The Consequences of Uncertain Debt Targets*

Alexander W. Richter       Nathaniel A. Throckmorton

April 19, 2013

Abstract

The proposals aimed at debt reduction, such as those leading up to the U.S. Budget Control Act of 2011, reveal significant differences in their targets. These proposals and the degree of political polarization make clear the uncertainty surrounding short-run tax rates and long-run debt targets. We employ a real business cycle model in which a representative Bayesian household learns about the long-run debt target in an endogenous tax rule. The household attempts to extract the long-run debt target from a noisy signal and uses their subsequent inferences of the target to estimate the probabilities that govern its evolution. Uncertainty about the long-run debt target is reflected in the income tax rate, which distorts the household’s consumption, labor, and saving decisions. Monte Carlo simulations of the model demonstrate that this type of uncertainty leads to two-sided risk. There are welfare losses at the median of the distribution and there is asymmetric bias towards welfare losses, but there is a possibility for welfare gains when the household overinvests in capital. When we extend the model to allow half of the fiscal adjustment to come through government spending instead of solely through taxes, the variance of the welfare distribution nearly doubles.

Keywords: Bayesian Learning; Uncertainty; Fiscal Policy; Welfare; Real business cycle model

JEL Classifications: D83; E32; E62; H68

--Richter, Department of Economics, Auburn University, 0332 Haley Center, Auburn, AL (arichter@auburn.edu); Throckmorton, Department of Economics, Indiana University, 100 S. Woodlawn, Wylie Hall 105, Bloomington, IN (nathrock@indiana.edu). We thank Eric Leeper and Todd Walker for helpful discussions.
1 INTRODUCTION

Independent analysis of the various proposals aimed at U.S. deficit reduction leading up to the Budget Control Act of 2011 (Pub.L. 112-25) reveal significant differences in their implied 10-year debt targets, with estimates ranging from roughly 63 to 76 percent of GDP in 2022 (figure 1a). Each of these proposals improves on the CBO’s grim projection in its 2012 Long-Term Budget Outlook, which forecasts that debt will reach 90 percent of GDP by 2022 when incorporating historically common and expected fiscal adjustments. However, over the next decade none of these proposals reduce the debt-to-GDP percentage to a level consistent with the CBO’s baseline scenario, which indicates that debt will fall to 58.5 percent of GDP when conditioning on current law.

The lack of clarity in these projections and their inconsistency with the post-World War II average U.S. debt-to-GDP percentage, which is roughly 45 percent, significantly adds to the uncertainty that always surrounds fiscal policy. Even in normal times the legislative process, elections, and government responses to unexpected crises creates uncertainty that is unavoidable with a democratically elected representative government. However, the current CBO projections, which predict unsustainable growth in government debt, create additional uncertainty about how Congress will respond. Unlike the debt run-up in the 1940s, which was caused by temporary war-related expenditures, the current fiscal problem is structural and requires legislative action.¹

¹Immediately following WWII many industrialized countries faced unprecedented debt-to-GDP percentages. For example, the debt-to-GDP percentage in the U.S., the U.K., Australia, Canada, and New Zealand were 122, 270, 92, 115, and 148 percent, respectively. Over the next quarter century (and in many cases sooner), all of these percentages fell below 60 percent and in most cases they fell well-below 50 percent (Abbas et al. (2011)). There are two main reasons for the sharp reduction. First, every one of these countries ran primary surpluses on average following the war. Second, and more importantly, average inflation exceeded 3.5 percent in all of these countries, which reduced real interest rates, while growth rates ranged from 2.5-5 percent. History suggests large debt reductions are possible, but given the current political gridlock and anemic growth in the U.S., debt reduction is very uncertain.
What makes the fiscal outlook so dangerous and the degree of fiscal uncertainty unprecedented is that political polarization has risen steadily since the 1950s and is currently at a record level (figure 1b). When political polarization is high, the differences in legislative proposals are more stark and the likelihood of last minute compromise that could result in several different outcomes is much greater. For example, during the 2011 debt ceiling debate and the run-up to the 2013 “fiscal cliff”, Congress considered several different resolutions that could have resulted in very different outcomes. In this sense, the lack of compromise is a type of fiscal noise that impacts households’ beliefs. The deficit reduction proposals provide evidence that Congress is aware of the structural deficit and its potential consequences. However, without a credible plan that commits to a debt target, households must form expectations over a range of long-run debt targets, which affects their beliefs about future tax rates, their optimal consumption/saving decisions, and welfare.

This paper uses a novel application of Bayesian learning to endogenize fiscal uncertainty within a dynamic stochastic general equilibrium (DSGE) framework. We use a real business cycle (RBC) model where a representative household learns about the long-run debt target set by the fiscal authority. Each year the fiscal authority sets a long-run debt target and a corresponding income tax rate consistent with the target, the current debt-to-GDP ratio, and discretionary policy. The long-run debt target is obscured by year-to-year fiscal noise (e.g. the legislative process, elections, responses to unexpected crises, etc.) that changes the household’s perception of the true target.

The household knows the underlying tax rule and observes the tax rates and debt-to-GDP ratios, but does not know the current long-run debt target because it is obscured by fiscal noise. Instead, the household forms beliefs about the long-run debt target with the Hamilton (1989) filter, and uses these beliefs to estimate the long-run debt target transition matrix with importance sampling [Gelman et al. (2004); Geweke (2005)]. In other words, given the household’s sequence of past beliefs about the long-run debt target, they infer what is the most likely transition matrix that generates the sequence. If the household’s beliefs are incorrect, then they are surprised by future tax rates that are inconsistent with their inference of the current long-run debt target. These surprises impose real consequences, since the tax rate is levied proportionally against income and distorts the household’s consumption, labor, and saving decisions.

To obtain a better sense of the degree of fiscal uncertainty, figure 2a plots each of the CBO’s baseline projections of the debt-to-GDP percentage (dashed lines) with the actual percentage superimposed (solid line). It is not surprising that over long horizons these projections are inaccurate. By law (Pub.L. 93-344, sec. 202(e)), the CBO’s projections are based on current law, which is a poor predictor of long-run policy. For example, the projections in the early-mid 1990s did not initially account for the Balanced Budget Act of 1997 (Pub.L. 10533) and in the early 2000s the projections did not account for the U.S. involvement in Iraq and Afghanistan or the Economic Growth and Tax Relief Reconciliation Act of 2001 (Pub.L. 10716). However, these projections make clear the alternative paths for the debt-to-GDP percentage that were possible. Without the Balanced Budget Act, the debt-to-GDP percentage would have likely risen in the late 1990s. Likewise, without significant and unanticipated war related expenses or tax cuts, the debt-to-GDP percentage would have likely fallen in the early 2000s. These policy changes indicate alternative debt targets and form a probability distribution that agents can use to base expectations of future policy changes.

Figure 2b plots the distribution of the difference between the CBO’s projection in any given year and the actual debt-to-GDP percentage. Over shorter horizons significant legislative changes are less likely to occur and, on average, the CBO’s projections are more accurate. The accuracy of the CBO’s projections over a 5-year time span suggest that changes in the debt target are infrequent.
To quantify the consequences of fiscal uncertainty, we compare the results from Monte Carlo simulations across different restrictions on the household’s information set. The benchmark is the full information case, where current and past long-run debt targets and the transition probabilities are known. In this case, there is no learning and the only sources of uncertainty are future debt targets and discretionary changes in the tax rate. We compare three limited information cases to this benchmark: 1. the long-run debt target is always unknown and the transition probabilities are known; 2. current and past long-run debt targets are known and the transition probabilities are unknown; and 3. both the long-run debt target and transition probabilities are always unknown.

The information set affects both the speed at which the household learns about the long-run debt target and transition probabilities and the convergence properties of the learning algorithm. The household’s ability to distinguish between changes in the long-run debt target and temporary fiscal noise shocks affects convergence. More fiscal noise creates greater uncertainty about the
current long-run debt target. Since their observations are always noisy, the percent of correct inferences approaches 100 percent only when the standard deviation of that noise is small. Furthermore, incorrect past inferences of the long-run debt target influence their estimate of the target’s transition matrix, and create feedback between incorrect inferences of the target and estimates of the transition matrix. Thus, the transition matrix asymptotically approaches a belief that is different from the true matrix. In the most restrictive information set (case 3), the household correctly infers only about 71 percent of the long-run debt targets and the norm distance between the estimated and true transition matrix is 22 percent of its maximum value after 250 years of learning.

Uncertainty stemming from limited information about the current long-run debt target and its transition probabilities influences the business cycle by regularly moving output by as much as 0.1 percent (and sometimes more) relative to the full information case, which is consistent with Fernandez-Villaverde et al. (2011). We also find that uncertainty about the long-run debt target leads to two-sided risk. There are welfare losses (relative to the full information case) at the median of the distribution and there is asymmetric bias towards welfare losses, but there is a possibility for welfare gains when the household overinvests in capital. We calculate that the 10-year loss in real GDP (in 2012 dollars) equals $372 billion and $57 billion at the 10th and 50th percentiles and a gain of $242 billion at the 90th percentile. When we extend the model to allow half of the fiscal adjustment to come through government spending instead of solely through taxes, the variance of the welfare distribution nearly doubles. Nevertheless, we caution that these values represent a floor on the costs of fiscal uncertainty for two main reasons. First, these numbers are relative to the full information case, which assumes future debt targets are unknown. Obviously, if these costs were relative to some type of “forward guidance” that the fiscal authority provided, the costs of the uncertainty would be much larger. Second, we have not accounted for any of the uncertainty surrounding discretionary government spending or entitlement programs. These represent an additional layer of fiscal uncertainty that is the subject of future research.

2 CONTACTS WITH THE LITERATURE

Several of the early papers that studied fiscal uncertainty in a dynamic general equilibrium framework permitted stochastic discrete shifts in policy, so that households place positive probability on recurring structural changes in policy [Aizenman and Marion (1993); Bizer and Judd (1989); Dotsey (1990)]. A key feature in some of the more recent papers is that households do not observe the current policy regime [Andolfatto and Gomme (2003); Davig (2004); Leeper and Zha (2003); Schorfheide (2005)]. Instead, households form inferences about the current regime using Bayesian updating. We build on this literature by assuming households do not know the transition probabilities governing regime change, which creates an additional uncertainty channel. Specifically, we set up a hidden Markov-switching model where the household’s information set is limited, so that both the current long-run debt target and transition probabilities are unknown.

Much of the adaptive learning literature models uncertainty in a similar way (see Evans and Honkapohja (2001) for an overview). Contrary to rational expectations, they relax the assumption that households have full information about the structure of the economy and monetary/fiscal policy behavior, and instead assume households use historical data to form beliefs about these unknown components. In our approach, the household knows the structure of the economy, but filters a noisy signal to infer the current long-run debt target and estimate its transition probabilities.

An incorrect inference affects the household’s current behavior in two ways. First, the deci-
sion rules used to calculate the potential marginal benefits of saving will differ from the rules that are based on the correct target. This happens because the probabilities governing policy changes, which influence the household’s expectations, are conditional on the wrong long-run debt target and, therefore, are not the probabilities they would have used with a correct inference. Second, incorrect inferences also alter current and future estimates of the transition probabilities. Furthermore, feedback occurs when both channels are open since the accuracy of the inference of the long-run debt target depends on the estimates of the transition probabilities. Learning operates through these channels to affect behavior and influence future tax rates, which endogenously respond to debt. Since the tax rate is levied proportionally against income, uncertainty about the long-run debt target and its dynamics affects the real economy.

The segment of the adaptive learning literature that allows for policy change generally assumes that the possible outcomes are not in the household’s information set [Eusepi and Preston (2012); Giannitsarou (2006)]. In this setting, agents may learn about a policy change that has not previously occurred. Bianchi and Melosi (2012) also allow households to discover unanticipated policy regimes in a model with monetary and fiscal policy switching, but where households update their beliefs using Bayesian learning. As an alternative, Evans et al. (2009) use a model with adaptive learning where the possible fiscal policy changes are in the household’s information set. In our work, we use a Bayesian learning approach where the household’s expectations are guided by the debt target proposals. The household behaves as a Bayesian updater drawing from a specific set of long-run debt targets and making estimates of the transition probabilities. In other words, the uncertainty is not over what outcomes are possible, but which one will prevail and for how long.

Another segment of the literature also models the uncertainty surrounding fiscal policy by assuming that households form expectations over a range of possible resolutions to the looming fiscal crisis [Davig and Leeper (2011); Davig et al. (2010, 2011); Richter (2012)]. This work makes clear that expectations about future policy significantly affect current private behavior and that a prolonged period of stagflation is a possible outcome of the uncertainty. The main drawbacks with this literature is that it requires the modeler to take a strong stance on the potential resolutions to the uncertainty and it assumes the household knows the current policy state and the true probability distributions governing the evolution of policy. We use a limited information approach, where the possible long-run debt targets are consistent with the current policy proposals.

Bloom (2009) captures a change in the uncertainty surrounding business conditions using a shock to the variance of the driving processes in a firm-level model. He notes that uncertainty may come from political shocks, of the kind we use to motivate this paper, which influence investment and hiring decisions of firms. Bloom et al. (2012) extend an RBC model with time varying volatility and find that uncertainty shocks move GDP by about 3 percent. Fernandez-Villaverde et al. (2011) use a similar approach to analyze fiscal uncertainty by allowing the volatilities of distortionary consumption, labor, and capital taxes and government consumption to vary across time. They find that a rise in uncertainty causes prolonged recessions. Specifically, a positive volatility shock increases the range of realizations of the fiscal policy instruments relative to normal times, which causes firms to reduce investment to decrease their exposure to potentially higher tax rates. While this approach provides a first pass at highlighting the consequences of fiscal uncertainty, the drawback is the source of the uncertainty is not grounded in the policy debate and is exogenous.

In our model, the optimal investment strategy changes when beliefs differ from the long-run debt target due to a noisy signal extraction problem, but the mechanism is endogenous. For example, if the household believes the fiscal authority is targeting a lower long-run debt target, when
in fact they are not, then they decrease investment to reduce their exposure to a higher expected short-run tax rate, in the same way they would respond to a positive volatility shock. The household also decreases labor because they expect to transition to a regime with a higher tax rate, which lowers the after tax wage rate. The combination of an unchanged tax rate and a reduction in capital and labor reduces tax revenue, which requires the government to issue more debt to finance its consumption. The increase in debt then leads to a self-fulfilling higher future tax rate, despite the long-run debt target remaining unchanged. Thus, our approach endogenizes the uncertainty surrounding the long-run debt target.

The remainder of the paper is organized as follows. Section 3 lays out the economic model, the signal-extraction problem the household faces, and the processes governing fiscal policy behavior. Section 4 calibrates the model and explains the solution and simulation procedures. Section 5 presents the results. We first illustrate the basic channels of the uncertainty and how incorrect affect the hold’s decisions. Then we show the speed at which the household learns about the debt target and the welfare/dollar equivalent consequences of the uncertainty using Monte Carlo simulations of the model. Section 6 concludes.

3 Model

A representative household chooses sequences \( \{c_t, n_t, k_t, i_t, b_t\}_{t=0}^{\infty} \) to maximize expected lifetime utility, given by,

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_{t+1}^{1-\sigma}}{1-\sigma} - \frac{n_{t+1}^{1+\eta}}{1+\eta} \right\},
\]

where \( \beta \in (0, 1) \) is the subjective discount factor, \( 1/\sigma \) is the intertemporal elasticity of substitution, \( 1/\eta \) is the Frisch elasticity of labor supply, \( c \) is consumption, and \( n \) is labor hours. These choices are constrained by

\[
c_t + i_t + b_t = (1 - \tau_t)(w_t n_t + r_t^k k_{t-1}) + r_{t-1} b_{t-1} + \bar{z},
\]

\[
k_t = i_t + (1 - \delta) k_{t-1},
\]

where \( w \) is the real wage rate, \( r^k \) is the real rental rate of capital, \( k \) is the capital stock, which depreciates at rate \( \delta \), \( b \) is a one-period real government bond with real return \( r \), \( \bar{z} \) is a fixed transfer from the government, and \( \tau \) is a proportional tax rate levied against capital and labor income. The representative household’s optimality conditions imply

\[
w_t(1 - \tau_t) = \chi n_t^\alpha e_t^\sigma,
\]

\[
1 = \beta E_t \{(c_t/c_{t+1})^\sigma[(1 - \tau_{t+1})r_{t+1}^k + 1 - \delta]\};
\]

\[
1 = \beta E_t \{(c_t/c_{t+1})^\sigma r_t\}.
\]

A perfectly competitive representative firm produces output according to \( y_t = a k_{t-1}^\alpha n_t^{1-\alpha} \), where \( a \) is technology and \( \alpha \in (0, 1) \). Each period the firm chooses \( \{k_{t-1}, n_t\} \) to maximize profits, given by \( y_t - w_t n_t - r_t^k k_{t-1} \). The optimality conditions imply

\[
r_t^k = \alpha y_t/k_{t-1},
\]

\[
w_t = (1 - \alpha)y_t/n_t.
\]
The fiscal authority finances a fixed amount of discretionary spending, $\bar{g}$, and transfers, $\bar{z}$, by levying proportional taxes on income and issuing a one-period real bond. The government’s flow budget constraint is given by

$$b_t + \tau_t (w_t n_t + r_t^k k_{t-1}) = r_{t-1} b_{t-1} + \bar{g} + \bar{z}. \quad (9)$$

The tax rate endogenously responds to fluctuations in the debt-to-output ratio according to

$$\tau_t = \bar{\tau}(s_t) + \gamma (b_{t-1}/y_{t-1} - \bar{y}(s_t)) + \nu_t,$$  

(10)

where $\bar{\tau}(s_t)$ is a state dependent intercept of the tax rule that is consistent with the stationary equilibrium implied by the prevailing long-run debt target, $\bar{y}(s_t)$.\footnote{The tax shock, $\nu \sim \mathcal{N}(0, \sigma^2)$, is a proxy for discretionary tax policy. It obscures changes in the intercept when the state is unknown since the tax rate, lagged debt-to-output ratio, and tax rule, which are all known, would reveal the state.} The household’s perception of the prevailing debt target, $by^*_t$, follows

$$by^*_t = \bar{y}(s_t) + x_t,$$ 

(11)

$$x_t = \rho x_{t-1} + \varepsilon_t,$$  

(12)

where $s \in \{1, \ldots, m\}$ is one of $m$ possible long-run debt target states. The distance between long-run debt targets, $\Delta = \bar{y}(s) - \bar{y}(s-1) > 0$ for all $s > 1$. The long-run debt target evolves according to an $m$-state Markov process with transition matrix, $P$, given by,

$$P = \begin{bmatrix}
    \Pr[s_t = 1 | s_{t-1} = 1] & \cdots & \Pr[s_t = m | s_{t-1} = 1] \\
    \vdots & \ddots & \vdots \\
    \Pr[s_t = 1 | s_{t-1} = m] & \cdots & \Pr[s_t = m | s_{t-1} = m]
\end{bmatrix} = \begin{bmatrix}
    p_{11} & \cdots & p_{1m} \\
    \vdots & \ddots & \vdots \\
    p_{m1} & \cdots & p_{mm}
\end{bmatrix},$$

where $0 \leq p_{ij} < 1$ and $\sum_{j=1}^{m} p_{ij} = 1$ for all $i \in \{1, \ldots, m\}$. $x_t$ is serially correlated fiscal noise, with persistence $\rho$ and disturbance $\varepsilon \sim \mathcal{N}(0, \sigma^2)$ where $E[\varepsilon_t \nu_t] = 0$.

The decomposition of the perception of the prevailing debt target, $by^*_t$, between its long-run component, $\bar{y}(s_t)$, and the short-run fiscal noise, $x_t$, is unknown to the household. This presents the household with a signal extraction problem that they must solve to make optimal decisions and form expectations conditional on their belief about the long-run debt target state, $s_t$.

The aggregate resource constraint is given by

$$c_t + i_t + \bar{g} = y_t.$$  

(13)

The transversality conditions omit explosive sequences for capital and debt and are given by

$$\lim_{T \to \infty} \beta^T E_t \{ (c_T)^{-\sigma} k_T \} = 0 \quad \text{and} \quad \lim_{T \to \infty} \beta^T E_t \{ (c_T)^{-\sigma} b_T \} = 0.$$  

A competitive equilibrium consists of prices, $\{w_t, r_t^k, r_t\}_{t=0}^\infty$, quantities, $\{c_t, n_t, k_t, i_t, y_t\}_{t=0}^\infty$, government policies, $\{\tau_t, b_t, \bar{g}, \bar{z}\}_{t=0}^\infty$, and exogenous sequences, $\{\varepsilon_t, \nu_t, s_t, x_t\}_{t=0}^\infty$, that satisfy the household’s and firm’s optimality conditions, the government’s budget constraint, the fiscal policy rules, the asset, labor, and goods markets’ clearing conditions, and the transversality conditions.
We embed a Bayesian learning algorithm into a simulation procedure that uses a global solution to the model in section 3 and study the household’s behavior across different assumptions on their information set, given in table 1. In cases 1 and 3, the household does not know the past or current long-run debt targets and infers the current state using the filter described in Hamilton (1989). In cases 2 and 3, the household does not know the transition matrix of the long-run debt target and uses importance sampling to estimate the probabilities following the procedures in Gelman et al. (2004) and Geweke (2005). Thus, the household forms expectations about future after-tax returns to capital to make optimal consumption/saving decisions, conditional on their information set.

A formal description of the information sets begins with writing the model compactly as

$$E_t[f(v_{t+1}, v_t)|\Omega^\ell_t] = 0,$$

where $f$ is vector-valued function that represents the structure of the model that is known to the household, $v_t = (k_{t-1}, b_{t-1}, r_{t-1}, y_{t-1}, c_t, n_t, \hat{\imath}_t, \tau_t, w_t, r^k_t)$, and $\Omega^\ell_t$ is the household’s information set in case $\ell \in \{0, 1, 2, 3\}$. With $m = 3$, the (required) information sets are defined as follows,

$$\begin{align*}
\Omega^0_t & \equiv \{P, p_{11}, p_{22}, p_{33}, s_t\} & \text{(Full Information)} \\
\Omega^1_t & \equiv \{P, p_{11}, p_{22}, p_{33}, \hat{s}_t, (by^*)^t\} & \text{(Case 1)} \\
\Omega^2_t & \equiv \{P, \hat{p}_{t,11}, \hat{p}_{t,22}, \hat{p}_{t,33}, s^t\} & \text{(Case 2)} \\
\Omega^3_t & \equiv \{P, \hat{p}_{t,11}, \hat{p}_{t,22}, \hat{p}_{t,33}, \hat{s}^t, (by^*)^t\}, & \text{(Case 3)}
\end{align*}$$

where $P \equiv \{\sigma, \eta, \chi, \delta, a, \alpha, \gamma, \bar{\tau}(1), \bar{\tau}(2), \bar{\tau}(3), b(1), b(2), b(3), \sigma_0^2, \sigma_1^2, \rho\}$ is the set of model parameters shared by all information sets, $z^t = \{z_i\}_{i=0}^T$ for $z \in \{by^*, s, \hat{s}\}$ are histories of observations required for the Hamilton filter and importance sampler, $\hat{s}_t$ is the inferred state from the Hamilton filter, and $\{\hat{p}_{t,11}, \hat{p}_{t,22}, \hat{p}_{t,33}\}$ are the time-$t$ estimates of the transition probabilities from the importance sampler. We assume the household knows the true transition matrix is symmetric such that $p_{ij} = (1 - p_{ii})/2$ and estimates only the diagonal entries. Since the estimates are time dependent in cases 2 and 3, the household must solve for the optimal policy functions every period, changing the weights on state dependent outcomes in expectation accordingly.

To initialize the learning algorithm, we generate a random sequence of $T$ debt target states, $\{s^T_t\}_{t=1}^T$, which determine the intercepts, $\{by(1) + \Delta(s_t - 1)\}_{t=1}^T$, of the debt target process, given in (11). There is also a random sequence of debt target shocks, which determines the amplitude of the serially correlated fiscal noise, $\{x^T_t\}_{t=1}^T$, and obscures the debt target state. Using its tax rule,

---

This specification of the tax rule, which assumes that taxes endogenously respond to government debt, has become fairly standard in the fiscal policy literature. It is supported by the empirical findings of Bohn (1998), who contends that the U.S. primary surplus is an increasing function of the U.S. debt-to-GDP ratio.
In cases 1 and 3, the fiscal noise surrounding the long-run debt target imposes a signal extraction problem that the household solves each period to form their beliefs about the state. The household observes the tax rate and lagged debt-to-GDP ratio at the beginning of each period. Given their perception of the debt target state, \( b_t^* \), and their prior belief about the transition matrix, the household applies the Hamilton (1989) filter to (11) and chooses the state with the highest likelihood to form their current belief (see appendix A.1 for details).

In case 1, the household’s prior belief about the transition matrix equals the true probabilities used to generate the sequence of debt target states. However, for cases 2 and 3 the household uses importance sampling from a Dirichlet distribution to update their prior belief about the probabilities (see appendix A.2 for details). At the beginning of the simulation, we assume the prior probabilities are uninformative, so the distribution of their inferences of the debt target states is uniform across all Monte Carlo simulations.

In cases 2 and 3, the household re-optimizes (i.e. solves for new policy functions) after updating their estimates of the probabilities, since the probabilities affect their expected marginal returns to capital. Given that the model permits discrete changes in the long-run debt target, which cause the tax rate to deviate far from its initial steady state, and that the household’s expectations of these sudden policy shifts are critical to their inferences, we solve the nonlinear model with a fixed-point projection method (with linear interpolation and Gauss Hermite quadrature) that yields a global solution [Richter et al. (2012)]. This solution technique simultaneously solves for the optimal policy functions at each point in the discretized state space. Specifically, it assumes the current policy functions hold at current and future periods and uses the Euler equations to back out the updated policy functions. The primary advantage of this procedure is that it does not require a nonlinear solver to find the current policies, which permits us to parallelize the simulations.

In all cases, the household makes their optimal choices conditional on their inference of the current debt target state, regardless if it is correct. The fiscal authority uses the updated debt-to-GDP ratio, its current debt target state, and the discretionary tax shock to set the tax rate in the next period. The household uses this new information when filtering and sampling to hopefully improve their understanding of the debt target state and the behavior of fiscal policy.

We calibrate the model at an annual frequency to study learning and the consequences of uncertainty over several decades. The baseline calibration, given in Table 2, is consistent with the RBC literature. The discount factor, \( \beta \), is set to equal 0.9615, which corresponds to a 4% annual real interest rate. The annual depreciation rate, \( \delta \), is set to equal 10 percent and the cost share of capital, \( \alpha \), is set to equal 0.33. We set the coefficient of relative risk aversion, \( \sigma \), to equal 1, implying log utility in consumption. The Frisch elasticity of labor supply, \( 1/\eta \), is set to equal 0.5,
which is consistent with the value used by the CBO and the findings of Chetty et al. (2011).

The leisure preference parameter, $\chi$, implies a steady state share of time spent working equal to 0.33 in the middle debt target state. The level of technology, $a$, is set so that output is equal to 1 in the middle debt target state. We fix $\chi$ and $a$ in the alternative states and solve for the implied steady state values of taxes, labor, and output that are consistent with the long-run debt targets.

The ratios of government expenditures/output and transfers/output are set to equal 8 percent and 9 percent, which match post-WWII U.S. averages. We assume there are three debt target states: low, mid, and high. They are set to equal 60, 75, and 90 percent of output, which correspond to the House Republican’s 2012 deficit reduction proposal (and the CBO’s Baseline projection), the President’s 2012 budget, and the CBO’s alternative fiscal scenario. The strength of the fiscal response to changes in debt, $\gamma$, is set to equal 0.25 so that a switch in the true long-run debt target takes the economy approximately 10 years to adjust to its new equilibrium in the full information case, which is consistent with Congress’s planning horizon. It also guarantees a sufficient response by the fiscal authority to ensure stable long-run debt dynamics.

Figure 3a demonstrates the speed of convergence of the debt-to-GDP percentage after a switch in the long-run debt target from mid (75) to low (60). The 2012 House proposal is provided for comparison. The other shocks have been turned off, but the hump-shape in the House proposal is possible given the appropriate changes in discretionary tax policy. When $\gamma = 0.25$ the speed of convergence matches closely with the House’s 10-year proposal. The implied 10-year target also matches closely. In the model, the long-run debt target is approached asymptotically, so there is a trade-off between matching the 10-year target and the speed of convergence. Thus, we further justify our choice of $\gamma$ by examining the short-run adjustment in the tax rate implied by the endogenous tax rule. A short-run adjustment of 3% is reasonable considering the 2012 American Taxpayer Relief Act already changes marginal rates by similar amounts (the top marginal income tax rate rose by 4.6 and the payroll tax rose by 2 percentage points), but not enough to achieve the low debt target. Thus, an increase in the average income tax rate of 3 percentage points, as shown in figure 3b, seems consistent with current deficit reduction proposals.

The true transition matrix is set so that each debt target state has an expected duration of 5 years ($p_{ii} = 0.8$) and there are equal probabilities of switching to the other states ($p_{ij} = 0.1$ for $i \neq j$). We choose five years to coincide with the persistence of current law (figure 2b) and the average number of years served by a president. We justify a symmetric transition matrix for two reasons. First, an asymmetric distribution would bias the results in one direction. We show there are consequences to fiscal uncertainty even with a symmetric distribution. Second, current policy does not indicate that one regime is more likely, due to the array of policy proposals.

We estimate the process governing the fiscal noise, (12), using the de-trended log of the House Polarization index shown in figure 1b from 1977 to 2011. We find that the process is persistent with $\rho$ equal to 0.86221. In the baseline calibration, the standard deviation, $\sigma_{\epsilon}$, is equal to 0.050655 so that the distance between the debt target intercepts, $\Delta$, is 1.5 times the standard deviation of (12). Under this calibration the household cannot easily distinguish between fiscal noise shocks and changes to the debt target state. For example, conditional on the mid debt target, there is at least a 45 percent chance that a fiscal noise shock yields a signal that is more than half way to the

---

$^3$Since 2007, the CBO has published its debt-to-GDP ratio projections based on their baseline and alternative fiscal financing scenarios. Comparing these projections to the actual debt-to-GDP path from 2007-2012 reveals significant error. Thus, it is possible that households are conditioning on an even wider set of possible debt targets. This is another reason that our results represent a floor on the costs of fiscal uncertainty.
neighboring long-run target. We compare our measures of learning and welfare to a low noise case, where \( \sigma_\varepsilon \) equals 0.025328 so that \( \Delta \) is 3 times the standard deviation of (12).

5 Results

We present three distinct sets of results. The first set is based on a unique sequence of debt target states, fiscal noise shocks, and discretionary tax shocks. It shows how uncertainty across the three alternative information sets, given in table 1, impacts the paths of key aggregate variables relative to the full information case and describes the sources of the differences in detail. The second and third sets of results are based on 5000 Monte Carlo simulations of the model. These results show two alternative measures of learning and the welfare consequences of the uncertainty across two levels of fiscal noise.

5.1 Uncertainty Channels

Figure 4 highlights the differences between the paths of output, consumption, capital, and labor hours in the three limited information cases as a percentage deviation from the full information case. The purpose of this exercise is to illustrate how limited information changes the household’s decisions relative to when they have full information. Uncertainty stemming from limited information about the current long-run debt target and its transition probabilities influences the business cycle by regularly moving output by as much as 0.1 percent (and sometimes more) relative to the full information case over the 100 year simulation. Thus, uncertainty about the long-run debt target explains a similar amount of the business cycle as Fernandez-Villaverde et al. (2011) attribute to stochastic volatility shocks to fiscal policy.

The paths shown in figure 4 for cases 1 (solid line) and 3 (dash-dotted line) contain sequences of spikes that correspond to intervals of incorrect inferences of the long-run debt target, which are shaded over the path of output in figure 5. A dark-shaded/blue (light-shaded/red) region means the inferred long-run debt target is higher (lower) than the true value. The direction is important to note because it has opposite implications for the household’s expectation of the future tax rate. If they believe the long-run debt target is higher (lower) than the truth, then they also believe the future
Figure 4: Paths of the endogenous variables as percentage deviations from the full information case. The paths for case 1 (solid line), case 2 (dashed line), and case 3 (dash-dotted line) are based on identical sequences of debt target states and fiscal noise shocks.

Figure 5: The path of output for case 1 (top) and case 3 (bottom) as a percent deviation from the full information case. The shaded regions indicate intervals of incorrect inferences of the long-run debt target. A dark-shaded/blue (light-shaded/red) region means that the inferred long-run debt target is higher (lower) than the true value. Both panels are based on identical sequences of debt target states and fiscal noise shocks.
short-run tax rate will decrease (increase) next period to allow debt to rise (fall) toward its long-run value. This is why output increases when the belief is above the truth (blue regions) and decreases when it's below the truth (red regions). The relatively higher volatility is due to the household using suboptimal decision rules to form expectations conditional on the wrong state. The magnitude of the deviations is determined by the distance between the debt target states ($\Delta$) and the strength of the fiscal authority’s response to changes in the long-run debt target ($\gamma$). Either a larger $\Delta$ or $\gamma$ would increase volatility. In case 1 after an incorrect inference(s), the household’s beliefs realign with the true long-run debt target and the paths of the endogenous variable converge back to the full information case. In case 3, this is not necessarily true because even if the household’s inference is correct, their beliefs about the transition matrix are incorrect.

The path for case 1 contains fewer incorrect inferences than case 3, which is generally true for any random sequence of states and fiscal noise shocks. There are two important conclusions from figure 5. First, the decisions deviate from the full information case in the same year as the incorrect inference and persist well beyond each interval of incorrect inference. This persistence is the result of the incorrect inference influencing future tax rates through a deviation of the current debt-to-GDP ratio from the full information case. Since the tax rate endogenously responds to the debt-to-GDP ratio, the time it takes output to converge back to the full information path is determined by the responsiveness of the tax rate, given by $\gamma$. A higher $\gamma$ would shorten these deviations from the full information case, but at the expense of higher volatility. In short, the fiscal authority’s planning horizon affects the tradeoff between the volatility and duration of the deviation from the full information case.

Second, the number and locations of the intervals of incorrect inference change between cases 1 and 3. Notice for this sequence that the fourth and fifth intervals of incorrect inference in case 1 are shorter than the ones that begin around the same periods in case 3. Also, there is a long interval of incorrect inference from periods 7 to 19 in case 3 that does not occur in case 1. Thus, feeding the estimates of the transition probabilities (rather than the true probabilities) into the Hamilton filter not only increases the number of periods with incorrect inferences, but it can also change the household’s qualitative conclusions of the prevailing long-run debt targets between cases 1 and 3.

The intuition behind the spikes is illustrated in figure 6, which shows nonlinear impulse responses in case 1 to an incorrect inference of the debt target state relative to the full information case in the absence of fiscal noise and discretionary tax shocks. The tax rate initially corresponds to the middle long-run debt target and the debt-to-GDP ratio is at its corresponding stochastic steady state. Consider the case when the household incorrectly infers a high debt target state in period 1 (solid line). Since the tax rate is based on the true debt target state and the past debt-to-GDP ratio, it does not immediately adjust in the first period. However, since the household believes future taxes will be lower, they increase their labor supply and substitute away from consumption in favor of investment in the first period. Also, they expect to be wealthier in the future and for consumption growth to increase, so the current interest rate must increase to clear the bond market. In period 2, the household realizes their inference of the debt target was incorrect, and decrease their labor supply and investment in anticipation of higher future taxes. Given that the tax rate fell, the higher interest rate from the previous period forces the fiscal authority to debt finance their deficit. In period 3, the tax rate increases in response to the higher level of debt, which further reduces labor and investment. Thus, the learning mechanism causes the endogenous variables to overshoot the stochastic steady state following an incorrect inference of the long-run debt target.

The deviations from the full information paths shown in figure 4 for case 2 (dashed line) are
Figure 6: Impulse responses to an incorrect inference of the debt target state relative to the full information case in the absence of fiscal noise and discretionary tax shocks. The true state is always the mid debt target. Percentages are plotted relative to the full information case. The limits on the vertical axis correspond to the extrema of the paths.

roughly as volatile as case 3. The combination of uncertainty about the state and probabilities in case 3 (dash-dotted line) appear to be the addition of cases 1 and 2. However, the estimates of the transition probabilities differ from the truth, which adversely affects the household’s ability to correctly infer the state using the Hamilton filter. Thus, the percent of periods with incorrect inferences of the long-run debt target is generally higher in case 3 than in case 1. The feedback between these incorrect inferences and the estimates of the transition probabilities cause the paths in case 3 to deviate in different ways than the addition of the paths for cases 1 and 2 imply.

Figure 7 summarizes the likelihood value associated with each debt target state. The square, x, and diamond markers correspond to the likelihood of being in the state with a high, middle, or low long-run debt target. The solid line corresponds to the largest likelihood value across all states in any given period. This represents a first pass at characterizing the uncertainty about the debt target state. The lower its value the more uncertain the household is that the current state is the one given by the highest likelihood value. The drawback with this measure of uncertainty is that it ignores the possibility of the other two states having non-zero likelihood values.

Figure 8 presents an alternative measure of uncertainty. Higher periods of uncertainty generally correspond to intervals of incorrect inference of the state even though the household might make a correct inference by chance in periods of high uncertainty. In each period, the Hamilton filter outputs the probability of being in each state conditional on the household’s perception of the debt target, \( \Pr[s_t = i | (y_t^*)] \). Since there are three states this is a 3-element vector. Absolute uncertainty is defined as the household assigning each state equal probability, \( \Pr[s_t = i | .] = 1/3 \)
Debt Target State Likelihoods

Figure 7: The likelihood of being in each state given by the Hamilton filter in case 3. The solid line represents the upper manifold, i.e., the highest likelihood across all 3 states. The square, x, and diamond markers correspond to the likelihood of being in the state with a high, middle, or low debt target intercept.

Fiscal Uncertainty Index (Case 1)

Fiscal Uncertainty Index (Case 3)

Figure 8: The uncertainty index is the norm difference between the vector of state likelihoods and absolute uncertainty (i.e., \( \left( \sum_{i=1}^{3} (P[s_t = i] - 1/3)^2 \right)^{1/2} \)), as a percentage of the maximum possible norm difference. Thus the uncertainty index ranges between 0, absolute certainty, and 1, absolute uncertainty. The top and bottom panels are based on identical sequences of debt target states and fiscal noise shocks.
for $i \in \{1, 2, 3\}$. In this case, we want the uncertainty index to equal 1. Absolute certainty is defined as the household assigning one state a probability of 1, $\Pr[s_t = i] = 1$ for some $i$. In this case, we want the uncertainty index to equal 0. The norm difference of the likelihood vector between absolute certainty and absolute uncertainty is at its maximum, $\text{norm}_{\text{max}} = \sqrt{(1 - 1/3)^2 + (0 - 1/3)^2 + (0 - 1/3)^2}$. Thus, we employ the following uncertainty index,

$$\frac{\text{norm}_{\text{max}} - \sqrt{(\Pr[s_t = 1] - 1/3)^2 + (\Pr[s_t = 2] - 1/3)^2 + (\Pr[s_t = 3] - 1/3)^2}}{\text{norm}_{\text{max}}}$$

which ranges from 0 (absolute certainty) to 1 (absolute uncertainty) as desired. The intervals of incorrect inferences are superimposed on this uncertainty index. Notice that the average level of uncertainty is greater in case 3 than in case 1. Periods of high uncertainty generally correspond to intervals of incorrect inferences, which alters the predicted business cycles relative to the full information case. However, it is possible that the household could be very confident of their inference when in fact they are incorrect if there is a sequence of unusually large fiscal noise shocks (e.g. periods 44-48). In this case, they are simply mistaking the fiscal noise shocks for a change in the long-run debt target, which causes them to be overconfident.

### 5.2 Measures of Learning

We use two measures, computed from 5000 Monte Carlo simulations of the model, to assess how quickly and how effectively the household learns about the state and dynamics of fiscal policy over time. The first measure is the percent of correctly inferred long-run debt targets per period in limited information cases 1 and 3, where the household does not know the sequence of long-run debt targets. The second measure is the norm difference between the long-run debt target transition probabilities estimated by the household and the true probabilities in limited information cases 2 and 3, where the household does not know the true probabilities. Both measures are available for case 3 since both uncertainty channels are open.

**Figure 9a** plots the percentage of correctly inferred long-run debt targets across time. In case
1, the household knows the long-run debt target transition probabilities, but we randomize their inference of the state in period 0 since they begin the simulation without any data to guide them. In case 3, the household does not know the transition probabilities and starts with an uninformative prior. Thus, they correctly infer the state in period 0 in one-third of the simulations in both limited information cases. Initially, the household in case 1 correctly infers the state in more of the simulations than in case 3 since they know the true transition probabilities. As time evolves, the case 3 household collects data on the long-run debt target, which improves their ability to correctly infer the state. The true probabilities are fed into the Hamilton filter in case 1 starting in period 1, whereas in case 3 the probabilities evolve from the uninformative prior from period 0, which alters the likelihood of each state. Therefore, there is a uniformly higher percentage of correctly inferred states in case 1 relative to case 3. By period 250, the measure converges to about 77% in case 1 compared to 72% in case 3, which reflects the interaction between the two uncertainty channels.

The household’s ability to correctly infer the debt target state is a function of distance between the long-run targets and the variance of the serially correlated noise. Figure 10a demonstrates this by comparing the percentage of correctly inferred debt target states in simulations where the volatility of fiscal noise is reduced from $\sigma_\varepsilon = 0.050655$ (high) to $\sigma_\varepsilon = 0.025328$ (low). High fiscal noise increases the difficulty of the signal extraction problem by changing the distribution of the long-run debt target signal. Plots of the distributions in figure 10b show a near normal distribution under high fiscal noise and a clearly trimodal distribution resulting from low fiscal noise. In the latter case, the household could fairly accurately infer the state even without the assistance of the Hamilton filter. With the distances between states large enough and a low enough variance, the percentage of correctly inferred states converges to over 95% after 100 periods. When faced with higher noise, the inference problem is more complicated, as the percentage falls to slightly above 70%. We only show the first 100 periods to emphasize the difference in the speed of convergence. Subject to high fiscal noise, the household correctly infers the state in 70% of the simulations by period 60, but achieves similar accuracy in less than 10 years when faced with low fiscal noise.
Figure 9b plots the norm difference between the true and estimated transition matrices as a percentage of the maximum norm difference, which captures how long it takes for the household’s beliefs to converge to the truth. This distance is initially high due to the uninformative priors, and converges slower than the first measure. Case 2 converges more quickly than case 3 since the household knows the true debt target state. The only reason it does not immediately converge to 0 in case 2 is because the shaping parameters in the Dirichlet distribution used in the importance sampler are representative of the proportional realizations of the debt target states (appendix A.2). Since the simulations are random, these shaping parameters will approach those corresponding to the data generating process only over a large sample. Thus, the norm difference will converge to 0 for a long enough simulation.

The initial increase in case 3 (figure 9b, solid line) happens through period 5, the average duration of the prevailing long-run debt target. If the initial inference of state is incorrect, then this biases the corresponding diagonal probability in the transition matrix to favor that state, while the other rows remain uninformative. This bias leads to a persistent incorrect inference. Once the household believes they observe a switch in the state, the estimate of the diagonal probability corresponding to staying in the previous (incorrect) state begins to correct itself. Thus, the estimates begin to converge toward the truth as the household receives more of the signal. However, the norm difference in case 3 converges to a transition matrix that is different from the truth, since their information set will always contain incorrect inferences of the state. When faced with lower fiscal noise, this measure will converge to a value closer to the truth. It converges even though the household’s past inferences of the state may be incorrect. In other words, the interaction of these wrong inferences with the shaping parameters of the Dirichlet distribution in the importance sampler does not prevent the household from improving their understanding of fiscal policy dynamics.

5.3 Welfare Distributions

In this section, we quantify the welfare consequences of fiscal uncertainty. First, we describe how to calculate welfare and show the distribution over time for a large number of simulations. Second, we compute 2012 dollar equivalent losses to GDP and consumption goods required to compensate the household for fiscal uncertainty.

Utilizing the welfare cost measure of Schmitt-Grohe and Uribe (2007), we quantify the welfare costs of uncertainty by thinking of the decisions made with limited information as alternative policies to the full information case. Thus, we solve for a $\lambda$ that satisfies

$$E_t W(c_t^{PI}, n_t^{PI}) = E_t W((1-\lambda)c_t^{FI}, n_t^{FI}),$$

where $W(c_t^X, n_t^X) \equiv \sum_{i=t}^{T-1} \beta^{i-t} u(c_i^X, n_i^X)$ and $X \in \{PI, FI\}$ indicate limited and full information sequences. $T$ is the simulation length, so $W$ is the time-$t$ present value of remaining simulation utility. Since the calculation is based on a Monte Carlo simulation, the length of the interval $[t, T]$ must ensure that the present value at $t$ of utility near the end of the simulation is close to zero. We choose $T = 500$, noting that $\beta^{500} \approx 3 \times 10^{-9}$. With $\sigma = 1$, the additively separable specification of utility yields

$$E_t W(c_t^{PI}, n_t^{PI}) = \frac{1 - \beta^{T-t}}{1 - \beta} \log(1 - \lambda) + E_t W(c_t^{FI}, n_t^{FI}).$$

Hence,

$$\lambda = 1 - \exp \left\{ \frac{1 - \beta}{1 - \beta^{T-t}} \left( E_t W(c_t^{PI}, n_t^{PI}) - E_t W(c_t^{FI}, n_t^{FI}) \right) \right\},$$
which represents the fraction of full information consumption goods in the interval \([t, T]\) required to equate the household’s present value of remaining simulation utility between the limited and full information cases. A positive (negative) \(\lambda\) represents a welfare cost (benefit) in the limited information case relative to the full information case.

Before analyzing the welfare distributions, it is important to make clear that fiscal uncertainty may lead to welfare benefits. In the full information case, the household knows the current debt target state but not the future state. This means the household must form expectations over three possible realizations to inform their optimal consumption/saving decision. When the debt target state is unknown, the household’s expectations are potentially misaligned with the full information case, which may lead to welfare benefits from upside risk. For example, if the household incorrectly infers a debt target at the beginning of the simulation, then it believes taxes will certainly be lower in the near future (allowing debt to rise) and will respond by saving more than the optimal choice implied by the full information case. If realizations of the future debt target state turn out to be exceptionally high (meaning taxes are exceptionally low) relative to what a full information household expects on average, then the limited information household will benefit from higher after-tax returns to capital in the form of higher utility. Over-accumulation of capital when the debt target state is unknown also increases future labor productivity. Thus, the limited information household does not need to supply as much labor to be as productive as the full information household, which allows them to enjoy more leisure. These two points imply that fiscal uncertainty imposes the type of two-sided risk that is briefly described in Fernandez-Villaverde et al. (2011).

Figure 11 shows the median, 10th, and 90th percentile bands of the distribution of welfare costs in case 3 in terms of the percent of full information consumption goods forgone over the remaining simulation and compares these distributions when fiscal noise is high (figure 11a) and low (figure 11b). For both cases, welfare volatility increases in the beginning, since the amount of uncertainty is greatest, as measured by the number of incorrect long-run debt target inferences and rapidly changing transition probability estimates. However, the present value of remaining
simulation utility under low fiscal noise is closer to the full information case. The percentile bands (dashed lines) taper toward zero as the influence of the uninformative prior in the importance sampler diminishes and the household’s inferences of the long-run debt target become more accurate. Notably, reducing the fiscal noise by half does not reduce the variance of welfare costs by half, and the distribution remains asymmetric with welfare benefits smaller in magnitude than the costs.

The potential welfare benefits are always less than the welfare costs for two reasons. First, the tax is proportionally levied against income, so the percentage reduction in the tax rate necessary to transition from a high to a low debt target is greater in magnitude than the percentage increase to transition from low to high. Second, and more importantly, since the prior belief of the transition matrix is uninformative, there is a high variance of the outcomes in the household’s expectations. This induces cautious behavior; households reduce investment to avoid potentially higher tax rates. In the 10th percentile, welfare costs reach their maximum of 0.14 and 0.12 percent of full information consumption goods for high and low fiscal noise around period 30, whereas the welfare benefits are noticeably smaller, −0.07 and −0.06 respectively. Thus, a reduction in fiscal noise does more to avoid potential welfare costs than it does to eliminate any benefits from uncertainty.

5.4 QUANTIFYING UNCERTAINTY  We look at two measures that quantify the costs of the uncertainty surrounding the long-run debt target. The first is the 2012 dollar equivalent loss in real GDP over the next 10 years. Second, once we establish 2012 dollar equivalent full information GDP, we use our calculated welfare costs to obtain a dollar equivalent amount of real consumption goods that would compensate the household for the next 10 years of uncertainty (assuming high fiscal noise). This would make them indifferent between case 3 and the full information case. Since there are a wide range of possibilities, we focus on the losses at the 10/50/90 percentiles across the 5000 simulations. This yields numbers we can compare to, for example, the $1.2 trillion (over 10 years) sequestration component of the Budget Control Act of 2011.

Figure 12 shows the distributions of output and consumption in case 3 as percent deviations
This section relaxes the assumption that changes in government spending. More specifically, government consumption now endogenously responds to lagged debt according to

$$g_t = \bar{g}(s_t) - \gamma_g(b_{t-1}/y_{t-1} - \bar{y}(s_t)) + \nu_t,$$

where $\bar{g}(s_t)$ is a state dependent intercept of the spending rule that is consistent with the stationary equilibrium implied by the prevailing long-run debt target, $\bar{y}(s_t)$. The government consumption

---

4We also performed similar calculations treating the real GDP forecasts in the CBO’s 2012 Long Term Budget Outlook as equivalent to full information (since they do not model fiscal uncertainty). This alternative approach yielded slightly larger costs at the 10/50 percentiles and slightly larger benefits at the 90 percentile.
shock, $\nu \sim N(0, \sigma^2_\nu)$, is a proxy for discretionary spending policy. This specification implies that when the debt target state is unknown, the long-run level of government spending is also unknown.

We set the tax and government consumption intercepts, $\bar{\tau}(s_t)$ and $\bar{g}(s_t)$, so that changes in tax revenue and spending each account for half of the adjustment necessary to service a change in long-run debt. To remain consistent with our baseline calibration, we set $\gamma_\tau$ and $\gamma_g$ so that the economy takes approximately 10 years to transition to its new long-run equilibrium following a switch in debt target state and so the speed of adjustment is consistent with the proposals (figure 13a). Along the transition path, changes in tax revenue and government spending each finance roughly half of the debt service (figure 13b).

Figure 14 shows nonlinear impulse responses to an incorrect inference of the debt target when both the income tax rate and government consumption respond to the lagged debt-to-GDP ratio. An incorrect inference of the debt target state means that the household believes future taxes and government consumption will adjust. Changes in expected taxes generate the same dynamics that appear in figure 6, except that the magnitude is smaller since taxes only finance part of the debt service. Changes in expected government consumption account for the remaining dynamics.

Adding uncertainty about the long-run level of spending amplifies the impulse responses relative to the case where only the tax rate is endogenous, since changes in expected government consumption have a larger effect than changes in expected taxes. To understand why, it is first important to note that the initial response of output to a government consumption shock in an RBC model is less than one, and considerably less than one if the shock is transitory [Campbell (1994), Baxter and King (1993)]. Thus, a positive government consumption shock reduces the amount of output available to the household, and they must reduce consumption and investment. However, if the household has foresight over future shocks, then an anticipated positive shock would lead them to reduce current consumption and investment to smooth consumption. We think about a change in the belief of the debt target as we do with foresight. If the household incorrectly infers a high debt target, when in fact it is mid, then they anticipate an increase in government consumption to meet the higher debt target, which crowds out future consumption. This anticipation leads them to...
Figure 14: Impulse responses to an incorrect inference of the debt target state relative to the full information case in the absence of fiscal noise, discretionary tax, and discretionary spending shocks. The true state is always the mid debt target. Percentages are plotted relative to the full information case. The limits on the vertical axis correspond to the extrema of the paths.

smooth consumption by increasing investment today, even though their inference is incorrect.

As the impulse responses indicate, allowing government spending to partially finance a change in the long-run debt target amplifies movements in all variables. Table 3 shows this results in real loss in GDP over 10 years of $842 billion and $200 billion at the 10th and 50th percentiles, a factor of 2.3 and 3.5 times the losses when only the tax rate is endogenous. Notably, there is a considerably larger upside of $403 billion, 1.7 times the best case outcome with only taxes. Government consumption is partly responsible for financing changes in the debt target, which increases the asymmetry over the multiplicative tax rate. Changes in expectations (due to the evolving estimates of the transition probabilities) are important for both future tax rates and government spending, and the presence of government spending results in a more asymmetric distribution.

6 Conclusion
This paper analyzes the effects of uncertainty about long-run debt targets on the business cycle and quantifies its welfare consequences. We extend a standard RBC model by introducing a fiscal authority that targets one of three possible long-run debt targets by varying a proportional tax on capital and labor income. The targets we choose are consistent with the alternative policy proposals. We assume the household understands the possible outcomes of fiscal policy but limit their information set, so there is uncertainty about the prevailing long-run debt target and the probabilities that govern its evolution. Our main conclusions are as follows:
Only Taxes

<table>
<thead>
<tr>
<th>Percentile</th>
<th>10</th>
<th>50</th>
<th>90</th>
<th>10</th>
<th>50</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP Change</td>
<td>−372</td>
<td>−57</td>
<td>242</td>
<td>−842</td>
<td>−200</td>
<td>403</td>
</tr>
<tr>
<td>Cons. Goods to Compensate</td>
<td>141</td>
<td>27</td>
<td>−70</td>
<td>268</td>
<td>73</td>
<td>−103</td>
</tr>
</tbody>
</table>

Table 3: 2012 dollar (in billions) equivalent 10-year consequences of fiscal uncertainty.

1. Fiscal uncertainty imposes two-sided risk. When the household overinvests in capital relative to the full information case and the future debt target state is fortuitously high, the household benefits in the form of higher welfare. Reducing fiscal noise results in a reduction of the median welfare costs across time. However, median welfare costs decrease less than one-for-one with a decrease in the standard deviation of the fiscal noise shock.

2. The feedback between incorrect inferences of the state and the estimates of the transition probabilities cause the paths in case 3 to deviate from the full information case in different ways than the addition of the paths for cases 1 and 2 would suggest. In case 3, uncertainty stemming from limited information about the current long-run debt target and its transition probabilities influences the business cycle by regularly moving output by as much as 0.1 percent (and sometimes more) relative to the full information case, which is consistent with Fernandez-Villaverde et al. (2011). Also, case 3 results in a median 10-year loss in real GDP (in 2012 dollars) of $57 billion, while households require $27 billion in consumption goods to compensate them for lost utility from the next 10 years of fiscal uncertainty. When we allow government spending to partially finance a switch in the debt target, the welfare distribution becomes more asymmetric, the variance doubles, and $73 billion in consumption goods would compensate them for lost utility at the median.

3. The household’s ability to learn is a function of the fiscal noise, which obscures the long-run debt target. More fiscal noise results in more incorrect inferences of the true long-run debt target and estimates of the transition probabilities that are further from the truth. Despite not knowing the transition probabilities, the household’s estimates converge toward the truth over time. They are always able to improve their understanding of fiscal policy dynamics, even when their information set contains incorrect inferences of the long-run debt target.

The main contribution of this paper is to model the uncertainty surrounding long-run debt targets by assuming that households have limited information about the goal of fiscal policy. This is motivated by the number and variety of actual policy proposals. What probability distribution should households place on these potential long-run targets, conditional on the degree of polarization in Congress? In such an environment, they may understand that passing a budget to achieve any one of the targets is difficult and affects the likelihood of whether a debt-to-GDP path like the CBO’s alternative fiscal scenario or the status quo occurs. Therefore, we propose that, while the outcomes are understood, the probability distribution of long-run debt targets is uncertain. Thus, households learn about fiscal policy as time unfolds. The welfare costs of this type of uncertainty increase as it becomes more difficult for households to accurately understand fiscal policy. We show that the fiscal authority can reduce the welfare costs by giving households a clearer signal about the true long-run debt target.
REFERENCES


25


A TECHNICAL APPENDIX

A.1 HAMILTON FILTER  In cases 1 and 3, the household does not know the debt target state. This section outlines the filter in Hamilton (1989), which is used by the household to infer the latent debt target state. The filter takes as input a vector of conditional probabilities,

\[ \Pr[s_{t-1} = i | by^*_t, \ldots, by^*_0], \]

where \( i \in \{1, \ldots, m\} \) with \( m \) realizations of the debt target (latent) state, \( s \). \( by^*_t \) are observations of the household’s perception of the debt target, given by,

\[ by^*_t = \overline{by} + \Delta(s_t - 1) + x_t, \]
\[ x_t = \rho x_{t-1} + \varepsilon_t, \]

where \( \Delta \geq 0 \) is the distance between debt target intercepts, \( \overline{by} \) is the smallest intercept, \( x_t \) is serially correlated noise, and \( \varepsilon \sim N(0, \sigma^2). \) The filter outputs the updated conditional probabilities,

\[ \Pr[s_t = j | by^*_t, \ldots, by^*_0], \]

where \( j \in \{1, \ldots, m\} \), and the conditional likelihood of \( by^*_t \),

\[ f(by^*_t|by^*_t-1, \ldots, by^*_0). \]

To apply the filter, the probability density of \( \varepsilon_t \) is normally distributed so that

\[ f(by^*_t|s_t = j, s_{t-1} = i, by^*_t-1, \ldots, by^*_0) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{\varepsilon_t^2}{2\sigma^2} \right), \]

where \( \varepsilon_t = (by^*_t - \overline{by} - \Delta(s_t - 1)) - \rho(by^*_{t-1} - \overline{by} - \Delta(s_{t-1} - 1)) \). The filter’s output is obtained with the following sequence of calculations for all \( i, j \):

1. Calculate the joint probability of \( (s_t = j, s_{t-1} = i) \) conditional on past \( by^* \),
   \[ \Pr[s_t = j, s_{t-1} = i | by^*_t-1, \ldots, by^*_0] = \Pr[s_t = j | s_{t-1} = i] \Pr[s_{t-1} = i | by^*_t-1, \ldots, by^*_0]. \]

2. Calculate the joint conditional density-distribution of \( by^*_t \) and \( (s_t = j, s_{t-1} = i) \),
   \[ f(by^*_t, s_t = j, s_{t-1} = i | by^*_t-1, \ldots, by^*_0) = f(by^*_t|s_t = j, s_{t-1} = i, by^*_t-1, \ldots, by^*_0) \Pr[s_t = j, s_{t-1} = i | by^*_t-1, \ldots, by^*_0]. \]

3. Calculate the likelihood of \( by^*_t \) conditional on its history,
   \[ f(by^*_t|by^*_t-1, \ldots, by^*_0) = \sum_{j=1}^{m} \sum_{i=1}^{m} f(by^*_t|s_t = j, s_{t-1} = i | by^*_t-1, \ldots, by^*_0). \]

4. Calculate the joint probabilities of \( (s_t = j, s_{t-1} = i) \) conditional on current and past \( by^* \),
   \[ \Pr[s_t = j, s_{t-1} = i | by^*_t, \ldots, by^*_0] = \frac{f(by^*_t, s_t = j, s_{t-1} = i | by^*_t-1, \ldots, by^*_0)}{f(by^*_t|by^*_t-1, \ldots, by^*_0)}. \]

5. Calculate the output by summing the joint probabilities over the realizations \( s_{t-1} \),
   \[ \Pr[s_t = j | by^*_t, \ldots, by^*_0] = \sum_{i=1}^{m} \Pr[s_t = j, s_{t-1} = i | by^*_t, \ldots, by^*_0]. \]

The household’s inference of the debt target state, \( \hat{s}_t \), equals the argmax of \( \Pr[s_t = j | by^*_t, \ldots, by^*_0] \).
A.2 Importance Sampler In cases 2 and 3, the household does not know the probabilities of the transition matrix. However, they know there is a time invariant transition matrix, \( P \), with stationary distribution, \( \pi \). This section outlines the importance sampler, which the household uses to estimate the transition probabilities. The likelihood of observing \( s^t \equiv \{s_i\}_{i=0}^t \) is

\[
p(s^t|\pi, P) = \left( \prod_{j=1}^{3} \pi_j(P)^{1j} \right) \left( \prod_{i=1}^{3} \prod_{j=1}^{3} p_{ij}^{n_{ij}} \right) = \prod_{j=1}^{3} \pi_j(P)^{1j} \prod_{i=1}^{3} p_{ij}^{n_{ij}} \prod_{j=1}^{3} p_{oij}^{n_{ij}},
\]

where \( \pi_j \) is the prior belief about the stationary distribution, \( 1_j \in \{0,1\} \) is an indicator for whether state \( j \) is occupied by the household at time \( 0 \), \( p_{ij} \) is the probability corresponding to row \( i \) and column \( j \) of the transition matrix, and \( n_{ij} \) is the number of observed transitions from states \( i \) to \( j \). Following Geweke (2005), the conjugate prior follows a Dirichlet distribution given by

\[
p(\theta) = \left[ \prod_{j=1}^{3} \Gamma \left( \sum_{i=1}^{3} a_{ij} \right) \right] / \left[ \prod_{i=1}^{3} \prod_{j=1}^{3} \Gamma(a_{ij}) \right] \left( \prod_{i=1}^{3} \prod_{j=1}^{3} p_{ij}^{a_{ij}-1} \right),
\]

where \( a_{ij} > 0 \) are the shaping parameters of the prior distribution, and \( \Gamma \) is the gamma function. The posterior density is given by the product of the likelihood function, (15), and the prior density, (16). Thus, dropping the constants of proportionality, the posterior density is

\[
p(P_t|s^t) \propto \left( \prod_{j=1}^{3} \pi_j(P_t)^{1j} \right) \left( \prod_{i=1}^{3} \prod_{j=1}^{3} p_{ij}^{a_{ij}+n_{ij}-1} \right),
\]

where \( s^t \) is the sequence of states the household believes they occupied. The number of observed transitions, which are calculated after inferring the current state, \( s_t \), determine the shaping parameters of the posterior distribution. Thus, as time unfolds the household’s estimates of the probabilities converge toward the true probabilities.

The posterior distribution, (17), does not correspond to any standard density function that we can sample from directly. However, the second component is a product of two independent Dirichlet probability density functions. Consequently, we utilize the following importance sampling algorithm to sample from this distribution. First, sample \( L \) draws from a Dirichlet distribution with parameters \( a_i + n_i \), where \( a_i = [a_{i1}, a_{i2}, a_{i3}] \) and \( n_i = [n_{i1}, n_{i2}, n_{i3}] \) for \( i = \{1,2,3\} \). To accomplish this, we follow the procedure outlined in Gelman et al. (2004, Appendix A). Draws are made from a Gamma distribution with parameters \( ((a_i + n_i)/2, 2) \), and then weighted by the sum of the draws corresponding to each state, \( i \). That is, using the draws, \( x_{ij}^\ell \), from a Gamma distribution, a draw from a Dirichlet distribution is \( \theta_{ij}^\ell = x_{ij}^\ell / \sum_{i=1}^{3} x_{ij}^\ell \). Second, the draws from the Dirichlet distribution are weighted by the coefficient of the posterior distribution, \( w_{\ell} \equiv \prod_{j=1}^{3} \pi_j(P_t^\ell)^{1j} \) for all \( \ell \in \{1,\ldots,L\} \), and then divided by the sum of the weights. Formally, the weighting procedure produces an estimate of the transition probabilities (draw from the posterior distribution),

\[
\hat{p}_{ij} = \frac{\sum_{\ell=1}^{L} w_{\ell} \theta_{ij}^\ell}{\sum_{\ell=1}^{L} w_{\ell}}.
\]

Alternatively, we could have drawn from the posterior distribution by applying the independence Metropolis-Hastings algorithm or acceptance sampling. Since we use a representative agent model, the efficiency gains from these alternative sampling algorithms is negligible.