Systemic Risk and Stability in Financial Networks

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Motivation

- Much recent interest in the relationship between systemic risk and network effects, mainly a consequence of the Financial Crisis.

  "In the current crisis, we have seen that financial firms that become too interconnected to fail pose serious problems for financial stability and for regulators. Due to the complexity and interconnectivity of today's financial markets, the failure of a major counterparty has the potential to severely disrupt many other financial institutions, their customers, and other markets."

  – Charles Plosser, 03/06/09

  "Interconnections among financial intermediaries are not an unalloyed good. Complex interactions among market actors may serve to amplify existing market frictions, information asymmetries, or other externalities."

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  The auto industry bailout.

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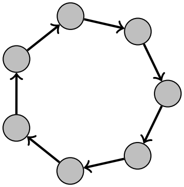
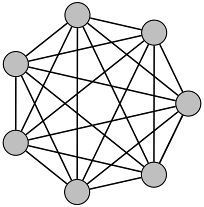
Systemic Risk and Networks?

- A common conjecture: More interbank connections enhance the resilience of the financial system to idiosyncratic shocks, whereas "sparser" network structures are more fragile.
    - Kiyotaki and Moore (1997)
    - Allen and Gale (2000)
    - Freixas, Parigi and Rochet (2000)





Systemic Risk and Networks?

- But also the opposite perspective: more densely connected financial networks are more prone to systemic risk: reminiscent of epidemics.
  - Vivier-Lirimont (2006)
  - Blume *et al.* (2011)
- In the context of input-output economies with linear interactions, sparsity is *not relevant*. Rather, it is the symmetry that matters.
  - Acemoglu *et al.* (2012)
- Which perspective?

This Paper

- A model of interbank lending and counterparty risk in financial networks.
- The form of interactions and magnitude of shocks are crucial for understanding systemic risk and fragility.
- For small shocks, sparsity implies fragility and interconnectivity implies stability.
- Phase transition: with larger shocks, the more complete networks become most fragile, whereas "weakly connected" networks become stable.
- Equilibrium financial networks may be inefficiently fragile.

Interpretation

- Related to a conjecture by Haldane (2009):

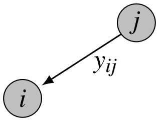
  "Interconnected networks exhibit a knife-edge, or tipping point, property. Within a certain range, connections serve as a shock-absorber. The system acts as a mutual insurance device with disturbances dispersed and dissipated [. . .] But beyond a certain range, the system can flip the wrong side of the knife-edge. Interconnections serve as shock-amplifiers, not dampeners, as losses cascade. The system acts not as a mutual insurance device but as a mutual incendiary device. Risk-spreading – fragility – prevails."

Related Literature

- Financial networks
  - Allen and Gale (2000), Freixas, Parigi and Rochet (2000), Gai, Haldane and Kapadia (2011)
    Caballero and Simsek (2013), Alvarez and Barlevi (2013)
    Elliott, Golub and Jackson (2013).
- Input-output networks
  - Jovanovic (1987), Long and Plosser (1993), Durlauf (1993), Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012), Bigio and La'O (2013)

A Minimalist Model of Financial Networks

- n risk-neutral financial institutions (banks)
- three dates: $t = 0, 1, 2$
- each bank has an initial capital k
- Banks lend to one another at $t = 0$ and write standard debt contracts in exchange.
  - to be repaid at $t = 1$
  - face values: $\{y_{ij}\}$
  - defines a financial network



- For now, we take the interbank commitments as given. (to be endogenized later)

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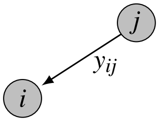

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A Minimalist Model of Financial Networks

- After borrowing, bank i invests in a project with returns at $t = 1, 2$.
  - random return of z_i at $t = 1$.
  - deterministic return of A at $t = 2$ (if held to maturity).
- Bank i's obligations:
  - interbank commitments $\{y_{ji}\}$
  - a more senior outside obligation of value $v > 0$.
- If the bank cannot meet its obligations, it defaults:
  - liquidates its project prematurely and gets ζA
  - costly liquidation: $\zeta < 1$
  - pays back its creditors on *pro rata* basis

Summary: Timing and Description of Events

- $t = 0$:
  - interbank lending happens
  - banks invest in projects
- $t = 1$:
  - short term returns $\{z_i\}$ are realized,
  - banks have to meet the interbank and outside obligations
  - any shortfall leads to default and forces costly liquidation
- $t = 2$:
  - remaining assets have their long-run returns realized.

Payment Equilibrium

- Focus on $t = 1$ with the financial network taken as given:
  - z_j: short-term returns.
  - c_j: cash.
  - y_j: total commitments of bank j to all other banks.
  - ℓ_j: liquidation amount
  - v: outside commitments.

$$x_{ij} = \begin{cases} y_{ij} & \text{if } c_j + z_j + \ell_j + \sum_s x_{js} \geq v + y_j \\ \frac{y_{ij}}{y_j}\left(c_j + z_j + \ell_j + \sum_s x_{js} - v\right) & \text{if } c_j + z_j + \ell_j + \sum_s x_{js} \in \left(v, v + y_j\right) \\ 0 & \text{if } c_j + z_j + \ell_j + \sum_s x_{js} \in \left(0, v\right) \end{cases}$$

Payment Equilibrium

- Let $Q = [y_{ij}/y_j]$ and $x_{ij} = q_{ij}x_j$.

$$x = \left[\min\{Qx + c + z + \ell, y\}\right]^+$$
$$\ell = \left[\min\{y - (Qx + c + z), \zeta A\}\right]^+$$

- Payment equilibrium: a fixed point $\{x, \ell\}$ of the above set of equations.

Proposition

A payment equilibrium exists and is generically unique.

- A generalization of the result of Eisenberg and Noe (2001).

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Notions of Fragility

- Focus on regular financial networks: $y_j = y$ for all j
- Also assume
  - $\zeta = 0$
  - $z_j \in \{a, a - \epsilon\}$
  - $c_j = 0$

Lemma

Conditional on the realization of a shock, the social surplus in the economy is equal to

$$W = na - \epsilon + (n - \sharp defaults)A.$$

- Number of defaults in the presence of one negative shocks
  - Resilience: maximum possible number of defaults
  - Stability: expected number of defaults

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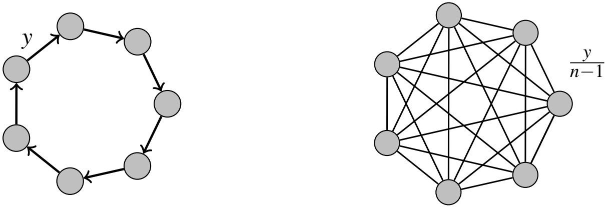
Small Shock Regime

Proposition

There exist ϵ^* and y^* such that for all $\epsilon < \epsilon^*$ and $y > y^*$,

(a) *the complete financial network is the most stable and resilient,*

(b) *the ring financial network is the least stable and resilient,*



Small Shock Regime

- γ-convex combination of two financial networks:

$$y_{ij} = (1-\gamma)y_{ij}^{\text{ring}} + \gamma y_{ij}^{\text{comp}}$$

Proposition

Suppose that $\epsilon < \epsilon^*$ and $y > y^*$.

(a) *the γ-convex combination of the ring and complete financial networks becomes more stable and resilient as γ increases.*

(b) *If there is no contagion in a given network, then there is no contagion in the γ-convex combination of that network with the complete network.*

Small Shock Regime

- Sparsity $\rightarrow$ fragility
  Interconnectivity $\rightarrow$ resilience
  Similar to Allen and Gale (2000) and Freixas *et al.* (2000)

- Intuition: the complete network reduces the impact of a given bank's failure on any other bank, whereas in the ring, all the losses are transferred to the next bank.

- In contrast to Acemoglu *et al.* (2012)
  A model of input-output economies with linear interactions.
  - with linear interactions, positive and negative shocks cancel out.
  - non-linearities of the debt contracts imply that defaults cannot be averaged out by "successes".

δ-Connected Financial Networks

- Financial network is δ-connected if there exists a subset M such that
  (a) $y_{ij} \leq \delta$ for all $i \in M$ and $j \notin M$.
  (b) $y_{ij} \leq \delta$ for all $i \notin M$ and $j \in M$.

- Financial network disconnected if $\delta = 0$.
- "weakly connected" if δ small.

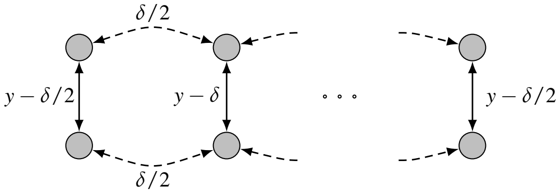
Large Shock Regime

Proposition

If $\epsilon > \epsilon^*$ and $y > y^*$, then

(a) *complete and ring networks are the least resilient and stable networks.*

(b) *for δ small enough, δ-connected networks are more stable and resilient than both.*



- Phase transition/Regime change:
  with large shocks, the complete is as fragile as the ring

Intuition

- Two absorption mechanisms:
  (i) The excess liquidity of non-distressed banks $a - v > 0$.
  (ii) The senior creditors of the distressed banks with claims v.

- The complete network:
  - utilizes (i) very effectively, more than any other network.
  - utilizes (ii) less than any other network.
  - when shocks are small, (i) can absorb all the losses.

- Weakly connected networks:
  - do not utilize (i) that much.
  - utilize (ii) very effectively.
  - with large shocks, networks that utilize (ii) more effectively are more stable.

Distance to Shock

- Normalize the interbank commitments: $q_{ij} = y_{ij}/y$.
- Distance to Shock: Suppose that bank i is hit with the shock:

$$m_{ji} = 1 + \sum_{k \neq j} q_{jk} m_{ki}$$

Proposition

Suppose that $\epsilon > \epsilon^*$ and let $m^* = y/(a-v)$.

(a) *If $m_{ji} < m^*$, then bank j defaults.*

(b) *If all banks default, then $m_{ji} < m^*$ for all j.*

- "More connectivity" (shorter distances) means more fragility.
- Network centrality not relevant.

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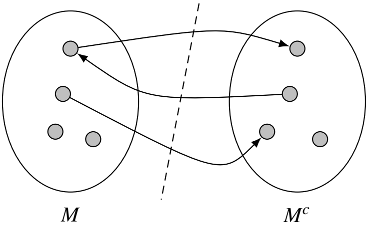
The Bottleneck Parameter

- Bottleneck parameter:

$$\phi = \min_{M \subseteq N} \sum_{\substack{i \in M \\ j \notin M}} \frac{y_{ij}}{|M||M^c|}$$

- How easy is it to “cut” the financial network into two components?
- Captures the extent of connectivity of the network.



- Example: for a δ-connected network: $\phi \leq \delta$.

Large Shock Regime

Proposition

Suppose that $\epsilon > \epsilon^*$ and that $y_{ij} = y_{ji}$.

(a) *If ϕ is high enough, then all banks default.*

(b) *If ϕ is low enough, then at least one bank does not default.*

- More interconnectivity implies more fragility
  - complete network has maximal bottleneck parameter: $\phi \leq \phi_{\text{comp}}$.
  - δ-connected networks have small bottleneck parameter: $\phi \leq \delta$.
  - γ-convex combinations with the complete network increase ϕ:

$$\phi(\gamma y + (1-\gamma) y_{\text{comp}}) \geq \phi(y).$$

Robust-Yet-Fragile Financial Networks

- Interconnected financial networks (in our model) are simultanously
  - very robust to small shocks
  - very fragile in the face of large shocks.

- Related to Haldane's conjecture:
  - The same features that make the network robust for a set of parameters, make it highly fragile for another.

Formation of Financial Networks

- Now consider date $t = 0$.
- Banks endowed with k units of capital and an investment opportunity
  - cannot invest their own funds *a la* Diamond (1982)
  - need to borrow from one another instead
  - exogenous limit k_{ij} on how much i can borrow from j
- Banks decide whether and how much to lend to one another.
  - they can hoard their cash instead.
  - they determine the interest rate they charge other banks.
- Banks can also borrow from outside financiers:
  - competitive
  - risk-neutral
  - have access to a linear technology with rate of return r.

Contracts

- Debt contracts with contingency covenants.
- Each bank and outside depositor posts a contract $\mathbf{R}_i = (R_{i1}, \ldots, R_{in})$
- R_{ij} a mapping from j's lending behavior to the interest rate.
  - Face value of the contract: $y_{ij} = \ell_{ij} R_{ij}(\ell_{j1}, \ldots, \ell_{jn})$.
- Banks with higher risk of default face higher interest rates.

Contracts

(1) All agents $i \in \{0, 1, \ldots, n\}$ simultaneously post contracts $\mathbf{R}_i$. If i cannot lend to bank j or decides not to do so, then $R_{ij} = \varnothing$.

(2) Given the set of contracts $(\mathbf{R}_0, \ldots, \mathbf{R}_n)$, each bank j decides on the amount that it borrows from agent i.

Solution Concept

Definition

A subgame perfect equilibrium if

(a) given the financial network, the PE determines the debt repayments,

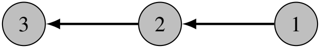
(b) given $\{\mathbf{R}_i\}$ the financial network is a NE of the lending subgame,

(c) no bank has an incentive to post a different contract.

- The interest rates are determined endogenously.
- The outside financiers are indifferent between lending and investing in their technology with return r.

Bilateral Efficiency: The 3-chain Financial Network

- Bank 2 can only borrow from 3 and bank 1 from 2.
- Suppose only bank 1 is subject to a shock with probability p.
- Lending by bank 2 to bank 1 exposes both 2 and 3 to a higher risk.



Proposition

The 3-chain financial network is efficient if and only if it is part of an equilibrium.

- Thanks to the covenants, all bilateral externalities are internalized.

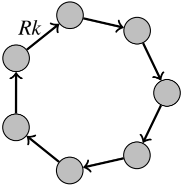
Financial Network Externality: Overlending

Proposition

Suppose $\epsilon < \epsilon^*$. There are $\underline{\alpha} < \bar{\alpha}$ such that the ring financial network

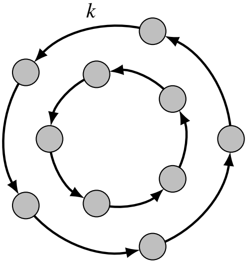
(a) *is part of an equilibrium if $\underline{\alpha}A < (r-1)k$.*

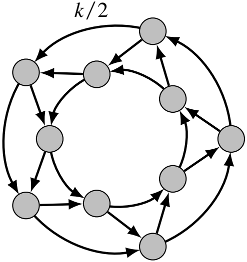
(b) *is socially inefficient if $(r-1)k < \bar{\alpha}A$.*



- Overlending in equilibrium due to a *financial network externality*.
- Intuition: when lending to one another, banks internalize the extra risk they impose on their creditors, but not on their creditors' creditors.
- The public good of financial stability is under-provided in equilibrium.

Financial Network Externality: Excessively Sparse Networks





Proposition

The double-ring financial network is an inefficient equilibrium.

- Banks do not internalize that denser connections reduce the extent of contagion.

Robust-Yet-Fragile Equilibrium Networks

- Consider the complete network
- small shock $\epsilon_\ell < \epsilon^*$ with probability $1 - p$.
- large shock $\epsilon_h > \epsilon^*$ with probability p.

Proposition

There exist constants $\bar{p} > 0$ and A large enough such that

(a) *If $p = 0$, the complete network is socially efficient.*

(b) *If $p > 0$, the complete network is socially inefficient.*

(c) *If $p < \bar{p}$, the complete network is part of an equilibrium.*

- In the presence of highly unlikely large shocks,
  - there is too much lending in equilibrium.
  - banks do not internalize the effect of large shocks on the network.

Summary

- A framework for studying the relationship between the structure of financial networks and the extent of contagion and cascading failures
- Small shocks: rings are most unstable and the complete network is the most stable.
- For larger shocks, there is a phase transition: complete network is the most unstable, and strictly less stable than weakly connected networks.
- Equilibrium financial networks may be inefficient, due to the presence of a financial network externality.