The Interaction Between Technology Adoption and Trade
When Firms are Heterogeneous

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Abstract

This paper develops a monopolistic competition model with heterogeneous firms to study the interaction between technology adoption and trade in a world of two countries facing different technology adoption costs. It shows that a reduction in the technology adoption cost in one country increases the productivity, induces more firms to adopt advanced technology, and improves welfare in this country, while decreasing the productivity, inducing more firms to switch back to old technology, and reducing welfare in the other country. Furthermore, although a reduction in transport costs always makes the country with lower adoption cost better off, it can hurt the other country.

JEL Classification: F1, L1, and O1
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1 Introduction

This paper studies the interaction between technology adoption and trade in a monopolistic competition model with heterogeneous firms. Firms have different productivity levels and those wishing to export face both variable and fixed costs of trade. Upon learning their productivity, firms can adopt a new, lower unit-cost technology; but adopting this technology requires an additional fixed investment. Upon characterizing the equilibrium of this open economy model with two countries facing different technology adoption costs, this paper addresses the effects of changes in technology adoption cost and transport costs on each economy.

The model predicts that a decline in the cost of technology adoption in one country makes this country better off: the overall productivity increases due to the exit of the least productive firms, technology improves as some firms adopt new technology, and welfare increases. In the other country, however, the productivity decreases due to the entry of the least productive firms, technology regresses as some firms switch back to old technology, and welfare declines. Furthermore, although a reduction in transport costs always makes the country with lower adoption cost better off, it can hurt the other country.

These results follow from the fact that a fall in technology adoption cost or transport costs asymmetrically affects the intensity of competition in two countries. A decline in technology adoption cost in one country (called country 1) intensifies the competition in this country, because increased demand for labor by the firms adopting advanced technology and the entry of more firms in response to higher returns from using advanced technology raises the real wage. Consequently, the least productive (domestic and foreign) firms exit this market. Increased competition in country 1 also induces the least productive foreign firms using advanced technology to switch back to old technology. This occurs because they can not cover their adoption costs with a reduction in their export revenues. As a result, demand for labor in country 2 decreases which in turn lowers the real wage and induces the least productive firms to enter.
Falling trade costs intensify competition in country 1 (assuming that technology adoption is cheaper there). This happens because new profit opportunities from the foreign market increase the incentives for more productive firms to export and/or adopt new technology. In country 2, however, firms face two competing forces: lowering trade costs increases each foreign firm’s export revenue, while an increased competition in country 1 reduces it. For example, if the second effect is dominant, then the competition in country 2 becomes less intense. Consequently, as in the previous case, the decreased real wage in this country induces least productive firms to enter (see Section 3 for more details).

This paper contributes to an emerging literature on firm heterogeneity, technology adoption, and trade. Yeaple (2005) investigates how the interaction between technology adoption and trade gives rise to firm heterogeneity. In his model, firms are ex-ante homogeneous. In equilibrium, firms that choose to export also choose to employ new technology and hire high-skilled workers, while non-exporters employ old technology and hire low-skilled workers. His model predicts that a reduction in transport costs (between two symmetric countries) induces firms to adopt the new cost-saving technology, reallocates workers from the old technology to the new, and raises the skill-premium. In this paper, unlike Yeaple’s model, workers are identical and firms are ex-ante heterogeneous in their productivity. Upon learning their productivity, each successful entrant decides which technology to use. Furthermore, technology adoption costs differ across countries. The results in this paper show that falling trade costs may actually cause a technical regress in the country where adoption cost is more expensive by forcing some firms to switch back to the old technology.

Using the insights from Yeaple (2005), Bustos (2010) incorporates technology choice in Melitz’s (2003) influential model of trade with heterogeneous firms. Her model predicts that the additional revenues obtained from falling trade costs induce exporters to upgrade technology. Using data on Argentinean firms, an empirical analysis of the model indeed shows that firms in industries facing higher reductions in Brazil’s tariffs increase their investment in technology faster. Given that she studies the impact of a trade liberalization
agreement between Brazil and Argentina, her model reasonably assumes that the costs of technology adoption in both countries are the same. How do these results change when technology adoption costs differ across countries? Addressing this question is particularly important in the context of trade between developed and developing countries. This paper shows that such an extension has two important consequences. First, having different costs for technology adoption across countries enables one to investigate firms' behavior in response to a change in the cost of technology adoption in one country. As discussed above, in such a situation, reduction in the cost of technology adoption in one country makes the other country worse off. Second, unlike the Bustos model, the analysis in this paper reveals that globalization has a different impact on each economy: a reduction in tariffs increase productivity and welfare, and induce exporters to upgrade their technology in the country with lower technology adoption cost, while they can have adverse effects on these variables in the other country.

In a recent paper, Atkeson and Burstein (2010) develop a model to analyze the welfare gains stemming from firms' decisions on exit, export, and process and product innovation, in response to a reduction in transport costs. They find that the effect of changes in these decisions on welfare is largely offset by the effect of changes in product innovation. Their study differs from mine in several aspects. Most notably, in their model, each firm's productivity changes over time due to idiosyncratic productivity shocks. In my model, each firm can improve its initial productivity only once, and this happens after paying technology adoption costs.

In two other related papers, Falvey et al. (2005) and Demidova (2008) study the effects of globalization on productivity and welfare in a monopolistic competition model with heterogeneous firms and technological asymmetries. Assuming that productivity distribution in one country is stochastically better than that in the other country, they show that falling trade costs improve the average productivity and welfare in the technologically advanced country while having ambiguous effects on these variables in the less-advanced one. Their
models do not include technology choice for firms, and consequently, they can not address the interaction between technology adoption and trade as this paper does. Furthermore, technological asymmetries in their models arise from the differences in underlying productivity distributions across countries, whereas in this paper the underlying productivity distributions across countries are the same and the technological asymmetries stem from the differences in the costs of technology adoption across countries.

This paper is also related to a large number of studies about technology adoption in industrial organization (IO) and economic growth literatures. The studies in the IO literature investigate the effects of firm size and the intensity of competition on the pace and pattern of technology adoption when adoption cost falls exogenously over time (see, e.g., Reinganum, 1981; Fudenberg and Tirole, 1985; and Götz, 1999, among many others). Elderington and McCalman (2008) extend the Götz model to trade between two symmetric countries, and they find that trade liberalization has a generally positive impact on the rate of technology adoption and firm-level productivity. Since adoption costs fall exogenously over time, firms in their model eventually adopt a new technology, and trade only affects the rate of technology diffusion. In my model, however, adoption costs do not change over time, and thus some firms stick to old technology forever. Furthermore, their model assumes that firms using the same technology are identical and the adoption costs are the same across countries. Consequently, unlike the model in this paper, their model predicts that trade liberalization has identical positive effects on both countries.

The studies on technology adoption in the growth literature, on the other hand, mainly focus on why adopting advanced technologies is difficult in the less developed countries (LDCs), and they offer several competing explanations. Atkinson and Stiglitz (1969) and Basu and Weil (1998), for example, emphasize the level of development (measured by capital-labor ratio) in technology adoption process, while Acemoglu and Zilibotti (2001) draw attention to the importance of the technology-skill mismatch arising from relative supplies of skills in LDCs. In a recent paper, Romer (2010) gives several illuminating ex-
amples about how rules/institutions play key roles in adopting new technologies (see also Parente and Prescott, 1994). This paper introduces some of these insights into the recent literature on trade with firm heterogeneity.\footnote{Based on the models from endogenous growth theory, a large literature investigates whether trade is a conduit for technology diffusion across countries (see Grossman and Helpman, 1991; Keller, 2004). However, these studies do not consider firm heterogeneity and firm-level dynamics in response to trade liberalization. They are aimed at analyzing the effects of trade on long-run growth. In this paper, product development is completely internalized by firms themselves and thus firms can not benefit from technologies developed by other (domestic and foreign) producers (i.e., there is no sustained growth). Extending the current set-up to a long-run growth model is left for future research.}

The rest of this paper is organized as follows. The next section introduces the model and characterizes its equilibrium in a global setting. Section 3 investigates how each economy responds to changes in technology adoption and trade costs, and section 4 concludes the paper.

\section{The Model}

I consider a world of two countries with equal sizes. Each country produces a homogenous good and a set of differentiated goods using only labor, which is inelastically supplied at its fixed aggregate level \( L \). Preferences are identical across countries.

\subsection{Consumers}

Preferences of consumers are modeled by the following utility function

\[ U = q_0^{1-\alpha} Q^\alpha, \quad 0 < \alpha < 1, \]  

where \( q_0 \) is the consumption of the homogeneous good and \( Q \) denotes the aggregate consumption index defined over a continuum of products indexed by \( \omega \)

\[ Q = \left[ \int_{\omega \in \Omega} q(\omega)^\rho d\omega \right]^{1/\rho}. \]  

The variable \( \Omega \) denotes the set of varieties available for consumption and \( q(\omega) \) denotes consumption of brand \( \omega \). The elasticity of substitution between any two brands is \( \sigma = 1/(1 - \rho) \), and I assume that \( 0 < \rho < 1 \) so that \( \sigma > 1 \).
Consumers spending an amount $E$ maximize their instantaneous utility by spending $(1 - \alpha)E$ amounts on the homogeneous good, and $\alpha E$ amounts on differentiated goods. It is well-known that the optimal quantity, $q(\omega)$, and expenditure, $r(\omega)$, levels for individual brands are given given by

$$
q(\omega) = Q[p(\omega)/P]^{-\sigma}, \quad r(\omega) = \alpha E[p(\omega)/P]^{1-\sigma},
$$

(3)

where $p(\omega)$ is the price of brand $\omega$, and $P$ is the aggregate price index associated with the aggregate consumption index $Q$ (i.e., $PQ = \alpha E$):

$$
P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}.
$$

(4)

2.2 Producers

The homogeneous good is produced in a competitive market with a constant returns to scale technology with one unit of labor producing one unit of output. The wage rate is normalized to one so that the price of the homogeneous good is also one. The homogeneous good is further assumed to be freely traded so that the wage rates in both countries are the same and equal to one.

Differentiated goods are produced by a continuum of monopolists, each choosing to produce a different variety. However, each variety can be produced using either of two types of technology, and production using low technology (denoted by $l$) is available to all successful entrants. In order to produce $q$ units of output using low technology, $\ell = f + q/\varphi$ units of labor are required, where $f$ denotes the fixed overhead cost of production measured in units of labor and $\varphi$ is firm the specific productivity level.

However, firms can upgrade their technology by reducing the marginal cost of production; but, technology upgrading (or adoption) requires an additional fixed cost. More specifically, production of $q$ units of output using advanced technology (denoted by $h$) requires $\ell_i = a_i f + q/b\varphi$ units of labor in country $i$, where $a_i > 1$ and $b > 1$. Without loss of generality, it is assumed that adopting advanced technology is cheaper in country 1,
i.e., $a_1 < a_2$. As mentioned in the introduction, the difference in technology adoption costs across countries can arise from the differences in the quality of institutions (which covers rule of law, effectiveness of governance, level of corruption, etc.), the level of infrastructure, and even human capital necessary to implement better technology.

Firms wishing to export face both variable and fixed costs. Variable costs are modeled in the standard iceberg transport costs: for one unit of a good to arrive, $\tau > 1$ units of a good must be shipped. Consistent with several empirical evidences (e.g., Tybout and Roberts, 1997), exporting firms must pay a fixed foreign market entry cost $F_x > 0$, which is measured in units of labor. This fixed cost covers the costs of setting a distribution network in the foreign market, modifying the product to meet the foreign market specifications as well as the costs of regulations imposed by the foreign government. The decision about exporting happens after the firm’s productivity is revealed.

Each incumbent firm faces a constant probability of death $\delta$ in each period as in the Melitz model. Having no uncertainty in the export market makes each firm indifferent between paying a one time investment cost $F_x$ and paying $f_x = \delta F_x$ in each period. Hereafter it is assumed that in each period exporters pay $f_x$ in addition to their overhead production cost $f$ or $a_i f$.

A firm operating in country $i$ with productivity $\varphi$ faces a demand curve described in (4). Profit maximizing behavior yields the following price rules in domestic and foreign markets:

\[
\begin{align*}
p_{idh}(\varphi) &= 1/(b\rho \varphi), & p_{ixh}(\varphi) &= \tau/(b\rho \varphi); \\
p_{idl}(\varphi) &= 1/(\rho \varphi), & p_{ixl}(\varphi) &= \tau/(\rho \varphi).
\end{align*}
\] (5)

With these pricing rules, the per-period profits earned from domestic sales, $\pi_{idv}(\varphi)$, and profits earned from export sales, $\pi_{ixv}(\varphi)$, are given by

\[
\begin{align*}
\pi_{idh}(\varphi) &= r_{idh}(\varphi)/\sigma - a_i f, & \pi_{ixh}(\varphi) &= r_{ixh}(\varphi)/\sigma - f_x, \\
\pi_{idl}(\varphi) &= r_{idl}(\varphi)/\sigma - f, & \pi_{ixl}(\varphi) &= r_{ixl}(\varphi)/\sigma - f_x,
\end{align*}
\] (6)

(7)

where $r_{idv}(\varphi)$ and $r_{ixv}(\varphi)$ represent the corresponding revenues obtained from sales in do-
mestic and foreign markets, respectively, and are calculated according to (3).

Furthermore, using these pricing rules in the demand and expenditure functions given by (3) yields

\[
\frac{q_{ijv}(\varphi_1)}{q_{ijv}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^\sigma, \quad \frac{r_{ijv}(\varphi_1)}{r_{ijv}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{-\sigma-1}, \quad \frac{r_{ijh}(\varphi)}{r_{ijl}(\varphi)} = b^{\sigma-1}, \quad \frac{r_{iixv}(\varphi)}{r_{iidv}(\varphi)} = \tau^{1-\sigma},
\]

where \(i = 1, 2; j = d, x; \) and \(v = h, l\). Thus, using the same technology, a more productive firm will have a lower price, will produce more output, and will earn a higher profit than a less productive firm.

There is an unbounded pool of prospective and ex-ante identical entrants. Firms face an initial investment of \(f_e > 0\) units of labor, which is thereafter sunk. Firms then draw their initial productivity parameter \(\varphi\) from a common distribution \(g(\cdot)\), which has positive support over \((0, \infty)\) and has a continuous cumulative distribution \(G\). Productivity draws are assumed to follow a Pareto distribution:

\[
G(\varphi) = 1 - \varphi^{-k},
\]

where \(k\) is the shape parameter. I further assume that \(k > \sigma + 1\) so that the distribution has a finite variance and the aggregate variables converge. The Pareto distribution has been widely used in recent trade literature and it makes the analysis more tractable. Furthermore, several studies have found that the distribution of firm sizes in the US closely follows a Pareto distribution (e.g., Cabral and Mata, 2003; Helpman et al., 2004).

After entry with a low productivity draw, a firm may decide to immediately exit and not produce. If a firm chooses to produce, it then decides which production technology to use. Since each firm’s productivity level does not change over time, its optimal per period profit will remain constant. Using \(v\)-type technology \((v = h, l)\), a firm in country \(i\) with productivity \(\varphi\) earns a per-period profit \(\pi_{iv}(\varphi) = \pi_{idv}(\varphi) + \max\{0, \pi_{iixv}(\varphi)\}\). Since each firm faces a constant probability of death \(\delta\) in each period, the market value of a typical incumbent is given by \(\nu_{iv}(\varphi) = \pi_{iv}(\varphi)/\delta\).
A firm upgrades its technology if and only if $\pi_{ih}(\varphi) \geq \pi_{il}(\varphi)$. The marginal firm that is indifferent between these two technologies is given by

$$\pi_{il}(\varphi_{ih}) = \pi_{ih}(\varphi_{ih}).$$

Thus, any firm with $\varphi \geq \varphi_{ih}$ uses advanced technology. This further implies that if there are some firms using advanced technology and serve only their domestic market, there will be no exporting firm using low technology.\(^2\)

However, using firm-level data from Argentina, Bustos (2010) shows the existence of exporting firms using low (or old) technology. In particular, she documents that firms are sorted into three groups: the least productive firms that use old technology and serve only the domestic market, a middle group of firms that uses old technology and exports, and the most productive firms that use the advanced technology and export. This classification is assumed to hold in this paper as well.

The production cutoff level for firms using low technology is given by $\pi_{idl}(\varphi_{il}) = 0$ and the export cutoff level for the firms using low technology is determined by $\pi_{ixl}(\varphi_{ixl}) = 0$. Using these zero profit conditions together with (7) yields\(^3\)

$$\varphi_{ixl} = A\varphi_{jl},$$

where $A = \tau(f_x/f)^{1/(\sigma-1)}$ and $i, j = 1, 2$. Since many empirical studies have shown that only a fraction of firms export (e.g., Bustos, 2010, among many others), it is assumed that the trade costs are sufficiently high so that both exporters and non-exporters co-exists, i.e., $\varphi_{ixl} > \varphi_{il}$ for $i = 1, 2$. This assumption, combined with (11), ensures that $A > 1$.

\(^2\)If there are firms using advanced technology and serve only their local markets, then the firm with $\varphi_{ih}$ is clearly one of them. By definition, no firm with $\varphi > \varphi_{ih}$ will be using the low technology. Suppose that there is a firm with $\varphi < \varphi_{ih}$ such that it uses low technology and exports. Since the fixed foreign-market entry cost is the same under both technologies, in this case, the firm with $\varphi_{ih}$ must also export. But then, this contradicts the initial supposition that the marginal firm serves only its domestic market.

\(^3\)The zero-profit cutoff conditions ensure that $r_{ixl}(\varphi_{ixl}) = \tau^{1-\sigma}r_{jxl}(\varphi_{ixl}) = \sigma f_x$ and $r_{jdl}(\varphi_{jl}) = \sigma f$. Using (7) implies that $r_{jxl}(\varphi_{ixl}) = (\varphi_{ixl}/\varphi_{jl})^{\sigma-1}r_{jxl}(\varphi_{jl}) = (\varphi_{ixl}/\varphi_{jl})^{\sigma-1}\sigma f$. Substituting $r_{jxl}(\varphi_{ixl})$ into $\tau^{1-\sigma}r_{jxl}(\varphi_{ixl}) = \sigma f_x$ yields (11). Furthermore, (11) implies that $(\varphi_{ixl}/\varphi_{jl})(\varphi_{ixl}/\varphi_{jl}) = A^2$. Since it is assumed that $\varphi_{ixl} > \varphi_{il}$, it follows that $A > 1$. 
The productivity cutoff level for technology adoption is found as follows. Since firms using advanced technology also export, using the profit functions (6) and (7) in (10) yields

\[ r_{idh}(\varphi_{ih}) + r_{ixh}(\varphi_{ih}) - r_{idl}(\varphi_{ih}) - r_{ixl}(\varphi_{ih}) = (a_i - 1)\sigma f. \]

Using the zero-profit cutoff conditions together with \(\varphi_{ixl} = A\varphi_{ijl}\) in the above equation yields

\[\varphi_{ih} = \left[ \frac{a_i - 1}{b_{ijl} - 1} \right]^{\frac{1}{\sigma - 1}} \left[ \varphi_{il}^{1 - \sigma} + (\tau\varphi_{jil})^{1 - \sigma} \right]^{\frac{1}{\sigma - 1}}. \] (12)

The technology switching cutoff level \(\varphi_{ih}\) in each country increases in the cost of implementing advanced technology (captured by \(a_i\)) and the production cutoff levels \(\varphi_{1l}\) and \(\varphi_{2l}\). To ensure that all firms using advanced technology export, it is further assumed that technology adoption cost (captured by the parameter \(a_i\)) is sufficiently high so that the productivity cutoff level for technology upgrading is always greater than the export cutoff level, i.e., \(\varphi_{ih} > \varphi_{ixl}\) for \(i = 1, 2\).5

The ex-post distribution of productivity levels for successful entrants, \(\mu_i\), is the conditional distribution of \(g(\varphi)\) on \([\varphi_{il}, \infty)\):

\[\mu_i(\varphi) = \begin{cases} \frac{g(\varphi)}{1 - G(\varphi_{il})} & \text{if } \varphi > \varphi_{il} \\ 0 & \text{otherwise.} \end{cases} \] (13)

The ex-ante probability that one of these successful firms will export is given by \(\zeta_{ix} = [1 - G(\varphi_{ixl})]/[1 - G(\varphi_{il})]\). By the law of large numbers, \(\zeta_{ix}\) also represents the ex-post fraction of incumbent firms that export.

Using the distribution function given by (9), the average profit of a successful entrant in country \(i\) is (see Appendix):

\[ \bar{\pi}_i = \frac{f}{\beta - 1} \left[ 1 + \left( \frac{f_x}{f} \right) \zeta_{ix} + (a_i - 1)\zeta_{ih} \right], \] (14)

4To see this, first note that \(r_{idh}(\varphi_{ih}) - r_{idl}(\varphi_{ih}) = (b_{ijl}^{\sigma - 1} - 1)r_{idl}(\varphi_{ih})\) and \(r_{ixh}(\varphi_{ih}) - r_{ixl}(\varphi_{ih}) = (b_{ijl}^{\sigma - 1} - 1)r_{ixl}(\varphi_{ih})\). Furthermore, \(r_{idl}(\varphi_{ih}) = (\varphi_{ih}/\varphi_{il})^{\sigma - 1}r_{idl}(\varphi_{il})\) and \(r_{ixl}(\varphi_{ih}) = (\varphi_{ih}/\varphi_{ixl})^{\sigma - 1}r_{ixl}(\varphi_{ixl})\). Using the zero-profit conditions gives (12).

5Substituting \(\varphi_{ijl} = \varphi_{ixl}/A\) into (12) and using \(\varphi_{ih} > \varphi_{ixl} > \varphi_{il}\) implies that \(a_i > b_{ijl}^{\sigma - 1} + (b_{ijl}^{\sigma - 1} - 1)f_x/f\).
where \( \beta = k/(\sigma - 1) > 1 \) and \( \zeta_{ih} \) denotes the ex-ante probability that one of the incumbents will adopt \( h \)-technology, i.e., \( \zeta_{ih} = [1 - G(\varphi_{ih})]/[1 - G(\varphi)] \).

Since the probability of successful entry is \( 1 - G(\varphi) \), the expected value of a firm is \( [1 - G(\varphi_{il})] \bar{\pi}_i / \delta \) is the average value of a successful entrant in country \( i \).

Setting this expected value equal to the initial fixed investment cost \( f_e \) gives the free-entry condition

\[
[1 - G(\varphi_{il})] \frac{\bar{\pi}_i}{\delta} = f_e.
\]

Finally, it is assumed that the trade between two countries is balanced. Let \( \theta_i \) denote the fraction of labor working in the differentiated good sector in country \( i \). Since the homogeneous good is produced under constant returns to scale at unit cost, the amount produced in country 1 is \( (1 - \theta_1)L \). Then \((1 - \theta_1)L - (1 - \alpha)E_1 \) represents the net export of the homogeneous good to country 2. In the balanced trade:

\[
R_{1x} - R_{2x} + (1 - \theta_1)L - (1 - \alpha)E_1 = 0,
\]

where \( R_{ix} \) is the aggregate revenue earned by firms in country \( i \) from exports. Since the world expenditure on the homogeneous good equals the world income generated in this sector, it follows that

\[
(1 - \alpha)E_1 + (1 - \alpha)E_2 = (1 - \theta_1)L + (1 - \theta_2)L.
\]

### 2.3 Equilibrium Analysis

Substituting (14) into the free-entry condition (15) and using (12) yields

\[
\varphi_{1l}^{-k} + \left( \frac{f_e}{f} \right)(A \varphi)_{1l}^{-k} + \frac{(b^\sigma - 1 - 1)\beta}{(a_1 - 1)^{\beta-1}} \left[ \varphi_{1l}^{1-\sigma} + (\tau \varphi_{2l})^{1-\sigma} \right]^{\beta} = \frac{\delta(\beta - 1)f_e}{f},
\]

\[
\varphi_{2l}^{-k} + \left( \frac{f_e}{f} \right)(A \varphi)_{2l}^{-k} + \frac{(b^\sigma - 1 - 1)\beta}{(a_2 - 1)^{\beta-1}} \left[ \varphi_{2l}^{1-\sigma} + (\tau \varphi_{1l})^{1-\sigma} \right]^{\beta} = \frac{\delta(\beta - 1)f_e}{f},
\]

where again \( \beta = k/(\sigma - 1) > 1 \). Equations (18) and (19) represent a system of two equations with two unknowns \( \varphi_{1l} \) and \( \varphi_{2l} \). Note that the country size \( L \) does not appear in any of
these equations. Moreover, each of the above equations represents an inverse relationship between $\varphi_{1l}$ and $\varphi_{2l}$.

There is a simple and intuitive way to characterize the solution to the above system. To this end, consider a completely symmetric case by setting $a_1 = a_2 = a$. Equations (18) and (19) represent two curves in ($\varphi_1$, $\varphi_2$) space, and these two curves intersect each other at the 45-degree line so that $\varphi_{1l} = \varphi_{2l} = \varphi_l$.\(^6\) At the intersection, the (negative of the) slope of the first curve is greater than the second curve’s as depicted in Figure 1.\(^7\) When technology adoption is cheaper in country 1, the first curve will be on the right of the original curve as shown by the dashed-curve in Figure 1. Hence, in the new equilibrium, the production cutoff level in country 1 is greater than that in country 2, i.e., $\varphi_{1l} > \varphi_{2l}$.

\(^6\)Recall that the Pareto distribution given by (9) is defined over $[1, \infty)$. Thus, the production level in each country can not be smaller than 1.

\(^7\)Totally differentiating (18) and then using $\varphi_{1l} = \varphi_{2l} = \varphi$ yields $|d\varphi_{2l}/d\varphi_{1l}| = \tau^{\sigma-1}(1 + \gamma)/(A^{1-k} + \gamma)$, where $A = \tau^{\sigma-1}f_x/f$ and $\gamma = (b^{\sigma-1} - 1)(b^{\sigma-1} - 1) / (a - 1)$. Since $\tau > 1$ and $A > 1$, it follows that $|d\varphi_{2l}/d\varphi_{1l}| > 1$. Similarly, totally differentiating (19) yields $|d\varphi_{2l}/d\varphi_{1l}| = (A^{1-k} + \gamma)/\tau^{\sigma-1}(1 + \gamma) < 1$. 

Figure 1: Characterization of the solution.
The complete characterization of the solution requires that the above two curves must intersect with each other. For a given $a_2$, let $\varphi_{1l}(a_2)$ denote the solution to equation (19) when $\varphi_{2l} = 1$. Similarly, let $\varphi_{2l}(a_2)$ denote the solution to equation (19) when $\varphi_{1l} = 1$. With these boundary values, let $\bar{a}_1(a_2)$ denote the solution to equation (18) when $(\varphi_{1l}, \varphi_{2l}) = (\varphi_{1l}(a_2), 1)$. Note that if $a_1 < \bar{a}_1(a_2)$, the first curve will not intersect the second one, and hence, there will be no solution to the above system. Similarly, let $\bar{a}_1(a_2)$ denote the solution to equation (18) when $(\varphi_{1l}, \varphi_{2l}) = (1, \varphi_{2l}(a_2))$. If $a_1 > \bar{a}_1(a_2)$, the first curve will not intersect the second one, and hence, there will again be no solution to the above system. Thus, the two curves intersect each other only if

$$a_1(a_2) \leq a_1 \leq \bar{a}_1(a_2).$$

(20)

The above condition states that the difference in technology adoption costs between two countries should not be too large. This condition is assumed to hold so that there is no complete specialization in production of differentiated goods in any country. Note that when $a_1 < \bar{a}_1(a_2)$, the differentiated goods will be produced only in country 1. When $a_1 > \bar{a}_1(a_2)$, country 2 will specialize in production of differentiated goods.

**Proposition 1.** Suppose that the condition in (20) is satisfied and $a_1 < a_2$. The system of equations represented by (18) and (19) has a unique solution $(\varphi_{1l}, \varphi_{2l})$ such that $\varphi_{1l} > \varphi_{2l}$.

The intuition behind this result is as follows. Firms using advanced technology have higher operating profits due to a lower marginal cost of production. Consequently, cheaper technology adoption in country 1 increases the incentive for firms to adopt advanced technology. Furthermore, higher potential returns from using better technology also induce entry of more firms in country 1. Increased demand for labor by firms adopting advanced technology and the new entrants bids up the real wage in country 1 more than that in country 2, and hence, more firms with low productivity exit in country 1.

Once the production cutoff levels are determined, one can solve for the remaining endogenous variables. Since $\varphi_{1l} > \varphi_{2l}$, equation (11) implies that $\varphi_{1xl} < \varphi_{2xl}$, which in turn
implies that the ex-post fraction of incumbents that export in country 1 is greater than that in country 2, i.e., $\zeta_{1x} > \zeta_{2x}$. Since the real wage is higher in country 1 (as discussed above), the aggregate price index in country 1 will be smaller than that in country 2 (recall that nominal wage is normalized to one). It follows that competition in country 1 is more intense than that in country 2. As a result, the export cutoff level for firms in country 1 will be lower than that in country 2.

Since adopting advanced technology is cheaper in country 1 than country 2, one can reasonably expect that the cutoff level for technology switching in country 1 is smaller than that in country 2. As shown in the appendix, this is indeed the case. This, combined with $\varphi_{1l} > \varphi_{2l}$, implies that the ex-post fraction of successful entrants that use advanced technology in country 1 is greater than that in country 2, i.e., $\zeta_{1h} > \zeta_{2h}$. The following corollary summarizes these results.

**Corollary 1.** Suppose that condition (20) is satisfied and $a_1 < a_2$. In equilibrium, the export and technology switching cutoff levels in country 1 are smaller than the corresponding cutoff levels in country 2 (i.e., $\varphi_{1xl} < \varphi_{2xl}$ and $\varphi_{1h} < \varphi_{2h}$). Furthermore, the ex-post fraction of firms that export and the ex-post fraction of firms that adopt advanced technology in country 1 are greater than those in country 2 (i.e., $\zeta_{1x} > \zeta_{2x}$ and $\zeta_{1h} > \zeta_{2h}$).

To find the aggregate expenditure $E_i$, I consider the steady-state case in which the per-period flow of successful entrants must be equal to the flow of incumbents who exit the market: $[1 - G(\varphi_{il})]M_{ie} = \delta M_i$, where $M_{ie}$ is the mass of entrants in country $i$. Then the aggregate amount of labor employed by prospective entrants is $L_{ie} = M_{ie}f_{ie} = \delta M_{ie}f_{ie}/[1 - G(\varphi_{il})] = M_i \bar{\pi}_i$, where the last equality follows from the free-entry condition (15). Thus the aggregate amount of labor devoted to R&D in each country equals the level of aggregate profits earned by all producers in that country. By the labor market clearing-condition, $L_0 + L_p + L_e = L_0 + L_p + \Pi = L$, where $L_0$ and $L_p$ denote the total amount of labor employed in the production of homogeneous and differentiated goods, respectively; and $\Pi$ is the level of aggregate profits earned by all differentiated-product producers. Furthermore,
the total spending on all goods must be equal to the total income $E_i = wL_0 + wL_p + wL_e = L$, since the wage rate is normalized to one.

Using $E = L$, the welfare per worker in each country is found as follows. Since the price of the homogeneous good is one, it follows that $q_{i0} = (1 - \alpha)E_i = (1 - \alpha)L$. The total expenditure on differentiated goods is $P_iQ_i = \alpha E_i = \alpha L$, and thus $Q_i = \alpha L P_i^{-1}$.

Substituting these into the utility function given by (1) implies that the welfare per worker in country $i$ is $W_i = (1 - \alpha)^{1-\alpha} \alpha^\alpha P_i^{-\alpha}$. The aggregate price index $P_i$ can be pinned down from the zero profit condition: $r_{id}(\varphi_{il}) = \alpha L (\rho \varphi_{il} P_i)^{\alpha - 1} = \sigma f$. Substituting $P_i$ into welfare per worker function yields

$$W_i = (1 - \alpha)^{1-\alpha} (\alpha \rho)^\alpha (\alpha L / \sigma f)^{1-\alpha} \varphi_{il}^{\alpha}.$$  \hspace{1cm} (21)

Thus per capita welfare increases in country size $L$ and the production cutoff level $\varphi_{il}$, and decreases in the fixed overhead cost $f$. Furthermore, $W_1 > W_2$, since $\varphi_{1l} > \varphi_{2l}$ from Proposition 1.

**Proposition 2.** Suppose that condition (20) is satisfied and $a_1 < a_2$. In the steady-state equilibrium, the welfare per worker in country 1 is greater than that in country 2, i.e., $W_1 > W_2$.

What is the pattern of the trade between these two countries? The crucial point here is that the existence of a freely traded homogeneous good can make the aggregate revenue earned by the differentiated-good sector differ from the aggregate expenditure on the differentiated goods, i.e., $R_i \neq \alpha L$. The following proposition characterizes the pattern of the trade between these two countries, when trade is balanced (see Appendix for proof).

**Proposition 3.** Suppose that condition (20) is satisfied and $a_1 < a_2$. In the steady-state equilibrium, country 1 exports only differentiated goods, whereas country 2 exports both homogeneous and differentiated goods.

Since the fraction of firms that adopt advanced technology is higher in country 1, and using the advanced technology generates higher profit, the average profit and revenue of a
successful entrant is higher in country 1.\(^8\) This makes the differentiated good sector more attractive in country 1, as prospective firms respond to higher average profit opportunities. In equilibrium, the aggregate revenue earned by differentiated-good producers in country 1 will be higher than that in country 2, i.e., \(R_1 > R_2\). Given that the total spending in each country is \(L\), the finding \(R_1 > R_2\) further implies that the revenue earned by the firms that produce the homogeneous good in country 1 is smaller than that in country 2. Since consumers in both countries love varieties and the trade is balanced between these countries, country 1 must export differentiated goods and import the homogeneous good; while country 2 must export both the homogeneous and differentiated goods.

3 Comparative Statics

The previous section has shown the country which has a lower cost of technology adoption is better off: it has a higher average productivity (by having higher production productivity cutoff level), uses (on average) better technology (by having a higher fraction of firms that use advanced technology), and has a higher welfare per worker. In this section, I will investigate how these endogenous variables respond to changes in the costs of technology adoption and trade.

3.1 Change in the Cost of Technology Adoption

Suppose that both countries are in their steady-state equilibrium as described in the previous section. To make analysis more interesting, consider a reduction in the cost of technology adoption in country 2. As shown in Figure 2, the curve associated with country 2 will shift upward and intersect country 1’s curve at point B. Thus, the production cutoff level decreases in country 1, while increasing in country 2. Using equation (11), it follows that the export cutoff level increases in country 1 and decreases in country 2. These, combined with the corresponding changes in the production cutoff levels, imply that the ex-post fraction

\(^8\)More precisely, using \(\varphi_{1l} > \varphi_{2l}\) in the free-entry condition (15) implies that the average profit (and hence, the average revenue) of a successful entrant in country 1 is higher than that in country 2.
of firms that export decreases in country 1, while increasing in country 2.

How about the effects of this reduction on the cutoff levels for technology adoption in both countries? As shown in the appendix, such a reduction increases the technology switching cutoff level in country 1, while decreasing it in country 2. Thus, the fraction of firms that adopt advanced technology decreases in country 1, while increasing in country 2.

Since the welfare per worker increases in productivity cutoff level $\varphi_{it}$, the welfare per worker decreases in country 1, while increasing in country 2. Completely opposite results will hold, if there is a reduction in the cost of technology adoption in country 1. The following proposition summarizes these results.

**Proposition 4.** Suppose that condition (20) is satisfied and $a_1 < a_2$. A reduction in the cost of technology adoption in one country increases the production cutoff level, the ex-post fraction of firms that export, the ex-post fraction of firms that adopt advanced technology, and the welfare per worker in this country; while decreasing the production cutoff level,
the ex-post fraction of firms that export, the ex-post fraction of firms that adopt advanced technology, and the welfare per worker in other country.

A reduction in technology adoption cost in country 2 enables more firms in this country to upgrade their technology. This, combined with entry of more firms in response to higher potential returns from using advanced technology, increases the demand for labor, and thus bids up the real wage in this country. A higher real wage intensifies competition and forces the least productive domestic and foreign firms to exit. Tougher competition in country 2 also induce some foreign firms to switch back to old technology due to the fact that they can not cover their adoption costs with a reduction in their export revenues. The demand for labor falls in country 1 as firms stop exporting or switch back to old technology. The decline in demand for labor in country 1 lowers the real wage and induces some of the least productive firms to enter the market, and hence, the production cutoff level decreases in country 1.

As stated above, tougher competition in country 2 forces some foreign firms to exit this country. Hence, the export cutoff level for firms in country 1 increases. On the other hand, since the competition in country 1 is not as tough as before, firms in country 2 now find the access to country 1 easier, and thus the export cutoff level for firms in country 2 decreases.

When the cost of technology adoption decreases in country 2, it becomes easier for firms to adopt advanced technology. It intuitively follows that the technology switching cutoff level will be lower in country 2. Recall that a firm upgrades its technology if the additional profits obtained from using advanced technology are not less than its investment cost (see equation (10)). There are two competing effects in country 1. On one hand, since the competition in country 1 is not as tough as before, each firm’s revenue increases (see the revenue function in (3)). On the other hand, since all firms using advanced technology are exporters and the competition in country 2 is now tougher, their export revenues from country 2 decline. Increased revenues in the domestic market will not be high enough to overcome the decline in their export revenues due to the existence of the new inefficient
domestic firms and the new exporters from country 2. Such firms find that the total profit obtained from domestic and export sales is not enough to cover the technology investment cost; and consequently, switch back to using low technology.

3.2 Trade Liberalization

This section studies the effects of trade liberalization through a reduction in transportation costs on productivity, welfare, and technology choice in each country. According to the equilibrium conditions given by (18) and (19), a reduction in the transportation cost $\tau$ shifts both curves to the right. Since technology adoption costs differ across countries, such a reduction is expected to have a different effect on production cutoff level in each country. As formally shown in the appendix, reduction in $\tau$ always increases the production cutoff level in country 1 (where technology adoption is cheaper), while it can increase or decrease the production cutoff level in country 2 as depicted in Figures 3.a and 3.b.

According to equation (11), a reduction in the transportation cost $\tau$ affects the export cutoff level through $A = \tau(f_x/f)^{1/(\sigma-1)}$ and $\varphi_{jl}$. Reduction in $A$ always dominates the ambiguity in $\varphi_{2l}$ so that the export cutoff level declines in country 1. However, the decline in $A$ may not be high enough to overcome the rise in $\varphi_{1l}$. Consequently, the export cutoff level in country 2 may increase (see Appendix). These, combined with the changes in production cutoff levels, imply that the ex-post fraction of firms that export increases in country 1, while it can increase or decrease in country 2.

The effects of this policy on technology choice in each country are similar to its effects on the export cutoff levels: the technology switching cutoff level in country 1 ($\varphi_{1h}$) decreases, whereas it can increase or decrease in country 2. Thus, the ex-post fraction of firms that adopt advanced technology in country 1 increases, while that in country 2 can increase or decrease.

Proposition 5. Suppose that condition (20) is satisfied and $a_1 < a_2$. Trade liberalization through a reduction in transportation cost $\tau$ increases the production cutoff level, the ex-post
a. Productivity cutoff increases in country 2.

b. Productivity cutoff decreases in country 2

Figure 3: Effects of a reduction in $\tau$ on the productivity cutoffs.
fraction of firms that export, the ex-post fraction of firms that adopt advanced technology, and the welfare per worker in country 1; while it has an ambiguous effect on these variables in country 2.

Bustos (2010), based on a symmetric model, finds that a reduction in the transportation cost increases the production cutoff level, the ex-post fraction of firms that export, the ex-post fraction of firms that adopt advanced technology, and the welfare per worker in both countries. The above proposition states that when countries have different technology adoption costs, the country where technology adoption is more expensive can be hurt as a result of trade liberalization. As briefly discussed in the introduction, the different effects of falling variable trade costs on these countries stem from the asymmetry in the degree of competition in two countries created by this policy.

Intuition for these results is simple. Further exposure to trade provides new profit opportunities to the more productive firms. The profit opportunities from export markets induce entry of more firms and create incentives for the more productive firms to export and/or adopt new technology. Since technology adoption is cheaper in country 1, the demand for labor by the more productive firms and the new entrants is higher in this country. Increased demand for labor bids up the real wage, intensifies competition, and forces the least productive firms to exit.

Exporters from country 2 face two competing forces. On one hand, a reduction in $\tau$ directly increases the revenue from export sales (see the revenue equation (3)). On the other hand, tougher competition in country 1 reduces each firm’s revenue obtained from sales in this country (see again the revenue equation (3)). If the direct effect of a reduction in $\tau$ dominates the second effect, then trade liberalization provides new profit opportunities to firms in country 2, which in turn increase the demand for the labor in this country. As in country 1, a higher wage rate forces the least productive firms to exit country 2, and hence, the production cutoff level in country 2 increases. However, if the second effect is dominant, then some foreign firms exit country 1, since their operating profits now can not meet the
foreign market fixed entry costs. A fall in the demand for labor by the exporters in country 2 lowers the real wage, makes competition less intense, and induces the least productive firms to enter the market. As a result, the production cutoff level decreases in country 2.

Since the new profit opportunities induce more firms in country 1 to export, the export cutoff level decreases in country 1. Tougher competition in country 1 reduces revenues of all firms in this country. However, in equilibrium, additional profits obtained from the sales in country 2 are high enough to cover the losses from the sales in the domestic market. The net positive increase in the overall profit enables some more productive firms to upgrade their technology and thus the technology cutoff level in country 1 also decreases. Since further exposure to trade may not provide new profit opportunities for firms in country 2, it has an ambiguous effect on the export and technology cutoff levels.

Before closing this section, note that trade liberalization can also happen through a reduction in the foreign market entry fixed cost $f_x$. An inspection of (18) and (19) reveals that lowering the fixed entry cost $f_x$ shifts both curves to the right as in the previous case depicted in Figures 3.a and 3.b: the production cutoff level increases in country 1, while that in country 2 can increase or decrease. However, the effects of this policy change on the technology cutoff levels are different from the previous case. In particular, a reduction in the fixed entry cost $f_x$ increases the technology switching cutoff levels in both countries (i.e., $d\varphi_{ih}/df_x < 0$).\(^9\) These results do not hinge on the asymmetry in technology adoption costs across countries. Even when countries face the same costs, lowering $f_x$ increases the technology cutoff levels in both countries (as is evident from equation (8) in Bustos, 2010).

As in the previous case, a reduction in $f_x$ still intensifies the competition in country 1, and thus each firm’s revenue from sales in this country declines. However, for some firms the profit obtained from the export market is not high enough to overcome the loss in domestic sales. Consequently, some firms switch back to old technology in country 1. Similarly, for some firms using advanced technology in country 2, declines in their profits from the sales

\(^9\)The formal proofs of the above claims are available upon request.
in country 1 dominate the changes in their profits in domestic market; and thus, they also switch back to old technology.

4 Conclusion

This paper has incorporated technology adoption choice in a model of trade with heterogeneous firms. There are two countries and the costs of adopting advanced technology are different across countries. Firms, upon observing their productivity levels, decide which technology to use and whether or not to export. The paper has then analyzed the interplay between technology adoption and trade liberalization. In particular, it has investigated how firms respond to the following two changes: (i) a decline in the cost of technology adoption in one country, and (ii) trade liberalization through a reduction in transport costs.

The analysis has shown that a decline in the cost of technology adoption in one country makes this country better off in the sense that it increases the production cutoff level, the ex-post fraction of firms that export, the ex-post fraction of firms that adopt advanced technology, and the welfare per worker; while making the other country worse off. Furthermore, because the costs of technology adoption differ across countries, trade liberalization has a different effect on each economy. In particular, a fall in the variable trade cost makes the country with lower technology adoption cost better off, while having an ambiguous effect on productivity, technology choice, and welfare in the other country.

In this paper, firms are heterogenous with respect to their productivity levels. Several empirical studies have also documented firm heterogeneity in the form of product-quality.\textsuperscript{10} Some recent papers have developed trade models based on firm-specific quality heterogeneity to study the effects of trade liberalization on the pattern of export prices and quality distributions (see, e.g., Baldwin and Harrigan, 2007; Kugler and Verhoogen, 2008; and Dinopoulos and Unel, 2009). One interesting direction is to incorporate quality heterogeneity along the lines of Baldwin and Harrigan (2007) into the present model. My preliminary

\textsuperscript{10}For example, Schott (2004), Hummels and Klenow (2005), Hallak (2006), and Manova and Zhang (2009) among many others.
analysis in this direction suggests that a decline in the cost of technology adoption in one country increases the average quality, improves the welfare, and induces firms to adopt advanced technology in this country; while having opposite effects on these variables in the other country.
Appendix

A. Calculations of the average revenue and the average profit

The average revenue of a successful entrant is given by

\[ \bar{r}_i = \int_{\bar{\phi}_{il}}^{\bar{\phi}_{ih}} r_{id}(\varphi) \mu(\varphi) d\varphi + \int_{\bar{\phi}_{ixl}}^{\bar{\phi}_{ixh}} [r_{ixl}(\varphi) + r_{ixh}(\varphi)] \mu(\varphi) d\varphi + \int_{\bar{\phi}_{ih}}^{\infty} [r_{ihl}(\varphi) + r_{ihh}(\varphi)] \mu(\varphi) d\varphi, \]

which can be rearranged as

\[ \bar{r}_i = \int_{\bar{\phi}_{il}}^{\bar{\phi}_{ih}} r_{id}(\varphi) \mu(\varphi) d\varphi + \int_{\bar{\phi}_{ih}}^{\infty} r_{ihl}(\varphi) d\varphi + \int_{\bar{\phi}_{ixl}}^{\bar{\phi}_{ixh}} r_{ixl}(\varphi) \mu(\varphi) d\varphi + \int_{\bar{\phi}_{ixh}}^{\infty} r_{ixh}(\varphi) \mu(\varphi) d\varphi. \]  (22)

The sum of the first two terms on the right hand side (RHS) represents the average revenue earned from domestic sales (\(\bar{r}_{id}\)), while the sum of the last two terms on the RHS represents the average revenues earned from sales in the export market (\(\bar{r}_{ix}\)). Using (8) together with the Pareto distribution given by (9), \(\bar{r}_{id}\) is given by

\[ \bar{r}_{id} = \sigma f_{ixl} \int_{\bar{\phi}_{il}}^{\bar{\phi}_{ih}} \left( \frac{\varphi}{\varphi_{il}} \right)^{\sigma-1} \mu(\varphi) d\varphi + \sigma f_{ixh} \int_{\bar{\phi}_{ih}}^{\infty} b^{\sigma-1} \left( \frac{\varphi}{\varphi_{il}} \right)^{\sigma-1} \mu(\varphi) d\varphi \]

where \(\beta = k/(\sigma - 1) > 1\). Using similar steps, it follows that

\[ \bar{r}_{ix} = \sigma f_{x} \int_{\bar{\phi}_{ixl}}^{\bar{\phi}_{ixh}} \left( \frac{\varphi}{\varphi_{ixl}} \right)^{\sigma-1} \mu(\varphi) d\varphi + \sigma f_{x} \int_{\bar{\phi}_{ixh}}^{\infty} b^{\sigma-1} \left( \frac{\varphi}{\varphi_{ixl}} \right)^{\sigma-1} \mu(\varphi) d\varphi \]

Substituting (23) and (24) into (22) yields

\[ \bar{r}_i = \left( \frac{\beta \sigma f}{\beta - 1} \right) \left[ 1 + \left( \frac{f_x}{f} \right) \zeta_{ix} + \left( b^{\sigma-1} - 1 \right) \left( \frac{\varphi_{ih}}{\varphi_{ixl}} \right)^{\sigma-1-k} \right] \]

where the second equality follows from \(\varphi_{ixl} = A \varphi_{jl}\) and (12). Using the profit functions in (5) and (6), one can easily show that the average profit is given by \(\bar{\pi}_i = \bar{r}_i / (\sigma \beta) = [1 + (f_x/f) \zeta_{ix} + (a_i - 1) \zeta_{ih}] f / (\beta - 1)\) as in equation (14). □
B. Proof of Corollary 1

The first part of Corollary 1 is shown in the text. To prove the second part, first note that substituting the average profit function (14) into the free-entry condition (15) yields

\[
\varphi_{il}^{-k} + (f_x/f)A^{-k}\varphi_{jl}^{-k} + (a_i - 1)\varphi_{ih}^{-k} = \delta(\beta - 1)f_x/f. \tag{26}
\]

(26)

Setting \(i = 1, 2\) (and hence, \(j = 2, 1\)), the above equation yields a system of two equations. Subtracting these two equations side by side and rearranging terms yields

\[
(1 - A^{-k}f_x/f)(\varphi_{2l}^{-k} - \varphi_{1l}^{-k}) + (a_2 - 1)\varphi_{2h}^{-k} - (a_1 - 1)\varphi_{1h}^{-k} = 0.
\]

Since \(k > \sigma - 1\) and \(\tau > 1\), it follows that \(A^k > A^{\sigma - 1} = \tau^{\sigma - 1}f_x/f > f_x/f\), and thus \(1 - A^{-k}f_x/f > 0\). In addition, using \(\varphi_{1l} > \varphi_{2l}\) ensures that \((1 - A^{-k}f_x/f)(\varphi_{2l}^{-k} - \varphi_{1l}^{-k}) > 0\).

Thus, the above equation implies

\[
(a_1 - 1)\varphi_{1h}^{-k} > (a_2 - 1)\varphi_{2h}^{-k} \Rightarrow \left(\frac{\varphi_{2h}}{\varphi_{1h}}\right)^k > \frac{a_2 - 1}{a_1 - 1} \Rightarrow \varphi_{2h} > \varphi_{1h},
\]

where the last inequality follows from \(a_2 > a_1\). Finally, combining the last inequality with \(\varphi_{1l} > \varphi_{2l}\) implies that \(\zeta_{1h} > \zeta_{2h}\). \(\square\)

C. Proof of Proposition 3

\(R_{ix} = \bar{r}_{ix}M_i\) where \(\bar{r}_{ix}\) is given by (24) and \(M_i\) is the mass of varieties produced in country \(i\). Since \(\theta_i\) is the fraction of labor used in the differentiated-good sector in country \(i\), it follows that \(\bar{r}_iM_i = R_i = \theta_iL\) and thus \(M_i = \theta_iL/\bar{r}_i\). Then the balanced trade condition (16) becomes

\[
\frac{\bar{r}_{1x}\theta_1}{\bar{r}_1} - \frac{\bar{r}_{2x}\theta_2}{\bar{r}_2} + \alpha - \theta_1 = 0, \tag{27}
\]

where we use \(E_1 = L\). Furthermore, substituting \(E_i = L\) in equation (17) implies that \(\theta_1 + \theta_2 = 2\alpha\), i.e., \(\theta_2 = 2\alpha - \theta_1\). Inserting this into (27) and rearranging the terms yields

\[
\theta_1 = \alpha \left[1 + \frac{z_1 - z_2}{1 - z_1 - z_2}\right],
\]

26
where $z_i \equiv \bar{r}_{ix}/\bar{r}_i$. Thus, to show that $\theta_1 > \alpha$ (which ensures that $\theta_1 > \theta_2$), it is enough to prove that $z_1 - z_2 > 0$ and $1 - z_1 - z_2 > 0$.

Note that the average revenue function (25) implies that $\bar{r}_1/\bar{r}_2 = (\varphi_{1l}/\varphi_{2l})^k$. Using this in $z_1 - z_2 > 0$ implies that $\varphi_{1l}^{-k}\bar{r}_{1x} > \varphi_{2l}^{-k}\bar{r}_{2x}$. Substituting (24) into this last inequality yields

$$\varphi_{1xl}^{-k} + (b^{\sigma - 1} - 1)\varphi_{1xl}^{-k}\left(\frac{\varphi_{1l}}{\varphi_{1xl}}\right)^{\sigma - 1 - k} > \varphi_{2xl}^{-k} + (b^{\sigma - 1} - 1)\varphi_{2xl}^{-k}\left(\frac{\varphi_{2l}}{\varphi_{2xl}}\right)^{\sigma - 1 - k}.$$

Since $\varphi_{1xl} < \varphi_{2xl}$, it follows that $\varphi_{1xl}^{-k} > \varphi_{2xl}^{-k}$. Thus, to prove that $z_1 > z_2$, it suffices to show that

$$\varphi_{1xl}^{-k}\left(\frac{\varphi_{1l}}{\varphi_{1xl}}\right)^{\sigma - 1 - k} > \varphi_{2xl}^{-k}\left(\frac{\varphi_{2l}}{\varphi_{2xl}}\right)^{\sigma - 1 - k} \iff \left(\frac{\varphi_{2l}}{\varphi_{1l}}\right)^{1 + k - \sigma} > \left(\frac{\varphi_{1xl}}{\varphi_{2xl}}\right)^{\sigma - 1}.$$

Since $\varphi_{2l} > \varphi_{1l}$ and $\varphi_{1xl} < \varphi_{2xl}$, the last inequality always holds (recall that $k + 1 > \sigma > 1$).

Note that $1 - z_1 - z_2 = \bar{r}_{1d}/\bar{r}_1 - \bar{r}_{2x}/\bar{r}_2$. Thus, $1 - z_1 - z_2 > 0$ if and only if $\bar{r}_{1d}/\bar{r}_{2x} > \bar{r}_1/\bar{r}_2$. Using $\bar{r}_1/\bar{r}_2 = (\varphi_{1l}/\varphi_{2l})^k$ together with $\bar{r}_{1d}$ and $\bar{r}_{2x}$ given by (23) and (24) implies that $1 - z_1 - z_2 > 0$ if and only if

$$1 + (b^{\sigma - 1} - 1)\left(\frac{\varphi_{1l}}{\varphi_{1l}}\right)^{\sigma - 1 - k} > \left(\frac{f_x}{f}\right)\left(\frac{\varphi_{1l}}{\varphi_{2l}}\right)^k + (b^{\sigma - 1} - 1)\left(\frac{f_x}{f}\right)\left(\frac{\varphi_{1l}}{\varphi_{2l}}\right)^k\left(\frac{\varphi_{2l}}{\varphi_{2l}}\right)^{\sigma - 1 - k}.$$

Substituting $\varphi_{2l} = A\varphi_{1l}$ into the above inequality yields

$$1 + (b^{\sigma - 1} - 1)\left(\frac{\varphi_{1l}}{\varphi_{1l}}\right)^{\sigma - 1 - k} > \left(\frac{f_x}{f}\right)A^{-k} + (b^{\sigma - 1} - 1)\tau^{1 - \sigma}\left(\frac{\varphi_{2l}}{\varphi_{1l}}\right)^{\sigma - 1 - k}.$$

Since $A^k > A^{\sigma - 1} > f_x/f$, it follows that $(f_x/f)A^{-k} < 1$. Furthermore, $\varphi_{2l} > \varphi_{1l}$ and $\tau^{1 - \sigma} < 1$ imply that $\varphi_{1l}^{\sigma - 1 - k} > \tau^{1 - \sigma}\varphi_{2l}^{\sigma - 1 - k}$. \qed

\textbf{D. Proof of Proposition 4}

Without loss of generality, assume that there is a reduction in $a_2$ as in the main text. The results related to production and export cutoff levels easily follow from Figure 2 and the discussion in the text. To show that $d\varphi_{1l}/da_2 < 0$, consider equation (12) with $(i, j) =
(1, 2). Taking logarithms of both sides of this equation and differentiating with respect to $a_2$ yields

$$\frac{d\varphi_{11}}{da_2} = \left( \frac{a_1 - 1}{b^{\sigma - 1} - 1} \right) \left( \frac{\varphi_{11}}{\varphi_{1h}} \right)^\sigma \frac{d\varphi_{1h}}{da_2} - \tau^{1 - \sigma} \left( \frac{\varphi_{11}}{\varphi_{2l}} \right)^\sigma \frac{d\varphi_{2l}}{da_2}. \quad (28)$$

Setting $i = 1$ and differentiating the equilibrium condition (26) with respect to $a_2$ yields

$$\varphi_{11}^{-1 - k} \frac{d\varphi_{11}}{da_2} + (f_s/f) A^{-k} \varphi_{2l}^{-1 - k} \frac{d\varphi_{2l}}{da_2} + (a_1 - 1) \varphi_{1h}^{-1 - k} \frac{d\varphi_{1h}}{da_2} = 0. \quad (29)$$

Substituting (28) into (29) and rearranging terms yields

$$\left[ \frac{1}{b^{\sigma - 1} - 1} + \zeta_{1h}^{1 - \frac{1}{\sigma}} \right] \frac{d\varphi_{1h}}{da_2} = \frac{\tau^{1 - \sigma} \varphi_{1h}^{\sigma}}{(a_1 - 1) \varphi_{2l}^{\sigma}} \left[ 1 - \zeta_{1x}^{1 - \frac{1}{\sigma}} \right] \frac{d\varphi_{2l}}{da_2},$$

where $\zeta_{1h}$ is the ex-post fraction of firms that adopt advanced technology, $\zeta_{ix}$ is the ex-post fraction of firms that export, and $\beta = k/(\sigma - 1) > 1$. Given that $0 < \zeta_{1x} < 1$ and $0 < \zeta_{1h} < 1$, the sign of the bracket on the RHS is always positive. Since $d\varphi_{2l}/da_2 < 0$, it follows that $d\varphi_{1h}/da_2 < 0$.

I will follow similar steps to prove $d\varphi_{2h}/da_2 > 0$. Consider now equation (12) with $(i, j) = (2, 1)$. Differentiating this equation with respect to $a_2$ yields

$$\frac{d\varphi_{11}}{da_2} = \left( \frac{a_2 - 1}{b^{\sigma - 1} - 1} \right) \left( \frac{\varphi_{2l}}{\varphi_{2h}} \right)^\sigma \frac{d\varphi_{2h}}{da_2} - \tau^{1 - \sigma} \left( \frac{\varphi_{2l}}{\varphi_{1l}} \right)^\sigma \frac{d\varphi_{1l}}{da_2} - \frac{\varphi_{2l} \varphi_{2h}^{1 - \sigma}}{(\sigma - 1)(b^{\sigma - 1} - 1)}. \quad (30)$$

Setting $i = 2$ and differentiating the equilibrium condition (26) with respect to $a_2$ yields

$$\varphi_{2l}^{-1 - k} \frac{d\varphi_{2l}}{da_2} + (f_s/f) A^{-k} \varphi_{1l}^{-1 - k} \frac{d\varphi_{1l}}{da_2} + (a_2 - 1) \varphi_{2h}^{-1 - k} \frac{d\varphi_{2h}}{da_2} - \frac{\varphi_{2h}^{-k}}{k} = 0. \quad (31)$$

Substituting (30) into (31) and rearranging terms yields

$$\left[ \frac{1}{b^{\sigma - 1} - 1} + \zeta_{2h}^{1 - \frac{1}{\sigma}} \right] \frac{d\varphi_{2h}}{da_2} = \frac{\tau^{1 - \sigma} \varphi_{2h}^{\sigma}}{(a_2 - 1) \varphi_{1l}^{\sigma}} \left[ 1 - \zeta_{2x}^{1 - \frac{1}{\sigma}} \right] \frac{d\varphi_{1l}}{da_2} + \frac{\varphi_{2h}}{(a_2 - 1)k} \left[ \frac{\beta}{b^{\sigma - 1} - 1} + \zeta_{2h}^{1 - \frac{1}{\sigma}} \right].$$

Given that $0 < \zeta_{2x} < 1$, the sign of the first bracket on the RHS is always positive. This, combined with $d\varphi_{1l}/da_2 > 0$, ensures that $d\varphi_{2h}/da_2 > 0$. \(\square\)

E. Proof of Proposition 5

Differentiating (12) with respect to $\tau$ yields

$$\frac{d\varphi_{1l}}{d\tau} = \frac{(b^{\sigma - 1} - 1) \varphi_{1l}^{\sigma}}{a_1 - 1} \left[ \varphi_{1l}^{-\sigma} \frac{d\varphi_{1l}}{d\tau} + \tau^{1 - \sigma} \varphi_{2l}^{-\sigma} \frac{d\varphi_{2l}}{d\tau} + \tau^{1 - \sigma} \varphi_{2l}^{-\sigma} \right]. \quad (32)$$
Differentiating the equilibrium conditions (26) with respect to $\tau$ and using (32) together with $\varphi_{ixl} = A\varphi_{jl}$ yields

$$
\frac{d\varphi_{il}}{d\tau} + \tau B_i \left[ \frac{\varphi_{il}}{\tau \varphi_{jl}} \right]^\sigma \frac{d\varphi_{jl}}{d\tau} + B_i \varphi_{jl} \left[ \frac{\varphi_{il}}{\tau \varphi_{jl}} \right]^\sigma = 0,
$$

(33)

where $B_i$ is defined as

$$
B_i = \frac{\varphi_{ixl}^{\sigma-k-1} + (b^{\sigma-1} - 1) \varphi_{ih}^{\sigma-k-1}}{\varphi_{il}^{\sigma-k} + (b^{\sigma-1} - 1) \varphi_{ih}^{\sigma-k}}.
$$

(34)

Since $\varphi_{ixl} > \varphi_{il}$ and $k > \sigma - 1 > 0$, it immediately follows that $B_i < 1$. Setting $i = 1, 2$ (and hence, $j = 2, 1$), (33) yields a system of two equations, which yields

$$
\frac{d\varphi_{1l}}{d\tau} = \left[ \frac{B_2 - (\tau \varphi_{1l}/\varphi_{2l})^{\sigma-1}}{1 - B_1 B_2 \tau^{2(1-\sigma)}} \right] B_1 \varphi_{1l} \tau^{1-2\sigma},
$$

(35)

$$
\frac{d\varphi_{2l}}{d\tau} = \left[ \frac{B_1 - (\tau \varphi_{2l}/\varphi_{1l})^{\sigma-1}}{1 - B_1 B_2 \tau^{2(1-\sigma)}} \right] B_2 \varphi_{2l} \tau^{1-2\sigma}.
$$

(36)

Since $B_i < 1$ and $\tau^{2(1-\sigma)} < 1$, it follows that $1 - B_1 B_2 \tau^{2(1-\sigma)} > 0$. The numerator of the first expression in the bracket on the RHS of (35) is negative, since $\varphi_{1l} > \varphi_{2l}$ and $B_2 < 1$. Thus, $d\varphi_{1l}/d\tau < 0$. However, since $B_1 < 1$ and $\varphi_{2l} < \varphi_{1l}$, the sign of (36) is ambiguous, i.e., $d\varphi_{2l}/d\tau \gtrless 0$.

Differentiating $\varphi_{1xl} = A\varphi_{2l}$ with respect to $\tau$ and using (36) yields

$$
\frac{d\varphi_{1xl}}{d\tau} = \frac{\varphi_{1xl}}{\tau} \left[ \frac{1 - B_2 (\varphi_{2l}/\tau \varphi_{1l})^{\sigma-1}}{1 - B_1 B_2 \tau^{2(1-\sigma)}} \right] > 0,
$$

where the inequality follows from $B_2 < 1$ and $\varphi_{2l} < \varphi_{1l}$. Similarly, differentiating $\varphi_{2xl} = A\varphi_{1l}$ with respect to $\tau$ and using (35) yields

$$
\frac{d\varphi_{2xl}}{d\tau} = \frac{\varphi_{2xl}}{\tau} \left[ \frac{1 - B_1 (\varphi_{1l}/\tau \varphi_{2l})^{\sigma-1}}{1 - B_1 B_2 \tau^{2(1-\sigma)}} \right] \gtrless 0.
$$

Finally, substituting (35) and (36) into (32) and rearranging the terms yields

$$
\frac{\tau}{\varphi_{1l}} \frac{d\varphi_{1l}}{d\tau} = \left[ \frac{b^{\sigma-1} - 1}{a_1 - 1} \right] \left[ \frac{\varphi_{1hl}}{\tau \varphi_{2l}} \right]^{\sigma-1} \left[ \frac{1 - B_2 (\varphi_{2l}/\tau \varphi_{1l})^{\sigma-1}}{1 - B_1 B_2 \tau^{2(1-\sigma)}} \right] [1 - B_1] > 0,
$$

$$
\frac{\tau}{\varphi_{2l}} \frac{d\varphi_{2l}}{d\tau} = \left[ \frac{b^{\sigma-1} - 1}{a_2 - 1} \right] \left[ \frac{\varphi_{2hl}}{\tau \varphi_{1l}} \right]^{\sigma-1} \left[ \frac{1 - B_1 (\varphi_{1l}/\tau \varphi_{2l})^{\sigma-1}}{1 - B_1 B_2 \tau^{2(1-\sigma)}} \right] [1 - B_2] \gtrless 0,
$$

where the inequalities follow from $B_i < 1$ and $\varphi_{2l} < \varphi_{1l}$. 29
References


