Appendix: Technical Details for Estimating Industry and Worker-Type Composition and Adjusting Measures of Hourly Earnings and Hours Worked

In this appendix, sections 1 and 2 provide the mathematical and technical details used to compositionally adjust hours worked and average hourly earnings measures from the US Bureau of Labor Statistics' (BLS) establishment survey data. Section 3 compares the compositional adjustments described here with comparable measures from the BLS and the Federal Reserve Bank of San Francisco. The section also shows the impact of controlling for industry and worker-type when constructing the compositional adjustment rather than controlling for industry only.

Section 1: Mathematical details

Let *i* denote a detailed North American Industry Classification System (NAICS) industry or industry group, let *j* denote a type of worker that is either "all workers," "production/nonsupervisory workers," or "nonproduction/supervisory workers." Let *t* denote a particular month. We define the following quantities:

$W_{i,j,t}$: Average hourly earnings ($s) for type *j* workers in industry *i* in month *t*.

$H_{i,j,t}$: Aggregate weekly hours worked for type *j* workers in industry *i* in month *t*.

$P_{i,j,t}$: Aggregate weekly payrolls ($s) worked for type *j* workers in industry *i* in month *t*.

The US Bureau of Labor Statistics has published data for these variables for all workers since March 2006 and production/nonsupervisory workers since January 1990. For supervisory workers who are not also production workers, we define the above variables in the obvious way:

$$P_{i,Sup,t} = P_{i,All,t} - P_{i,Prod,t}\,, H_{i,Sup,t} = H_{i,All,t} - H_{i,Prod,t}, W_{i,Sup,t} = \frac{P_{i,Sup,t}}{H_{i,Sup,t}}$$

We also define

W_t^{BLS}: Published average hourly earnings ($s) for all private nonfarm payroll workers in month *t*.

H_t^{BLS}: Published aggregate weekly hours worked for all private nonfarm payroll workers in month *t*.

P_t^{BLS}: Published aggregate weekly payrolls ($s) for all private nonfarm payroll workers in month *t*.

$h_{i,j,t} = \frac{H_{i,j,t}}{\sum_{i,j} H_{i,j,t}}$: Month *t* share of aggregate hours for type *j* workers in industry *i*.

$\Delta h_{i,j,t} = h_{i,j,t} - h_{i,j,t-1}$: Monthly change in share of aggregate hours for type *j* workers in industry *i*.

Then, one can decompose one plus the monthly growth rate of average aggregate monthly earnings as

$$(1)\ \frac{W_t^{BLS}}{W_{t-1}^{BLS}} = \frac{\frac{P_t^{BLS}}{H_t^{BLS}}}{\frac{P_{t-1}^{BLS}}{H_{t-1}^{BLS}}} \approx \frac{\frac{\sum_{i,j} P_{i,j,t}}{\sum_{i,j} H_{i,j,t}}}{\frac{\sum_{i,j} P_{i,j,t-1}}{\sum_{i,j} H_{i,j,t-1}}} = Q_t^W \sqrt{\left(\frac{\sum_{i,j} W_{i,j,t} H_{i,j,t}}{\sum_{i,j} W_{i,j,t-1} H_{i,j,t}}\right)\left(\frac{\sum_{i,j} W_{i,j,t} H_{i,j,t-1}}{\sum_{i,j} W_{i,j,t-1} H_{i,j,t-1}}\right)}$$

where the composition (quality) adjustment factor Q_t^W is defined as

$$(2) Q_t^W = \sqrt{\left(1 + \frac{\sum_{i,j} W_{i,j,t-1} \Delta h_{i,j,t}}{\sum_{i,j} W_{i,j,t-1} h_{i,j,t-1}}\right)\left(1 + \frac{\sum_{i,j} W_{i,j,t} \Delta h_{i,j,t}}{\sum_{i,j} W_{i,j,t} h_{i,j,t-1}}\right)}$$

The approximation symbol in equation (1) is a consequence of the rounding conventions that the BLS uses when constructing measures of aggregate hours worked.[1] The square root term in the right-hand side of equation (1) is the wage-based analog to the Fisher ideal price index that the US Bureau of Economic Analysis (BEA) uses to construct the price index for personal consumption expenditures (PCE), where wages are analogous to consumption prices and hours worked are analogous to real spending on a particular category of goods or services.[2] One can see that the composition adjustment factor in equation (2) will be identical to one if shares of hours worked are constant across industry groups and worker types. It is always the case that $\sum_{i,j} \Delta h_{i,j,t} = 0$. During recessions, and especially in April 2020, higher-wage workers have tended to have $\Delta h_{i,j,t} > 0$ and lower wage workers have tended to have $\Delta h_{i,j,t} < 0$. In such cases, $Q_t^W > 1$ and one plus the monthly growth rate of published average hourly earnings will be larger than the Fisher ideal wage term in equation (1). Because of the approximation symbol in (1), we define composition adjusted wage growth as

$$(3)\ \frac{W_t^{QAdj}}{W_{t-1}^{QAdj}} = \left(\frac{1}{Q_t^W}\right)\frac{W_t^{BLS}}{W_{t-1}^{BLS}}$$

One can also decompose growth in the published measure aggregate hours worked as

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[1] In particular, for any particular industry or industry group, the BLS constructs published aggregate weekly hours worked in that industry (group) as the product of the published number of workers and the published estimate of average weekly hours worked **after** rounding average hours to one decimal place. This method implies that for a given subindustry decomposition of a particular industry, the sum of the published levels of aggregate hours worked and aggregate payrolls will not equal the sum of published aggregate hours worked or aggregate payrolls of the larger industry, even with nonseasonally adjusted data.

[2] The BLS's Employment Cost Index and the Australian Bureau of Statistics' Wage Price Index are both conceptually similar to the Consumer Price Index. See https://www.bls.gov/opub/mlr/2001/09/art1full.pdf and https://www.abs.gov.au/websitedbs/D3310114.nsf/home/Wage+Price+Indexes+FAQs .

$$(4)\ \frac{H_t^{BLS}}{H_{t-1}^{BLS}} \approx \frac{\sum_{i,j} H_{i,j,t}}{\sum_{i,j} H_{i,j,t-1}} = \left(\frac{1}{Q_t^W}\right)\sqrt{\left(\frac{\sum_{i,j} W_{i,j,t-1} H_{i,j,t}}{\sum_{i,j} W_{i,j,t-1} H_{i,j,t-1}}\right)\left(\frac{\sum_{i,j} W_{i,j,t} H_{i,j,t}}{\sum_{i,j} W_{i,j,t} H_{i,j,t-1}}\right)}$$

Again, because of the approximation due primarily to rounding BLS conventions, we define growth in composition-adjusted aggregate weekly hours worked as

$$(5)\ \frac{H_t^{QAdj}}{H_{t-1}^{QAdj}} = Q_t^W \frac{H_t^{BLS}}{H_{t-1}^{BLS}}$$

With these definitions, weekly payrolls growth is related to growth in composition-adjusted wages and hours worked in a straightforward way:

$$(6)\left(\frac{P_t^{BLS}}{P_{t-1}^{BLS}}\right) = \left(\frac{H_t^{BLS}}{H_{t-1}^{BLS}}\right)\left(\frac{W_t^{BLS}}{W_{t-1}^{BLS}}\right) = \left(\frac{H_t^{QAdj}}{H_{t-1}^{QAdj}}\right)\left(\frac{W_t^{QAdj}}{W_{t-1}^{QAdj}}\right)$$

The composition-adjustment factors included in the Excel file (also linked from the blog post) have been chained together across months and indexed to equal 1.00 in February 2020. For example:

$$(7) Q_{Feb20}^{Website} = 1.0, \frac{Q_t^{Website}}{Q_{t-1}^{Website}} = Q_t^W$$

Section 2: Further bookkeeping details

Since published data for detailed industries for production/nonsupervisory workers are available going back to January 1990, we repeat the steps above for just these workers to get a longer time series than the all-workers version. We partition the private nonfarm sector into 253 industries where complete monthly hours and earnings data are available for production workers back to 1990. Hours, payrolls, and hourly earnings data for three of these industries are derived as the residual from data for other industries.[3] In 243 of these 253 industries, nonproduction supervisory workers' share of total compensation and total hours worked is at least 2 percent over the entire period of March 2006 to the present, whether or not seasonally adjusted data are used. For these industries, we split the total industry-level hours and earnings data into production/nonsupervisory and nonproduction/supervisory groups. For the remaining 10 industries, we do not split hours and earnings into these two groups of workers. We also do the decomposition above separately, with the BLS's seasonally adjusted data as well as their data that are not seasonally adjusted.

For most detailed NAICS industry codes outside of the manufacturing sector between the 3-digit and 6-digit level, month *t* labor market data are not released until month *t*+1 total nonfarm payroll employment data are released early in month *t*+2. However, month *t* labor market data for 2-digit NAICS codes and 3-digit NAICS manufacturing codes are released concurrently with

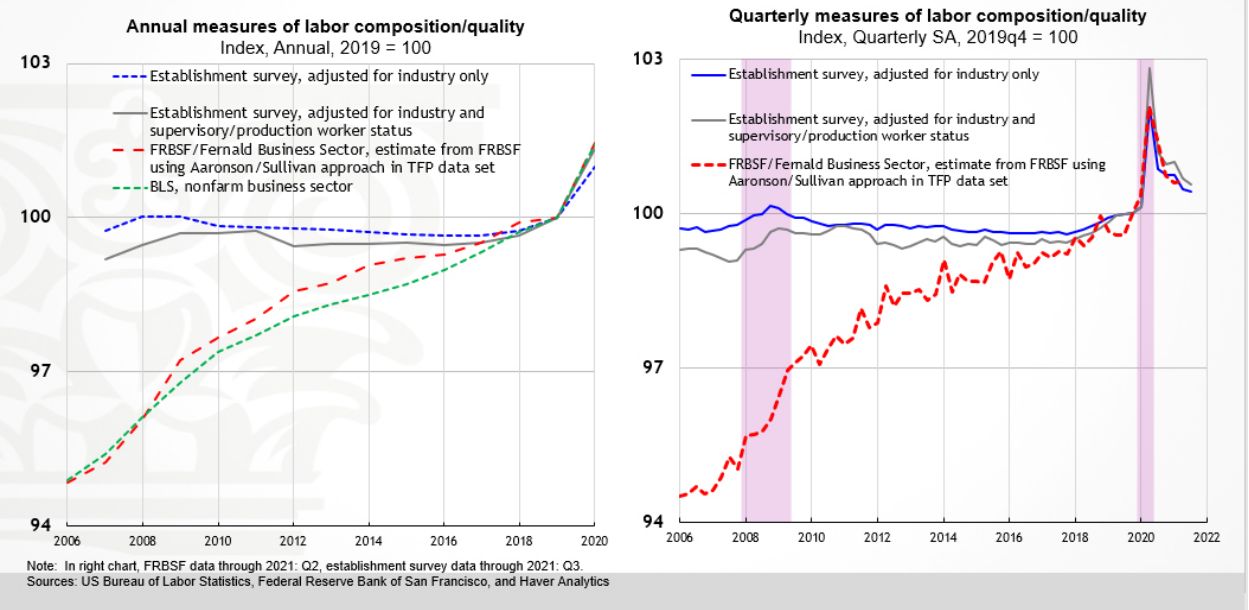
[3] These are Water+Rail+Pipelineline+Scenic and sightseeing+postal service transportation (NAICS 482, 483, 486, 488, and 491); Educational services (NAICS 61) and the Monetary Authorities combined with Lessors of Nonfinancial Intangible Assets (NAICS 521 and 533).

month t total nonfarm payroll employment. Therefore, for the most recent month of labor market data, which we denote by T, we construct an alternative labor-composition adjustment factor $Q_T^{W,32}$ using 32 NAICS industries to partition private nonfarm payroll employment into "production and nonsupervisory" workers and nonproduction/supervisory workers in each of these 32 industries. For month T, we use $Q_T^{W,32}$ in place of Q_T^W in equations (3), (5) and (7).

Section 3: Comparison of labor composition adjustment with BLS and San Francisco Fed and impact of adjusting by worker type

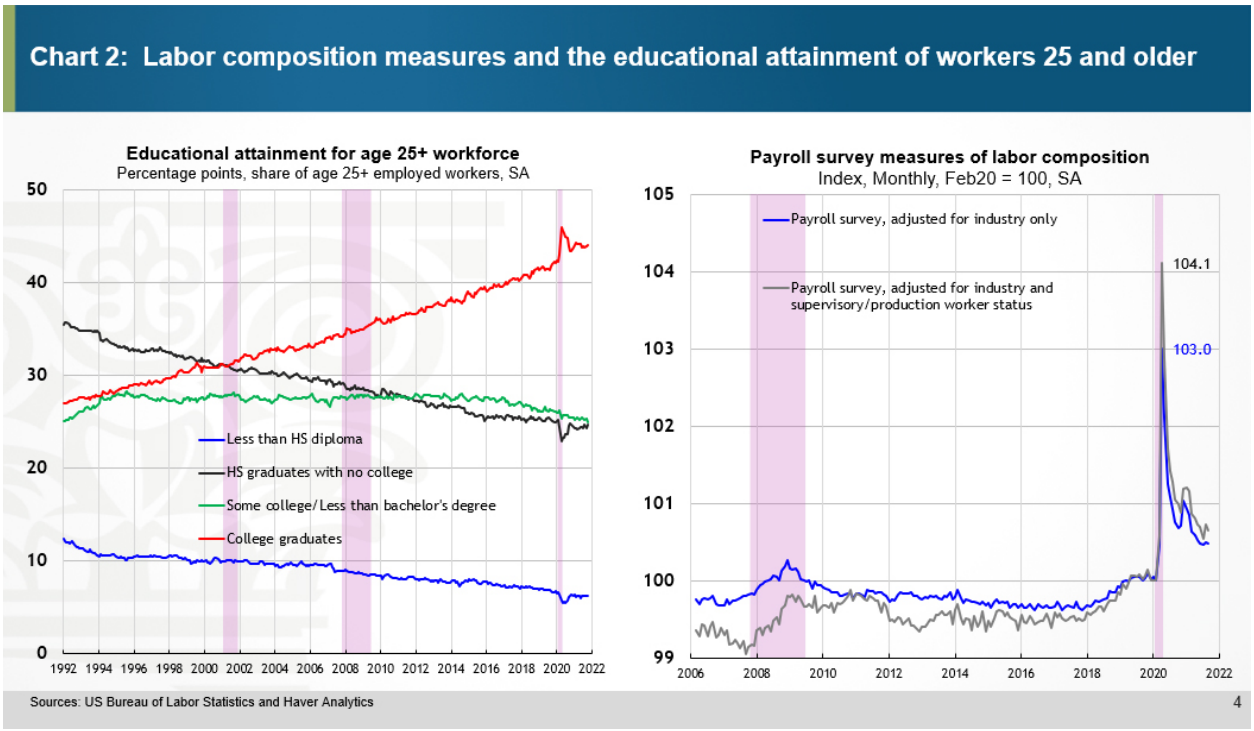
The BLS includes an annual measure of labor composition as part of its multifactor productivity release, while the Federal Reserve Bank of San Francisco includes quarterly measures in its total factor productivity database. Two of these series, which the BLS and San Francisco Fed use to adjust hours worked in productivity measurements, are plotted in the left and right panels of chart 1 alongside aggregated variants calculated with monthly establishment survey data.

Chart 1: Measures of labor composition or quality used to adjust wages and hours worked



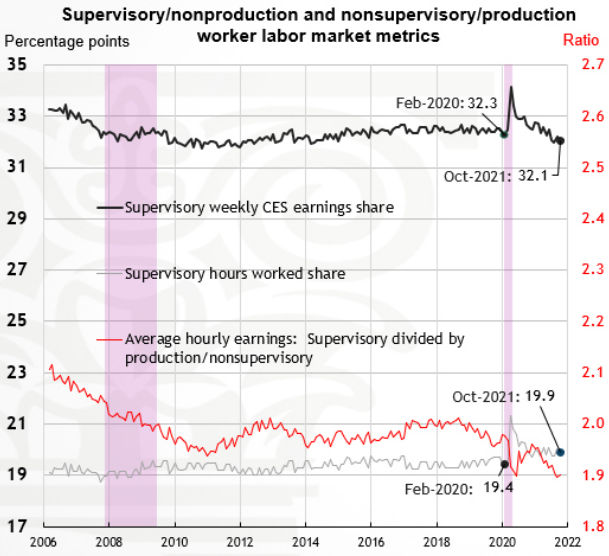
As described above, the gray lines in chart 1 are constructed using wage and hours data for 253 industry groups, all but 10 of which are further split into production/nonsupervisory and nonproduction/supervisory employee groups. Although the BLS does not directly publish earnings and hours worked data for the latter group of employees, we can estimate them using the published data for all employees and "production and nonsupervisory" employees. The blue lines in chart 1 are calculated using a finer set of 317 industry groups without further splitting by worker type. We can see that the labor composition measures constructed with the establishment survey did not trend up as sharply from the mid-2000s to 2019 as the BLS and San Francisco Fed measures did, likely because the latter measures directly use educational attainment as a dimension of labor-composition change, while the construction in this chart does not. The figure in the left-hand side of chart 2 shows that the share of the workforce age 25 and up with a college

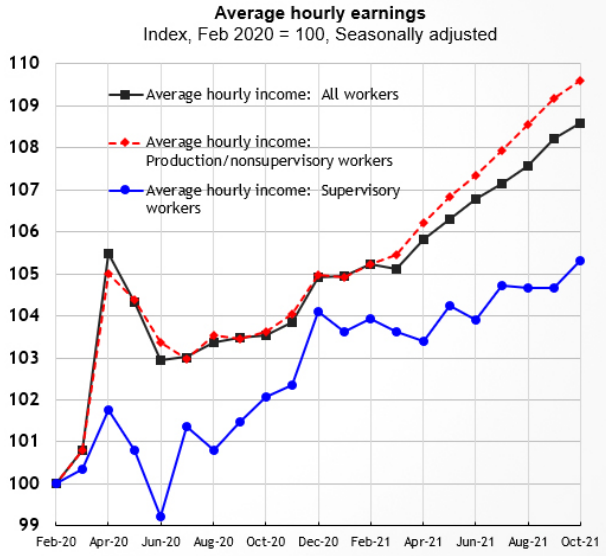
degree has trended up fairly steadily during the last 30 years, while the share with only a high school degree and the share without a high school degree have both trended down during the same timeframe.



The right-hand figures in charts 1 and 2 show that the measure of labor composition using the supervisory/production worker distinction spiked up more in 2020 than the measure using only industry employment differences. Chart 3 illustrates why this may be the case. In the figure in the left panel of the chart, we can see that although the hours worked by private nonproduction/supervisory employees are only one-fifth as large as hours worked by "production and nonsupervisory" employees, their total weekly payrolls are nearly one-third larger and their average hourly earnings are about twice as large. From February 2020 to April 2020, hours worked and dollar payrolls of "production and nonsupervisory" employees spiked relative to the same measures for "production and nonsupervisory" employees. The figure in the right panel of chart 3 shows that this relative spike was meaningful enough that average hourly earnings for all employees grew more from February 2020 to April 2020 than average hourly earnings grew for *either* "production and nonsupervisory" employees or for nonproduction/supervisory employees. Since February 2020, average hourly earnings for "production and nonsupervisory" employees have increased 4.3 percentage points more than average hourly earnings for nonproduction/supervisory employees.

Chart 3: Labor market metrics by supervisory/production worker status





Sources: US Bureau of Labor Statistics and Haver Analytics