Firm-to-Firm Trade

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- The Ultimate Goal: Understanding the impact of interfirm trade, particularly outsourcing, on labor market outcomes

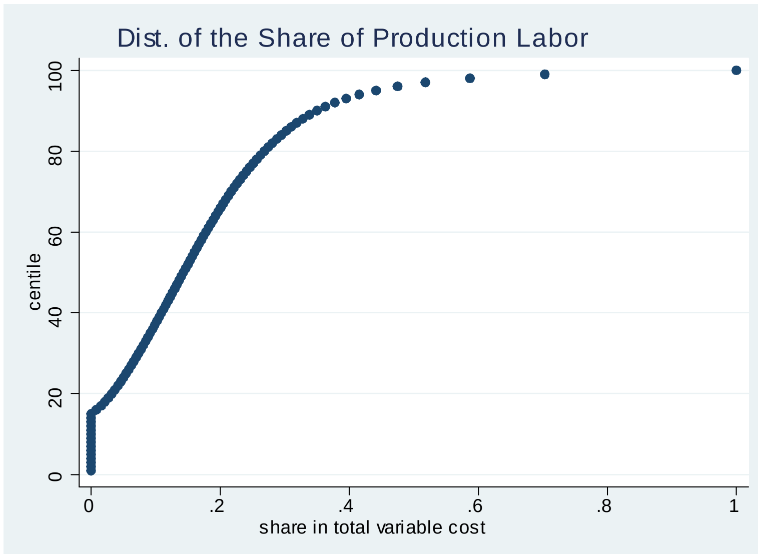
- Achieving it requires coming to grips with enormous heterogeneity in how firms produce and who they sell to

- Previous work this decade: accommodating producer heterogeneity into general equilibrium analysis

- But producers' demand and their technologies continue to be treated as monolithic

Observation 1

- Huge heterogeneity in firm input and purchasing decisions
- Example: French manufacturing firms' labor shares and unskilled labor shares in gross production


Dist. of the Share of Production Labor
centile
0
20
40
60
80
100
0
.2
.4
.6
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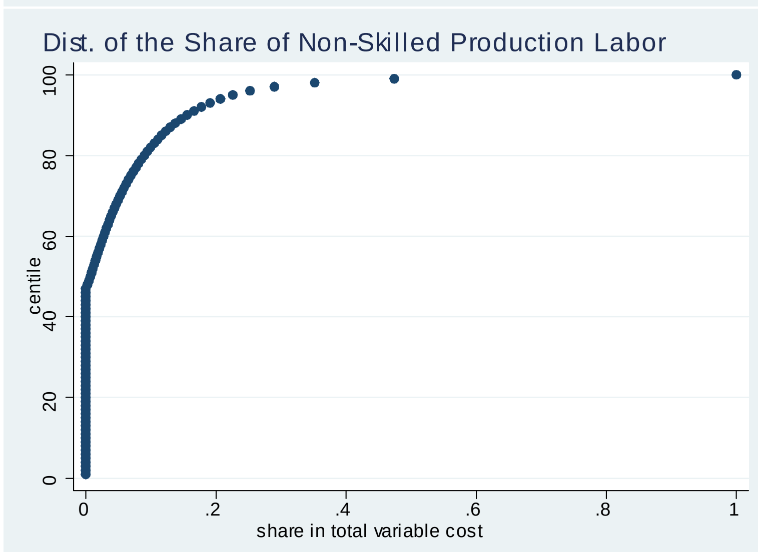
1
share in total variable cost


Dist. of the Share of Non-Skilled Production Labor
centile
0
20
40
60
80
100
0
.2
.4
.6
.8
1
share in total variable cost

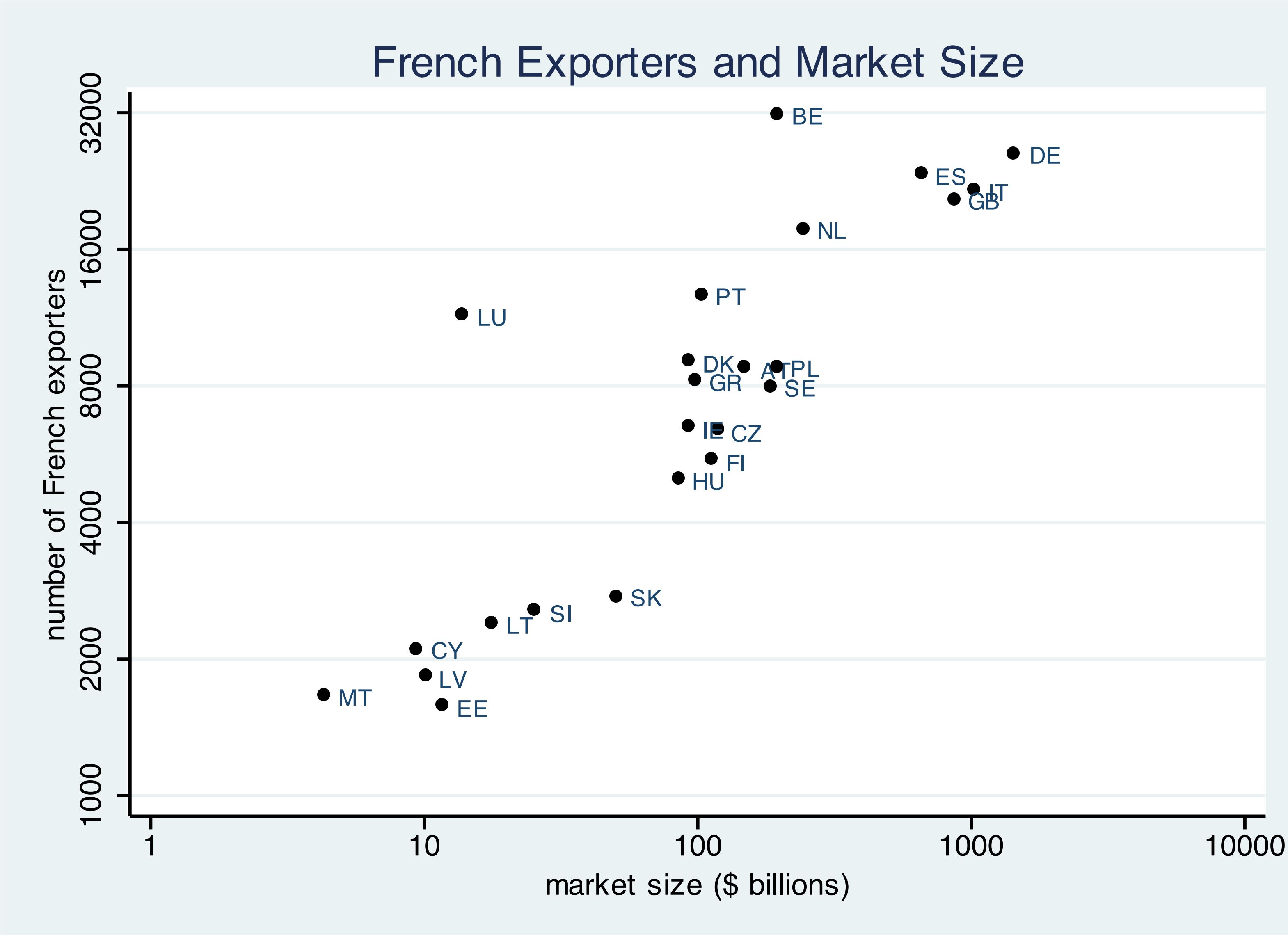
- But standard models have a common production function with common shares across firms, at least within a sector

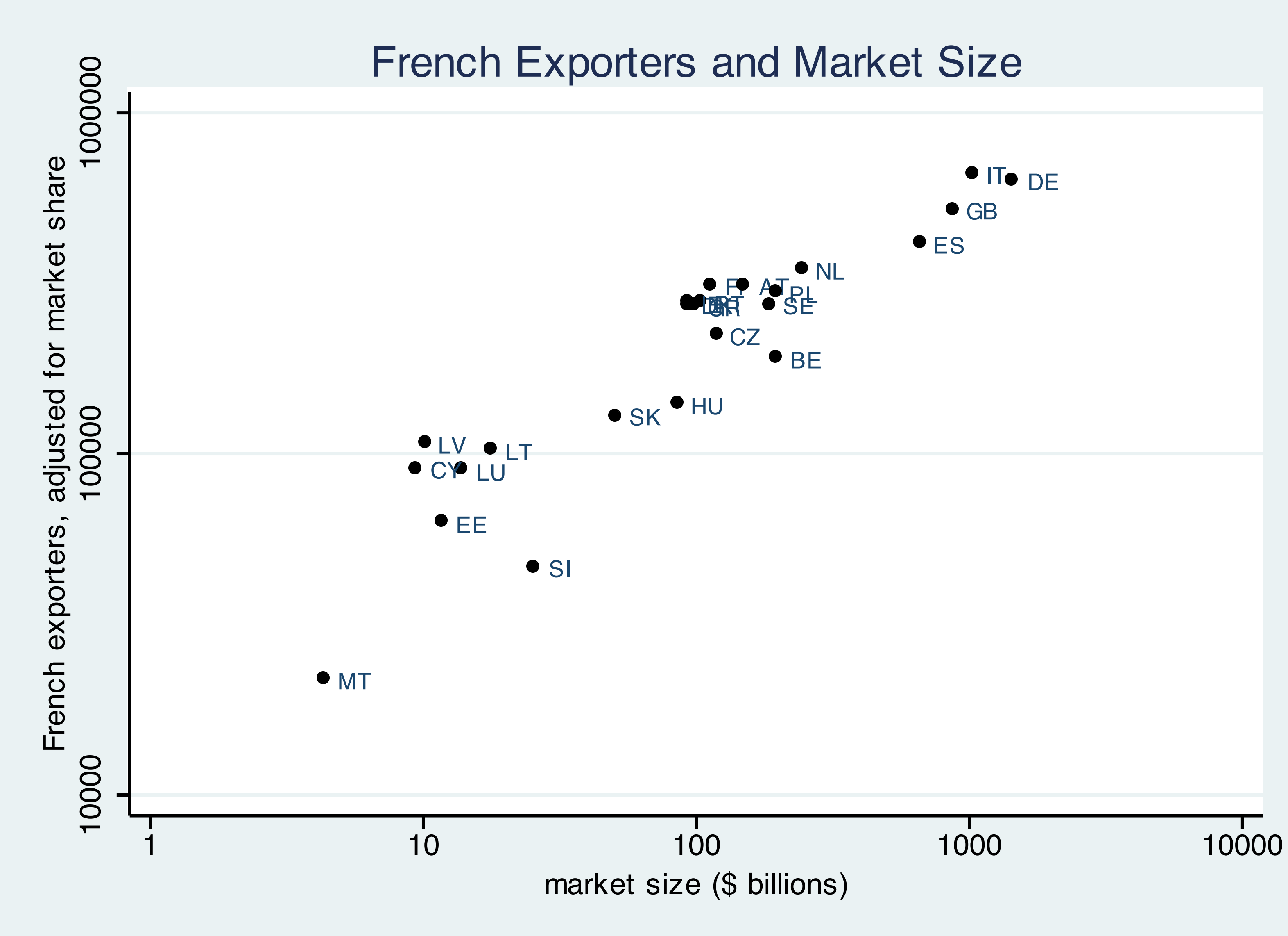
Observation 2

- Huge heterogeneity in the number of buyers a firm has and in their sales to a given buyer

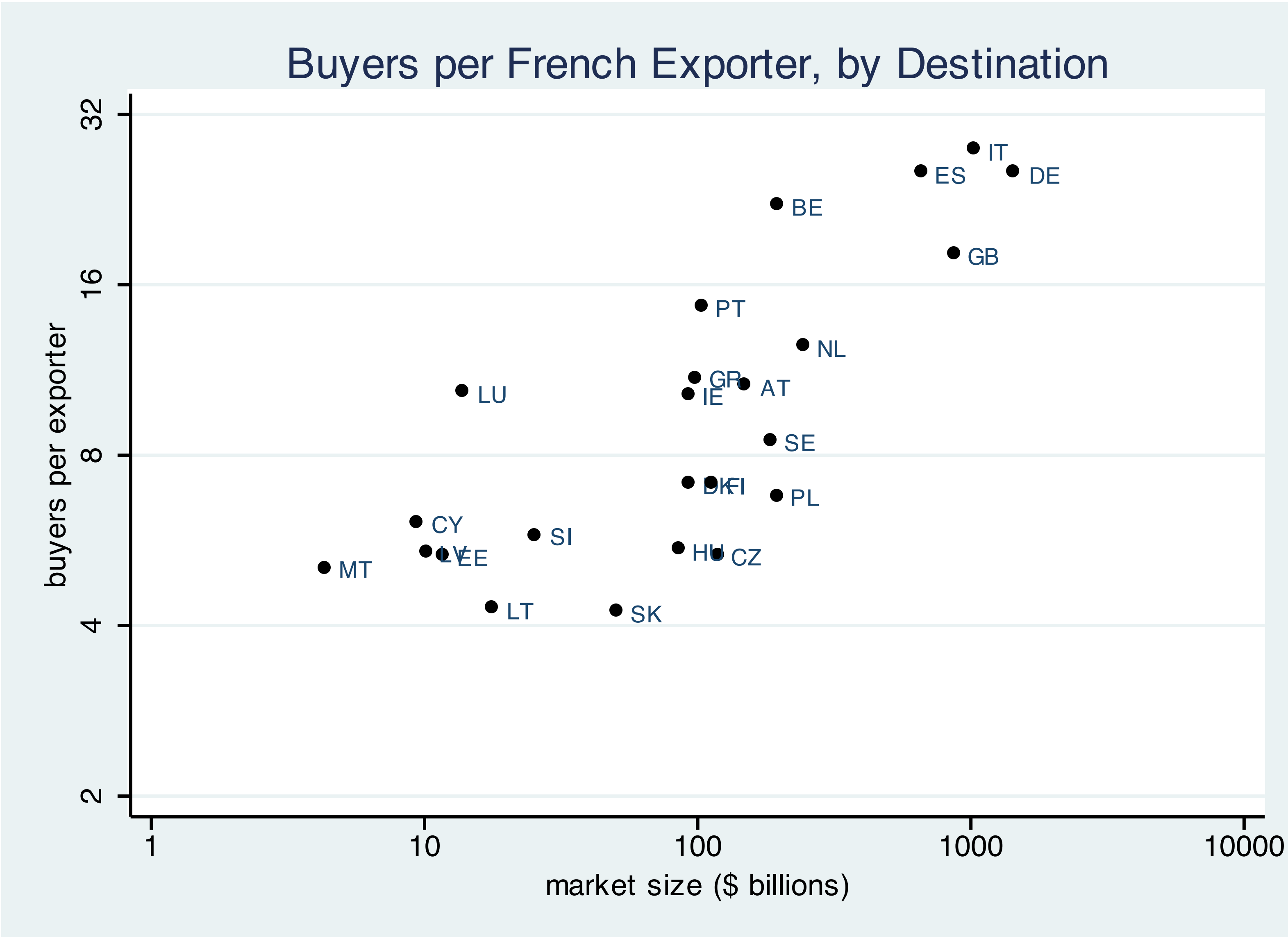
- But as with exporters, there are clear patterns

- Some EU evidence


French Exporters and Market Size
number of French exporters
1000
2000
4000
8000
16000
32000
market size ($ billions)
1
10
100
1000
10000
BE
DE
ES
IT
GB
NL
PT
LU
DK
AT
PL
GR
SE
IE
CZ
FI
HU
SK
SI
LT
CY
LV
MT
EE



French Exporters and Market Size
French exporters, adjusted for market share
market size ($ billions)
10000
100000
1000000
1
10
100
1000
10000
IT
DE
GB
ES
NL
FI
AT
PL
SE
CZ
BE
HU
SK
LV
LT
CY
LU
EE
SI
MT



Buyers per French Exporter, by Destination
buyers per exporter
32
16
8
4
2
1
10
100
1000
10000
market size ($ billions)
IT
ES
DE
BE
GB
PT
NL
GR
AT
LU
IE
SE
DK
FI
PL
CY
SI
LV
EE
MT
HU
CZ
LT
SK

Customers per French Exporter

	Destination Market			
	Lithuania	Denmark	UK	Germany
Market Size ($billions)	18	94	882	1480
Customers per Exporter:				
Mean	4.2	7.1	17.9	24.9
Percentiles:				
25th	1	1	1	2
50th	2	2	3	4
75th	4	5	9	12
90th	9	12	25	35
95th	15	21	48	70
99th	40	77	224	329

Data are for 2005.

But standard models have a representative buyer

Can we also incorporate the heterogeneity and granularity we observe in the customs and firm-level production data into a GE framework?

Progress so Far

- Continuum at the aggregate. Otherwise general equilibrium analysis is problematic

- But outcomes for individual firms and households are granular

Related Literature

- Much

- But particularly Oberfield (2013) on upstream and downstream linkages

- Garetto (2013) with head-to-head competition between workers and intermediates

- Relates to buyer-seller networks as in Chaney (2013) and in Eaton, Eslava, Krizan, Jinkins, and Tybout (2014) but we are static and GE

Model Basics

Standard Ricardian Assumptions

- N countries which we index by i (when selling) or n (when buying)

- Country i has an endowment of L_i^l workers of type l

- Workers are the final buyers

Buyers (Households or Firms)

- $k = 1, .., K$ **purposes** (a household's **needs** or a firm's **tasks**)

- Cobb-Douglas share α_k for households or β_k for firms

- Purpose k can be fulfilled with a good from a particular firm or with a type of labor

Production Function

Output of firm j in country i:

$$Q_i(j) = z_i(j) \prod_{k=1}^{K} b_k^{-1} \left(\frac{m_{k,i}(j)}{\beta_k} \right)^{\beta_k}$$

$z_i(j)$ efficiency, $k = 1, 2, .., K$, tasks, b_k a constant (to go away later), β_k share

- Type of labor appropriate for task k, $l(k)$
- Wage for task k, $w_{k,i} = w_{l(k),i}$
- Firm labor productivity at task k, $q_{i,k}(j)$
- Nash bargaining with an all-powerful buyer $\longrightarrow$ unit cost pricing
- Hence no profits or fixed costs

- Unit cost to perform task k:

$$c_{k,i}(j) = \mathsf{min}\left\{\frac{w_{k,i}}{q_{k,i}(j)}, c_{k,i}^{\mathsf{min}}(j)\right\}$$

- Firm j's cost to deliver in n:

$$c_{ni}(j) = \frac{d_{ni}}{z_i(j)} \prod_{k=1}^{K} \left(\frac{c_{k,i}(j)^{\beta_k}}{b_k}\right)$$

Distributional Assumptions

1. Measure of potential producers in country i with efficiency $Z \geq z$:

$$\mu_i^Z(z) = T_i z^{-\theta}$$

2. Worker productivity performing a task for a given producer Q distributed:

$$F(q) = \mathrm{Pr}[Q \leq q] = e^{-q^{-\phi}}$$

3. Measure of producers who can supply country i at a unit cost below c is:

$$\mu_i(c) = \Upsilon_i c^{\theta}$$

   where $\theta > 0$ and $\Upsilon_i \geq 0$.

1,2 are exogenous. We will derive 3

Matching Buyers and Sellers

- Match intensity between a seller with cost c and a buyer for task k in n is:

$$e_{k,i}(c) = \lambda_{k,i} c^{-\varphi}$$

- $\lambda_{k,i}$ is a parameter governing how easy it is for sellers and buyers to find each other for task k in market i

- $0 \leq \varphi < \theta$ gives an advantage to low cost sellers

- A buyer in i for task k encounters a number of suppliers with unit cost below c that is distributed Poisson with parameter

$$
\begin{aligned}
\rho_{k,i}(c) &= \int_0^c e_{k,i}(x)d\mu_i(x) \\
&= \int_0^c \lambda_{k,i}x^{-\varphi}\theta\Upsilon_i x^{\theta-1}dx \\
&= \frac{\theta}{\theta-\varphi}\lambda_{k,i}\Upsilon_i c^{\theta-\varphi}
\end{aligned}
$$

Distribution of Lowest Cost

- From Poisson, probability of no input with unit cost $C \leq c_k$:

$$\text{exp}\left[-\rho_{k,i}(c_k)\right]$$

- Probability no worker can perform the task at cost $C \leq c_k$ is $F(w_{k,i}/c_k)$.

- Distribution of the lowest cost to fulfill task k:

$$G_{k,i}(c_k) = 1 - F(w_{k,i}/c_k)e^{-\rho_{k,i}(c_k)}$$

- Assume:

$$\theta - \varphi = \phi$$

to give us:

$$G_{k,i}(c_k) = 1 - e^{-\Xi_{k,i} c_k^{\phi}}$$

where

$$\begin{aligned} \Xi_{k,i} &= \nu_{k,i} + w_{k,i}^{-\phi} \\ \nu_{k,i} &= \frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i \end{aligned}$$

- Hire workers with probability $\upsilon_{k,i} = w_{k,i}^{-\phi} / \Xi_{k,i}$

- Purchase intermediate with probability $1 - \upsilon_{k,i} = \nu_{k,i} / \Xi_{k,i}$

Labor Shares

- With a continuum of producers $\upsilon_{k,i}$ is the aggregate share of labor in performing task k in country i.

- Aggregate share of labor of type l in total production costs:

$$\beta_i^l = \sum_{k \in \Omega_l} \beta_k \upsilon_{k,i}$$

- Overall labor share in production costs:

$$\beta_i^L = \sum_l \beta_i^l.$$

- Labor share depends on wages and other factors through $\upsilon_{k,i}$.

The Aggregate Cost Distribution

- Measure of suppliers from n who can deliver to i at a unit cost below c:

$$\begin{aligned}
\mu_{ni}(c) &= T_i d_{ni}^{-\theta} c^{\theta} \prod_k \int_0^{\infty} b_k^{\theta} c_k^{-\theta\beta_k} dG_{k,i}(c_k) \\
&= T_i d_{ni}^{-\theta} c^{\theta} \prod_k \int_0^{\infty} b_k^{\theta} c_k^{-\theta\beta_k} \phi \Xi_{k,i} c_k^{\phi-1} \exp\left(-\Xi_{k,i} c_k^{\phi}\right) dc_k \\
&= T_i d_{ni}^{-\theta} c^{\theta} \prod_k \Xi_{k,i}^{\tilde{\beta}_k} \\
&= T_i \Xi_i d_{ni}^{-\theta} c^{\theta}
\end{aligned}$$

where:

$$\begin{aligned}
\tilde{\beta}_k &= \frac{\theta}{\phi}\beta_k \\
\Xi_i &= \prod_{k=1}^{K} \left(\Xi_{k,i}\right)^{\tilde{\beta}_k} \\
b_k &= \left[\Gamma(1 - \widetilde{\beta}_k)\right]^{-1/\theta}
\end{aligned}$$

- Summing across sources i:

$$\mu_n(c) = \sum_{i=1}^{\mathcal{N}} \mu_{ni}(c) = \Upsilon_n c^{\theta}$$

the cost distribution of suppliers posited above, where we now have:

$$\Upsilon_n = \sum_i T_i \Xi_i d_{ni}^{-\theta},$$

- The Υ_n's solve the system of equations:

$$\Upsilon_n = \sum_i T_i d_{ni}^{-\theta} \prod_k \left(\frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i + w_{k,i}^{-\phi} \right)^{\tilde{\beta}_k}$$

  for $n = 1, 2, ..., \mathcal{N}$ (given wages)

- Blackwell's sufficient conditions for solution

- Homogeneity of Υ_n's in all T_i's and all $1/\lambda_{k,i}$'s

- Trade shares:

$$\pi_{ni} = \frac{T_i \Xi_i d_{ni}^{-\theta}}{\sum_{i'} T_{i'} \Xi_{i'} d_{ni'}^{-\theta}}$$

The Aggregate Production Function

$$
\begin{aligned}
Q_i &= \prod_{k=1}^{K} \left[\tilde{\gamma} \left(L_{k,i} \right)^{\phi/(\phi+1)} + (1-\tilde{\gamma}) \left(I_{k,i} \right)^{\phi/(\phi+1)} \right]^{\beta_k(\phi+1)/\phi} \\
\tilde{\gamma} &= \frac{1}{1+\Gamma(1+1/\phi)^{\phi/(1+\phi)}}
\end{aligned}
$$

where $L_{k,i}$ is the labor used in task k in country i and $I_{k,i}$ are intermediates used for task k in country i.

Consumer side

- analogous to firms
- shares α_k instead of β_k.
- Expected expenditure function for utility V

$$Y(V, P_n^C) = V P_n^C.$$

- Price index

$$P_n^C = \prod_{k=1}^{K} \left(\Xi_{k,n} \right)^{-\tilde{\alpha}_k}$$

Aggregate Equilibrium I: Production

- With balanced trade final spending X_n^C is labor income:

$$X_n^C = \sum_{l=1}^{L} w_n^l L_n^l = \sum_{k=1}^{K} w_{k,n} L_{k,n}.$$

- Total production:

$$Y_i = \sum_{n=1}^{\mathcal{N}} \pi_{ni} \left[\Phi_n^C X_n^C + \Phi_n^I Y_n \right]$$

$$\Phi_n^C = 1 - \alpha_n^L, \quad \Phi_n^I = 1 - \beta_n^L$$

Leontief Algebra

$$\boldsymbol{Y} = \boldsymbol{\Pi}\left(\boldsymbol{\Phi}^{C}\boldsymbol{X}^{C} + \boldsymbol{\Phi}^{I}\boldsymbol{Y}\right)$$

$$\boldsymbol{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_{\mathcal{N}} \end{bmatrix}, \quad \boldsymbol{X}^C = \begin{bmatrix} X_1^C \\ X_2^C \\ \cdot \\ \cdot \\ \cdot \\ X_{\mathcal{N}}^C \end{bmatrix}$$

$$\mathbf{\Phi}^j = \begin{bmatrix} \Phi_1^j & 0 & \cdots & 0 & 0 \\ 0 & \Phi_2^j & \cdots & 0 & 0 \\ . & . & . \quad . \quad . & . & . \\ . & . & . \quad . \quad . & . & . \\ . & . & . \quad . \quad . & . & . \\ 0 & 0 & \cdots & \Phi_{\mathcal{N}-1}^j & 0 \\ 0 & 0 & \cdots & 0 & \Phi_{\mathcal{N}}^j \end{bmatrix} \quad j = C, I$$

$$\mathbf{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{21} & \cdots & \pi_{\mathcal{N}-1,1} & \pi_{\mathcal{N}1} \\ \pi_{12} & \pi_{22} & \cdots & \pi_{\mathcal{N}-1,2} & \pi_{\mathcal{N}2} \\ \vdots & \vdots & \vdots\ \vdots\ \vdots & \vdots & \vdots \\ \pi_{1,\mathcal{N}-1} & \pi_{2,\mathcal{N}-1} & \cdots & \pi_{\mathcal{N}-1,\mathcal{N}-1} & \pi_{\mathcal{N},\mathcal{N}-1} \\ \pi_{1\mathcal{N}} & \pi_{2\mathcal{N}} & \cdots & \pi_{\mathcal{N}-1,\mathcal{N}} & \pi_{\mathcal{N}\mathcal{N}} \end{bmatrix}$$

Solving for $\boldsymbol{Y}$:

$$\mathbf{Y} = (\boldsymbol{I}_{\mathcal{N}} - \boldsymbol{\Pi}\boldsymbol{\Phi}^{I})^{-1}\boldsymbol{\Pi}\boldsymbol{\Phi}^{C}\boldsymbol{X}^{C}$$

where $\boldsymbol{I}_{\mathcal{N}}$ is the $\mathcal{N} \times \mathcal{N}$ identity matrix.

Aggregate Outcomes II: The Labor Market

$$w_i^l L_i^l = \alpha_i^l X_i^C + \beta_i^l Y_i.$$

Simulating Aggregate Outcomes

Table 1: Baseline Parameter Settings for Simulation

Parameter	symbol	value
Pareto parameters:		
efficiency distribution	theta	5
price distribution	phi	3
Technology level per person	T_i/L_i	3.6
World labor force	L	1
Labor by type (fractions of labor force):	L^l	
service		0.6
production		0.4
Iceberg trade cost	d	1.2
Tasks, by type:		
service tasks:		
number of tasks	K	4
total share	beta	0.4
production tasks:		
number of tasks	K	12
total share	beta	0.6
Task shares in consumption (same as for production)	alpha	
Outsourcing parameters:	lambda	
service		0
production		0.2

Table 2: Aggregate Results of Simulation

	Country Size					
	L=0.001	L=0.009	L=0.09	L=0.2	L=0.3	L=0.4
Production value added:						
Share of GDP	0.133	0.133	0.134	0.135	0.135	0.135
Share of gross production	0.27	0.27	0.26	0.24	0.23	0.22
Fraction of production tasks outsourced:	0.55	0.55	0.57	0.60	0.62	0.64
Import share of production	1.00	0.98	0.79	0.61	0.48	0.39
Wage:						
service	0.91	0.91	0.96	1.01	1.06	1.11
production	0.93	0.93	0.92	0.91	0.91	0.90
Skill premium (service/production)	0.98	0.98	1.04	1.11	1.17	1.23
Real wage:						
service	1.39	1.40	1.46	1.54	1.61	1.67
production	1.42	1.42	1.41	1.39	1.37	1.36
Welfare (real per capita consumption)	1.40	1.41	1.44	1.48	1.51	1.54

1. Production value added does not include service tasks (i.e. purchased services)

2. Wage is normalized so that labor income of the World is 1

Table 3: Aggregate Results with Different Trade Costs

	Trade Cost (small country, L=.009)					Trade Cost (large country, L=0.3)				
	10.00	1.80	1.20	1.05	1.00	10.00	1.80	1.20	1.05	1.00
Production value added:										
Share of GDP	0.01	0.05	0.13	0.13	0.12	0.12	0.12	0.14	0.13	0.12
Share of gross production	0.59	0.50	0.27	0.17	0.14	0.34	0.32	0.23	0.16	0.14
Fraction of prod. tasks outsourced:	0.02	0.16	0.55	0.72	0.77	0.44	0.47	0.62	0.73	0.77
Import share of production	0.00	0.83	0.98	0.99	0.99	0.00	0.11	0.48	0.65	0.70
Wage:										
service	0.67	0.62	0.91	1.14	1.25	0.91	0.93	1.06	1.18	1.25
production	1.48	1.17	0.93	0.73	0.63	1.14	1.11	0.91	0.72	0.63
Skill premium (service/production)	0.46	0.53	0.98	1.56	1.97	0.79	0.84	1.17	1.63	1.97
Real wage:										
service	0.75	0.85	1.40	2.02	2.44	1.18	1.23	1.61	2.10	2.44
production	1.66	1.61	1.42	1.30	1.24	1.48	1.47	1.37	1.28	1.24
Welfare (real per capita cons.)	1.12	1.16	1.41	1.73	1.96	1.30	1.33	1.51	1.77	1.96

1. Production value added does not include service tasks (i.e. purchased services)

2. Wage is normalized so that labor income of the World is 1

Implications for Individual Producers

I. The Distribution of Final Buyers

- Seller with cost c in market n encounters a number of final customers for need k that is Poisson with parameter:

$$e_{k,n}(c)L_n = \lambda_{k,n}c^{-\varphi}L_n.$$

- Makes a sale with probability $e^{-\Xi_{k,n}c^{\phi}}$

- So the number of final sales for need k is Poisson with parameter:

$$\eta^C_{k,n}(c) = \lambda_{k,n}L_n c^{-\varphi}e^{-\Xi_{k,n}c^{\phi}},$$

- Total final sales is Poisson with parameter:

$$\eta_n^C(c) = \sum_{k=1}^{K} \eta_{k,n}^C(c).$$

II. The Distribution of Intermediate Buyers

- M_n denotes the measure of active producers in country n (determined below)

- Seller with cost c in market n encounters a number of intermediate customers for task k that is Poisson with parameter:

$$e_{k,n}(c)M_n = \lambda_{k,n}c^{-\varphi}M_n.$$

- Makes a sale with probability $e^{-\Xi_{k,n}c^{\phi}}$

- So the number of intermediate sales for need k is Poisson with parameter:

$$\eta^I_{k,n}(c) = \lambda_{k,n}M_n c^{-\varphi}e^{-\Xi_{k,n}c^{\phi}},$$

- Total intermediate sales is Poisson with parameter:

$$\eta_n^I(c) = \sum_{k=1}^{K} \eta_{k,n}^I(c).$$

III. The Distribution of Total Buyers

- Number of buyers for a firm selling in n at cost c is distributed Poisson with parameter:

$$\eta_n(c) = \eta_n^C(c) + \eta_n^I(c) = (L_n + M_n)\, c^{-\varphi} \sum_{k=1}^{K} \lambda_{k,n} e^{-\Xi_{k,n} c^{\phi}}.$$

- Worldwide number of buyers for a producer in i is distributed Poisson with parameter:

$$\begin{aligned} \eta_i^W(c) &= \sum_{n=1}^{\mathcal{N}} \eta_n(c d_{ni}) \\ &= \sum_{n=1}^{\mathcal{N}} (L_n + M_n)\,(d_{ni})^{-\varphi}\, c^{-\varphi} \sum_{k=1}^{K} \lambda_{k,n} e^{-\Xi_{k,n} (d_{ni})^{-\phi} c^{\phi}}. \end{aligned}$$

IV. The Measures of Producers and Sellers

- Measure of producers solves:

$$\begin{aligned} M_i &= \int_0^\infty (1 - e^{-\eta_i^W(c)}) d\mu_{ii}(c) \\ &= T_i \Xi_i \int_0^\infty (1 - e^{-\eta_i^W(c)}) \theta c^{\theta - 1} dc. \end{aligned}$$

- Requires solving for a fixed point since $\eta_i^W(c)$ itself depends on the measure of customers for intermediates M_n in each market n.

- Measure of firms selling in n, N_n

$$N_n = \int_0^\infty (1 - e^{-\eta_n(c)}) d\mu_n(c)$$
$$= \Upsilon_n \int_0^\infty (1 - e^{-\eta_n(c)}) \theta c^{\theta - 1} dc.$$

- Measure of exporters to i from n

$$N_{ni} = \pi_{ni} N_n.$$

Simulating Firm-Level Outcomes

Table 4: Firm-Level Results of Simulation

	Country Size					
	L=0.001	L=0.009	L=0.09	L=0.2	L=0.3	L=0.4
Measures of firms:						
producing	0.01	0.05	0.64	1.65	2.59	3.47
selling	0.04	0.34	2.21	3.72	4.62	5.27
Measures normalized by Labor:						
producing	5.7	5.8	7.1	8.2	8.6	8.7
selling	42.4	37.7	24.5	18.6	15.4	13.2
Fraction of firms:						
exporting						
selling domestically	0.02	0.15	0.71	0.88	0.92	0.93
Mean # customers per firm:	1.03	1.19	2.28	3.58	4.65	5.62
Size distribution (percentiles):						
25th	1	1	1	1	1	1
50th	1	1	1	2	2	2
75th	1	1	2	3	4	5
90th	1	2	4	7	10	12
95th	1	2	6	12	17	21
99th	2	4	15	30	43	55

Table 5: Firm-Level Results with Different Trade Costs

	Trade Cost (small country, L=.009)					Trade Cost (large country, L=0.3)				
	10.00	1.80	1.20	1.05	1.00	10.00	1.80	1.20	1.05	1.00
Measures of firms:										
producing	0.00	0.03	0.05	0.04	0.03	7.62	6.36	2.59	1.44	1.08
selling	0.00	0.07	0.34	0.33	0.28	7.62	7.16	4.62	2.98	2.28
Measures normalized by Labor:										
producing	0.4	3.4	5.8	4.5	3.6	25.4	21.2	8.6	4.8	3.6
selling	0.4	8.1	37.7	36.5	31.3	25.4	23.9	15.4	9.9	7.6
Fraction of firms:										
exporting										
selling domestically	1.00	0.41	0.15	0.09	0.08	1.00	1.00	0.92	0.73	0.63
Mean # customers per firm:	1.01	1.06	1.19	1.29	1.36	5.48	5.25	4.65	5.11	5.62
Size distribution (percentiles):										
25th	1	1	1	1	1	1	1	1	1	1
50th	1	1	1	1	1	2	2	2	2	2
75th	1	1	1	1	1	5	5	4	5	5
90th	1	1	2	2	2	12	12	10	11	12
95th	1	2	2	3	3	21	20	17	19	21
99th	2	2	4	5	5	53	50	43	49	55

Implications

- A continuum of potential producers and workers
- Thus simple expressions for aggregate such as total consumption, production, and trade flows
- But individual producers buy from and sell to only a finite number of customers which can vary enormously from firm to firm (like Klette and Kortum, 2004).

- Individual firms are heterogeneous in size for four reasons:

1. As in Melitz, BEJK, or EKK I, in terms of underlying efficiency.

2. The efficiency of the input suppliers they hook up with.

3. How many buyers they hook up with.

4. How big those buyers are.

- Heterogeneity in unit cost, as in the Melitz, BEJK, or EKK can't explain why an efficient producer would ever skip over a market that a less efficient producer from the same source would sell in.

- It also can't explain why two producers from the same source don't sell in the same proportion in the markets where both do sell.

- EKK I resorted to market-specific taste and entry cost shocks for an explanation.

- Larger markets attract more sellers, but not proportionately more, in line with evidence on entry

- BEJK could not do this at all while Melitz requires entry costs that vary with market size with a particular elasticity (EKK).