Microfoundations of Inflation Persistence in the New Keynesian Phillips Curve
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Abstract: This paper proposes a dynamic stochastic general equilibrium model that endogenously generates inflation persistence. We assume that although firms change prices periodically, they face convex costs that preclude optimal adjustment. In essence, the model assumes that price stickiness arises from both the frequency and size of price adjustments. The model is estimated using Bayesian techniques, and the results strongly support both sources of price stickiness in the U.S. data. In contrast with traditional sticky price models, the framework yields inflation inertia, a delayed effect of monetary policy shocks on inflation, and the observed “reverse dynamic” correlation between inflation and economic activity.

JEL classification: E0, E31

Key words: inflation persistence, Phillips curve, sticky prices, convex costs

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1 Introduction

The standard New Keynesian Phillips curve (NKPC) based on the optimizing behavior of price setters in the presence of nominal rigidities is mostly built on the models of staggered contracts of John B. Taylor (1979, 1980), and Guillermo Calvo (1983), and the quadratic adjustment cost model of Julio Rotemberg (1982). The framework is broadly used in the analysis of monetary policy, with price rigidity as the main transmission mechanism through which monetary policy impacts the economy - when firms face difficulties in changing some prices, they may respond to monetary shocks by changing instead their production and employment levels.

Although the NKPC has some theoretical appeal, there is a growing literature on its empirical shortcomings regarding the ability to match some stylized facts on inflation dynamics and the effects of monetary policy. In particular, the standard NKPC models have been criticized due to the failure to generate inflation persistence. Accordingly, although the price level responds sluggishly to shocks, the inflation rate does not. In addition, these models do not yield the result that monetary policy shocks first impact output, and subsequently cause a delayed and gradual effect on inflation. Fuhrer and Moore (1995) and Nelson (1998), among others, suggest that in order for a model to fully explain the time series properties of aggregate inflation and output it requires that not only the price level but also the inflation rate be sticky.

In response to those critiques, this paper proposes a microfounded theoretical model that endogenously generates inflation persistence as a result of optimizing behavior from agents. In addition, a result from the model is that monetary policy shocks first impact economic activity, and subsequently inflation but with a long delay, reflecting inflation inertia. The model is also able to capture the cross-dynamic correlation between inflation and output gap.

We consider that firms face two sources of price rigidities, related to both the inability to change prices frequently and to the cost of sizeable adjustments. Calvo (1983)’s staggered

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price setting has been the most frequently used framework in the literature to derive the NKPC, with a fraction of firms completely adjusting their prices to the optimal level at discrete time intervals in response to changes in various costs. Another popular framework is Rotemberg (1982) in which firms set prices to minimize deviations from the optimal price subject to quadratic frictions of price adjustment. While both the Calvo pricing and the quadratic cost of price adjustment are designed to model sticky prices, the former is related to the frequency of price changes, while the latter is associated with the size of price changes. Costs of price adjustment might arise from managerial costs (information gathering, decision making) and customer costs (negotiation, communication, ‘fear of upsetting customers’, etc.). A recent extensive literature on microdata shows pervasive evidence of both infrequent and small price adjustments.2

We combine staggered price setting and quadratic costs of price adjustment in a unified framework. The solution of the model implies that prices are not continuously adjusted and that firms that are able to change prices do not fully adjust them due to convex costs of adjustment. Inflation persistence is endogenously generated as a result of this double price stickiness setting.

Several authors have proposed alternative NKPC models that can account for some of the empirical facts on inflation and output. The most popular ones are extensions of Calvo’s staggered prices or contracts based on sticky information or backward rule-of-thumbs. Gregory N. Mankiw and Ricardo Reis (2002) propose a model in which information is costly and, therefore, disseminated slowly. Each period a fraction of the firms sets prices based on outdated information, whereas the other fraction sets the optimal price path based on new information about the state of the economy. In this framework prices adjust continuously but information does not. The sticky-information model is consistent with inflation

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persistence. Whereas some recent papers find empirical support for sticky information (e.g. Christopher D. Carroll 2003, Peter J. Klenow and Jonathan L. Willis 2007), others find evidence that firms review their prices more often than the frequency of price changes (e.g. Silvia Fabiani, Martine Druant, Ignacio Hernando, Claudia Kwapił, Bettina Landau, Claire Loupias, Fernando Martins, Thomas Y. Math, Roberto Sabbatini, Harald Stahl, and Ad C. J. Stokman 2005, Olivier Coibion 2010). A more disputable empirical implication of this model is the result that prices change frequently, contradicting widespread micro-data studies. The evidence found across countries and different data sources is that firms keep prices unchanged for several months (e.g. Mark Bils and Peter Klenow 2004, Angeloni et al. 2006, Alvarez 2008, Emi Nakamura and Jon Steinsson 2008, Klenow and Malin 2010, among several others).

Alternatively, some widely used models assume that a fraction of the firms changes its prices according to a backward looking rule of thumb. In Gali and Gertler (1999) firms set new prices based on past prices of other firms with a correction for recent inflation, rather than based on profit maximization. Lawrence J. Christiano, Martin Eichenbaum, Charles L. Evans (2005), Jon Steinsson (2003), and Frank Smets and Rafael Wouters (2003, 2007), among others, assume that part of the firms adjusts their prices by automatic indexation to past inflation. These models explain inflation inertia as they incorporate a lagged inflation term into the resulting hybrid NKPC. A common feature in these models is the arbitrary role given to past inflation as at least some agents are backward-looking in the process of setting prices and, as a result, firms do not reoptimize prices each given period.

The hybrid NKPC models, such as the one proposed by Gali and Gertler (1999), are generally criticized for being ad-hoc as they lack a theory of firms to motivate their backward-looking behavior (see e.g. Jeremy Rudd and Karl Whelan 2007, Woodford 2007, Cogley and Sbordone 2008, Luca Benati 2008, etc.). The indexation models of Christiano et al (2005), Steinsson (2003), and Smets and Wouters (2003) have more structure as a fixed share of random Calvo-type firms could set their prices optimally each period while the rest changes according to past aggregate inflation. Thus, in their framework the lagged inflation term is derived from firms’ decisions. However, these modified hybrid NKPC models - as well as the sticky information models - imply that prices are adjusted continuously, contrarily to the Calvo-based model of Gali and Gertler (1999) in which there is a constant probability that
a firm will change its price in a given period. Thus, the general indexation models are also not supported by pervasive micro-data evidence of price stickiness (Jeremy Rudd and Karl Whelan, 2006). Notice that continuously price updating is an implication of many NKPC models including Ricardo Reis (2006), Christiano et al (2005), Smets and Woulters (2003), Rotemberg (1982), Sharon Kozicki and Peter A. Tinsley (2002), among many others.

In our proposed double sticky price model, the Phillips curve is derived from a dynamic stochastic general equilibrium (DSGE) model, and relates current inflation to inflation expectations, lagged inflation, and real marginal cost or output gap. In contrast to the Calvo-cum-indexation models by Gali and Gertler (1999) and Christiano et al (2005), the lagged inflation term is endogenously generated in a forward-looking framework - since price stickiness arises from both the size and frequency of price adjustments, current price depends on lagged price twice and, hence, a lagged inflation term is endogenously generated from the optimizing behavior of the firms. Thus, inflation is backward looking without assuming a backward-looking component in price setting. That is, agents remain forward looking and follow an optimizing behavior. Further, in contrast to the general indexation models and sticky information models, prices are not continuously adjusted in the proposed model. The new Phillips curve based on dual stickiness nests the standard NKPC as a special case (Calvo pricing or quadratic cost) and offer an alternative to the ad-hoc hybrid NKPC and the sticky information Phillips curve.

The small-scale DSGE model is estimated using Bayesian techniques. Empirical results indicate that the parameter estimates associated with the two types of price stickiness are highly significant, supporting the proposed model. In addition, the estimates closely match extensive micro-data evidence regarding the frequency of price changes and the size of price adjustment.

Common ways in the literature to evaluate the model success in capturing inflation dynamics are by examining the estimated parameter associated with lags of inflation in the Phillips curve, the autocorrelation functions for inflation, and the impulse response functions. First, we find evidence of intrinsic inflation persistence as the coefficients associated with lags of inflation in the Phillips curve are positive and highly significant. Second, the autocorrelation function for the estimated inflation is high and decay gradually, also indicating that inflation is highly persistent. The estimated inflation autocorrelation closely tracks
the observed data.

The model provides a theoretical foundation on inflation inertia, which in turn has an important role in matching the data. We provide evidence on the importance of the lagged inflation term in generating the observed cross-correlation between inflation and the output gap. John B. Taylor and Michael Woodford (1999) consider as a yardstick of a success of monetary models their ability to generate the “reverse dynamic” cross-correlation between output gap and inflation. Accordingly, the proposed model yields the “reverse dynamic” result that current output gap tends to be positively related with future inflation, whereas past inflation tends to be negatively associated with current output gap.

Finally, the response of inflation to a contractionary monetary policy shock is marked different between the standard NKPC and the proposed model. In the standard sticky price model, the impact of a monetary policy shock on inflation is immediate. In the proposed double sticky price model, by contrast, the response of inflation is gradual, displaying significant inertia, which is more consistent with outside lags regarding the impact of monetary policy on inflation, and substantial econometric evidence. Such an effect is produced because the new price chosen at discrete time intervals is only partially adjusted due to quadratic costs. The double price stickiness model is, thus, favored over several dimensions compared to the standard NKPC model.

The paper is organized as follows. Section 2 sets up the model and derives the implied new Phillips curve. Section 3 introduces the associated small-scale dynamic general equilibrium model. Section 4 reports the estimation results, and Section 5 concludes.

2 Firms’ Problems and the Phillips Curve

We assume that the economy has two types of firms: a representative final goods-producing firm and a continuum of intermediate goods-producing firms.

2.1 The Final Goods-Producing Firm

The final goods-producing firm purchases a continuum of intermediate goods, $Y_i$, at input prices, $P_i$, indexed by $i \in [0, 1]$. The final good, $Y_t$, is produced by bundling the intermediate
goods:

\[ Y_t = \left[ \int_0^1 Y_{it}^{1/\lambda_f} \, di \right]^{\lambda_f} \tag{1} \]

where \( 1 \leq \lambda_f < \infty \). The final-good-producing firm chooses \( Y_{it} \) to maximize profit in a perfectly competitive market taking both input \( (P_{it}) \) and output prices \( (P_t) \) as given, solving the following problem:

\[
\max P_t \left[ \int_0^1 Y_{it}^{1/\lambda_f} \, di \right]^{\lambda_f} - \int_0^1 P_{it} Y_{it} \, di \tag{2} 
\]

subject to the technology described in (1). The first order condition of the final goods-producing firm implies that:

\[
Y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\lambda_f/\left(\lambda_f - 1\right)} \tag{3} \]

where \( \lambda_f/\left(\lambda_f - 1\right) \) measures the constant price elasticity of demand for each intermediate good. The relationship between the prices of the final and intermediate goods can be obtained by integrating (3):

\[
P_t = \left[ \int_0^1 P_{it}^{1/(1-\lambda_f)} \, di \right]^{1-\lambda_f} \tag{4} 
\]

Equation (4) is derived from the fact that the final goods-producing-firm earns zero profits. The final good price can be interpreted as the aggregate price index.

### 2.2 The Intermediate Goods-Producing Firm

The most common way in the literature to derive the NKPC is based on Calvo’s (1983) price setting, in which a fraction \( (1 - \theta) \) of firms resets prices to optimize profit while the remaining firms keep their prices unchanged. In the Calvo economy, as implied by equation (4), the aggregate price level evolves according to:

\[
P_t = \left[ (1 - \theta)\tilde{P}_{it}^{1/(1-\lambda_f)} + \theta P_{t-1}^{1/(1-\lambda_f)} \right]^{1-\lambda_f}, \tag{5} 
\]

where \( \tilde{P}_t \) denotes the optimal price set by the intermediate good-producing firms. The fraction of firms that reoptimizes their price at time \( t \) choose the same price in equilibrium, thus \( \tilde{P}_{it} = \tilde{P}_t \) for all \( i \).\(^3\). Since individual prices are optimized infrequently, the aggregate

\(^3\)See e.g. Michael Woodford (1996) and Tack Yun (1996).
price level also adjusts sluggishly, making current price level depend on its own lag.

Another popular way of introducing nominal rigidities is to assume that prices are sticky because firms face costs of changing their prices. Rotemberg (1982) proposes that firms face quadratic cost of price adjustment reflecting convex, variable costs. Several dimensions of managerial and customer relations might imply variable costs of price adjustments. Since these costs increase with the magnitude of price adjustments, firms are constrained with respect to the size of price changes. We assume that each intermediate goods-producing firm faces a quadratic adjustment cost of adjusting its nominal price given by:

$$QAC = \frac{c}{2} \left( \frac{\bar{P}_t - \bar{P}_{t-1}}{P_t - P_{t-1}} \right)^2 Y_t.$$  \hspace{1cm} (6)

Equation (6) implies that it is costly for current individual price to deviate from past price level, which makes prices sticky.

The aggregate price level also moves slowly due to the sluggish adjustment in individual prices. In this respect, both the Calvo pricing and the quadratic cost of price adjustment models are similar since they result in sticky prices. However, they yield different implications with respect to the frequency and size of price adjustment. While the Calvo model is associated with the frequency of price changes, the quadratic cost of price adjustment model is related to firms’ decisions about the magnitude of price adjustment. The Calvo pricing implies that firms change prices infrequently due to staggered contracts and fixed costs of changing prices. On the other hand, the size of price adjustment is more closely related to variable costs such as managerial costs (information gathering, decision making costs), customer costs (negotiation, communication costs) and managers’ ‘fear of antagonizing customers’ (Julio Rotemberg 1982, 2005).

The intermediate goods-producing firm maximizes real profit from selling its output in a monopolistically competitive market assuming that price is fixed with the Calvo probability \(\theta\) in any given period. In addition, firms face a quadratic adjustment cost in adjusting their prices. The firm chooses \(\bar{P}_t\) to maximize:

$$E_t \sum_{k=0}^{\infty} (\theta \beta)^k \left[ \left( \frac{\bar{P}_t - mc_{t+k}}{\bar{P}_{t+k}} \right) Y_{t+k} \right] - \frac{c}{2} \left( \frac{\bar{P}_t - \bar{P}_{t-1}}{P_t - P_{t-1}} \right)^2 Y_t$$ \hspace{1cm} (7)
subject to the demand function described by equation (3) and by equation (5). \(mc_t\) represents the real marginal cost of labor. Firm \(i\)'s profit depends on \(\tilde{P}_t\) as long as it cannot re-optimize its price. The average duration of price contracts is calculated as \(1/(1 - \theta)\) in the Calvo model. When the quadratic cost of price adjustment is zero, the above problem leads to the standard NKPC based on the assumption that firms completely adjust their prices whenever they reset them.

The first order condition of the constrained maximization problem given in (7) can be rewritten in terms of real values as follows:

\[
E_t \sum_{k=0}^{\infty} (\theta \beta)^k [(\hat{p}_t \tilde{X}_{tk} - m_{ct+k})(\hat{p}_t \tilde{X}_{tk})^{-a}Y_{t+k}] - c (\hat{p}_t - \hat{p}_{t-1}) Y_t
\]

where \(\tilde{X}_{tk} \equiv 1/\pi_{t+1} \pi_{t+2} ... \pi_{t+k}, \hat{p}_t \equiv \hat{P}_t/P_t\) and \(a \equiv \lambda_f/(\lambda_f - 1)\). Log-linearization of (8) gives rise to:

\[
E_t \sum_{k=0}^{\infty} (\theta \beta)^k [(\hat{p}_t + \hat{X}_{tk} - \hat{m}_{ct+k})] = \frac{c}{1-a} (\hat{p}_t - \hat{p}_{t-1})
\]

where \(\hat{p}_t, \hat{X}_{tk},\) and \(\hat{m}_{ct+k},\) respectively, denote the log-deviation of \(\hat{p}_t, \hat{X}_{tk},\) and \(\hat{m}_{ct+k},\) from their steady state values. The equality in equation (9) results from the trade-off between the marginal cost (r.h.s) and the marginal benefit (l.h.s.) from changing prices after considering future inflation and real marginal cost. A rise in prices has a positive effect on profit, whereas an increase in future inflation or in real marginal cost of labor has a negative impact. The marginal cost of adjusting prices associated with variable costs increases with the size of the price adjustment.

Log-linearizing the Calvo pricing described by equation (5) yields the following equation:

\[
\hat{p}_t = \frac{\theta}{1-\theta} \hat{\pi}_t.
\]

Thus, \(\hat{p}_t - \hat{p}_{t-1} = [\theta/(1-\theta)][\hat{\pi}_t - \hat{\pi}_{t-1}]\). Plugging this into equation (9), rearranging the terms, and deleting the hat on the variables for convenience yield:

\[
\pi_t = \Lambda_1 E_t \pi_{t+1} + \Lambda_2 \pi_{t-1} + \lambda m_{ct}
\]

where \(\Lambda_1 \equiv \eta/\tau, \Lambda_2 \equiv \kappa/\tau, \lambda \equiv (1-\theta\beta)/\tau, \tau \equiv (\theta/(1-\theta) + (1+\theta\beta)\kappa), \eta \equiv \theta\beta(1/(1-\theta) + \kappa), \kappa \equiv c(1-\theta\beta)\theta/(a-1)(1-\theta).\)
The derivation of the new Phillips curve reveals how the two sources of price stickiness considered endogenously generate a lagged inflation term. Since price stickiness arise from both the size and frequency of price adjustments, current price depends on lagged price twice and, hence, a lagged inflation term is endogenously generated from the optimizing behavior of the firms.

The proposed model nests the standard sticky price NKPC as a particular case. When the quadratic cost of price adjustment is zero, our double price sticky NKPC model collapses into the standard NKPC of the form:

\[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \theta \beta)}{\theta} m c_t. \]  

The NKPC is often criticized due to the failure to generate the observed inflation persistence and hump-shaped responses of inflation to monetary policy shocks. These shortcomings arise because the inflation rate change quickly due to the role of inflation expectations. A lagged inflation term as derived in equation (11) plays an important role in overcoming these problems as shown in Section 4.

3 A Small Scale DSGE Model

We consider a small scale DSGE model consisting of three equations: the IS curve, the Phillips curve, and the Taylor rule. The IS curve is derived from maximizing the expected present discounted value utility function, \( U(C_t, 1 - N_t) = \frac{C_t^{1-1/\sigma}}{1-1/\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \), subject to the budget constraint, \( C_t + \frac{B_t}{P_t} = \left( \frac{W_t}{P_t} \right) (N_t) + (1 + i_t - 1)(\frac{B_{t-1}}{P_t}) + \Pi_t \), where \( C_t \) is the composite consumption good, \( N_t \) is hours worked, \( \Pi_t \) is real profits received from firms, and \( B_t \) is the nominal holdings of one-period bonds that pay a nominal interest rate \( i_t \). The IS curve is:

\[ y_t = E_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + \varepsilon^y_t \]  

where \( y_t \) and \( i_t \) denote the output gap and the nominal interest rate, respectively. The disturbance term, \( \varepsilon^y_t \), is the preference shock, which is assumed to follow the AR(1) process, \( \varepsilon^y_t = \delta \varepsilon^y_{t-1} + \nu^y_t \) with \( \nu^y_t \sim N(0, \sigma^2_y) \).

\[ \text{see Walsh (2003) for more details.} \]
\[ \pi_t = \Lambda_1 E_t \pi_{t+1} + \Lambda_2 \pi_{t-1} + \lambda_1 y_t + \varepsilon_{\pi_t}^\pi \]  

(14)

where \( \lambda_1 = \left( \frac{1}{\sigma} + \varphi \right) \lambda \). We denote the disturbance term \( \varepsilon_{\pi_t}^\pi \) as the cost-push shock, which follows a normal distribution, \( N(0, \sigma_{\pi}^2) \). The Taylor rule is of the form:

\[ i_t = \rho i_t + (1 - \rho) (\alpha_{\pi} E_t \pi_{t+1} + \alpha_y y_t) + \varepsilon_{i_t}^i \]  

(15)

where the interest rate shock \( \varepsilon_{i_t}^i \) follows a normal distribution, \( N(0, \sigma_{i_t}^2) \), and \( \rho \) measures the degree of interest rate smoothing in monetary policy. We assume that policy makers are forward-looking in stabilizing inflation. However, they conduct monetary policy in response to current economic activity. The Federal Reserve's responses to inflation and the output gap are determined by the parameters \( \alpha_{\pi} \) and \( \alpha_y \).

4 Bayesian Estimation of the DSGE Model

4.1 Data and Priors

In order to estimate the DSGE model, we employ the output gap measure from the Congressional Budget Office (CBO), the effective Federal Funds Rate obtained from the Federal Reserve Bank of Saint Louis, and we use the implicit GDP deflator from the Bureau of Labor Statistics to calculate the inflation rate. The data range from 1960:1 to 2007:4.

The priors on the model parameters are summarized in Table 1. We set the parameter that measures the degree of market power of each intermediate goods-producing firm, \( a \), to 6. This implies a steady state markup of price over marginal cost of 20%, as in Julio J. Rotemberg and Michael Woodford (1992) and Peter Ireland (2001). We also set \( \beta \) and \( \varphi \) to be 0.99 and 1.5, respectively, as commonly assumed in the literature. We use 40,000 draws to estimate the DSGE model, but only start calculating posterior features after 20,000 draws. The Metropolis-hastings algorithm is applied to obtain the maximum likelihood estimates.

4.2 Estimation Results

The estimated monetary policy parameters \( \rho, \alpha_{\pi}, \) and \( \alpha_y \) are similar to the ones reported in the literature. The parameter measuring the degree of interest rate smoothing is estimated
to be 0.72. The estimate of $\alpha_\pi$ associated with the Fed’s response to inflation expectations is 1.72, whereas the parameter related to the response of the Fed to the output gap is estimated to be 0.48.

The parameters associated with both Calvo-type price stickiness or Rotemberg (1982)’s quadratic costs are highly statistically significant. Thus, the estimation results indicate that the null hypothesis of no price rigidities with respect to the frequency and size of price adjustment is rejected, supporting the double sticky price NKPC model.

**Frequency of Price Adjustments**

The posterior mean of $\theta$, a measure of the degree of nominal rigidity, is estimated to be 0.75, which implies that only a quarter of the firms is able to reset their prices to optimize profit while the remaining keep their prices unchanged. The estimate also implies that the average length of time between price changes is 4 quarters.

This finding is in accord with microdata evidence that there is substantial price stickiness. Klenow and Malin (2010) report that prices change on average at least once a year, rather than being continuously adjusted. Alvarez (2008) investigates firms in 18 countries and, based on surveys of the distribution of price changes, also finds that prices generally change around once a year, with a median of 11.1 months. The median duration of price changes is found to be only 20% for consumer prices, and 21% for producer prices. Alvarez et al (2006) find that price changes are even less common in the Euro area. On average, in a given month only 15.1% of prices change, and the average length of time between price changes is from 4 to 5 quarters. These figures mean that price adjustment in the euro area is less frequent than in the US. Finally, Eichenbaum, Jaimovich and Rebelo (2008) propose a method to measure sticky reference prices among shorter-lived new prices, and find that they change only every 11.1 months. Mark Bils, Peter J. Klenow, and Benjamin A. Malin (2009) generalize their definition of reference prices for the U.S. CPI and find that the weighted median duration of reference prices is 10.6 months.

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5Klenow and Malin (2010) find that there is heterogeneity in the price changes across sectors. In particular, durables change prices more frequently than nondurables and services; and nondurables change prices more often than services.
6The reference price for each UPC as defined by Eichenbaum et al (2009) corresponds to the modal price in each quarter using weekly price data from a large U.S. supermarket chain. Bils, Klenow, and Malin (2009) define the reference price in each month as the most common price of an item in the 13-month window.
The results from our proposed model regarding the frequency of price adjustments are, thus, very much in agreement with microdata evidence, as well as with the estimated average duration of price spells. On the other hand, models that predict continuous (100%) price changes such as general indexation models or sticky information models diverge from the pervasive low frequency of price adjustment observed in every country and across different data sources.

**Size of Price Adjustments**

We find that the magnitude of adjustment costs is large and statistically significant. The posterior mean of the parameter associated with the quadratic adjustment cost, $c$, is estimated to be 171.0, with the 95% confidence interval ranging from 143.2 to 197.4. The results imply that when firms adjust prices, the size of the adjustments is small. This is in accord with microdata evidence on the size of price changes. The general empirical finding is that price changes are mostly smaller than the size of aggregate inflation (see e.g. Dhyne et al. 2005, Alvarez et al (2006), Klenow and Kryvstov 2008, etc.) More specifically, Klenow and Kryvtsov (2008) show that in the U.S. 44% of consumer price changes are smaller than 5%, 25% are smaller than 2.5%, and 12% are smaller than 1%, in absolute value. Vermeulen et al. (2007) study the Euro area and find that a quarter of producer price changes is smaller than 1% in absolute value, and that the mean price change is only 4%.

**4.3 Dynamic Correlation Between the Output Gap and Inflation**

Taylor and Woodford (1999) stress that the ability to characterize the “reverse dynamic” relationship between output gap and inflation is a yardstick of success of monetary models. In this respect, it is worth investigating whether the double sticky price model is able to replicate this observation, and the role of the lagged inflation term in generating the dynamic cross-correlation between the two variables. Figure 1 displays the correlation between inflation and output gap obtained from the estimates of our proposed DSGE model reported in Table 1 (line with squares), the baseline DSGE model estimated assuming that the quadratic adjustment cost is zero (line with triangles), and the observed correlation between inflation and output gap (line with circles). As highlighted by Gali and Gertler (1999), current centered on the current month.
output gap tends to be positively related with future inflation, whereas past inflation tends to be negatively associated with current output gap. Figure 1 shows that our DSGE model performs well in replicating the observed dynamic correlation between inflation and the output gap – a rise in current inflation is associated with a subsequent decline output gap, and a rise in current output gap signals a rise in future inflation. Figure 1 also shows that the implications change substantially when the model is reestimated with the constraint of no quadratic costs. In particular, the baseline NKPC model fails to predict the observed reverse dynamic correlation as it implies that lagged output gap (inflation) is positively associated with future inflation (output gap). In addition, the contemporaneous correlation between these series obtained from the baseline model is very high, in contrast to the actual data. This evidence indicates that the lagged inflation term in our model plays a crucial role in accounting for the output-inflation dynamics.

4.4 Impulse Response and Autocorrelation Functions

This section investigates whether the estimated DSGE model is able to generate plausible impulse response functions and inflation persistence. The impulse response functions are generated using the estimates from Table 1 and are plotted in Figure 2. We also plot the implied autocorrelation functions of inflation by the double sticky price DSGE model in Figure 3.

Figure 2 displays the impulse response function of inflation, output gap, and interest rate to an one-standard-deviation cost-push shock (first column), a preference shock (second column), and an interest rate shock (third column), for the proposed NKPC model (solid curve) and the baseline NKPC that assume no price adjustment cost (dashed curve).

As seen in the first column the cost-push shock leads to an increase in inflation in both models. However, the response of inflation dies off more gradually in the proposed model, around 10 quarters after the shock. Inflation persistence in the double sticky NKPC model is much more consistent with reality. On the other hand, the baseline NKPC yields the counterfactual result that inflation displays very little inertia. Further, the model predicts that the Federal Reserve raises interest rate in response to higher inflation, which leads to a decrease in the output gap, in contrast with the baseline NKPC. The shock has the largest impact on output gap after around 4 quarters.
The effects of a preference shock on the variables are shown in the second column for both models. The shock results in an increase in inflation and in the output gap and, therefore, in the interest rate. In response to an increase in inflation and in the output gap, the Federal Reserve raises interest rate to stabilize the economy. Differences between the models are very apparent when we examine the response of inflation. In the baseline NKPC model, although prices are sticky, inflation does not exhibit persistence. The largest impact on inflation takes place one quarter after the shock in this model. In contrast, inflation’s response in the proposed NKPC model is much more gradual and persistent, with the largest impact occurring 9 quarters later. This is because price setters not only change their prices infrequently, they do not completely readjust their prices when the opportunity arises due to convex costs.

It is well-known in the literature that the standard NKPC fails to generate a hump-shaped response of inflation to a monetary policy shock. The third column shows the estimated effect of a one-standard-deviation contractionary monetary policy shock on both models. The policy shock has a positive effect on interest rate and a negative impact on inflation and output. Inflation falls instantly in response to this shock in the standard NKPC model, displaying no inertia. By contrast, we observe a delayed, more gradual response of inflation following the policy shock in the proposed NKPC. The largest impact on inflation occurs after 5 quarters. Thus, the double sticky price model is more in accord with the persistence in inflation as found in the data.

We further investigate whether the proposed model is able to match the observed inflation persistence by examining the autocorrelation function. Figure 3 presents the autocorrelation function for inflation generated by the DSGE model and by the data. The model-implied autocorrelation function closely matches the observed inflation persistence in the data. Fuhrer and Moore (1995) stress the inability of the standard NKPC model to generate inflation inertia, displaying an autocorrelation function that dies off quickly, which goes against the empirical evidence on inflation dynamics. As discussed in the literature (e.g., Gali and Gertler 1999), inflation persistence plays an important role in explaining costly disinflation. Thus, the double sticky price model is able to match both the high autocorrelation of inflation as well as the gradual response of inflation to monetary policy shocks.
4.5 Robustness of the Results

In this section, we reestimate the DSGE model using an extended version of the IS curve. We also reestimate the model for subsamples to check the robustness of the results.

Extended IS Curve

We consider the results for the following form of the IS curve:

\[ y_t = \mu_y E_t y_{t+1} + (1 - \mu_y) y_{t-1} - \sigma (i_t - E_t \pi_{t+1}) + \varepsilon^y_t. \]  

Equation (16) nests (13) as a special case. The rationale for including the lagged output gap can be found, for example, in Jeffrey C. Fuhrer’s (2000) habit in consumption model. Fuhrer (2000) shows that this extension generates more persistence in the implied series.

Table 2 reports estimation results when we replace equation (13) with (16). The results obtained are very similar to the ones reported in Table 1. The posterior mean estimate of \( \theta \) is 0.77, implying that the average duration between price changes is again about 4 quarters. The posterior mean of \( c \) is estimated to be 166.5, ranging from 140.3 to 195.1. Thus, the presence of the two sources of price stickiness is supported once again. The estimate of \( \mu_y \) is 0.70, indicating that the contribution of expectations to output gap dynamics is relatively more important than lagged output gap. The other common parameters are also stable across the different specifications.

Subsample Results

We divide the sample between the sub-periods before and after the early 1980s and reestimate the proposed DSGE model. The sub-sample estimates of the DSGE model are reported in Table 3, for 1960Q1 to 1979Q4 and for 1983Q1 to 2007Q4.

The parameter \( \theta \) does not change much across sub-samples. It is estimated to be 0.73 for the pre-1979 period and 0.79 for the post-1983 period. On the other hand, the coefficient associated with quadratic adjustment costs is estimated to be higher in the post-1983 era. The estimate of \( c \) is 99.7 for the first sample and 126.8 for the second sample. This change in the parameter \( c \) across sub-samples implies that the convex costs of changing prices is higher during the more recent low inflation period. Overall, the presence of double price stickiness is again confirmed by the data, showing that our results are robust across samples.
Turning to the parameters of the Taylor rule, the estimates measuring the degree of the interest rate smoothing are lower in the first sample period. The finding indicates a drastic change in the estimate of $\alpha_\pi$ showing that the Fed has responded more aggressively to expected future inflation in the second sample period. These results are consistent with Clarida, Gali, and Gertler (2000) and a vast subsequent literature. On the other hand, the response of the Fed to current economic activity is estimated to be slightly lower in the second sample, compared to the first one. Finally, the estimated standard deviations of the shocks are smaller in the second sample, which is consistent with the increased stabilization of the economy during the Great Moderation period.

5 Conclusion

One of the most popular ways to generate inflation persistence in the literature is to assume that a fraction of firms reset their prices by automatic indexation to past period’s inflation rate. General automatic indexation model have been criticized for the lack of microfoundations backing the introduction of the lagged inflation term in the Phillips curve.

This paper proposes a model in which a lagged inflation term is endogenously generated from microfounded optimization behavior from the agents. Prices are sticky due to infrequent price changes and to convex costs of price adjustments. The estimated results on the frequency and size of price changes closely match extensive microdata evidence. The double sticky price NKPC model satisfactorily explains the observed dynamic behavior between output gap and inflation. In addition, the model provides a theoretical foundation for the resulting inertial inflation, and yields the result that monetary policy shocks have a delayed, gradual impact on inflation. Such an effect is produced because even the chosen new price at discrete time intervals is only partially adjusted due to quadratic costs. Overall, these results together indicate that the double sticky price model with both staggered prices and costs of adjustment applied to the price level is in closer agreement with the data than standard sticky NKPC models.
References


### Table 1: Estimation Results - Double Sticky Price DSGE model

<table>
<thead>
<tr>
<th>parameters</th>
<th>prior dist.</th>
<th>prior mean</th>
<th>prior st.dev.</th>
<th>posterior mean</th>
<th>95% of confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>beta</td>
<td>0.5</td>
<td>0.10</td>
<td>0.75</td>
<td>[0.69, 0.82]</td>
</tr>
<tr>
<td>$c$</td>
<td>normal</td>
<td>30</td>
<td>30.0</td>
<td>171.0</td>
<td>[143.2, 197.4]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>invg</td>
<td>1</td>
<td>$\infty$</td>
<td>0.16</td>
<td>[0.13, 0.18]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>beta</td>
<td>0.7</td>
<td>0.05</td>
<td>0.72</td>
<td>[0.69, 0.76]</td>
</tr>
<tr>
<td>$\alpha_{\pi}$</td>
<td>normal</td>
<td>1.5</td>
<td>0.25</td>
<td>1.72</td>
<td>[1.58, 1.86]</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>normal</td>
<td>0.5</td>
<td>0.1</td>
<td>0.48</td>
<td>[0.33, 0.62]</td>
</tr>
<tr>
<td>$\delta_y$</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.94</td>
<td>[0.91, 0.97]</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
<td>0.73</td>
<td>[0.66, 0.80]</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
<td>0.19</td>
<td>[0.15, 0.22]</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
<td>1.36</td>
<td>[1.23, 1.48]</td>
</tr>
</tbody>
</table>

Bayesian estimation results for the proposed DSGE model using equation (13) as the IS curve. The log likelihood is -971.9. Estimates are obtained from the full sample period from 1960Q1 to 2007Q4.
### Table 2: Estimation Results - Extended Double Sticky Price DSGE model

<table>
<thead>
<tr>
<th>parameters</th>
<th>prior dist.</th>
<th>prior mean</th>
<th>prior std. dev.</th>
<th>posterior mean</th>
<th>posterior 95% of confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.77</td>
<td>[0.70, 0.83]</td>
</tr>
<tr>
<td>$c$</td>
<td>normal</td>
<td>30</td>
<td>30</td>
<td>166.5</td>
<td>[140.3, 195.1]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>invg</td>
<td>1</td>
<td>$\infty$</td>
<td>0.14</td>
<td>[0.12, 0.17]</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
<td>0.70</td>
<td>[0.63, 0.78]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>beta</td>
<td>0.7</td>
<td>0.05</td>
<td>0.70</td>
<td>[0.66, 0.74]</td>
</tr>
<tr>
<td>$\alpha_{\pi}$</td>
<td>normal</td>
<td>1.5</td>
<td>0.25</td>
<td>1.80</td>
<td>[1.66, 1.93]</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>normal</td>
<td>0.5</td>
<td>0.1</td>
<td>0.46</td>
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</tr>
<tr>
<td>$\delta_y$</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.94</td>
<td>[0.91, 0.97]</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
<td>0.75</td>
<td>[0.67, 0.82]</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
<td>0.17</td>
<td>[0.14, 0.21]</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
<td>1.36</td>
<td>[1.23, 1.49]</td>
</tr>
</tbody>
</table>

Bayesian estimation results for the proposed DSGE model using equation (16) as the IS curve. The log likelihood is -973.1. Estimates are obtained from the full sample period from 1960Q1 to 2007Q4.

### Table 3: Estimation Results of the Double Sticky Price DSGE model: Subsample Analysis

<table>
<thead>
<tr>
<th>parameters</th>
<th>pre-1979 estimate</th>
<th>post-1983 estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>posterior mean</td>
<td>95% of confidence interval</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.73</td>
<td>[0.65, 0.82]</td>
</tr>
<tr>
<td>$c$</td>
<td>99.7</td>
<td>[70.9, 128.3]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.23</td>
<td>[0.17, 0.28]</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>0.65</td>
<td>[0.56, 0.75]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.66</td>
<td>[0.61, 0.72]</td>
</tr>
<tr>
<td>$\alpha_{\pi}$</td>
<td>1.33</td>
<td>[1.22, 1.44]</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>0.47</td>
<td>[0.33, 0.61]</td>
</tr>
<tr>
<td>$\delta_y$</td>
<td>0.92</td>
<td>[0.87, 0.96]</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
<td>0.91</td>
<td>[0.77, 1.05]</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>0.21</td>
<td>[0.15, 0.27]</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>0.82</td>
<td>[0.70, 0.93]</td>
</tr>
</tbody>
</table>

Loglikelihood: -404.2, -395.8

Bayesian estimation results for the proposed DSGE model using equation (13) as the IS curve. Estimates cover the sample period from 1960Q1 to 1979Q4 and 1983Q1 to 2007Q4.
Figure 1: Dynamic Correlation between Output Gap and Inflation

Correlation( output gap(t), inflation(t+k) )

-10 -8 -6 -4 -2 0 2 4 6 8 10

Correlation

Data

Upper bound for a 95% CI

Lower bound for a 95% CI

Model

Baseline model
Figure 2: Impulse Response Functions
Figure 3: Autocorrelation Function