The Budgetary and Welfare Effects of Tax-Deferred Retirement Saving Accounts

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Abstract

We extend a heterogeneous-agent dynamic general-equilibrium overlapping-generations model and analyze the budgetary, macroeconomic, and welfare effects of tax-deferred retirement saving accounts similar to the U.S. 401(k) plans. We solve the model for an equilibrium transition path under several different government financing assumptions. If we ignored the budgetary cost, tax-deferred accounts would surely increase saving and total output, and make all age cohorts on average better off. However, if the cost was financed by cutting transfer payments or increasing income tax rates, tax-deferred accounts would likely hurt the current and near future households, although the long-run macroeconomic effect would still be positive.

JEL Classification Numbers: D91, E62, H31.

Key Words: 401(k) plans; IRA; dynamic general equilibrium; heterogeneous agents.

1 Introduction

Tax-deferred retirement saving accounts, such as U.S. 401(k) plans and individual retirement accounts (IRAs), are expected to have a sizable effect on individual life-cycle saving, thus aggregate wealth accumulation, through the tax-favored properties. However, few literature has analyzed the budgetary cost and the

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welfare effect of introducing these accounts. Like most other government programs, tax-deferred accounts are not self financing. Although we can expect some positive impacts on the aggregate economy, a large part of tax benefits households receive from these accounts must be financed eventually by either cutting government expenditure, or increasing other tax revenue, or both. Moreover, the short-run cost of newly introduced tax-deferred accounts is expected to be much higher than the long-run cost. At the beginning of the policy change, many working-age households contribute to tax-deferred accounts (and pay less taxes), but few retired households withdraw from these accounts (and pay more taxes). Without considering the budgetary cost and taking the government financing assumptions into account, we cannot fully evaluate the effects of tax-deferred retirement saving accounts.

In the present paper, we extend a standard dynamic general-equilibrium overlapping-generations (OLG) model by implementing tax-deferred accounts, and we analyze the possible budgetary, macroeconomic, and welfare effects of introducing 401(k)-type tax-deferred accounts to the economy. Households in the model economy are heterogeneous with respect to age, working ability, and asset holdings in regular taxable accounts and tax-deferred accounts. Households receive idiosyncratic working ability shocks each year and choose optimal consumption, labor supply, and savings in these two accounts. We solve the heterogeneous-agent OLG model for an equilibrium transition path to deal with both the transition cost and the long-run cost of the tax-deferred accounts, and we show the macroeconomic and welfare implications across time and generations.

The main questions of the present paper are how much stylized tax-deferred accounts increase national wealth and total output, how large the short-run and long-run budgetary costs are, and how these accounts change the welfare of current and future households in a dynamic general equilibrium setting. Both individual and macroeconomic effects depend critically on how the government finances the budgetary cost of the tax-deferred accounts. In our policy experiments, to satisfy the government inter-temporal budget constraint, we assume the following four financing assumptions: (a) cutting government consumption (which is not in the household’s utility function) to balance the budget in each period, (b) cutting government lump-sum transfer payments in each period, (c) increasing the marginal income tax rates in each period, and (d) increasing the marginal tax rates once at the time of policy change and increasing the government debt gradually.¹ We show how each financing assumption affects the macroeconomic, budgetary, and welfare

¹Government consumption is considered to be a waste in the model economy, since it has no effect on households’ utility. Lump-sum transfers are proxies of government provided goods and services that are perfect substitutes of other private goods and services. Thus the first two financing assumptions, (a) and (b), represent the two polar cases of cutting government expenditure.
implications of the policy change.

The tax-deferred retirement saving accounts we analyze in this paper have the following properties. Contributions to the tax-deferred accounts are income tax deductible, capital income generated in the accounts are not taxable, and withdrawals from the accounts are income taxable. Contributions are capped by the annual contribution limit and labor income, whichever is smaller. There are 10% early withdrawal penalties if households are aged 59 or younger. The main advantage of the tax-deferred accounts is the reduction of lifetime income tax burden through deferring tax payments and smoothing taxable income (or equivalently, smoothing marginal income tax rates). The main disadvantage is the lower liquidity due to early-withdrawal penalties. Thus, to analyze these positive and negative effects, the model economy has to be equipped with heterogeneous households, idiosyncratic income shocks, a progressive income tax, and a liquidity constraint.

The tax-deferred accounts explained above provide two kinds of tax saving effects to households. First, households are allowed to defer part of their income tax payments from working periods to retirement periods. Delaying tax payments will decrease the present value of lifetime tax payments for the newborn households even if the government tax revenue in each period is unchanged. Second, when the individual income tax is progressive, households are allowed to smooth the marginal income tax rates and reduce the lifetime income tax payments because the marginal tax rates are on average higher when households are working, and lower when households are retired. This tax-saving effect is even larger when the economy is growing and the government adjusts the income tax brackets to avoid the real bracket creep.

The previous empirical papers estimate how much tax-deferred retirement saving accounts increase national saving. However, the predictions on the net saving effect of tax-deferred accounts differ widely. For example, Venti and Wise (1990) estimate that the majority of IRA contributions represent net new saving; Gale and Scholz (1994) show that raising the annual IRA contribution limit would have resulted in little increase in national saving; Poterba, Venti, and Wise (1995) find little evidence that 401(k) contributions substitute for other forms of personal saving; Attanasio and DeLeire (2002) find that at most 9% of IRA contributions represented net additions to national saving; Benjamin (2003) estimates that about one half

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2 In the U.S. 401(k) plans, 10% early withdrawal penalties are applied to the distributions before reaching age 59 and 1/2.
3 A representative-agent stochastic OLG model does not work for this paper, since average households face liquidity constraints much less likely except for the very early stage of their life. An OLG version of Krusell-Smith (1998) type growth model with both aggregate and idiosyncratic shocks would work better. However, adding at least three more state variables make the computation of transition paths impractical, since we cannot use the linear-quadratic approximation in the economy with liquidity constraints and precautionary savings.
of 401(k) balances represent new private savings, and about one quarter of 401(k) balances represent new national savings; and Gale (2005) reviews the literature and concludes that only a relatively small portion of the existing wealth accumulated in tax-deferred accounts represents a net addition of private savings.

To the best of our knowledge, İmrohoroğlu, İmrohoroğlu, and Joins (1998) are the first ones that numerically analyze the long-run effect of tax-deferred accounts on individual saving and aggregate economy in a dynamic general-equilibrium setting. İmrohoroğlu et al. construct a heterogeneous-agent OLG model with inelastic labor supply, a flat-rate income tax, a liquidity constraint, and unemployment shocks; and they show that approximately 9% of IRA contributions constitutes incremental saving. They also suggest that the effect of tax-deferred accounts would likely be small, because these accounts do not affect the rate of return on incremental saving for households whose originally intended saving was above the annual contribution limit of tax-deferred accounts.

This is not necessarily true for an economy with a progressive income tax. The first-order conditions of the household’s optimization problem (see Appendix) indicate that contributions blow the limit has the direct effect on saving; in addition, any positive contributions to the tax-deferred accounts reduce the marginal labor income tax rate and increase the ratio of consumption to leisure; and any positive future contributions reduce the future marginal capital income tax rate and increase current saving in the regular taxable accounts. For this reason, it is important to assume endogenous labor supply and a progressive income tax system in the model economy. More recently, working concurrently with the present paper, Kitao (2009) extends İmrohoroğlu et al. (1998) by introducing endogenous labor supply, idiosyncratic wage shocks, and a progressive income tax. The present paper develops a heterogeneous-agent OLG model closer to the one constructed by Kitao.

The main difference between the present paper and the two papers mentioned above is that we solve the model for an equilibrium transition path to analyze both the long-run (permanent) effect and the short-run (transition) effects of introducing tax-deferred accounts. This is very important for the policy assessment. The transition cost of introducing tax-deferred accounts can be significantly higher and, by changing the timing of taxation, the tax-deferred accounts will possibly make the current households worse off, even if the future households will be better off in the long run. In the present paper, we first calibrate the heterogeneous-agent OLG model to the U.S. economy without tax-deferred retirement saving accounts. This baseline economy is assumed to be in a steady-state equilibrium, i.e., it is on a balanced growth path. Then we introduce the tax-deferred accounts described above to the economy, solve the model for equilibrium transition paths
under four different financing assumptions, and evaluate the individual and aggregate effects of tax-deferred accounts both in the short run and in the long run.

The main findings are as follows. If the government financed the budgetary cost of tax-deferred accounts by cutting its consumption, the policy change would increase both capital stock (national wealth) and total output (GDP) throughout the transition path, and it would make all households—both current and future households—on average better off. If the government cut its transfer payments to households instead, the tax-deferred accounts would increase capital stock and total output even further; however, it would make current and near future households worse off, although future households would likely be better off in the long run. If the government raised the marginal income tax rates proportionally in each period, the macroeconomic effect would be worse, i.e., it would be negative in the short run and much smaller positive in the long run. Yet, the welfare effect under the marginal tax rate increase would be better than that under the transfer payment cut. Finally, if the government increased the marginal income tax rates once at the time of policy change and increased its debt gradually to spread the transition cost to the future households, the macroeconomic effect would be much smaller, but the welfare losses of current and near future households would be minimized.

The rest of the paper is laid out as follows: Section 2 describes the heterogeneous-agent overlapping-generations model with taxable and tax-deferred saving accounts, Section 3 shows the calibration of the baseline economies, Section 4 explains the effects of introducing 401(k) type tax-deferred accounts to the economy with a progressive income tax, Section 5 shows the effects of two alternative tax-deferred accounts, Section 6 examines the robustness of the model predictions by assuming two alternative baseline economies, and Section 7 concludes the paper. Appendix explains the computational algorithms to solve the household optimization problem and to solve the model for an equilibrium transition path.

2 The Model Economy

The economy consists of a large number of overlapping-generations households, a perfectly competitive representative firm with constant-returns-to-scale technology, and a government with a commitment technology.
2.1 The Households

The households are heterogeneous with respect to their age, \( i = 1, \ldots, I \), working ability, \( e \in E \), which follows the first-order Markov process, and beginning-of-period asset holdings — regular taxable assets, \( a_1 \in A_1 \), and tax-deferred assets, \( a_2 \in A_2 \). In period, \( t = 0, \ldots \), which is a year in this model economy, each household receives an idiosyncratic working-ability shock, \( e \), and chooses its optimal consumption, \( c \), working hours, \( h = 1 - l \), and end-of-period asset holdings, \( a'_1 \) and \( a'_2 \), in taxable and tax-deferred accounts.

The individual state variables are \((a_1, a_2, e, i)\). Let \( \Omega_t \) be a time series of vectors of factor prices and government policy variables that describes a future path of the aggregate economy,

\[
\Omega_t = \{r_s, w_s, C_{G,s}, \tr_{LS,s}, \varphi_s, \tau_{P,s}, \tr_{SS,s}, W_{G,s}, s_{2,s}^{\text{max}}\}_{s=t}^{\infty},
\]

where \( r_t \) is the interest rate; \( w_t \) is the average wage rate; \( C_{G,t} \) is government consumption; \( \tr_{LS,t} \) is the uniform lump-sum transfer; \( \varphi_t = (\varphi_{0,t}, \varphi_{1,t}, \varphi_{2,t})^T \) is the vector of individual income tax function parameters; \( \tau_{P,t} \) is the flat payroll tax rate for social security; \( \tr_{SS,t} \) is the uniform pay-as-you-go social security benefit; \( W_{G,t} \) is the government net wealth; and \( s_{2,t}^{\text{max}} \) is the annual contribution limit of the tax-deferred accounts.

Let \( v_i(a_1, a_2, e; \Omega_t) \) be the value function of a heterogeneous household of age \( i \) in period \( t \). The household’s optimization problem is

\[
(1) \quad v_i(a_1, a_2, e; \Omega_t) = \max_{c,t,s_2} \left\{ u(c, l) + \beta \phi_i E \left[ v_{i+1}(a'_1, a'_2, e'; \Omega_{t+1}) \mid e \right] \right\}
\]

subject to

\[
(2) \quad a'_1 = \frac{1}{(1 + \mu)\phi_i} \left[ (1 + r_t)a_1 + (1 - \tau_{P,t})w_t e(1 - l) - \tau_{I,t}(r_t a_1, w_t e(1 - l), s_2; \varphi_t) 
\right. 
\]

\[
+ \left. 1_{\{i \geq I_R\}} \tr_{SS,t} + \tr_{LS,t} - c - s_2 \right] \geq 0,
\]

\[
(3) \quad a'_2 = \frac{1}{(1 + \mu)\phi_i} \left[ (1 + r_t)a_2 + s_2 \right] \geq 0,
\]

\[
(4) \quad c > 0, \quad 0 < l \leq 1, \quad -(1 + r_t)a_2 \leq s_2 \leq \min(s_{2,t}^{\text{max}}, w_t e(1 - l)),
\]

\[\text{Let } S_t = \{x_t(a_1, a_2, e, i), W_{G,t}\} \text{ be the state of the economy, where } x_t(a_1, a_2, e, i) \text{ is the distribution of households, and let } \\
\Psi_t = \{C_{G,s}, \tr_{LS,s}, \varphi_s, \tau_{P,s}, \tr_{SS,s}, W_{G,s}, s_{2,s}^{\text{max}}\}_{s=t}^{\infty} \text{ be the government policy schedule. Then, the household’s value function is shown as } v_i(a_1, a_2, e; S_t, \Psi_t), \text{ and the factor prices and government policy variables are shown as } r_s(S_t, \Psi_t), w_s(S_t, \Psi_t), \varphi_s(S_t, \Psi_t), \text{ and so on, for } s \geq t. \text{ However, it is impossible to solve the model of this form because the dimension of } S_t \text{ is infinite.} \]
In this paper we avoid this curse of dimensionality problem by replacing \((S_t, \Psi_t)\) with \(\Omega_t\). Since we do not assume aggregate shocks in the model economy, the time series \(\Omega_t\) is deterministic and perfectly foreseeable, thus it will suffice to find the fixed point of \(\Omega_t\) to solve the model economy for an equilibrium transition path.
where \( l \) is leisure hours; \( s_2 \) is the contribution to the tax-deferred accounts; \( u(c, l) \) is a period utility function; \( \tilde{\beta} \) is the growth-adjusted time discount factor; \( \phi_i \) is the survival rate at the end of age \( i \); \( E[\cdot] \) is an expected value operator; \( e' \) is the working ability in the next period; \( \mu \) is the labor-augmenting productivity growth rate; \( \tau_{I,i}(\cdot; \varphi_i) \) is the individual income tax function with parameters \( \varphi_i \);

\[ 1_{\{\cdot\}} \] is the indicator function that returns 1 if the condition in \( \{\cdot\} \) is satisfied and 0 otherwise; and \( I_R \) is the exogenous retirement age.

The individual income tax depends on taxable capital income, labor income, and the contribution to the tax-deferred accounts, which is possibly negative. Capital income generated in the tax-differed accounts, \( r_t a_2 \), is not taxable in period \( t \).

To express a balanced growth path by a steady-state equilibrium, individual variables other than leisure hours are normalized by the long-run growth rate \( 1 + \mu \). We assume taxable assets and tax-deferred assets are both nonnegative, so that the early-withdrawal penalty of tax-deferred accounts affects the household’s portfolio decision. For simplicity we also assume a perfect annuities market in the economy, where the price of one period actuarially fair annuity is \( \phi_i \).\(^7\)

Solving the above problem for \( c, l, \) and \( s_2 \) for all possible states, we obtain the household’s decision rules, \( c_i(a_1, a_2, e; \Omega_t), l_i(a_1, a_2, e; \Omega_t), \) and \( s_{2,i}(a_1, a_2, e; \Omega_t) \).\(^8\) The other decision rules are also obtained as \( h_i(a_1, a_2, e; \Omega_t) \equiv 1 - l_i(a_1, a_2, e; \Omega_t) \), and

\[
a'_1,i(a_1, a_2, e; \Omega_t) \equiv \frac{1}{(1 + \mu) \phi_i} \left[ (1 + r_t) a_1 + (1 - \tau_{P,t}) w_t e h_i(a_1, a_2, e; \Omega_t) \right. \\
- \tau_{I,i}(\eta_1 a_1, w_t e h_i(a_1, a_2, e; \Omega_t), s_{2,i}(a_1, a_2, e; \Omega_t); \varphi_i) \\
+ \left. 1_{\{i \geq I_R\}} \tau_{SS,t} + \tau_{LS,t} - c_i(a_1, a_2, e; \Omega_t) - s_{2,i}(a_1, a_2, e; \Omega_t) \right],
\]

\[
a'_{2,i}(a_1, a_2, e; \Omega_t) \equiv \frac{1}{(1 + \mu) \phi_i} \left[ (1 + r_t) a_2 + s_{2,i}(a_1, a_2, e; \Omega_t) \right].
\]

Let \( x_{i,t}(a_1, a_2, e) \) be the growth-adjusted population density of households of age \( i \) in period \( t \), and let \( X_{i,t}(a_1, a_2, e) \) be the corresponding cumulative distribution function. We assume that households enter the

\(^5\)The individual income tax function is age dependent because of the early withdrawal penalty and the catch-up contribution limit of tax-deferred accounts.

\(^6\)We also need to assume the elasticity of intra-temporal substitution of consumption for leisure to be unity to make the model consistent with a growth economy.

\(^7\)This assumption is not as strong as what it might look. When bequests are intentional (warm glow) and the marginal value of leaving bequests, \( v'(z') \), coincides with the marginal value of saving, \( v_{1,i+1}(a_1, a_2', e'; \Omega_{i+1}) \), the first-order condition implies that the household’s optimal end-of-period wealth, \( a_1' \), will be identical to that in the economy with a perfect annuities market and without an intentional bequest motive.

\(^8\)We form a complementarity problem with the first order conditions and the constraints (2)-(4) and solve it by a Newton type nonlinear equation solver for each individual state and \( \Omega_i \). See Appendix for the detail of the computational algorithm.
economy with no assets, i.e., \( a_1 = a_2 = 0 \), and that the growth-adjusted population of age 1 households is normalized to unity,

\[
\int_{A_1 \times A_2 \times E} dX_{1,t}(a_1, a_2, e) = \int_{E} dX_{1,t}(0, 0, e) = 1.
\]

Let \( \pi_i(e' \mid e) \) be the transition probability density of working ability from \( e \) at age \( i \) to \( e' \) at age \( i + 1 \). Let \( \nu \) be the time-invariant population growth rate. Then, the law of motion of the growth-adjusted population distribution is

\[
x_{i+1,t+1}(a'_1, a'_2, e') = \frac{1}{1 + \nu} \int_{A_1 \times A_2 \times E} \mathbf{1}_{\{a'_1 = a'_1, a_2, e; \Omega_t\}, a'_2 = a'_2, (a_1, a_2, e; \Omega_t)} \pi_i(e' \mid e) dX_{i,t}(a_1, a_2, e).
\]

The growth-adjusted private taxable wealth, \( W_{1,t} \), and tax-deferred wealth, \( W_{2,t} \), are

\[
W_{1,t} = \sum_{i=1}^{t} \int_{A_1 \times A_2 \times E} a_1 dX_{i,t}(a_1, a_2, e), \quad W_{2,t} = \sum_{i=1}^{t} \int_{A_1 \times A_2 \times E} a_2 dX_{i,t}(a_1, a_2, e).
\]

In a closed economy, capital stock (national wealth), \( K_t \), and labor supply in efficiency units, \( L_t \), are

\[
K_t = W_{1,t} + W_{2,t} + W_{G,t},
\]

\[
L_t = \sum_{i=1}^{t} \int_{A_1 \times A_2 \times E} \epsilon_{i}(a_1, a_2, e; \Omega_t) dX_{i,t}(a_1, a_2, e).
\]

### 2.2 The Firm

In each period, the representative firm chooses the capital input, \( \tilde{K}_t \), and efficiency labor input, \( \tilde{L}_t \), to maximize its profit, taking factor prices, \( r_t \) and \( w_t \), as given, i.e.,

\[
\max_{\tilde{K}_t, \tilde{L}_t} F(\tilde{K}_t, \tilde{L}_t) - (r_t + \delta) \tilde{K}_t - w_t \tilde{L}_t,
\]

where \( F(\cdot) \) is a constant-returns-to-scale production function, and \( \delta \) is the depreciation rate of capital. The profit maximizing conditions are

\[
F_K(\tilde{K}_t, \tilde{L}_t) = r_t + \delta, \quad F_L(\tilde{K}_t, \tilde{L}_t) = w_t,
\]
and the factor markets clear when $K_t = \bar{K}_t$ and $L_t = \bar{L}_t$.

2.3 The Government

We assume for simplicity that the social security pension system is pay-as-you-go, the payroll tax rate is flat without any tax cap, and the retirement benefit is uniform for all households aged $i \geq I_R$, where $I_R$ is exogenous retirement age. In the model economy, the payroll tax rate is fixed at the same level and the benefit is determined endogenously so that the social security budget is always balanced. The government’s social security payroll tax revenue, $T_{P,t}$, is

$$T_{P,t} = \tau_{P,t} w_t L_t,$$

and the individual social security benefit for $i \geq I_R$, $tr_{SS,i,t}$, is

$$tr_{SS,t} = \left( \sum_{i=I_R}^{I} \int_{A_1 \times A_2 \times E} dX_{i,t}(a_1, a_2, e) \right)^{-1} T_{P,t}.$$  

The government’s income tax revenue is

$$T_{I,t} = \sum_{i=1}^{I} \int_{A_1 \times A_2 \times E} \tau_{I,i} (r_t a_1, w_t e h_i(a_1, a_2, e; \Omega_t), s_{2,i}(a_1, a_2, e; \Omega_t); \varphi_t) dX_{i,t}(a_1, a_2, e),$$

the aggregate lump-sum transfer expenditure is

$$T_{R_{LS,t}} = \sum_{i=1}^{I} \int_{A_1 \times A_2 \times E} dX_{i,t}(a_1, a_2, e),$$

and the law of motion of government wealth is

$$W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)} \left[ (1+r_t)W_{G,t} + T_{I,t} - C_{G,t} - T_{R_{LS,t}} \right].$$

Note that aggregate variables are normalized by both the long-run productivity growth rate, $1+\mu$, and the population growth rate, $1+\nu$, so that the balanced growth path of the economy is obtained as a steady state equilibrium.

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9If a policy change increases the labor income of working-age households, other things being equal, elderly households will also be better off through the increased social security benefit under this assumption.
2.4 Recursive Competitive Equilibrium

The recursive competitive equilibrium of this model economy is defined as follows.

**Definition Recursive Competitive Equilibrium:** Let \((a_1, a_2, e, i)\) be the individual state of households. A time series of factor prices and the government policy variables,

\[\Omega_t = \{r_s, w_s, C_{G,s}, tr_{LS,s}, \varphi_s, \tau_{P,s}, tr_{SS,s}, W_{G,s}, s^\text{max}_{2,s}\}_{s=t}^\infty,\]

the value functions of households, \(\{v_i(a_1, a_2, e; \Omega_s)\}_{s=t}^\infty\), the decision rules of households,

\[\{c_i(a_1, a_2, e; \Omega_s), l_i(a_1, a_2, e; \Omega_s), s_{2,i}(a_1, a_2, e; \Omega_s), a'_{1,i}(a_1, a_2, e; \Omega_s), a'_{2,i}(a_1, a_2, e; \Omega_s)\}_{s=t}^\infty,\]

and the distribution of households, \(\{x_{i,s}(a_1, a_2, e)\}_{s=t}^\infty\), are in a recursive competitive equilibrium if, for all \(s = t, \ldots, \infty\), each household solves the optimization problem (1)-(4), taking \(\Omega_s\) as given; the firm solves its profit maximization problem (5)-(6); the government policy schedule satisfies (7)-(11); and the goods and factor markets clear.

The economy is in a steady-state equilibrium (thus, on a balanced growth path) if, in addition,

\[x_{i,s+1}(a_1, a_2, e) = x_{i,s}(a_1, a_2, e)\text{ and } \Omega_{s+1} = \Omega_s \text{ for all } s = t, \ldots, \infty.\]

3 Calibration

Tables 1 and 2 summarize the main parameter values and corresponding target values. We create three baseline economies on the balanced growth path without tax-deferred accounts. In the main baseline economy, we assume a progressive individual income tax with effective marginal tax rate up to 30% and the coefficient of relative risk aversion, \(\gamma\), equals to 3.0. In the first alternative baseline economy, the progressive income tax is replaced with a flat 15.19% income tax, keeping the relative size of tax revenue to output, \(T_1/Y\), at the same level. In the second alternative baseline economy, we reduce the coefficient of relative risk aversion parameter to 1.5 from 3.0.

In all of the three baseline economies, the capital-output ratio, \(K/Y\), is targeted to 2.7 by choosing the time discount factor, \(\beta\); the life cycle wealth-output ratio, \((W_1 + W_2)/Y\), is 1.8 by assuming there is additional private wealth, \(W_3\), determined by motives other than life cycle and precautionary saving.
motives against working ability shocks;\(^{10}\) the wage rate, \(w\), is normalized to 1.0 by choosing the total factor productivity, \(A\); the interest rate, \(r\), is set at 5.0\% by adjusting the depreciation rate of capital, \(\delta\); and the income tax revenue-output ratio, \(T_I/Y\), is targeted to 12\% by adjusting one of the income tax function parameters.\(^{11}\)

\(^{10}\)Saving motives that could enhance private wealth accumulation but not considered in this paper are altruistic bequests, entrepreneurship, and precautionary motives against health and other expenditure shocks. Without introducing “other” private wealth, in the presence of pay-as-you-go social security pensions, the time discount factor that generates \(K/Y = 2.7\) observed in the data will be greater than one, which will generate a steeply increasing age-consumption profile and over-emphasize the importance of life cycle savings.

\(^{11}\)Federal individual income tax revenue is 8.1\% as a percentage of GDP in 2008. However, the sustainable level of income tax revenue is considered to be 10.0\% of GDP, since the corresponding numbers are 9.6\% and 10.3\% in 1999 and 2000, respectively, when the federal budget is balanced (Congressional Budget Office, 2009). Adding state and local individual income tax revenue, which is on average 2.0\% of GDP in years during the past 10 years (Census Bureau, 2009), we assume individual tax revenue to be 12\% as a percentage of GDP in the baseline economy.

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**Table 1: Common Parameters and Policy Variables in the Baseline Economies**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Notes</th>
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</thead>
<tbody>
<tr>
<td>Share parameter of consumption (\alpha)</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>Share parameter of capital stock (\theta)</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Depreciation rate of capital stock (\delta)</td>
<td>0.61</td>
<td>(r = 0.050) in the baseline</td>
</tr>
<tr>
<td>Labor-augmenting productivity growth rate (\mu)</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>Population growth rate (\nu)</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>Auto correlation parameter (\rho)</td>
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<td></td>
</tr>
<tr>
<td>Standard deviation of temporary shock (\sigma_e)</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Total factor productivity (A)</td>
<td>0.9528</td>
<td>(w = 1.0) in the baseline</td>
</tr>
<tr>
<td>Annual contribution limit (s_2^{\text{max}})</td>
<td>0.0</td>
<td>No tax-deferred accounts</td>
</tr>
<tr>
<td>Lump-sum transfers (tr_{LS})</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Government net wealth (W_G)</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Social security payroll tax rate (\tau_P)</td>
<td>0.10</td>
<td>OASI tax rate (2 \times 0.053/1.053)</td>
</tr>
</tbody>
</table>

**Table 2: Other Parameter Values in the Baseline Economies**

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Main</td>
</tr>
<tr>
<td>Time discount factor (\beta)</td>
<td>0.9782</td>
</tr>
<tr>
<td>Growth adjusted discount factor (\tilde{\beta})</td>
<td>0.9657</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion (\gamma)</td>
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</tr>
<tr>
<td>Income tax function parameters $\varphi_0$</td>
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</tr>
<tr>
<td>$\varphi_1$</td>
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</tr>
<tr>
<td>$\varphi_2$</td>
<td>1.319</td>
</tr>
<tr>
<td>Social security benefits (tr_{SS})</td>
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</tr>
</tbody>
</table>
3.1 Utility and Production Functions

To make the model economy consistent with a growth economy, the period utility function is assumed
to be one of Cobb-Douglas and constant relative risk aversion:

\[
u(c, l) = \frac{(e^\alpha l^{1-\alpha})^{1-\gamma}}{1-\gamma} = \frac{(e^{\alpha}(1-h)^{1-\alpha})^{1-\gamma}}{1-\gamma}.
\]

The share parameter of consumption, \(\alpha\), is set at 0.36, following the real business cycle literature (e.g., Cooley and Prescott, 1995), the coefficient of relative risk aversion, \(\gamma\), is assumed to be 3.0 in the main baseline economy, and it is later reduced to 1.5. The labor-augmenting productivity growth rate, \(\mu\), is set at 0.018, which is equal to the average annual growth rate for real GDP per capita for 1997-2007 in the U.S. (Bureau of Labor Statistics, 2009). The growth-adjusted time discount factor is calculated as

\[\tilde{\beta} = \beta(1 + \mu)^{\alpha(1-\gamma)}.
\]

The production function is also one of Cobb-Douglas with constant-returns-to-scale technology:

\[F(K, L) = AK^\theta L^{1-\theta}.
\]

The share parameter of capital, \(\theta\), is set at 0.30, which is consistent with macroeconomic statistics, the depreciation rate of capital stock, \(\delta\), is 0.061 so that the interest rate is 5.0%, and the total factor productivity scalar, \(A\), is adjusted so that the average wage rate, \(w\), is normalized to 1.0 in the baseline economies.\(^{12}\)

3.2 Demographics

We assume each household enters the economy at the beginning of age 21 and possibly lives up to the end of age 100. The model age \(i = 1\) is real age 21, and the model age \(i = I = 80\) is real age 100. For simplicity, we assume all households retire at the beginning of age 65, or model age \(i = I_R = 45\), and start receiving social security benefits. The population growth rate, \(\nu\), is set at 0.01. The end-of-age survival rates, \(\phi_i\), are calculated from the 2003 male mortality rates in Social Security Administration (2007) and shown in Table 3. Under these assumptions, the growth-adjusted population in the model economy is 41.9308, and the population of age 65 or older is 7.8877, when the population of age 21 is normalized to unity.

\(^{12}\)The Cobb-Douglas production function implies \(r + \delta = \theta/(K/Y) = 0.30/2.7 = 0.111; K/L = (K/Y)/(1 - \theta) = 2.7/0.7 = 3.8571;\) and \(A = w/[(1-\theta)(K/L)^\theta] = 1.0/(0.7 \times 3.8571^{0.3}) = 0.9528.\)
Table 3: Survival Rates at the End of Each Age

<table>
<thead>
<tr>
<th>Age</th>
<th>Survival Rate</th>
<th>Age</th>
<th>Survival Rate</th>
<th>Age</th>
<th>Survival Rate</th>
<th>Age</th>
<th>Survival Rate</th>
</tr>
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<td>41</td>
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<td>82</td>
<td>0.913187</td>
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<tr>
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<td>43</td>
<td>0.996721</td>
<td>63</td>
<td>0.984117</td>
<td>83</td>
<td>0.904158</td>
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<tr>
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<td>84</td>
<td>0.894091</td>
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<tr>
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<td>0.996110</td>
<td>65</td>
<td>0.981249</td>
<td>85</td>
<td>0.882983</td>
</tr>
<tr>
<td>26</td>
<td>0.998664</td>
<td>46</td>
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<td>66</td>
<td>0.979553</td>
<td>86</td>
<td>0.870830</td>
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<tr>
<td>27</td>
<td>0.998687</td>
<td>47</td>
<td>0.995422</td>
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<td>0.857617</td>
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<td>0.998684</td>
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<td>0.973546</td>
<td>89</td>
<td>0.827908</td>
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<td>0.994304</td>
<td>70</td>
<td>0.971096</td>
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<td>0.793646</td>
</tr>
<tr>
<td>32</td>
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<td>0.993406</td>
<td>72</td>
<td>0.965378</td>
<td>92</td>
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<tr>
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<td>80</td>
<td>0.928313</td>
<td>100</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Source: Author’s calculation from the 2003 male mortality rates in Table 4.C6 from Social Security Administration (2008). The survival rate at the end of age 100 is replaced with 0.

3.3 The Working Ability Process

The individual working ability, $e_i$, at model age $i$ before the retirement in the model economy is

$$\ln e_i = \ln \bar{e}_i + \ln z_i,$$

where $\bar{e}_i$ is the average working ability at age $i$, and the persistent shock, $z_i$, follows an AR(1) process,

$$\ln z_i = \rho \ln z_{i-1} + \epsilon_i, \quad \ln z_0 = 0,$$

where $\epsilon_i \sim N(\mu_\epsilon, \sigma_\epsilon^2)$. The unconditional expected value of $z_i$ is normalized to unity. The auto-correlation parameter, $\rho$, is assumed to be 0.95, and the standard deviation, $\sigma_\epsilon$, is set at 0.25. The log deviation from the
mean, $\ln z_i$, is also normally distributed and the variance is increasing in age,

$$
\mu_{\ln z_i} = -\frac{1}{2}\sigma_{\ln z_i}^2, \quad \sigma_{\ln z_i}^2 = \sum_{j=1}^{i} \rho_{\ln z_i}^{2(i-1)} \sigma_{\epsilon}^2 = \frac{1 - \rho_{\ln z_i}^{2i}}{1 - \rho_{\ln z_i}^2} \sigma_{\epsilon}^2.
$$

We construct the average working ability, $\bar{e}_i$, for ages between 21 and 64 from the 2005 median earnings of male workers in Social Security Administration (2008). Because the median earnings are not shown for all ages in the table, we smooth out the raw data by taking the 5-year moving average and an additional 3-year moving average. We discretize the log persistent shock, $\ln z_i$, into 11 levels by using Gauss-Hermite quadrature nodes with $\mu_{\ln z_i}$ and $\sigma_{\ln z_i}^2$, then generate 5 levels of $\ln z_i$ by combining 4 nodes in each tail distribution into one node.\(^\text{13}\) The 5 nodes of $\ln z_i$ at age 21 ($i = 1$) are (0.5885, 0.7684, 0.9692, 1.2225, 1.5964), the nodes at age 51 ($i = 31$ when labor income is at its peak) are (0.1539, 0.3552, 0.7355, 1.5233, 3.5154), and the corresponding weights are $\pi = (0.0731, 0.2422, 0.3694, 0.2422, 0.0731)$. Table 4 shows the period age-working ability profile in the model economy. The age-working ability profile in the table is that of cross section in each period. The cohort age-working ability is tilted upward by $$(1 + \mu_{\ln z_i})$$ for all $i < I_R$, the Markov transition matrix, $\Pi_i = [\pi(e_{i+1}^k | e_i^j)]$, with $\rho = 0.95$ is calculated as

$$
\Pi_i = \begin{pmatrix}
0.8979 & 0.1021 & 0.0000 & 0.0000 & 0.0000 \\
0.0308 & 0.8902 & 0.0790 & 0.0000 & 0.0000 \\
0.0000 & 0.0518 & 0.8964 & 0.0518 & 0.0000 \\
0.0000 & 0.0000 & 0.0790 & 0.8902 & 0.0308 \\
0.0000 & 0.0000 & 0.0000 & 0.1021 & 0.8979 \\
\end{pmatrix}
$$

### 3.4 The Government

The progressive income tax function is an extended version of Gouveia and Strauss (1994):

$$
\tau_{I,t}(r_t a_1, w_t e(1 - l), s_2; \varphi_t) \equiv \tilde{\tau}_{I,t}(y) = \varphi_{0,t} \left[ y - \left( y^{-\varphi_{1,t}} + \varphi_{2,t} \right)^{-1/\varphi_{1,t}} \right] - 1_{\{i < 40\}} 0.1 \min(s_2, 0),
$$

where $y \equiv r_t a_1 + w_t e(1 - l) - s_2 \geq 0$ is a taxable income. The taxable income is the sum of taxable capital income and labor income less contribution to tax-deferred accounts. The parameter $\varphi_{0,t}$, which is the effective marginal tax rate when $y$ goes to infinity, is set at 0.30 or 30% in the baseline economies with

---

\(^{13}\)See Judd (1998) for the general calculation of Guss-Hermite quadrature.
Table 4: Individual Working Abilities by Age

<table>
<thead>
<tr>
<th>Age ((i + 20))</th>
<th>(e_i)</th>
<th>(e_i^1)</th>
<th>(e_i^2)</th>
<th>(e_i^3)</th>
<th>(e_i^4)</th>
<th>(e_i^5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>0.3186</td>
<td>0.1875</td>
<td>0.2448</td>
<td>0.3088</td>
<td>0.3895</td>
<td>0.5086</td>
</tr>
<tr>
<td>22</td>
<td>0.3825</td>
<td>0.1811</td>
<td>0.2616</td>
<td>0.3604</td>
<td>0.4965</td>
<td>0.7173</td>
</tr>
<tr>
<td>23</td>
<td>0.4523</td>
<td>0.1825</td>
<td>0.2833</td>
<td>0.4155</td>
<td>0.6092</td>
<td>0.9457</td>
</tr>
<tr>
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<td>0.7299</td>
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</tr>
<tr>
<td>25</td>
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</tr>
<tr>
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<td>59</td>
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<td>0.1706</td>
<td>0.3977</td>
<td>0.8310</td>
<td>1.7363</td>
<td>4.0476</td>
</tr>
<tr>
<td>60</td>
<td>1.0809</td>
<td>0.1617</td>
<td>0.3772</td>
<td>0.7887</td>
<td>1.6489</td>
<td>3.8468</td>
</tr>
<tr>
<td>61</td>
<td>1.0086</td>
<td>0.1506</td>
<td>0.3516</td>
<td>0.7355</td>
<td>1.5387</td>
<td>3.5923</td>
</tr>
<tr>
<td>62</td>
<td>0.9206</td>
<td>0.1372</td>
<td>0.3206</td>
<td>0.6710</td>
<td>1.4046</td>
<td>3.2812</td>
</tr>
<tr>
<td>63</td>
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<td>0.1216</td>
<td>0.2842</td>
<td>0.5952</td>
<td>1.2465</td>
<td>2.9134</td>
</tr>
<tr>
<td>64</td>
<td>0.7124</td>
<td>0.1059</td>
<td>0.2477</td>
<td>0.5189</td>
<td>1.0871</td>
<td>2.5422</td>
</tr>
</tbody>
</table>

Source: Author’s calculation from the 2005 median earnings of male workers by age group in Table 4.B6 from Social Security Administration (2008). The population weighted average of working abilities is normalized to 1.0.
a progressive income tax. The parameter $\varphi_{1,t}$, which determines the shape of the marginal tax rate curve, is set at 0.839, the average of the estimated parameters in years 1979-89. Following Conesa, Kitao, and Kruger (2009), we adjust the parameter $\varphi_{2,t}$ so that the relative size of income tax revenue to output is equal to our target value, 0.12, in our baseline economies.\(^{14}\) The second term of the progressive income tax function is the early withdrawal penalty of tax-deferred accounts. We assume a 10% penalty in addition to the individual income tax on $s_2 < 0$ when the household is younger than age 60 or model age $i = 40$.

The social security pension system is pay-as-go and uniform, and the social security budget is separated from the rest of the government budget. The flat payroll tax rate, $\tau_{P,t}$, is assumed to be 10% for all period $t$, which is approximately equal to the current effective OASI tax rate, $2 \times 0.053/1.053 = 0.1007$, below the tax cap. The uniform social security benefit per household, $tr_{SS,t}$, is calculated in each period to satisfy the pay-as-you-go condition.

In the baseline economies, the government net wealth, $W_{G,t}$, and the lump-sum transfer, $tr_{LS,t}$, are both assumed to be 0, and the social security budget is balanced. Thus,

$$C_{G,t} = (1 + r_t)W_{G,t} + T_{I,t} - TR_{LS,t} - (1 + \mu)(1 + \nu)W_{G,t+1} = T_{I,t},$$

i.e., the government consumption, which has no effect on the household utility, is equal to its income tax revenue. In the policy experiments with tax deferred accounts, we will obtain $C_{G,t}$, $tr_{LS,t}$, $T_{I,t}(\varphi_{0,t})$, or $W_{G,t+1}$ endogenously.

4 Introducing Tax-Deferred Retirement Saving Accounts

In this section, we analyze the long-run (steady-state) and short-run (transition) effects of introducing tax-deferred retirement saving accounts to the main baseline economy with a progressive income tax and $\gamma = 3.0$. We assume the economy is in the initial steady-state equilibrium (the baseline economy) in period 0, and the government changes the annual contribution limit to the tax-deferred accounts, $s_{2\text{max}}$, from 0 to 0.1765 permanently at the beginning of period 1. Because labor income per working-age household is 0.3530 in the baseline economy, the new level of $s_{2\text{max}}$ corresponds to 50.0% of the average labor income, which is roughly consistent with the U.S. 401(k) plans. The annual contribution limits of 401(k) plans in

\(^{14}\)In the main baseline economy, the average labor income of age 21-64 households is 0.3530. The average and marginal income tax rates at the average labor income (with no capital income) are 12.2% and 18.5%, respectively. When the household income is twice as large, the average and marginal tax rates are 16.8% and 23.3%, respectively.
2008 and 2009 are $15,500 and $16,500, which are 48.9% and 51.6%, respectively, of the average earnings of production and nonsupervisory workers in 2008 (Bureau of Labor Statistics, 2009). As explained in Section 3, we assume an 10% early withdrawal penalty for households younger than age 60.

4.1 Government’s Financing Assumptions

Introducing tax-deferred accounts is not free from the government’s perspective. The individual income tax revenue would decline both in the short run and in the long run. Thus, to close the government’s inter-temporal budget constraint, we assume the following four government financing policies: (a) cutting government consumption, $C_{G,t}$, each period to balance the budget; (b) cutting government (lump-sum) transfer payments, $TR_{LS,t}$, each period instead; (c) increasing marginal income tax rates, $\varphi_{0,t}$, proportionally each period instead; finally, (d) increasing marginal tax rates once in period 1 and changing government net wealth, $W_{G,t}$, each period so that the government budget is just sustainable.

The four financing rules are summarized as follows:

(a) $C_{G,t} \leftarrow C_{G,t} = T_{I,t}(\varphi_{0,t}), \quad TR_{LS,t} = W_{G,t} = 0$

(b) $tr_{LS,t} \leftarrow TR_{LS,t} = T_{I,t}(\varphi_{0,t}) - C_{G,t}, \quad W_{G,t} = 0$

(c) $\varphi_{0,t} \leftarrow T_{I,t}(\varphi_{0,t}) = C_{G,t}, \quad TR_{LS,t} = W_{G,t} = 0$

(d) $\varphi_{0}, W_{G,t+1} \leftarrow \lim_{t \to \infty} W_{G,t+1} = W_{G}, \quad TR_{LS,t} = 0$

where $W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)} \left[(1+r_t)W_{G,t} + T_{I,t}(\varphi_{0}) - C_{G,t} - TR_{LS,t}\right]$ and $W_{G,1} = 0$.

In the model economy, government consumption is not in the utility function of households, thus, it is simply a waste. Cutting effective income tax rates financed by removing part of wastes would surely improve the overall welfare of the economy. Thus, any positive welfare implication under financing assumption (a) should be taken with caution. However, the policy experiment with this assumption is important, because it would provide the first-order budgetary cost and macroeconomic effect of tax-deferred accounts, i.e., how much income tax revenue would be reduced if the other government policy variables (except for the endogenous social security benefit level) were kept at the baseline levels.

Financing assumption (b), cutting transfer payments uniformly, is based on the following setup: the government expenditure is not a waste but a perfect substitute of other consumption goods, and all house-
holds are benefited equally from the government provided goods and services. Under this assumption, macroeconomic variables such as capital stock and labor supply tend to be larger due to the negative income effect. With this secondary effect, the decline in income tax revenue would be relatively small. In a realistic economy, the macroeconomic and welfare effects would probably be somewhere between those under assumption (a) and assumption (b).

Financing assumption (c), increasing marginal tax rates proportionately would reflect both the income effect and the substitution effect. The macroeconomic variables would likely be smaller (worse), thus the budgetary cost would be higher under this assumption compared to the previous ones. Tax-deferred accounts would reduce the marginal tax rates of households that make contributions to the accounts. However, if the budgetary cost was financed by increasing the statutory income tax rates, the policy change would increase the marginal tax rates of those who do not make any contributions, such as young households with binding borrowing constraints and most elderly households.

Financing assumptions (a), (b), and (c) are all budget neutral in each period, i.e., the government net wealth stay at the same level as that in the baseline economy. However, the budgetary cost of tax-deferred accounts is much higher in the short run than in the long run as we will see below. In other words, there is an additional transition cost of introducing tax-deferred accounts in the short run before many elderly households start withdrawing money from the accounts and paying additional income taxes. Financing assumption (d), raising marginal tax rates proportionally once in period 1 and increasing government debt gradually, intends to spread the transition cost to all generations and reduce the burden on the current households. The one time tax increase, which is larger than that in the final steady state in (c), would also indicate the truer cost of tax-deferred accounts.

4.2 The Welfare Measure

We measure the welfare gains or losses of newborn (age 21, \(i = 1\)) households at the beginning of \(t = 1, \ldots, \infty\) by the uniform percent changes, \(\lambda_{1,t}\), in the baseline consumption path that would make their expected lifetime utility equivalent with the expected utility after the introduction of tax-deferred accounts; i.e., \(\lambda_{1,t}\) satisfies

\[
E_{t-1} \left[ \sum_{i=1}^{I} \beta^{i-1} \left( \prod_{j=1}^{i-1} \phi_j \right) u \left( c_{t+i-1}, h_{t+i-1} \right) \right]
\]
\[ E_{-1} \left[ \sum_{i=1}^{I} \tilde{\beta}^{i-1} \left( \prod_{j=1}^{i-1} \phi_j \right) u \left( (1 + \lambda_{1,t}) c_{i,i-1}, l_{i,i-1} \right) \right], \]

and when \( u(c, l) = \left( \frac{e^\alpha l^{1-\alpha} l^{-\gamma}}{1-\gamma} \right), \) the equivalent variation percent change, \( \lambda_{1,t}, \) is calculated as

\[
\lambda_{1,t} = \left( \frac{E_{t-1} v_1(a_1, a_2, e; \Omega_t)}{E_{-1} v_1(a_1, a_2, e; \Omega_0)} \right)^{\frac{1}{\alpha(1-\gamma)}} - 1 = \left( \frac{\sum_j \pi(e^j) v_1(0, 0, e^j; \Omega_t)}{\sum_j \pi(e^j) v_1(0, 0, e^j; \Omega_0)} \right)^{\frac{1}{\alpha(1-\gamma)}} - 1.
\]

Recall that \( s_{2,t}^{\max} = 0 \) in the policy schedule \( \Omega_0 \) and \( s_{2,t}^{\max} = 0.1765 \) in the policy schedule \( \Omega_t \) for all \( t \geq 1. \)

Similarly, we calculate the welfare changes of households of age \( i \) with state \((a_1, a_2, e)\) at the time of policy change \((t = 1)\) by the uniform percent changes, \( \lambda_{i,1}(a_1, a_2, e), \) required in the baseline consumption path so that the rest of the lifetime value would be equal to the rest of the lifetime value after the policy change; i.e., \( \lambda_{i,1}(a_1, a_2, e) \) satisfies

\[
E_1 \left[ \sum_{k=i}^{I} \tilde{\beta}^{k-i} \left( \prod_{j=i}^{k-1} \phi_j \right) u \left( c_{k,1+k-i}, l_{k,1+k-i} \right) \right] = E_0 \left[ \sum_{k=i}^{I} \tilde{\beta}^{k-i} \left( \prod_{j=i}^{k-1} \phi_j \right) u \left( (1 + \lambda_{i,1}(a_1, a_2, e)) c_{k,k-i}, l_{k,k-i} \right) \right],
\]

and it is calculated as

\[
\lambda_{i,1}(a_1, a_2, e) = \left( \frac{v_i(a_1, a_2, e; \Omega_1)}{v_i(a_1, a_2, e; \Omega_0)} \right)^{\frac{1}{\alpha(1-\gamma)}} - 1.
\]

Then, the cohort-average welfare changes in period 1, \( \lambda_{i,1}, \) is obtained as the unconditional expectation, which is equivalent with the population weighted average, of the equivalent variation percent changes, i.e.,

\[
\lambda_{i,1} = \int_{A_1 \times A_2 \times E} \lambda_{i,1}(a_1, a_2, e) dX_{i,1}(a_1, a_2, e) \quad \text{for } i = 2, \ldots, I.
\]

Note that \( \lambda_{i,1} \) for \( i = 1, \ldots, I \) shows the cohort-average welfare changes of all current households alive at the time of policy change, and \( \lambda_{1,t} \) for \( t = 2, \ldots, \infty \) shows the cohort-average welfare changes of all future/unborn households.
Table 5: The Long-Run Effects of Tax-Deferred Accounts in the Economy with a Progressive Income Tax and $\gamma = 3.0$ (% changes from the baseline economy)

<table>
<thead>
<tr>
<th>Financing Assumption</th>
<th>Run 1 (a)</th>
<th>Run 1 (b)</th>
<th>Run 1 (c)</th>
<th>Run 1 (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Stock (National Wealth)</td>
<td>12.8</td>
<td>13.7</td>
<td>11.4</td>
<td>4.7</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>0.3</td>
<td>1.1</td>
<td>-0.6</td>
<td>-1.5</td>
</tr>
<tr>
<td>Total Output (GDP)</td>
<td>3.9</td>
<td>4.7</td>
<td>2.9</td>
<td>0.3</td>
</tr>
<tr>
<td>Income Tax Revenue</td>
<td>-8.8</td>
<td>-7.8</td>
<td>0.0</td>
<td>4.2</td>
</tr>
<tr>
<td>Private Consumption</td>
<td>4.2</td>
<td>3.6</td>
<td>1.4</td>
<td>-0.7</td>
</tr>
<tr>
<td>Working Hours</td>
<td>0.2</td>
<td>1.7</td>
<td>-0.4</td>
<td>-1.3</td>
</tr>
<tr>
<td>Welfare of Age 21 Households</td>
<td>3.5</td>
<td>0.5</td>
<td>2.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>-17.6</td>
<td>-17.4</td>
<td>-17.0</td>
<td>-9.4</td>
</tr>
<tr>
<td>Average Wage Rate</td>
<td>3.6</td>
<td>3.6</td>
<td>3.5</td>
<td>1.9</td>
</tr>
<tr>
<td>Private Wealth</td>
<td>12.8</td>
<td>13.7</td>
<td>11.4</td>
<td>15.6</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>-8.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Lump-Sum Transfers+1</td>
<td>0.0</td>
<td>-7.9</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Income Tax Rates</td>
<td>0.0</td>
<td>0.0</td>
<td>11.6</td>
<td>18.7</td>
</tr>
<tr>
<td>Government Wealth+2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-10.9</td>
</tr>
<tr>
<td>Taxable Wealth Share+3</td>
<td>15.4</td>
<td>15.6</td>
<td>12.5</td>
<td>10.0</td>
</tr>
<tr>
<td>Tax-Deferred Wealth Share+3</td>
<td>84.6</td>
<td>84.4</td>
<td>87.5</td>
<td>90.0</td>
</tr>
<tr>
<td>Average Tax Rate after Retirement+4</td>
<td>10.6</td>
<td>10.6</td>
<td>11.8</td>
<td>12.7</td>
</tr>
<tr>
<td>Capital Stock (Tax Adjusted)</td>
<td>5.6</td>
<td>6.5</td>
<td>3.3</td>
<td>-4.7</td>
</tr>
<tr>
<td>$\Delta$Capital Stock /$\Delta$Tax-Deferred</td>
<td>19.0</td>
<td>20.2</td>
<td>16.7</td>
<td>6.3</td>
</tr>
<tr>
<td>$\Delta$Capital Stock (adj.) /$\Delta$Tax-Deferred (adj.)</td>
<td>9.4</td>
<td>10.7</td>
<td>5.5</td>
<td>-7.3</td>
</tr>
</tbody>
</table>

The contribution limit, $s^m_{\max}$, is set at 50% of average baseline earnings. \(^1\) The change as a percentage of the baseline tax revenue. \(^2\) The change as a percentage of the baseline capital stock. \(^3\) The share in the life-cycle and precautionary wealth $W_1 + W_2$. \(^4\) Raw numbers in percent. The average tax rate after retirement is 6.1% in the baseline economy.

### 4.3 Long-Run Effects on Macro Economy and Welfare

Table 5 shows the long-run effects of introducing tax-deferred accounts financed with (a) government consumption cuts, (b) lump-sum transfer payment cuts, (c) raising marginal income tax rates, and (d) raising marginal tax rates once and increasing debt gradually. To obtain the result of the last column, (d), we need solve the model for an equilibrium transition path because we do not know the level of government net wealth (debt) otherwise. However, we can calculate the other 3 long-run (steady-state) results, (a)-(c), without solving the model for transition paths.

Run 1 (a) assumed that the government cut its consumption (waste) to finance the tax-deferred accounts.
Contributions to the tax-deferred accounts would reduce the marginal income tax rates in the economy with a progressive income tax (the substitution effect) and would decrease the tax payments by the households (the income effect). It turned out that the former effect was larger than the latter, and labor supply would increase by 0.3% in the long run, compared to the baseline economy. Since capital income generated in the tax-deferred accounts was not taxable until its withdrawal, the overall after-tax rate of return would go up (the substitution effect) and would shift the tax payments from the working period to the retirement period. These two effects were stronger than the income effect from the tax cut, and capital stock would increase by 12.8%, and total output would increase by 3.9%.

Income tax revenue would decline by 8.8% in the long run, and government would have to decrease its consumption by the same percentage to balance the budget. Since government consumption is not in the utility function of households, introducing tax-deferred accounts with cutting government consumption would surely improve the overall welfare of the economy. The age 21 “newborn” households would be better off by 3.5% from the baseline economy. The share of regular taxable wealth would be 15.4%, and the share of tax-deferred wealth would be 84.6%. The difference in the shares between two accounts is overstated, however, because wealth in the tax-deferred accounts is before tax and includes future tax payments, thus its purchasing power is lower than wealth in taxable accounts of the same level (see Gale, 2005). The average income tax rate for households aged 65 or older is 6.1% in the baseline economy, but the rate would rise to 10.6% in the long run. When the tax-deferred wealth was adjusted by this tax rate, capital stock would increase only by 5.6%. The net contribution rate of the tax-deferred accounts to national wealth—the ratio of the increase in national wealth to the increase in tax-deferred wealth—would be 19.0% before the tax adjustment and 9.4% after the tax adjustment.

Run 1 (b) assumed that the government cut lump-sum transfer payments (or equivalently, introduced lump-sum taxes) to balance the budget. The main difference from Run 1 (a) is the negative income effect of cutting lump-sum transfers. Thus labor supply would increase more by 1.1% in the long run. Because cutting transfer payments to elderly households would increase the optimal wealth level at the time of retirement, capital stock would increase by 13.7% (6.5% if it was tax-adjusted), and total output would increase by 4.7%. Since the positive macroeconomic effect would be larger, the overall budgetary cost would be smaller in Run 1 (b). The government would have to reduce transfer payments by 7.9% as a percentage of the baseline income tax revenue. The share of tax-deferred wealth would be 84.4%. Despite the larger macroeconomic effect relative to Run 1 (a), the age 21 households would be better off only by 0.5%, because households
would have to work more, and the risk sharing effect of progressive taxation would be reduced.

Run 1 (c) assumed that the government increased the marginal income tax rates proportionally to balance the budget. The main difference from Run 1 (b), is the substitution effect due to the changes in the marginal tax rates. Labor supply would decrease by 0.6% in the long run. Since the after-tax rate of return to savings in the taxable accounts would decline, capital stock would increase at a lower rate, 11.4% (3.3% if it was tax adjusted), and total output would increase by 2.9%. With this macroeconomic feedback, the government would have to increase the marginal (and average) income tax rates by 11.6%. Households would allocate more wealth to the tax-deferred accounts when the marginal income tax rates were higher. So, the share of tax-deferred wealth would be 87.5%, significantly higher than those under the previous two assumptions. Although the macroeconomic effect was lower than that in Run 1 (b), the age 21 households would be better off by 2.1%.

Run 1 (d) assumed that the government increased the marginal income tax rates proportionally once and increased government debt gradually period by period, so that the combination of these two financings would make the government budget just sustainable. See Appendix for the computational algorithm of this one time tax increase. The main difference from Run 1 (c) is that the economy includes government debt, which would be 10.9% in the long run as a percentage of baseline capital stock. To finance not only the long-run cost of tax-deferred accounts but also the debt service, \(-r W_G\), the government would have to increase the marginal income tax rates by 18.7% in Run 1 (d). Although private wealth would increase by 15.6%, national wealth (capital stock) would increase only by 4.7% because of the government debt. If it was adjusted by the average income tax rate, 12.7%, after retirement, capital stock would actually decrease by 4.7%. The higher marginal tax rates would decrease labor supply by 1.5%, but total output would increase by 0.3%. Due to the higher marginal tax rates, the share of tax-deferred wealth would be even higher and reach 90.0%. The age 21 households would be better off slightly by 0.1%.

Overall, introducing tax-deferred accounts would likely generate positive effects on total output and wealth accumulation in the long run. However, to what extent the households would be better off depends on the financing assumption of the government. If tax-deferred accounts were financed by cutting government expenditure or increasing marginal tax rates in each period, the long-run welfare of households would on average be improved. However, if part of the transition costs were spread to the future households by increasing government debt, the long-run welfare improvement would be very small even if it was positive.
4.4 Long-Run Effects on Life-Cycle Behaviors

![Graphs showing long-run effects of tax-deferred accounts by age](image)

Figure 1: The Long-Run Effects of Tax-Deferred Accounts by Ages

Figure 1 shows the long-run effects of introducing tax-deferred accounts by age, which correspond to runs in Table 5. In each of the 6 charts, the solid black line shows the baseline economy, the dashed blue line shows Run 1 (a), the long-dashed red line shows Run 1 (b), the short-dashed green line shows Run 1 (c), and the solid navy blue line shows Run (d).

Private consumption would be the highest if the government cut its consumption (waste), and it would be the second highest if the government reduced lump-sum payment transfers. Consumption before the
retirement would be about the same level as that of the baseline economy if the government increased the marginal tax rates in each period. However, it would be lower than the baseline level if the government also increased its debt gradually to spread the transition cost to the future households. Private consumption would likely be higher after the retirement due to larger life-cycle saving. The effect of tax-deferred accounts on working hours would be fairly small. Working hours would increase slightly if the government cut consumption or transfer payments and decrease if the government raised the marginal income tax rates.

Taxable wealth in the baseline economy would be peaked at age 58 before the retirement. If tax-deferred accounts were introduced, the households would allocate a large portion of private wealth to the tax-deferred accounts. Taxable wealth would be peaked at age 49 in Runs (a)-(b) and age 52 in Runs (c)-(d), because taxable wealth reflects the precautionary savings against the lifetime labor income uncertainty (the wage shocks), which decreases after age around 50. Tax-deferred wealth would be peaked at age 60 in all 4 runs. Some households would continue savings until age 65 but many others would start withdrawing money from the tax-deferred accounts at age 60, when the 10% early withdrawal penalty was removed. Some households even start withdrawing from the tax-deferred accounts before reaching age 60.

In the baseline economy, the average income tax payment is peaked at age 51 when the average labor income is the highest. The peak is unchanged after the policy change in all 4 cases. The income tax payment would tend to be lower when households were aged 60 or younger, and the tax payment would be significantly higher after age 60 and after the retirement. Tax-deferred accounts would shift the income tax payments to the later years of life. Even if the government annual tax revenue was unchanged as in Run (c), the present value of lifetime tax payments of age 21 households would be smaller, thus the “newborn” households would be on average better off by the tax deferral.

In our model economy, households do not save at all for the first 5 years, because their wages are assumed to be too low compared to the later years. The contribution to the tax-deferred accounts would be peaked at age 43 in Runs 1 (a)-(b), age 39 in Run 1 (c), and age 38 in Run 1 (d). Interestingly, many households would start withdrawing money from the tax-deferred accounts before age 60. Net contribution to the tax-deferred accounts would become negative at age 58 in Runs 1 (a)-(b), age 57 in Run 1 (c), and age 55 in Run 1 (d). Because we only assume working ability shocks in the model economy, and the marginal income tax rates are low when households receive negative shocks, it is still optimal for some households to withdraw money from the tax-deferred accounts and pay the 10% penalty. When we assume a flat rate income tax instead of a progressive income tax, however, the net contribution to the tax-deferred accounts
would be on average distinctly positive for households aged 59 or younger.

### 4.5 Transition Effects of Introducing Tax-Deferred Accounts

As explained in Section 1, considering an equilibrium transition path is important to analyze the budgetary cost and welfare effect of introducing tax-deferred accounts. First, when tax-deferred accounts were newly introduced, current households would shift their preexisting wealth from the regular taxable accounts to the tax-deferred accounts to reduce and defer their tax payments and attain the optimal allocation between two accounts. The duration of this adjustment would depend on the annual contribution limit, \( s_2^{\text{max}} \), to the tax-deferred accounts. Second, for the first several decades after the policy change, working-age households would contribute to the tax-deferred accounts and pay lower income taxes, but a smaller number of retired households would withdraw wealth from their tax-deferred accounts and pay higher taxes, because the initial elderly households do not have the tax-deferred accounts. Since the budgetary cost would be higher and possible macroeconomic effects would be less than full in the short run, with the exception of financing assumption (a), the current households would relatively be worse off by the introduction of tax-deferred accounts.

Figure 2 shows the transition effects of introducing tax-deferred retirement saving accounts. As explained above, we assumed that the economy was in the initial steady state equilibrium (i.e., on the balanced growth path) without tax-deferred accounts in year (period) 0, and that the government introduced tax-deferred accounts at the beginning of year 1. In each of these 8 charts, the dashed blue line shows the percent changes from the baseline economy in Run 1 (a), cutting government consumption, the long-dashed red line shows the results of Run 1 (b), cutting lump-sum transfers, the short-dashed green line shows the results of Run 1 (c), increasing the marginal income tax rates, and the solid navy blue line shows the results of Run 1 (d), increasing the marginal tax rates once and government debt gradually.

Under the financing assumption (a), income tax revenue would decline by 24.8% in the first year and 19.1% on average during the first 20 years. The government would have to cut its consumption (waste) by the same amount to balance the budget. Labor supply would change little. Capital stock and output would increase gradually to the long-run steady-state levels. Due to the increase in disposable income, private consumption would jump by 1.8% in the first year, then it would increase gradually to its new steady-state level. The increase in private (life cycle) wealth as a percentage of increase in tax-deferred wealth would be 13.1% in the first year. The average welfare would improve in all age cohorts at the time of policy change.
Increased capital stock and stable labor supply would make the interest rate lower and the average wage rate higher. Although the lowered interest rate would reduce the capital income of elderly households, the increased social security benefits through the higher wage rate would make them on average better off. The initial working age households would be better off by tax savings and tax deferral effects of the tax-deferred accounts, and future households would be better off in addition by the positive macroeconomic effect.

Under the assumption (b), income tax revenue would decrease by 23.3% in the first year and 17.6% on average for the first 20 years. The government would have to reduce lump-sum transfers by the same amount, which would hurt the current and near future households significantly. Labor supply would jump by 2.7% in the first year by the negative income effect, then it would decrease gradually to its new steady-state level. Total output would also jump by 1.9% in the first year, then capital stock and output would increase gradually to the long-run steady-state levels. Private consumption would initially decrease by 0.8%, then it would also increase gradually to its long-run level. The ratio of the increase in private wealth to the increase in tax-deferred wealth would be 14.7% in the first year. With the lump-sum transfer cut, due to the increased lifetime income risk and inequality, the average welfare would decline for all households alive at the time of the policy change. The average welfare of age 21 households would also be below its baseline level until year 24.

Under the assumption (c), the government would have to increase the marginal income tax rates by 52.7% in the first year and 32.0% on average for the first 20 years. Labor supply would decrease drastically by 7.0% in the first year, although it would recover as the marginal tax rates would decline. Capital stock would also decrease for the first 3 years before increasing gradually. Total output would decrease by 4.9% in the first year, then it would increase to its new steady-state level. Private consumption would also decline by 4.8% in the first year, and it would be below the baseline level until year 29. Since private wealth would decrease initially even though tax-deferred wealth would increase, the ratio of these two changes would be -8.9% in the first year. Again, the current households would be on average worse off in all age cohorts with the marginal income tax rate increase. The average welfare change of age 21 households would be negative until year 9. However, the welfare losses under the assumption (c) are smaller than those under the assumption (b) for all age cohorts.

Finally, under the assumption (d), the government would have to increase the marginal income tax rates by 18.7% in the first year and increase its debt gradually. Since the budgetary cost is higher in the short run, income tax revenue would decline by 14.4% in the first year and 8.6% on average for the first 20 years.
The income tax revenue would be below the baseline level until year 23. Labor supply would decrease by 1.5% then stay about the same level in the transition path. Capital stock would slightly decrease for the first 5 years, then it would increase gradually to the long-run steady-state level. Total output would decline by 1.0% in the first period and would be slightly above the baseline level after 28 years. Private consumption would decline by 1.2% in year 1 and 2.0% in year 13, then it would increase gradually but stay below its baseline level. The increase in private wealth and capital stock as a percentage of increase in tax-deferred wealth would be 9.9% and -1.1%, respectively, in the first year. The welfare losses of the current households would be significantly smaller compared to those under assumptions (b) and (c). However, except for those aged between 44 and 60, current households would be on average worse off.

5 Tax-Deferred Accounts under Alternative Assumptions

In our main policy experiments, Runs 1 (a)-(d), the annual contribution limit to the tax-deferred accounts was set at 50% of the average labor income in the baseline economy, and a 10% early withdrawal penalty was applied for households aged 59 or younger. In this section, we analyze the effects of introducing tax-deferred accounts under two alternative assumptions: adding the “catch-up” contribution limit for households aged 50 or older (Section 5.1) and removing the early withdrawal penalty for all households (Section 5.2). The first assumption would make the tax-deferred accounts in the model economy closer to the U.S. 401(k) plans. The second assumption would evaluate the effect of allowing households to borrow from their own tax-deferred accounts at the market interest rate with a simpler calculation.

5.1 Tax-Deferred Accounts with Catch-Up Contributions

The annual contribution limit, $s_{2}^{\text{max}}$, to the tax-deferred accounts was set at 0.1765 or 50% of average baseline labor income. In this section, we increase the contribution limit for households aged 50 or older to 0.2353 or 66.7% of average baseline earnings. Since the catch-up contribution limits in 2008 and 2009 are $5,000 and $5,500, respectively, an additional 16.7% of average labor income is roughly consistent with the current U.S. 401(k) plans. We solve the model under the same four financing assumptions to satisfy the government inter-temporal budget constraint. Table 6 shows the long-run effects of introducing tax-deferred accounts with catch-up contributions.\footnote{Due to limits of space, we will show only the long-run results of Runs 2-5 in Tables 6-9. More detailed computational results of these runs will be provided in Excel files upon request.}
Table 6: The Long-Run Effects of Tax-Deferred Accounts with Catch-Up Contributions in the Economy with a Progressive Income Tax and $\gamma = 3.0$ (% changes from the baseline economy)

<table>
<thead>
<tr>
<th>Financing Assumption</th>
<th>2 (a) Cut</th>
<th>2 (b) Cut</th>
<th>2 (c) Inc</th>
<th>2 (d) Inc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Government Consumption</td>
<td>Lump-Sum Transfers</td>
<td>Income Tax Rates</td>
<td>Tax Rates and Debt</td>
</tr>
<tr>
<td>Capital Stock (National Wealth)</td>
<td>13.0</td>
<td>14.1</td>
<td>11.9</td>
<td>4.6</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>0.3</td>
<td>1.2</td>
<td>-0.5</td>
<td>-1.5</td>
</tr>
<tr>
<td>Total Output (GDP)</td>
<td>3.9</td>
<td>4.9</td>
<td>3.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Income Tax Revenue</td>
<td>-8.9</td>
<td>-7.9</td>
<td>0.0</td>
<td>4.4</td>
</tr>
<tr>
<td>Private Consumption</td>
<td>4.3</td>
<td>3.7</td>
<td>1.6</td>
<td>-0.7</td>
</tr>
<tr>
<td>Working Hours</td>
<td>0.2</td>
<td>1.7</td>
<td>-0.4</td>
<td>-1.4</td>
</tr>
<tr>
<td>Welfare of Age 21 Households</td>
<td>3.6</td>
<td>0.5</td>
<td>2.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>-17.8</td>
<td>-17.8</td>
<td>-17.6</td>
<td>-9.2</td>
</tr>
<tr>
<td>Average Wage Rate</td>
<td>3.6</td>
<td>3.7</td>
<td>3.6</td>
<td>1.8</td>
</tr>
<tr>
<td>Private Wealth</td>
<td>13.0</td>
<td>14.1</td>
<td>11.9</td>
<td>16.1</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>-8.9</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Lump-Sum Transfers*1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Income Tax Rates*2</td>
<td>0.0</td>
<td>0.0</td>
<td>11.7</td>
<td>19.1</td>
</tr>
<tr>
<td>Government Wealth*2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-11.5</td>
</tr>
</tbody>
</table>

The contribution limit, $s_{2}^{\text{max}}$, is 50% of average baseline earnings for aged 49 or younger and 66.7% of average earnings for aged 50 or older. *1 The change as a percentage of the baseline tax revenue. *2 The change as a percentage of the baseline capital stock.

The effect of adding the catch-up contribution limit to households aged 50 or older is small. In Runs 2 (a)-(c), the budgetary cost of the tax-deferred accounts would be only 0.0-0.1% higher compared to Runs 1 (a)-(c). Labor supply would be 0.0-0.1% higher, capital stock would be 0.2-0.4% higher, and total output would be 0.0-0.2% higher compared to the previous experiments without the catch-up contributions. So long as households are rational and forward looking, the additional contribution limit for households near the retirement is almost unnecessary, though it would make the age 21 households better off slightly by 0.0-0.1% in the long run. Run 2 (d) is a little different, however. Because the short run cost of tax-deferred accounts with catch-up contributions is higher, the government would have to increase the marginal tax rates by 19.1%, 0.4 percentage points higher than that in Run 1 (d), and the government debt would be 11.5% of the baseline capital stock. Capital stock would be 0.1% lower, and the welfare gain of age 21 households would be reduced to 0.0% from 0.1% in Run 1 (d).
5.2 Tax-Deferred Accounts without Early Withdrawal Penalties

In the United States, households are usually allowed to borrow money from their own tax-deferred accounts. Thus they do not necessarily have to withdraw wealth from the accounts and pay both income taxes and penalties. We did not consider this possibility in our policy experiments, because it would require at least one more state variable of households in the model economy. In this section, we analyze the effect of tax-deferred accounts without early withdrawal penalties instead. Borrowing from the tax-deferred accounts at the market interest rate and withdrawing wealth without paying penalties are roughly equivalent if the constraint on the annual contribution limit is not binding. Table 7 shows the long-run effects of tax-deferred accounts without early withdrawal penalties.

Removing early withdrawal penalties has a larger effect on macro economy and social welfare. Since tax revenue from the penalty was removed and the households were more willing to contribute their money to the tax-deferred accounts, the long-run budgetary cost would be 1.5-1.7% higher in Runs 3 (a)-(b) compared to the main experiments, Runs 1 (a)-(b). The government would have to increase the marginal income tax

Table 7: The Long-Run Effects of Tax-Deferred Accounts without Early Withdrawal Penalties in the Economy with a Progressive Income Tax and $\gamma = 3.0$ (% changes from the baseline economy)

<table>
<thead>
<tr>
<th>Financing Assumption</th>
<th>3 (a) Cutting Government Consumption</th>
<th>3 (b) Cutting Lump-Sum Transfers</th>
<th>3 (c) Increasing Income Tax Rates</th>
<th>3 (d) Increasing Tax Rates and Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Stock (National Wealth)</td>
<td>14.2</td>
<td>15.3</td>
<td>12.3</td>
<td>5.3</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>0.0</td>
<td>1.1</td>
<td>-1.0</td>
<td>-2.0</td>
</tr>
<tr>
<td>Total Output (GDP)</td>
<td>4.1</td>
<td>5.1</td>
<td>2.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Income Tax Revenue</td>
<td>-10.5</td>
<td>-9.4</td>
<td>0.0</td>
<td>3.9</td>
</tr>
<tr>
<td>Private Consumption</td>
<td>4.5</td>
<td>3.8</td>
<td>1.1</td>
<td>-1.0</td>
</tr>
<tr>
<td>Working Hours</td>
<td>-0.2</td>
<td>1.6</td>
<td>-1.0</td>
<td>-2.0</td>
</tr>
<tr>
<td>Welfare of Age 21 Households</td>
<td>4.2</td>
<td>0.4</td>
<td>2.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>-19.8</td>
<td>-19.6</td>
<td>-18.8</td>
<td>-11.0</td>
</tr>
<tr>
<td>Average Wage Rate</td>
<td>4.1</td>
<td>4.0</td>
<td>3.9</td>
<td>2.2</td>
</tr>
<tr>
<td>Private Wealth</td>
<td>14.2</td>
<td>15.3</td>
<td>12.3</td>
<td>15.9</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>-10.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Lump-Sum Transfers*1</td>
<td>0.0</td>
<td>-9.4</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Income Tax Rates</td>
<td>0.0</td>
<td>0.0</td>
<td>14.0</td>
<td>21.1</td>
</tr>
<tr>
<td>Government Wealth*2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-10.6</td>
</tr>
</tbody>
</table>

The contribution limit, $s_{\text{max}}^{\text{max}}$, is 50% of average baseline earnings. *1 The change as a percentage of the baseline tax revenue. *2 The change as a percentage of the baseline capital stock.
rates 2.4% more in Runs 3 (c)-(d). Interestingly, the long-run government debt would be slightly smaller in Run 3 (d). Labor supply would be 0.0-0.5% lower, and capital stock would be 0.6-1.6% higher. Total output would be 0.2-0.4% higher in Runs 3 (a)-(b) but 0.1-0.2% lower in Runs (c)-(d). The long-run welfare gain of age 21 households would be larger in Runs 3 compared to Runs 1 except for Run 3 (b), where the government cut lump-sum transfers, because the liquidity constraint due to the tax-deferred accounts was relaxed.

6 Tax-Deferred Accounts in the Alternative Baseline Economies

When tax-deferred accounts were introduced to the economy, households would be able to reduce the present value of lifetime tax payments in two ways. First, households would reduce the present value of payments by delaying the tax payments from working years to retired years by using the tax-deferred accounts. Second, households would reduce the tax payments by contributing to the tax-deferred accounts when the marginal tax rate was high (before the retirement) and withdrawing money from the accounts when the marginal tax rate was low (after the retirement). However, if the income tax rate was flat, the latter effect of tax-deferred accounts would disappear, i.e., the budgetary cost of the tax-deferred accounts would be lower, and the macroeconomic and welfare effects would be smaller.

The degree of household risk aversion would also affect the size of tax-deferred accounts with early withdrawal penalties. If households were less risk averse, they would need less precautionary savings in the regular taxable accounts and allocate more wealth in the tax-deferred accounts. This would also affect the budgetary cost and the macroeconomic and welfare effects of tax-deferred accounts. In this section, we analyze the effect of introducing tax-deferred accounts in the alternative baseline economy with a 15.19% flat rate income tax (Section 6.1) and the economy in which the coefficient of relative risk aversion, $\gamma$, is 1.5 (Section 6.2).

6.1 Baseline Economy with a Flat Rate Income Tax

In the economy with a flat rate income tax, since its tax saving effect was smaller, households would contribute less to the tax deferred accounts, thus the budgetary cost for the government would be lower. To examine how much the progressivity of the tax system is important for the tax-deferred accounts, we replace the progressive income tax with a flat rate income tax. The flat tax rate, 15.19%, is chosen so that the ratio
Table 8: The Long-Run Effects of Tax-Deferred Accounts in the Economy with a Flat Rate Income Tax and $\gamma = 3.0$ (% changes from the baseline economy)

<table>
<thead>
<tr>
<th>Financing Assumption</th>
<th>4 (a)</th>
<th>4 (b)</th>
<th>4 (c)</th>
<th>4 (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting Government Consumption</td>
<td>7.5</td>
<td>7.7</td>
<td>6.9</td>
<td>2.5</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>0.1</td>
<td>0.4</td>
<td>0.1</td>
<td>-0.3</td>
</tr>
<tr>
<td>Total Output (GDP)</td>
<td>2.2</td>
<td>2.5</td>
<td>2.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Income Tax Revenue</td>
<td>-2.8</td>
<td>-2.5</td>
<td>0.0</td>
<td>2.8</td>
</tr>
<tr>
<td>Private Consumption</td>
<td>2.0</td>
<td>1.8</td>
<td>1.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Working Hours</td>
<td>0.2</td>
<td>0.8</td>
<td>0.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>Welfare of Age 21 Households</td>
<td>2.1</td>
<td>0.9</td>
<td>1.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>-10.8</td>
<td>-10.6</td>
<td>-10.0</td>
<td>-4.2</td>
</tr>
<tr>
<td>Average Wage Rate</td>
<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Private Wealth</td>
<td>7.5</td>
<td>7.7</td>
<td>6.9</td>
<td>8.7</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>-2.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Lump-Sum Transfers$^1$</td>
<td>0.0</td>
<td>-2.5</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Income Tax Rates</td>
<td>0.0</td>
<td>0.0</td>
<td>3.1</td>
<td>6.3</td>
</tr>
<tr>
<td>Government Wealth$^2$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-6.2</td>
</tr>
</tbody>
</table>

The contribution limit, $s_{2}^{\max}$, is 50% of average baseline earnings. The flat income tax rate is set at 15.19%.

$^1$ The change as a percentage of the baseline tax revenue.  
$^2$ The change as a percentage of the baseline capital stock.

The contribution limit, $s_{2}^{\max}$, is 50% of average baseline earnings. The flat income tax rate is set at 15.19%. The budgetary cost of introducing these accounts would also be smaller. The government would have to reduce its consumption by only 2.8% in Run 4 (a), reduce lump-sum transfers by 2.5% in Run 4 (b), increase the flat income tax rate by 3.1% in Run 4 (c), or increase the tax rate by 6.3% at the time of policy change and increase the debt by 6.2% in the long run as a percentage of baseline capital stock in Run 4 (d). The effect on the macro economy would also be smaller. Capital stock would increase by 6.9-7.7% in Runs (a)-(c) and 2.5% in Run (d). Total output would increase by 2.0-2.5% in Runs (a)-(c) and 0.5% in Run (d).
Table 9: The Long-Run Effects of Tax-Deferred Accounts in the Economy with a Progressive Income Tax and $\gamma = 1.5$ (% changes from the baseline economy)

<table>
<thead>
<tr>
<th>Financing Assumption</th>
<th>5 (a)</th>
<th>5 (b)</th>
<th>5 (c)</th>
<th>5 (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting Government Consumption</td>
<td>13.3</td>
<td>14.1</td>
<td>11.7</td>
<td>6.2</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>0.2</td>
<td>1.1</td>
<td>-0.7</td>
<td>-1.4</td>
</tr>
<tr>
<td>Total Output (GDP)</td>
<td>3.9</td>
<td>4.8</td>
<td>2.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Income Tax Revenue</td>
<td>-9.4</td>
<td>-8.4</td>
<td>0.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Private Consumption</td>
<td>4.3</td>
<td>3.6</td>
<td>1.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>Working Hours</td>
<td>0.2</td>
<td>1.8</td>
<td>-0.5</td>
<td>-1.3</td>
</tr>
<tr>
<td>Welfare of Age 21 Households</td>
<td>3.8</td>
<td>1.0</td>
<td>2.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>-18.4</td>
<td>-18.0</td>
<td>-17.6</td>
<td>-11.2</td>
</tr>
<tr>
<td>Average Wage Rate</td>
<td>3.8</td>
<td>3.7</td>
<td>3.6</td>
<td>2.3</td>
</tr>
<tr>
<td>Private Wealth</td>
<td>13.3</td>
<td>14.1</td>
<td>11.7</td>
<td>17.1</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>-9.4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Lump-Sum Transfers</td>
<td>0.0</td>
<td>-8.5</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Income Tax Rates</td>
<td>0.0</td>
<td>0.0</td>
<td>12.6</td>
<td>18.7</td>
</tr>
<tr>
<td>Government Wealth</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-10.9</td>
</tr>
</tbody>
</table>

The contribution limit, $s_{2}^{\text{max}}$, is 50% of average baseline earnings. *1 The change as a percentage of the baseline tax revenue. *2 The change as a percentage of the baseline capital stock.

Except for Run (b), the average welfare of age 21 households would be smaller or worse when the income tax rate was flat. Although the budgetary cost was lower, the households would actually be worse off by 0.3% under the financing assumption (d).

### 6.2 Baseline Economy with Less Risk-Averse Households

With the 10% early withdrawal penalty, wealth in the tax-deferred accounts would be less liquid compared to wealth in regular taxable accounts. Thus, households would keep part of their life-cycle savings in the regular taxable accounts. The size of this precautionary saving would depend on the degree of the household’s risk aversion. Here, we reduce the coefficient of relative risk aversion, $\gamma$, in the utility function to 1.5 from 3.0 and examine its effect. As explained before, we recalculate the time discount factor and the social security benefit level so that this baseline economy is consistent with the main baseline economy with $\gamma = 3.0$. Because the average labor income per working-age household is 0.3571 in this new economy, we set the annual contribution limit, $s_{2}^{\text{max}}$, at 0.1785, so that it is 50% of the average baseline earnings. Table 9
shows the long-run effect of tax-deferred accounts in the economy with less risk averse households.

Households would allocate more wealth in the tax-deferred accounts, and the budgetary cost would be higher by 0.6-1.0% in Runs 5 (a)-(c) compared to Runs 1 (a)-(c). Interestingly, under the financing assumption (d), the sizes of the income tax increase and long-run debt increase would be unchanged from Run 1 (d). Labor supply would be about the same level as that in Runs 1 (a)-(d), capital stock would be 0.3-0.5% higher in Runs (a)-(c) and 1.5% higher in Run (d), and total output would be about the same level in Runs (a)-(c) and 0.5% higher in Run (d). The average welfare gain of age 21 households would be larger by 0.2-0.5% in Runs (a), (b), and (d) but smaller by 0.1% in Run (c). Overall, the effect of tax-deferred accounts would not change significantly even if we reduced the coefficient of risk aversion by half.

7 Concluding Remarks

In the present paper, we have analyzed the budgetary, macroeconomic, and welfare effects of introducing 401(k)-type tax-deferred retirement saving accounts to the heterogeneous-agent OLG economy. Our policy experiments show that the budgetary cost of the tax-deferred accounts is significant. For example, the government would have to cut its transfer payments by 7.9% or raise the marginal income tax rates proportionally by 11.6% in the long run, and the budgetary cost would be much larger in the short run. Although national wealth and total output would increase and the future households would likely be better off in the long run, introducing the tax-deferred accounts would make the current and near future households worse off. Even if the transition cost was spread to the future households by increasing government debt, most households alive at the time of the policy change would be on average worse off.

One of the reasons why introducing tax-deferred accounts would not improve the overall welfare is that the policy change would be more favorable to high-income households than low-income households, especially when the income tax was progressive. Because high-income households are more likely to have sufficient wealth for their retirement, there is probably room to improve the welfare effect of tax-deferred accounts by shifting the tax benefits from high-income households to low-income and less-wealthy households. We will leave this topic, the optimal design of tax-deferred accounts, for our future research.

Although the OLG model developed in our analyses is fairly detailed, to use this model for the “dynamic scoring” of tax-deferred accounts, we need to make a few more refinements to the model economy. First, the present paper assumes that all households would have an access to the tax-deferred accounts. However,
according to Purcell (2009), only 66% of full-time workers and 36% of part-time workers in the U.S. were offered a retirement plan at work in 2007. Thus, the projected percentage of the firms that would offer a tax-deferred retirement saving plan must be considered. Second, the present paper abstracts from other tax-deferred and tax-favored assets, such as real estates, life insurance, defined benefit pension plans. With the other tax-favored investment opportunities, households would contribute less to the 401(k)-type accounts than what the model predicted.

A supplemental contribution of the present paper is that we have constructed a dynamic general equilibrium OLG model with heterogeneous households with two assets, and we have solved the model for equilibrium transition paths efficiently by a desktop PC. The structure of the model is relatively simple. So, we can easily modify the model to solve other household decision problems between two assets, such as housing assets and financial assets, risky assets and risk-free assets, social security individual accounts and other private assets, and cash and other less-liquid assets. The model and solution algorithm shown in Appendix will be useful for future analyses of government fiscal policy changes.

A Computational Algorithms

We solve the household’s optimization problem recursively from age \( i = I \) to age \( i = 1 \) by discretizing the asset spaces, \( A_1 = A_2 = [0, \infty) \), into 50 levels each, \( \hat{A}_1 = \hat{A}_2 = \{a^1, a^2, \ldots, a^{50}\} \), and the working ability space, \( E = [0, \infty) \), into 5 levels, \( \hat{E} = \{e^1, e^2, \ldots, e^5\} \). In this appendix, we first explain the algorithm to solve the household’s two-period problem (the Bellman equation) for each individual state node, \((a_1, a_2, e) \in \hat{A}_1 \times \hat{A}_2 \times \hat{E}\), taking \( \Omega_t \) as given. Then, we explain the algorithm to solve the model for an equilibrium transition path in which the government changes the marginal income tax rates, \( \phi_0 \), once in period \( t = 1 \) and adjusts the government net wealth gradually for \( t = 1, \ldots, \infty \). For the numerical methods to solve a Kuhn-Tucker condition for household’s optimal decision, see Judd (1998) and Miranda and Fackler (2002). For more general algorithm to compute a steady-state equilibrium and an equilibrium transition path, see Conesa and Krueger (1999) or Nishiyama and Smetters (2007).
A.1 Algorithm to Solve the Household Problem

We solve the household’s optimization problem backward from \( i = I \) to \( i = 1 \), assuming the terminal value \( v_{I+1}(a_1, a_2, e; \Omega_{t+1}) = 0 \). The household’s problem at age \( i \) in period \( t \) in Section 2 is modified to

\[
(13) \quad v_i(a_1, a_2, e; \Omega_t) = \max_{c,l,s_2} \left\{ u(c, l) + \tilde{\beta} \phi_i E[v_{i+1}(a_1', a_2', e'; \Omega_{t+1}) | e] \right\}
\]

subject to

\[
(14) \quad 0 < c \leq c_{\text{max}}, \quad 0 < l \leq 1 \equiv l_{\text{max}},
\]

\[
 s_2 \text{min} = -(1 + r_t) a_2 \leq s_2 \leq \min(s_{2,t}^{\text{max}}, w_t e(1 - l)) \equiv s_{2,t}^{\text{max}}(l),
\]

where

\[
 c_{\text{max}} = (1 + r_t) a_1 + (1 - \tau_{P,t}) w_t e(1 - l) - \tau_{I,i}(r_t a_1, w_t e(1 - l), s_2; \varphi_t) + 1_{\{i \geq I_R\}} t r s_{SS,t} + t r s_{LS,t} - s_2,
\]

\[
a_1' = \frac{1}{(1 + \mu) \phi_i} \left[ c_{\text{max}} - c \right], \quad a_2' = \frac{1}{(1 + \mu) \phi_i} \left[ (1 + r_t) a_2 + s_2 \right].
\]

The non-negativity conditions of assets, \( a_1' \geq 0 \) and \( a_2' \geq 0 \), are satisfied by the conditions \( c \leq c_{\text{max}} \) and \( s_2 \geq s_{2,t}^{\text{min}} \) in (14).

Let the objective function be

\[
f(c, l, s_2) = u(c, l) + \tilde{\beta} \phi_i E[v_{i+1}(a_1', a_2', e'; \Omega_{t+1}) | e].
\]

Then, the first-order conditions for an interior solution are

\[
(15) \quad f_c(c, l, s_2) = u_c(c, l) - \frac{\tilde{\beta}}{1 + \mu} E[v_{1,i+1}(a_1', a_2', e'; \Omega_{t+1}) | e] = 0,
\]

\[
(16) \quad f_l(c, l, s_2) = u_l(c, l) - u_c(c, l) w_t e[1 - \tau_{I,2,i}(r_t a_1, w_t e(1 - l), s_2; \varphi_t) - \tau_{P,t}] = 0,
\]

\[
(17) \quad f_s(c, l, s_2) = \frac{\tilde{\beta}}{1 + \mu} E[v_{2,i+1}(a_1', a_2', e'; \Omega_{t+1}) | e] - u_c(c, l) [1 + \tau_{I,3,i}(r_t a_1, w_t e(1 - l), s_2; \varphi_t)] = 0,
\]

where \( v_{k,i}(a_1, a_2, e; \Omega_t) \) and \( \tau_{I,k,i}(r_t a_1, w_t e(1 - l), s_2; \varphi_t) \) are the marginal value and marginal income tax
functions with respect to the $k$-th argument. Equations (15) and (17) are the Euler equations and (16) is the marginal rate of substitution condition.

The income tax function is age dependent due to the early withdrawal penalty. The marginal income tax of the contribution to tax-deferred accounts, $\tau_{I,3,i}(\cdot, \cdot, s_2; \varphi_t)$, is negative and nondecreasing. Thus, $f_c(c, l, s_2)$, $f_l(c, l, s_2)$, and $f_s(c, l, s_2)$ are all decreasing in $c$, $l$, and $s_2$, respectively. With the inequality constraints (14), the Kuhn-Tucker conditions of the household’s problem are expressed as the following nonlinear complementarity problem,

$$
\begin{array}{ll}
f_c(c, l, s_2) = 0 & \text{if } 0 < c < c^{\max}, > 0 \text{ if } c = c^{\max}, \\
f_l(c, l, s_2) = 0 & \text{if } 0 < l < l^{\max}, > 0 \text{ if } l = l^{\max}, \\
f_s(c, l, s_2) < 0 & \text{if } s_2 = s_2^{\min}, = 0 \text{ if } s_2^{\min} < s_2 < s_2^{\max}(l), > 0 \text{ if } s_2 = s_2^{\max}(l),
\end{array}
$$

which is expressed more compactly as the nonlinear system of equations,

$$
\begin{align}
\min \left\{ \max \begin{bmatrix}
  f_c(c, l, s_2) \\
  f_l(c, l, s_2) \\
  f_s(c, l, s_2)
\end{bmatrix}, \begin{bmatrix}
  \varepsilon - c \\
  \varepsilon - l \\
  s_2^{\min} - s
\end{bmatrix}, \begin{bmatrix}
  c^{\max} - c \\
  l^{\max} - l \\
  s_2^{\max}(l) - s_2
\end{bmatrix} \right\} = 0,
\end{align}
$$

where $\varepsilon$ is a small positive number. Following Miranda and Fackler (2002), we replace the $\min(u, v)$ and $\max(u, v)$ operators with

$$
\phi^-(u, v) \equiv u + v - \sqrt{u^2 + v^2}, \quad \phi^+(u, v) \equiv u + v + \sqrt{u^2 + v^2},
$$

respectively, to make the above system of equations differentiable without altering the solutions. We solve equation (18) with a nonlinear equation solver, NEQNF, in the Fortran IMSL library.

Once we solve the problem, we also calculate the value and the marginal values with respect to $a_1$ and $a_2$. The marginal values are

$$
\begin{align}
v_{1,i}(a_1, a_2, e; \Omega_t) &= u_c(c_i(a_1, a_2, e; \Omega_t), l_i(a_1, a_2, e; \Omega_t)) \times \\
&\quad \left[ 1 + r_t(1 - \tau_{I,1,i}(r_t a_1, w_t e h_i(a_1, a_2, e; \Omega_t), s_2,i(a_1, a_2, e; \Omega_t); \varphi_t)) \right], \\
v_{2,i}(a_1, a_2, e; \Omega_t) &= u_c(c_i(a_1, a_2, e; \Omega_t), l_i(a_1, a_2, e; \Omega_t)) \times \\
&\quad \left[ 1 + r_t(1 - \tau_{I,1,i}(r_t a_1, w_t e h_i(a_1, a_2, e; \Omega_t), s_2,i(a_1, a_2, e; \Omega_t); \varphi_t)) \right],
\end{align}
$$

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and these are used to solve the optimization problem of age $i - 1$ in period $t - 1$.

The first-order conditions and the envelope conditions imply the following effects of tax-deferred accounts on households’ behavior: In the economy with a progressive income tax,

- by (17) and (20), the tax-deferral and the marginal tax rate smoothing effects would encourage working-age households to save more if the originally intended saving (in the absence of tax-deferred accounts) was below the contribution limit; in addition,

- by (16), any positive contribution to the tax-deferred accounts would reduce the marginal labor income tax rate thus increase the ratio of consumption to leisure;

- by (15) and (19), any positive future contribution would reduce the future marginal capital income tax rate thus increase current saving in the regular taxable accounts.

In the economy with a flat income tax,

- by (17) and (20), the tax-deferral effect would encourage working-age households to save more if the originally intended saving was below the contribution limit.

A.2 Algorithm to Find a Sustainable One-time Tax Increase

We assume that the economy is in the steady-state equilibrium, which is on the balanced growth path, in period 0 and that the government introduces tax-deferred retirement saving accounts at the beginning of period 1. Let $T$ be a sufficiently large integer such that the economy is said to reach its new steady-state equilibrium within $T$ periods. We set $T = 150$ and solve the model for an equilibrium transition path by the following algorithm:

1. Set the initial guess of the capital-labor ratio and the government policy variables,

$$\Omega_1^0 = \{(K/L)_s, C_{G,s}, \tau_{LS,s}^0, \varphi^0, \tau_{P,s}^0, \tau_{SS,s}^0, W_{G,s}^0, s_{2,s}^{max,0} \}_{s=1}^T,$$

where $\varphi^0$ is the time-invariant marginal income tax parameter, and $W_{G,1}^0 = W_{G,0}$. 

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2. Compute the final steady-state equilibrium in period $T$, assuming $W_{G,T}$ is endogenous, i.e.,

$$W^0_{G,T} = \frac{C^0_{G,T} + T R^0_{LS,T} - T I_T(\varphi^0)}{1 + r_T - (1 + \mu)(1 + \nu)},$$

and calculate the marginal value functions, $\{v_{k,i}(a_1, a_2, e; \Omega^0_T)\}_{i=1}^I$ for $k = 1$ and 2.

3. For $t = T - 1, T - 2, \ldots, 1$, solve backward the household’s problem to obtain the decision rules, $\{d_i(a_1, a_2, e; \Omega^0_t)\}_{i=1}^I$ and the marginal value functions, $\{v_{k,i}(a_1, a_2, e; \Omega^0_t)\}_{i=1}^I$ for $k = 1$ and 2, recursively.

4. For $t = 1, 2, \ldots, T - 1$, compute forward the capital-labor ratio, $(K/L)_{t+1}^1$, the government net wealth, $W_{G,t+1}^1$, and the distributions of households, $\{x_{i,t+1}(a_1, a_2, e)\}_{i=1}^I$, recursively, using the decision rules obtained in Step 3.

5. If $W^1_{G,T}$ and $W^0_{G,T}$ are close enough, replace $\varphi^1$ with $\varphi^0$ and go to Step 6. Otherwise, update $\varphi^1$ and return to Step 4.

6. If $\Omega^1_1 = \{(K/L)_{s}^1, C^1_{G,s}, T R^1_{LS,s}, \varphi^1, \tau^1_{P,s}, T R^1_{SS,s}, W^1_{G,s}, s_{2,s}^{\max,1}\}_{s=1}^T$ and $\Omega^0_1$ are close enough, stop. Otherwise, update $\Omega^1_1$ by using $\Omega^1_1$ and return to Step 2. (Most policy variables are exogenous thus unchanged during the iterations.)

In Step 5, we update $\varphi^1$ by using the ratio of the present discount sum of government expenditure (net of the initial government wealth) to the present discount sum of income tax revenue,

$$\varphi^1 \leftarrow \eta \frac{\sum_{t=1}^{\infty} \left( \prod_{s=1}^{t} (1 + r_s)^{-1} \right) [(1 + \mu)(1 + \nu)]^{t-1} (C_{G,t} + T R_{LS,t}) - W_{G,1}^1}{\sum_{t=1}^{\infty} \left( \prod_{s=1}^{t} (1 + r_s)^{-1} \right) [(1 + \mu)(1 + \nu)]^{t-1} T_I(t, \varphi^1) \varphi^1 + (1 - \eta) \varphi^1},$$

where $\eta$ is a dampening parameter.

References


Figure 2: The Transition Effects of Introducing Tax-Deferred Accounts