Sovereign Risk and Bank Balance Sheets: 
The Role of Macroprudential Policies

Emine Boz
International Monetary Fund

Pablo N. D’Erasmo
Federal Reserve Bank of Philadelphia

C. Bora Durdu
Federal Reserve Board

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Abstract

This paper explores the effect of macroprudential policies on bank balance sheets, sovereign default risk, aggregate fluctuations and welfare. We build a model of defaultable sovereign debt that incorporates endogenous production and a banking sector with a rich balance sheet. The model captures the observed regularities on bank credit, bank holdings of sovereign bonds and the cyclical properties of sovereign bonds and their spreads. A sovereign default amplifies the business cycle since the default reduces the value of bank assets and constrains the level of credit in the economy. In counterfactual experiments, we analyze the implications of changes in capital requirements as proposed in Basel III. Our findings suggest that increasing risk-weighted capital requirements as well as introducing leverage ratios (i.e., capital requirements based on total assets as opposed to risk-weighted assets) in addition to capital requirements improve welfare.

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1 Introduction

In light of the recent financial crisis in Europe, sovereign default risk and its linkages with the domestic banking sector climbed to the top of the global economic agenda. One of the main links between banks and sovereigns runs through the banks’ holdings of sovereign bonds that a sovereign can default upon leading to losses for banks and hampering their lending to the private sector. The terms with which lending to the private sector takes place affect investment and output which in turn influences sovereign’s debt and default dynamics. These linkages between sovereign default risk and bank balance sheets may strengthen in the run up to a crisis as the quantity and quality of private lending opportunities weaken leading banks to hold more sovereign securities.¹

The nature and the strength of sovereign and bank balance sheet nexus can best be understood in conjunction with bank regulation. This is because the specific way in which bank regulation is formulated can distort the banks’ private lending and sovereign bond holding decisions. For example, Sturzenegger and Zettelmeyer (2006) explain that during the run up to the Russian default of 1998, Russian banks accumulated large stocks of sovereign bonds with the returns from these bonds reaching 30 percent of total income in the first quarter of 1998. They argue that this behavior was partly due to the favorable treatment of these securities under the existing regulatory framework.

Striking evidence for the sovereign debt and bank balance sheet nexus is provided by the observed correlations between sovereign spreads, bank lending to the private sector and the interest rates of these loans in the peripheral European countries. The rise of sovereign spreads in these countries, plotted in Figure 1 coincided with a decline in the volume of loans extended to nonfinancial corporations and an increase in the interest rates associated with these loans. As Figure 2 shows, corporate sector loan interest rates started to rise gradually in 2006, peaking in early 2009 (left panel) while net loans as a share of GDP declined rapidly in early 2009 (right panel), coinciding with the peak of the interest rates. These negative trends reversed in late-2009 perhaps due to various policy interventions. Even so, the reversal appears to have been temporary considering the more recent increases in the interest rate coupled with the decline in credit.

The apparent link between sovereign risk and banking sector stresses is not specific to the recent European experience. Reinhart and Rogoff (2010) document the strong link between banking crises and sovereign default across many advanced and emerging countries. They show that banking crises most often either precede or coincide with sovereign debt crises.

¹Using aggregate data, Asonuma, Bakhache and Hesse (2014) document that the banks’ sovereign domestic debt holdings increased significantly during the recent crisis.
For the cases of Russia and Argentina, Sturzenegger and Zettelmeyer (2006) mention the disruptions in the financial sector around the default episodes of these countries in 1998 and 2001, respectively.

This paper provides a framework to understand the sovereign risk and bank balance sheets nexus and quantify how regulatory changes in the banking sector such as capital requirements affect macroeconomic fluctuations, the risk of sovereign default and the behavior of bank lending. To do so, we build a quantitative model that incorporates households, firms, a domestic bank, international lenders, and a sovereign. The sovereign’s problem is in the spirit of Eaton and Gersovitz (1981) in that it maximizes the households’ welfare and strategically chooses to default or not on its debt. Households own the firms and the bank, supply labor to productive firms and make deposits to the financial sector. Firms have access to a risky technology and borrow from the bank to be able to operate this technology.

The asset side of bank’s balance sheet consists of holdings of sovereign domestic bonds and corporate loans both of which are risky. We assume deposit insurance which creates a moral hazard problem, potentially leading the bank to engage in excessive risky lending. This moral hazard problem provides a rationale for the use of capital requirements, limiting the bank’s lending not to exceed a certain fraction of risk-weighted assets. The bank is a monopolist in the domestic corporate credit market but acts as a competitive player in the sovereign debt market.

The bank’s balance sheet and sovereign risk jointly affect macroeconomic fluctuations. The heightened risk of a potential sovereign default constrains the bank’s ability to extend
credit to the firms. This happens through the capital requirement—as mentioned earlier, the capital requirement limits the size of the bank loans to a multiple of its equity. In our baseline scenario, the capital requirement follows closely that in Basel II where the sovereign bond receives a zero-weight in the computation of the risk weighted assets. A reduction in the loan supply hampers production of the firms that require bank loans to operate their projects. Hence, output falls, which, in turn, increases sovereign default risk further.

After following the Basel II framework for the positive part of our analysis, we examine several modifications to the capital requirements that are motivated by the discussions that took place during the design of Basel III. As detailed in BCBS (2009a,b), Basel III increases the capital adequacy ratios and introduces a leverage ratio that requires capital to be greater than a fraction of total assets. Our main counterfactual captures a case like Basel III where the capital requirement is tighter and a leverage ratio is imposed on the bank. We find....

Other counterfactuals help understand the role of national discretion to assign a zero-risk weight to domestic sovereign bonds. To do so, we compare scenarios with a higher capital requirement on risk-weighted assets where the sovereign bonds continue to receive a zero weight with the ones where this weight is positive.

This paper connects various strands of the literature. The first is the one on sovereign debt following the Eaten-Gersovitz tradition that focuses mostly on building models with endogenous sovereign default with the aim of accounting for emerging markets business cycle characteristics, e.g., Aguiar and Gopinath (2006), Arellano (2008), and Mendoza and Yue (2008), among others. Our difference from this literature is that we incorporate a domestic
banking sector and study how this banking sector interacts with the sovereign borrowing and default decisions. Assuming that the sovereign can only default indiscriminately on all bonds regardless of who holds them, our framework significantly enriches the default decision by making it not only dependent on the total stock of outstanding bonds but also on the fraction held by the domestic bank.

Our analysis of bank capital requirements on macroeconomic fluctuations is in the spirit of Van den Heuvel (2002) and Van den Heuvel (2008) in that we also assume a regulatory bank capital requirement and analyze its macroeconomic implications. Similar to Van den Heuvel (2008), we justify the presence of the capital requirement via moral hazard arising due to deposit insurance and capture the potential costs of these requirements that impose a limit on the fraction of assets that can be financed through deposits.

Our work is also related to a more recent line of papers on sovereign debt and financial sector. Bolton and Jeanne (2010) study sovereign risk in connection with the recent waive of sovereign debt crises in Europe. These authors emphasize cross border spillovers of sovereign risk in an environment with financial integration. Our paper is related to theirs in that we also explore the importance of banks in sovereign default risk. Bocola (2013) studies the link between sovereign default and the banking sector in a model where default risk is exogenous but the structure of the credit market is richer. Gennaioli, Martin and Rossi (2010) study the interaction between sovereign default and domestic bank lending in a more theoretical model. They consider a sovereign that borrows to finance a public sector project and domestic banks hold sovereign debt in order to reduce the sovereign’s incentives to default because the sovereign internalizes the worsening of domestic bank balance sheets. Differently from these papers, we build a quantitative framework in which capital requirements are at the center of the analysis. Finally, Sosa Padilla (2012) studies the relationship between sovereign default and bank credit. The main differences of our work from that of Sosa Padilla are that we introduce risky corporate loans and capital requirements. These additional features allow us to study how bank risk-taking behavior and sovereign default risk interact. Moreover, we are able to analyze the effect of important regulatory changes proposed in the Basel accords and how their implications for the incentives of financial intermediaries and the government.

On the empirical front, two studies greatly support the particular mechanism we focus on to capture the interaction between sovereign risk and the bank balance sheets using bank level data, i.e. the importance of bank’s holdings of government securities. First, using data for a large set of countries, Gennaioli, Martin and Rossi (2014) show that banks that have larger stocks of sovereign securities in the run-up to a default end up cutting their loans.

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2 In this line of papers, Basu (2009) and Basu (2010) are among the first to model a sovereign’s default decision and its connections to the domestic banks’ holdings of the debt.
to the private sector the most during default. Second, Baskaya and Kalemli-Ozcan (2014) use Turkish data and use a novel identification method by studying an exogenous shock (1999 earthquake) to sovereign default risk. They show that those banks that held more sovereign bonds right before the earthquake were affected more than others, cutting their lending to the private sector. In related work, Acharya et al. (2014) show that the sovereign crisis effects the real economy through firms’ financing from banks which is in line with our modeling framework.

2 Environment

The economy is populated by a continuum of households, a continuum of firms, a bank, a government and a continuum of international lenders. The bank intermediates resources between households, firms and the government. The government issues sovereign bonds in domestic and international markets and cannot commit to repay.

The economy is driven by a technology shock $z_t \in \mathbb{Z}$ that follows a Markov process with transition matrix $F(z_t, z_{t+1}) = \Pr(z_{t+1}|z_t)$. We assume that every period $t$ is divided in two sub-periods. In the first sub-period, agents in the economy make decisions with $z_t$ as part of their state space. The second sub-period starts with the realization of $z_{t+1}$ and decisions are made by incorporating this variable to the state space. As it will become clear in the description of the environment, this timing assumption is convenient for the exposition of the problem as well as to obtain the solution of the model.

2.1 Households

Households are infinitely lived and maximize the expected present discounted value of utility given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{(c_t - h_t^\eta)^{1-\sigma}}{1-\sigma} \right]$$

where $E_0$ is the expectation operator, $\beta \in (0,1)$ denotes the subjective discount factor, $c_t$ denotes consumption, $h_t$ labor supply and $\sigma$ is the coefficient of relative risk aversion.

Households are endowed with a unit of labor that they can supply at wage rate $w_t$. They are also endowed with $\bar{d}$ units of a nonstorable perishable good that they can deposit in a domestic bank with a risk-free return $r^b$ or with international financial intermediaries with return $r$. Feasibility implies that $d^b + d^i \leq \bar{d}$ that is, the total deposits in a domestic bank $d^b$ and with financial intermediaries $d^i$ cannot be larger than $\bar{d}$. The ownership of productive
firms is uniformly distributed across households. They can also hold divisible shares of the bank and we normalize the number of shares to one. Shares $S_{t+1}$ are traded at the end of the period at price $P_t$ after dividends are paid. Households pay lump-sum taxes equal to $T_t$ to the government. We do not restrict the sign of $T_t$, so that whenever $T_t < 0$, the government makes lump-sum transfers.

Households’ aggregate budget constraint can be written as

$$c_t + P_t S_{t+1} = w_t h_t + (1 + r_b) d_t^b + (1 + r) d_t^i + (P_t + \Pi_t^b) S_t + \pi_t^+ - T_t,$$

where $\Pi_t^b$ denotes dividend payments from the banking sector and $\pi_t^+$ denotes dividend payments from the ownership of firms.

2.2 Corporate Sector

Firms in the corporate sector live for one period and have access to an investment project with technology given by $f(z_{t+1}, a_{it}, h_{it}) = z_{t+1} a_{it} h_{it}^\alpha$ that combines aggregate productivity $z_{t+1}$, idiosyncratic productivity $a_{it}$ for firm $i$, and labor $h_{it}$. The production function displays decreasing returns in the variable input $h$ so that $0 < \alpha < 1$. In order to operate the project, the firm needs to borrow one unit of investment from the bank at interest rate $r^f$.

The aggregate technology shock $z \in Z$ evolves as a Markov process, $F(z', z) = \Pr(z_{t+1} = z' | z_t = z)$. Idiosyncratic productivity, $a$, is drawn from a continuous distribution $A$ with support in $A = \{a, \bar{a}\}$. For those firms that choose to invest, the return of the project at the end of the period (i.e., after the realization of $z_{t+1}$) is

$$f(z_{t+1}, a_{it}, h_{it}) = \begin{cases} z_{t+1} a_{it} h_{it}^\alpha & \text{with prob } p(z_{t+1}) \\ 0 & \text{with prob } 1 - p(z_{t+1}) \end{cases}$$

in the successful and unsuccessful state respectively. The success probability $p(z_{t+1})$ is a function of aggregate productivity and we assume that $p(z_{t+1})$ is increasing in $z_{t+1}$, i.e., $p(z^1) > p(z^2)$ for any $z^1 > z^2$. This captures the cyclical features of firm exit and failure. This assumption also implies that, conditional on idiosyncratic productivity $a_{it}$, while firms are ex-ante identical they are ex-post heterogeneous because a fraction of them fail. Finally, we assume that there is limited liability and normalize the firms’ outside option to zero.

Firms maximize expected profits and labor decisions are made after the realization of $z_{t+1}$. Only those who can make positive expected profits will choose to invest and, conditional on investment, only those with who can make ex-post positive profits will operate the technology (i.e. hire workers, pay salaries and the principal and interest rate on the loan). That
implies that those operating the project will be a subset of those choose to invest and were successful. This fraction will be a function of \( p(z_{t+1}) \) as well as \( z_t \) and equilibrium prices \( w_t \) and \( r^\ell_t \). We derive this expression explicitly below but for future reference we denote it by \( p^+(z_t, w_t, r^\ell_t, z_{t+1}) \).

### 2.3 Domestic Banking Sector

We assume that there is one large domestic bank that extends loans to firms \( \ell_t \geq 0 \), and receives deposits from the households \( d^b_t \geq 0 \). The bank can also purchase government bonds \( b^\text{gov}_{t+1} \) at price \( q_t \). The bank has monopoly power in the domestic loan market, but it acts competitively in the sovereign debt market. The former assumption on the bank’s monopoly power in the domestic loan market assumption is motivated by the high degree of concentration of the banking sector in European countries. The latter assumption is based on the observation that sovereign debt is traded in secondary markets and while in many countries the majority of debt is hold domestically a large fraction is still in foreign hands. Since the bank is a monopoly in the domestic loan market, it internalizes the impact of its loan supply \( \ell_t \) on the equilibrium loan interest rate \( r^\ell_t \). Moreover, it will be optimal to set the interest rate paid on deposits to the risk free rate, \( r^b_t = r \). This implies that, in equilibrium, the households are indifferent between depositing their funds to the domestic and to the international investors. We assume that the bank’s demand for deposits determine the equilibrium value of deposits taken by the bank and the residual is deposited to international lenders.

In the early sub-period, after receiving payments from sovereign bonds due \( b_t \), banks will extend loans and accumulate new government bonds. Lending to the corporate sector and the government can be financed by the internal funds generated by deposits and the service of government debt, or with external funds, \( \tilde{s}_t \). Our convention is that \( \tilde{s}_t < 0 \) denotes seasoned equity issuance and \( \tilde{s}_t > 0 \) denotes the bank is retaining funds. We assume that equity issuance entails costs so that the bank will never find it optimal to simultaneously pay out dividends and to issue equity. In doing so, we follow the tradition in the corporate finance literature, e.g. Hennessy and Whited (2005), and assume a quadratic cost function. Formally, assuming \( s \) denotes the net payout to equity holders, total issuance costs equal

\[
\phi(\tilde{s}) = I_{[\tilde{s} < 0]}[\phi_0(-\tilde{s}) + \phi_1 \tilde{s}^2].
\]

After the realization of \( z_{t+1} \) (i.e., during the second sub-period), the fraction of successful projects is realized, the bank receives the proceeds from its lending activities and pays backs deposits. If necessary, it issues equity \( s_t \) to cover negative profits and continues. Issuing equity at this stage also entails costs \( \phi(s) \).
We assume that the government can default on its debt and after default the government will be prevent from issuing new bonds for a stochastic number of periods. This implies that the bank’s optimization problem will be different depending on whether the sovereign bond market is open or not. If the bond market is open, the bank’s feasibility constraint at the beginning of the period is given by

\[ \tilde{s}_t = d^b_t + b_t - \ell_t - q_t b_{t+1}, \] (4)

where \( \tilde{s}_t \) is the net payments to shareholders. Using the bank balance sheet identity, equity after loans have been extended can be defined as

\[ e_t = \ell_t + q_t b_{t+1} - d^b_t. \] (5)

If the bond market is closed, the feasibility constraint becomes

\[ \tilde{s}_t = d^b_t - \ell_t, \] (6)

and equity equals \( e_t = \ell_t - d^b_t \).

Consistent with the Basel Accords, we assume that the bank is subject to a risk-weighted capital requirement constraint. That is

\[ e_t \geq \varphi (\ell_t + \omega q_t b_{t+1}), \] (7)

where \( \varphi \in [0, 1] \) is the minimum fraction of total assets that the value of equity can take. The parameter \( \omega \) denotes the weight on sovereign bonds—assumed to be zero in our baseline scenario in line with the preferential treatment of euro area sovereigns in Basel II. In one of our counterfactual experiments we analyze the case for a minimum leverage ratio. In this environment, this entails to setting \( \omega \) to 1.

At the end of the period (i.e., after the realization of the aggregate productivity), the value of net equity issuance by the bank is given by

\[ s_t = p^+(z_t, w_t, r^\ell_t, z_{t+1})(1 + r^\ell_t)\ell_t - (1 + r^b_t)d^b_t + \tilde{s}_t - \varphi(\tilde{s}_t). \] (8)

Then, the net dividend payment to shareholders at the end of the period is

\[ \Pi^b_t = s_t - \varphi(s_t). \] (9)
2.4 Government

The government maximizes households’ utility. It borrows or saves using one period non-contingent bonds denoted by $B_{t+1} \in \mathbb{R}$. Debt is issued in a competitive sovereign debt market at a discounted price, $q_t$. The domestic bank as well as international lenders participate in the market for sovereign debt. We denote sovereign’s borrowing by $B_{t+1} < 0$ and savings by $B_{t+1} > 0$. Any losses (or proceeds) from borrowing and lending in the sovereign debt market are funded through lump-sum taxes $T_t$.

If the government borrows, it receives $q_t B_{t+1}$ units of current period goods and promises to deliver $B_{t+1}$ units of the following period good. The government cannot commit to repay its outstanding debt. At the beginning of the period, before the credit markets open, it can choose to default, $D_t = 1$, or not, $D_t = 0$. In case of default, the government is excluded from borrowing and lending for a stochastic number of periods. More specifically, after a default, with probability $\mu$ the government will regain access to the credit markets and with probability $1 - \mu$, it will stay in financial autarky. As introduced earlier, $x_t = 0$ denotes periods when the government is in good credit standing and it has access to borrowing or saving and $x_t = 1$ denotes periods when it is excluded from the credit markets. Note that as it is standard in the sovereign debt literature we assume an exogenous period of exclusion. However, we depart from this literature in that we do not impose any exogenous output cost of default.

If the government chooses not to default, its budget constraint is

$$T_t = q_t B_{t+1} - B_t. \quad (10)$$

During financial autarky, the government’s budget constraint becomes $T_t = 0$.

2.5 International Lenders

International lenders are risk-neutral and have unlimited access to funds at interest rate equal to $r \geq 0$. International lenders act competitively and invest in sovereign bonds. They also receive deposits, $d^i$, from the households. Since the government is can not commit to repay its debt, the equilibrium price function will depend on the default probability $\lambda_t$.

Expected profits on a loan of size $B_{t+1}$ at price $q_t$ are equal to

$$\Omega_t = -q_t (-B_{t+1}) + \frac{(1 - \lambda_t)}{(1 + r)} (-B_{t+1}), \quad (11)$$

where $\lambda_t$ denotes the expected probability of government default in period $t$. In expected
terms international lenders’ profits are zero.

2.6 Timing of Events

Each period is divided into two sub-periods.

Initial sub-period:

1. Period $t$ starts. The state space is $\{b_t, B_t, z_t\}$. Firms productivity is realized.

2. If credit markets are open, the government chooses to default or not ($D_t = \{0, 1\}$).
   - If $D_t = 0$, the government chooses $B_{t+1}$ and the bank chooses $b_{t+1}$ taking as given the schedule of bond prices $q_t$. The sovereign debt market remains open.
   - If $D_t = 1$, the government is in financial autarky and no bonds are issued. The sovereign debt market closes and will be open next period with probability $\mu$.

3. The bank collects deposits $d_t$ from households, decide on the amount of loans $\ell_t$ and the value of net equity issuance $\tilde{s}_t$.

4. Firms choose whether to invest and demand a loan or not. Loan demand and supply determine the loan interest rate $r^\ell$.

5. Households select the amount of deposits to take to the domestic bank.

Final sub-period:

1. $z_{t+1}$ is realized and the fraction of successful projects $p(z_{t+1})$ is determined.

2. Firms with successful projects decide whether to operate the technology and demand labor or not. The fraction of loan defaults $1 - p^+$ is determined.

3. Household decide on how much labor to supply. Labor demand and supply determine the equilibrium wage rate $w$.

4. Total output is determined.

5. Bank profits are realized and the bank decides on the amount of equity to issue $s_t$, which in turn determines the net payments to shareholders $\Pi_b^t$.

6. Government transfers are determined.

7. Households receive government transfers, wages, payments from the bank and the corporate sector and consume.

8. Period $t$ ends.
3 Equilibrium

This section describes the solution of the problem of the different agents in the economy and defines the equilibrium concept. We use recursive notation in that variables with subscript \( t \) are denoted without time subscript and those with a subscript \( t + 1 \) are denoted with primes.

3.1 Households’ Problem

In equilibrium, the bank drives down the interest rate on deposits \( r^b \) to \( r \), so that households are indifferent between placing their endowment in the domestic bank or international lenders. Essentially, the bank chooses how much of the available deposits to take and the rest is absorbed by the international lenders.

Recall that consumption, labor and the accumulation of shares decisions are made after the realization of \( z' \). The problem of the households is

\[
V(z, S) = \max_{\{d^b, d^i, h, S', c\}} \beta E_{z'|z} \left[ \frac{(c - h^s)}{\eta} \right]^{1-\sigma} + V(z', S')
\]

subject to

\[
\begin{align*}
d^b + d^i & \leq \bar{d} \quad \text{(12)} \\
c + P S' & = \bar{w} h + (1 + r^b) d^b + (1 + r) d^i + (P + \Pi^b) S + \pi - T. \quad \text{(13)}
\end{align*}
\]

The first order condition for labor yields the following relationship between aggregate labor supply and the wage rate:

\[
h^s = \bar{w}^{\frac{1}{\eta-1}} \quad \text{(14)}
\]

The first order condition for \( S_{t+1} \) is:

\[
P \cdot \left( c - \frac{h^s}{\eta} \right)^{-\sigma} = \beta \cdot E_{z'|z'} \left[ \left( c' - \frac{h'^s}{\eta} \right)^{-\sigma} \cdot \left( \Pi^b + P' \right) \right].
\]

Reorganizing the expression above yields the following standard expression:

\[
P = E_{z'|z'} \left[ \bar{R} \cdot (\Pi^b + P') \right], \quad \text{(15)}
\]

where \( \bar{R} = \beta E_{z'|z'} \left[ \left( c' - \frac{h'^s}{\eta} \right)^{-\sigma} / \left( c - \frac{h^s}{\eta} \right)^{-\sigma} \right] \) is the stochastic discount factor. The bank discounts the flow of dividends using this discount factor.
3.2 Firms’ Problem

At the beginning of the period, after the realization of \(a\), at observed \(z\) and given factor prices, firm \(i\) maximizes its expected profits given by

\[
\pi(a, z, w, r^\ell) = E \left[ \max \left\{ \max_{h \geq 0} \{z' a h^\alpha - wh - r^\ell \}, 0 \right\} \right]
\]  

(16)

where \(r^\ell\) is the interest paid out for a unit of loan from the bank. The expectation is taken over the aggregate productivity shock and the idiosyncratic success/failure of the project.

At the beginning of the period, a given firm will choose to invest in the project if expected profits are greater than or equal to zero. This implies that there is an idiosyncratic productivity threshold above which a firm decides to invest by borrowing from the bank. We denote this threshold by \(a^*(z, w, r^\ell)\). This threshold is the solution to

\[
\pi(a^*, z, w, r^\ell) = 0.
\]  

(17)

After the realization of \(z'\), only a fraction \(p(z')\) of those firms that invested will be successful. A successful firm chooses to operate and hire workers if

\[
\pi^+(a, z', w, r^\ell, z') = \max_{h \geq 0} \{z' a h^\alpha - wh - r^\ell \} \geq 0.
\]

In an interior solution, the labor first order condition for this problem implies that the individual labor demand is

\[
h(a, w, z') = \left[ \frac{z' a^\alpha}{w} \right]^{\frac{1}{1-\alpha}}.
\]  

(18)

Using the solution to this problem, we can derive a new idiosyncratic productivity threshold \(\hat{a}\) such that a firm is indifferent between operating the project or not. This threshold is the solution to

\[
\pi^+(\hat{a}, z', w, r^\ell) = 0.
\]

(19)

It is important to note that if the realization of \(z'\) is high enough, \(\hat{a} < a^*\). For this reason, the relevant productivity threshold (i.e. the one that can be used to derive the fraction of firms that effectively operate) is given by

\[
a^+(z, w, r^\ell, z') = \max\{a^*(z, w, r^\ell), \hat{a}(z', w, r^\ell)\}.
\]

Using the (ex-ante) investment threshold \(a^*(z, w, r^\ell)\) we can derive the aggregate loan de-
mand:
\[ \ell^d(z, w, r^\ell) = \int_{a^*(z, w, r^\ell)}^{\bar{a}} dA(a) = 1 - A(a^*(z, w, r^\ell)). \] (19)

It is evident from this analysis that after the realization of the aggregate shock \( z' \), a fraction of firms that invested in the project will choose not to operate and thus default on their loans. This fraction combines the exogenous fraction of firms that fail \( 1 - p(z') \) with the fraction of firms that even after being successful choose to default because at their level of productivity and the realization of the aggregate shock \( z' \) their maximum level of profits is non-positive. More specifically, the total fraction of firms that repays the loan after choosing to invest equals
\[ p^+(z, w, r^\ell, z') = p(z') \frac{1 - A(a^+(z, w, r^\ell, z'))}{1 - A(a^*(z, w, r^\ell))}. \] (20)

In case of a favorable realization of \( z' \), we have that \( a^+(z, w, r^\ell, z') = a^*(z, w, r^\ell) \), so failure is given completely by the function \( p(z') \). On the other hand, when the realization of \( z' \) is low enough \( a^+(z, w, r^\ell, z') > a^*(z, w, r^\ell) \) and a component of firm default is endogenous. Note also that there is an important link between the loan interest rate and firm default since both \( a^+(z, w, r^\ell, z') \) and \( a^*(z, w, r^\ell) \) are increasing functions of \( r^\ell \).

Using the (ex-post) operating threshold \( a^+(z, w, r^\ell, z') \) we can derive the aggregate labor demand:
\[ H^d(z, w, r^\ell, z') = p(z') \int_{a^+(z, w, r^\ell, z')}^{\bar{a}} h(a, w, z') dA(a). \] (21)

Assuming a Uniform distribution for the idiosyncratic productivity shocks, \( A = U[a, \bar{a}] \), aggregate labor demand can be written as
\[ H^d(z, w, r^\ell, z') = p(z') \left[ z' \frac{\alpha}{w} \right]^{1-\alpha} \int_{a^+(z, w, r^\ell, z')}^{\bar{a}} a^{1-\alpha} \frac{da}{\bar{a} - a} \] (22)
\[ = p(z') \left[ z' \frac{\alpha}{w} \right]^{1-\alpha} \left[ \frac{z^{2-\alpha}}{\alpha^{1-\alpha}} - a^+(z, w, r^\ell, z')^{2-\alpha} \right] \frac{1}{\bar{a} - a} \left( \frac{1 - \alpha}{2 - \alpha} \right). \] (23)

Using the aggregate loan supply equation \( h^s = w^{1-\frac{1}{\eta-1}} \) we can derive the equilibrium wage \( w \) for given \( \{z, z', r^\ell\} \). In particular, in equilibrium
\[ w^{\frac{1}{\eta-1}} = p(z') \left[ z' \frac{\alpha}{w} \right]^{1-\alpha} \left[ \frac{z^{2-\alpha}}{\alpha^{1-\alpha}} - a^+(z, w, r^\ell, z')^{2-\alpha} \right] \frac{1}{\bar{a} - a} \left( \frac{1 - \alpha}{2 - \alpha} \right). \] (24)
In this setting, aggregate output is given by
\[
y(z, w, r^\ell, z') = p(z') \int_{a^+} ah(a, w, z')^\alpha a^{1-\alpha} dA(a),
\]
\[
= p(z') (z')^{1-\alpha} \int_{a^+} ah(a, w, z') a^{1-\alpha} dA(a),
\]

3.3 Bank’s Problem

In this section, we present the bank’s problem. The relevant state variables are the bank’s own bond holdings \(b\), the level of aggregate productivity \(z\), the total amount of debt issued by the government \(B\), and whether the sovereign debt market is open or not. The bank takes as given the policy functions of the government.

Before the government decides to default or not, the value of the bank for a given level of bank bond holdings \(b\), the total stock of government debt \(B\), and the level of productivity \(z\) is
\[
W(b, B, z) = D(b, B, z)W^{D=1}(z) + (1 - D(b, B, z))W^{D=0}(b, B, z),
\]
where \(D(b, B, z)\) is the default decision of the government. \(W^{D=1}(z)\) denotes the value of the bank if the government defaults and \(W^{D=0}(b, B, z)\) denotes the value of the bank if the government does not default.

When the government does not default, the bank optimization problem can be summarized as follows:
\[
W^{D=0}(b, B, z) = \max_{\ell, d^b \in [0, \bar{d}], \bar{\nu}, \bar{s}, s} E_{\{z'|z\}} \left[ \tilde{R} \left( \Pi^b(s) + W(b', B', z') \right) \right]
\]
s.t.
\[
\tilde{s} = d^b + b - \ell - q(b', B', z)b',
\]
\[
e = \ell + q(b', B', z)b' - d^b,
\]
\[
e \geq \varphi (\ell + \omega q(b', B', z)b'),
\]
\[
s = p^+(z, w, r^\ell, z')(1 + r^\ell)\ell - (1 + r^b)d^b + \tilde{s} - \phi(\tilde{s}),
\]
\[
\Pi^b(s) = s - \phi(s),
\]
\[
\ell = \ell^d(z, w, r^\ell)
\]
where \(\tilde{R}\) denotes the endogenous stochastic discount factor, equation (27) corresponds to resources available at the beginning of the period, equation (28) defines equity from the balance sheet identity, equation (29) is the capital requirement constraint, equation (30)
denotes the available resources after the realization of the shocks, equation (31) is the net dividend payment and equation (32) is the loan market clearing condition.

Note that even though the bank acts competitively in the sovereign debt market, it internalizes the fact that the default decisions of the government are affected by the bank bond holdings, so the bond price changes with $b'$. The amount of loans extended to firms determines the equilibrium loan interest rate.

When the government defaults on its debt or if the period starts with the government in financial autarky, the problem of the bank is

$$W^{D=1}(z) = \max_{\ell, d \in [0, \bar{d}], \tilde{s}, s} E \left[ \tilde{R} \left( \Pi^b(s^+) + \mu W^{D=0}(0, 0, z') + (1 - \mu) W^{D=1}(z) \right) \right]$$

s.t.

$$\tilde{s} = d^b - \ell,$$ (34)

$$e = \ell - d^b,$$ (35)

$$e \geq \varphi \ell,$$ (36)

$$s = p^+(z, w, r^\ell, z')(1 + r^\ell)\ell - (1 + r^b)d^b + \tilde{s} - \phi(s^-),$$ (37)

$$\Pi^b(s^+) = s^+ - \phi(s^+),$$ (38)

$$\ell = \ell^d(z, w, r^\ell)$$ (39)

When the government is in default, the only sources of funds for the bank are the deposit and equity issuance. The lack of funds through the government bond market has implications for the loan market equilibrium and also features as one of the endogenous costs of default.

3.4 Government’s Problem

To formulate the government optimization problem, we first derive the level of consumption in default and non-default states that enter into the government budget constraint. We first define consumption in the default state. Aggregate consumption in default state is

$$c = wh^* + \bar{d}(1 + r^b) + \pi + \Pi^b$$

$$= y(z, w, r^\ell, z') - (1 - p^+(z, w, r^\ell, z'))\ell + \bar{d} + (\bar{d} - d^b)r^b - \phi(\tilde{s}) - \phi(s).$$
Similarly, aggregate consumption in the non-default state is
\[
c = wh^* + \bar{d}(1 + r^b) + \Pi^b + T
\]
\[
= y(z, w, r^\ell, z') - (1 - p^+(z, w, r^\ell, z'))\ell + \bar{d} + (\bar{d} - d^b)r^b
\]
\[
-\phi(\bar{s}) - \phi(s) + (B + b) - q(b', B', z)(B' + b').
\]

We can now formulate the government optimization problem recursively as a function of state variables, \{b, B, z\}, and the availability of access to the credit markets. The amount of bonds held by the bank, \(b\), needs to be carried around as a state variable since households own the bank and the firms and household consumption is affected by flow of bank and firm profits, which, in turn, is a function of \(b\), among other variables.

At the beginning of the period, if the government is in good credit condition, the government decides whether to default or not (\(D = 1\) or \(D = 0\)). The value at state \{b, B, z\} is given by
\[
V(b, B, z) = \max_{D \in \{0, 1\}} \{V^{D=0}(b, B, z), V^{D=1}(z)\},
\]
where \(V^{D=0}(b, B, z)\) is the value if the government chooses to pay back and remain in the credit market and \(V^{D=0}(z)\) is the continuation value if the government defaults.

If the government chooses not to default, it can issue new bonds and its maximization problem can be formulated as follows:
\[
V^{D=0}(b, B, z) = \max_{B'} \beta E \left\{ U(c, h) + V(b', B', z') \right\}
\]
\quad \text{s.t.}
\[
c = y(z, w, r^\ell, z') - (1 - p^+(z, w, r^\ell, z'))\ell + \bar{d} + (\bar{d} - d^b)r^b
\]
\[
-\phi(\bar{s}) - \phi(s) + (B + b) - q(b', B', z)(B' + b').
\]

where it is understood that \(h\) is consistent with household maximization and \(b', s\) and \(s'\) are consistent with bank’s optimization problem.

The value function when the government chooses to default is given by:
\[
V^{D=1}(z) = E \beta \left\{ U(c, h) + \left[ \mu V^{D=0}(0, 0, z') + (1 - \mu) V^{D=1}(z') \right] \right\},
\]
\quad \text{s.t.}
\[
c = y(z, w, r^\ell, z') - (1 - p^+(z, w, r^\ell, z'))\ell + \bar{d} + (\bar{d} - d^b)r^b - \phi(\bar{s}) - \phi(s).
\]
As in the state of non-default, \( h \) is consistent with household maximization and \( b', s^- \) and \( s^+ \) are consistent with bank’s optimization problem. While in autarky, the country may regain access to external markets with an exogenous probability \( \mu \). When the economy returns to financial markets, it does so with no debt, \( B = 0 \) and \( b = 0 \), and with a continuation value of \( V^{D=0}(0,0,z) \).

The solution to the government optimization problem provides a debt policy function \( B'(b,B,z) \) and the optimal default decision rule \( D(b,B,z) \). The default policies determine a default set \( \Gamma(b,B) \) defined as the set of values for the productivity such that default is optimal given the level of debt held by the bank \( b \) and the total stock of debt issued by the government \( B \),

\[
\Gamma(b,B) = \{ z \in \mathcal{Y} : \ D(b,B,z) = 1 \}. \tag{45}
\]

Using \( \Gamma(b,B) \) we can compute the default probability of the government

\[
\lambda(b',B',z) = \int_{z' \in \Gamma(b',B')} F(dz',z). \tag{46}
\]

### 3.5 International Lender’s Problem and Equilibrium Bond Price

International lenders make zero expected profit on each of the contracts offered to the government. This implies that the equilibrium bond price is given by:

\[
q(b',B',z) = \frac{1 - \lambda(b',B',z)}{(1 + r)}. \tag{47}
\]

### 3.6 Definition Recursive Competitive Equilibrium

A Recursive Competitive Equilibrium is a set of value functions, decision rules and prices \( P, r^b, r^\ell, w, q \) such that:

1. Households value function and decision rules are consistent with the solution to the household problem.

2. Firms decision rules and operating thresholds are consistent with firm’s optimization.

3. Bank’s value function and decision rule are consistent with the solution to the bank’s problem.

4. The value function as well as the government’s default and borrowing decision rules are consistent with the solution to the government problem.
5. At price $P(b, B, z, z')$, households demand for shares equals supply, i.e., $S(b, B, z, z') = 1$.

6. The wage rate $w(b, B, z)$ clears the labor market.

7. The loan interest rate $r^L(b, B, z)$ clears the loan market.

8. The schedule of bond prices $q(b', B', z)$ is consistent with the zero profit condition of international lenders.

9. The government budget constraint is satisfied.

4 Quantitative Analysis

4.1 Calibration

We set the model period to one-year and calibrate the parameters using Spanish data. We choose Spain mainly because it was at the center of the recent European financial crisis and it also has a financial sector greatly exposed to the sovereign debt.

We assume that aggregate productivity follows an AR(1) process:

$$\log(z_t) = \bar{\pi}(1 - \rho_z) + \rho_z \log(z_{t-1}) + \epsilon_t,$$

with $|\rho_z| < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon)$. Once the parameters of the process are estimated from the data, we discretize it using the method outlined in Tauchen (1986) using five points in the set $\mathcal{Z} = \{z_{min}, \ldots, z_{max}\}$. We set $p(z)$ as an equally spaced grid between $p(z_{min})$ and $p(z_{max})$ and calibrate these two values to the data. We allow the cost of equity issuance to differ across states with access to international credit markets and without it. We denote the parameters of these functions $\phi_0^{D\in\{0,1\}}$ and $\phi_1^{D\in\{0,1\}}$. Furthermore, we assume that these are linear functions so we set $\phi_1^D = 0$ for $D = 0, 1$.

We divide the parameters of the model into two sets. The first set includes parameters that can be pinned down directly from the data. This set includes the following parameters:

$$\Theta^1 = \{\sigma, \beta, \mu, \eta, \bar{\pi}, \alpha, r, r^b, \rho, p(z_{min}), p(z_{max}), \varphi, \omega\}.$$

The second set is calibrated so that model-implied behavior of certain aggregates—that we further discuss below—remain closely aligned with their counterparts in the data. This set
includes the following parameters:

$$\Theta^2 = \{\sigma_\epsilon, \bar{d}, \phi^D_0 = 0, \phi^D_0 = 1, \underline{a}, \overline{a}\}.$$  

We first discuss how we pin down the parameters in $\Theta^1$. Following existing studies in the literature, the risk aversion parameter $\sigma$ is set to 2. The discount factor $\beta$ is set to 0.96. The reentry probability $\mu$ is set to 0.25 consistent with the observed periods of exclusion being relatively short in recent sovereign defaults (this would also be consistent with a potential sovereign default by a European economy). The curvature parameter of the labor supply, $\eta$, is set to 1.30—within the range in Greenwood, Hercowitz and Huffman (1988) and more recently in Kean and Rogerson (2012). We normalize $\mu_z$ so that the average output is equal to one along the equilibrium path. The labor share in output, $\alpha$ is set to 0.66. The risk-free interest rate, $r$, is set to 2 percent, calculated using average real government bond yields of Germany in 1999–2012. The interest rate paid on deposits, $r^b$ is also set to 2 percent consistent with the equilibrium of the model. The capital requirement coefficient, $\varphi$, is 4 percent, which is the Tier 1 capital requirement under Basel II and $\omega$ is set to zero to reflect the fact that before Basel III implementation, Euro Area sovereign bonds received zero weight.

The autocorrelation of TFP, $\rho$ is set to 0.51 based on an AR(1) estimation of Spanish multi-factor productivity data from OECD in 1984–2011 period. We do not set $\sigma_\epsilon$ to the value estimated using this procedure because we do not model capital accumulation and investment. Doing so would imply an output variability in the model that is significantly less than that in the data. Hence, we include $\sigma_\epsilon$ in $\Theta^2$ and calibrate it to match the model-implied output variability in the data.

Finally, we calibrate the maximum and minimum values of probability of firm success, $p(z_{max})$ and $p(z_{min})$. To do so, we use the impaired loans to total loans ratio for the largest five banks in Spain when ranked using total assets from Bankscope for the 2004–2012 period. The impaired loans to total loans ratio in our model corresponds to $1 - p(z)$. We use the average of this ratio in the data for the 2004–2007 period (i.e., “non-crisis” years) to set the value of $1 - p(z_{min})$ and the average during the 2008–2012 period (i.e., “crisis” years) to set the value of $1 - p(z_{max})$. The values in the data are 0.008 and 0.04 for the “non-crisis” and “crisis” period.

We now discuss how we pin down the parameters in $\Theta^2$. We choose these parameter values to minimize the distance between a set of data moments and model moments. Through an identification strategy, we were able to link which parameter has more impact on which particular model-implied moment, hence we were able to identify a set of target moments as
We further discuss below.

We set $\sigma_\varepsilon$ to match the volatility of output in Spain which is 2.58 percent based on HP-detrended IFS data on real output over the 1980–2012 period. We identify the bank in our model with the top 5 banks in Spain (when sorted by assets) during the period 1999–2013 (all available data sample in Bankscope). The value of total deposits ($\bar{d}$) is set to match the median ratio of deposits to loans that equals 95.69% (computed as deposits and short term funding over total loans).

The equity issuance cost parameter when the economy is in good credit standing with international lenders ($\phi_{D=0}$) is one of the main determinants of the portfolio composition of the banking sector and is calibrated to the bank equity capital to asset ratio. We find that the median value of total bank equity capital to asset ratio for the top 5 banks in Spain over the 1999–2013 period is 12.33%. The parameter $\phi_{D=1}$ controls the cost of external funding (beyond domestic deposits) for the banking sector during a default event. We set it using average sovereign spreads in Spain. Spreads are calculated using the average observed Spanish spreads (relative to the risk-free German bonds with the same maturity) since 1979, which yields 149 bps. This estimate also lies between the highest and lowest spread observed in the post-1999 period (i.e., after the adoption of the Euro).

Finally, we calibrate $\alpha$ and $\pi$ that control the elasticity of loan demand thus determining the expected return on corporate loans and in place the demand for sovereign bonds to match the loans to asset ratio at the bank level and the ratio of bank bond holdings to total sovereign debt. We observe that the median value of total gross loans to total asset ratio for the top 5 banks in Spain is 63.69%. The ratio of domestic banks’ holdings of sovereign debt securities to the total stock of securities of the general government is from the data set provided by Arslanalp and Tsuda (2012). We compute the average of this ratio for 2001-2013 and find 43%.

Table 1 presents the parameter values and the targets.

---

3 The selection of time period over which to compute mean bond spreads is not a straightforward task. Government bond yields in Spain follow a U-shaped trajectory over a sample that starts in 1979—the beginning year of the most comprehensive sample in IFS. During post-1999 period, the spreads of Spanish bonds over German bond (deflated by respective country inflation rates) appear minuscule until 2009. Hence the post-1999 period essentially implies 11 years of almost zero spreads and three years of larger spreads; with an average of less than 50 bps. Thus, focusing only on the post-1999 period might be misleading.
Table 1: Parameter Values and Targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-Aversion</td>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>Reentry probability</td>
<td>$\mu$</td>
<td>0.25</td>
</tr>
<tr>
<td>Labor Supply Elasticity</td>
<td>$\eta$</td>
<td>1.30</td>
</tr>
<tr>
<td>Avg. Aggregate Productivity</td>
<td>$\tau$</td>
<td>2.41</td>
</tr>
<tr>
<td>Labor Share Ouptut</td>
<td>$\alpha$</td>
<td>0.66</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>$r$</td>
<td>0.02</td>
</tr>
<tr>
<td>Deposit interest rate</td>
<td>$r^b$</td>
<td>0.02</td>
</tr>
<tr>
<td>Autocorrelation $z$</td>
<td>$\rho$</td>
<td>0.54</td>
</tr>
<tr>
<td>Min. value failure prob.</td>
<td>$p(z_{\text{min}})$</td>
<td>0.96</td>
</tr>
<tr>
<td>Max. value failure prob.</td>
<td>$p(z_{\text{max}})$</td>
<td>0.99</td>
</tr>
<tr>
<td>Capital Requirement</td>
<td>$\varphi$</td>
<td>0.04</td>
</tr>
<tr>
<td>Risk-weight</td>
<td>$\omega$</td>
<td>0.00</td>
</tr>
<tr>
<td>Std. Dev. TFP (%)</td>
<td>$\sigma_\varepsilon$</td>
<td>2.56</td>
</tr>
<tr>
<td>Max. value deposits</td>
<td>$\bar{d}$</td>
<td>0.28</td>
</tr>
<tr>
<td>Equity issuance cost</td>
<td>$\phi_{D=0}^0$</td>
<td>0.20</td>
</tr>
<tr>
<td>Equity issuance cost</td>
<td>$\phi_{D=1}^0$</td>
<td>0.18</td>
</tr>
<tr>
<td>Min value productivity</td>
<td>$a$</td>
<td>0.20</td>
</tr>
<tr>
<td>Max value productivity</td>
<td>$\bar{a}$</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Note: Parameters above the line correspond to parameters in set $\Theta^1$ (i.e., those set from independent targets). Parameters below the line correspond to parameters in set $\Theta^2$ (i.e., set to minimize the distance between the targets in the model and in the data) and set to be internally consistent. Data are from Bankscope and IFS.

Table 2 presents a comparison between the model moments and targets and shows that, at this stage, the model does a reasonably good job in matching the standard deviation of output, the equity to asset ratio and the average spreads in Spain. However, the model overpredicts the loan-to-asset ratio and the ratio of bank holdings of sovereign bonds to total outstanding government bonds.
Table 2: Parameter Values and Targets

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev. Output</td>
<td>2.58</td>
<td>3.13</td>
</tr>
<tr>
<td>Deposit to Loan Ratio</td>
<td>95.69</td>
<td>94.83</td>
</tr>
<tr>
<td>Loans to Asset Ratio</td>
<td>63.69</td>
<td>84.62</td>
</tr>
<tr>
<td>Bank Equity to Asset Ratio</td>
<td>12.33</td>
<td>19.13</td>
</tr>
<tr>
<td>Bank bonds to Gov. Bond ratio</td>
<td>43.00</td>
<td>79.15</td>
</tr>
<tr>
<td>Avg. spreads in Spain</td>
<td>1.49</td>
<td>1.96</td>
</tr>
</tbody>
</table>

5 Quantitative Results

In this section, we describe the dynamics of the model. We also present a series of results to show that the model captures the interaction between sovereign risk and the banking sector and delivers dynamics broadly consistent with the euro area data.

5.1 Model Dynamics

We start with a description of the link between the banking sector and production. Figure 3 shows the loan demand (Panel (i)) and the idiosyncratic productivity threshold above which projects are operated as a function of the interest rate (Panel (ii)) for different values of aggregate TFP shock, z.
Panel (i) of Figure 3 reveals that the loan interest rate is decreasing in the loans demanded by the firms. Panel (ii) of the same figure shows that, for a given level of aggregate productivity $z$, as interest rate increases, the fraction of entrepreneurs that decide to operate its technology decreases (since the productivity threshold increases). This results in a lower loan demand, as shown in Panel (i), but also in a lower labor demand and higher equilibrium wages.

Several forces determine the supply of loans as well as the demand for deposits and sovereign bonds held by the banking sector. We begin with the analysis of the bank’s problem when the government is in default. This is because in default, the bank’s decision is simpler since the bank only chooses the amount of loans and how to finance them. Figure 4 shows the expected dividend function $\Pi(z, z')$ (as a function of $\ell(r^\ell, z)$ for different values of $z$) together with the loan demand and the capital requirement constraint faced by the bank. The circles in the figure denote the unconstrained maximum for each case.
Panel (i) shows that the unconstrained objective of the bank has a clear maximum. Note that since loans mature within a period, this is effectively the objective function of the bank. The shape of the function is determined by the expected return on the loans relative to the cost of financing those loans (deposits plus external financing). Given that, in the default state, the sovereign bond market is closed, the capital requirement constraint becomes $\ell \geq \frac{d}{1 - \varphi}$. It is evident from Panel (ii) that the unconstrained optimum is below the amount of loans required to satisfy this constraint at the requested deposits. That implies that in the default state, the capital requirement constraint will be binding and the loans will equal $\ell = \frac{d}{1 - \varphi}$. The intuition is simple, in the unconstrained case, the bank will collect deposits and external funds to equalize its cost with the expected return on loans. The capital requirement constraint imposes a lower bound on loans that prevents the bank from attaining this maximum.

We now move on to the case under no default. The loan decision in this case is linked to the dynamic choice of sovereign bonds. A higher demand for sovereign bonds requires more demand deposits and external funding—affecting the cost of financing for loans as well. A higher demand for sovereign bonds also provides a larger cushion for the bank at the beginning of the next period, since the bank will have more bank capital then. Moreover, higher current holdings of bonds reduces the need for external financing impacting the trade-
offs of the optimal loan supply and bond demand. To illustrate the problem of the bank, Figure 5 shows the expected dividend function (similar to the objective function in a static problem) evaluated at $z_M$, $B = B_M$ and $b = b_M$ (average productivity, government bonds and bank bond holdings in equilibrium) for different values of $b'$. More specifically, we show $b' \in \{b_L, b_M, b_H\}$ for low, medium and high bond demand.

Figure 5: Dividends and Loan Interest Rates

Panel (i): $E[\text{Div}(z_M, z', b_M)]$

Panel (ii): Loan Demand ($r(\ell, z)$) and Cap. Req.

Panel (i) of Figure 5 shows that the shape of the objective function is similar across different values of $b'$ but the level is affected by the amount of external funds necessary to finance the different bond demands. The static maximization would imply that the maximum could be attained with the lowest bond demand but this abstracts from the dynamic considerations (relaxing capital requirements and external financing constraints in future periods). We also note in Panel (ii) that as the demand for sovereign bond increases, the capital requirement constraint becomes more loose as reflected in the vertical dashed shifting to the left. For this parametrization, the unconstrained maximum can be attained even when the bank chooses the lowest possible value of bonds (i.e., $b' = b_L$).

In order to provide intuition on how sovereign bond holdings affect the optimal choices of the bank, as in the previous figures, Figure 6 presents the expected dividend function evaluated at $z_M$, $B = B_M$ and $b' = b_M$ (average productivity, government bonds and bank
bond demand in equilibrium) for different values of current bond holdings $b \in \{b_L, b_M, b_H\}$, where, as before, $\{b_L, b_M, b_H\}$ denote low, medium and high bond holdings.

Figure 6: Bank Policy for Median Level of Bond Holdings

Figure 6 shows that the shape of the objective function is not greatly affected by changes in $b$. However, the level of the profit function is increasing in $b$. This again is the result of the reduction in the need to collect external funds in order to finance the optimal amount of loans. This contrasts with the objective function being decreasing in $b'$. The static and dynamic trade-off together with the comparison of the expected returns of both assets determine the supply for loans and the demand for government bonds.

One important aspect of our model is the role of credit in amplifying the costs of a sovereign default. Figure 7 presents the optimal loan supply in default and non-default states for different productivity levels (with the non-default function evaluated at $b = 0$).
The bottom panel of Figure 7 shows that the loan supply is increasing in $z$ for the non-default case but remains flat for the default state. The loan supply does not respond to changes in $z$ because the capital requirement constraint is binding in equilibrium during default. The figure makes it clear that when productivity is at its average level or above the average level, a sovereign default would imply a reduction in credit and an increase in interest rates on loans to firms (relative to the case under default). At low productivity levels, the opposite is true. It is important to note that this just illustrates the decision rules of the bank. We analyze in more detail the model dynamics during default along the equilibrium path in the next two sections.

To explain the portfolio choice of the bank in more detail, Figure 8 presents the optimal loan supply and optimal bond demand (Panel (i) and (ii) respectively) of the bank evaluated at $z_M$ for different values of $B$ as a function of $b$. 
Figure 8 shows that the loans are decreasing in government bond holdings while the bond demand is increasing. This is directly linked to the expected return on both assets. As the level of government debt increases, the risk of default also increases reducing the price of sovereign bonds (i.e., increasing the sovereign spread) and thus inducing a higher demand for sovereign bonds. We also note that the demand for government bonds is decreasing in bank bond holdings. As the stock of bonds increases, the shadow value of an extra unit of bond decreases, thus reducing its demand.

After analyzing the bank problem in detail, we now move on to presenting the dynamics of the sovereign’s problem. The solution to the sovereign’s problem is similar to that of standard models of sovereign default with the difference that output costs of default, in our model, are endogenous (via the credit channels) and the addition that defaulting on sovereign debt implies reducing the wealth of the banking sector in the domestic economy (as opposed to a default only on international lenders). Figure 9 presents the default decision of the government $D(b, B, z)$ evaluated at $z_M$. 

Figure 8: Loans and Bank Bond Demand as a Function of Bank Holdings of Government Bonds
Figure 9: Default Set

Figure 9 shows that as in other quantitative models of sovereign debt, default is more likely to be chosen at higher levels of government debt. In addition, the sovereign is less likely to default when the banks hold more of its bonds—this is one of the key aspects in our analysis. For a given level of government debt $B$, the government is less likely to default, the higher is the bond level in the domestic banking sector. This result arises because the government takes into account the costs that a default would impose on the bank; the static cost (arising from the destruction of domestic wealth as well as the changes in credit supply) plus costs in the future periods due to the bank being forced to operate using only deposits and external funding to issue loans.

Finally, to illustrate the costs of different debt levels and further shed light on the behavior of supply of bonds and demand of bonds from the domestic banking sector, Figure 10 shows the price schedule for sovereign debt $q(b, B, z)$ evaluated at $z_M$. The result that the government is less likely to default when the bank holds more of its bonds can also be seen in this figure. A given level of government debt can be financed at a lower interest rate (higher $q$), the larger the bond holdings of the bank.
5.2 Main Results

We now turn to the simulation results and the statistical properties of the model. We first highlight the performance of the model in accounting for some important stylized facts on the behavior of government bonds and the bank’s portfolio. Table 3 lists the key moments in the first column, and lists the corresponding moments implied by the model in the second column. The first line reports the ratio of the bank’s holdings of government bonds relative to

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[q^b/(\ell + qb^\prime)]$</td>
<td>11.56</td>
<td>19.59</td>
</tr>
<tr>
<td>$E[B/y]$</td>
<td>46.28</td>
<td>12.84</td>
</tr>
<tr>
<td>$\rho(b/y, y)$</td>
<td>-0.85</td>
<td>-0.05</td>
</tr>
<tr>
<td>$\rho(B/y, y)$</td>
<td>0.92</td>
<td>0.04</td>
</tr>
<tr>
<td>$\rho(\ell, y)$</td>
<td>0.36</td>
<td>0.81</td>
</tr>
<tr>
<td>$\rho(r^\ell, y)$</td>
<td>0.68</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Notes: This table summarizes the long-run moments in the data and in our simulations. All the mean values, denoted as $E[\cdot]$, and the standard deviation of output, $\sigma(y)$, are reported in percent.
the total bank assets. The model-implied ratio of 15 1/4 percent appears reasonably close to 11 1/2 percent in the data. The next two lines in the table compare the behavior of the mean total government debt to output, $E[B/y]$ in the model to that in the data. In examining the behavior of government debt, we focus on data for the gross general government debt from IFS, which include both domestic and foreign debt. Unlike most existing studies in the recent literature, we do not restrict ourselves to only external debt since, in our framework, debt is held both by domestic and foreign agents. However, similar to those studies, our model supports a fairly small amount of government debt in equilibrium compared to the data, 46 1/4 percent in the data and around 7 percent in the model.

The next two moments are the correlations of total sovereign debt to output ratio and bank’s bond holdings to output ratio with output. These two correlations suggest that both in the data, and in the model, the sovereign issues more debt in bad times, consistent with the recent surge of debt during the crisis. At the same time, the bank holds more of the government bonds in bad times. Although the model performs well qualitatively, from a quantitative perspective, the model falls short of generating the strong correlations observed in the data.

In our framework, the bank finds it optimal to reallocate its asset portfolio which consists of loans to firms and government bond holdings, such that it gives more weight to government bonds in bad times. This is because firms’ demand for loans goes down, leaving the bank with smaller monopoly profits and also the return on the sovereign bond goes up. Such portfolio reallocation is evident in the negative correlation between bank’s bond holding and output ($\rho(b/y, y) = -0.05$) and a positive correlation between loans and output ($\rho(\ell, y) = 0.81$).

The interest rates on loans to firms, $r_\ell$ is highly, positively correlated with output both in the data and in the model. In the data, we take the interest rate on loans to nonfinancial corporations up to 1-yr from IFS (available starting in 2003:q1) and deflate it with the CPI inflation. Absent any policy intervention, in a downturn, one may expect a decline in credit to be accompanied by an increase in the interest rate, implying a negative correlation between output and the interest rate. The data, however, suggests the opposite relationship between these two series, which might, partly, be due to policy interventions. The interest rate rises and peaks in 2008:q4, which is the peak of the economic boom and then falls until

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4Sovereign debt to output ratio in the data is total gross general government debt as explained in the calibration section. For bank’s bond holdings, we take domestic banks’ holdings of domestic government securities from the data set provided by Arslanalp and Tsuda (2012).

5Broner, Erce, Martin and Ventura (2013) also document an increase in home bias in sovereign bond markets during turbulent times.

6The data correspondent for $\ell$ is the net claims on the private sector of nation-wide residency depository corporations from IFS. We take the first difference and look at the correlation between the flow of loans and output.
2010:q3. The interest rate picks up again afterwards during the most turbulent times for Spain. This pattern suggests that the policy can achieve procyclical interest rates generally with the exception of during more severe crises episodes such as the recent one. Hence, the data suggest overall a procyclical interest rate. Our model also has the same prediction but for, potentially, different reasons than those that drive the observed patterns in the data as we do not model interest rate policy. In our model, the monopoly profit maximization of the bank delivers the result that when the demand for loans is high, the bank chooses to extend more loans at a higher price. So, not only do they extract more profits through larger loans but also they charge a higher price. Given a highly concentrated banking system in Spain, such a mechanism may be in place in real world also. However, policy interventions discussed above is likely to be playing a role as well.

Loans and interest rates are negatively correlated. This is evident in Equation ??, which suggests that the threshold productivity increases with the interest rates on the loans to firms. Loans are simply determined by the threshold productivity with a higher threshold productivity implying fewer firms deciding to operate their projects and the aggregate demand for labor being smaller.

When the market for sovereign bonds is open, for given productivity, the larger bank bond holdings ($b$) are associated with smaller loans. A closer look at Equations 27–29 reveals why this is the case. For a given source of funds, the banks start the period with, i.e., $b + d$ an expansion of these funds to finance a larger asset portfolio, i.e. $\ell + qb'$ requires new equity issuance. Since the equity issuance is costly and it is the total assets that is constrained by costly equity issuance, when the bank finds it profitable to extend large loans to the firms, it also finds it optimal to cut back on the purchases of sovereign bonds. Note that if issuing new equity were costless, such a trade-off would not be present.

Combining the bank’s balance sheet identity in Equation 28 with the capital requirement in Equation 29 and the fact that $\omega = 0$ in our baseline case helps further understand the link between firm loans and bank’s sovereign bond holdings:

$$(1 - \varphi)\ell + qb' \geq d.$$  

The above equation suggests that since equity is defined as the difference between assets and liabilities in the balance sheet, it is not just the loans to firms that matter but also the holdings of sovereign bonds are important. One key difference between the two is that an extra term $(1 - \varphi)$ appears in front of loans. This is because each unit of extra loan increases the equity requirement by $\varphi$ but there is no such mechanism for bond holdings in
the baseline scenario. During default, the above expression becomes:

\[(1 - \varphi)\ell \geq d.\]

Given our calibration that ensures that the capital requirement binds for sure during exclusion, loans are simply \(\ell = d \ast (1 - \varphi)\).

On the sovereign’s side, ceteris paribus, the sovereign has weaker incentives to default when the stock of sovereign bonds is smaller and when the bank holds more bonds. The former result is standard in sovereign default models due to the benefit from defaulting being smaller if the amount of debt that is defaulted on is smaller. The latter result is novel in our framework that arises because the sovereign internalizes the fact that when the domestic bank holds more of these bonds, its balance sheet deteriorate more, having a larger negative effect on its lending to the firms and output. Such internalization is evident in Figure 10, which shows that the price of sovereign bonds is higher when the bank holds more of them.

Having understood the model dynamics for given productivity, we can analyze the response of the economy to fluctuations in productivity. First, remember that lower aggregate productivity shocks coincide with a smaller fraction of the firms that succeed. Hence, fewer firms operate their project and demand for loans is smaller. As a result, the bank extends fewer loans while demanding more sovereign bonds. This implies a trade-off for the sovereign when it makes its default decision. On the one hand, lower productivity and output strengthens its incentives to default to avoid low levels of consumption, on the other hand, larger bond holdings by the bank weakens its default incentives as explained above. In equilibrium, we find that the former effect is stronger and the sovereign is more likely to default during low productivity periods.

5.3 Default Event Analysis

In this section, we aim to take a closer look at the behavior of our model around the time of default. To do so, we identify default events in our time series simulations and compute the average of all macroeconomic variables across all of the default events. We restrict the window to four periods before and four periods after the default decision which gives a total of nine periods including period zero, when the default is chosen. Remember that \(\mu\), the probability of reentry for the sovereign, is calibrated to 0.25 which implies an average duration of 4 periods of exclusion. Hence, on average, right at the end of the event study window, the sovereign will regain access to the sovereign debt market.

Figure 11 plots the behavior of consumption, labor, TFP, output and loans around default events as percentage deviations from the unconditional mean of each variable. Looking at
TFP, note that it takes about a 5 percent decline in TFP to trigger a default. Given the persistence of these shocks, TFP remains below its long run mean throughout the rest of the period we explore. Labor follows closely the behavior of TFP. Even though firms decide whether to operate their technology or not and also how much labor to hire before they observe $z$, which is realized at the end of the period, they will have observed the contemporaneous value of $z$ in the beginning of the period. Hence the contemporaneous values of TFP and labor are highly correlated during default events, as well as, in normal times.

Figure 11: Dynamics around Default: Macroeconomic Aggregates

Consumption and output decline about 3 percent in the period before and at the time of default. The declines at date $-1$ are due to the timing structure of our model. Values of output and consumption are not solely determined by the contemporaneous values of the shock but by the two subsequent values. Output and consumption at date $-1$ are functions of TFP at date $-1$ since labor, firm entry and loan decisions are made based on the value of $z_{-1}$ but also $z_0$ that is realized at the end of period $-1$. As a result, the large decline in TFP at date 0 leads to declines in consumption and output at date $-1$.

Models timing structure is also the reason why it appears as if the model lacks amplification of TFP shocks, i.e., a 5 percent decline TFP lowers contemporaneous output by less than 5 percent. The largest decline in TFP that occurs at time 0 determines the productivity
at which the output at $-1$ gets produced. Since the labor, entry/exit and loan decisions were made based on the realization of the shock at $-1$ which was not low, the output that is computed for date $-1$ shows a decline that is smaller than the decline of TFP that occurs in date 0. Similarly, at date 0, at the time of default, even though labor, firm entry/exit and loans show large declines, the productivity at which output is produced is based on $z_1$, which has significantly reverted back to its mean.

Finally, at the time of default, loans decline by about 1 to 1 1/2 percent and this decline is protracted. Loans decline not only because firms’ demand for loans falls but also because the bank’s supply is lower. Having observed the large negative aggregate TFP shock in period 0, fewer firms decide to operate their technology. On the bank’s side, the sovereign’s decision to default at the beginning of period 0 leads the capital adequacy requirement to bind, limiting the size of the loans that can be extended without having to pay the cost of issuing new equity. In equilibrium, loans decline after the sovereigns decision to default while the interest rate on these loans fall as shown in Figure 12.

Figure 12: Dynamics around Default: Interest Rates

Figure 13 reveals that during the run-up to a default, the sovereign issues more bonds while both the bank and international lenders buy more of them. International lenders holdings are calculated as the difference between the total stock of sovereign bonds and the
banks holdings, i.e., \((-B - b)/y\) (solid red line). In the run-up, as the sovereign issues more bonds, the price of the bond falls, as shown as an increase in the sovereign bond interest rate plotted in Figure 12. With more or less unchanged demand for loans and interest rates on lending to firms, the bank finds it optimal to buy more of the sovereigns bonds that yield a higher payoff. The bank, however, does not buy up all of the additional stock of sovereign bonds and hence the international lenders holdings of the bond also increases.

Figure 13: Dynamics around Default: Holdings of Sovereign Bonds

5.4 Normative Analysis: Capital Requirements

We now analyze counterfactual scenarios to examine the implications of the changes in the capital requirement. Table 4 summarizes our findings. The first column shows the baseline results. In the second column, the capital requirement coefficient, \(\varphi\), is set to 0.06, which is also the value in Basel III. The third column keeps the capital requirement at the baseline value of 0.04 but the right hand side of the constraint is modified to include bank’s sovereign bond holdings in risk-weighted-assets with a corresponding weight \(\omega\) of 100 percent. This case with \(\omega\) of 100 percent corresponds to a so-called “leverage ratio,” which captures total assets without any risk weighting. The remaining columns, columns 4-6, include both a capital adequacy requirement and a leverage ratio. We modify the model slightly to impose
an inequality constraint in addition to the one in the baseline framework:

\[ e_t \geq \varphi^{lev} (\ell_t + q_tb_{t+1}) . \]

Table 4: Counterfactual Experiments: Changes in the Capital Requirement

<table>
<thead>
<tr>
<th>Moment</th>
<th>Baseline</th>
<th>( \varphi = 0.06 )</th>
<th>( \varphi = 0.04 )</th>
<th>( \varphi = 0.06 )</th>
<th>( \varphi = 0.06 )</th>
<th>( \varphi = 0.06 )</th>
<th>( \varphi^{lev} = 0.04 )</th>
<th>( \varphi^{lev} = 0.04 )</th>
<th>( \varphi^{lev} = 0.04 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank Loans / Assets %</td>
<td>84.23</td>
<td>84.20</td>
<td>84.54</td>
<td>84.20</td>
<td>84.20</td>
<td>84.20</td>
<td>84.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r^L ) %</td>
<td>23.740</td>
<td>23.725</td>
<td>23.742</td>
<td>23.725</td>
<td>23.725</td>
<td>23.725</td>
<td>23.725</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b/B ) %</td>
<td>82.47</td>
<td>90.39</td>
<td>79.41</td>
<td>90.39</td>
<td>90.39</td>
<td>90.39</td>
<td>89.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B/y ) %</td>
<td>12.84</td>
<td>11.96</td>
<td>13.05</td>
<td>11.96</td>
<td>11.96</td>
<td>11.96</td>
<td>11.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Def. Probability %</td>
<td>0.973</td>
<td>1.159</td>
<td>1.251</td>
<td>1.159</td>
<td>1.159</td>
<td>1.159</td>
<td>1.109</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma(c) ) %</td>
<td>1.76</td>
<td>1.62</td>
<td>1.80</td>
<td>1.62</td>
<td>1.62</td>
<td>1.62</td>
<td>1.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha(b,B,z) ) %</td>
<td>0.0342</td>
<td>-0.0050</td>
<td>0.0341</td>
<td>0.0342</td>
<td>0.0342</td>
<td>0.0346</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table summarizes some key moments in the baseline, the first column, and with alternative parameterizations of the capital requirement in the subsequent columns. All the mean values, denoted as \( E[\cdot] \), with the exception of \( E[d] \), \( E[\ell] \), \( E[s^-] \) are reported in percent. The standard deviation of output, \( \sigma(y) \), and the standard deviation of consumption, \( \sigma(c) \) are also reported in percent.

By working with two constraints, we can mimic more closely Basel III which has a capital adequacy requirement and a separate leverage ratio. The last three columns of the table have the same values for \( \varphi \) and \( \varphi^l \), so going from column 4 to column 6, the only change is the increase in the weight assigned to the sovereign bond in the capital adequacy requirement.

We find that a tighter capital requirement, keeping everything else unchanged, improves welfare by 0.0342 percent. This can be seen by comparing the baseline scenario with column 2. Even though the mean spreads and default probability are slightly higher in column 2 compared to baseline, overall, consumption is somewhat less variable under column 2. Since the bank needs to set aside more equity when it extends loans to firms, it shifts its portfolio towards holding more sovereign bonds.

Switching to just a leverage ratio requirement and eliminating the capital adequacy requirement marginally reduces welfare. As one would expect, a leverage ratio requirement makes sovereign bonds less attractive for the bank. The bank on average holds less capital and sovereign defaults more often.

Comparison of columns 4-6 suggest that welfare is increasing in the risk weight assigned
to the sovereign bonds in the capital requirement \((\omega)\).\(^7\) In fact, of all the counterfactuals we look at, the highest welfare is attained in the last column where the capital adequacy requirement is tightest and in addition, a leverage ratio requirement is imposed.

Overall, our results suggest that Basel III would improve welfare compared to Basel II as the former imposes a tighter capital adequacy requirement introduces a leverage ratio requirement. That said, there is room for further improvement by eliminating the preferential treatment of sovereign bonds by assigning them a zero weight in the calculation of risk weighted assets.

6 Conclusion

With the recent financial crisis in Europe, sovereign debt default and its relationship with the banking sector climbed to the top of the global economic agenda. This paper proposes a model to examine the link between sovereign risk and banking sector stresses. The model captures the procyclical bank credit and countercyclical bank holdings of sovereign bonds. Since the sovereign defaults indiscriminately, bank losses due to a default hampers its lending to firms, thereby, generating an endogenous cost of default.

Using the model, we quantify how regulatory changes in the banking sector (such as capital requirements) affect macroeconomic fluctuations, the risk of sovereign default and the probability of a banking crisis. Our preliminary findings suggest that both the introduction of leverage ratios and increasing the capital requirement on risk weighted assets where sovereign bonds are assigned a zero weight improve welfare. Further welfare improvement can be achieved by doing away with the practice of assigning a zero risk weight to sovereign bonds.

\(^7\)Note that the last column effectively has only one constraint because with \(\omega = 1\), the capital adequacy requirement becomes a leverage ratio requirement and the capital adequacy requirement always binds before the leverage ratio requirement does \((\varphi > \varphi^d)\).
References


