Persistence of Shocks, Employment and Wages

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Abstract

This paper develops and estimates a structural model of the firm that combines both employment and wage decisions and distinguishes between permanent and transitory shocks to profitability. Workers are paid a share of the value of the marginal worker as in Stole and Zwiebel (1996) and it is costly to adjust the number of workers. The model predicts that transitory shocks have a strong impact on wages and little effect on employment while permanent shocks have a strong effect on employment and little effect on wages. Structural estimation of the model on a panel of French firms shows that it is consistent with the data. Only a small amount of adjustment costs are needed to reproduce observed job reallocation and inaction rates. Permanent shocks are the most important source of output fluctuations. Ignoring wages flexibility leads to substantial bias, in particular in the estimation of the curvature of the profit function.

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1 Introduction

Data on individual firms reveals an enormous amount of heterogeneity in firm-level productivity and the importance of idiosyncratic shocks. Davis and Haltiwanger (1999) document that over 10% of existing jobs are destroyed each year, and approximately the same amount are created within the same period. More recently, Hsieh and Klenow (2009) find that equalizing the marginal productivity of labor and capital across plants would boost aggregate total factor productivity by around 30–40% in the U.S. manufacturing sector and even more in China or India. These results suggest we should study in more detail, the impediments to reallocation of resources from low to high productivity firms. A natural candidate to explain the absence of factor reallocation is adjustment costs. These costs may be technological, e.g. reduced efficiency during the period of adjustment, or they may be institutional, such as employment protection legislation.

While there is a substantial literature studying labor adjustment costs, most of this literature assumes that wages are not linked to idiosyncratic shocks. This is a critical assumption, since as noted by Bertola and Rogerson (1997), wage flexibility can alter the incentives to adjust the workforce. For example, following a negative shock, a firm may not have to reduce its workforce if wages go down sufficiently.

Also, firms face both permanent and transitory shocks to profitability and may well adopt different strategies depending on the persistence of the shocks. The persistence and variance of shocks are crucial parameters that determine the benefits of reallocating resources across firms and there is little agreement on their values (See Gourio (2008) and Midrigan and Xu (2009)).

This paper develops and estimates a structural model of the firm that combines both employment and wage decisions and distinguishes between permanent and transitory shocks to business conditions. Specifically, I show that the combined assumptions of (1) decreasing return to labor, (2) Nash bargaining with multiple workers as in Stole and Zwiebel (1996) and, (3) costly employment adjustment, imply that transitory shocks have a strong impact on wages and little effect on employment while permanent shocks have a strong effect on employment and little effect on wages.

The firm produces with decreasing returns to labor and is subject to transitory and permanent shocks to its profitability. A transitory shock only changes today’s profits while a permanent shock changes both today’s profits and expected future profits. The wage is negotiated every year and workers are paid a share of the value of marginal worker as in Stole and Zwiebel (1996). Finally, adjusting the level of employment is costly. The intuitive mechanism at work is as follows: if there is a shock that raises the marginal productivity of labor, and this shock is expected to last a long time, then the firm pays the cost of hiring additional workers. Since the marginal product of labor is decreasing, this offsets the shock, so that the marginal worker is not much more valuable than before. Therefore the wage does not rise much, but there is a substantial rise in employment. In the case of a transitory shock, it is not worthwhile to add more workers, because it will be costly.

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1See Hamermesh (1993) and Bond and Reenen (2006) for exhaustive surveys of the literature.
to decrease employment after the shock expires. In this case the marginal worker is more valuable this period, so the wage rises. There is not much change in employment in this case.

Each of the above three assumptions is central for the results to hold. First, if the return to labor is constant instead of decreasing, the model is similar to a search model with a linear technology and Nash bargaining. Adding a worker to the firm has no impact on the productivity of the other workers. As result, persistent and transitory shocks have the same impact on wages. Second, if wages are set competitively instead of bargained over the marginal surplus, the model is similar to the model of Bentolila and Bertola (1990) and to the model of Hopenhayn and Rogerson (1993) where shocks to idiosyncratic firms productivity have no impact on wages unless the shocks are correlated across firms. Third, in a frictionless labor market, employment fully adjusts to shocks. Then, labor productivity and wages are constant. Idiosyncratic shocks have no impact on wages and affect employment independently of their persistence.

I use a panel of French firms to test the model’s predictions. Using a simple econometric specification, I confirm the key predictions of the model: transitory shocks to output have a strong effect on wages while permanent shocks have a very small effect on wages. Also, output and wages dynamics can be adequately represented by the sum of a permanent component and a serially uncorrelated component that represents both transitory shocks and measurement errors. This is not the case for employment dynamics whose patterns are consistent with the presence of adjustment costs. Permanent shocks explain the majority of variation in output growth, explaining 60% of the variance in output growth. In contrast, transitory shocks and measurement errors are the main source of variation in wages growth, explaining 85% of wages growth.

To assess the model quantitatively, I estimate the structural parameters - labor adjustments costs, worker bargaining power and the sources of dispersion of the observed variables - by simulated method of moments. The estimation results indicate that the model is remarkably consistent with the data. Relatively modest adjustment costs (less than a month of wages) can reproduce the data well. This conflicts with the perceived sclerosis of the French labor market. I estimate a low curvature of the profit function of about 1/3. I also show that ignoring wage flexibility would lead to substantial bias in estimating this curvature: the curvature would be estimated to be equal to more than 1/2. I estimate that each worker receives 38% of the marginal surplus and that the value of home production is slightly lower than the minimum wage. The variance of the permanent shock to profitability is 50% higher than the variance of the transitory shock. Permanent shocks explain a larger fraction of the variance of output growth than the fraction they explain of profitability growth. This is because employment fluctuations amplify permanent shocks more than transitory shocks. The variance of measurement errors in employment, wages and output are all significant as has usually been found in other studies that have used different methodologies. The variance of measurement error in wages is very high. This may be explained by the fact that variations in labor

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2See Mortensen and Pissarides (1999) or Rogerson et al. (2005) for surveys.
productivity are the only source of wages fluctuations in the model.

**Connections with Existing Literature**  The wage setting mechanism used in this paper is borrowed from Stole and Zwiebel (1996) and Smith (1999). They generalize the Nash bargaining solution to a setting with downward-sloping labor demand. This has been used in a similar context by Bertola and Caballero (1994), Acemoglu and Hawkins (2010), Elsby and Michaels (2008) and Fujita and Nakajima (2009). These papers use a framework similar to the one used in this paper, and focus on incorporating firm size into search models, the effect of aggregate shocks on wages and the unemployment-volatility puzzle. I focus on the impact of idiosyncratic shocks of different persistence and I estimate the model, while they use calibration.

Adjustment costs are used in many fields of economics to explain a wide range of facts. In these models, the calibration procedure uses some measure of dismissal costs to assign values to the adjustment costs parameters. However, this practice is not entirely satisfactory because adjustment costs have an implicit component that is intrinsically difficult to measure (Hamermesh and Pfann (1996)). In addition, there are many regulations that can hardly be summarized by dismissal costs. Indeed the estimation results point out that relatively modest adjustment costs (less than a month of wages) can reproduce the data well. This conflicts with the perceived sclerosis of the French labor market.

The estimation of a structural model of labor demand dynamics links with the work of Cooper et al. (2005), Rota (2004), Aguirregabiria and Alonso-Borrego (2009) and Trapeznikova (2009). Rota (2004) estimates a labor demand model with fixed costs using Hotz and Miller’s (1993) estimator. Aguirregabiria and Alonso-Borrego (2009) distinguish between fixed-term contracts and indefinite-term contract and they evaluate the effect of reforms to fixed-term contracts in Spain. Cooper et al. (2005) and Trapeznikova (2009) allow firms to choose both employment and hours. Despite the strong protection embodied in the permanent contracts that prevail in France, I estimate adjustment costs to be relatively modest. This is consistent with the idea that firms can destroy jobs at relatively low cost by not-renewing short-term contracts. In the dataset used in the current paper, variations in hours do not seem to drive the relation between wages and shocks to profitability.

Most estimated models of labor demand have so far neglected the distinction between less and more persistent variations in firm productivity, and it is customary to assume that firm productivity follows a stationary autoregressive process of order one. Despite the presence of some descriptive studies, with the exception of Gourio (2008) who focuses on investment, there are no other estimated structural models. This contrasts with the various dynamic models for individual workers

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3A non-exhaustive list includes the behavior of gross job flows and aggregate employment over the business cycle (Campbell and Fisher (2000), Veracierto (2008)), the impact of firing costs on productivity and employment (Bentolila and Bertola (1990), Hopenhayn and Rogerson (1993)), the micro-foundations of aggregate employment adjustment (King and Thomas (2006)).

4Measures of dismissal costs are reported for example in Heckman et al. (2000).

5For example, Franco and Philippon (2007) document the importance of permanent shocks for firm dynamics.
that have been proposed in the earnings dynamics literature.\textsuperscript{6}

Some empirical papers investigate the relationship between wages and profits but none of them consider this question simultaneously with that of employment flexibility. Using matched employer-employee data from Italy, Guiso et al. (2005) find that wages are sensitive to permanent output shocks to but not to transitory output shocks. My paper studies the joint responses of wages and employment while they focus on the wages of workers that stay with the same firm. Also my paper is able to distinguish between profitability shocks and output shocks while they only consider the reduced-form effect of output shocks that combine profitability shocks and the corresponding response of employment. The significant effect of a permanent shock is higher than the effect I find here, but the effect remains small. The non-significant effect of transitory shocks is more puzzling. Understanding the difference between the impact of transitory shocks on worker level wages (estimated in their paper) with the impact on firm level wages (estimated in this paper) is an important question left for future research.

There is an interesting parallel between the results of this paper and Topel (1986). Using a dynamic spatial equilibrium model, he finds that wages respond more to transitory shocks in local labor markets than to permanent shocks. This is because a positive permanent shock generates in-migration to that market which has a negative effect on local wages.

**Organization of the Paper** The remainder of the paper proceeds as follows. Section 2 develops the theoretical framework and examines the impact of transitory and permanent shocks on wages and employment. Section 3 conducts a reduced-form empirical investigation. Section 4 develops the methodology for the structural estimation of the parameters. Section 5 presents the results of the structural estimation. Section 6 concludes.

## 2 A Model of Labor Demand

This section presents a model of the firm that combines both employment and wage decisions and distinguishes between permanent and transitory shocks to business conditions. The firm produces with decreasing returns to labor and is subject to transitory and permanent shocks to its profitability. The wage is negotiated every year and workers are paid a share of the value of marginal worker (following Stole and Zwiebel (1996)). There is a constant cost to creating or destroying jobs. The model shows that transitory and permanent shocks have different effects on wages and employment.

### 2.1 Framework

Time is discrete and runs forever. The risk neutral firm produces an homogeneous good using labor $n$. Every period it faces two independent sources of profitability variations: 1. a transitory

\textsuperscript{6}See McCurdy (1982); Abowd and Card (1989); Meghir and Pistaferri (2004); Blundell et al. (2008).
shock $\epsilon^T \sim \mathcal{N}\left(-\frac{1}{2}\sigma^2 T, \sigma^2 T\right)$ that is serially uncorrelated and I.I.D. across firms and time and 2. a permanent shock $\epsilon^P \sim \mathcal{N}\left(-\frac{1}{2}\sigma^2 P, \sigma^2 P\right)$ that is serially uncorrelated and I.I.D. across firms and time such that the permanent component of profitability $A_t$ evolves over time $t$ as

$$\log A_{t+1} = \log A_t + \epsilon^P_t$$

The revenue function is $(e^{\epsilon^T} A)^{1-\alpha} n^{\alpha}$ where $0 < \alpha < 1$ reflects decreasing returns-to-labor and/or market power. A firm may have a high profitability $e^{\epsilon^T} A$ because it has market-power and/or because of the higher quality of its production. I do not have information on output prices so I cannot disentangle these two effects.\(^7\)

Combined with the wage function $w(A, \epsilon^T, n)$, it gives profits $\pi$ as a function of $A, \epsilon^T$ and $n$:

$$\pi(A, \epsilon^T, n) = e^{\epsilon^T} A^{1-\alpha} n^\alpha - w(A, \epsilon^T, n)n$$

There is a constant cost $c$ that is paid for every job destroyed and similarly a constant cost $c$ for every job created. I consider net rather than gross adjustment costs: adjustment costs are associated with job destruction but not with workers flows. Thus adjustment costs are unrelated to the identities of the workers who fill these positions. This choice is determined by data availability. I do not consider fixed adjustment costs since it creates many technical difficulties. I do not consider convex adjustment costs since they generate smoothing which does not appear in the data. The model also assumes no entry or exit. This is for tractability and because I do not have data on entry/exit.

At the beginning of each period the timing of events is:

- A manager knows his past employment level ($n_t$), current level of profitability ($A_t$) and the transitory shock $\epsilon^T$.
- Given $(A, \epsilon^T, n)$ the manager creates or destroys jobs ($d_t$). This affects production in the current period.
- A firm and its workers bargain over current period wages, $w(A, \epsilon^T, n + d)$.
- Production takes place.

The manager’s problem is to choose a state contingent sequence of employment to maximize the present discounted value of current and future profits, given the previous level of employment, the current state of profitability and the transitory shock. The parameter $\beta$ represents the rate at which the manager discounts profits in future periods, and is in $(0, 1)$. Define the value function at

\(^7\)Formally, the firm has a production function $Bn^a$ with productivity $B$, employment $n$ and $a$ is a parameter. It faces an iso-elastic demand curve with elasticity $\eta$: $CP^{-\eta}$. These can be combined into a revenue function: $BC^{1-\frac{\eta}{\eta}}n^{a(1-\frac{\eta}{\eta})}$. 

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period 0, \( V^*(A_0, \epsilon^T_0, n_{-1}) \) as the present discounted value of current and future profits given initial productivity, \( A_0 \), lagged employment, \( n_{-1} \), and initial transitory shock, \( \epsilon^T_0 \):

\[
V^*(A_0, \epsilon^T_0, n_{-1}) = \sup_{n_t, t \geq 0} E \left\{ \sum_{t=0}^{\infty} \beta^t \left[ (e^T_t A_t)^{1-\alpha} n_t^2 - w_t(A_t, \epsilon^T_t, n_t) - (n_t - n_{t-1})^+ - \overline{\epsilon}(n_t - n_{t-1})^- \right] | A_0, n_{-1} \right\}
\]

where \( x^+ = x \) if \( x \) is positive and zero otherwise and \( x^- = -x \) if \( x \) is negative and zero otherwise.

Under standard conditions, \( V^*(A, \epsilon^T, n) \) is the unique solution to Bellman’s equation:

\[
V(A, \epsilon^T, n) = \max_d \left\{ e^{(1-\alpha)\epsilon^T} A^{1-\alpha} (n + d)^{\alpha} - w(A, \epsilon^T, n + d)(n + d) - c_d^+ - \overline{c}_d^- \right. \\
+ \beta E \left[ V(Ae^{\epsilon^P}, \epsilon^T, n + d) \right] \}
\]

where \( E \) is an expectation over the joint distribution of \( \epsilon^P \) and \( \epsilon^T \).

### 2.2 Employment Policy

Given the wage function, determined later on, and given \( (A, \epsilon^T, n) \), the optimal choice \( d \), of creating and destroying jobs, must satisfy the first order conditions:

\[
\alpha e^{(1-\alpha)\epsilon^T} A^{1-\alpha} (n + d)^{\alpha - 1} - w(A, \epsilon^T, n + d) - w(A, \epsilon^T, n + d)(n + d) - \overline{c} + \beta E(V_n(Ae^{\epsilon^P}, \epsilon^T, n + d)) \leq 0
\]

with equality if \( d > 0 \), and

\[
\alpha e^{(1-\alpha)\epsilon^T} A^{1-\alpha} (n + d)^{\alpha - 1} - w(A, \epsilon^T, n + d) - w(A, \epsilon^T, n + d)(n + d) + \overline{c} + \beta E(V_n(Ae^{\epsilon^P}, \epsilon^T, n + d)) \leq 0
\]

with equality if \( d < 0 \).

The optimal choice for next period employment \( n'(A, \epsilon^T, n) \), given the state \( (A, \epsilon^T, n) \), reads

\[
n'(A, \epsilon^T, n) = \begin{cases} 
\overline{\pi}(A, \epsilon^T) & \text{if } n > \overline{\pi}(A, \epsilon^T) \\
n & \text{if } \overline{\pi}(A, \epsilon^T) < n < \overline{\pi}(A, \epsilon^T) \\
\underline{\pi}(A, \epsilon^T) & \text{if } n < \underline{\pi}(A, \epsilon^T)
\end{cases}
\]
where the two targets function $\pi$ and $\overline{\pi}$ are defined as:

$$
\alpha \left( e^T A \right)^{1-\alpha} \pi(A, e^T) - (\alpha - 1) + \beta E(V_n(Ae^P, e^T, \overline{\pi}(A, e^T)))
$$

$$
= w(A, e^T, \pi(A, e^T)) + w_n(A, e^T, \overline{\pi}(A, e^T))\overline{\pi}(A, e^T) - \pi
$$

Optimality requires the firm to create and destroy jobs as needed to keep the marginal value of labor in the closed interval $[-\overline{\pi}, \overline{\pi}]$. The optimal decision rule consists of two targets that are consistent with the Euler equation. If the level of employment at the beginning of a period lies between the two targets, it is not worth hiring/firing and the firm stays put until the next period. The optimization problem has a sequential nature: 1. choosing a target for employment and 2. whether to hire/fire or stay put. And there is no smoothing: if the manager decides to adjust, he directly jumps to the target without additional smoothing and independently of lagged employment. Figure 1 plots optimal employment given shocks.

![Figure 1: Optimal Decision rule conditional on Shocks](image)

Labor productivity is higher during expansions than contractions. This is because the firm is not creating (destroying) as much jobs as it would in a frictionless labor market. The effect of adjustment costs on labor productivity is ambiguous. Following a positive (negative) shock, the firm hires (fires) relatively less which leads to a higher (lower) productivity of labor during expansion (recession) periods.
2.3 Bargaining and Wages

I adopt the wage bargaining solution of Stole and Zwiebel (1996) and Smith (1999) which generalizes the Nash solution to a setting with downward-sloping labor demand. This approach has been used recently in a similar context by Acemoglu and Hawkins (2010), Elsby and Michaels (2008) and Fujita and Nakajima (2009).

The firm cannot commit to long term contracts and costless renegotiation takes place every period. Workers are homogeneous and the current wage rate is the outcome of a sequence of bilateral negotiations between the firm and its employees, where each employee is regarded as the marginal worker. Wages are then the outcome of a Nash bargaining game over the marginal surplus. Workers and the firm each receive a given fraction, \( \gamma \) and \( 1 - \gamma \), of the marginal surplus.

A job-seeker's threat point is the value achieved during the prospective employment period by disclaiming the current job opportunity and continuing to search, that is, the unemployment value \( U \). Wages are set after employment has been determined. Thus, hiring costs are sunk at the time of wage-setting. The firm's threat point is the value achievable by destroying a job, that is, the cost of destroying a job \(-\bar{r}\). The solution takes the form:

\[
(1 - \gamma) [W(A, \epsilon^T, n) - U] = \gamma [V_n(A, \epsilon^T, n) + \bar{r}]
\]

where \( W(A, \epsilon^T, n) \) is the value of being employed in a firm with current state \((A, \epsilon^T, n)\).\(^8\)

While employed, a worker receives a flow payoff equal to the bargained wage minus taxes \( w(A, \epsilon^T, n)(1 - \tau) \) where \( \tau \in (0, 1) \) is a constant tax rate that captures the difference between labor costs for the firm and the actual wage perceived by the worker. A worker loses her job with some probability \( s \) next period:\(^9\):

\[
W(A, \epsilon^T, n) = (1 - \tau) w(A, \epsilon^T, n) + \beta E \left[ sU + (1 - s)W(A', \epsilon'^T, n') \right]
\]

where the expectation is taken over the distribution of shocks.

An unemployed worker receives flow payoff \( b \), which represents unemployment benefits, the value of leisure and home production. She finds a job with probability \( f \).

\[
U = b + E [(1 - f) U + fW(A, \epsilon^T, n)]
\]

In the Appendix, I show that the solution to Equation 8 is:

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\(^8\)An alternative would be to assume that delaying producing is the outside option of both the employer and worker. When delaying production, workers receive flow payoff \( b \) and the firm forgoes the productivity of the marginal worker. The solution takes the form \( (1 - \gamma) (1 - \tau) w(A, \epsilon^T, n) - b] = \gamma [\pi_n(A, \epsilon^T, n)] \). The formulation complicates the numerical solution of the model because the constraint \((1 - \tau) w(A, \epsilon^T, n) \geq b \) may be binding.

\(^9\)It will turn out to be unnecessary to characterize \( s \) because the value of working in a firm that is downsizing will be equal to the value of unemployment.
\[ w(A, \epsilon^T, n) = \frac{1}{1 - \tau} ((1 - \gamma) b + \bar{\epsilon} \gamma (1 - \beta (1 - f)) + \xi \beta \gamma f) + \frac{\gamma \alpha}{1 - \tau - \gamma(1 - \alpha)} (e^T A)^{1-\alpha} n^{\alpha-1} \]  

(9)

Wages are a function of labor productivity. Since \( \alpha < 1 \), employment and wages are inversely related conditional on the TFP level. As in Stole and Zwiebel (1996), firms have an incentive to “over-hire”: by increasing employment, firms can strategically push bargained wages down.

The relationship between wages and the curvature of the profit function \( \alpha \) is ambiguous. The less concave is demand and the production technology, the lower the rate of diminishing returns as workers are added and, as a result, these workers are able to negotiate a larger share of the surplus. On the other hand, the less concave is demand and the production technology, the larger the size of the firm conditional on \( A \) and the lower the productivity of labor.

2.4 The Differential Impact of Transitory and Permanent Shocks

In Equation 9, permanent and transitory shocks enter symmetrically. Yet, they have a different impact on wages because of the behavior of employment. In this section, I explore the mechanisms underlining the differential impact of transitory and permanent shocks on wages.

A transitory shock only increases today’s profits while a permanent shock increases both today’s profits and future profits. This can be seen formally by looking at the first-order conditions for a firm creating jobs:

\[ \alpha e^{(1-\alpha)T} A^{1-\alpha}(n + d)^{\alpha-1} - w(A, \epsilon^T, n + d) - w_n(A, \epsilon^T, n + d) (n + d) + \beta E(V_n(Ae^{\epsilon^P}, e^{\epsilon^T}, n + d)) = \xi \]  

(10)

The transitory shock \( \epsilon^T \) only appears on the derivative of the one-period profit function. Conversely, the permanent level of profitability \( A \) appears in the derivative of both the one-period profit function and the value function.

Figure 2 plots the left hand side of Equation 10 for different values of the transitory shock \( \epsilon^T \) and the permanent shock \( \epsilon^P \). The left-hand side of Equation 10 is labelled the marginal value of labor: the increase in the value of the firm following a marginal increase in employment. When varying the transitory (permanent) shock, the permanent (transitory) shock is set to zero.

When shocks are positive, the marginal value of employment is always much higher for a permanent shock than for a transitory shock of the same magnitude. And when shocks are negative, the opposite result holds. Hence the benefits of creating or destroying jobs following a transitory shock are on average smaller than when the shock is permanent.

If there is a positive transitory shock, the benefits of adding workers is relatively low because it
is costly to create jobs and it will be costly to destroy jobs after the shock expires. Then, the firm decides to create or destroy few jobs. The marginal worker is more valuable this period and so the wage rises.

If there is a positive permanent shock, the firm pays the cost to create additional jobs. Since there are decreasing returns to labor, this offsets the shock, so that the marginal worker is not much more valuable than before the shock. Thus the wage does not rise much but there is a substantial rise in employment.

The argument follows the same logic for negative shocks. Note that labor productivity is equal to output divided by employment such that the variations in labor productivity are smaller for a permanent shock compared to a transitory shock of the same magnitude.

There are three central assumptions for the results to hold: (1) decreasing return to labor, (2) Nash bargaining with multiple workers as in Stole and Zwiebel (1996) and, (3) costly employment adjustment. First, if the return to labor is constant instead of decreasing, the model is similar to a search model with a linear technology and Nash bargaining.\textsuperscript{10} Adding a worker to the firm has no impact on the productivity of the other workers. As result, persistent and transitory shocks have the same impact on wages. Second, if wages are set competitively instead of bargained over the marginal surplus, the model is similar to the model of Bentolila and Bertola (1990) and to the model of Hopenhayn and Rogerson (1993) where shocks to idiosyncratic firms productivity have no impact on wages unless the shocks are correlated across firms. Third, in a frictionless labor market, employment fully adjusts to shocks. Then, labor productivity and wages are constant. Idiosyncratic

\textsuperscript{10}See Mortensen and Pissarides (1999) or Rogerson et al. (2005) for surveys.
shocks have no impact on wages and affect employment independently of their persistence.

To illustrate the last point, the red line in Figure 2 shows the marginal value of labor without adjustment costs. It does not depend on the persistence of the shocks. This is because in Equation 10, the term that differs between transitory and permanent shock $E(V_a(Ae^{\epsilon'}, e^{I'}, n + d))$ is always set to zero in the absence of adjustment costs. Employment reacts similarly to both kind of shocks. Because employment fully adjusts to shocks, labor productivity is constant. Wages, being a function of labor productivity, are also constant. This shows that some form of adjustment costs are necessary for permanent and transitory shocks to have a differential effect on wages and employment.

2.5 Comparative Statics

I simulate the model using reasonable parameters values. Specifically, I examine how labor productivity, wages, the reallocation rate and the inaction rate, defined as the fraction employment growth lower than 2.5% in absolute value, change when adjustment costs and worker bargaining power change. Figure 3 reports the results. The blue line represents the model’s outcomes with $c \in [0, 0.02]$ and the red line represents the model’s outcomes with $\gamma \in [0, 0.5]$. The horizontal green line represents the model’s outcomes using the structural parameter estimates.

Adjustment costs slow down the process of job reallocation: the reallocation rate increases from 0.1041 to 0.2294 and the inaction rate decreases from 37.80% to 2.81% when I remove adjustment costs. Adjustment costs have a small and positive impact on wages and labor productivity.

The Nash bargaining parameter $\gamma$ is an indicator of wage flexibility. At the limit when $\gamma = 0$, wages are constant and equal to the value of home production $b$. A higher $\gamma$ means that wages are more dependent on business conditions. Wages decrease more upon realization of a negative labor-demand shock, and increase more upon realization of a positive shock. When $\gamma = 0$, the reallocation rate increases from 0.1041 to 0.1282. Intuitively, following a positive shock, wages remain constant while they would have risen if workers had some bargaining power. Firms then have a greater incentive to adjust employment.

Surprisingly, labor productivity is a non-monotonic function of worker bargaining power. As $\gamma$ increases, there are two effects on labor productivity. First, for any level of labor productivity wages are higher which reduces profits and labor demand. Second, the firm has an incentive to “over-employ”: by hiring more workers the firm can reduce worker productivity and wages. If wages are not related to firm productivity, this incentive disappears and firms set the marginal product of labor equal to the value of home production. As $\gamma$ increases, the incentive to “over-employ” increases and depresses labor productivity. From the model simulation, the second effect dominates for low values of $\gamma$ and the first effect dominates for higher values of $\gamma$.

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11I do this before detailing the estimation of the parameter values which are presented in Section 5. Note that I ignore the effect of variations in $c$ or $\gamma$ on the probability of finding a job $f$. It explains about 1% of wage levels and does not affect wages dynamics. It is thus unlikely to change the results.
Figure 3: Comparative Statics
Wages are an increasing function of $\gamma$ and the relation is convex because of the behavior of labor productivity explained in the previous paragraph.

3 Data and Descriptive Statistics

This section describes the data sources. Then, I present some justifications for the assumption that profitability in the model is the sum of a random walk and a I.I.D. process. Finally, I show that the data are consistent with the model’s key prediction: transitory shocks to value-added have a strong effect on wages, but permanent shocks have only a small effect on wages.

3.1 Data Source

The data used is the “Real Normal Profits” (“Benefices Reels et Normaux” or BRN). It consists of declarations completed annually by firms with a turnover of more than 3.5 million francs (1992 threshold) liable for income tax in respect of “Industrial and Commercial Profits” (“Benefices Industriels et Commerciaux” or BIC). It corresponds to the profits declared by firms whose commercial, industrial or craft-work activity is carried out for lucrative purposes (60% of the firms, 94% of the turnover). The data cover the period 1994-2000.

The employment variable is defined as full-time equivalent employment, that is total hours worked divided by average annual hours worked in full-time jobs. The observed variable includes some part-time and temporary workers and rounding can be expected. Every year, a firm reports an average of the number of employees at the end of each quarter in that year. Output is measured by value-added which is defined in the dataset as the difference between production and intermediate consumption net of all variations in stocks. Average labor compensation per firm is calculated as the ratio of the total real wage bill and the number of full-time equivalent employees over the year. I drop firms with less than 100 employees. This is to avoid the effect of size-dependent policies which is especially important when the size of 10 and 50 employees are crossed. The empirical strategy cannot accommodate these effects and they are investigated in a separate paper (Gourio and Roys (2011)). I trimmed all variables at the .99-quantile and the .01-quantile. I end up with 53,585 firm-year observations, 25,354 in the manufacturing sector, 3,920 in the construction section, 8,470 in the trade sector and 15,841 in the service sector.

3.2 Modelling Shocks: Univariate Series

This section investigates the presence of permanent shocks, transitory shocks and measurement errors in output, employment and wages. It will provide some justification for assuming that

---

12I also estimated a model where wages were computed as the ratio of the total real wage bill and the total number of hours without finding much quantitative differences in the parameters estimates. Variations in hours worked do not seem to drive the results.
profitability is the sum of a permanent component and a transitory component.

Suppose the logarithm of value added, employment or wages, say $\log Y$, can be decomposed into a permanent component $P$ and mean-reverting transitory component $\nu$. The process for each firm $i$ is

$$\log Y_{it} = Z_{it}\varphi_t + P_{it} + \nu_{it}$$  \hspace{1cm} (11)

where $t$ indexes time and $Z$ is a set of observable characteristics. I allow for a calendar year effect $\varphi_t$.

The permanent $P_{it}$ follows a martingale process of the form

$$P_{it} = P_{it-1} + \zeta_{it}$$  \hspace{1cm} (12)

where $\zeta_{it}$ is serially uncorrelated. The transitory component $\nu_{it}$ follows an $MA(q)$ process, where the order $q$ is to be established empirically:

$$\nu_{it} = \sum_{j=0}^{q} \theta_j \epsilon_{it-j}$$  \hspace{1cm} (13)

with $\theta_0 = 1$.

The log of the variable net of predictable individual components $y_{it} = \log Y_{it} - Z_{it}\varphi_t$ is in first differences:

$$\Delta y_{it} = \zeta_{it} + \Delta \nu_{it}$$  \hspace{1cm} (14)

Assume that $\zeta_{it}$ and $\nu_{it}$ are uncorrelated at all leads and lags. Assume the variance of the innovations are constant over time. Then the parameters to estimate are $\sigma^2_{\zeta}, \sigma^2_{\epsilon}, q, \theta_1, \ldots, \theta_q$.

Identification of these parameters is straightforward. If $\nu$ is an $MA(q)$ process, $\text{cov}(\Delta \nu_t, \Delta \nu_{t+s})$ is zero whenever $s \geq q + 1$. Then those covariances identifies $q, \theta_1, \ldots, \theta_q$ and $\sigma^2_{\zeta}$.

The key moment condition that identifies the variance of the permanent shock is:

$$E \left[ \Delta y_t \left( \sum_{j=-(1+q)}^{(1+q)} \Delta y_{t+j} \right) \right] = \sigma^2_{\zeta}$$  \hspace{1cm} (15)

It was derived by Meghir and Pistaferri (2004). This exploits the structure of the $MA$ process to cancel terms out.

To describe the structure of transitory shocks, I estimate the autocovariances of $\Delta y_{it}$ using standard methods (Abowd and Card (1989)). The test statistic equals the squared-autocovariance divided by their respective variance. It is distributed as a Chi-Square distribution, with a degree of freedom equal to the number of time periods available for estimation. Table 1 presents estimates of
the autocovariances up to order three along with the test of zero restrictions for the null hypothesis that \( \text{cov}(\Delta y_t, \Delta \nu_{t+s}) = 0 \) with \( 1 \leq s \leq 3 \).

Table 1 indicates that only first order correlations are present for residual wage and output growth. The statistical implication is that \( q = 0 \). Then for both wage and output the transitory component follows a \( MA(0) \) process: it is i.i.d. Table 1 also indicate that residual employment growth follows a different pattern: autocovariances of employment are significant at all orders. Further they are positive which suggests that employment growth is not well represented by the sum of a permanent component and a \( MA(q) \) process. As will be investigated in the next subsection and in the structural estimation of the model, observed employment growth is consistent with the existence of adjustment costs.

To test for the absence of permanent shocks, the test statistic is equal to the pooled estimate of the variance of the permanent shock divided by its standard error. It is asymptotically (for large \( N \)) distributed as a standard normal. The standard error is computed using the block bootstrap procedure (see Hall and Horrowitz (1996)). In this way I account for serial correlation of arbitrary form, heteroskedasticity, as well as for the use of pre-estimated residuals.

The results are reported in Table 2. For the whole sample, the variance of a permanent output shock is estimated to be 0.0457 (with a bootstrap standard error of 0.0016) for output, and 0.0036 (with a bootstrap standard error of 0.0003) for wages. The hypothesis of no permanent shocks is strongly rejected in both case. In all sectors, permanent shocks are the main source of variations of output, explaining around 60% of the total variance of output growth residuals, while transitory shocks are the main source of variation of wages, explaining around 85% of the total variance of wage growth residuals.

### 3.3 Joint Dynamics of Output and Wages

This section examines a central prediction of the theoretical model: a transitory shock to output has a stronger effect on wages than a permanent shock. Building on the previous section, (unexplained) growth of output is

\[
\Delta y_{it} = \zeta^y_{it} + \Delta \nu^y_{it} + \Delta r^y_{it}
\]

Assume (unexplained) growth of wages is

\[
\Delta w_{it} = \tau \zeta^y_{it} + \phi \Delta \nu^y_{it} + \Delta r^w_{it} + \zeta^w_{it}
\]

where \( \zeta^y_{it} \) is a permanent wage shock independent of output and \( \nu^y_{it} \) is wage measurement error. Permanent productivity shocks \( \zeta^y_{it} \) have a permanent impact on wages with a loading factor of \( \tau \), transitory productivity shocks \( \nu^y_{it} \) have an impact on wages with loading factor of \( \phi \in [0,1] \). This econometric specification is borrowed from Blundell et al. (2008). They analyze the response of
<table>
<thead>
<tr>
<th>Order</th>
<th>Full sample</th>
<th>Manufacturing</th>
<th>Construction</th>
<th>Trade</th>
<th>Services</th>
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<tr>
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<td>0.0007</td>
<td>0.0020</td>
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<td>0.0018</td>
</tr>
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</table>

Table 1: Test of the Hypothesis of Zero Autocovariances
Full Sample Manufacturing Construction Trade Services

Output
Pooled Estimate 0.0457 0.0429 0.0415 0.0457 0.0513
(0.0016) (0.0025) (0.0047) (0.0038) (0.0030)
Fraction of the Variance 60.29% 58.78% 57.36% 61.98% 62.27%

Wages
Pooled Estimate 0.0036 0.0018 0.0020 0.0034 0.0069
(0.0003) (0.0003) (0.0004) (0.0006) (0.0009)
Fraction of the Variance 13.11% 9.72% 13.91% 16.35% 14.44%

Table 2: Variance of Permanent Shocks

consumption to changes in income. It is possible to point identify \( \sigma^2_{\zeta y}, \sigma^2_{\zeta w} \) and \( \tau \):\(^{13}\)

\[
\sigma^2_{\zeta y} = E[\Delta y_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})]
\]

\[
\tau = \frac{E[\Delta w_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})]}{E[\Delta y_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})]}
\]

\[
\sigma^2_{\zeta w} = E[\Delta w_t(\Delta w_{t-1} + \Delta w_t + \Delta w_{t+1})] - \tau^2 \sigma^2_{\zeta y}
\]

It is not possible to separately identify measurement error in output from transitory shocks to output. Only the sum is identified:

\[
\sigma^2_{\nu y} + \sigma^2_{r y} = -E(\Delta y_t \Delta y_{t+1})
\]

\( \phi \) is not point-identified. However, a lower bound is given by:

\[
\phi \geq \frac{E(\Delta w_t \Delta y_{t+1})}{E(\Delta y_t \Delta y_{t+1})} = \frac{\phi \sigma^2_{\nu y}}{\sigma^2_{\nu y} + \sigma^2_{r y}}
\]

Table 3 displays the pooled estimates. It also displays the variance decomposition of wage growth. The estimates of \( \tau \) are not statistically significant: the hypothesis of insensitivity of wages to permanent output shocks cannot be rejected. And the effect is economically small: a 10% permanent output shock induces a 1.68% permanent change in wages over the whole sample.

Conversely, wages respond strongly to a transitory shock to value added. Because of the presence of measurement errors, I can only provide a lower bound at this stage. But the loading factor is larger than 58% over the whole sample. There are differences in the transmission of transitory

\(^{13}\)Theoretical Moments are derived in Appendix A.
shocks by sector: the transmission is the highest in the services sector (92.07%) and the lowest in the construction sector (38.35%).

3.4 Labor Adjustment

Figure 4 shows the range of variation in production and employment. Throughout the period, on average, the rate of change in employment was zero or close to zero for about 15% of the firms. Hence employment growth rates display high spikes around zero, compared to the smooth patterns of sales variations observed.

Table 4 reports the distribution of net employment growth at the firm level. Two facts stand out from the distribution of job reallocation: 1. there is a significant amount of relatively small net employment adjustment: 26% of firms have a variation of employment of less than 2.5%. 2. these small adjustments are complemented by significant bursts of job creation and destruction: around 40% of firms either contract or expand employment by more than 10% in a given year.

These results give some support for the assumption of linear labor adjustment costs. While this is a simplification, it can reproduce observed patterns of employment at the firm level.
Figure 4: Distribution of Output and Employment Growth Rates
4 Structural Estimation: Framework

Estimating the parameters using Euler equation techniques is not adequate here because corner solutions are relatively important: around 15% of the observations correspond to zero adjustment. This means that selection bias is likely to be severe. Pakes (1994) proposed estimating the structural parameters using some modified Euler equations which take into account the number of periods between two consecutive interior solutions. But there remain several limitations to using Euler equations\textsuperscript{14}. The most important problems are the short time dimension (\(T = 7\)) and the substantial measurement error in observed variables.

Exploiting the discrete decision (whether to adjust the employment level or not) would be another possibility. For example, Rota (2004) estimates a labor demand model with fixed costs using Hotz and Miller’s (1993) estimator. The former is suited to discrete decision processes (the decision to adjust or not). The continuous decision (how much to invest) is estimated non-parametrically. There are two important limitations to this approach. First, identification of the structural parameters would rely entirely on the discrete decision to adjust employment. This problem is further compounded by the fact that important measurement errors can be expected in recorded employment. Second firm profitability \(A_t\) is not observed. This is a problem since it is the main determinant of both employment and wages. A first stage would be needed to recover \(A_t\) by first estimating a production function. And again it would be difficult to separate true variations in profitability from measurement errors.

As an alternative, I use simulated method of moments.

4.1 Measurement Errors

The employment variable is defined as full-time equivalent employment, that is total hours worked divided by average annual hours worked in full-time jobs. The observed variable includes some part-time and temporary workers and heavy rounding can be expected. It is an average of the number of employees in the firm that ignores the flows during the year. Output is measured by value-added which is defined in the dataset as the difference between production and intermediate consumption net of all variations in stocks. From the literature on structural estimates of productivity (see for example the recent survey of Ackerberg et al. (2007)), confusing true variation in productivity and measurement error in output would over estimate the degree of profitability dispersion over both time and firms.

I explicitly introduce measurement error into the simulated moments to mimic the bias these impute into the actual data moments. I incorporate measurement error in employment, output and wages into the simulation by multiplying these variables by, respectively, \(mn_{it}, mo_{it}, mw_{it}\). I assume measurement errors are i.i.d across firms and time and follow a log-normal distribution with mean

\textsuperscript{14}See for example the discussion in Alan and Browning (2009).
zero and standard deviations given by, respectively, $\sigma_{MRN}, \sigma_{MRO}, \sigma_{MRW}$.

### 4.2 Identification

I show in this section how the parameters of the model affect features of employment, output and wages at the firm level leaving the exact description of the estimator to the next section. A simulated method of moments estimator needs observed moments that are a well-behaved function of the structural parameters. It considers a set of auxiliary parameters that are statistics of the data. It defines a binding function $\Psi(\theta)$ that maps the structural parameters $\theta$ to the auxiliary parameters. The model is identified if the Jacobian of the binding function $\nabla \Psi(\theta)$ is of full-rank. The problem is not trivial because $\Psi$ does not admit a closed form.

Average labor productivity and wages identify $\alpha$ in the production function and the value of home-production $b$. To understand the mechanism at work, consider a firm facing no adjustment costs $c = 0$ and with a constant wage $\gamma = 0$. Its decision consists of equalizing at every period the marginal productivity of labor with wages $w = b$. Simple calculations show that labor productivity is then:

$$p = \frac{O}{n} = \frac{A^{1-\alpha}n^\alpha}{n} = \frac{b}{\alpha}$$

Obviously, a high wage rate reduces employment and increases labor productivity. A high curvature of the profit function pushes firms to expand which reduces labor productivity. In a dynamic framework, this is still true but labor productivity has a non-trivial dynamics due to adjustment costs.

Workers bargaining power $\gamma$ affects the correlation between output growth and wages growth. It has a negative impact on the reallocation rate: when wages are more flexible, firms hire (fire) less when productivity goes up (down) because wages simultaneously go up (down).

To separately identify adjustment costs $c$ from the variance of idiosyncratic shocks, consider the reallocation rate $E(\Delta \log n)$, which measures the number of jobs created or destroyed, and the inaction rate $E(I \{\log n = \log n_{-1}\})$, which counts periods where employment stays constant. Adjustment costs decrease the propensity to create and destroy jobs: the reallocation rate is low when adjustment costs are high. Similarly, adjustment costs increase inaction. The volatility of shocks has the opposite effect on these two moments: a highly volatile environment triggers more job reallocation and less inaction. Also the variance of labor productivity $E(\Delta \log p)^2$ increases when either $c$ or $\sigma$ increase.

To separately identify the variance of permanent and transitory shocks, I use the statistic $E \left( \sum_{j=-1}^{1} y_{it+j} \right)$ where $x$ and $y$ are employment growth, wages growth or output growth as in Section 3.3. It captures the correlation between $x$ and $y$ that comes from permanent shocks and not transitory shocks. $E(x_{it}y_{it-1})$ captures the correlation between $x$ and $y$ that comes from transitory shocks and not permanent shocks. To separately identify measurement error in output $\sigma_{MRO}$ from true transitory variation in profitability $\sigma_{\epsilon T}$, I use the fact that measurement errors in output
do not affect the reallocation rate nor the correlation between wages and labor productivity while transitory shocks to profitability do. The approach is similar for measurement errors in employment and wages.

4.3 Simulated Method of Moments

I describe how I recover the vector of structural parameters \( \theta = (\alpha, b, c, \gamma, \sigma_{eP}, \sigma_{eT}, \sigma_{MRN}, \sigma_{MRO}, \sigma_{MRW}) \).\(^\text{15}\)

I selected a set of sample auxiliary parameters \( \hat{\Psi} \) that are statistics of the data. For an arbitrary value of \( \theta \), I use the structural model to generate \( S \) simulated datasets and compute simulated auxiliary parameters \( \Psi^s(\theta) \). The parameters estimates \( \hat{\theta} \) are then derived by searching over the parameter space to find the parameter vector that minimizes the criterion function:

\[
\hat{\theta} = \arg \min_{\theta \in \Theta} \left( \hat{\Psi} - \frac{1}{S} \Psi^s(\theta) \right)' W \left( \hat{\Psi} - \frac{1}{S} \Psi^s(\theta) \right)
\]

where \( W \) is a weighting matrix. I use the identity matrix as using the optimal weighting matrix is fraught with a well-known small sample bias problem (See Altonji and Segal (1996)). This procedure will generate a consistent estimate of \( \theta \). The minimization is performed using numerical techniques. The variance of the estimator is estimated by bootstrap. I use 1000 bootstrap repetitions.

4.4 Numerical Solution

The full details of the numerical approach are in the Appendix. I use a collocation method and B-splines to approximate the unknown functions.\(^\text{16}\) Approximating the derivative of the value function \( V_n(A, n + d, \epsilon_T) \) is challenging: it has two kinks at the unknown thresholds. However to find the optimal policy and simulate the model, it is sufficient to know the expected values \( E \left[ V_n(Ae^{\epsilon_P}, n, \epsilon_T) \right] \) as a function of \((A, n)\) and where the expectation is taken with respect to the joint distribution of permanent and transitory shocks. This expectation is a smooth function of \((A, n)\): the convolution of any integrable function and a normal density is analytic (see Lehmann (1959)).

Figure 5 plots \( E \left[ V_n(Ae^{\epsilon_P}, n + d, \epsilon_T) \right] \) as a function of \( n \) for a fixed \( A \). For low (high) values of employment, the firm creates (destroys) jobs and the marginal value of the firm is equal to the costs of creating (destroying) jobs. For intermediate values, the firm is inactive and the marginal value of employment is a decreasing function of employment. The figure displays clearly that the kinks at the threshold values are smoothed out by the expectation operator.

\(^{15}\)I assume symmetric adjustment costs: \( c = \xi = \zeta \) because the identification of asymmetries is difficult.

\(^{16}\)See Judd (1998).
Figure 5: $E \left[ V_n(Ae^{\epsilon n}, n + d, \epsilon T) \right]$ as a function of $n$ and for a fixed $A$

5 Structural Estimation: Results

Some parameters are not estimated and are set off the model. The discount factor $\beta$ is set to 0.95 so that the annual real interest rate is 5 percent. The probability of finding a job $f$ is set to 0.35 which is an average of the annual transition from unemployment to employment observed over the period of observations (the data are taken from the “Enquete Emploi”, a survey of about 1/300th of the French population, conducted annually by INSEE, the French National Statistical Institute). The ratio between the labor costs for the employer and the remuneration perceived by the worker $1 - \tau$ is set to 0.62 (this number is an average in France over the period of observations).

5.1 Structural Parameters

Table 5 reports the results of the estimation of the full model and a model with $\gamma = 0$.

The estimated value of $\alpha$ is 0.3214. This is lower than estimates typically obtained from production function estimation. The reason for this is that the curvature of the production function $\alpha$ is not the labor share of value added. This is because the labor share depends on workers bargaining power $\gamma$. To understand the intuition consider a firm that faces no adjustment costs and no taxes: $c = \tau = 0$. There the labor share is

$$\frac{w_n}{\alpha} = \frac{\alpha}{1 - \gamma(1 - \alpha)}$$

In the full model, this above expression would not hold, but the labor share would still be increasing in both $\alpha$ and $\gamma$. The estimation of the constrained model gives a more usual value of the production function’s curvature: $\hat{\alpha} = 0.5525$. Because wages are constant in the constrained model,
it corresponds to the labor share. Using a related structural model, Rota (2004) finds even lower coefficients in the range 0.11 – 0.13 with a panel of Italian firms. In the literature, $\gamma$ is often calibrated to $2/3$ which corresponds to the share of labor in value-added in national accounts. Also it is sometimes estimated using production function estimation techniques (see Ackerberg et al. (2007) for a survey) which typically gives estimates close to the labor share in value-added.

Adjustment costs per worker are estimated to be equal to 4.77% of the average workers wage. Combined with an observed job reallocation rate of 10.96%, the estimate implies that adjustment costs incurred by firms average about 52.28% of the wage bill per worker. This also means that a firm has to pay an extra-year of wages for every twenty jobs created or destroyed. These numbers are lower than what could be expected given the stringent labor market regulation in France. An examination of the data on worker flows provides an explanation. There are two types of regular employment contracts in France: indefinite-term contracts (CDI) and fixed-duration contracts (CDD). Almost 60% of the exit from employment can be attributed to the end of fixed-duration contracts which are by definition flexible. Less than 7% of the exits from employment are made through a layoff procedure and less than 1% through a layoff for economic reasons. Hence by using fixed-duration contracts, French firms destroy jobs at relatively low costs. CDDs are also the most common method of hiring: more than 2/3 of all hires are through CDDs. Rota (2004) estimates the median level of fixed (not linear) costs to be around 15 months labor cost in Italy. Using Compustat,

Table 5: Structural Parameters Estimates - Standard-Errors are given in parentheses below the point estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Full Model</th>
<th>Model with $\gamma = 0$</th>
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Bloom (2009) estimates linear adjustment costs in the U.S. of about 1.8% of annual wages, and a fixed cost of around 2.1% of annual revenue with no quadratic adjustment costs. Aguirregabiria and Alonso-Borrego (2009) find that the linear component is the most important part of labor adjustment costs which is around 15% of a worker’s annual salary for hiring and around 50% for firing. All these papers assume wages do not change with idiosyncratic shocks.

The estimated variance of a permanent shock is $\hat{\sigma}^2_{\epsilon^P} = 0.0306$. This is larger than the estimated variance of a transitory shock $\hat{\sigma}^2_{\epsilon^T} = 0.0232$. This says that the variance of permanent shocks is 50% higher. A variance decomposition of profitability growth reveals that 43.78% of profitability variance can be attributed to permanent shocks. This can seem at odds with the findings of Section 3.3 where 60.29% of the variance of output growth can be attributed to permanent shocks. This is actually consistent with the idea that employment reacts more to permanent shocks. The variance of output growth may be written:

$$\text{Var} (\Delta \log o) = (1 - \alpha)^2 (\sigma^2_{\epsilon^P} + 2\sigma^2_{\epsilon^T}) + \alpha^2 \text{Var} (\Delta \log n) + 2\sigma^2_{\text{MRO}} + (1 - \alpha) \alpha (2\text{Cov} (\Delta \log n, \epsilon^P) + 2\text{Cov} (\Delta \log n, \Delta \epsilon^T))$$

The empirical results suggest that even though transitory shocks to profitability explains a larger fraction of profitability variance, they explain a smaller fraction of output variance because employment variations amplify permanent shocks more than transitory shocks.

I find that workers bargaining power over the marginal surplus is $\hat{\gamma} = 0.3824$. Using the same firm dataset matched with a worker dataset and a different methodology, Cahuc et al. (2006) find bargaining power in the range $0.15 - 0.62$ depending on a worker’s skill category. They warn though that these numbers are biased upward because they do not account for on-the-job-search and between-firm competition for employed workers.

I estimate the per-period utility of being unemployed $b$ to be 799 euros per month which is close to but lower than the after-tax minimum wage during that period (about 900 euros per month). The model implies a replacement rate of 59.74% which is relatively close to the actual replacement rate in France which is around 55%.

The variance of measurement errors in employment, wages and output are all significant. Consider the following decomposition. Let $\log x = \log x^{\text{model}} + 2\text{MR}$ where $x$ is observed employment, wages or output and $\text{MR}$ is measurement error. Then $\text{Var} (\Delta \log x) = \text{Var} (\Delta \log x^{\text{model}}) + 2\sigma^2_{\text{MR}}$ and the variance explained by the model is the ratio $\frac{\text{Var} (\Delta \log x^{\text{model}})}{\text{Var} (\Delta \log x)}$. According to the model’s estimates, the model explains 98.69%, 87.30% and 29.79%, respectively, of the variance of employment growth, output growth and wages growth. Measurement errors in output are important. This has been reported in the literature on the structural estimation of production functions. Estimated measurement errors in wages are large. This number is surprising because wages are typically a better recorded variable. In the model, the only source of variations in wages is labor productivity.
Moments Data Model \( \gamma = 0 \)

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
<th>( \gamma = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E\left(\frac{n}{w}\right))</td>
<td>0.3143 (0.0019)</td>
<td>0.3049</td>
<td>0.3057</td>
</tr>
<tr>
<td>( E\left(\frac{n}{o}\right))</td>
<td>0.1532 (0.0006)</td>
<td>0.1698</td>
<td>0.1700</td>
</tr>
<tr>
<td>( E\left(\frac{w}{n}\right))</td>
<td>0.5632 (0.0053)</td>
<td>0.5584</td>
<td>0.5578</td>
</tr>
<tr>
<td>Reallocation</td>
<td>0.1096 (0.0009)</td>
<td>0.1106</td>
<td>0.1065</td>
</tr>
<tr>
<td>Inaction</td>
<td>0.1523 (0.0018)</td>
<td>0.1536</td>
<td>0.1514</td>
</tr>
</tbody>
</table>

| \( E\left(\Delta \log n \Delta \log n\right)\) | 0.0349 (0.0013) | 0.0278 | 0.0260 |
| \( E\left(\Delta \log o \Delta \log o\right)\) | 0.0758 (0.0029) | 0.0682 | 0.0723 |
| \( E\left(\Delta \log w \Delta \log w\right)\) | 0.0274 (0.0012) | 0.0247 | 0.0000 |
| \( E\left(\Delta \log n \Delta \log n_{-1}\right)\) | 0.0003 (0.0003) | 0.0030 | 0.0048 |
| \( E\left(\Delta \log o \Delta \log o_{-1}\right)\) | -0.0120 (0.0013) | -0.0162 | -0.0174 |
| \( E\left(\Delta \log w \Delta \log w_{-1}\right)\) | -0.0115 (0.0007) | -0.0122 | 0.0000 |

| \( E\left(\Delta \log n \sum_{1}^{1} \Delta \log n_{i}\right)\) | 0.0317 (0.0013) | 0.0338 | 0.0356 |
| \( E\left(\Delta \log o \sum_{1}^{1} \Delta \log o_{i}\right)\) | 0.0457 (0.0017) | 0.0360 | 0.0378 |
| \( E\left(\Delta \log w \sum_{1}^{1} \Delta \log w_{i}\right)\) | 0.0036 (0.0003) | 0.0002 | 0.0000 |
| \( E\left(\Delta \log w \Delta \log o\right)\) | 0.0146 (0.0012) | 0.0136 | 0.0000 |
| \( E\left(\Delta \log n \Delta \log o\right)\) | 0.0227 (0.0011) | 0.0363 | 0.0299 |
| \( E\left(\Delta \log w \Delta \log n\right)\) | -0.0065 (0.0006) | 0.0050 | 0.0000 |
| \( E\left(\Delta \log w \Delta \log o_{-1}\right)\) | -0.0069 (0.0008) | -0.0094 | 0.0000 |
| \( E\left(\Delta \log n \Delta \log o_{-1}\right)\) | 0.0051 (0.0005) | 0.0039 | 0.0061 |
| \( E\left(\Delta \log n \Delta \log w_{-1}\right)\) | 0.0038 (0.0003) | 0.0006 | 0.0000 |
| \( E\left(\Delta \log n \sum_{1}^{1} \Delta \log w_{i}\right)\) | -0.0029 (0.0004) | 0.0004 | 0.0000 |
| \( E\left(\Delta \log o \sum_{1}^{1} \Delta \log o_{i}\right)\) | 0.0277 (0.0011) | 0.0346 | 0.0364 |
| \( E\left(\Delta \log o \sum_{1}^{1} \Delta \log w_{i}\right)\) | 0.0008 (0.0004) | 0.0007 | 0.0000 |

| \( E\left(\Delta \log w \Delta \log o_{-1}\right)\) | 0.5805 (0.0584) | 0.5811 | 0.0000 |
| \( E\left(\Delta \log o \sum_{1}^{1} \Delta \log w_{i}\right)\) | 0.0168 (0.0087) | 0.0188 | 0.0000 |
| \( E\left(\Delta \log o \sum_{1}^{1} \Delta \log o_{i}\right)\) | 0.0168 (0.0087) | 0.0188 | 0.0000 |

| \( E\left(\Delta \log w \Delta \log o_{-1}\right)\) | 0.5805 (0.0584) | 0.5811 | 0.0000 |
| \( E\left(\Delta \log o \sum_{1}^{1} \Delta \log w_{i}\right)\) | 0.0168 (0.0087) | 0.0188 | 0.0000 |
| \( E\left(\Delta \log o \sum_{1}^{1} \Delta \log o_{i}\right)\) | 0.0168 (0.0087) | 0.0188 | 0.0000 |

Table 6: Auxiliary Parameters

In reality, several factors outside the model presumably contribute to observed wages variation, such as on-the-job search and match specific shocks...

### 5.2 Auxiliary Model

The moments I use to estimate the model are listed in Table 6. The standard deviations are obtained through bootstrapping on the original panel. Column 2 and Column 3 in Table 6 report the simulated moments for, respectively, the full model and the constrained model with \( \gamma = 0 \). Most moments were discussed in Section 3.

Overall the model fits the data well. The model can reproduce the average value of labor productivity, wages and the labor share. Despite the low curvature of the profit function \( \hat{\alpha} = 0.3214, \)
the model predicts a labor share of 0.5584 which is close to the labor share in the data 0.5632. This is because, as explained above, the labor share also depends on worker’s bargaining power $\hat{\gamma} = 0.3824$. The curvature of the profit function is noticeably higher under the constrained model where $\gamma = 0$. I find that $\hat{\alpha} = 0.5525$. Even with this restriction I find that the model is still able to match the labor share: the model predicts a labor share equal to 0.5578. This suggests the potential bias when estimating production and wages are set through a rent-sharing mechanism.

The model can reproduce the observed reallocation rate ($0.1096$ in the model and $0.1106$ in the data) and inaction rate ($0.1536$ in the model and $0.1523$ in the data). This result is particularly striking given that estimated adjustment costs are modest. Adjustment costs are found to be less than one month of wage per job created or destroyed.

The model correctly predicts a large impact of transitory profitability shocks on wages: $E(\Delta w \Delta o_{-1})$ is too low in the model $-0.0094$ compared to the data $-0.0069$. A measure of the transmission of transitory profitability shocks into wages $E(\Delta w \Delta o_{-1}) / E(\Delta o_{-1})$ is almost exactly matched: $0.5805$ in the data and $0.5811$ in the model. This is because the model predicts a little too high transitory variance of output: $E(\Delta o_{-1})$ is equal to $-0.0120$ in the data versus $-0.0162$ in the model.

The model also correctly predicts a weak impact of permanent profitability shocks on wages: $E(\Delta \log o \sum_{-1}^{t-1} \Delta \log w_{i})$ is equal to $0.0008$ in the data and $0.0007$ in the model. A measure of the transmission of permanent profitability shocks into wages $E(\Delta \log o \sum_{-1}^{t-1} \Delta \log w_{i}) / E(\Delta \log o \sum_{-1}^{t-1} \Delta \log o_{j})$ is $0.0168$ in the data compared to $0.0188$ in the model. The model underpredicts the variance of permanent output shocks: $0.0360$ versus $0.0457$ in the data. This is because there is a trade-off between reproducing this variance and the variance of permanent employment shocks: the variance of permanent employment shocks $E(\Delta \log n \sum \Delta \log n_{i})$ is equal to $0.0317$ in the data and $0.0338$ in the model.

The contemporaneous and lagged covariances between output and employment growth are explained well by the model: $E(\Delta \log n \Delta \log o)$ is equal to $0.0277$ in the data and $0.0363$ in the model; $E(\Delta \log n \Delta \log o_{-1})$ is equal to $0.0051$ in the data and $0.0039$ in the model; $E(\Delta \log o \sum_{-1}^{t-1} \Delta \log n_{i})$ is equal to $0.0277$ in the data and $0.0346$ in the model. Similarly the model fits well the contemporaneous and lagged covariances between output and wage growth: $E(\Delta \log w \Delta \log o)$ is equal to $0.0146$ in the data and $0.0136$ in the model. However, the model does not do as well in matching the covariances between wages and employment. These correlations are weakly positive in the model while the data suggests a negative correlation: $E(\Delta \log n \Delta \log w)$ is equal to $-0.0065$ in the data versus $0.0050$ in the model. $E(\Delta \log n \Delta \log w_{-1})$ is equal to $0.0038$ in the data and $0.0006$ in the model while $E(\Delta \log n \sum_{-1}^{t-1} \Delta \log w_{i})$ is equal to $-0.0029$ in the data and $0.0004$ in the model. Increasing the variance of measurement error in employment reconciles the model and the data for these moments. While these would allow the model to fit the above covariances well, it would substantially worsen the fit of the inaction rate. When the measurement error in employment increases, the fraction of employment variation smaller than $0.025\%$ decreases noticeably. The introduction of quadratic adjustment costs could allow us to simultaneously match both of these statistics. This
is because quadratic adjustment costs provides an incentive for the firm to make only small adjustments to their labor force. However this would substantially complicate the numerical solution of the model.

6 Conclusion

This paper offers a structural framework to analyze the impact of permanent and transitory shocks to profitability on wages and employment at the firm-level. The firm produces with decreasing returns to labor and is subject to transitory and permanent shocks to its profitability. The wage is negotiated every year and workers are paid a share of the value of marginal worker (following Stole and Zwiebel (1996)). Finally, adjusting the level of employment is costly. The model predicts that transitory shocks have a strong impact on wages and little effect on employment while permanent shocks have a strong effect on employment and little effect on wages.

I find support for this mechanism using a panel of French firms: transitory shocks to output have a strong effect on wages but permanent shocks have only a small effect on wages. I structurally estimate the model by simulated method of moments. The model fits the data well. The estimation results point to adjustment costs being modest (less than a month of wages). These small adjustment costs can however reproduce the data well. This is surprising given the perceived sclerosis of the French labor market. Permanent shocks are the most important source of output fluctuations. Ignoring wages flexibility leads to substantial bias, particularly in the estimation of the curvature of the profit function.

There are a range of future extensions of this paper. An important extension is to allow for worker heterogeneity. This is natural because employment and compensation of heterogeneous workers may fluctuate differently. For instance, a firm is probably more reluctant to fire a worker with highly specific and valuable human capital. Following a negative shock, the worker and the firm may then agree on a temporary wage cut to avoid human capital losses. Similarly, the model abstracts from capital and hours choices and it would be interesting to investigate the joint dynamics of different inputs following shocks of different persistence. A second type of extensions would consider more general profitability processes, for instance, a time-varying variance (as in Bloom (2009) or Meghir and Pistaferri (2004)) or firm-specific differences in profitability growth rates (as in Guvenen (2007)). This would provide a more complete characterization of firm risks and the consequences for firms input choices. All these extensions are left for future research.
References


A Theoretical Moments for Section 3.3

This section describes the theoretical moments used for the estimation in Section 3.3.

\[
\begin{align*}
E[\Delta y_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})] &= \sigma^2_{\zeta_y} \\
E(\Delta y_t \Delta y_t) &= \sigma^2_{\zeta_y} + 2(\sigma^2_{\nu y} + \sigma^2_{\nu y}) \\
E(\Delta y_t \Delta y_{t+1}) &= -\left(\sigma^2_{\nu y} + \sigma^2_{\nu y}\right)
\end{align*}
\]

and \(E(\Delta y_t \Delta y_{t+s}) = 0, s \geq 2\). Similarly,

\[
\begin{align*}
E[\Delta w_t(\Delta w_{t-1} + \Delta w_t + \Delta w_{t+1})] &= \tau^2 \sigma^2_{\zeta_y} + \sigma^2_{\zeta_w} \\
E(\Delta w_t \Delta w_t) &= \tau^2 \sigma^2_{\zeta_y} + 2\left(\phi^2 \sigma^2_{\nu y} + \sigma^2_{\nu w}\right) + \sigma^2_{\zeta_w} \\
E(\Delta w_t \Delta w_{t+1}) &= -\left(\phi^2 \sigma^2_{\nu y} + \sigma^2_{\nu w}\right)
\end{align*}
\]

and \(E(\Delta w_t \Delta w_{t+s}) = 0, s \geq 2\). The covariance between output growth and wage growth is:

\[
\begin{align*}
E(\Delta w_t \Delta y_t) &= \tau \sigma^2_{\zeta_y} + 2\phi \sigma_{\nu y} \\
E(\Delta w_t \Delta y_{t+1}) &= -\phi \sigma^2_{\nu y} \\
E[\Delta w_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})] &= \tau \sigma^2_{\zeta_y}
\end{align*}
\]

and \(E(\Delta w_t \Delta y_{t+s}) = 0, |s| \geq 2\).

B Theoretical Model

This section describes the solution of the wages function and the numerical solution of the model. The technical appendix of Campbell and Fisher (2000) provides a rigorous treatment of a similar dynamic programming problem. I refer the reader to this technical appendix for the results on the existence of a solution and the properties of the policy function that apply to the current framework with some minor changes in the notation.

B.1 Bargaining

Wages depend on the marginal value of labor \(V_n\) which can be decomposed:

\[
V_n(A, n, \epsilon^T) = \begin{cases} \\
\frac{\bar{\xi}}{\alpha} (\epsilon^T A)^{1-\alpha} n^{\alpha-1} - w(A, \epsilon^T, n) - w_n(A, \epsilon^T, n) n + \beta E \left[V_n(A\epsilon^P, \epsilon^T, n + d)\right] & \text{if } n < \underline{n}(A, \epsilon^T) \\
\frac{\bar{\pi}}{\alpha} (\epsilon^T A)^{1-\alpha} n^{\alpha-1} - w(A, \epsilon^T, n) - w_n(A, \epsilon^T, n) n + \beta E \left[V_n(A\epsilon^P, \epsilon^T, n + d)\right] & \text{if } \underline{n}(A, \epsilon^T) < n < \bar{n}(A, \epsilon^T) \\
\frac{\bar{\pi}}{\alpha} (\epsilon^T A)^{1-\alpha} n^{\alpha-1} - w(A, \epsilon^T, n) - w_n(A, \epsilon^T, n) n + \beta E \left[V_n(A\epsilon^P, \epsilon^T, n + d)\right] & \text{if } n > \bar{n}(A, \epsilon^T)
\end{cases}
\]
Let $W(A, e^T, n)$ be the value of employment in a firm of size $n$ with state $(A, e^T, n)$. The worker’s surplus in an expanding firm is:

$$W(A, e^T, \bar{w}(A, e^T)) - U = \frac{\gamma}{1 - \gamma} (V_n(A, e^T, \bar{w}(A, e^T)) + \bar{c})$$

$$= \frac{\gamma}{1 - \gamma} (\zeta + \bar{c})$$

Similarly, in a contracting firm:

$$W(A, e^T, \bar{w}(A, e^T)) - U = \frac{\gamma}{1 - \gamma} (V_n(A, e^T, \bar{w}(A, e^T)) + \bar{c})$$

$$= 0$$

Upon finding a job, the new job must be in a firm which is creating jobs. The value to a worker of unemployment is:

$$U = b + \beta (1 - f)U + \beta f E \left[ W \left( A, e^T, \bar{w}(Ae^{\epsilon'}, e^T) \right) \mid n < \bar{w}(Ae^{\epsilon'}, e^T) \right]$$

Then,

$$(1 - \beta)U = b + \beta f \frac{\gamma}{1 - \gamma} (\zeta + \bar{c})$$

The value of employment is:

$$W(A, e^T, n) = (1 - \tau)w(A, e^T, n) + \beta \left[ W \left( Ae^{\epsilon'}, e^T, \bar{w}(Ae^{\epsilon'}, e^T) \right) \mid n < \bar{w}(Ae^{\epsilon'}, e^T) \right]$$

$$+ \beta \left[ W \left( Ae^{\epsilon'}, e^T, n \right) \mid \bar{w}(Ae^{\epsilon'}, e^T) < n < \bar{w}(Ae^{\epsilon'}, e^T) \right]$$

$$+ \beta \left[ U \mid n > \bar{w}(Ae^{\epsilon'}, e^T) \right]$$

Rearranging terms, gives:

$$W(A, e^T, n) = (1 - \tau)w(A, e^T, n) + \beta U + \beta \frac{\gamma}{1 - \gamma} P \left( n < \bar{w}(Ae^{\epsilon'}, e^T) \right) (\zeta + \bar{c})$$

$$+ \beta \frac{\gamma}{1 - \gamma} E \left[ V_n \left( Ae^{\epsilon'}, e^T, n \right) + \bar{c} \mid \bar{w}(Ae^{\epsilon'}, e^T) < n < \bar{w}(Ae^{\epsilon'}, e^T) \right]$$

It follows that workers surplus is:

$$W(A, e^T, n) - U = (1 - \tau)w(A, e^T, n) - b + \beta \frac{\gamma}{1 - \gamma} \left( P \left( n < \bar{w}(Ae^{\epsilon'}, e^T) \right) - f \right) (\zeta + \bar{c})$$

$$+ \beta \frac{\gamma}{1 - \gamma} E \left[ V_n \left( Ae^{\epsilon'}, e^T, n \right) + \bar{c} \mid \bar{w}(Ae^{\epsilon'}, e^T) < n < \bar{w}(Ae^{\epsilon'}, e^T) \right]$$

This must equal $\frac{\gamma}{1 - \gamma} \left[ V_n(A, e^T, n) + \bar{c} \right]$. Then,
\[(1 - \tau)w(A, \epsilon^T, n) = (1 - \gamma) b + \gamma \alpha \left( e^{tT}A \right)^{1-\alpha} n^{\alpha-1} - \gamma w_n(A, \epsilon^T, n)n + \bar{\gamma} \gamma (1 - \beta (1 - f)) + \xi \beta \gamma f \]

Solving this differential equation gives:

\[w(A, \epsilon^T, n) = \frac{1}{1 - \tau} \left( (1 - \gamma) b + \bar{\gamma} \gamma (1 - \beta (1 - f)) + \xi \beta \gamma f \right) + \frac{\gamma \alpha}{1 - \tau - \gamma (1 - \alpha)} \left( e^{tT}A \right)^{1-\alpha} n^{\alpha-1} \]

Note that \(w(A, \epsilon^T, n)\) is homogeneous of degree 0 in \((A, n)\).

### B.2 Homogeneity of the Value Function

\(V\) is homogeneous of degree 1 in \(A\) and \(n\). To see this, define \(y = \frac{n}{\lambda} A\). Consider \(\lambda \geq 0\).

\[V(\lambda A_0, \epsilon^T, \lambda n - 1) = \lambda A_0 \sup_{y, t \geq 0} E \sum_{t=0}^{\infty} \beta^t \frac{A_0}{A_0} \left\{ e^{tT(1-\alpha)} y^\alpha - w(y, \epsilon^T) y - \xi \left( y - \frac{y - e^{\epsilon^T}}{e^t} \right) \right\} \]

\[= \lambda V(A_0, \epsilon^T, n - 1) \]

with the constraint that \(\frac{y}{e^{\epsilon^T}} = \frac{n - 1}{\lambda} A\). Define \(v(x, \epsilon^T) = V(1, \frac{n - 1}{\lambda} A, \epsilon^T)\) with \(x = \frac{n - 1}{\lambda} A\). \(v\) satisfies the Bellman equation:

\[v(x, \epsilon^T) = \max_y \left\{ e^{(1-\alpha)x} y^\alpha - w(y, \epsilon^T) y - \xi (y - x)^+ - \bar{\xi} (y - x)^- + \beta E \left[ e^{x^p} v( ye^{-x^p}, \epsilon^T') \right] \right\} \]

Define

\[d(y) = E \left[ e^{x^p} v( ye^{-x^p}, \epsilon^T') \right] \]

d\(y\) is written as the convolution of a normal density and a integrable function. Such a convolution is analytic from a well known property of the exponential family of distributions (see Theorem 9 in Lehmann (1986)). Therefore \(d(y)\) has derivatives of all order. Define

\[d_y(y) = E \left[ v_y( ye^{-x^p}, \epsilon^T') \right] \]
The optimal policy can be rewritten:

\[ y(x, \epsilon^T) = \begin{cases} 
  y(\epsilon^T) & \text{if } x < y(\epsilon^T) \\
  x & \text{if } y(\epsilon^T) < x < \overline{y}(\epsilon^T) \\
  \overline{y}(\epsilon^T) & \text{if } x > \overline{y}(\epsilon^T) 
\end{cases} \]

with

\[
\frac{\alpha(1-\tau-\gamma)}{1-\tau-\gamma(1-\alpha)} e^{(1-\alpha)e^\tau} \frac{y(\epsilon^T)}{\overline{y}(\epsilon^T)^{\alpha-1}} - \frac{1}{1-\tau} ((1-\gamma) b + \overline{\gamma}(1-\beta(1-f)) + \xi \beta \gamma f) - \xi + \beta \hat{d}_y(y(\epsilon^T)) = 0
\]

\[
\frac{\alpha(1-\tau-\gamma)}{1-\tau-\gamma(1-\alpha)} e^{(1-\alpha)e^\tau} \frac{y(\epsilon^T)}{\overline{y}(\epsilon^T)^{\alpha-1}} - \frac{1}{1-\tau} ((1-\gamma) b + \overline{\gamma}(1-\beta(1-f)) + \xi \beta \gamma f) + \overline{\gamma} + \beta \hat{d}_y(\overline{y}(\epsilon^T)) = 0
\]

C Numerical Solution

I use a collocation method as described in Judd (1998). Let \( \hat{d}_y(y;p) \) be the function used to approximate \( d_y(y;p) \) with \( p \) a vector of parameters. I assume it can be written as a linear combination of a set of \( P \) known linearly independent basis function \( B_1, \ldots, B_n \),

\[
\hat{d}_y(y;p) = \sum_{i=1}^{P} p_i B_i(y)
\]

whose basis coefficients \( p_1, \ldots, p_P \) are to be determined. I use splines as basis-function. I apply standard Chebyshev interpolation nodes and Gaussian quadrature to the contraction mapping that define \( d_y(y;p) \). Details of the numerical solution are available upon request.