Banks, Credit Market Frictions, and Business Cycles

Ali Dib†
Bank of Canada
April 2, 2010

Abstract
This paper proposes a fully micro-founded framework that incorporates an active banking sector into New Keynesian DSGE models with financial frictions. Then, it evaluates the role and importance of banks’s behavior and financial shocks in the U.S. business cycles. The banking sector consists of two types of heterogenous banks that offer different banking services and interact in an interbank market. Loans are produced using interbank borrowing and bank capital subject to the bank capital requirement condition. Banks have monopoly power, set nominal deposit and prime lending rates, choose their portfolio compositions and their leverage ratio, and can endogenously default on fractions of their interbank borrowing and bank capital returns. Also, the model includes unconventional monetary policy shocks. Overall, an active banking sector amplifies real effects of supply-side shocks, while it dampens effects of demand-side and financial shocks on real variables. The presence of an active banking sector reduces the impacts of financial shocks, lowers macroeconomic volatilities, and improves social welfare. Moreover, expansionary unconventional monetary policies reduce negative impacts of financial crises.

JEL classification: E32, E44, G1
Keywords: Banks; Interbank market; Bank capital; Credit; Financial shocks; Monetary policy.

*I am grateful to Ron Alquist, Ricardo Caballero, Lawrence Christiano, Carlos de Resende, Brigitte Desroches, Andrea Gerali, Sharon Kozicki, Robert Lafrance, Philipp Maier, Federico Mandelman, Virginia Queijo von Heideken, Julio Rotemberg, Eric Santor, Lawrence Schembri, Jack Selody, Moez Souissi, Skander van Den Heuvel, seminar participants at the Bank of Canada, the 2009 NBER/Philadelphia Fed. Workshop on “Methods and Applications for the DSGE Models”, MIT, IMF, Federal Reserve Bank of Richmond, Reserve Bank of Australia, Australian National University, University of Ottawa, and and participants at the BIS/ECB workshop “Monetary Policy and Financial Stability,” the BoC/IMF workshop on “Economic Modeling and the Financial Crisis,” Canadian Economic Association, and Computational for Economic and Finance, for their comments and discussions. The views expressed in this paper are those of the author and should not be attributed to the Bank of Canada.

†International Economic Analysis Department, Bank of Canada. 234 Wellington St. Ottawa, ON. K1A 0G9, Canada. Email: ADib@bankofcanada.ca, Phone: 1-613-782 7851, Fax: 1-613-782 7658.
1. Introduction

The recent global financial crisis has underscored the need to develop DSGE models with real-financial linkages and an active banking sector. Such a model would allow an empirical evaluation of banks’ role and behavior in the transmission and propagation of supply and demand shocks, and an assessment of the importance of financial shocks, originating in the banking sector, as a source of business cycle fluctuations. The banking sector, however, has been ignored in most DSGE models developed in the literature, except some very recent papers.\(^1\) Moreover, in the literature, financial frictions are usually modeled only on the demand side of the credit market using either the Bernanke, Gertler and Gilchrist (1999) financial accelerator mechanism (BGG, hereafter) or the Iacovello (2005) framework.\(^2\) In light of the ongoing financial crisis, real-financial linkages have become the focus of attention.

This paper proposes a microfounded framework that incorporates an active banking sector, interbank market, bank capital, and a credit market into a DSGE model with a financial accelerator à la BGG (1999).\(^3\) The model is calibrated to the U.S. economy and used to evaluate the role of profit-maximizing banks in business cycles and in the transmission and propagation of shocks to the real economy, to assess the importance of financial shocks in explaining macroeconomic fluctuations, and to examine the potential role of unconventional monetary policies (quantitative and qualitative monetary easing) in offsetting the real impacts of the financial crisis.

The paper is related to the following studies: Goodhart, Sunirand and Tsomocos (2006), Christiano, Motto and Rostagno (2009), Cúrdia and Woodford (2009a,b), de Walque, Pierrard and Rouabah (2009), and Gerali, Neri, Sessa and Signoretti (2009). In contrast to the previous studies that examine the role of bank capital in the business cycle fluctuations, this paper introduces bank capital to satisfy the bank’s capital requirement condition as in Basel II Accords, which is a pre-condition to operate and make loans to entrepreneurs.\(^4\) Therefore, this allows

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\(^1\)For example, Cúrdia and Woodford (2009a,b), Gertler and Karida (2009), and Zhang(2009).

\(^2\)For example, Carlstrom and Fuerst (1997), Cespedes, Chang and Velasco (2004), Elekdag, Justiniano and Tchakarov (2006), and Christensen and Dib (2008).

\(^3\)This framework is fully microfounded in the sense that all banks maximize profits and take optimal decisions under different constraints.

\(^4\)For example, Holmstrom and Tirole (1997), Meh and Moran (2009), Markovic (2006), Goodfriend and McCallum (2007), and others.
bank capital to be an attenuation mechanism of real effects of demand and financial shocks. For instance, an increase in borrowing demand by entrepreneurs, induced by a drop of policy rate, forces banks to increase their leverage ratio and/or bank capital holdings. Higher leverage ratio and/or higher bank capital imply higher marginal costs of raising bank capital and, thus, higher marginal costs of producing loans that imply a higher lending prime rate. This increases external financing costs of entrepreneurs and erodes a part of initial entrepreneurs’s demand to finance their new investment.

Our basic model is a DSGE model for a closed economy similar to Christensen and Dib (2008), which is based on BGG (1999). The key addition to this model is the supply-side of the credit market, which is introduced in the form of an active banking sector with an interbank market. The model incorporates an optimizing banking sector with two types of monopolistically competitive banks: “savings banks” and “lending banks”. It works as follows: Banks supply different banking services and the two types of banks interact in the interbank market. They have the monopoly power when setting nominal deposit and prime lending rates (subject to quadrating adjustment costs). Savings banks collect deposits from workers, set nominal deposit rates, and choose the composition of their portfolio (composed of risk-free assets and risky interbank lending) to maximize profits. Lending banks borrow from savings banks on the interbank market and receive bank capital from bankers to satisfy the bank capital requirement condition, which imposes a minimum level of bank capital must be held to provide loans to entrepreneurs. Lending banks can receive, if needed, liquidity injections from the central bank and/or swap a fraction of their loans (risky assets) for government bonds (risk-free assets).

Following Goodhart et al. (2006), we assume endogenous strategic or necessary defaults on bank capital and interbank borrowing, optimally chosen by lending banks; however, when defaulting, banks pay expected convex penalties in the next period. In addition, banks optimally choose their leverage ratio, that is, the ratio of loans to bank capital subject to the maximum leverage ratio imposed by regulators. We assume the presence of convex gains of holding bank capital in excess of the required level. This implies that variations in the banks’ leverage ratio directly affect the marginal cost of raising bank capital. Therefore, movements in the banks’

5The two different banks are necessary to generate heterogeneity, which in turn leads to an interbank market where different banks can interact.
leverage ratio may amplify or dampen the effects of the business cycles, as pointed out by Fostel and Geanakopolos (2008) and Geanakopolos (2009).6

During normal times, the central bank conducts monetary policy following a standard Taylor rule: The central bank adjusts short-term nominal interest rates in response to inflation and output changes. However, during crisis periods, the central bank can use unconventional (quantitative and qualitative) monetary policies by injecting newly created money to the banking system and/or swapping a fraction of banks’ loans for government bonds.7 Through these channels, the central bank can serve as the lender of last resort to lending banks during crisis.

In this framework, the banking sector affects credit market conditions and, thus, the real economy through the following channels: (1) variations in bank capital and bank capital price expectations; (2) monopoly power in setting nominal deposit and lending interest rates with nominal rigidities implies that interest rate spreads vary over business cycles;8 (3) the optimal allocation of the banks’ portfolio between interbank lending (risky-assets) and risk-free asset holdings (government bonds); (4) the optimal choice of the banks’ leverage ratio that is subject to the bank capital requirement condition; (5) the default risk channels that arise from endogenous strategic or necessary defaults on interbank borrowing and bank capital returns; and (6) marginal costs of raising bank capital.

The economy is subject to two supply shocks (technology and investment-efficiency shocks) three demand shocks (monetary policy, government spending, and preferences shocks), two financial shocks (riskiness and financial intermediation process), and two quantitative and qualitative monetary easing shocks. Supply and demand shocks are commonly used in the literature; however, financial shocks require some explanation. Riskiness shocks are modeled as shocks to the elasticity of the risk premium that affect the external finance costs of entrepreneurs. They are meant to represent shocks to the standard deviation of the entrepreneurial distribution, as in Christiano et al. (2009), shocks to agency costs paid by lending banks to monitor entrepreneurs’ output, and/or shocks to entrepreneurs’ default threshold.9 These shocks may be

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6The cost of bank capital depends on the bank’s capital position. If banks hold excess bank capital, the marginal cost of raising bank capital on the market is lower, since banks are well capitalized.
7Quantitative easing, which is associated with newly created money, expands banks’ balance sheets; while swapping banks’ assets for government bonds changes only banks’ assets compositions.
8See Cúrdia and Woodford (2009a) for the importance of time-varying spreads on monetary policy.
9As shown in Bernanke et al. (1999), the elasticity of the external finance premium to the entrepreneurs’ leverage ratio depends on the standard deviation of the entrepreneurial distribution, the agency cost parameter,
interpreted as exogenous changes in the confidence of banks with credit risks in their borrowers and/or the overall health of the economy, thus affecting external costs of entrepreneurial borrowing. Shocks to financial intermediation process are exogenous events that affect the credit supply of lending banks. They may represent technological advances or disruptions in the intermediation process, or approximate perceived changes in creditworthiness.\textsuperscript{10} Finally, quantitative and qualitative monetary easing shocks are used by the central bank to provide liquidity to the banking system and to enhance banks’ conditions.

The model is successful in reproducing most of the salient features of the U.S. economy: key macroeconomic volatilities, autocorrelations, and correlations with output. Moreover, the presence of an active banking sector is welfare improving. This results from banks sharing risks with entrepreneurs by offering non-contingent debt contracts. Also, bank capital acts to attenuate the real effects of demand and financial shocks, as banks effectively unsure workers’ consumption. Thus, the main role of banks in this economy is to reduce the negative impact of uncertainty in the presence of different structural shocks, particularly financial shocks. At the same time, financial shocks cause business cycles, as disturbances in the banking sector may be a source of macroeconomic fluctuations. We also find that bank leverage is procyclical, indicating that banks are willing to extend more loans during booms and tend to restrict their supply of credit during recessions.

The paper proceeds as follows. Section 2 presents the model. Section 3 discusses the parameter calibration. Section 4 discusses the empirical results. Section 5 concludes.

2. The Model

The economy is inhabited by two types of households (workers and bankers). The banking sector consists of two types of heterogenous monopolistically competitive banks. We call them “savings” and “lending” banks to indicate that they offer different banking services, but interact in an interbank market. As in BGG (1999), the production sector consists of entrepreneurs, capital producers, and retailers. Finally, there is a central bank and a government.

\textsuperscript{10}Advances in financial engineering, credit rationing, and highly sophisticated methods for sharing risk are examples of intermediation process shocks.
2.1 Households

2.1.1 Workers

Workers derive utility from total consumption, \( C^w_t \); real money balances, \( M^c_t \); and leisure, \( 1 - H_t \), where \( H_t \) denotes hours worked. The workers’ preferences are described by the following expected utility function:

\[
V^w_0 = E_0 \sum_{t=0}^{\infty} \beta^t w_u(C^w_t, M^c_t, H_t).
\]  

(1)

The single-period utility is

\[
u(\cdot) = e_1 \left( \frac{C^w_t}{C^w_{t-1}} \right)^{1-\gamma w} + \frac{\varphi (M^c_t)^{1-\nu}}{1-\nu} + \frac{\eta (1-H_t)^{1-\zeta}}{1-\zeta},
\]

(2)

where \( \varphi \in (0, 1) \) is a habit formation parameter; \( \gamma w \) is a positive parameter denoting the workers’ risk aversion and the inverse of the elasticity of intertemporal substitution of consumption; \( \nu \) denotes the money-interest elasticity; and \( \zeta \) is the inverse of the elasticity of intertemporal substitution of leisure. The parameters \( \varphi \) and \( \eta \) measure the weight on real cash balances and leisure in the utility function, respectively. \( e_t \) is a taste shock that follows an AR(1) process.

The representative worker enters period \( t \) with \( D_{t-1} \) units of real deposits in savings banks and \( M^c_{t-1} \) units of real money balances held outside of banks, which do not earn interest. Deposits pay the gross nominal interest rate \( R^D_t \) set by savings banks between \( t \) and \( t+1 \). During period \( t \), workers supply labour to the entrepreneurs, for which they receive real labor payment \( W_t H_t \), (\( W_t \) is the economy-wide real wage). Furthermore, they receive dividend payments, \( \Pi^R_t \), from retail firms, as well as a lump-sum transfer from the monetary authority, \( T_t \), and pay lump-sum taxes to government, \( \tilde{T}^w_t \). Workers allocate their funds to private consumption \( C^w_t \), real money holdings \( M^c_t \), and real deposits, \( D_t \). Their budget constraint in real terms is

\[
C^w_t + M^c_t + D_t \leq W_t H_t + \frac{R^D_{t-1} D_{t-1}}{\pi_t} + \frac{M^c_{t-1}}{\pi_t} + \Pi^R_t + T_t - \tilde{T}^w_t,
\]

(3)

where \( \pi_{t+1} = P_{t+1}/P_t \) is the gross inflation rate. A representative worker household chooses \( C^w_t, M^c_t, H_t, \) and \( D_t \) to maximize its expected lifetime utility, Eq. (1), subject to the single-period utility function, Eq. (2), and the budget constraint, Eq. (3). The first-order of this optimization problem are in Appendix A.

\footnote{In this economy, \( R^D_t \) is different from the rate of return on government bonds.}
2.1.2 Bankers

Bankers (bank owners) own the two types of banks, from which they receive profits. They consume, have access to the non-contingent government bond market, and accumulate bank capital supplied to lending banks to satisfy the bank capital requirement for a contingent bank capital return. It is assumed that bankers’ preferences depend only on consumption and are given by

\[ V_0^b = E_0 \sum_{t=0}^{\infty} \beta_t u \left( C_t^b \right). \]  

The single-period utility function is

\[ u(\cdot) = \frac{e_t}{1 - \gamma_b} \left( \frac{C_t^b}{(C^b_{t-1})^{\gamma_b}} \right)^{1 - \gamma_b}, \]  

where \( \gamma_b \) is a positive structural parameter denoting bankers’ risk aversion and the inverse of the elasticity of intertemporal substitution. \( e_t \) denotes a preference shock which follows an AR(1) process.

Bankers enter period \( t \) with \((1 - \delta_{t-1})Z_{t-1}\) units of bank capital stock, whose price is \( Q_t^Z \) in period \( t \), where \( \delta_{t-1}^Z \) is a banks’ default on bank capital occurring at the end of the period \( t - 1 \), and \( Z_t \) is total claims (bank equities or shares) held by bankers. Bank capital pays a contingent gross nominal return rate \( R_t^Z \) between \( t - 1 \) and \( t \). Bankers also enter period \( t \) with \( B_{t-1} \) units of real government bonds that pay the gross risk-free nominal interest rate \( R_t \) between \( t \) and \( t + 1 \). During period \( t \), bankers receive profit payments, \( \Pi_t^{sb} \) and \( \Pi_t^{lb} \) from saving and lending banks, and pay lump-sum taxes to government, \( \tilde{T}_t^b \). They allocate these funds to consumption \( C_t^b \), real government bonds \( B_t \), and real bank capital acquisition \( Q_t^Z Z_t \). We assume quadratic adjustment costs to alter the bank’s capital stock.\(^{12}\) Bankers’ budget constraint in real terms is

\[ C_t^b + Q_t^Z Z_t + B_t = \frac{R_{t-1} B_{t-1}}{\pi_t} + (1 - \delta_{t-1}^Z) \frac{R_t^Z Q_t^Z Z_{t-1}}{\pi_t} - \frac{\chi Z}{2} \left( \frac{\pi_t Z_t}{Z_{t-1}} - \pi \right)^2 Q_t^Z Z_t + \Pi_t^{sb} + \Pi_t^{lb} - \tilde{T}_t^b. \]  

\(^{12}\)We interpret these adjustment costs as costs paid to brokers or the costs of collecting information about the banks’ balance sheet.
A representative banker chooses $C^b_t$, $B_t$, and $Z_t$ in order to maximize its expected lifetime utility Eq.(4), subject to Eq.(5) and the budget constraint, Eq.(6). The first-order conditions for this optimization problem are:

\[ e_t \left( \frac{C^b_t}{(C^b_{t-1})^{\gamma_b}} \right)^{1-\gamma_b} - \beta_b \varphi E_t \left[ e_{t+1} \left( \frac{C^b_{t+1}}{(C^b_{t})^{\gamma_b}} \right)^{1-\gamma_b} \right] = C^b_t \lambda^b_t; \quad (7) \]

\[ \frac{\lambda^b_t}{R_t} = \beta_b E_t \left[ \frac{\lambda^b_{t+1}}{\pi_{t+1}} \right]; \quad (8) \]

\[ \beta_b E_t \left\{ \frac{\lambda^b_{t+1} Q^Z_{t+1}}{\pi_{t+1}} \left[ (1 - \delta_t^Z) R^Z_{t+1} + \chi_Z \left( \frac{\pi_{t+1} Z_{t+1}}{Z_t} - \pi \right) \left( \frac{\pi_{t+1} Z_{t+1}}{Z_t} \right)^2 \right] \right\} = \lambda^w_t Q^Z_t \left[ 1 + \chi_Z \left( \frac{\pi_t Z_t}{Z_{t-1}} - \pi \right) \frac{\pi_t Z_t}{Z_{t-1}} \right]; \quad (9) \]

where $\lambda^b_t$ is the Lagrangian multiplier associated with the bankers’ budget constraint.

Eq.(7) determines the marginal utility of banker’s consumption. Eq.(8) relates the marginal rate of substitution to the real interest rate on bonds. Finally, Eq.(9) corresponds to the optimal dynamic evolution of the bank capital stock.

Combining conditions (8) and (9) yields the following condition relating return on bank capital $R^Z_t$ to the risk-free interest rate on government bonds $R_t$:

\[ E_t \left\{ \frac{Q^Z_{t+1}}{Q^Z_t} \left[ (1 - \delta_t^Z) R^Z_{t+1} + \chi_Z \left( \frac{\pi_{t+1} Z_{t+1}}{Z_t} - \pi \right) \left( \frac{\pi_{t+1} Z_{t+1}}{Z_t} \right)^2 \right] \right\} = R_t \left[ 1 + \chi_Z \left( \frac{\pi_t Z_t}{Z_{t-1}} - \pi \right) \frac{\pi_t Z_t}{Z_{t-1}} \right]. \quad (10) \]

This condition leads to three channels, through which changes in bank capital affect the real economy: (1) the price expectation channel, which arises from expectations of capital gains or losses from holding bank capital shares, due to expected changes in the price of bank capital $E_t \left( Q^Z_{t+1}/Q^Z_t \right)$. This channel implies that the efficient market hypothesis does not hold in the short run, (2) the adjustment cost channel, which is a result of the information asymmetry between bankers and banks. It implies changes in current and expected stocks of bank capital given by the terms $\chi Z (\cdot)$. The presence of adjustment costs is necessary to reduce the information asymmetry and the adjustment costs are interpreted as costs to
enter into the bank capital market. Finally, the default risk channel arises from the existence of the probability of default on bank capital repayment, \( \delta_Z^t > 0 \), decided by the lending banks. This default probability is counter-cyclical. Therefore, movements in bank capital, caused by macroeconomic fluctuations, have direct impacts on bank capital accumulation and consequently on credit supply conditions.

2.2 Banking sector

The banking sector consists of two types of heterogeneous profit-maximizing banks: Savings and lending banks.

2.2.1 Savings banks

Savings banks refer to all banks that are net lender in the interbank market. There is a continuum of savings banks, operating in a monopolistically competitive environment when collecting deposits, \( D_t \), from workers. We assume that all deposits are fully insured. Each bank \( j \in (0, 1) \) sets the deposit interest rate \( R^D_{j,t} \) paid on deposits and chooses the optimal allocation of its portfolio between lending a fraction \( s_{j,t} \) of deposits on the interbank market, \( \tilde{D}_{j,t} = s_{j,t}D_{j,t} \), (interbank lending) to lending banks, and investing the fraction \( (1 - s_{j,t}) \) in risk-free assets, \( B^b_{j,t} \), that are government bonds that pay the policy rate \( R_t \). Each period, there is a probability \( \delta^D_t \) that lending banks default on their interbank borrowing. When investing in interbank lending, savings banks pay quadratic costs to monitor lending banks. The interbank rate, \( R^I^B_t \), is endogenously determined to clear the interbank market. Table 1 displays the balance sheet of the \( j \)'th savings bank.\(^{13}\)

<table>
<thead>
<tr>
<th>Table 1: Savings bank's balance sheet</th>
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</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
</tr>
<tr>
<td>Interbank lending: ( \tilde{D}_{j,t} )</td>
</tr>
<tr>
<td>Government bonds: ( B^b_{j,t} )</td>
</tr>
</tbody>
</table>

Given monopolistic competition and the imperfect substitution between deposits, the \( j \)'th savings bank faces the following deposit supply function, that is increasing in the relative

\(^{13}\)Note that \( \tilde{D}_{j,t} = s_{j,t}D_{j,t} \) and \( B^b_{j,t} = (1 - s_{j,t})D_{j,t} \) where \( s_{j,t} \in (0, 1) \).
deposit interest rate across period. As in Gerali et al. (2009), the individual deposit supply is

\[ D_{j,t} = \left( \frac{R_{j,t}^D}{R_t^D} \right)^\vartheta D_t, \]  

(11)

where \( D_{j,t} \) is deposits supplied to bank \( j \), while \( D_t \) denotes total deposits in the economy; and \( \vartheta_D > 1 \) is the elasticity of substitution between different types of deposits.\textsuperscript{14} Also, there is a quadratic adjustment cost of intertemporally varying the deposit interest rate. This rigidity allows an interest rate spread that evolves over the cycle. We assume adjustment costs à la Rotemberg (1982), given by

\[ Ad_{j,t}^{R^D} = \frac{\phi_{R^D}}{2} \left( \frac{R_{j,t}^D}{R_{j,t-1}^D} - 1 \right)^2 D_t, \]  

(12)

where \( \phi_{R^D} > 0 \) is an adjustment cost parameter. The optimization problem of the \( j^{th} \) savings bank is

\[ \max_{s_{j,t}, R_{j,t}^I} E_0 \sum_{t=0}^{\infty} \beta^t \lambda^b_\ell \left\{ \left[ s_{j,t} R_{t}^{IB} (1 - \delta_D) + (1 - s_{j,t}) R_t - R_{j,t}^D \right] D_{j,t} - \frac{\chi_s}{2} (s_{j,t} D_{j,t})^2 - Ad_{j,t}^{R^D} \right\}, \]

subject to (11) and (12). Because bankers are the sole owners of banks, the discount factor is the stochastic process \( \beta^t \lambda^b_\ell \), where \( \lambda^b_\ell \) denotes the marginal utility of bankers’ consumption.\textsuperscript{15}

The term \( s_{j,t} R_{t}^{IB} (1 - \delta_D) + (1 - s_{j,t}) R_t \) is the weighted average return of investing in interbank lending and in holding government bonds. The term \( 0.5 \chi_s (s_{j,t} D_{j,t})^2 \) represents quadratic monitoring costs of lending out on the interbank market, where \( \chi_s > 0 \) is a parameter determining the steady-state level of these costs.

\textsuperscript{14}This supply function is derived from the definition of aggregate supply of deposits, \( D_t \), and the corresponding deposit interest rate, \( R_t^D \), in the monopolistic competition framework, as follows:

\[ D_t = \left( \int_0^1 D_{j,t}^{1+\delta_D} d_j \right)^{\frac{1}{1+\delta_D}} \text{ and } R_{t}^D = \left( \int_0^1 R_{j,t}^{D,1+\delta_D} d_j \right)^{\frac{1}{1+\delta_D}}, \]

where \( D_{j,t} \) and \( R_{j,t}^D \) are the supply and deposit interest rate faced by each savings bank \( j \in (0,1) \).

\textsuperscript{15}Savings banks take \( \delta^s_t \) as given when maximizing their profits.
The interbank rate is endogenously determined as:

\[ R_{IBt} = R_t + \alpha \chi_s D_t \left( 1 - \delta_t D_t \right), \]  
(15)

where \( \alpha \) is a parameter determining the steady-state spread between interbank and policy rates, \( R_{IBt} - R_t \). This spread is increasing in the default probability on interbank lending and marginal costs of monitoring interbank lending, \( \chi_s D_t \). In normal time, it is almost constant, and it equals zero in the absence of defaults on interbank lending and monitoring costs.\(^{16}\)

Condition (13) describes interbank lending supplied by the savings banks; it states that the fraction \( s_{j,t} \) of deposits allocated to interbank lending is decreasing in the probability of default on interbank lending and in the policy rate, while it is increasing in the interbank rate and total deposits. An increase in \( s_{j,t} \) leads to an expansion in credit supply. Condition (14) defines the deposit interest rate, \( R_{Dt} \), as a mark-down of the weighted average return of savings banks’ assets.\(^{17}\)

Thus, rising riskiness of interbank lending (a higher \( \delta_t D_t \)) encourages savings banks to increase their risk-free holdings and to reduce their interbank lending. Also, an increase in the interbank rate, the return rate on risk-free assets, reduces interbank lending supply. An increase in total deposits expands interbank lending, leading to an expansion in credit supply conditions.

This framework adds two channels through which savings banks’ behavior affects credit supply conditions and the real economy. First, by setting deposit return rates in a monopolistically competitive market, combined with the nominal rigidity of deposit rates, savings banks
influence the intertemporal substitution of consumption across periods and thus facilitate con-
sumption smoothing.\textsuperscript{18} Second, by optimally dividing deposits between interbank lending and
risk-free asset holding, savings banks affect credit supply conditions by expanding or tightening
credit market conditions.

\textbf{2.2.2 Lending banks}

There is a continuum of lending banks, indexed by \( j \in (0, 1) \), that operate in a monopolistically
competitive market to provide loans to entrepreneurs. The \( j \)'th lending bank borrows \( \tilde{D}_{j,t} \) from
savings banks on the interbank market and demands bank capital \( Z_{j,t} \) from bankers, paying the
bank capital price \( Q^Z_t \) and a non-contingent return rate \( R^Z_{t+1} \).\textsuperscript{19} We assume that bank capital
is held by lending banks as government bonds that pay the risk-free rate \( R_t \). Each lending bank
\( j \) can receive liquidity injections from the central bank, \( m_{j,t} \), (quantitative monetary easing).
Also, if needed, bank \( j \) may swap a fraction of its loans for government bonds, \( x_{j,t} \), from the
central bank in the crisis time (qualitative monetary easing). Through these channels, the
central bank can serve as lender of last resort to lending banks in times of crisis.

Each lending bank has monopoly power when setting its prime lending rate \( R^L_{j,t} \) subject to
quadratic adjustment costs. The bank \( j \) may also decide to default on some of its interbank
borrowing and bank capital payments. Defaults can be either strategic or mandatory (when
a bank cannot afford to repay their debt). In addition, lending banks optimally choose their
leverage ratio (ratio of loans to bank capital), taking into account the maximum ratio imposed
by the regulators. We assume that having a lower leverage ratio that the regulatory limit
entails quadratic gains for the bank. These gains directly affect the marginal cost of raising
bank capital in the financial market.

To produce loans \( L_{j,t} \) for entrepreneurs, the lending bank \( j \) uses interbank borrowing, \( \tilde{D}_{j,t} \),
plus liquidity injection received from the central bank (quantitative monetary easing), \( m_{j,t} \),
and the total market value of its bank capital \( Q^Z_t Z_{j,t} \), plus any swapping of assets with the
central bank, \( x_{j,t} \). We assume that banks use the following Leontief technology to produce

\textsuperscript{18}Since the marginal rate of substitution equals the deposit rate, the sluggishness in this rate affects the
intertemporal substitution between current and future consumption.

\textsuperscript{19}In this economy, interbank borrowing is always equal to interbank lending, so that \( \tilde{D}_t = s_t D_t \).
loans:

\[
L_{j,t} = \min \left\{ \tilde{D}_{j,t} + m_{j,t}; \kappa_{j,t} (Q^Z_{t} Z_{j,t} + x_{j,t}) \right\} \Gamma_{t},
\]

(16)

where \( \kappa_{j,t} \leq \bar{\kappa} \) is the bank \( j \)'s optimally chosen leverage ratio and \( \bar{\kappa} \) is the maximum leverage ratio imposed by regulators.\(^{21}\) When \( \kappa_{j,t} < \bar{\kappa} \), the bank \( j \) accumulates bank capital beyond the required level. The variable \( \Gamma_{t} \) represents a shock to the intermediation process affecting credit supply “loan production”. \( \Gamma_{t} \) is a shock to the balance sheet of lending banks. It represents exogenous factors and approximates perceived changes in creditworthiness. Technological advances in the intermediation process are another source of variation in \( \Gamma_{t} \).\(^{22,23}\) It is assumed that \( m_{t}, x_{t}, \) and \( \Gamma_{t} \) evolve according to AR(1) processes.\(^{24}\)

Using Leontief technology to produce loans implies perfect complementarity between interbank borrowing and bank capital. This complementarity allows bank capital to be an attenuation mechanism of demand and financial shocks, as loans expand requires a higher leverage ratio and/or an increase in bank capital holding. Therefore, the marginal costs of raising bank capital increase, leading to rising marginal costs of producing loans and thus rising borrowing costs for entrepreneurs. These extra costs dampen (partly) the initial demand for borrowing and investment. Furthermore, the marginal cost of producing loans is simply the sum of the marginal cost of interbank borrowing and that of raising bank capital. The latter is adjusted by the Bank’s leverage ratio. Table 2 shows the \( j \)'th lending bank’s balance sheet in period \( t \).

<table>
<thead>
<tr>
<th>Table 2: Lending bank’s balance sheet</th>
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\(^{20}\)Leontief technology implies perfect complementarity between deposits and bank capital when producing loans and satisfies the bank capital requirement condition.\(^{21}\) Note that \( \kappa_{j,t} \) is the ratio of bank’s loans to its bank capital. Therefore, it is the inverse of the bank capital ratio.\(^{22}\) The process of loan evaluation certainly has evolved over time, through stochastic technological advances in information services. These variations may represent changes in total factor productivity in the intermediation process. Advances in computational finance and sophisticated methods of sharing risk are examples of this shock.\(^{23}\) This shock may reflect lending banks’ perception of the risk in the economy. Banks may underevaluate (overevaluate) risk during booms (recessions). This exogenously increase (decrease) loan supply. During booms, \( \Gamma_{t} > 1 \), so it is a credit easing shock, while during recession \( \Gamma_{t} < 1 \) means a credit rationing shock.\(^{24}\) The steady state values of \( m_{t} \) and \( x_{t} \) are zero, while that of \( \Gamma_{t} \) is equal to unity.
Note that exchanges loans for government bonds, \( x_{j,t} \), simply changes the composition of the bank lending assets. In contrast, liquidity injections, \( m_{j,t} \), and shocks to financial intermediation, \( \Gamma_t \), affect the total values of lending banks balance sheet, implying balance sheet expansion or contraction.

At each period, the lending bank \( j \) sets the prime lending rate, \( R^L_{j,t} \), as a mark-up of the marginal cost of producing loans, plus the marginal costs of adjusting this nominal rate across periods. Bank \( j \) also optimally chooses \( \kappa_{j,t} \) subject to the bank capital requirement condition imposed by regulators, \( \bar{\kappa} \), and the defaults on bank capital and interbank borrowing, \( \delta^Z_{j,t} \) and \( \delta^D_{j,t} \), respectively. As in Gerali et al. (2009), the adjustment costs associated with changes in prime lending rates are modelled à la Rotemberg (1982) and given by

\[
Ad^R_{j,t} = \frac{\phi_{RL}}{2} \left( \frac{R^L_{j,t}}{R^L_{j,t-1}} - 1 \right)^2 L_t, \tag{17}
\]

where \( \phi_{RL} > 0 \) is an adjustment cost parameter. In addition, when choosing \( \kappa_{j,t} < \bar{\kappa} \), there are quadratic gains since banks are well-capitalized because the marginal cost of raising bank capital in financial markets depends on the bank’s capital position. These gains are modelled using the following function: \( 0.5 \chi_{\kappa} ( (\bar{\kappa} - \kappa_{j,t}) Q^Z_t Z_{j,t}/\bar{\kappa} )^2 \), where \( \chi_{\kappa} > 0 \) is a parameter determining the steady-state value of \( \kappa_t \). When \( \kappa_{j,t} = \bar{\kappa} \), the bank’s leverage ratio meets the required level exactly, and there are no gains associated with it. However, when \( \kappa_{j,t} < \bar{\kappa} \), the bank leverage ratio is below the requirement and banks are well-capitalized. Well-capitalized banks have lower costs of raising capital. Thus, the optimal choice of the banks’ leverage ratio affects the costs of lending directly through its impact on bank capital raising costs. \(^{25}\)

The lending bank optimization problem is to choose \( \kappa_{j,t}, \delta^D_{j,t}, \delta^Z_{j,t}, \) and \( R^L_{j,t} \). The lending

\(^{25}\)Equation (24) hereafter displays the relation between the marginal cost of loans and the cost of raising bank capital.
banks’ profit maximization problem is

\[
\max_{\{R_{j,t}, \kappa_{j,t}, \delta_{D,j,t}, \delta_{Z,j,t}\}} \ E_0 \sum_{t=0}^{\infty} \beta^t_b \lambda^b \left\{ R_{j,t}^L L_{j,t} - (1 - \delta_{Z,j,t}) R_{t+1}^B \tilde{D}_{j,t} - R_t m_{j,t} - \left[ (1 - \delta_{Z,j,t}) R_{t+1}^Z - R_t \right] Q_t^Z Z_{j,t} \right. \\
- \frac{\chi_{DD}}{2} \left( \frac{\delta_{D,j,t-1} \tilde{D}_{j,t-1}}{\pi_t} \right)^2 - \frac{\chi_{DZ}}{2} \left( \frac{\delta_{Z,j,t-1} Q_t^Z Z_{j,t-1}}{\pi_t} \right)^2 \\
+ \frac{\chi_{Z}}{2} \left( \frac{\bar{k} - \kappa_{j,t-1} Z_{j,t}}{\bar{k}} \right)^2 - (R_{j,t}^L - R_t) x_{j,t} - Ad_{j,t}^L \left\} ,
\]

subject to (16), (17), and the following demand function for loans:

\[
L_{j,t} = \left( \frac{R_{j,t}^L}{R_t^L} \right)^{-\vartheta_L} L_t,
\]

where \( \vartheta_L > 1 \) is the elasticity of substitution between different types of loans that provided by different lending banks.\(^{26}\) The discount factor is given by the stochastic process \( \beta^t_b \lambda^b \), where \( \lambda^b \) denotes the marginal utility of consumption of bankers—the owners of the lending banks.

The terms \( R_{t} m_{j,t} \) represents the cost of liquidity injections received from the central bank, while \( \left[ (1 - \delta_{Z,j,t}) R_{t+1}^Z - R_t \right] Q_t^Z Z_{j,t} \) denotes the net cost of bank capital, which depends on payment of non-defaulted fraction net of the return from holding bank capital as government bonds, \( B_t^b \). The terms \( 0.5 \chi_{DD} \left( \delta_{D,j,t-1} \tilde{D}_{j,t-1} / \pi_t \right)^2 \) and \( 0.5 \chi_{DZ} \left( \delta_{Z,j,t-1} Q_t^Z Z_{j,t-1} / \pi_t \right)^2 \) are increasing in the defaults on interbank borrowing and bank capital that occurred during the previous period.

The terms \( (R_{j,t}^L - R_t) x_{j,t} \) denote the effects of qualitative monetary easing shocks on bank’s profits, where \( R_{j,t}^L - R_t \) is the cost of swapping a fraction of loans for government bonds.

The first-order conditions of this optimization problem, with respect to \( \kappa_{j,t}, \delta_{D,j,t}, \delta_{Z,j,t} \), and

\(^{26}\)This demand function is derived from the definition of aggregate demand of loans, \( L_t \), and the corresponding prime lending rate, \( R_{t+1}^L \), in the monopolistic competition framework, as follows:

\[
L_t = \left( \int_0^1 \frac{L_{j,t}^{-1/\vartheta_L}}{\bar{k}_t} dj \right)^{1/\vartheta_L} \quad \text{and} \quad R_t^L = \left( \int_0^1 R_t^{L_{j,t}^{-1/\vartheta_L}} dj \right)^{1/\vartheta_L} ,
\]

where \( L_{j,t} \) and \( R_{j,t}^L \) are the loan demand and lending rate faced by each lending bank \( j \in (0, 1) \).

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\( R_{j,t} \) are:

\[
\kappa_{j,t} = \bar{\kappa} \left( 1 - \frac{\Gamma_t (R^L_{j,t} - 1)}{\chi \kappa Q^Z_t Z_{j,t}} \right); \quad (19)
\]

\[
\delta^{D}_{j,t} = E_t \left[ \frac{R_t \pi_{t+1}}{\chi \delta D_{j,t}} \right]; \quad (20)
\]

\[
\delta^Z_{j,t} = E_t \left[ \frac{R_t \pi_{t+1}}{\chi \delta Q^Z_t Z_{j,t}} \right]; \quad (21)
\]

\[
R^L_{j,t} = 1 + \frac{\vartheta}{\vartheta - 1} (\zeta_{j,t} - 1) - \frac{\phi_{RL}}{\vartheta - 1} \left( \frac{R^L_{j,t}}{R^L_{j,t-1}} - 1 \right) \frac{R^L_{j,t-1}}{R^L_{j,t-1}} \right) + \frac{\beta \phi_{RL}}{\vartheta - 1} E_t \left[ \left( \frac{R^L_{j,t+1}}{R^L_{j,t}} - 1 \right) \frac{R^L_{j,t+1}}{R^L_{j,t}} \right], \quad (22)
\]

where

\[
\zeta_{j,t} = \Gamma_t^{-1} \left[ R^{IB}_t + \left( R^Z_{t+1} - R^{IB}_t - (R^L_{j,t} - 1) \frac{\bar{\kappa} - \kappa_{j,t}}{\kappa} \right) Q^Z_t Z_{j,t} \right], \quad (24)
\]

is the marginal cost of producing loans. In addition, the Leontief technology implies the following implicit demand functions of interbank borrowing and bank capital:

\[
L_{j,t} = \Gamma_t (\tilde{D}_{j,t} + m_{j,t}); \quad (25)
\]

\[
L_{j,t} = \Gamma_t \kappa_{j,t} (Q^Z_t Z_{j,t} + x_{j,t}). \quad (26)
\]

Eq. (19) describes leverage ratios, \( \kappa_{j,t} \), as a function of different macroeconomic variables. It shows that it is decreasing in the return rate of loans, \( R^L_{j,t} \), and financial intermediation shocks, \( \Gamma_t \); whereas it is increasing in bank capital and bank capital prices. Eq.(20) indicates that default on interbank borrowing increases in expected inflation and the policy rate, while it decreases in total interbank lending. (An increase in expected inflation reduces future default penalty payments.) In Eq.(21), default on bank capital increases in expected inflation and the policy rate, while it decreases as the total value of bank capital rises. Eq.(23) relates the prime lending rate, \( R^L_{j,t} \), to the marginal cost of producing loans, \( \zeta_{j,t} \), and to current costs and future gains of adjusting the prime lending rate.

Eq.(24) indicates that the marginal cost of producing loans depends on the cost of interbank borrowing, \( R^{IB}_t \), and the shadow price of using capital to satisfy the capital requirement.
condition. In this case, the marginal cost of bank capital is equal to the difference between \( R_{t+1}^Z \) and \( R_t \), the risky return paid on bank capital and the risk-free return on holding bank capital as government bonds, and the marginal benefit of holding bank capital in excess of the required level.\(^{27}\)

### 2.3 Production sector

#### 2.3.1 Entrepreneurs

The entrepreneurs’ behavior follows BGG (1999). Entrepreneurs, who manage firms that produce wholesale goods, are risk neutral and have a finite expected horizon for planning purposes. The probability that an entrepreneur will survive until the next period is \( \nu \). This assumption ensures that entrepreneurs’ net worth (the firm equity) is never sufficient to self-finance new capital acquisitions, so they issue debt contracts to finance their desired investment expenditures in excess of net worth.

At the end of each period, entrepreneurs purchase capital, \( K_{t+1} \), that will be used in the next period at the real price \( Q_t^K \). Capital acquisition is financed partly by their net worth, \( N_t \), and by borrowing \( L_t = Q_t^K K_{t+1} - N_t \) from lending banks.

The entrepreneurs’ demand for capital depends on the expected marginal return and the expected marginal external financing cost at \( t+1 \), \( E_{t+1}F \), which equals the real interest rate on external (borrowed) funds. Optimization guarantees that

\[
E_{t+1}F = E_t \left[ \frac{r_{t+1}^K + (1 - \delta)Q_{t+1}^K}{Q_t^K} \right], \tag{27}
\]

where \( \delta \) is the capital depreciation rate. The expected marginal return of capital is given by the right-side terms of (27), where \( r_{t+1}^K \) is the marginal productivity of capital at \( t+1 \) and \( (1 - \delta)Q_{t+1}^K \) is the value of one unit of capital used in \( t+1 \).

BGG solve a financial contract that maximizes the payoff to the entrepreneur, subject to the lender earning the required rate of return. BGG show that—given parameter values associated with the cost of monitoring the borrower, characteristics of the distribution of entrepreneurial returns, and the expected life span of firms—their contract implies an external finance premium, \(^{27}\)

\[
\text{If } \kappa_{j,t} = \bar{\kappa}, \text{ then } \zeta_{j,t} = \Gamma^{-1}_{t} \left[ R_t^{BG} + \bar{\kappa}^{-1}Q_t^{\bar{\kappa}} (R_{t+1} - R_t) \right].
\]
\( \Psi(\cdot) \), that depends on the entrepreneur's leverage ratio. The underlying parameter values determine the elasticity of the external finance premium with respect to the firm leverage.

In our framework, the marginal external financing cost is equal to an external finance premium plus the gross real prime lending rate. Thus, the demand for capital should satisfy the following optimality condition:

\[
E_t F_{t+1} = E_t \left[ \frac{R^L_t}{\pi_{t+1}} \Psi(\cdot) \right],
\]

where \( E_t \left( \frac{R^L_t}{\pi_{t+1}} \right) \) is an expected real prime lending rate (with \( R^L_t \) set by the lending bank and depends on the marginal cost of making loans) and the external finance premium is given by

\[
rp_t \equiv \Psi(\cdot) = \Psi \left( \frac{Q^K_t K_{t+1}}{N_t} ; \psi_t \right),
\]

with \( \Psi'(\cdot) < 0 \) and \( \Psi(1) = 1 \), and \( \psi_t \) represents an aggregate riskiness shock.

The external finance premium, \( \Psi(\cdot) \), depends on the borrower’s equity stake in a project (or, alternatively, the borrower’s leverage ratio). As \( \frac{Q^K_t K_{t+1}}{N_t} \) increases, the borrower increasingly relies on uncollateralized borrowing (higher leverage) to fund the project. Since this raises the incentive to misreport the outcome of the project, the loan becomes riskier, and the cost of borrowing rises.

In particular, the external finance premium is assumed to have the following functional form

\[
rp_t \equiv \Psi(\cdot) = \left( \frac{Q^K_t K_{t+1}}{N_t} \right)^{\psi_t},
\]

where \( \psi_t \) is a time-varying elasticity of the external finance premium with respect to the entrepreneurs’ leverage ratio. Following Christiano et al. (2009), we assume that \( \psi_t \) is an aggregate riskiness shock that follows an AR(1) process. BGG (1999) show that this elasticity, \( \psi > 0 \), depends on the standard deviation of the distribution of the entrepreneurs’ idiosyncratic shocks, the agency cost and the entrepreneurs’ default threshold. Therefore, a positive shock to \( \psi_t \) may result from exogenous increases in the distribution of the entrepreneurs’ idiosyncratic shocks, the agency costs, and/or the entrepreneurs’ default threshold. The result is a rise in \( \psi_t \) and thus in the external finance premium.\(^{29}\)

\(^{28}\)When loans riskiness increases, the agency costs rise and the lender’s expected losses increase. A higher external finance premium paid by successful entrepreneurs offsets these higher losses.

\(^{29}\)A positive shock to the standard deviation widens the entrepreneurs’ distribution, so lending banks are unable to distinguish the quality of the entrepreneurs.
Aggregate entrepreneurial net worth evolves according to

\[ N_t = \nu V_t + (1 - \nu) g_t, \]

where \( V_t \) denotes the net worth of surviving entrepreneurs net of borrowing costs carried over from the previous period, \( 1 - \nu \) is the share of new entrepreneurs entering the economy, and \( g_t \) is the transfer or “seed money” that new entrepreneurs receive from entrepreneurs who exit.\(^{30}\) \( V_t \) is given by

\[ V_t = \left[ F_t Q_{t-1}^K K_t - E_{t-1} F_t (Q_{t-1}^K K_t - N_{t-1}) \right], \]

where \( F_t \) is the ex post real return on capital held in \( t \), and

\[ E_{t-1} F_t = E_{t-1} \left[ \frac{R_t^{L^t}}{\pi_t} \psi \left( \frac{Q_{t-1}^K K_t}{N_{t-1}} \right) ; \psi_{t-1} \right] \]

is the cost of borrowing (the interest rate in the loan contract signed in time \( t - 1 \)). Earnings from operations in this period become next period’s net worth. In our formulation, borrowers sign a debt contract that specifies a nominal interest rate.\(^{31}\) The loan repayment (in real terms) will then depend on the ex post real interest rate. An unanticipated increase (decrease) in inflation will reduce (increase) the real cost of debt repayment and, therefore, will increase (decrease) entrepreneurial net worth.

To produce output \( Y_t \), the entrepreneurs use \( K_t \) units of capital and \( H_t \) units of labor following a constant-returns-to-scale technology:

\[ Y_t \leq A_t K_t^\alpha H_t^{1-\alpha}, \quad \alpha \in (0, 1), \]

where \( A_t \) is a technology shock common to all entrepreneurs and it assumed to follow a stationary an AR(1) process. Each entrepreneur sells his output in a perfectly competitive market for a price that equals his nominal marginal cost. The entrepreneur maximizes profits by choosing \( K_t \) and \( H_t \) subject to the production function (33). See Appendix A for entrepreneurs’ first-order conditions.

\(^{30}\)The parameter \( \nu \) will affect the persistence of changes in net worth.

\(^{31}\)In BGG, the contract is specified in terms of the real interest rate.
2.3.2 Capital producers

Capital producers use a linear technology, subject to an investment-specific shock $\Upsilon_t$, to produce capital goods $K_{t+1}$, sold at the end of period $t$. They use a fraction of final goods purchased from retailers as investment goods, $I_t$, and the existing capital stock to produce new capital goods. The new capital goods replace depreciated capital and add to the capital stock. The disturbance $\Upsilon_t$ is a shock to the marginal efficiency of investment. Since $I_t$ is expressed in consumption units, $\Upsilon_t$ influences the amount of capital in efficiency units that can be purchased for one unit of consumption. Capital producers are also subject to quadratic investment adjustment costs specified as $\chi I_t^2 (I_t I_t - 1)^2 I_t$.

The capital producers’ optimization problem, in real terms, consists of choosing the quantity of investment $I_t$ to maximize their profits, so that:

$$\max_{I_t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta_t \lambda_t \{ Q_t^K [ \Upsilon_t I_t - \frac{\chi_t}{2} (\frac{I_t}{I_{t-1}} - 1)^2 I_t ] - I_t \}.$$ (34)

Thus, the optimal condition is

$$\frac{1}{Q_t^K} = \Upsilon_t - \chi I_t \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \beta_t \chi I_t \mathbb{E}_t \left[ \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \frac{Q_{t+1}^K}{Q_t^K} \frac{\lambda_{t+1}}{\lambda_t} \right].$$ (35)

which is the standard Tobin’s $Q$ equation that relates the price of capital to marginal adjustment costs. Note that in the absence of investment adjustment costs, capital price $Q_t^K$ is constant and equals 1. We introduce investment adjustment costs in the model to allow for capital price variability, which contributes to the volatility of entrepreneurial net worth.

The quantity and price of capital are determined in the capital market. The entrepreneurial demand curve for capital is determined by equations (28) and (A.5), whereas the supply of capital is given by equation (35). The intersection of these curves gives the market-clearing quantity and price of capital. Capital adjustment costs slow down the response of investment to different shocks, which directly affects the price of capital.

Furthermore, the aggregate capital stock evolves according to

$$K_{t+1} = (1 - \delta)K_t + \Upsilon_t I_t - \frac{\chi_t}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 I_t,$$ (36)

where $\delta$ is the capital depreciation rate, and the shock $\Upsilon_t$ follows an AR(1) process.
2.3.3 Retail firms

The retail sector is used to introduce nominal rigidity into this economy. Retail firms purchase the wholesale goods at a price equal to their nominal marginal cost, and diversify them at no cost. They then sell these differentiated retail goods in a monopolistically competitive market. Following Calvo (1983) and Yun (1996), we assume that each retailer cannot reoptimize its selling price unless it receives a random signal. The constant probability of receiving such a signal is \((1 - \phi_p)\); and, with probability \(\phi_p\), the retailer \(j\) must charge the same price at the preceding period, indexed to the steady-state gross rate of inflation, \(\pi\). At time \(t\), if the retailer \(j\) receives the signal to reoptimize, it chooses a price \(\tilde{P}_t(j)\) that maximizes the discounted, expected real total profits for \(l\) periods.

2.4 Central bank and government

2.4.1 Central bank

We assume that the central bank adjusts the interbank rate, \(R_t\), which is also the nominal risk-free interest rate, in response to deviations of inflation, \(\pi_t\), and output, \(Y_t\), from their steady-state values. Thus monetary policy evolves according to the following Taylor-type policy rule:

\[
\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\varphi} \left(\frac{Y_t}{Y}\right)^{\varphi_Y} \exp(\varepsilon_{Rt})
\]

where \(R\), \(\pi\), and \(Y\) are the steady-state values of \(R_t\), \(\pi_t\), and \(Y_t\), respectively; and \(\varepsilon_{Rt}\) is a monetary policy shock normally distributed with zero mean and standard deviation \(\sigma_R\).

During financial crisis periods, the central bank can use unconventional monetary policies: quantitative and/or qualitative monetary easing shocks, \(m_t\) and \(x_t\). Therefore, it can inject liquidity into the banking system and/or swap a fraction of banks loans for risk-free assets (government bonds) used to enhance banks’ capital position.

2.4.2 Government

Each period, the government buys a fraction of the final good \(G_t\), reimburses its last period contracted debt, and makes interest payments. We assume that the government runs a balanced budget financed with lump-sum taxes, \(\tilde{T}_t^w + \tilde{T}_t^b\). Therefore, government’s budget in real terms
is

\[ G_t + \left[ B_{t-1}^{sb} + B_{t-1}^{lb} \right] R_{t-1} / \pi_t = B_t + B_t^{sb} + B_t^{lb} + \tilde{T}_t^w + \tilde{T}_t^b \]

(38)

where \( B_t^{sb} = (1 - s_t) D_t \) and \( B_t^{lb} = Q_t^Z Z_t + \tilde{m}_t \) are government bonds held by saving and lending banks, respectively. We assume that government spending \( G_t \) follows an AR(1) process.

2.5 Markets clearing

Under Ricardian equivalence, government bonds held by bankers are equal to zero, so \( B_t = 0 \) in equilibrium. The real aggregate money stock, \( M_t \), is composed of workers’ money demand: deposit and cash balances and money injections from quantitative monetary easing shocks, such that \( M_t = D_t + M_t^w + m_t \). The newly created money is transferred to workers, so that \( T_t = M_t - M_{t-1} / \pi_t \).

Lastly, the resource constraint implies that \( Y_t = C_t^w + C_t^b + I_t + G_t + \omega_t \), where \( \omega_t \) represents the default penalties minus the gains of excess bank capital holdings. Total consumption, \( C_t \), is simply the sum of workers and bankers consumption. Thus, \( C_t = C_t^w + C_t^b \).

2.6 Shock processes

A part from monetary policy shock, \( \varepsilon_{Rt} \), which is a zero-mean i.i.d. shock with a standard deviation \( \sigma_R \), the other structural shocks follow AR(1) processes:

\[ \log(X_t) = (1 - \rho_X) \log(X) + \rho_X \log(X_{t-1}) + \varepsilon_{Xt}, \]

(39)

where \( X_t = \{ A_t, \Upsilon_t, \epsilon_t, G_t, \psi_t, \Gamma_t, x_t, m_t \} \), \( X > 0 \) is the steady-state value of \( X_t \), \( \rho_X \in (-1, 1) \), and \( \varepsilon_{Xt} \) is normally distributed with zero mean and standard deviation \( \sigma_X \).

3. Calibration

We calibrate the model’s parameters to capture the key features of the U.S. economy for the period 1980Q1–2008Q4. Table 3 reports the calibration values. The steady-state gross domestic inflation rate, \( \pi \), is set equal to 1.0075, which is the historical average in the sample. The discount factors, \( \beta_w \) and \( \beta_b \), are set to 0.9979 and 0.9943 to match the historical averages of nominal deposit and risk-free interest rates, \( R_t^D \) and \( R_t^L \) (see Table 4 for the steady-state values of some key variables). The risk aversion parameters in workers’ and bankers’ utility functions,
\( \gamma_w \) and \( \gamma_b \), are set to 3 and 2, respectively, as we assume that workers are more risk averse than bankers. Assuming that workers allocate one third of their time to market activities, we set \( \eta \), the parameter determining the weight of leisure in utility, and \( \zeta \), the inverse of the elasticity of intertemporal substitution of labour, to 0.996 and 1, respectively. The weight of real cash balances in the workers utility function, \( \pi \), is set to 0.0003, so that, in the steady-state equilibrium, real cash balances is about one tenth of the money stock, matching the ratio of \( M_1 \) to \( M_2 \) that observed in the data. The parameter \( \nu \) is set to 4, implying a money-interest elasticity of 0.25. The habit formation parameter, \( \varphi \), is set to 0.65, as estimated in Christiano et al. (2009).

The capital share in the production, \( \alpha \), and the capital depreciation rate, \( \delta \), are set to 0.33 and 0.025, respectively; values commonly used in the literature. The parameter measuring the degree of monopoly power in the retail goods market \( \theta \) is set to 6, which implies a 20 percent markup in the steady-state equilibrium. The parameters \( \vartheta_D \) and \( \vartheta_L \) that measure the degrees of monopoly power of saving and lending banks are set equal to 2.9 and 2.91, respectively. These values are set to match the historical averages of deposit and prime lending rates, \( R^D \) and \( R^L \), (see Table 4.)

The nominal price rigidity parameter, \( \phi_p \), in the Calvo-Yun contract setting is set to 0.75, implying that the average price remains unchanged for four quarters. This value is estimated in Christensen and Dib (2008) for the U.S. economy and commonly used in the literature. The parameters of the adjustment costs of deposit and prime lending interest rates, \( \phi_{RD} \) and \( \phi_{RL} \), are respectively set to 40 and 55 to match the standard deviations (volatilities) of deposit and prime lending rates to those observed in the data.

Monetary policy parameters \( \varrho_n \) and \( \varrho_Y \) are set values of 1.2 and 0.05, respectively. These values satisfy the Taylor principle. The standard deviation of monetary policy shock, \( \sigma_R \), is given the usually estimated value of 0.006.

The investment and bank capital adjustment cost parameters, \( \chi_I \) and \( \chi_Z \), are set to 8 and 70, respectively. This is to match the relative volatilities of investment and loans (with respect to output) to those observed in the data. Similarly, the parameter \( \chi_s \), which determines the ratio of bank lending to total assets held by the savings banks \( s_t \), is set to 0.0075, so that the steady-state value of \( s_t \) is equal to 0.82, which corresponds to the historical ratio observed
in the data. The parameter $\chi_\kappa$ is set to 2.41, so that the steady-state value of the bank’s leverage ratio, $\kappa$, is equal to 12, which matches the historical average observed in the U.S. data.

Based on the Basel II minimum required bank capital ratio of 8%, we assume that the maximum imposed bank leverage, $\bar{\kappa}$, is 12.5. Similarly, we calibrate $\chi_{\delta \psi}$ and $\chi_{\delta Z}$, the parameters determining total costs of banks’ defaults on interbank borrowing and bank capital, so that these probabilities of defaults are equal to 1.6% in annual terms. (See Table 3).

Following BGG (1999), the steady-state leverage ratio of entrepreneurs, $1 - N/K$, is set to 0.5, matching the historical average. The probability of entrepreneurial survival to next period, $\nu$, is set at 0.9833; while $\psi$, the steady-state elasticity of the external finance premium, is set at 0.05, the value used by BGG and close to the one estimated by Christensen and Dib (2008).

We calibrate the shocks’ process parameters either using values in previous studies or estimated values. The parameters of technology, preference, and investment-specific shocks are calibrated using the estimated values in Christensen and Dib (2008). To calibrate the parameters of government spending process, we use an OLS estimation of government spending in real per capita terms. (See Appendix B.) The estimated values of $\rho_G$, the autocorrelation coefficients, is 0.81; while the estimated standard errors, $\sigma_G$, is 0.0166.

To calibrate the parameters of the riskiness shock process $\psi_t$, we set the autocorrelation coefficient $\rho_\psi$ at 0.83, the estimated value in Christiano et al. (2009), while the standard error $\sigma_\psi$ is set to 0.05 to match the volatility of the external risk premium to that observed in the data, measured the difference between Moody’s BAA yield corporate bond yields and the 3Month T-bill rate. We set the autocorrelation coefficient and the standard error of financial intermediation process $\rho_\Gamma$ and $\sigma_\Gamma$ to 0.8 and 0.003, respectively. These values are motivated by the potential persistence and low volatility of this financial shock.

Finally, we set the autocorrelation coefficients of quantitative and qualitative monetary easing shocks, $\rho_m$ and $\rho_x$, equal to 0.5, and their standard deviations, $\sigma_m$ and $\sigma_x$, to 0.

32 In the data, the ratio of total government securities held by banks to their assets, $1 - s$, is 0.18.
33 This is because the maximum bank leverage ratio is simply the inverse of the minimum required bank capital ratio, which is 8% in Basel II Accords.
34 Christensen and Dib (2008) estimate $\psi$ at 0.046 for the U.S. economy.
35 Future work consists of estimating the model’s structural parameters using either a maximum likelihood procedure, used in Christensen and Dib (2008), Ireland (2003) and Dib(2003), or a Bayesian approach used in Christiano et al. (2009), Dib et al. (2008), Elkdag et al. (2006), Queijo von Heideken (2009), and others.
4. Empirical results

4.1 Impulse responses

To assess the contribution of the banking sector in our baseline model, we plot the impulse responses of key macroeconomic variables to the structural shocks in two models: (1) the full model with the banking sector (baseline, hereafter) and (2) a model with only a financial accelerator mechanism (FA model, hereafter). Figures 1 to 6 display the impulse responses to shocks to technology, monetary policy, riskiness, financial intermediation, and quantitative and qualitative monetary easing. Each variable’s response is expressed as the percentage deviation from its steady-state level.

As indicated before, the banking sector can act as both amplifying or dampening mechanism. The amplification is illustrated in Figure 1, which shows that the effects of a 1% positive technology shock on output and investment are amplified in the model with the full banking sector. There is a significant amplification of investment and larger effects on the external finance premium and loans. Nevertheless, following the technology shock nominal interest rates and inflation fall, but by much less in the model with the banking sector. The decline in inflation increases the real cost of repaying existing debt, creating a debt-deflation effect which pushes down net worth. Lower net worth increases the external finance premium and demand for loans. As a result, the response of loans to the technology shock are larger in the model with the banking sector, indicating that firms need further external funds to finance their capital acquisitions.

Figure 1 also shows that following a positive technology shock, the bank leverage ratio decreases on impact, before moving persistently above its steady-state level. Bank capital values increases persistently after the shock, and for a longer time. Note also that after a positive technology shock, both deposit and prime lending rates decrease, but less than the policy rate. This is due to the presence of the adjustment costs of changing both rates, which implies partial pass-through of policy rate variations to deposit and prime lending rates.

Both defaults on interbank borrowing and bank capital decrease on impact, and they are very persistent. We also note that, following a technology shock, the fraction of deposits allocated by savings banks to the interbank market increases persistently, easing credit supply. This fact is explained by the lower policy rate (which is the return on risk-free assets) and lower
defaults on interbank borrowing. See also Figure 7 in Appendix D for the impulse responses to a 1% investment-efficiency shock.

Figure 2, which plots the responses to a negative 1% monetary policy shock, illustrates that the banking sector can dampen macroeconomic fluctuations. In response to this shock, the nominal interest rate falls sharply and output and investment rise for few quarters. In the model with the full banking sector, net worth rises less after a monetary policy easing, because of capital prices rise less. Therefore, the external finance premium falls by more, reflecting the decrease in firm leverage, and leading to a higher cost of lending. The higher funding cost of purchasing new capital depresses the demand for investment, and the expected price of capital increases slightly above its steady-state value. The presence of the banking sector implies a significant dampening of the impacts of monetary policy shocks on output, investment, net worth, capital prices, and loans, as the responses of these variables in the FA model are almost twice as large as in the baseline model, and persist for longer.

Figure 2 also shows that a monetary policy easing shock moves deposit and prime lending rates in opposite directions: the deposit rate decreases slightly, but persistently, while the prime lending rate rises on impact, before falling below its steady-state value. The bank leverage ratio falls on impact, before increasing one period later. The probability of defaulting on interbank borrowing increases after a positive monetary policy shock, while the supply of interbank lending increases on impact, before persistently dropping below its steady-state level (see also Figure 8 in Appendix D for the impulse responses to a 1% government spending shock.)

Figure 3 displays the impulse responses to a 10% riskiness shock. This shock may be interpreted as an exogenous increase in the degree of riskiness in the entrepreneurial sector. It is generated by an increase in the standard deviation of the entrepreneur distribution, or by an increase in agency costs paid by banks to monitor entrepreneurs in efforts to reduce information asymmetry. In response to this shock, output, investment, net worth, and prices of capital fall persistently below their steady-state levels in both models. Consumption, however, responds positively to the riskiness shock in the short terms, before decreasing at longer horizons. On the other hand, inflation and the policy rate move in opposite directions after the shock, increasing in the baseline model, while falling in the FA model.

The impact of the riskiness shocks in the FA model is much larger, implying that the
banking sector plays a substantial role in dampening the negative effects of riskiness shocks on the economy. Note also that the external finance premium rises in response to riskiness shock, while loans temporarily decline, before jumping above their steady-state levels. Banks react to this shock by increasing their leverage ratio slightly on impact, before persistently reducing it, implying the accumulation of excess bank capital. In addition, after this riskiness shock, both bank defaults increase, while the share of interbank lending in total deposits decreases, reflecting precautionary actions taken by banks.

Figure 4 shows the impulse responses to a 1% positive financial intermediation shock. This is a positive shock to "loan production", leading to rising credit supply without varying the inputs used in the loan production function. Following this shock, loans rise on impact, but fall persistently a few quarters later. At the same time, output, investment, net worth, and prices of capital positively respond to this shock. Nevertheless, inflation and policy rate decrease sharply. We note also that the bank leverage ratio is procyclical, and the exogenous expansion raises defaults on interbank borrowing and bank capital, reducing the interbank lending supply.

Note that the external finance premium and deposit and prime lending rates respond negatively to the shock. The instantaneous decline in the prime lending rate is larger than that of the policy rate. This is to accommodate the excess loan supply generated by the positive financial intermediary shock.

Figure 5 displays the impulse responses to a 1% quantitative monetary easing shock, $m_t$, a positive money injection in the interbank market. This shock gradually increases output, investment, and net worth, while inflation, the policy rate, and the external finance premium decline. We note that loans increase in the short term, but fall in the long run. Thus, firms with sound net worth will borrow less to finance their capital acquisition. As a consequence, they reduce their demand for credit in the medium term. Also, banks respond to this shock by increasing their leverage ratio and reducing their loanable funds as the fraction of deposits lent out on the interbank market persistently declines.

Banks also reduce their prime lending rates to accommodate the impact of this expansionary monetary shock. Interestingly, defaults rise after this expansionary shock. This reflects the changes in the confidence level of the economic agents with respect to the future riskiness and the health of the economy that results from the easing of monetary conditions.
Finally, Figure 6 displays the impulse responses to a 1% positive qualitative monetary easing shock, $x_t$, in which the central bank swaps a fraction of banks loans for government bonds used to enhance the bank capital holdings. This shock affects output and investment only marginally. It leads, however, to higher inflation and policy rates. This shock also reduces the bank leverage ratio and increases both defaults in the economy. Note also that interbank lending decreases following the increase in the risk-free asset return rate, the policy rate. Also, the marginal cost of producing loans increases because of the increases in the cost of the two factors.

Overall, the presence of an active banking sector, as proposed in this model, amplifies and propagates the impact of the supply-side shocks—technology and investment-efficiency shocks—on output and investment and the price of capital. However, the banking sector has dampening effects in the transmission and propagation of demand-side shocks—monetary policy, preference, and government spending shocks—on real variables. Moreover, the presence of the banking sector strongly dampens the effects of financial shocks on real variables, in particular those of the riskiness shock.

4.2 Volatility and autocorrelations
We consider the model-implied volatilities (standard deviations), relative volatilities, and correlations with output of the main variables of interest. Table 5 reports the standard deviations and relative volatilities of output, investment, consumption, loans, and the external finance premium from the data, and for the two simulated models. The standard deviations are expressed in percentage terms. All the model-implied moments are calculated using all the shocks.

Column 3 in Table 5 displays standard deviations, relative volatilities, and unconditional autocorrelations of the actual data. Columns 4–5 reports simulations with the baseline and FA models, respectively. In the data, Panel A shows that the standard deviation of output is 1.27, investment is 6.15, while consumption is 1.06. Loans have a standard deviation of 4.21. The external finance premium, however, is very less volatile; its standard deviation is only 0.38. Also, Panel B shows that investment and loans are 4.84 and 3.31 times as

36 In the data, all series are HP-filtered before calculating their standard deviations as well as their unconditional correlations with output.
volatile as output, while consumption and the external finance premium are less volatile than output, with relative volatilities of 0.83 and 0.30, respectively. The data also show, in Panel C, that output, investment, loans, and the external finance premium are highly persistent, with autocorrelation coefficients larger than 0.8; while consumption is moderately autocorrelated, with an autocorrelation coefficient of 0.73.

The simulation results show that in the model with an active banking sector, all volatilities, except that of the loans, are close to those in the data. The FA model, in which the banking sector is absent, overpredicts all the volatilities, except that of loans. This feature is common in standard sticky-price models. The baseline model is also very successful at matching the relative volatility of most of the variables. In contrast, the FA model underpredicts the relative volatilities of consumption and loans, but overpredicts that of the external finance premium.

Panel C in Table 5 displays the unconditional autocorrelations of the data and of the key variables generated by the two simulated models. In general, both models show larger autocorrelations in output, investment, consumption and loans than those observed in the data. Both models match the autocorrelation in the external finance premium very well. Interestingly, the baseline model is successful in reproducing negative correlations of the external risk premium, and banks’ defaults on interbank borrowing and bank capital with output. Moreover, the model shows that banks’ leverage ratio and the share of interbank lending in total deposits are procyclical (positively correlated with output). Thus, during boom periods savings banks and lending banks expand their interbank lending and credit supply. This helps in reducing the external finance costs of entrepreneurs and push further investment and output.

4.3 Welfare analysis

This section assesses welfare implications of including a banking sector in our baseline model by comparing it to the FA model. Using utility-based unconditional welfare calculations, we assess the welfare costs using different combinations of the model’s structural shocks: all shocks, supply, demand, and financial shocks. We compute a weighted average of households’ (workers and bankers) steady-state consumption that they are willing to pay to avoid the loss associated with the presence of uncertainty, driven by the shocks in the economy (see Appendix C for further details of the welfare calculation).
Table 7 reports the main findings. Welfare cost is measured in percentage of steady-state total consumption. In all cases, the implied welfare costs of business cycles fluctuations in terms of deterministic steady state are lower in the model with the banking sector. For example, using all shocks, the implied welfare cost is 0.14% of consumption in the deterministic steady state, compared to 0.69% in the FA model. Interestingly, the welfare cost caused by financial shocks is 0.103% of the steady-state of total consumption in the model with the banking sector, while it jumps to 0.47% in the model with only financial accelerator. In addition, the presence of the banking sector almost offsets the negative impacts of supply and demand shocks, as the welfare costs are only 0.001% and 0.051%; whereas, they are 0.08 and 0.135 in the FA model, respectively. Welfare costs associated with financial shocks are about 73% of the total welfare costs in the baseline model, and drop to 0.68% in the model without the banking sector. The effect of the banking sector on welfare, therefore, depends on the nature of the shocks.

Thus, the presence of an active banking sector reduces the impact of the financial shocks, lowers macroeconomic volatilities, and improves social welfare. These results can be explained by the fact that optimizing banks help consumption smoothing by optimally allocating resources between consumption and investment, in effect, offering insurance against business cycles fluctuations.

5. Conclusion

The global financial crisis has highlighted the need for DSGE models with real-financial linkages. This has become a key issue in the leading-edge research agenda in central banks and academia. This paper proposes a microfounded framework that incorporates a banking sector, credit market, and interbank market into a DSGE model to evaluate the role of an active banking sector in business cycles and the contribution of financial shocks to the U.S. economy fluctuations.

Financial frictions are modeled using both the demand and supply sides of credit market. We use the financial accelerator à la BGG (1999) to model the demand-side of credit market. The supply-side of credit market consists of two types of heterogeneous banks that offer different banking services and interact in an interbank market. This model provides rich and rigorous framework to address monetary and financial stability issues. The model includes demand-
and supply-sides of credit market and thus allows for policy simulation analysis of factor such as: (1) bank capital regulations; (2) expectations of bank capital prices; (3) endogenous bank defaults on interbank borrowing and bank capital; (4) interest rate spreads resulting from the monopoly power of banks when setting deposit and prime lending rates; (5) optimal choice of banks’ portfolio compositions; and (6) optimal choice of banks’ leverage ratios.

The model reproduces salient features of the U.S. economy, reproducing volatilities of key macroeconomic variables and their correlations with output, and the pro-cyclical banks’ leverage ratio. Banks can affect credit supply conditions and the transmission of real effects of shocks to the economy. Also, financial shocks explain a large fraction of business cycles. We also show that an active banking sector with sticky deposit interest rate is welfare improving. This result is robust to simulations with different shocks. Thus, the main role banks play in this economy is to reduce the effects of uncertainty.

The model can be used to address policy and financial stability questions, such as a zero-bound on interest rate, bank capital requirement regulations, and efficiency versus stability of the banking system. Future work will consist of estimating the model’s structural parameters, incorporating credit to households, and extending the framework to an open economy model.
References


Table 3: Parameter Calibration: Baseline model

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<th>Preferences</th>
<th>( \beta_w = 0.9979 )</th>
<th>( \beta_b = 0.9943 )</th>
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<td>( \phi_L = 6.3 )</td>
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<td>( \phi_{RL} = 55 )</td>
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### Table 4: Steady-state values and ratios: Baseline model

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<td>$\pi$</td>
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<td>$R$</td>
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<td>$rp$</td>
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<td>$\kappa$</td>
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<td>$\delta^D$</td>
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#### A. Steady-state values

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<td>$C^b/Y$</td>
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Table 5: Standard deviations and relative volatilities: (Data 1980:1–2008:4)

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<th>Definitions</th>
<th>Data</th>
<th>Baseline</th>
<th>FA</th>
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<td><strong>A. Standard deviations (in %)</strong></td>
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<tr>
<td>$Y_t$</td>
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Table 6: Correlations with output (Data 1980:1–2008:4)

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<td>$s_t$</td>
<td>Share of interbank lending</td>
<td>+</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>$\kappa_t$</td>
<td>bank leverage ratio</td>
<td>+</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>$\delta^D_t$</td>
<td>default on interbank borrowing</td>
<td>-</td>
<td>-0.35</td>
<td></td>
</tr>
<tr>
<td>$\delta^Z_t$</td>
<td>default on bank capital</td>
<td>-</td>
<td>-0.27</td>
<td></td>
</tr>
</tbody>
</table>
Table 7: Welfare analysis

| Welfare costs in terms of steady state of total consumption as percentage. |
|---|---|---|---|---|---|
| | All of the shocks | Supply shocks | Demand shocks | Financial shocks: Risk, $\psi_t$ | All |
| **A. Baseline model** | | | | | |
| Welfare cost (in % of the steady state of consumption) | 0.1406 | 0.0097 | 0.0511 | 0.0622 | 0.1030 |
| **B. FA model** | | | | | |
| Welfare cost (in % of the steady state of consumption) | 0.6953 | 0.0785 | 0.1346 | 0.4710 |
**Figure 1:** Responses to a 1% Positive Technology Shock

- Output
- Investment
- Consumption
- Inflation
- Policy rate
- Net worth
- Price of capital
- Risk premium
- Net worth
- Leverage ratio
- Value of bank capital
- Share of Interb. lending
- Deposits
- Prime lending rate
- Default on Interb. borrowing
- Default on bank capital

Legend: FA model - Baseline model

**Figure 2:** Responses to an Expansionary Monetary Policy Shock

- Output
- Investment
- Consumption
- Inflation
- Policy rate
- Net worth
- Price of capital
- Risk premium
- Net worth
- Leverage ratio
- Value of bank capital
- Share of Interb. lending
- Deposit rate
- Prime lending rate
- Default on Interb. borrowing
- Default on bank capital

Legend: FA model - Baseline model
Figure 3: Responses to a 10% Increase in the Riskiness Shock

Figure 4: Responses to a 1% Positive Financial Intermediation Shock
Figure 5: Responses to a 1% Positive Quantitative Monetary Easing Shock, $m_t$

Figure 6: Responses to a 1% Positive Qualitative Monetary Easing Shock, $x_t$
Appendix A: First-Order Conditions

A.1. Workers’ first-order conditions

The first-order conditions of the workers optimization problem are:

\[
\begin{align*}
\varepsilon_t \left( \frac{C_{w t}^w}{(C_{t-1}^w)^{2}} \right)^{1-\gamma_w} - \beta_w \varphi E_t & \left[ \varepsilon_{t+1} \left( \frac{C_{t+1}^w}{(C_t^w)^{2}} \right)^{1-\gamma_w} \right] = C_t^w \lambda_w^t; \quad (A.1) \\
\varpi_t (M_t^c) & = \lambda_w^t \left(1 - \frac{1}{R_t^d}\right); \quad (A.2) \\
\eta_t (1 - H_t) & = \lambda_w^t W_t; \quad (A.3) \\
\frac{\lambda_w^t}{R_t^d} & = \beta_w E_t \left( \frac{\lambda_{t+1}^w}{\pi_{t+1}} \right); \quad (A.4)
\end{align*}
\]

where \( \lambda_w^t \) is the Lagrangian multiplier associated with the budget constraint.

A.2. Entrepreneurs’ first-order conditions

The first-order conditions of the entrepreneurs’ optimization problem are:

\[
\begin{align*}
\gamma_t K_t & = \alpha \xi_t Y_t; \quad (A.5) \\
W_t & = (1 - \alpha) \xi_t \frac{Y_t}{H_t}; \quad (A.6) \\
Y_t & = A_t K_t^\alpha H_t^{1-\alpha}, \quad (A.7)
\end{align*}
\]

where \( \xi_t > 0 \) is the real marginal cost.

A.3. The retailer’s optimization problem

The retailer’s optimization problem is

\[
\max_{\{P_t(j)\}} E_0 \left[ \sum_{t=0}^{\infty} (\beta_t \phi_p)^{t+1} \lambda_{t+1}^w \Pi_t^R(j) \right], \quad (A.8)
\]
subject to the demand function\textsuperscript{37}

\[ Y_{t+1}(j) = \left( \frac{\tilde{P}(j)}{P_{t+1}} \right)^{-\theta} Y_{t+1}, \quad (A.9) \]

where the retailer’s nominal profit function is

\[ \Pi_{t+1}(j) = \left( \pi^l \tilde{P}(j) - P_{t+1}\xi_{t+1} \right) Y_{t+1}(j)/P_{t+1}. \quad (A.10) \]

The first-order condition for \( \tilde{P}(j) \) is

\[ \tilde{P}(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta_w \phi_p)^l \lambda_{t+1}^w Y_{t+1}(j) \xi_{t+1}}{E_t \sum_{l=0}^{\infty} (\beta_w \phi_p)^l \lambda_{t+1}^w Y_{t+1}(j) / P_{t+1}}. \quad (A.11) \]

The aggregate price is

\[ P_t^{1-\theta} = \phi_p(\pi P_{t-1})^{1-\theta} + (1 - \phi_p) \tilde{P}_t^{1-\theta}. \quad (A.12) \]

These lead to the following equation:

\[ \hat{\pi}_t = \beta_w E_t \hat{\pi}_{t+1} + \frac{(1 - \beta_w \phi_p)(1 - \phi_p)}{\phi_p} \hat{\xi}_t, \quad (A.13) \]

where \( \xi_t \) is the real marginal cost, and variables with hats are log deviations from the steady-state values (such as \( \hat{\pi}_t = \log(\pi_t / \pi) \)).

\textsuperscript{37}This demand function is derived from the definition of aggregate demand as the composite of individual final output (retail) goods and the corresponding price index in the monopolistic competition framework, as follows:

\[ Y_{t+1} = \left( \int_0^1 Y_{t+1}(j) \frac{\phi_p - 1}{\phi_p} dj \right)^{\frac{\phi_p - 1}{\phi_p}} \quad \text{and} \quad P_{t+1} = \left( \int_0^1 P_{t+1}(j) \frac{1}{\phi_p} - \frac{1}{1-\phi_p} \right)^{\frac{1}{\phi_p}}, \]

where \( Y_{t+1}(j) \) and \( P_{t+1}(j) \) are the demand and price faced by each individual retailer \( j \in (0, 1) \).
Appendix B: Data

1. Loans are measured by Commercial and Industrial Loans of all Commercial Banks (BUS-LOANS), quarterly and seasonally adjusted;

2. The external finance premium is measured by the difference between Moody’s BAA corporate bond yields and 3-Month Treasury Bill (TB3MS);

3. Inflation is measured by quarterly changes in GDP deflator ($\Delta \log(GDPD)$).

4. Prime lending rate is measured by Bank Prime Loan Rate (MPRIME);

5. Monetary policy rate is measured by the 3-Month Treasury Bill (TB3MS);

6. Deposit rate is measured by weighted average of the rates received on the interest-bearing assets included in M2 (M2OWN);

7. Real cash balances is measured by real M1 money stock per capita;

8. Real money stock is measured by real M2 money stock per capita;

9. Output is measured by real GDP per capita;

10. Total Consumption is measured by Personal Consumption Expenditures (PCEC);

11. Investment is measured by Gross Private Domestic Investment (GPDI);

12. Government spending is measured by output minus consumption and investment (GDP - PCEC - GPDI).
Appendix C: Welfare calculation

We use the second approximation procedure to calculate welfare. We define the present discounted value of the workers and bankers utility as:

\[ V^w_t = \sum_{t=0}^{\infty} u(C^w_t, M^e_t, H_t), \]

and

\[ V^b_t = \sum_{t=0}^{\infty} u(C^b_t), \]

where \( u(\cdot) \) denotes the single-period utility of workers and bankers.

Let \( \lambda^w_{ss} \) and \( \lambda^b_{ss} \) be the marginal utility of workers and bankers’ at the deterministic steady state. Therefore, the weighted average welfare cost is

\[ \frac{\Delta C}{C_{ss}} = \frac{C_{stoch} - C_{ss}}{C_{ss}} = a \frac{V^w_{stoch} - V^w_{ss}}{\lambda^w_{ss} C_{ss}} + (1 - a) \frac{V^b_{stoch} - V^b_{ss}}{\lambda^b_{ss} C_{ss}}, \]

where \( stoch \) and \( ss \) denote stochastic and deterministic steady-state means of total consumption, workers and bankers’ consumption, discounted values of utility, and the marginal utilities; the parameter \( a \) is the share of workers’ consumption in the total consumption, so that

\[ a = \frac{C^w_{ss}}{C^w_{ss} + C^b_{ss}}. \]
Appendix D: Impulse Responses

Figure 7: Responses to a 1% Positive Investment-Efficiency Shock

Figure 8: Responses to a 1% Positive Government Spending Shock