} \equiv [y_m, \mathbf{y}_f, \mathbf{y}_\pi, \mathbf{y}_h]^\top \quad ,$$

where y_m is the rate of return on the world market portfolio, $\mathbf{y}_f$ is the $(L \times 1)$ vector of logarithmic changes of the rates of appreciation of the measurement currency, $\mathbf{y}_\pi$ is the $(L + 1) \times 1$ vector of innovations in the inflation rates and $\mathbf{y}_h$ is the $(H \times 1)$ vector of demands hedging against variations in the investment opportunity set $(K = 2L + H + 2)$. When estimating the I-CAPM (PPP), the II-CAPM (SPOT), and the II-CAPM (PPP), the

[20] When using the IS–CAPM, translating returns into a new currency and measuring excess returns relative to the new currency risk-free rate would leave the intercept term equal to zero. With regard to the I–CAPM and II-CAPM, the new currency foreign exchange premium would be replaced by the old currency exchange risk premium. Nonetheless, the introduction of conditioning information and the expansion of the set of primitive securities to include managed portfolios might be affected by the choice of the measurement currency. Hence, the pricing implications delivered by alternative asset pricing models might differ according to the reference currency considered. Indeed, Dumas and Solnik (1995) find that the choice of the measurement currency does not affect the pricing implications of the international CAPM.

underlying risk premia are computed using hedging-portfolio analysis. It is now possible to formulate the four international asset pricing models in the order described above as follows:

$$
\begin{aligned}
v_y^1 &= 1 - y_m \delta_m \\
v_y^2 &= 1 - y_m \delta_m - \mathbf{y}_\pi^\top \delta_\pi \\
v_y^3 &= 1 - y_m \delta_m - \mathbf{y}_f^\top \delta_f \\
v_y^4 &= 1 - y_m \delta_m - \mathbf{y}_\pi^\top \delta_\pi - \mathbf{y}_h^\top \delta_h \ .
\end{aligned}
$$

When inflation rates in each country are non random, v_y^4 can be rewritten as

$$v_y^5 = 1 - y_m \delta_m - \mathbf{y}_f^\top \delta_f - \mathbf{y}_h^\top \delta_h \ \ ,$$

where δ_m, δ_f, δ_π, and δ_h are commensurable coefficient vectors of unconditional risk premia. Let $\beta_m^z \equiv E(\mathbf{r}^z y_m)$, $\beta_\pi^z \equiv E(\mathbf{r}^z \mathbf{y}_\pi^\top)$, $\beta_f^z \equiv E(\mathbf{r}^z \mathbf{y}_f^\top)$, and $\beta_h^z \equiv E(\mathbf{r}^z \mathbf{y}_h^\top)$ denote the (arrays of) the unconditional betas associated with the economic variables $\mathbf{y}$.

Using (27), mean excess returns can be written, in order, as

$$
\begin{aligned}
E(\mathbf{r}^z - r_f \iota) &= \beta_m^z \delta_m && (29) \\
E(\mathbf{r}^z - r_f \iota) &= \beta_m^z \delta_m + \beta_\pi^z \delta_\pi && (30) \\
E(\mathbf{r}^z - r_f \iota) &= \beta_m^z \delta_m + \beta_f^z \delta_f && (31) \\
E(\mathbf{r}^z - r_f \iota) &= \beta_m^z \delta_m + \beta_\pi^z \delta_\pi + \beta_h^z \delta_h \qquad (E(\mathbf{r}^z - r_f \iota) = \beta_m^z \delta_m + \beta_f^z \delta_f + \beta_h^z \delta_h) \ . && (32)
\end{aligned}
$$

As in the domestic setting, the composition of the restricted global tangency portfolio can be estimated by exactly identified GMM.

III. Data

This section describes the data used in the empirical analysis. Data are monthly and are expressed in percentage per month. The reluctance to using daily data in portfolio weights estimation is mainly attributed to the nonsynchronous trading of securities. Since many securities trade infrequently, accurate calculation of returns over an interval as short as a day is difficult. Specifically, covariance bias is greatest when one security or group of securities trades very frequently while the other trades very infrequently.

A. Decile, Bond, and Factor Portfolios

The period considered is March 1959 through December 1996 for stock returns, bond returns, and macroeconomic/financial factors. I use decile portfolio returns on NYSE-, AMEX-, and NASDAQ-listed stocks. Ten *size* stock portfolios are formed according to size deciles on the basis of the market value of equity outstanding at the end of the previous year. If a capitalization was not available for the previous year, the firm was ranked based on the capitalization on the date with the earliest available price in the current year. The returns are value-weighted averages of the firms's returns, adjusted for dividends. The securities with the smallest capitalizations are placed in portfolio one. The partitions on the CRSP file include all securities, excluding ADRs, that were active on NYSE-AMEX-NASDAQ for that year.

In addition to stock returns, I include bond portfolio returns in the analysis. The bond portfolios are comprised of a long-term government bond and a long-term corporate bond. The long-term government and corporate bonds are provided by Ibbotson Associates. The 1–month Treasury Bill (TB) rate pertains to a bill with at least 1 month to maturity and performs as the riskless asset in the analysis (Ibbotson Associates, SBBI module). All rates of return are nominal.

The subsequent empirical analysis is based on a set of six factors which have been previously used in tests of single and multi-beta models.[21] XEW represents the equally-weighted NYSE-AMEX-NASDAQ index return (CRSP). CG denotes the logarithm of the monthly gross growth rate of per capita real consumption of nondurable goods and services. The series used to construct consumption data are from CITIBASE. Monthly real consumption of nondurables and services are the GMCN and GMCS series deflated by the corresponding

[21] See, for example, Fama and French (1993), Chen, Roll, and Ross (1986), Burmeister and McElroy (1988), Ferson and Harvey (1991, 1999), Downs and Snow (1994), Kirby (1998), Balduzzi and Robotti (2000).

deflator series GMDCN and GMDCS. Per capita quantities are obtained by using data on resident population, series POPRES. HB3 is the 1–month return of a 3–month Treasury bill less the 1–month return of a 1–month bill (CRSP, Fama Treasury Bill Term Structure Files). PREM represents the yield spread between Baa and Aaa rated bonds (Moody's Industrial from CITIBASE). HML is a zero-investment book to market portfolio (from Fama-French online research data). SMB is a zero-investment size portfolio (from Fama-French online research data).

Variables that are statistically significant in multi-variate predictive regressions of means and volatilities perform as instruments in the analysis. The set of instruments includes a constant and the lagged values of the following three variables: i) XEW; ii) DIV, the monthly dividend yield on the Standard and Poor's 500 stock index (CITIBASE); and iii) INFL, the monthly rate of inflation (Ibbotson Associates). I choose these variables as a proxy for the information that investors use to set prices in the market.

B. Individual Stocks and Factor Portfolios

The period considered is February 1962 through October 1998 for stock returns and factor portfolios. I use holding period stock returns (including dividends) of firms listed in the Dow Jones Industrial Average (DJIA). Specifically, this set of stocks includes all the stocks in the DJIA that have monthly return data since April 1961 (22 stocks) plus eight other blue-chip stocks.[22] This set of stocks is chosen to mimic a portfolio manager's variance minimization (or tracking error minimization) problem, because portfolio managers tend to trade blue-chip stocks for their higher liquidity. All stock returns are from CRSP and most of them are traded on the New York Stock Exchange.

The economic/financial factors include the following four variables: i) MARKET repre-

[22] The tickers of the 22 stocks that are currently in the DJIA are: T, ALD, AA, BA, CAT, C, KO, DIS, DD, EK, XON, GE, GM, HWP, IBM, IP, JNJ, MRK, MMM, MO, PG, UTX. The other eight blue-chip stocks are: BS (Bethlehem Steel), CHV (Chevron), CL (Colgate Palmolive), F (Ford), GT (Goodyear tire and rubber), S (Sears, Roebuck & Co.), TX (Texaco), and UK (Union Carbide).

sents the DJIA monthly returns (from CRSP); [23] ii) CG is the logarithm of the monthly gross growth rate of per-capita real consumption of nondurable goods and services (from CITIBASE); iii) HML is a zero-investment book to market portfolio (from Fama-French online research data); iv) SMB is a zero-investment size portfolio (from Fama-French online research data).

In choosing the instruments, I consider a set of variables which have been previously used in studies of stock-return predictability. The set of instruments includes: i) a constant; ii) the lagged value of the DJIA stock market index; and iii) the lagged value of the earnings-price ratio.[24]

C. International Stocks and Factor Portfolios

The period considered is April 1970 through October 1998 for stock returns and economic variables and March 1970 through September 1998 for instrumental variables. Data are monthly. The starting and ending dates for the sample are dictated by macroeconomic and financial data availability. The universe of equities includes the Morgan Stanley Capital International (MSCI) national equity indices. The nominal returns are denominated in U.S. dollars and are calculated with dividends. All indices have a common basis of 100 in December 1969. The indices are constructed using the Laspeyres method which approximates value weighting.[25] U.S. dollar returns are calculated by using the closing European interbank currency rates from MSCI. The focus of the empirical analysis is on the four countries with

[23]Alternative market indexes were considered as well: i) the NYSE aggregate stock index (from CRSP); and ii) the EW DJIA index (from CRSP) formed by the 30 stocks included in the analysis. The DJIA stock index does slightly better than the two indexes mentioned above in maximizing the in-sample square Sharpe ratio of the tangency portfolio.

[24]The earnings-price ratio is the ratio between annual earnings per share and monthly prices per share. Prices and earnings are equally weighted averages across firms included in the sample. Prices per share are from CRSP and earnings per share are from COMPUSTAT (data item 58). Specifically, the price-earnings ratio is formed by dividing stock prices from July of year t to June of year t+1 by earnings of year t-1. The way in which I construct these data closely follows Fama-French (1992).

[25]See MSCI Methodology & Index Policy for a detailed description of MSCI's indices and properties.

the largest market capitalization: United States; United Kingdom; Japan; and Germany.[26]

In this study, I use inflation rates, spot exchange rates, and the rate of return on the world market portfolio as economic factors. Consumer price indices are from International Financial Statistics (IFS) and are denominated in local currency. Spot exchange rates are from MSCI. The world equity market index is a value–weighted combination of the country returns tracked by MSCI. Logarithmic changes in the spot exchange rates (SPOT) are used as a proxy for foreign exchange risk. Inflation risk is described by an ARIMA(0,1,1) for inflation (INFL). The global market risk proxy is provided by the level of world equity returns (WLDMK). Note that the variable INFL represents unexpected inflation and that the variable SPOT represents innovations in the exchange rate assuming that spot rates follow a random walk. In choosing the set of instruments, I focus on a set of variables which have been previously used in tests of international stock-return predictability.[27] The instruments include a constant, and the following five variables: i) DINFLUS is the lagged difference in the U.S. monthly rate of inflation (IFS); ii) EURO represents the one–month Eurodollar deposit rate (DRI) and performs as the conditionally nominal risk–free asset in my analysis; iii) USDIVYLD denotes the U.S. monthly dividend yield (MSCI) in excess of the 1-month Eurodollar deposit rate. Specifically, the monthly dividend yield is equal to 1/12 of the ratio between the previous year dividend and the index at the end of each month;[28] iv) WLDMK denotes the lagged value of the world stock market monthly returns (MSCI); v) DEFPREM denotes the U.S. default premium as given by the return difference between Moody's Baa-rated and Aaa-rated bonds (SBBI Yearbook).

[26] As of 1996, the market capitalization weight for these countries is 76.2% of the market capitalization world.

[27] See, for example, Ferson and Harvey (1993), Dumas and Solnik (1995), and De Santis and Gérard (1998).

[28] The following formula allows me to calculate the monthly yield as:

$$\text{Index with dividend}_t = \text{Index with dividend}_{\,t-1} \times \frac{\text{Index without dividend}_t}{\text{Index without dividend}_{t-1}} \times \left[1 + \frac{\text{annual yield}_t}{12}\right] .$$

IV. Empirical Results

First, in this study I empirically identify constant and time-varying tangency portfolio weights and Sharpe ratios with associated standard errors. Second, I provide an illustration of the role that the economic restrictions from single-index and multi-index equilibrium models in the presence of conditioning information can play in portfolio selection. Empirical results are presented for four universes of assets in increasing order of aggregation: 30 individual stocks; 10 size-sorted portfolios; 10 size-sorted portfolios and two bond portfolios; and four country equity portfolios.

A. Constant and Time-Varying Tangency Portfolio Weights

Panels A, B, C, and D of Table I contain estimates of the scaled unconditional weights of a tangency portfolio with associated Sharpe ratios.[29] Short sales and leveraging are allowed. Weight estimation is performed by exactly-identified GMM. Standard errors are computed by

[29] I use a χ^2 test of overidentifying restrictions to understand if the assumption of a constant rate of return on the riskless asset significantly affects the estimates of the weights of the tangency portfolio. Specifically, I test whether the set of estimates delivered by equation (3) also satisfies the same set of orthogonality conditions with the covariance matrix of asset excess returns being replaced by the covariance matrix of asset returns. This latter condition corresponds to the solution of an equivalent optimization problem with unconditional moments being replaced by their conditional counterparts. The GMM test is performed using individual stocks, decile portfolios, decile and bond portfolios, and international equities. When individual stocks from DJIA are considered (February 1962 through October 1998), the J-test of overidentifying restrictions is $\chi^2_{(30)} = 16.630$ with p-value equal to 0.98. For NYSE-AMEX-NASDAQ size sorted portfolios (March 1959 through December 1996), the J-test of overidentifying restrictions is $\chi^2_{(10)} = 14.833$ with p-value equal to 0.14. Augmenting the set of decile portfolios with a long-term government bond and a long-term corporate bond (March 1959 through December 1996) provides a J statistic which is $\chi^2_{(12)} = 21.663$ with p-value equal to 0.04. Specifically, when fixed-income securities are added to the portfolio of assets, I do not reject the existence of a constant rate of return on the risk-free asset at 1% level. Finally, for international equity portfolios (April 1970 through October 1998), the J-test of overidentifying restrictions is $\chi^2_{(4)} = 6.019$ with p-value equal to 0.19. Hence, in general, using the covariance matrix of asset returns instead of the covariance matrix of asset excess returns does not significantly affect the estimates of the underlying weights.

the delta method. Consider, for example, decile and bond portfolios. Weights for the full 38-year period are shown as well as weights for a 28-year and 10-year subperiod (corresponding to the periods pre- and post-1987 stock market crash). The estimates contain several extreme short and long positions. Many weights change dramatically between the two subperiods. For example, for aggregate stock and bond portfolios, in the first subperiod the optimal weight in a long-term government bond, r_{11}, is a long position of 130.9 percent, but in the second subperiod the optimal weights in r_{11} is a short position of 315.3 percent. Over the full 38-year period, the standard errors of the estimates are large. Not surprisingly, the standard errors are generally larger in the two subperiods. Individual stocks and international equities display a similar behavior. The result of such statistical imprecision is that most of the weight estimates are not statistically different from zero at the standard significance level of 0.05. Hence, data provide little information for domestic and international portfolio construction. On the contrary, the unconditional Sharpe ratios of the domestic and global tangency portfolios are reasonable in magnitude and statistically significant at any confidence level. This result is encouraging in light of the fact that a portfolio manager aims to maximize the out-of-sample performance of the average Sharpe ratio of a given portfolio, partially disregarding the precision of the estimates of the weights. Moreover, there is no obvious link between the precision of the estimates of the weights and the performance of the tangency portfolio. Future research should investigate whether higher precision and lower variability of the weight estimates necessarily translate into a better out-of-sample performance of a given portfolio.

Figures 1, 2, and 3 display the time-varying behavior of the scaled portfolio weights associated with the ten NYSE-AMEX-NASDAQ size-sorted portfolios, a long-term government and a long-term corporate bond over the whole sample period March 1959 through December 1996. Dynamic strategies are constructed using a constant, the lagged values of stock market returns, inflation, and dividend yield. Weights estimation is performed by exactly-identified GMM with mean asset excess returns constrained by a CAPM. Exact 95% confidence bounds are computed. Portfolio weights vary over time, are relatively stable but imprecisely estimated. Imposing the CAPM restrictions on excess expected returns eliminates extreme short and long positions in each asset but does not consistently increase the precision of the estimates. The time-varying behavior of tangency portfolio weights captures several important short positions in the 70s and in the 80s. In particular, all the weights in the risky assets have negative peaks after the 1987 stock market crash.

B. Finite Sample Properties of the Tangency Portfolio Weights

In this section, I use a bootstrap experiment to investigate the small-sample properties of constant and time-varying standardized tangency portfolio weights. This exercise allows me to recover the whole distribution of the tangency portfolio weights without assuming normality and independence of asset returns. Moreover, this exercise shows how the distribution of the underlying weights changes when I constrain mean excess returns according to a specific equilibrium asset pricing model. This characterization of the small-sample properties of the weights of the tangency portfolio with and without constraints is new in the asset pricing literature and complements the existing asymptotic findings as well as the small-sample findings based on normality and independence of asset returns. I also study the small-sample properties of time-varying portfolio weights explicitly accounting for the patterns of autocorrelation of the conditioning variables included in the analysis. In the following empirical analysis, I consider the ten NYSE-AMEX-NASDAQ decile portfolios, a long-term goverment bond and a long-term corporate bond. TB performs as the riskless asset return. XEW represents the return on the market portfolio. The lagged values of XEW, DIV, and INFL are used as a proxy for the information that investors use to set prices in the market. I denote with N = 12 the total number of risky assets and with T = 454 the total number of observations.

B.1. Bootstrapping Constant Tangency Portfolio Weights

Denote with $\mathbf{y}_t = [\mathbf{r}_t, r_{ft}, r_{mt}]^\top$ $(t = 1, \ldots, T)$ my set of data. I draw B = 2000 bootstrap samples, each of size T, from the empirical distribution function of the observed data. This resampling is done with replacement by first generating a pseudo-random number from the

$U(0,1)$ distribution and using it to generate a random number k that takes on the values $1,\ldots,T$ with equal probability. Then I set $\mathbf{y}_j^\star(i)$, the j^{th} observation of the i^{th} bootstrap sample, equal to $\mathbf{y}_k$. Repeating this operation T times yields a complete bootstrap sample, $\mathbf{y}^\star(i)$. I then calculate the vector of weights $\omega^\star(\mathbf{y}^\star(i))$ and store the result. The whole operation is then repeated for $i=1,\ldots,B$ bootstrap samples, at the end of which I have B statistics $\omega^\star(\mathbf{y}^\star(i))$. Note that imposing the CAPM restrictions on mean excess returns also involves resampling with replacement from r_m, the rate of return on the market, and running a time series regression on the reshuffled data. The results of this exercise are summarized as follows: i) The standardized unconstrained tangency portfolio weights exhibit extreme short positions and significantly high variation across bootstrap samples. Moreover, the distribution of these weights, not reported in the paper, is clearly non-normal and has fat tails. ii) As shown in Figure IV, when I impose a CAPM on mean excess returns, the distribution of the standardized constrained weights looks roughly normal. As shown in Table II, short positions in the underlying assets disappear and the sampling variability of the computed weights is smaller, suggesting that constrained weights can be estimated more precisely than their unconstrained counterparts.[30] This bootstrap experiment also suggests that unconstrained portfolio weights are not normally distributed in small samples and that the finite sample properties of the tangency portfolio weights can not be properly addressed using the traditional t and F statistics.

B.2. Bootstrapping Time-Varying Tangency Portfolio Weights

In this section, I set up a bootstrap experiment for studying the small-sample properties of the time-varying weights given by equation (14). To account for the patterns of autocorrelation of $\mathbf{z}_t$, I model the set of conditioning variables as a vector autoregression of order one, VAR(1):

$$\mathbf{z}_t = \mathbf{a} + \mathbf{z}_{t-1}\mathbf{B} + \mathbf{u}_t \ , \tag{33}$$

[30]The statistical properties of the asymptotic constrained estimates of the tangency portfolio weights, not reported in the paper, support the results delivered by the bootstrap exercise.

where $\mathbf{u}_t \sim \text{IID}(\mathbf{0}, \Omega)$. Define the $(J \times 1)$ vector of residuals $\hat{\mathbf{u}}_t = (\mathbf{z}_t - \hat{\mathbf{a}} - \mathbf{z}_{t-1}\ \hat{\mathbf{B}})$ for $t = 2, \ldots, 1000$. A bootstrap sample $\mathbf{z}_1^\star, \ldots, \mathbf{z}_{1000}^\star$ is created by sampling $\mathbf{u}_2^\star, \ldots, \mathbf{u}_{1000}^\star$ with replacement from the residuals, and then letting $\mathbf{z}_1^\star = \mathbf{z}_1$ and $\mathbf{z}_t^\star = \hat{\mathbf{a}} + \hat{\mathbf{B}}\mathbf{z}_{t-1}^\star + \hat{\mathbf{u}}_t^\star$, $t = 2, \ldots, 1000$. Denoting with $\mathbf{y}_t^\star = [\mathbf{r}_t^\star, r_{ft}^\star, r_{mt}^\star, \mathbf{z}^\star]^\top$ $(t = 1, \ldots, T)$ my strings of reshuffled data, I construct N=12 standardized time-varying weights by taking the average values of the corresponding series across bootstrap samples. I sort the bootstrap weights in ascending order and I simply pick the 25^{th} and the 975^{th} values for each of the 454 time periods, so that exactly 2.5% of the bootstrap replications yielded $\omega^\star(i)$'s below the lower limit and 2.5% yielded $\omega^\star(i)$'s above the upper limit of the confidence interval. The results of this exercise are summarized as follows: i) Extreme short positions in the underlying assets arise when computing unconstrained weights. Bootstrap confidence bounds are consistently large and uninformative. ii) When I use a CAPM to constrain mean excess returns, weights become relatively stable, their bounds are significantly tighter than in the unrestricted case but still uninformative at a 5% level. Figure 5 generally confirms the previous findings based on asymptotic considerations.

In summary, this small-sample exercise sheds some light on the properties of constant and time-varying tangency portfolio weights, suggesting that the in-sample and out-of-sample performance of the tangency portfolio might be significantly affected by the presence of extreme positions in the underlying assets. Moreover, the use of well-shaped weights in portfolio performance evaluation might not consistently improve the risk-return trade off of the tangency portfolio, as shown in the following empirical section.

C. The Performance of the Tangency Portfolio

This section analyzes the in-sample and out-of-sample performance of the tangency portfolio for different combinations of restricted and unrestricted mean excess returns. The corresponding Sharpe ratios are then compared across passive and active investment strategies. Among the passive strategies, the "EW10" ("EW30") strategy invests equal dollar amounts in $N = 10$ $(N = 30)$ size decile portfolios (DJIA stocks) every month. The "EW4" strategy invests equal dollar amounts in $N = 4$ country portfolios every month. Among the active strategies, I consider portfolios that incorporate conditioning information and portfolios restricted using single-index and multi-index asset pricing models. Portfolios based on no-short sale constraints are used as an additional benchmark.

A monthly series of expected return estimates and optimal portfolio weights is computed using a "rolling sample approach" for windows of $T = 60$ and $T = 120$ months. Consider, for example, the first data set formed by decile and bond portfolios. The first sample period for the 120-month window is March 1959 through February 1969. The decile and bond returns from that period are used to estimate expected returns and calculate portfolio weights, and the return on the resulting portfolio is recorded for March 1969. The 120-month window is then rolled forward so that March 1959 is dropped and March 1969 is added to the sample, and the portfolio return is recorded for April 1969. This process is continued until a time series of observations through December 1996 is constructed. The same approach is used for the 60-month window, except that the March 1959 through February 1964 period is used for the initial sample.

Unrestricted and restricted in-sample and ex-post Sharpe ratios are then constructed using the estimates of the unrestricted and restricted mean excess returns on the original assets and on the proposed dynamic strategies respectively. With regard to domestic stocks, the restricted estimates of the mean excess returns are those delivered by the CAPM, the C-CAPM, and the Fama-French three- and five-factor models. For international equities, the restricted estimates of the mean excess returns are those delivered by the IS–CAPM, the I–CAPM (PPP), the I–CAPM (SPOT), and the II-CAPM in presence of inflation risk and currency risk, respectively. In Tables III, IV, V, and VI the in-sample numbers are calculated over the same time period as the corresponding out-of-sample ones in the same panel.

C.1. Decile and Bond Portfolios

Panels A, B, and C of Table III display the in-sample and out-of-sample Sharpe ratios of the unrestricted and restricted tangency portfolios. First, when I consider a 60-month rolling window, the strategy of imposing no short sale constraints on the tangency portfolio weights dominates any other active or passive strategy.[31] The C–CAPM represents the only exception to this empirical regularity providing, in presence of conditioning information, an out-of-sample average Sharpe ratio of 0.155. In general, efficiently using conditioning information available in the market does not improve the out-of-sample performance of the tangency portfolio. The Fama-French performs worse than the C–CAPM and the market CAPM.

Second, when I consider a 120-month rolling window, active investment strategies that use conditioning information consistently outperform passive investment strategies and strategies based on no-short sale constraints. My results complement the findings of Whitelaw (1997).[32] Again, the Fama–French performs poorly in comparison with the other models.

Finally, across windows, fully constraining mean excess returns according to a partial equilibrium model does not improve the out-of-sample performance of the tangency portfolio. On the contrary, partially constraining mean excess returns (which corresponds to using values of λ between 0 and 1) effectively improves the out-of-sample risk-return trade-off.

[31] In the empirical analysis, I impose no-short sale constraints on constant tangency portfolio weights only. Appendix C shows how to impose no-short sale constraints on time-varying tangency portfolio weights. Results, not reported in the paper, indicate that the combination of conditioning information and weight non-negativity does not lead to an improved out-of-sample performance of the tangency portfolio.

[32] Whitelaw (1997) documents predictable time variation in stock market Sharpe ratios. Predetermined financial variables are used to estimate both the conditional mean and volatility of equity returns, and these moments are combined to estimate the conditional Sharpe ratios. In sample, estimated conditional Sharpe ratios show substantial time variation that coincides with the variation in ex-post Sharpe ratios and with the phases of the business cycle. Using a 120-month rolling window, Whitelaw identifies periods in which the ex-post Sharpe ratio is approximately three times larger than its full-sample value. Moreover, relatively naive market-timing strategies that exploit stock return predictability can generate Sharpe ratios more than 70% larger than a buy-and-hold strategy.

C.2. Decile Portfolios

Panels A, B, and C of Table IV display the in-sample and out-of-sample risk-return trade-off of the unrestricted and restricted tangency portfolios. First, across windows, efficiently using conditioning information available in the market improves the out-of-sample performance of the tangency portfolio. However, the decline of the ex-post Sharpe ratio compared to the in-sample Sharpe ratio is substantial. The Fama-French, the C–CAPM and the market CAPM perform along the same lines.

Second, across windows, fully constraining mean excess returns according to a partial equilibrium model does not improve the out-of-sample performance of the tangency portfolio. On the contrary, partially constraining mean excess returns (which corresponds to using values of λ between 0 and 1) effectively improves the out-of-sample risk-return trade-off.

Finally, dynamic investment strategies are superior to all passive strategies and to strategies that impose no-short sale constraints on the tangency portfolio weights.

C.3. Individual Stocks

Panels A, B, and C of Table V display the in-sample and out-of-sample properties of the unrestricted and restricted tangency portfolios. First, across windows, the strategy of imposing no-short sale constraints on the tangency portfolio weights dominates any other active or passive strategy. Specifically, the efficient use of the conditioning information available in the market does not improve the out-of-sample performance of the tangency portfolio. This latter result is consistent with the low patterns of predictability of stock returns at an individual level and with the additional noise due to the introduction of more assets in the analysis. Second, the Fama-French, the C–CAPM, and the market CAPM perform poorly. Finally, the passive strategy of investing equal dollar amounts in the underlying securities delivers results that are very similar to those produced by imposing no-short sale constraints on the tangency portfolio weights.

C.4. International Stocks

Panels A, B, and C of Table VI display the in-sample and out-of-sample Sharpe ratios of the unrestricted and restricted tangency portfolios. First, when a 60-month rolling window is used, dynamic strategies perform better than passive strategies. The opposite happens when a 120-month rolling window is used. In this latter case, the strategy of imposing no-short sale constraints on the tangency portfolio weights dominates any other active or passive strategy. Second, across windows, fully constraining mean excess returns according to a partial equilibrium model does not improve the out-of-sample performance of the tangency portfolio. Partially constraining mean excess returns (which corresponds to using values of λ between 0 and 1) slightly improves the out-of-sample risk-return trade-off. All the proposed international asset pricing models perform approximately in the same way with the exception of the I–CAPM (SPOT) which performs poorly.

V. Conclusions

The generality of the GMM approach proposed in this paper allows me to estimate constant and time-varying tangency portfolio weights and to test for their statistical significance. Most importantly, I investigate the in-sample and out-of-sample performance of unrestricted and restricted tangency portfolios in presence of conditioning information available to the investor and in presence of restrictions on mean excess returns imposed by competing asset pricing models. The main findings of this study are summarized as follows: First, as documented in the literature, the estimates of the tangency portfolio weights are extremely volatile and imprecise. Using an asset pricing-model to constrain mean asset returns eliminates extreme short positions in the underlying securities, but does not consistently improve the precision of the estimates of the weights. Second, fully restricting mean excess returns according to single-index and multi-index asset pricing models generally worsens the out-of sample performance of the tangency portfolio. On the contrary, tangency portfolios based on linear combinations of unrestricted and restricted mean excess returns perform well in the out-of-sample analysis. Third, passive investment strategies and/or strategies that impose no-short sale constraints on the tangency portfolio weights often dominate dynamic strategies. Dynamic investment strategies are not always effective in improving the out-of-sample performance of the tangency portfolio.

Specifically, the empirical findings of this paper show that economic and/or financial variables that in-sample significantly affect the mean and the variance of stock and bond returns cannot necessarily be used to improve the out-of-sample performance of the tangency portfolio.

Several other issues warrant further study. Future work should investigate the robustness of these results to different sets of assets and different frequencies. Finally, statistical constraints on the second moments of asset returns might be useful as well in improving the performance of the tangency portfolio.

Appendix A

This section shows that the squared Sharpe ratio of the tangency portfolio is a non-decreasing function of the number of underlying asset returns. Denote with (r, R) the augmented set of asset returns including the N original assets and S additional securities. The S additional securities can be either traded assets or non-traded assets or any combination of traded and non-traded assets. Consider the following regression of the S excess returns $(R - r_f 1_S)$ on the excess returns of the N benchmark assets, $(r - r_f 1_N)$:

$$R - r_f 1_S = \alpha_J + \beta(r - r_f 1_N) + u \ , \tag{34}$$

with $E(u) = E(ur) = 0$. The vector of Jensen's alphas is equal to $\alpha_J = \alpha + (\beta 1_N - 1_S) r_f$, where $\alpha = \mu_R - \beta\mu_r$ and $\beta = \Sigma_{Rr}\Sigma_{rr}^{-1}$. Define the tangency portfolio quantities $A \equiv 1^\top \Sigma^{-1} 1$, $B \equiv \mu^\top \Sigma^{-1} 1$, and $C \equiv \mu^\top \Sigma^{-1} \mu$. For the set r, these variables will be denoted as A_r, B_r, and C_r, while the absence of subscripts implies that these variables refer to the larger set (r, R). Notice that

$$\Sigma^{-1} = \begin{pmatrix} \Sigma_{rr} & \Sigma_{rR} \\ \Sigma_{Rr} & \Sigma_{RR} \end{pmatrix}^{-1} = \begin{pmatrix} \Sigma_{rr}^{-1} + \beta^\top \Sigma_{uu}^{-1} \beta & -\beta^\top \Sigma_{uu}^{-1} \\ -\Sigma_{uu}^{-1}\beta & \Sigma_{uu}^{-1} \end{pmatrix} . \tag{35}$$

Hence, $A = A_r + (\beta 1_N - 1_S)^\top \Sigma_{uu}^{-1} (\beta 1_N - 1_S)$, $B = B_r + \alpha^\top \Sigma_{uu}^{-1} (1_S - \beta 1_N)$, and $C = C_r + \alpha^\top \Sigma_{uu}^{-1} \alpha$. It can be easily shown that, for a given risk-free rate, the Sharpe ratio of a mean-variance efficient portfolio can be written as

$$Sh_r = (C_r - 2B_r r_f + A_r r_f^2)^{1/2} \ . \tag{36}$$

A similar expression also holds for the larger set of assets (r, R) with correspondent Sharpe ratio equal to Sh. Hence,

$$Sh^2 = Sh_r^2 + \alpha_J^\top \Sigma_{uu}^{-1} \alpha_J \ . \tag{37}$$

Thus, the change in maximum attainable squared Sharpe ratios equals the inner product of the vector of Jensen's alphas weighted by the inverse of the covariance matrix of u.[33]

[33]This result can also be found in Jobson and Korkie (1984).

Appendix B

This section shows how to combine Merton's (1973) intertemporal CAPM with the international asset pricing model in presence of deviations from PPP proposed by Adler and Dumas (1983). The continuous-time portfolio selection problem of a representative investor can be stated as follows:[34]

$$\text{Max} \;\; E \int_t^T V(C, P, s) \;\; ds \; , \tag{38}$$

where $C = C(W, P, y_k, t)$ denotes nominal consumption expenditures, P is the price level index, V is a function homogeneous of degree zero in C and P expressing the instantaneous rate of indirect utility, and y_k is a state variable that affects utility through nominal consumption. Following Merton (1969), the wealth dynamics can be written as

$$dW = [\sum_{i=1}^{N} w_i(\mu_i - r_f) + r_f]W dt - C dt - \sum_{i=1}^{N} w_i \sigma_i dz_i \; , \tag{39}$$

where $\mathbf{w} = \{w_i\}$ is $(N+1) \times 1$ vector of weights, μ_i is the instantaneous expected nominal rate of return on security i expressed in a reference currency, σ_i is the instantaneous standard deviation of the nominal rate of return on security i, r_f is the risk-free rate expressed in a reference currency and dz_i is the white noise of a standard Wiener process. Denoting with $J(W, P, y_k, t)$ the maximum value of (38) subject to (39), the Bellman principle states that total expected rate of increase of this function must be identically zero, so that

$$\begin{aligned} 0 \;\; = \;\; & \underset{(C, \mathbf{w})}{\text{Max}} \{ V(C, P, y_k, t) + J_t + J_W[-C + W(\sum_{i=0}^{N} w_i(\mu_i - r_f) + r_f)] + J_{y_k}\alpha \\ & + J_P P \pi + \frac{1}{2} J_{WW} W^2 \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} + \frac{1}{2} J_{yy} s^2 + \frac{1}{2} J_{PP} \sigma_\pi^2 P^2 + J_{WP} W P \sum_{i=1}^{N} w_i \sigma_{i\pi} \\ & + J_{y_k W} W \sum_{i=1}^{N} w_i \sigma_{i y_k} + J_{y_k P} \sigma_{y_k \pi} \} \; , \end{aligned} \tag{40}$$

where π is the inflation rate in each country expressed in local units, σ_{ij} are the instantaneous covariances of the nominal rates of return on the various securities, σ_π^2 is the instantaneous variance of the inflation rate, α is the mean value of the state variable y_k, $\sigma_{i\pi}$ is the covariance between security i and the inflation rate π, and $\sigma_{y_k\pi}$ is the covariance between the

[34] See Appendix in Adler and Dumas (1983) for a detailed explanation of the necessary assumptions.

state variable y_k and the inflation rate π.[35] Moreover, the homogeneity of degree zero of the function V implies that $J(W, P, y_k, t)$ and $C(W, P, y_k, t)$ that satisfy (40) must be homogeneous of degree zero in W and P: $J_P \equiv -(W/P)J_W$, $J_{PW} \equiv (-1/P)J_W - (W/P)J_{WW}$, $J_{PP} = 2(W/P^2)J_W + (W/P)^2 J_{WW}$. Hence, (40) can be rewritten as

$$\begin{aligned}
0 \quad = \quad & \underset{(C,\mathbf{w})}{\text{Max}} \{V(C, P, y_k, t) + J_t + J_W[-C + W(\sum_{i=0}^{N} w_i(\mu_i - r_f) + r_f)] + J_{y_k}\alpha - W\pi J_W \\
& + \frac{1}{2}J_{WW}W^2 \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} + \frac{1}{2}J_{yy}s^2 + W J_W \sigma_\pi^2 + \frac{1}{2}\sigma_\pi^2 W^2 J_{WW} - J_W W \sum_{i=1}^{N} w_i \sigma_{i_\pi} \\
& - W^2 J_{WW} \sum_{i=1}^{N} w_i \sigma_{i_\pi} + J_{y_k W} W \sum_{i=1}^{N} w_i \sigma_{iy_k} + J_{y_k P}\sigma_{y_k \pi}\} \ . \qquad (41)
\end{aligned}$$

Taking the first order conditions of (41) with respect to C and $\mathbf{w}$, then

$$V_C \quad = \quad J_W \qquad (42)$$

and

$$0 \quad = \quad J_W(\mu_i - r_f) + W J_{WW} \sum_{j=1}^{N} w_j \sigma_{ij} - J_W \sigma_{i_\pi} - W J_{WW} \sigma_{i\pi} + J_{y_k W}\sigma_{iy_k} \ , \qquad (43)$$

Solving (41) for the optimal portfolio of risky assets of investor l directly in vector notation

$$\mathbf{w}^l = \alpha^l \mathbf{\Sigma}_{rr}^{-1}[E(\mathbf{r} - r_f\mathbf{1})] + (1 - \alpha^l)\mathbf{\Sigma}_{rr}^{-1}\mathbf{s}_{r\pi}^l + \beta^l \mathbf{\Sigma}_{rr}^{-1}\mathbf{s}_{ry_k}^l \ , \qquad (44)$$

where $\mathbf{s}_{ry_k}$ and $\mathbf{s}_{r\pi}$ are the vectors of covariances between each country's security returns and the k-th state variable and level of inflation respectively, $\alpha^l \equiv -\frac{J_W^l}{J_{WW}^l W^l}$ and $\beta^l \equiv (\alpha^l)^2 \times \frac{J_{Wy_k}^l}{W^l}$.

Let $\mathbf{w}_\tau$ denote the $N \times 1$ vector of unscaled weights of the tangency portfolio:

$$\mathbf{w}_\tau \quad = \quad \mathbf{\Sigma}_{rr}^{-1}[E(\mathbf{r} - r_f\mathbf{1})] \ . \qquad (45)$$

Let $\mathbf{w}_{yt}^l$ and $\mathbf{w}_{\pi t}^l$ denote the $N \times 1$ vectors of unscaled weights hedging against variations in the investment opportunity set and against deviations from PPP, respectively, so that

$$\mathbf{w}_y^l \quad = \quad \mathbf{\Sigma}_{rr}^{-1}\mathbf{s}_{ry_k}^l \qquad (46)$$

and

$$\mathbf{w}_\pi^l \quad = \quad \mathbf{\Sigma}_{rr}^{-1}\mathbf{s}_{ry}^l \ . \qquad (47)$$

[35] See Chapter 13 of Ingersoll (1987) for a definition of the dynamics of the state variables.

Equation (44) can be rewritten as

$$\mathbf{w}^l = \alpha^l \mathbf{w}_\tau + (1-\alpha^l)\mathbf{w}_\pi^l + \beta^l \mathbf{w}_y^l \ . \tag{48}$$

Aggregating over countries and defining $\alpha^m = \frac{\sum_{l=1}^{L+1} \alpha^l W^l}{\sum_{l=1}^{L+1} W^l}$, $\alpha_\pi^l = \frac{(1-\alpha^l)W^l}{\sum_{l=1}^{L+1} W^l}$, $\alpha_y^l = \frac{\beta^l W^l}{\sum_{l=1}^{L+1} W^l}$,

$$\mathbf{w}_m = \alpha^m \mathbf{w}_\tau + \sum_{l=1}^{L+1} \alpha_\pi^l \mathbf{w}_\pi^l + \sum_{l=1}^{L+1} \alpha_y^l \mathbf{w}_y^l \ . \tag{49}$$

Hence the tangency portfolio, which prices all security returns, is a combination of the global market portfolio and the portfolios hedging against deviations from PPP and movements in the investment opportunity set of an international investor.

Appendix C

The investor's problem with and without conditioning information can be extended to account for non-negativity (solvency) constraints. Consider the investor's problem with conditioning information. The unscaled time-varying tangency portfolio weights delivered by the set of orthogonality conditions (12) in presence of weight non-negativity coincide with the standardized weights delivered by the following constrained optimization problem:

$$\max_{\alpha_z} E[\mathbf{r} \otimes \mathbf{z} - (\mathbf{1} \otimes \mathbf{z}) r_f]^\top \alpha_z \ , \tag{50}$$

subject to

$$\frac{1}{2}\alpha_z^\top Var(\mathbf{r}^z)\alpha_z \quad = \quad \sigma^2 \tag{51}$$

$$\text{and}$$

$$\sum_{s=1}^{J} \alpha_{z,[s+(i-1)J]} z_s \quad \geq \quad 0 \ , \tag{52}$$

for $i = 1, \ldots, N$ and σ^2 representing the variance of the tangency portfolio.

The non-negativity constraints in (52) can be represented in matrix notation as follows. Let $\mathbf{A_z}$ be the $(N \times NJ)$ matrix $\mathbf{A}_z = \mathbf{I} \otimes \mathbf{z}$, where $\mathbf{I}$ is an $(N \times N)$ identity matrix. Then, the constraints in (52) can be rewritten as $\mathbf{A}_z \alpha_z \geq \mathbf{0}$.

References

[1] Adler, Michael, and Bernard Dumas, 1983, International portfolio choice and corporation finance: A synthesis, *Journal of Finance,* 38, 925-984.

[2] Aït-Sahalia, Yacine, and Michael W. Brandt, 2001, Variable selection for portfolio choice, *Journal of Finance* 56, 1297-1351.

[3] Ayadi, Mohamed A., and Lawrence Kryzanowski, 2001, Assessing portfolio performance using asset pricing kernels, Working Paper, Concordia University.

[4] Balduzzi, Pierluigi, and Cesare Robotti, 2000, Hedging Portfolios and Tests of Asset Pricing Models, Boston College, work in progress.

[5] Bansal, Ravi, and Campbell R. Harvey, 1997, Dynamic trading strategies, Working Paper, Duke University.

[6] Bawa, Vijay S., Stephen J. Brown, and Roger W. Klein, 1979, *Estimation Risk and Optimal Portfolio Choice*, North Holland.

[7] Best, Michael J., and Robert Grauer, 1991, On the sensitivity of mean-variance efficient portfolios to changes in asset means: Some analytical and computational results, *Review of Financial Studies* 4, 315-342.

[8] Black, Fischer, and Robert Litterman, 1992, Global portfolio optimization, *Financial Analysts Journal*, September-October 1992.

[9] Bossaerts, Peter, and Pierre Hillion, 1999, Implementing statistical criteria to select return forecasting models: What do we learn?, *Review of Financial Studies* 12, 405-428.

[10] Britten-Jones, Mark, 1999, The sampling error in estimates of mean–variance efficient portfolio weights, *Journal of Finance* 54, 655-671.

[11] Chan, Luis K. C., Jason Karceski, and Josef Lakonishok, 1999, On portfolio optimization: Forecasting covariances and choosing the risk model, *Review of Financial Studies* 12, 937-974.

[12] Cohen, G., G. Hawawini, S. Maier, R. Schartz, and D. Whitcome, 1983, Friction in the trading process and the estimation of systematic risk, *Journal of Financial Economics* 12, 263-278.

[13] De Santis Giorgio, and Bruno Gerard, 1998, How big is the premium for currency risk?, *Journal of Financial Economics* 49, 375-412.

[14] Downs, David H., and Karl N. Snow, 1994, Sufficient conditioning information in dynamic asset pricing, Working paper, University of North Carolina.

[15] Duffie, Darrell, and Henry R. Richardson, 1991, Mean-variance hedging in continuous time, *Annals of Applied Probability* 1, 1-15.

[16] Dumas, Bernard, and Bruno Solnik, 1995, The world price of foreign exchange risk, *Journal of Finance* 50, 445-479.

[17] Elton, E. J., and M. J. Gruber, 1973, Estimating the dependence structure of share prices - Implications for portfolio selection, *Journal of Finance* 28, 1203-1232.

[18] Fama, Eugene F., 1996, Multifactor portfolio efficiency and multifactor asset pricing, *Journal of Financial and Quantitative Analysis* 31, 441-465.

[19] Fama, Eugene F., and Kenneth French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427-465.

[20] Fama, Eugene F., and Kenneth French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

[21] Ferson, Wayne E., and Campbell R. Harvey, 1993, The risk and predictability of international equity returns, *Review of Financial Studies*, 6, 527-566.

[22] Ferson, Wayne E., and Andrew F. Siegel, 1997, The efficient use of conditioning information in portfolios, Working Paper, University of Washington, Seattle.

[23] Ferson, Wayne E., and Andrew F. Siegel, 1998, Optimal Moment Restrictions on Stochastic Discount Factors, Working Paper, University of Washington, Seattle.

[24] Ferson, Wayne E., and Campbell Harvey, 1991, The variation in economic risk premiums, *Journal of Political Economy* 99, 385-415.

[25] Ferson, Wayne E., and R. Korajczyk, 1995, Do arbitrage pricing models explain the predictability of stock returns?, *Journal of Business* 68, 309-349.

[26] Frost, Peter A., and James E. Savarino, 1986, An empirical Bayes approach to efficient portfolio selection, *Journal of Financial and Quantitative Analysis* 21, 293-306.

[27] Frost, Peter A., and James E. Savarino, 1988, For better performance: Constrain portfolio weights, *Journal of Portfolio Management* 14, 29-34.

[28] Gallant, A. Ronald, Lars P. Hansen, and George Tauchen, 1990, Using conditional moments of asset payoffs to infer the volatility of intertemporal marginal rates of substitution, *Journal of Econometrics* 45, 141-179.

[29] Gibbons, M., S. Ross, and J. Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121-1152.

[30] Grauer, Frederick L.A., Robert H. Litzenberger, and Richard E. Stehle, 1976, Sharing rules and equilibrium in an international capital market under uncertainty, *Journal of Financial Economics*, 3, 233-256.

[31] Handa, Puneet, and Ashish Tiwari, 2001, Does stock return predictability imply improved asset allocation and performance? Evidence from the US stock market (1954-1998), Unpublished Manuscript, University of Iowa.

[32] Hansen, Lars Peter, and Ravi Jagannathan, 1991, Implications of security market data for models of dynamic economies, *Journal of Political Economy* 99, 225-262.

[33] Hansen, Lars Peter, and Ravi Jagannathan, 1997, Assessing specification errors in stochastic discount factors models, *Journal of Finance* 52, 557-590.

[34] Hansen, Lars Peter, and S. F. Richard, 1987, The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models, *Econometrica* 55, 587-613.

[35] Haugen, Robert A., 1997, *Modern Investment Theory* (Prentice Hall, Upper Saddle River, N.J.).

[36] Ingersoll, J., 1987, Theory of financial decision making, Rowman&Littlefield, Totowa, NJ.

[37] Jagannathan, Ravi, and Tongshu Ma, Three methods for improving the precision in covariance matrix estimation, 2000, *Unpublished Manuscript*, Northwestern University and University of Utah.

[38] Jobson, J. D., and Bob Korkie, 1980, Estimation for Markowitz efficient portfolios, *Journal of the American Statistical Association* 75, 544-554.

[39] Jobson, J. D., and Bob Korkie, 1984, On the Jensen measure and marginal improvements in portfolio performance: A note, *Journal of Finance* 39, 245-251.

[40] Jorion, Philippe, 1985, International portfolio diversification with estimation risk, *Journal of Business* 58, 259-278.

[41] Jorion, Philippe, 1986, Bayes-Stein estimation for portfolio analysis, *Journal of Financial and Quantitative Analysis* 21, 279-292.

[42] Jorion, Philippe, 1991, Bayesian and CAPM estimators of the means: Implications for portfolio selection, *Journal of Banking and Finance* 15, 717-727.

[43] Kirby, Chris, 1998, The restrictions on predictability implied by rational asset pricing models, *Review of Financial Studies* 11, 343-382.

[44] Lamont, Owen, Economic Tracking Portfolios, 1999, Working Paper, University of Chicago and NBER.

[45] Ledoit, Olivier, 1995, Linear-quadratic estimation of the covariance matrix under general asymptotics, Finance Working Paper #25-95, The Anderson School, UCLA.

[46] Ledoit, Olivier, 1997, Improved estimation of the covariance matrix of stock returns with an application to portfolio selection, Finance Working Paper #6-97, Anderson School, UCLA.

[47] MacKinlay, A. Craig, and Ľuboš Pástor, 2000, Asset pricing models: Implications for expected returns and portfolio selection, *Review of Financial Studies*, 13, 883-916.

[48] Markowitz, H. M., 1952, Portfolio selection, *Journal of Finance* 7, 77-91.

[49] Merton, Robert C., 1973, An intertemporal capital asset pricing model, *Econometrica*, 41, 867-887.

[50] Merton, Robert C., 1980, On estimating the expected return on the market: An exploratory investigation, *Journal of Financial Economics* 8, 323-361.

[51] Novomestky, Frederick, and John L. Kling, 2000, Portfolio analysis with bayesian dynamic linear models and conditioning information, Unpublished Manuscript, Polytechnic University.

[52] Pástor, Ľuboš, 1999, Portfolio Selection and Asset Pricing Models, Unpublished Manuscript, The Wharton School, University of Pennsylvania.

[53] Pesaran, M. Hashem, and Allan Timmermann, 1995, Predictability of stock returns: Robustness and economic significance, *Journal of Finance* 50, 1201-1228.

[54] Robotti, Cesare, 2000, The Price of Inflation and Foreign Exchange Risk in International Equity Markets, *Unpublished Manuscript*, Boston College.

[55] Sercu, P., 1980, A generalization of the international asset pricing model, *Reveu de l' Association Francaise de Finance*, 1, 91-135.

[56] Solnik, Bruno H., September 1974, An international market model of security price behavior, *Journal of Financial and Quantitative Analysis*, 537-554.

[57] Stevens, Guy V.G., 1998, On the inverse of the covariance matrix in portfolio analysis, *Journal of Finance* 53, 1821-1827.

[58] Whitelaw, Robert F., 1997, Time-varying Sharpe ratios and Market Timing, Working Paper S/97/29, New York University, Salomon Center.

Table I

Estimates of an Unconditional Tangency Portfolio

This table contains estimates of the scaled unconditional weights of a tangency portfolio with relative Sharpe ratios. Weights are in percentage form. T-statistics are reported in parentheses. The weights are estimated by exactly-identified GMM. The unconditional Sharpe ratios and the composition of the tangency portfolio are jointly estimated. Short sales and leveraging are allowed.

Panel A: Decile and Bond Portfolios

Time Periods	1959.3 - 1996.12	1959.3 - 1986.12	1987.1 - 1996.12
Assets	Weights	Weights	Weights
r_1	110.360 (2.144)	98.688 (1.486)	515.549 (0.369)
r_2	26.879 (0.535)	106.125 (1.169)	−229.508 (−0.341)
r_3	−113.128 (−1.488)	−193.597 (−1.373)	−173.058 (−0.310)
r_4	4.171 (0.063)	80.836 (0.846)	−563.174 (−0.363)
r_5	−72.444 (−0.960)	−154.491 (−1.180)	231.390 (0.305)
r_6	−82.469 (−0.976)	−98.923 (−0.821)	−97.223 (−0.195)
r_7	−37.004 (−0.442)	−41.306 (−0.341)	−208.868 (−0.298)
r_8	131.526 (1.328)	176.928 (1.128)	177.735 (0.287)
r_9	82.874 (0.896)	119.119 (0.840)	353.211 (0.343)
r_{10}	−10.362 (−0.268)	−49.765 (−0.727)	68.865 (0.247)
r_{11}	110.391 (2.954)	130.883 (2.630)	−315.259 (−0.238)
r_{12}	−50.797 (−1.333)	−74.496 (−1.383)	340.340 (0.298)
Sharpe Ratios	0.291 (7.438)	0.281 (6.024)	0.480 (6.552)

Panel B: Decile Portfolios

Time Periods	1959.3 - 1996.12	1959.3 - 1986.12	1987.1 - 1996.12
Assets	Weights	Weights	Weights
r_1	280.928 (2.150)	260.526 (1.428)	431.122 (1.203)
r_2	61.876 (0.487)	233.187 (1.165)	−225.752 (−0.782)
r_3	−290.570 (−1.461)	−535.891 (−1.281)	−173.363 (−0.567)
r_4	2.688 (0.016)	214.699 (0.777)	−517.325 (−0.919)
r_5	−183.044 (−0.956)	−429.087 (−1.201)	220.561 (0.578)
r_6	−195.660 (−0.916)	−231.110 (−0.703)	−76.639 (−0.195)
r_7	−100.574 (−0.473)	−117.518 (−0.360)	−257.298 (−0.588)
r_8	341.299 (1.324)	479.754 (1.086)	245.427 (0.541)
r_9	213.858 (0.943)	328.882 (0.898)	257.716 (0.579)
r_{10}	−30.795 (−0.302)	−163.442 (−0.768)	135.550 (0.869)
Sharpe Ratios	0.278 (6.178)	0.266 (5.056)	0.463 (5.077)

Panel C: International Stocks

Time Periods	1970.4 - 1998.10	1970.4 - 1986.12	1987.1 - 1998.10
Assets	Weights	Weights	Weights
US	43.056 (1.178)	−25.849 (−0.308)	99.182 (1.861)
UK	14.010 (0.488)	8.530 (0.231)	43.031 (0.752)
JAP	20.343 (0.766)	109.195 (1.390)	−37.577 (−0.925)
GER	22.590 (0.722)	8.124 (0.166)	−4.636 (−0.120)
Sharpe Ratios	0.132 (2.419)	0.190 (2.829)	0.244 (2.712)

Panel D: 30 DJIA Stocks

Time Periods	1962.2 - 1998.10	1962.2 - 1986.12	1987.1 - 1998.10
Assets	Weights	Weights	Weights
AA	−3.643 (−0.153)	−33.301 (−0.407)	5.298 (0.323)
XON	−16.074 (−0.543)	−38.477 (−0.387)	−8.090 (−0.430)
ALD	38.465 (1.563)	82.858 (0.961)	7.592 (0.420)
BA	−11.280 (−0.522)	−3.347 (−0.067)	−12.508 (−0.584)
BS	14.879 (0.749)	11.217 (0.236)	19.748 (1.020)
CAT	8.931 (0.584)	27.371 (0.625)	2.055 (0.108)
C	7.145 (0.336)	−8.133 (−0.153)	10.308 (0.556)
CL	10.067 (0.707)	32.302 (0.688)	3.626 (0.271)
CHV	35.351 (1.170)	61.041 (0.732)	27.235 (1.026)
DD	−5.551 (−0.210)	−24.826 (−0.368)	−12.165 (−0.547)
DIS	19.161 (1.095)	51.082 (0.885)	3.846 (0.198)
F	−16.503 (−0.603)	−36.319 (−0.470)	−1.028 (−0.052)
EK	70.435 (1.720)	182.776 (1.081)	25.476 (0.682)
GM	0.940 (0.033)	−2.500 (−0.0358)	15.540 (0.569)
GE	13.333 (0.726)	47.276 (0.726)	−4.056 (−0.301)
GT	−17.041 (−0.721)	−58.250 (−0.664)	0.854 (0.054)
HWP	−10.711 (−0.433)	9.899 (0.157)	−22.515 (−1.190)
IBM	12.386 (0.514)	24.522 (0.428)	0.414 (0.016)
IP	28.456 (1.064)	57.177 (0.741)	19.617 (0.832)
JNJ	−32.402 (−1.078)	−73.669 (−0.822)	−1.737 (−0.071)
KO	−22.686 (−0.705)	−79.211 (−0.714)	9.439 (0.399)
MMM	4.327 (0.213)	16.453 (0.365)	−22.049 (−0.904)
MO	−28.287 (−1.622)	−74.996 (−1.015)	−12.032 (−1.065)
MRK	−0.801 (−0.027)	23.340 (0.321)	−2.619 (−0.100)
PG	−3.365 (−0.163)	15.530 (0.273)	−1.567 (−0.090)
UTX	4.863 (0.237)	16.912 (0.338)	−1.866 (−0.075)
UK	25.136 (1.050)	31.938 (0.484)	16.918 (0.816)
TX	−27.577 (−1.082)	−62.999 (−0.779)	−6.892 (−0.362)
T	−9.335 (−0.311)	−92.661 (−0.797)	27.641 (1.082)
S	11.380 (0.590)	−3.003 (−0.055)	13.513 (0.942)
Sharpe Ratios	0.301 (6.106)	0.312 (5.217)	0.418 (5.085)

Table II

Bootstrap Experiment Using Constant Tangency Portfolio Weights

This table contains means and standard deviations across bootstrap replications of the constant scaled weights of the tangency portfolio. The set of assets includes the ten NYSE-AMEX-NASDAQ size deciles, a long-term government bond and a long-term corporate bond. Panel A reports means and standard deviations for the unconstrained case. Panel B reports the same statistics for the CAPM-based case. Weights are in percentage form.

Panel A: Unconstrained Weights

Assets	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}
Means	116.49	19.18	-107.48	-12.62	-60.30	-92.27	-8.89	121.59	70.77	-8.27	108.23	-46.43
Std	8.57	4.39	10.77	4.11	8.24	6.79	9.08	12.55	13.10	3.70	3.26	3.60

Panel B: Constrained Weights

Assets	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}
Means	10.13	11.94	7.48	9.43	13.20	9.03	6.35	8.82	18.46	5.58	-2.01	1.59
Std	0.01	0.02	0.03	0.03	0.04	0.04	0.04	0.04	0.03	0.02	0.03	0.02

Table III

Ex-Post and In-Sample Performances of the Tangency Portfolios

(Decile and Bond Portfolios)

This table reports the ex-post and in-sample Sharpe ratios (SH) of the tangency portfolios for different values of λ. Risky assets include the 10 decile portfolios, a long-term government bond and a long-term corporate bond. The parameter of risk aversion is set equal to one. Weights estimation is performed by exactly-identified GMM with (TV) and without (C) conditioning information. The set of instruments includes a constant and the lagged values of XEW, DIV and INFL. t-statistics (in parentheses) are obtained by the delta method. The third and fifth columns of each panel report the in-sample Sharpe ratios of the tangency portfolios. The fourth and sixth columns of each panel report the out-of-sample Sharpe ratios of the tangency portfolios. The in-sample numbers are calculated over the same time-period of the corresponding out-of-sample ones in the same panel. The second row of the table reports the in-sample and out-of-sample Sharpe ratios obtained under no-short sales constraints. The out-of-sample quantities are computed using a 60-month and a 120-month rolling window, respectively.

Panel A: CAPM

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.190 (3.932)	0.145	0.147 (2.797)	0.064
$\lambda = 1, \quad C$	SH	0.345 (7.307)	0.124	0.315 (6.172)	0.157
$\lambda = 1, \quad TV$	SH	0.621 (13.573)	0.027	0.640 (12.578)	0.248
$\lambda = 0.75, \quad C$	SH	0.339 (7.156)	0.129	0.305 (5.988)	0.154
$\lambda = 0.75, \quad TV$	SH	0.619 (13.752)	0.027	0.634 (12.688)	0.249
$\lambda = 0.5, \quad C$	SH	0.317 (6.549)	0.136	0.280 (5.395)	0.146
$\lambda = 0.5, \quad TV$	SH	0.607 (13.763)	0.028	0.616 (12.618)	0.250
$\lambda = 0.25, \quad C$	SH	0.255 (5.010)	0.138	0.217 (3.975)	0.115
$\lambda = 0.25, \quad TV$	SH	0.535 (11.775)	0.029	0.530 (10.698)	0.251
$\lambda = 0, \quad C$	SH	0.126 (2.436)	0.092	0.090 (1.626)	0.005
$\lambda = 0, \quad TV$	SH	0.118 (2.287)	0.041	0.081 (1.462)	0.019

Panel B: C-CAPM

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.190 (3.932)	0.145	0.147 (2.797)	0.064
$\lambda = 1,\ \ C$	SH	0.345 (7.307)	0.124	0.315 (6.172)	0.157
$\lambda = 1,\ \ TV$	SH	0.621 (13.573)	0.027	0.640 (12.578)	0.248
$\lambda = 0.75,\ \ C$	SH	0.342 (7.215)	0.121	0.311 (6.081)	0.159
$\lambda = 0.75,\ \ TV$	SH	0.621 (13.571)	0.029	0.640 (12.582)	0.251
$\lambda = 0.5,\ \ C$	SH	0.333 (6.952)	0.114	0.302 (5.834)	0.160
$\lambda = 0.5,\ \ TV$	SH	0.620 (13.553)	0.033	0.639 (12.576)	0.255
$\lambda = 0.25,\ \ C$	SH	0.293 (5.909)	0.092	0.262 (4.890)	0.151
$\lambda = 0.25,\ \ TV$	SH	0.614 (13.384)	0.044	0.634 (12.462)	0.256
$\lambda = 0,\ \ C$	SH	0.028 (0.552)	-0.030	0.007 (0.129)	0.063
$\lambda = 0,\ \ TV$	SH	0.010 (0.193)	0.155	0.014 (0.253)	0.114

Panel C: Augmented Fama-French

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.190 (3.932)	0.145	0.147 (2.797)	0.064
$\lambda = 1,\ \ C$	SH	0.345 (7.307)	0.124	0.315 (6.172)	0.157
$\lambda = 1,\ \ TV$	SH	0.621 (13.573)	0.027	0.640 (12.578)	0.248
$\lambda = 0.75,\ \ C$	SH	0.340 (7.145)	0.034	0.315 (6.126)	0.162
$\lambda = 0.75,\ \ TV$	SH	0.587 (13.490)	0.008	0.615 (12.709)	0.190
$\lambda = 0.5,\ \ C$	SH	0.308 (6.347)	-0.034	0.293 (5.573)	0.133
$\lambda = 0.5,\ \ TV$	SH	0.490 (12.014)	-0.016	0.525 (11.666)	0.096
$\lambda = 0.25,\ \ C$	SH	0.257 (5.192)	-0.064	0.255 (4.724)	0.100
$\lambda = 0.25,\ \ TV$	SH	0.386 (9.739)	-0.031	0.525 (11.666)	0.012
$\lambda = 0,\ \ C$	SH	0.202 (4.030)	-0.078	0.210 (3.845)	0.078
$\lambda = 0,\ \ TV$	SH	0.301 (7.431)	-0.037	0.343 (7.825)	-0.040

Table IV

Ex-Post and In-Sample Performances of the Tangency Portfolios

(Decile Portfolios)

This table reports the ex-post and in-sample Sharpe ratios (SH) of the tangency portfolios for different values of λ. Risky assets include the 10 decile portfolios. The parameter of risk aversion is set equal to one. Weights estimation is performed by exactly-identified GMM with (TV) and without (C) conditioning information. The set of instruments includes a constant and the lagged values of XEW, DIV, and INFL. t-statistics (in parentheses) are obtained by the delta method. The third and fifth columns of each panel report the in-sample Sharpe ratios of the tangency portfolios. The fourth and sixth columns of each panel report the out-of-sample Sharpe ratios of the tangency portfolios. The "EW10" strategy invests equal dollar amounts in $N = 10$ NYSE-AMEX-NASDAQ stocks every month. The in-sample numbers are calculated over the same time-period of the corresponding out-of-sample ones in the same panel. The second row of the table reports the in-sample and out-of-sample Sharpe ratios obtained under no-short sales constraints. The out-of-sample quantities are computed using a 60-month and a 120-month rolling window, respectively.

Panel A: CAPM

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.176 (3.597)	0.155	0.127 (2.400)	0.048
EW10	SH	0.126 (2.434)	0.087	0.091 (1.630)	−0.009
$\lambda = 1,\ \ C$	SH	0.331 (7.058)	0.156	0.304 (6.025)	0.165
$\lambda = 1,\ \ TV$	SH	0.558 (13.165)	0.163	0.568 (12.048)	0.268
$\lambda = 0.75,\ \ C$	SH	0.325 (6.896)	0.160	0.294 (5.819)	0.161
$\lambda = 0.75,\ \ TV$	SH	0.556 (13.281)	0.164	0.562 (12.087)	0.268
$\lambda = 0.5,\ \ C$	SH	0.303 (6.284)	0.164	0.269 (5.201)	0.150
$\lambda = 0.5,\ \ TV$	SH	0.542 (13.113)	0.166	0.541 (11.841)	0.268
$\lambda = 0.25,\ \ C$	SH	0.244 (4.812)	0.156	0.207 (3.813)	0.113
$\lambda = 0.25,\ \ TV$	SH	0.470 (10.722)	0.171	0.455 (9.550)	0.265
$\lambda = 0,\ \ C$	SH	0.126 (2.440)	0.091	0.090 (1.628)	0.003
$\lambda = 0,\ \ TV$	SH	0.121 (2.342)	0.099	0.085 (1.521)	−0.005

Panel B: C-CAPM

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.176 (3.597)	0.155	0.127 (2.400)	0.048
EW10	SH	0.126 (2.434)	0.087	0.091 (1.630)	-0.009
$\lambda = 1, \quad C$	SH	0.331 (7.058)	0.156	0.304 (6.025)	0.165
$\lambda = 1, \quad TV$	SH	0.558 (13.165)	0.163	0.568 (12.048)	0.268
$\lambda = 0.75, \quad C$	SH	0.328 (6.962)	0.154	0.301 (5.930)	0.165
$\lambda = 0.75, \quad TV$	SH	0.558 (13.164)	0.164	0.568 (12.048)	0.270
$\lambda = 0.5, \quad C$	SH	0.320 (6.704)	0.148	0.292 (5.683)	0.160
$\lambda = 0.5, \quad TV$	SH	0.558 (13.162)	0.166	0.568 (12.046)	0.274
$\lambda = 0.25, \quad C$	SH	0.282 (5.712)	0.125	0.254 (4.772)	0.137
$\lambda = 0.25, \quad TV$	SH	0.557 (13.152)	0.167	0.567 (12.039)	0.274
$\lambda = 0, \quad C$	SH	0.024 (0.479)	-0.001	0.006 (0.111)	0.033
$\lambda = 0, \quad TV$	SH	-0.023 (-0.462)	0.068	-0.020 (-0.372)	0.167

Panel C: Fama-French

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.176 (3.597)	0.155	0.127 (2.400)	0.048
EW10	SH	0.126 (2.434)	0.087	0.091 (1.630)	-0.009
$\lambda = 1, \quad C$	SH	0.331 (7.058)	0.156	0.304 (6.025)	0.165
$\lambda = 1, \quad TV$	SH	0.558 (13.165)	0.163	0.568 (12.048)	0.268
$\lambda = 0.75, \quad C$	SH	0.306 (6.272)	0.150	0.287 (5.444)	0.169
$\lambda = 0.75, \quad TV$	SH	0.547 (12.637)	0.165	0.561 (11.615)	0.271
$\lambda = 0.5, \quad C$	SH	0.242 (4.841)	0.130	0.236 (4.346)	0.158
$\lambda = 0.5, \quad TV$	SH	0.493 (10.663)	0.169	0.511 (9.922)	0.275
$\lambda = 0.25, \quad C$	SH	0.155 (3.111)	0.086	0.164 (3.020)	0.121
$\lambda = 0.25, \quad TV$	SH	0.338 (6.862)	0.178	0.363 (6.666)	0.274
$\lambda = 0, \quad C$	SH	0.076 (1.519)	0.027	0.096 (1.776)	0.067
$\lambda = 0, \quad TV$	SH	0.100 (2.020)	0.110	0.130 (2.406)	0.079

Table V

Ex-Post and In-Sample Performances of the Tangency Portfolios

(30 DJIA Stocks)

This table reports the ex-post and in-sample Sharpe ratios (SH) of the tangency portfolios for different values of λ. Risky assets include the 30 DJIA stocks. The parameter of risk aversion is set equal to one. Weights estimation is performed by exactly-identified GMM with (TV) and without (C) conditioning information. The set of instruments includes a constant, the lagged value of MARKET, and the lagged value of the price-earning ratio. t-statistics (in parentheses) are obtained by the delta method. The third and fifth columns of each panel report the in-sample Sharpe ratios of the tangency portfolios. The fourth and sixth columns of each panel report the out-of-sample Sharpe ratios of the tangency portfolios. The "EW30" strategy invests equal dollar amounts in $N = 30$ Dow Jones stocks every month. The in-sample numbers are calculated over the same time-period of the corresponding out-of-sample ones in the same panel. The second row of the table reports the in-sample and out-of-sample Sharpe ratios obtained under no-short sales constraints. The out-of-sample quantities are computed using a 60-month and a 120-month rolling window, respectively.

Panel A: CAPM

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.234 (4.415)	0.127	0.230 (3.996)	0.128
EW30	SH	0.154 (2.870)	0.115	0.157 (2.687)	0.124
$\lambda = 1$, C	SH	0.299 (5.628)	-0.069	0.300 (5.165)	-0.035
$\lambda = 1$, TV	SH	0.580 (11.260)		0.558 (10.014)	-0.025
$\lambda = 0.75$, C	SH	0.299 (5.620)	-0.067	0.300 (5.160)	-0.034
$\lambda = 0.75$, TV	SH	0.580 (11.258)		0.558 (10.017)	-0.025
$\lambda = 0.5$, C	SH	0.299 (5.603)	-0.062	0.300 (5.147)	-0.031
$\lambda = 0.5$, TV	SH	0.580 (11.251)		0.558 (10.019)	-0.024
$\lambda = 0.25$, C	SH	0.297 (5.532)	-0.048	0.299 (5.091)	-0.022
$\lambda = 0.25$, TV	SH	0.579 (11.208)		0.559 (10.007)	-0.024
$\lambda = 0$, C	SH	0.151 (2.824)	$+0.052$	0.159 (2.718)	$+0.022$
$\lambda = 0$, TV	SH	0.148 (2.749)		0.157 (2.644)	$+0.011$

Panel B: C-CAPM

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.234 (4.415)	0.127	0.230 (3.996)	0.128
EW30	SH	0.154 (2.870)	0.115	0.157 (2.687)	0.124
$\lambda = 1,\ C$	SH	0.299 (5.628)	-0.069	0.300 (5.165)	-0.035
$\lambda = 1,\ TV$	SH	0.582 (10.978)		0.574 (10.100)	0.043
$\lambda = 0.75,\ C$	SH	0.299 (5.627)	-0.073	0.300 (5.164)	-0.035
$\lambda = 0.75,\ TV$	SH	0.580 (10.917)		0.571 (10.026)	0.044
$\lambda = 0.5,\ C$	SH	0.299 (5.625)	-0.080	0.300 (5.161)	-0.035
$\lambda = 0.5,\ TV$	SH	0.576 (10.742)		0.566 (9.826)	0.045
$\lambda = 0.25,\ C$	SH	0.299 (5.619)	-0.097	0.300 (5.152)	-0.032
$\lambda = 0.25,\ TV$	SH	0.554 (9.827)		0.540 (8.866)	0.047
$\lambda = 0,\ C$	SH	-0.015 (-0.286)	-0.105	-0.024 (-0.435)	-0.006
$\lambda = 0,\ TV$	SH	0.013 (0.250)		0.004 (0.067)	0.043

Panel C: Fama-French

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.234 (4.415)	0.127	0.230 (3.996)	0.128
EW30	SH	0.154 (2.870)	0.115	0.157 (2.687)	0.124
$\lambda = 1,\ C$	SH	0.299 (5.628)	-0.069	0.300 (5.165)	-0.035
$\lambda = 1,\ TV$	SH	0.580 (11.260)		0.558 (10.014)	-0.025
$\lambda = 0.75,\ C$	SH	0.299 (5.643)	-0.079	0.299 (5.164)	-0.034
$\lambda = 0.75,\ TV$	SH	0.582 (11.277)		0.558 (10.019)	-0.025
$\lambda = 0.5,\ C$	SH	0.293 (5.564)	-0.066	0.291 (5.068)	-0.031
$\lambda = 0.5,\ TV$	SH	0.578 (11.249)		0.554 (9.975)	-0.025
$\lambda = 0.25,\ C$	SH	0.256 (4.926)	-0.057	0.252 (4.444)	-0.022
$\lambda = 0.25,\ TV$	SH	0.541 (10.758)		0.518 (9.514)	-0.025
$\lambda = 0,\ C$	SH	0.097 (1.881)	-0.002	0.095 (1.691)	$+0.009$
$\lambda = 0,\ TV$	SH	0.169 (3.283)		0.169 (3.051)	-0.006

Table VI

Ex-Post and In-Sample Performances of the Tangency Portfolios (International Stocks)

This table reports the ex-post and in-sample Sharpe ratios (SH) of the tangency portfolios for different values of λ. Risky assets include equity returns on four country indexes: US, UK, Japan, Germany. The parameter of risk aversion is set equal to one. Weights estimation is performed by exactly-identified GMM with (TV) and without (C) conditioning information. The set of instruments includes a constant, DINFLUS, EURO, USDIVYLD, WLDMK, and DEFPREM. t-statistics (in parentheses) are obtained by the delta method. The third and fifth columns of each panel report the in-sample Sharpe ratios of the tangency portfolios. The fourth and sixth columns of each panel report the out-of-sample Sharpe ratios of the tangency portfolios. The "EW4" strategy invests equal dollar amounts in $N = 4$ international stocks every month. The in-sample numbers are calculated over the same time-period of the corresponding out-of-sample ones in the same panel. The second row of the table reports the in-sample and out-of-sample Sharpe ratios obtained under no-short sales constraints. The out-of-sample quantities are computed using a 60-month and a 120-month rolling window, respectively.

Panel A: IS–CAPM

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.162 (2.540)	0.098	0.180 (2.480)	0.148
EW4	SH	0.154 (2.466)	0.046	0.165 (2.328)	0.134
$\lambda = 1, \quad C$	SH	0.162 (2.540)	0.060	0.180 (2.480)	0.129
$\lambda = 1, \quad TV$	SH	0.336 (6.038)	0.102	0.356 (5.494)	0.093
$\lambda = 0.75, \quad C$	SH	0.162 (2.549)	0.057	0.181 (2.488)	0.130
$\lambda = 0.75, \quad TV$	SH	0.334 (5.994)	0.101	0.356 (5.476)	0.097
$\lambda = 0.5, \quad C$	SH	0.162 (2.556)	0.051	0.181 (2.495)	0.128
$\lambda = 0.5, \quad TV$	SH	0.327 (5.802)	0.100	0.352 (5.313)	0.103
$\lambda = 0.25, \quad C$	SH	0.162 (2.558)	0.042	0.181 (2.498)	0.120
$\lambda = 0.25, \quad TV$	SH	0.297 (5.049)	0.095	0.320 (4.623)	0.118
$\lambda = 0, \quad C$	SH	0.160 (2.555)	0.029	0.179 (2.496)	0.102
$\lambda = 0, \quad TV$	SH	0.157 (2.509)	0.015	0.177 (2.464)	0.123

Panel B: I-CAPM (PPP)

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.162 (2.540)	0.098	0.180 (2.480)	0.148
EW4	SH	0.154 (2.466)	0.046	0.165 (2.328)	0.134
$\lambda = 1,\ C$	SH	0.162 (2.540)	0.060	0.180 (2.480)	0.129
$\lambda = 1,\ TV$	SH	0.336 (6.038)	0.102	0.356 (5.494)	0.093
$\lambda = 0.75,\ C$	SH	0.161 (2.538)	0.029	0.179 (2.472)	0.123
$\lambda = 0.75,\ TV$	SH	0.331 (5.843)	0.107	0.347 (5.222)	0.102
$\lambda = 0.5,\ C$	SH	0.160 (2.535)	0.009	0.178 (2.463)	0.102
$\lambda = 0.5,\ TV$	SH	0.314 (5.363)	0.114	0.321 (4.648)	0.112
$\lambda = 0.25,\ C$	SH	0.160 (2.530)	−0.002	0.177 (2.452)	0.086
$\lambda = 0.25,\ TV$	SH	0.274 (4.463)	0.120	0.267 (3.720)	0.121
$\lambda = 0,\ C$	SH	0.159 (2.524)	−0.009	0.175 (2.441)	0.076
$\lambda = 0,\ TV$	SH	0.199 (3.165)	0.112	0.179 (2.507)	0.118

Panel C: I-CAPM (SPOT)

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.162 (2.540)	0.098	0.180 (2.480)	0.148
EW4	SH	0.154 (2.466)	0.046	0.165 (2.328)	0.134
$\lambda = 1,\ C$	SH	0.162 (2.540)	0.060	0.180 (2.480)	0.129
$\lambda = 1,\ TV$	SH	0.336 (6.038)	0.102	0.356 (5.494)	0.093
$\lambda = 0.75,\ C$	SH	0.138 (2.237)	0.044	0.148 (2.125)	0.111
$\lambda = 0.75,\ TV$	SH	0.322 (5.890)	0.096	0.341 (5.412)	0.091
$\lambda = 0.5,\ C$	SH	0.105 (1.759)	0.023	0.108 (1.601)	0.085
$\lambda = 0.5,\ TV$	SH	0.273 (4.994)	0.085	0.285 (4.588)	0.085
$\lambda = 0.25,\ C$	SH	0.075 (1.256)	−0.001	0.072 (1.071)	0.056
$\lambda = 0.25,\ TV$	SH	0.152 (2.639)	0.050	0.151 (2.346)	0.063
$\lambda = 0,\ C$	SH	0.049 (0.832)	−0.025	0.042 (0.633)	0.030
$\lambda = 0,\ TV$	SH	−0.004 (−0.061)	-0.076	−0.004 (−0.063)	−0.008

Panel D: II–CAPM (PPP)

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.162 (2.540)	0.098	0.180 (2.480)	0.148
EW4	SH	0.154 (2.466)	0.046	0.165 (2.328)	0.134
$\lambda = 1,\ C$	SH	0.162 (2.540)	0.060	0.180 (2.480)	0.129
$\lambda = 1,\ TV$	SH	0.336 (6.038)	0.102	0.356 (5.494)	0.093
$\lambda = 0.75,\ \ C$	SH	0.160 (2.494)	0.057	0.183 (2.493)	-0.011
$\lambda = 0.75,\ \ TV$	SH	0.335 (5.987)	0.110	0.358 (5.503)	0.092
$\lambda = 0.5,\ \ C$	SH	0.157 (2.450)	0.050	0.184 (2.492)	-0.023
$\lambda = 0.5,\ \ TV$	SH	0.325 (5.761)	0.120	0.349 (5.337)	0.089
$\lambda = 0.25,\ \ C$	SH	0.155 (2.413)	0.048	0.183 (2.493)	-0.027
$\lambda = 0.25,\ \ TV$	SH	0.301 (5.270)	0.130	0.321 (4.865)	0.082
$\lambda = 0,\ \ C$	SH	0.153 (2.381)	0.047	0.183 (2.480)	-0.029
$\lambda = 0,\ \ TV$	SH	0.262 (4.493)	0.135	0.274 (4.065)	0.069

Panel E: II–CAPM (SPOT)

Strategies	Statistics	In-sample	60-month window	In-sample	120-month window
No Short Sales	SH	0.162 (2.540)	0.098	0.180 (2.480)	0.148
EW4	SH	0.154 (2.466)	0.046	0.165 (2.328)	0.134
$\lambda = 1,\ C$	SH	0.162 (2.540)	0.060	0.180 (2.480)	0.129
$\lambda = 1,\ TV$	SH	0.336 (6.038)	0.102	0.356 (5.494)	0.093
$\lambda = 0.75,\ \ C$	SH	-0.151 (-2.419)	0.011	-0.163 (-2.301)	0.050
$\lambda = 0.75,\ \ TV$	SH	0.338 (6.098)	0.106	0.367 (5.670)	0.099
$\lambda = 0.5,\ \ C$	SH	-0.153 (-2.442)	-0.003	-0.166 (-2.332)	0.035
$\lambda = 0.5,\ \ TV$	SH	0.330 (5.988)	0.112	0.370 (5.750)	0.107
$\lambda = 0.25,\ \ C$	SH	-0.154 (-2.448)	-0.008	-0.167 (-2.340)	0.029
$\lambda = 0.25,\ \ TV$	SH	0.302 (5.484)	0.115	0.348 (5.478)	0.109
$\lambda = 0,\ \ C$	SH	-0.154 (-2.451)	-0.011	-0.167 (-2.344)	0.027
$\lambda = 0,\ \ TV$	SH	0.247 (4.408)	0.106	0.288 (4.492)	0.098

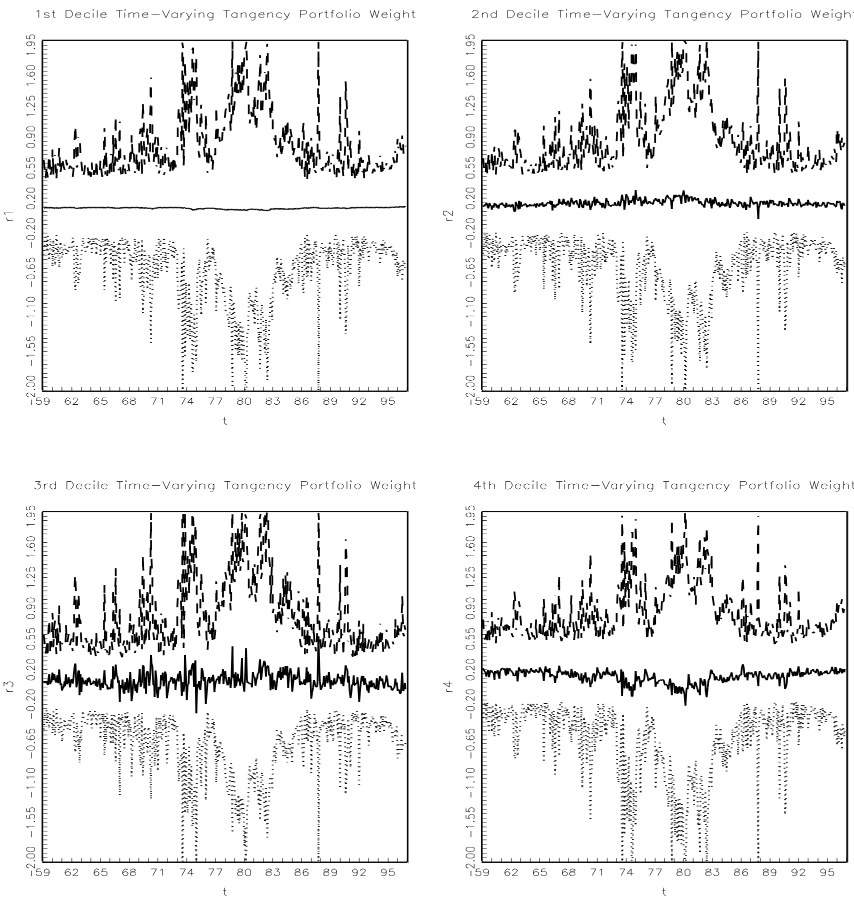

Figure 1. Time Varying Weights. The figure displays the time-varying behavior of the scaled tangency portfolio weights associated with the first four NYSE-AMEX-NASDAQ size-sorted portfolios. The vector of state variables includes a constant and the lagged values of XEW, INFL, and DIV. Weights estimation is performed by exactly-identified GMM with mean asset returns constrained by a CAPM. Dotted lines represent 95% exact confidence bounds.

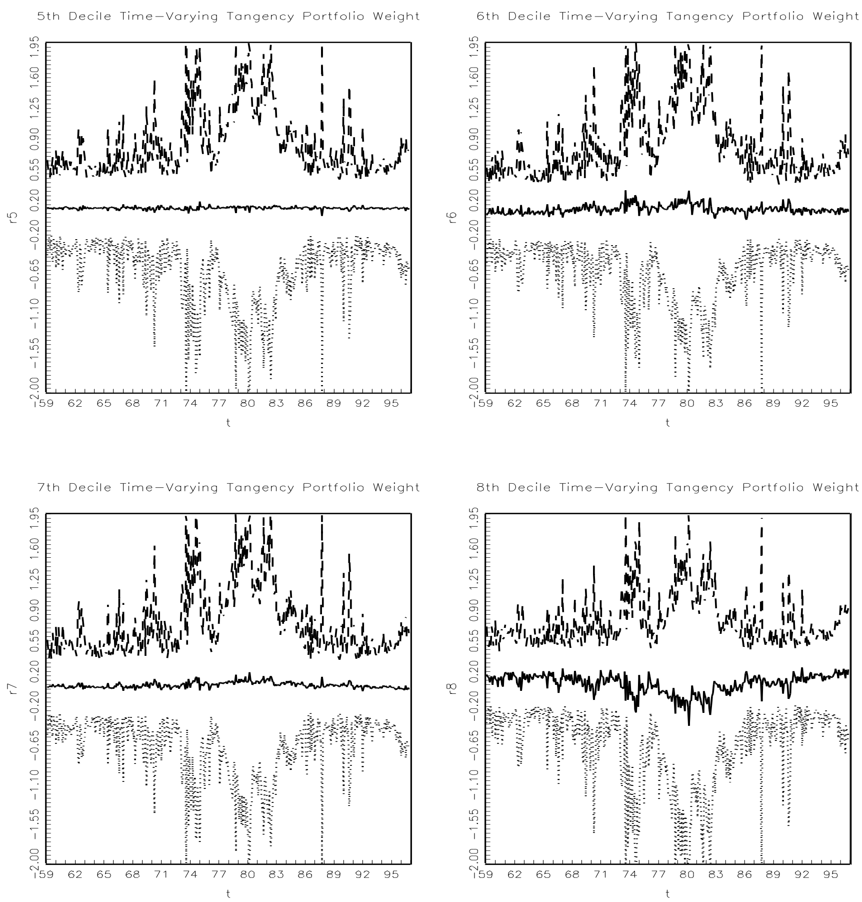

Figure 2. Time Varying Weights. The figure displays the time-varying behavior of the scaled tangency portfolio weights associated with the 5th to 8th NYSE-AMEX-NASDAQ size-sorted portfolios. The vector of state variables includes a constant and the lagged values of XEW, INFL, and DIV. Weights estimation is performed by exactly-identified GMM with mean asset returns constrained by a CAPM. Dotted lines represent 95% exact confidence bounds.

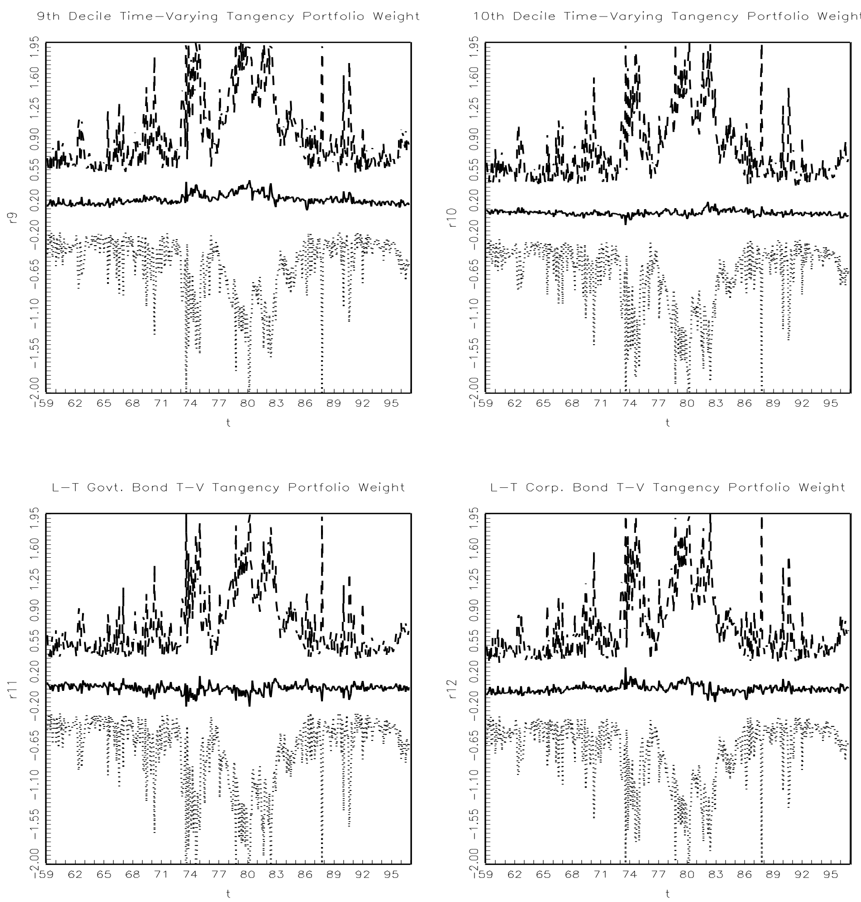

Figure 3. Time Varying Weights. The figure displays the time-varying behavior of the scaled tangency portfolio weights associated with the 9th and 10th NYSE-AMEX-NASDAQ size-sorted portfolios, a long-term government, and a long-term corporate bond. The vector of state variables includes a constant and the lagged values of XEW, INFL, and DIV. Weights estimation is performed by exactly-identified GMM with mean asset returns constrained by a CAPM. Dotted lines represent 95% exact confidence bounds.