Sovereign Default: The Role of Expectations[*]

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Abstract

The standard model of sovereign default, as in Aguiar and Gopinath (2006) or Arellano (2008), is consistent with multiple equilibrium interest rates. Some of those equilibria resemble the ones identified by Calvo (1988) where default is likely because rates are high, and rates are high because default is likely. The model is used to simulate equilibrium movements in sovereign bond spreads that resemble sovereign

debt crisis. It is also used to discuss lending policies similar to the ones announced by the European Central Bank in 2012.

1   Introduction

This paper is on the origins of sovereign debt crises. Are sovereign debt crises caused by bad fundamentals, alone, or, instead, do expectations play an independent role? The main point of the paper is that, indeed, both fundamentals and expectations can play important roles. High interest rates can be triggered by self confirming expectations, but those high rates are more likely when debt levels are relatively high. This can help explain the large and abrupt increases in spreads during sovereign debt crises, particularly in countries that have accumulated large debt levels, as in the recent European experience. It can also justify the policy response by the European Central Bank, to be credited for the equally large and abrupt reduction in sovereign spreads.

The literature on sovereign debt crises is ambiguous on the role of expectations. In a model with rollover risk, Cole and Kehoe (2000) have established that sunspots can play a role, that is strenghtened by bad fundamentals. Using a different mechanism, Calvo (1988) also shows that there are both low and high interest rate equilibria. The reason for the multiplicity in Calvo is that, while interest rates may be high because of high default probabilities, it is also the case that high interest rates induce high default probabilities. This gives rise to equilibria with high rates/likely default and low rates/unlikely default. In contrast with the results in those models, in the standard quantitative model of sovereign default as in Aguiar and Gopinath (2006) or Arellano (2008) there is a single low interest rate equilibrium.

In this paper, we take the model of Aguiar and Gopinath (2006) and

Arellano (2008), that build on Eaton and Gersovitz (1988) and make minor changes in the modelling choices concerning the timing of moves by debtors and creditors, and the actions that they may take. In so doing, we are able to produce expectation-driven movements in interest rates. The reason for those movements is the one identified by Calvo (1988), and more recently analyzed in related, independent work by Lorenzoni and Werning (2013). The change in the modelling choices is minor since direct evidence cannot be used to discriminate across them. Even if there is no direct evidence, there is ample indirect evidence provided by large and abrupt movements in spreads, apparently unrelated to fundamentals, during sovereign debt crises. .

Our theoretical exploration of self-fulfilling equilibria in interest rate spreads is motivated by two particular episodes of sovereign debt crises. The first is the Argentine crisis of 1998-2002. Back in 1993, Argentina had regained access to international capital markets, but the average country risk spread on dollar denominated bonds for the period 1993-1999, relative to the US bond, was 7%. The debt to GDP ratio, was roughly 35%, very low by international standards, and the average yearly growth rate of GDP was around 5%. Still, the Argentine government defaulted in 2002, after 4 years of a long recession. Notice that a 7% spread on a 35% debt to GDP ratio amounts to almost 2,5% of GDP on extra interest payments per year.[1] Accumulated over the 1993-1999 period, this is 15% of GDP, or almost half the debt to GDP ratio of Argentina in 1993. An obvious question arises: Had Argentina faced lower interest rates, would it have defaulted in 2002?

The second episode is the recent European sovereign debt crisis that started in 2010 and receded substantially since the policy announcements by the European Central Bank (ECB) in September 2012. The spreads on Italian and Spanish public debt, very close to zero since the introduction of the euro and until April 2009, were higher than 5% by the summer of 2012,

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[1] This calculation unrealistically assumes one period maturity bonds only. Its purpose is just to illustrate the point in a simple way.

when the ECB announced the program of Outright Monetary Transactions (OMTs). They were considerably higher in Portugal, and specially in Ireland and Greece. With the announcement of the OMTs, according to which the central bank stands ready to purchase euro area sovereign debt in secondary markets, the spreads in most of those countries slid down to less than 2%, even though the ECB did not actually intervene. The potential self-fulfilling nature of the events leading to the high spreads of the summer 2012 was explicitly used by the president of the ECB to justify the policy.

The model in this paper is of a small open economy with a random endowment, very similar to the structure in Aguiar and Gopinath (2006) or Arellano (2008) which follow up on Eaton and Gersovitz (1981). A representative agent can borrow noncontingent and cannot commit to repay. There is a penalty for defaulting. Foreign creditors are risk neutral so that the average return from lending to this economy taking into account the probability of default has to be equal to the risk-free international rate of interest. The timing and action assumptions are the following: In the beginning of the period, given the level of debt gross of interest and the realization of the endowment, the borrower decides whether to default. If there is default, the endowment is forever low. Otherwise, creditors move first and offer their limited funds at some interest rate. The borrower moves next and borrows from the low rate creditors up to some total optimal debt level. In equilibrium the creditors all charge the same rate, which is the one associated with the probability of default for the optimal level of debt chosen by the country. With these timing assumptions, there are multiple interest rate equilibria. High interest rates can generate high default rates which in turn justify high interest rates. In equilibria such as these, there is a sense in which interest rates can be "too high".

With this timing, when deciding how much to borrow, the borrower takes the interest rate as given. This does not mean that the borrower behaves as a small agent. Even if it takes current prices as given, it still takes into

account the effects of its current choices on future prices. The borrower is just not benefiting from a first mover advantage. A similar timing assumption in Bassetto (2005), also generates multiple Laffer curve equilibria. In Bassetto, if the government were to move first and pick the tax, there would be a single low tax equilibrium. Instead, if households move first and supply labor, there is also a high tax equilibrium. Bassetto convincingly argues that the assumption that the government is a large agent is unrelated to the timing of the moves.

The timing assumptions in Aguiar and Gopinath (2006) and Arellano (2008) are such that the borrower moves first, before the creditors. They also assume the borrower chooses the debt level at maturity including interest payments.[2] Creditors move next and respond with a schedule that specifies a single interest rate for each level of debt gross of interest. By choosing the debt at maturity, gross of interest, the borrower is able to select a point in the schedule, therefore pinning down the interest rate. It follows that there is a single equilibrium. The first mover advantage allows the borrow to coordinate the creditors actions on the low interest rate equilibrium.[3]

An alternative structure has the same sequence of moves, except that the borrower chooses current debt instead of debt at maturity. This is an important restriction, that prevents the borrower from taking advantage of moving first.[4] The interest rate schedule will then be a function of current debt, rather than debt at maturity. In this case, there will in general be multiple schedules.[5] Given current debt, if the interest rate is high, so is

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[2]In Eaton and Gersovitz (1981) the country chooses the level of debt net of interest payments.

[3]If the borrower as a first mover were to pick the interest rate then it would be possible to coordinate the actions of the creditors, and there would be a single low interest rate equilibrium.

[4]With the alternative timing, that the creditors move first, there are multiple interest rate equilibria regardless of the actions of the borrower.

[5]In Eaton and Gersovitz (1981), even if that is the assumption on the actions of the country, they dismiss the multiplicity by assumption (this is discussed in Section 2.3).

debt at maturity, and therefore the probability of default is also high. This is the spirit of the analysis in Calvo (1988).

Current debt in Calvo (1988) is exogenous, but debt at maturity is not since it depends on the endogeneously determined interest rate. If the borrower were to choose debt at maturity, given current debt, the interest rate would be pinned down, and, again, there would be a single equilibrium. Lorenzoni and Werning (2013) analyze a dynamic version of Calvo's model with exogenous public deficits, and argue against the possibility of the government choosing debt at maturity. For that, they build a game with an infinite number of subperiods, and assume that the government cannot commit not to reissue debt in those subperiods. As a result, the government is unable to select a point on the interest rate schedule.

As mentioned above, the reason for expectation-driven, high interest rate, equilibria, in these models is different from the one in Cole and Kehoe (2000). Still, in that set up it is the timing of moves that is crucial to generate multiplicity. In Cole and Kehoe, there is multiplicity when the choice of how much debt to issue takes place before the decision to default. In that case, it may be individually optimal for the creditors not to roll over the debt, which amounts to charging arbitrarily high interest rates. This may induce default, confirming the high interest rates. In our model there is no rollover risk because the decision of default is at the beginning of the period. Still, a similar timing assumption to the one in Cole and Kehoe generates the multiplicity. As creditors move first, it can be individually optimal to ask for high rates. That will induce a high probability of default, confirming the high rates.

Most of the theoretical analysis in this paper is done in a two period version of the model where the intuitions are very clear. We discuss the relevance of the alternative timing and action assumptions. The model is first solved with our prefered timing according to which the borrower behaves as a price taker. The solution can be derived very simply using a demand curve

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of debt by the borrower and a supply curve of funds by the creditors. In general there are multiple intersections of the demand and the supply curve. These are all potential equilibria, but some are more compelling than others.

For standard distributions of the endowment, the high rate equilibria have properties that make them fragile to reasonable refinements. Those high rates can be in portions of the supply curve in which the rates decrease with an increase in the level of debt. If that is the case, then the total gross service of the debt also descreases with an increase in the level of debt. For those high rates, creditors also jointly benefit from lowering interest rates, because of their effect on probabilities of default. These are all features of the high rate equilibria in Calvo (1988). We, instead, consider bimodal distributions of the endowment, with good and bad times. With those distributions, there are low and high rate equilibria, equally robust, for the same level of debt. The set of equilibria has the feature that for low levels of debt there is only one equilibrium. Interest rates are low and increase slowly with the level of debt. As debt becomes relatively high, then there are both low and high rate equilibria. Eventually, for higher levels of debt, there is a single high rate equilibrium.

In the region where the interest rates are unnecessarily high, policy can be effective in selecting a low rate equilibrium. A large lender can accomplish the missing coordination, by lending up to a maximum amount at a penalty rate. In equilibrium only private creditors would be lending. This may help understand the role of policies such as the OMTs introduced by the ECB, following the announcement by its president that it would do "whatever it takes" to avoid a sovereign debt crisis in the euro area.

The paper also includes a quantitative section with a dynamic model in which a sunspot variable is introduced that triggers coordination on high or low interest rates. To stay closer to the quantitative literature, and also for simplicity in the computations, we consider the standard timing in the literature that has the borrower move first and face an interest rate schedule.

In order to have multiplicity, the schedules are in terms of debt net of interest. The model is shown to be consistent with a sovereign debt crisis unraveling, in particular when debt is relatively large. We find this exercise to be useful, but there are clear weaknesses.

The simulations of the multi-period model are not calibration exercises. It is not clear how some of the modelling choices can be disciplined by the data. There are free choices in the timing or action assumptions, in assumptions on the distribution of the endowment, and in the sunspot. We still find that the exercise can be useful in understanding sovereign debt crises and the policies that may address them.

A final comment: As mentioned, this paper is closely related to Lorenzoni and Werning (2013), even if there are some important differences. They study a model where fiscal policy is exogenous. We instead characterize equilibria with optimal debt choices. Our main focus is on the importance of timing and action assumptions for multiple interest rate equilibria to arise. By exposing the importance of those assumptions we argue for the empirical relevance of that multiplicity. Along similar lines, Lorenzoni and Werning analyze games that also provide support for multiplicity. The main difference between the two papers is that Lorenzoni and Werning consider long maturity debt, and focus the analysis on equilibria with debt dilution, while we do not. In our set up, the multiplicity is closer to the one analyzed by Calvo (1988) - it arises with only short term debt. We emphasize the role of large debt levels and the plausibility of long periods of stagnation as possible drivers of the multiplicity.

2 A two period model

It is useful to analyze first the case of a simple two period model, where analytical results can be derived and some of the features of the model can be seen clearly. In particular it is easier to understand in the two period

model what drives the multiplicity of spreads and default probabilities that resembles the result in Calvo (1988).

We analyze a two-period, endowment economy populated by a representative agent that draws utility from consumption in each period and by a continuum of risk neutral foreign creditors. Each creditor has limited capacity, but there are enough of them so that there is no constraint on the aggregate credit capacity. The period utility function of the representative agent, U, is assumed to be strictly increasing, strictly concave and to satisfy standard Inada conditions. The endowment is assumed to be equal to 1 in the first period. That is the lower bound of the support of the distribution of the endowment in the second period. Indeed, uncertainty regarding future outcomes is described by a stochastic endowment $y \in [1, Y]$, with density $f(y)$ and corresponding cdf $F(y)$. The outstanding initial level of debt is assumed to be zero, and, in period one, the representative agent can borrow b in a non contingent bond in international financial markets. The risk neutral gross international interest rate is R^*. In period two, after observing the realization of the shock, the borrower decides either to pay the debt gross of interest, Rb, or default. If there is default, consumption is equal to the lower bound of the endowment process, 1. Note that there may be contingencies under which the borrower chooses to default, the interest rate charged by foreign creditors, R, may differ from the risk free rate R^*.

The timing of moves is the following: In the first period each creditor $i \in [0, 1]$ offers the limited funds at gross interest rate R_i. The borrower moves next and picks the level of debt $b = \int_0^1 b_i di$, where b_i is how much is borrowed from each creditor. In the second period, the borrower decides whether to default fully or to pay the debt in full.

The borrower decides to default if and only if $U\left(y - \int_0^1 b_i R_i di\right) \leq U(1)$, or

$$y \leq 1 + \int_0^1 b_i R_i di.$$

In order for creditors to make zero profits in equilibrium the interest rates

they charge will have to be the same, $R_i = R$. Assuming the country borrows the same amount from each creditor, default happens whenever

$$y \leq 1 + bR,$$

which defines a default threshold for output. The probability of default is then $F[1 + bR]$.

Since creditors are risk neutral, the expected return of lending to the borrower in this economy must be the same as R^*, so

$$R^* = R[1 - F(1 + bR)]. \tag{1}$$

This defines a locus of points (b, R) such that each point solves the problem of the creditors, which can be interpreted as a supply curve of funds. The mapping from debt levels to interest rates is a correspondence, since, in general for each b there are multiple Rs that satisfy equation (1). Multiple functions can be built with the points of the correspondence. We call those functions interest rate schedules.

The optimal choice of debt by the borrower is the one that maximizes utility

$$U(1 + b) + \beta \left[F(1 + bR)U(1) + \int_{1+bR}^{Y} U(y - bR)f(y)dy \right]. \tag{2}$$

subject also to an upperbound restriction on the maximum level of debt. Absent this condition, the optimal choice would be to borrow an arbitrarily large amount and default with probability one. The supply of debt would be zero in equilibrium.

The marginal condition, for an interior solution, is

$$U'(1 + b) = R\beta \int_{1+bR}^{Y} U'(y - bR)f(y)\, dy. \tag{3}$$

The optimal choice of debt for a given interest rate defines a locus of points (b, R) that can be interpreted as a demand curve for funds. The possible equilibria will be the points where the demand curve intersects the supply curve above described by (1).

An equilibrium in this economy can then be defined as:

Definition 1 An equilibrium is an interest rate $\widetilde{R}$ and a debt level $\widetilde{b}$ such that: Given $\widetilde{R}$, $\widetilde{b}$ maximizes (2); and (ii) the arbitrage condition (1) is satisfied.

2.1 Multiple equilibria

As mentioned above, there are in general multiple equilibria in this model, low rate equilibria, and high rate equilibria that resemble the multiple equilibria in Calvo (1988).

The supply curve defined implicitly by (1) is analyzed now. For that purpose, it is useful to define the function for the expected return on the debt,

$$h\left(R; b\right) = R\left[1 - F\left(1 + bR\right)\right],$$

that must be equal to the riskless rate, R^*. For $R = 0$, $h\left(0; b\right) = 0$. If the distribution of the endowment has a bounded support, for R high enough, if $1 + bR \geq Y$, then $h\left(R; b\right) = 0$. For standard distributions, the function $h\left(R; b\right)$ is concave, so that there are at most two solutions of $R^* = h\left(R; b\right)$.

In the case of the uniform distribution it is straightforward to obtain the solutions of $R^* = h\left(R; b\right)$, so that the supply curve can be described analytically. Let the distribution of the endowment process be the uniform, $f\left(y\right) = \frac{1}{Y-1}$, so that $F\left(y\right) = \frac{y-1}{Y-1}$. Then, from (1), the equilibrium interest rates must satisfy

$$R = \frac{1 \pm \left(1 - 4\frac{R^*b}{Y-1}\right)^{\frac{1}{2}}}{2\frac{b}{Y-1}},$$
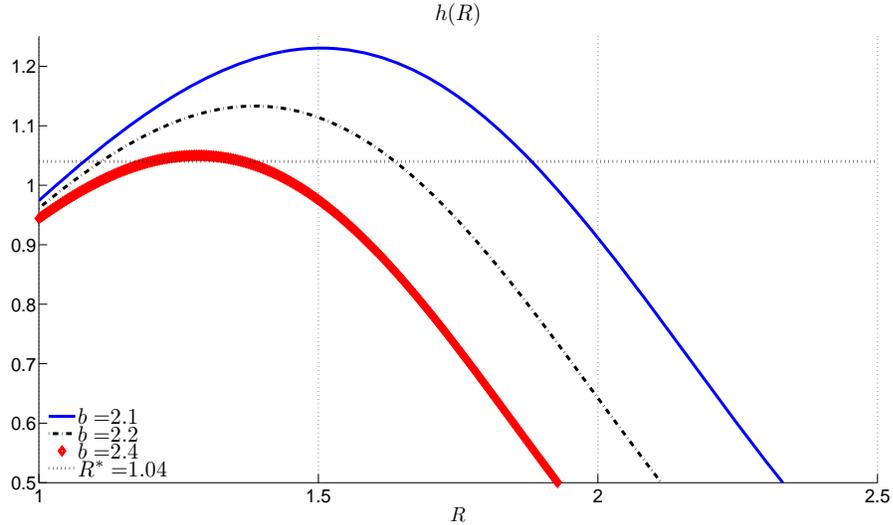
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Figure 1:   Expected return $h(R; b)$

provided $1 - 4\frac{R^*b}{Y-1} \geq 0$. The maximum level of debt consistent with an equilibrium with borrowing is given by $b^{\max} = \frac{Y-1}{4R^*}$. Below this value of debt, for each b, there are two possible levels of the interest rate.

In Figure 1, the curve $h(R; b)$ is depicted against R, where F is the cumulative normal. An increase in b shifts the curve h, downwards, so that the solutions for b are closer to each other. The second derivative of $h(R; b)$ is negative when $2f(1 + bR) \geq -f'(1 + bR)bR$. The function $h(R; b)$ does not have to be everywhere concave. This depends on the cumulative distribution $F(1 + bR)$.[6] We discuss below conditions for the non concavity of the function $h(R; b)$.

Figure 2 plots the solutions for R of equation (1) for each level of debt, also for the normal distribution.

The supply curve of Figure 2 has two monotonic schedules. For lower values of the interest rate, there is a flat schedule that is increasing in b.

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[6]In the appendix we further characterize conditions for concavity and study several commonly used distributions.
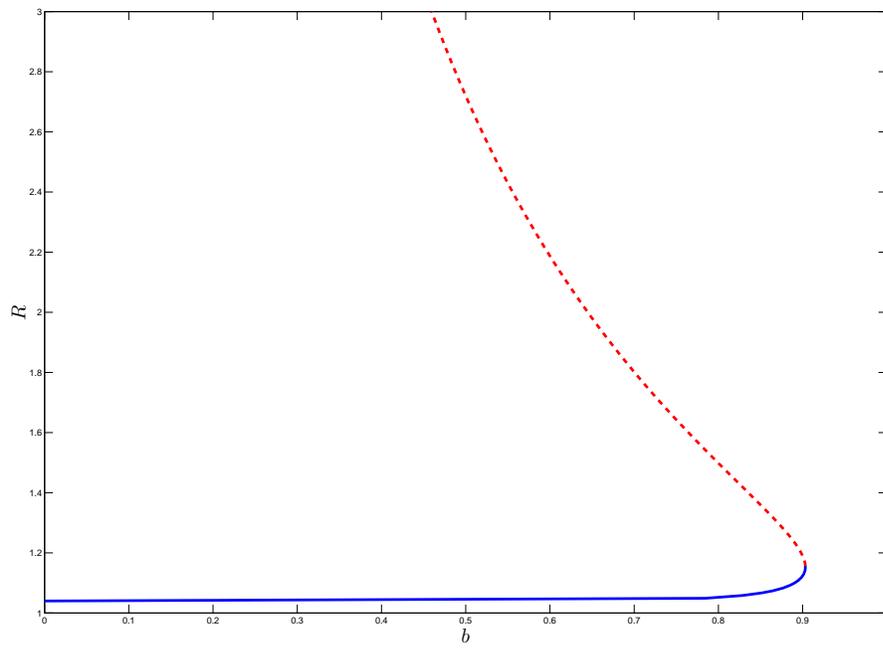
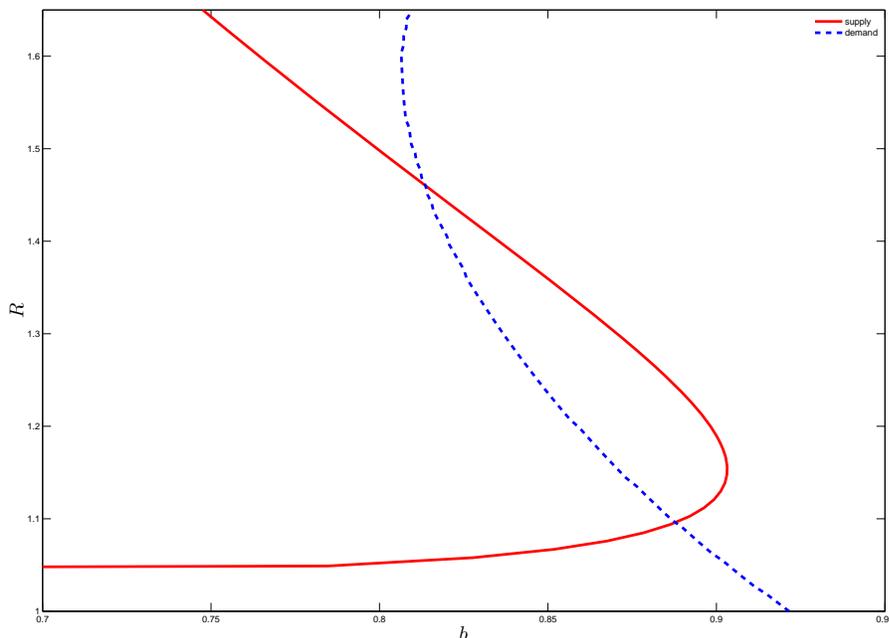Figure 2: Interest rate schedules

Figure 3: Supply and demand curves

There is also a steeper decreasing schedule for higher values of the interest rate.

The equilibrium must also be on a demand curve for the borrower, obtained from the solution of the problem defined in (2). Figure 3 below depicts the two curves, the supply and the demand curve.

The points on the decreasing schedule have particularly striking properties. For those points in the supply curve, not only does the interest rate go down with the level of debt, b, but the gross service of the debt, Rb, also decreases with the level of debt, b. To see this, notice that from (1), R increases in the level of Rb. The points on the decreasing schedule are fragile as candidates for equilibria in the following sense. Consider a perturbation of a point $(\widehat{R}, \widehat{b})$ in that schedule, that consists of the same interest rate, but

a slightly lower value for the debt $(\widehat{R}, \widehat{b} - \varepsilon)$. This point would lie below the schedule. At the point $(\widehat{R}, \widehat{b} - \varepsilon)$, the interest rate is the same as in $(\widehat{R}, \widehat{b})$, but the debt lower, so the probability of default is also lower. Thus, profits for the creditors are higher than at $(\widehat{R}, \widehat{b})$, where profits are zero. With positive profits, there would be an incentive to cut down prices and capture a larger share of the market. The incentives to further decrease rates remain while profits are positive, so one could imagine that the process would continue till the interest rate is the one in the increasing schedule, where profits are zero. Any further cuts in interest rates would imply negative profits.[7]

One could then hope that a reasonable refinement would rule out the possibility of a high rate equilibrium on the decreasing schedule; the equilibrium would therefore be unique. As we now show, such hopes are not realized.

2.1.1 A distribution with good and bad times

Equation (1) may have more than two solutions for R, for a given b, depending on the distribution of the endowment process.[8] One case in which there can be multiple increasing schedules is when the distribution combines two normal distributions, a distribution for good times and a distribution for bad times.

Consider two independent random variables, y^1 and y^2, both normal with different mean, μ^1 and μ^2, respectively, and the same standard deviation, σ. Now, let the endowment in the second period, y, be equal to y^1 with probability p, and equal to y^2 with probability $1 - p$.

If the two means, μ^1 and μ^2, are sufficiently apart, then (1) has four solutions, for some values of the debt, as Figure 4 shows. The correspondence between levels of debt and R, as solutions to the arbitrage equation above,

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[7]Additional, more formal arguments are provided in Lorenzoni and Werning (2013).

[8]In appendix 5.1, sufficient conditions are provided on the density so there are only two solutions.Conditions under which more than two solutions are likely to arise are also described.
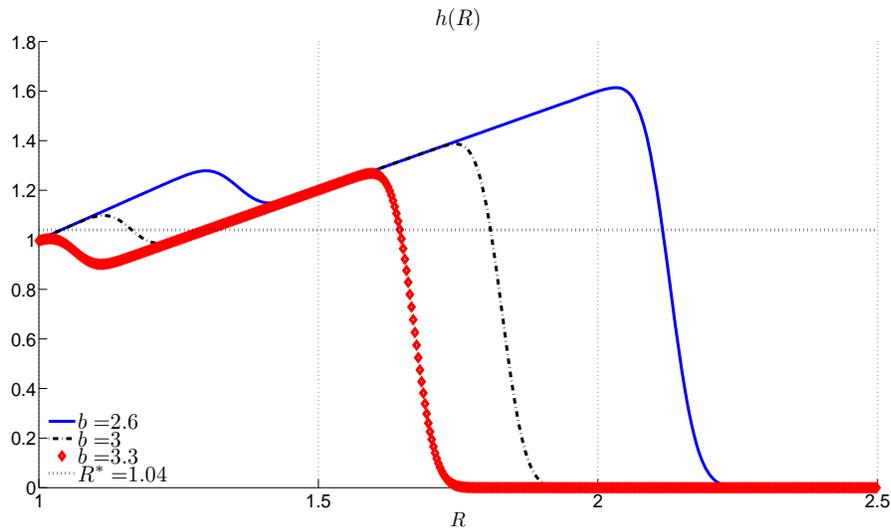
Figure 4:   Expected return for the bimodal distribution

is plotted in Figure 4, in which $p = 0.8$, $\mu^1 = 6$, $\mu^2 = 4$, $\sigma = 0.1$.[9] Clearly, there are debt levels for which there are only two solutions, so there is only one increasing schedule. But for intermediate levels of debt, the equation has four solutions and therefore multiple increasing schedules.

The supply curve for this case of the bimodal distribution is depicted in Figure 5, below.

This means that, even if one is restricted not to consider equilibria on decreasing schedules, the model may still exhibit multiplicity. The demand curve for this model is depicted in Figure 6 below. Notice that the multiplicity on the increasing schedules arises for relatively high levels of debt. That is a necessary condition from the supply curve that only exhibits multiplicity for relatively high values of debt, but the demand also has to be relatively large.

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[9]The relatively high probability and the average severity of a disaster can be thought of as the relatively frequent, long periods of stagnation.
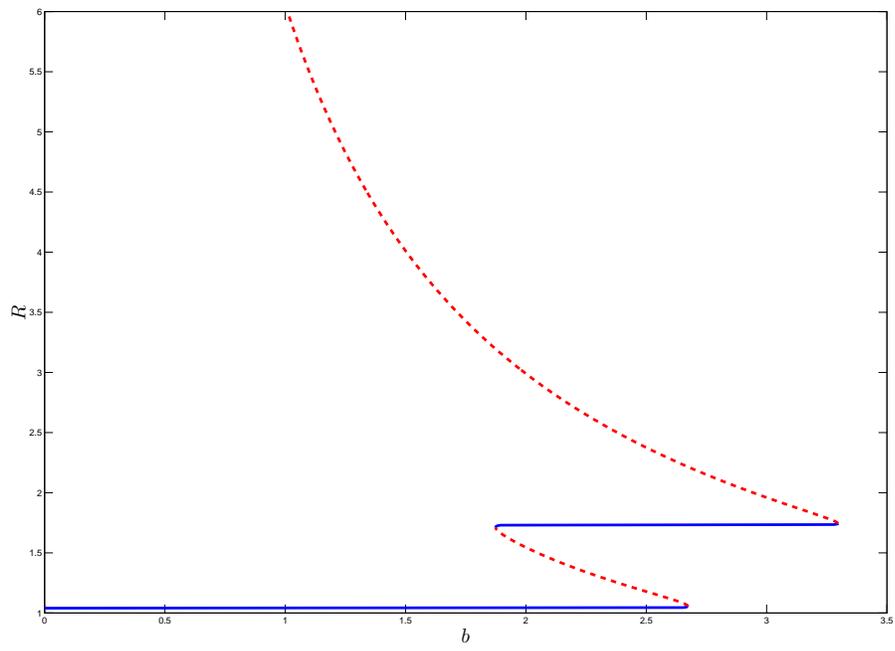
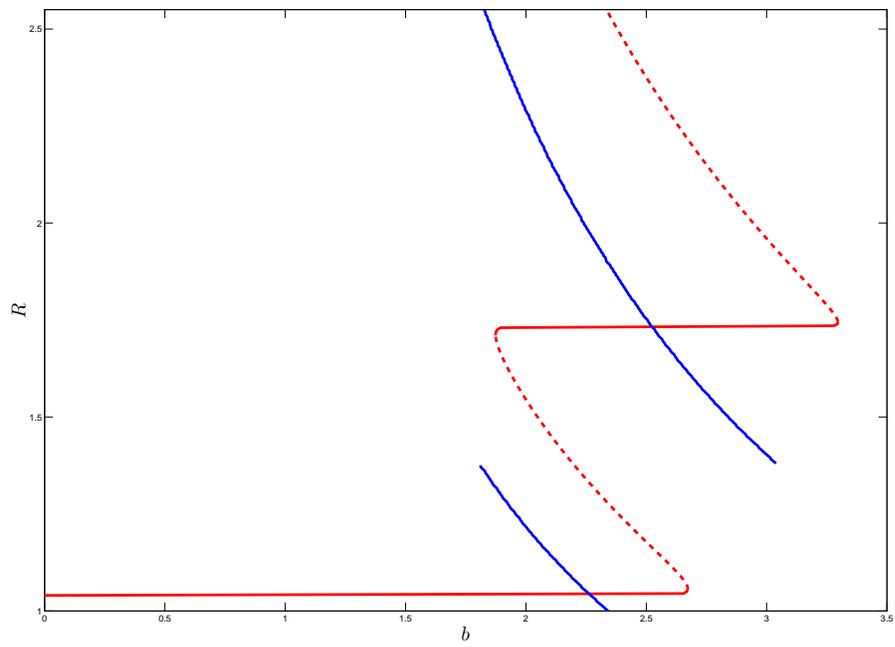Figure 5: Interest rate schedules for the bimodal distribution

Figure 6: Supply and demand for the bimodal distribution

Perturbing the distribution   If the debt level is relatively large, multiple equilibria are more likely to arise. This is the case with the bimodal distribution analyzed above, but it is particularly so, if the distribution is perturbed in the following way. Consider a perturbation $g(y)$ of the uniform distribution, so that the density would be $f(y) = \frac{1}{Y-1} + \gamma g(y)$, with $\int_1^Y g(y)dy = 0$. In particular the function g can be $g(y) = \sin ky$, with $k = \frac{2\pi}{Y-1}N$, where N is a natural number.[10]

If $N = 0$, the distribution is uniform, so there is a single increasing schedule. If $N = 1$, there is a single full cycle added to the uniform distribution. The amplitude of the cycle (relative to the uniform distribution) is controlled by the parameter γ. The number of full cycles of the $\sin ky$ function added to the uniform, is given by N. As $\gamma \to 0$, so does the perturbation.

Given a value for γ, the closer the debt to its maximum value, the larger the degree of multiplicity. The equation $\frac{1}{R} - \frac{1}{R^*}\left[1 - \frac{1+bR}{Y-1} - \gamma \sin kbR\right] = 0$ has more than two solutions for R, for γ that can be made arbitrarily small, as long as b is close enough to b^{max}. On the other hand, if b is lower than b^{max}, there is always a $\gamma > 0$, but small enough, such that there are only two zeros to the function above. An illustration is presented in Figure 7, for two levels of the debt and for two values of γ.

As can be seen, when the debt is low, a positive value of γ is not enough to generate multiplicity, but multiplicity arises as the level of the debt goes up.

Note that if γ is small, it may take a very long series to identify it in the data. Thus, it is hard to rule out this multiplicity based on calibrated versions of the distribution of output if the debt is close enough to its maximum.[11]

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[10]The uniform distribution is used only as an example.

[11]This resembles the result in Cole and Kehoe (2000), where the fraction of short term debt affects the chances of multiplicity.
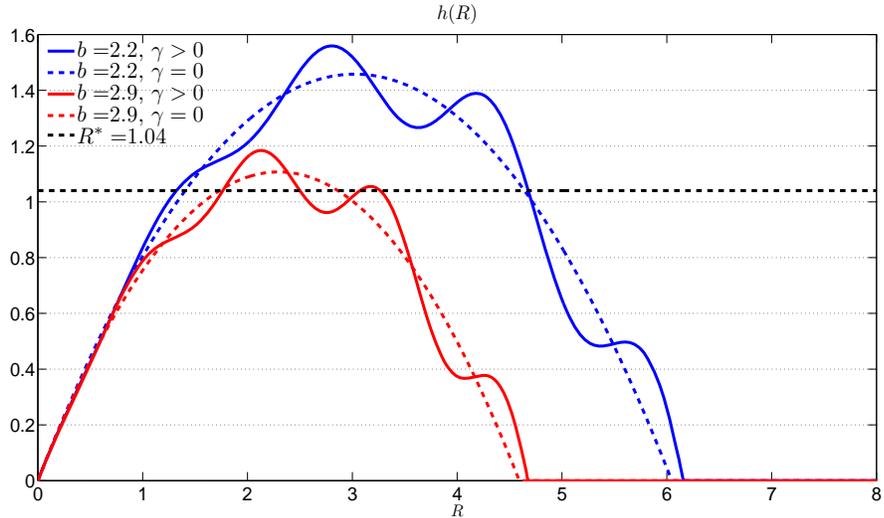
$h(R)$

Figure 7:   Perturbing the uniform distribution

2.2   Policy

To illustrate the effects of policy, the case of the bimodal distribution depicted in Figure 6 is considered. The extensions to other cases are straightforward.

Consider there is a new agent, a foreign creditor that can act as a large lender, with deep pockets.[12] This large lender can offer to lend to the country, at a policy rate R^P, any amount lower than or equal to a maximum level b^P. It follows that there cannot be an equilibrium with an interest rate larger than R^P.

Now, let us imagine that b^P and R^P are the debt level and interest rate corresponding to the maximum point of the low (solid line) increasing schedule in Figure 6. In this case, the only equilibrium is the point corresponding to the intersection of demand and supply on the low interest rate, increasing schedule. In addition, the amount borrowed from the large lender is zero. The reason the equilibrium interest rate is lower than the one offered by the

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[12]If the borrower was a small agent, rather than a sovereign, any creditor could possibly play this role.

large lender is that at that interest rate R^P and for debt levels strictly below b^P, there would be profits.

Notice that the large lender cannot offer to lend any quantity at the penalty rate. Whatever is the rate, the level of lending offered has to be limited by the points on the supply curve, otherwise, the borrower may borrow a very high amount and then default.

2.3   Current debt versus debt at maturity

The borrower in the model analyzed above chooses current debt. Would it make a difference if the borrower were to choose debt at maturity, gross of interest? We now consider an alternative game in which the timing of the moves is as before, but now the borrower chooses the value of debt at maturity that we denote by a, rather than the amount borrowed, b. Are there still multiple equilibria in this set up? The answer is yes. With this timing of moves, there are multiple interest rate equilibria whether the government chooses the amount borrowed b, or the amount paid back a. This is a relevant question, because in the models of Calvo (1988) and Arellano (2008) it is that assumption, of whether the borrower chooses b or a, that is key to have uniqueness or multiplicity of equilibria, as will be discussed later.[13]

Again, here, the creditors move first and offer the limited funds at gross interest rate R_i, $i \in [0, 1]$. The borrower moves next and picks the level of debt at maturity $a = \int_0^1 a_i di$. As before, the rate charged by each creditor will have to be the same in equilibrium. In the second period, the borrower defaults if and only if $y \leq 1 + a$. Arbitrage in international capital markets implies that

$$R^* = R\left[1 - F\left(1 + a\right)\right]. \tag{4}$$

The locus of points (a, R) defined by (4), that we interpret as a supply curve of funds, is monotonically increasing (which is not the case for the

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[13]The key for the different results is the timing assumption, as clarified in section 3.

supply curve in b and R defined in (1)).

The utility of the borrower is

$$U(1 + \frac{a}{R}) + \beta \left[F(1 + a)U(1) + \int_{1+a}^{Y} U(y - a)f(y)dy \right]. \qquad (5)$$

where $\frac{1}{R}$ is the price of one unit of a as of the first period. The marginal condition is

$$U'(1 + \frac{a}{R}) = R\beta \int_{1+a}^{Y} U'(y - a)f(y)\,dy. \qquad (6)$$

The locus of points (a, R) defined by the solution of this maximization problem can be interpreted as a demand curve for funds. There are again multiple intersection points of this demand curve with the supply curve. Provided the choice of a is interior, those points are the solutions of the system of two equations, (4) and (6), but those are the exact same two equations (1) and (3) that determine the equilibrium outcomes for R and b for $a = Rb$.

Figure 8 plots the supply curves for (b, R) and (a, R) defined in (1) and (4), respectively, for the normal distribution. It also plots the demand curves defined in (6) and (3), for the logarithmic utility function. With the timing assumed so far, whether the borrower chooses debt net or gross of interest is inessencial.

3 Timing of moves and multiplicity: Related literature

The timing of moves assumed above, with the creditors moving first, amounts to assuming that the borrower in this two period game takes the current price of debt as given.[14] The more common assumption in the literature is that

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[14]In the dynamic game the contemporaneous price is taken as given, but not so the future prices.
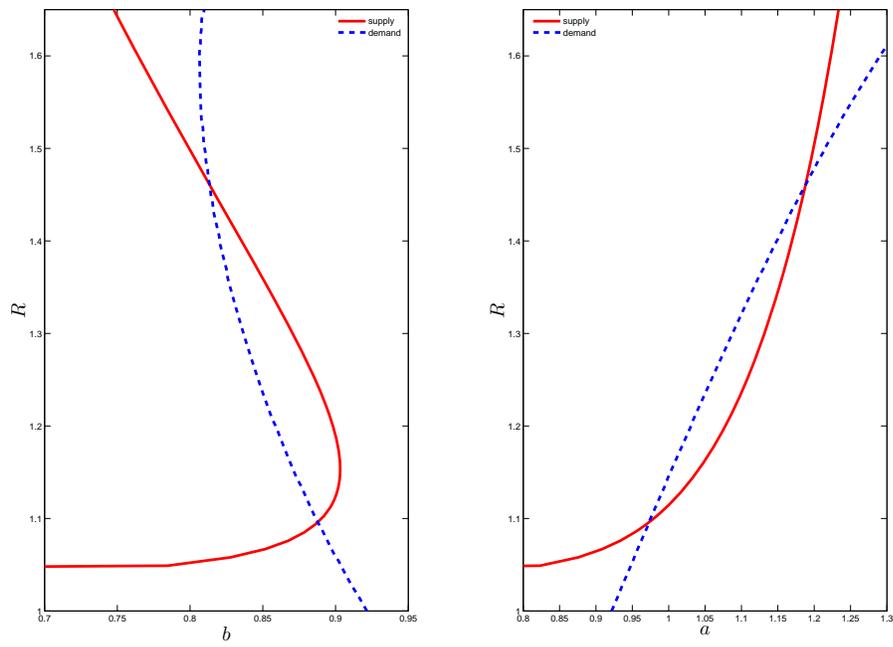
Figure 8: b and a schedules

the borrower moves first, choosing debt levels b or a, facing a schedule of interest rates as a function of those levels of debt, $R = R(b)$ or $R = \frac{1}{q(a)}$, depending on whether the choice is on b or a, respectively.

Suppose the schedule the borrower faces is $q(a)$ corresponding to the supply curve derived from (4) and depicted in the right hand panel of Figure 8. This is a monotonically increasing function. Since the borrower can choose a it is always going to choose in the low R/low a part of the schedule. The borrower is also going to take into account the monopoly power in choosing the level of a. These are the assumptions in Aguiar and Gopinath (2006) and Arellano (2008). The equilibrium is unique.

Suppose now that the borrower faces the full supply curve as depicted in Figure 2 with an increasing low rate schedule and a decreasing high rate schedule. Then by picking b the borrower is not able to select the equilibrium outcome.[15] There are multiple possible interest rates that make creditors equally happy. The way this can be formalized, as in Calvo (1988),[16] is with multiple interest rate functions $R(b)$, which can be the low rate increasing schedule or the high rate decreasing one. Any other combination of those two schedules is also possible. The borrower is offered one schedule of the interest rate as a function of the debt level b and chooses debt optimally given the schedule.

In summary, the assumption on the timing of moves is a key assumption to have multiple equilibria or a single equilibrium. If the creditors move first, there are multiple equilibrium interest rates and debt levels, and they are the same equilibria whether the borrower chooses current debt or debt at maturity. Instead, if the borrower moves first, and chooses debt at maturity, as in Aguiar and Gopinath (2006) and Arellano (2008), there is a single

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[15]Trivially, it is still possible to obtain uniqueness in the case where the borrower faces the supply curve in R and b defined by (1). If the borrower picks R, then it is able to select directly the low rate equilibrium. That is essentially what happens when the borrower faces the schedule $R(a)$ and picks a.

[16]In Calvo (1988) debt is exogenous.

equilibrium. Choosing debt at maturity amounts to picking the probability of default, and therefore also the interest rate. Finally if the borrower moves first and chooses the current level of debt, given an interest rate schedule defined as a one-to-one mapping from b to R, then the equilibrium will depend on the schedule and there is a continuum of equilibrium schedules. This is the approach in Calvo (1988). It is also the approach that we will follow in the dynamic computations in the next section.

Lorenzoni and Werning (2013) Lorenzoni and Werning (2013) use a dynamic, simplified version of Calvo (1988)'s model, in which the borrower is a government with exogenous deficits or surpluses. In a two-period version, there is an exogenous deficit in the first period $-s^h$, with $s^h > 0$. In the second period, the surplus is stochastic, $s \in \left[-s^h, S\right]$, with density $f(s)$ and corresponding cdf $F(s)$. In order to finance the deficit in the first period the government needs to borrow $b = s^h$. In the second period, it is possible to pay back the debt if $s \geq bR$, where R is the gross interest rate charged by foreign lenders.

The creditors are competitive; they must make zero profits. It follows that $R^* = R\left(1 - F\left(bR\right)\right)$. If we were to have written $q = \frac{1}{R}$, and $a = bR$, the condition would be $R^* = \frac{1}{q}\left(1 - F\left(a\right)\right)$. As before, it is possible to use these equations to obtain functions $R\left(b\right)$, using the first equation, and $q\left(a\right)$ using the second equation. These would be the two classes of schedules that were identified in the analysis above, when the government moves first. For the normal distribution, the schedules $R\left(b\right)$ and $q\left(a\right)$ will look like the supply curves in Figure 8. There are multiple equilibrium schedules $R\left(b\right)$. There's the good, increasing schedule and the bad, decreasing schedule, and there is a continuum of other schedules with points from any of those two schedules. The government that borrows $b = s^h$, may have to pay high or low $a = R\left(b\right)b$, depending on which schedule is being used, with the corresponding probabilities of default.

What if the schedule, instead, is $q(a)$? The schedule is unique, but there are multiple points in the schedule that finance b. The government that borrows $q(a) a = s^h$, can do it with low a and low $\frac{1}{q}$, or with high a and high $\frac{1}{q}$. If the government is able to pick a, then implicitly it is picking the interest rate. Lorenzoni and Werning (2013) use an interesting argument for the inability of the government to pick the debt level a. For that they write down a game in which they divide the period into an infinite number of subperiods, and do not allow for commitment in reissuing debt within the period. In that model the government takes the price as given. The intuition is similar to the durable good monopoly result. In our model, the large agent also takes the price as given due to the timing assumption.

Even if there are multiple equilibria, with high and low interest rates, the high interest rate equilibria that Lorenzoni and Werning focus on are of a different type. They assume that debt is long term and characterize high rate equilibria with debt dilution. Because, we assume debt is only short term, those equilibria are not in this model.

Eaton-Gersovitz (1981) In the model in Eaton and Gersovitz (1981) the borrower moves first, so it is key whether the equilibrium schedule is in b or a. In our notation they consider a schedule for $R(b)$. To be more precise, they assume that $a = \overline{R}(b)$, where $\overline{R}(b) = R(b) b$. Their equation (8) can be written using our notation as $\left[1 - \lambda\left(\overline{R}(b)\right)\right] \overline{R}(b) = R^* b$, where λ is the probability of default that depends on the level of debt at maturity. This is equivalent to

$$[1 - \lambda(R(b) b)] R(b) = R^*,$$

which is analogous to equation (1) in our model. As seen above there are multiple schedules in this case.

For the case of the uniform or normal distributions, there is both an increasing and a decreasing schedule $R(b)$. In that case, $\overline{R}(b) = R(b) b$ first goes up with b, and then goes down. Eaton and Gersovitz dismiss the

decreasing schedule by assuming that $R(b)b$ cannot go down when b goes up. This amounts to excluding decreasing schedules by assumption.[17]

4   The infinite period model: Numerical exploration

In this section, we lay out a dynamic version of the model and solve it numerically to illustrate its ability to reproduce some features of the data. We consider the more standard timing in which the borrower moves first. In order for there to be a role for sunspots, the borrower chooses the current debt, rather than debt at maturity. Having the borrower choose a point on the interest rate schedule makes the computations considerably simpler, but a more important reason to use this alternative timing is that it keeps the analysis closer to the literature that has computed equilibria with sovereign debt crises in models without a role for sunspots, as in Aguiar and Gopinath (2006) and Arellano (2008).

As before, time is discrete and indexed by $t = 0, 1, 2, \ldots$. The endowment y has bounded support, given by $[y_{\min}, y^{\max}] \subset \mathbb{R}_+$, and follows a Markov process with conditional distribution $F(y'|y)$.[18] At the beginning of every period, after observing the endowment realization y, the borrower can decide whether to repay the debt or to default. Upon default, the borrower is permanently excluded from financial markets and the value of the endowment becomes $y^d \in \mathbb{R}_+$ forever.

The period utility function, $U(c)$, is assumed to be strictly increasing, strictly concave and to satisfy standard Inada conditions. Let

$$V^{aut} = \frac{U(y^d)}{1 - \beta}. \tag{7}$$

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[17]See proof of Theorem 3 in Eaton and Gersovitz (1981).

[18]Assuming a bounded support simplifies the multiplicity discussion below, but it is not essential.

be the utility after default.[19]

We allow for a sunspot variable s that takes values in $S = \{1, 2, ..., N\}$ and has a Markovian distribution, with $p_{ii} = p$ and $p_{ij} = \frac{1-p}{N-1}$, with $i, j \in S$.

The borrower chooses the current debt b,[20] given an interest rate schedule that may have high rates or low rates, depending on the realization of the sunspot variable s.

Default rules   We restrict attention to equilibria with default rules defined by a threshold. Thus, default is assumed to follow a threshold $\underline{y}(\omega, y, s)$, such that the optimal rule is to pay the debt as long as $y' \geq \underline{y}(\omega, y, s)$ and default otherwise.[21]

The case with two schedules   We analyze the case with two possible schedules. The distribution is the bimodal distribution studied above. The sunspot variable can take two possible realizations, $s = 1, 2$, with transition probabilities, $p_{11} = p_{22} = p$ and $p_{12} = p_{21} = 1 - p$.

The value for the borrower, after deciding not to default, is given by value functions for $s = 1$ and $s = 2$, $V(\omega, y, 1)$ and $V(\omega, y, 2)$, and schedules also

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[19]Note that the value of autarky is independent of the state previous to default. This is because we assume that following default, the endowment is y^d, which simplifies the analysis.

[20]If, instead, the country were to choose debt at maturity, there would be a single positively sloped schedule including low interest points and high interest points, for the same b'. The agent would choose low interest given the level of current debt, b'. This is the setup in Arellano (2008) in which the country is able to prevent interest rates from being unnecessarily high.

[21]This is the case with positive autocorrelation of the endowment process.

for $s = 1$ and $s = 2$, $R(b', y, 1)$ and $R(b', y, 2)$, satisfying

$$V(\omega, y, 1) = \max_{c, b', \omega'} \left\{ U(c) + \beta \mathbb{E}_{y'} \left[\begin{array}{c} p \max \{ V(\omega', y', 1), V^{aut} \} \\ +(1-p) \max \{ V(\omega', y', 2), V^{aut} \} | y \end{array} \right] \right\} \quad (8)$$

subject to

$$c \leq \omega + b'$$
$$\omega' = y' - b' R(b', y, 1)$$
$$b' \leq \underline{b}$$

and

$$V(\omega, y, 2) = \max_{c, b', \omega'} \left\{ U(c) + \beta \mathbb{E}_{y'} \left[\begin{array}{c} p \max \{ V(\omega', y', 2), V^{aut} \} \\ +(1-p) \max \{ V(\omega', y', 1), V^{aut} \} | y \end{array} \right] \right\} \quad (9)$$

subject to

$$c \leq \omega + b'$$
$$\omega' = y' - b' R(b', y, 2)$$
$$b' \leq \underline{b}$$

Wealth ω is used as a state variable (instead of current debt) because it reduces the dimensionality of the state space.[22] The borrowing limit is important. Since the borrower always receives a unit of consumption for every unit of debt issued, it could always postpone default by issuing more debt. This is ruled out by imposing the constraint on the maximum amount of debt.

The interest rate schedule $R(b', y, s)$ is a function of the amount of debt because default probabilities depend on this, and the interest rate reflects the likelihood of this event. It is also a function of current output because, since the endowment follows a Markov process, it contains information about future default probabilities.

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[22] If we were to keep current debt b as a state, we would also need to know the previous period interest rate that is a function of the debt level in the previous period.

Default follows a threshold $\underline{y}(b', y, s, s')$, such that the optimal rule is to pay the debt as long as $y' \geq \underline{y}(b', y, s, s')$ and default otherwise.[23] If in state $s = 1, 2$, the threshold for default is the level of y' such that

$$V^{aut} = V(\omega', y', s') = V(y' - b'R(b', y, s), y', s').\qquad(10)$$

Creditors offer their amount of funds, as long as the expected return is R^*. The arbitrage condition for the risk free creditors, in state 1, is

$$R^* = R(b', y, 1)\left[p\left(1 - F\left(\underline{y}(b', y, 1, 1)\right)\right) + (1 - p)\left(1 - F\left(\underline{y}(b', y, 1, 2)\right)\right)\right]\qquad(11)$$

and in state 2,

$$R^* = R(b', y, 2)\left[p\left(1 - F\left(\underline{y}(b', y, 2, 2)\right)\right) + (1 - p)\left(1 - F\left(\underline{y}(b', y, 2, 1)\right)\right)\right]\qquad(12)$$

Equilibrium An equilibrium is given by functions

$$V(\omega, y, s), c(\omega, y, s), b'(\omega, y, s), R(b'(\omega, y, s), y, s), \underline{y}(b', y, s, s')$$

such that,

1. given $V(\omega', y', s')$, $\underline{y}(b', y, s, s')$ solves (10).

2. given $R(b'(\omega, y, s), y, s)$, $V(\omega, y, s)$, $c(\omega, y, s)$, $b'(\omega, y, s)$ solve (8) and (9).

3. Conditions (11), (12) are satisfied.

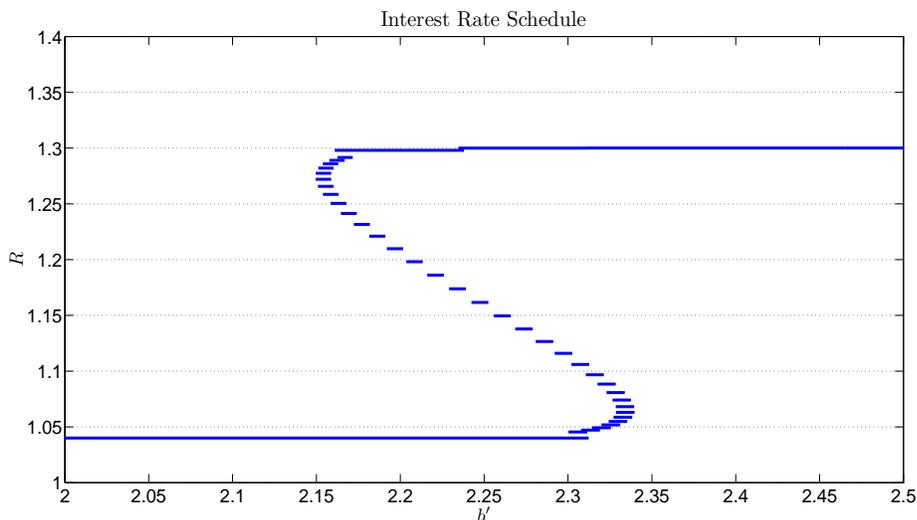[23] This is the case with positive autocorrelation of the endowment process.

Figure 9:

4.1 Simulating a sovereign debt crisis (Incomplete and Preliminary)

In this section, we report simulations of the dynamic model, by exploring the multiplicity of the type generated by the bimodal distribution discussed in Section 2.1.1. The parameter values are chosen to generate multiplicity, not by calibrating the model to data. Parameters are as follows: The discount factor is $\beta = 0.96$, preferences have constant relative risk aversion with parameter $\gamma = 2$ and the international risk free rate is $R^* = 1.04$.

The two normal distributions have mean 6 and 4, and a common standard deviation of 0.1. The probability of drawing from the bad (mean 4) distribution is $\pi = 0.2$. Finally, the probability of facing the bad (high interest rate) sunspot is 0.1.

Figure 9 shows the possible equilibrium schedules when defined as a function of b', so that Calvo-type equilibria can be expected. The Figure shows that for these parameter values there are two increasing schedules for values
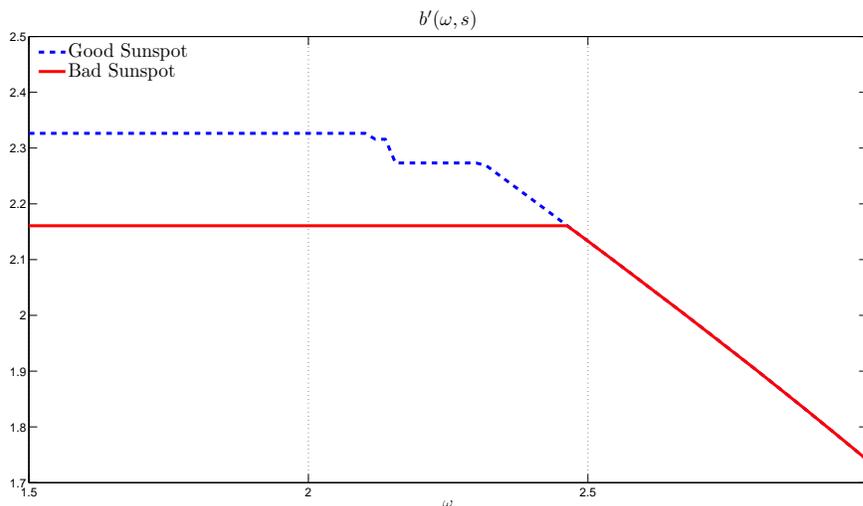
Figure 10:

of the debt between 2.10 and 2.35. A special feature of the increasing schedule is the apparently flat sections.[24] This is the result of having two normal distributions with relatively large differences in means, and very small standard deviations. Note that the "good" distribution has most of the mass between 5.8 and 6.2, so that, if the threshold is below 5.8, increases in the threshold have a negligible effect on the probability of default, so they barely affect the interest rates.

The same pattern is evident in Figure 10, where the interest rate schedule is plotted for the good realization of the sunspot (so the best rate is always selected) and the bad realization (so the worst rate is always selected). As can be seen, the rates are barely sensitive to the level of the debt up to a point, in which they dramatically increase, to become almost flat again. Notice that the multiplicity obtained is still quantitatively off the charts. The country risk spreads mentioned in the introduction (Argentina 2001 and Spain and Italy in 2012) are much smaller than the second equilibrium
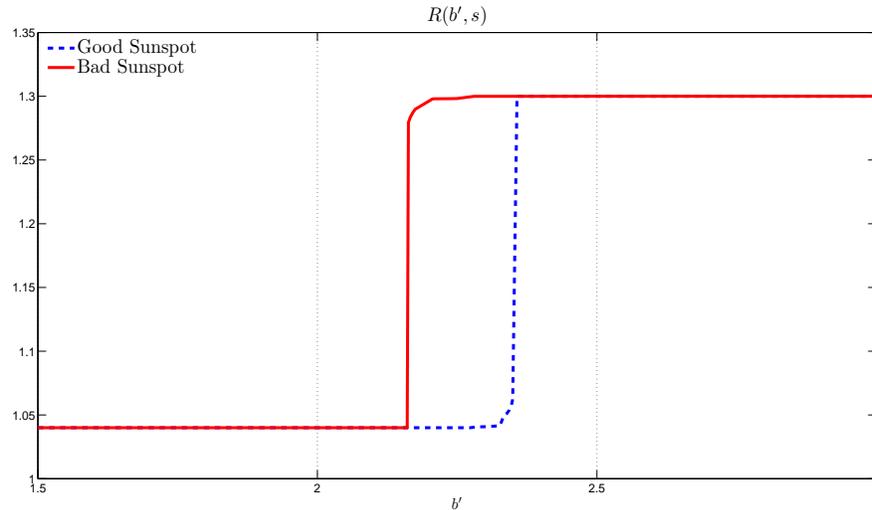
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[24]The schedules are not excatly flat..

Figure 11:

obtained, with a rate of 30%. This could be corrected by experimenting with the distribution of the endowment. In analyzing Figure 12 below, we will mention an alternative that looks more promising to us.

In Figure 11, the policy functions for the behavior of the debt are plotted. When the level of initial assets is relatively large, the relationship between initial assets and debt is negative, and it is independent of the value of the sunspot. The reason is that if initial wealth is large, the probability of default is zero, so the realization of the sunspot does not matter. But once the optimal debt level gets around 2.15, the interest rate is very sensitive to the debt level, if the sunspot is in the bad state. Thus, in that state, the country stops borrowing, to avoid those high interest rates. Thus, the red line is flat for values lower than 2.15. On the other hand, if the sunspot is good, the optimal debt keeps going up, to the point in which the interest rate starts being very sensitive to the debt level, as Figure 11 makes clear. A lesson from this plot is that the country will optimally keep borrowing but only in the good state of the sunspot.
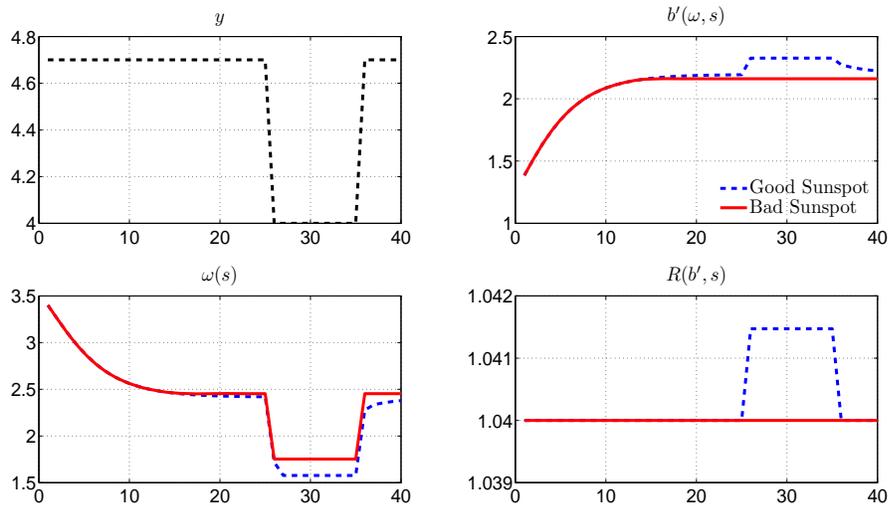
33

Figure 12:

This feature is also exhibited in the simulated path of the economy that Figure 12 displays.

In the upper left panel of Figure 12, a particular realization of the endowment is depicted. It was chosen to be constant for several periods, to experience a sudden and temporary drop that lasts for ten periods before it recovers the original value. The initial value of the wealth, ω, is chosen to be close to 3.5. Two cases are plotted, one in which the value of the sunspot is always good, and the other where the value of the sunspot is always bad.

The value of the endowment is relatively low, so the country chooses to increase its debt (right upper panel) and therefore ω goes down. Initially, the wealth is so high, that there is a single schedule, so there is no difference between the good and the bad values of the sunspot. Eventually, the debt tends to converge to a certain value, that is higher in the case the sunspot is good, for the reasons discussed on Figure 11. Interestingly, the behavior during the recession depends critically on the realization of the sunspot. If the sunspot realization is bad, the borrower adjusts consumption in the

same amount of the recession, without changing the debt, so the probability of default does not change. In this case, there is no effect of the recession on the country risk. On the other hand, if the realization of the sunspot is good, the borrower does increase marginally the debt, which therefore increases the interest rate by a very small amount. Once the recession is over, the borrower slowly brings back the debt level to the value it had before the recession.

Two features of the simulation are promising. First, the recession creates small increases in interest rates at the same time that countries are increasing the values for the debt. Second, interest rates may go up even if the realization of the sunspot is good. Thus, the possibility of a bad realization of the sunspot at debt levels where there are multiple increasing schedules implies changes in the interest rates or modest magnitudes, that depend on the probability of the sunspot. This avenue seems promising in trying to understand sovereign debt crisis.

5   Concluding remarks

In models with sovereign debt default, interest rates are high because default probabilities are high. The object of this paper is to investigate conditions under which the reverse is also true, that default probabilities are high because interest rates are high. This means that there can be equilibrium outcomes in which interest rates are unnecessarily high, and in which policy arrangements can bring them down. This exploration is motivated by the recent sovereign debt crisis in Europe, but it is also motivated by a literature that does not seem to be consensual on this respect. Indeed, while Eaton and Gersovitz (1981) claim that there is a single equilibrium, Calvo (1988) using a similar structure shows that there are both high and low interest rate equilibrium schedules. Aguiar and Gopinath (2006) and Arellano (2008) building on Eaton and Gersovitz, modify an important assumption on the choice of debt by the large player and find a single equilibrium. We show that small

changes in timing assumptions and actions of agents, that cannot be directly justified by empirical evidence, can explain these conflicting results.

Assumptions on whether the country chooses the debt net of interest payments or gross of those payments, or whether the borrower moves first or the creditors do, are not assumptions that can be obtained directly from empirical evidence. But there is indirect evidence. The multiplicity of equilibria that arises under some of those assumptions is consistent with the large and abrupt movements in interest rates that are observed in sovereign debt crises, while the single equilibrium is not.

We also simulate a dynamic version of the model, in which a sunspot variable can induce high frequency movements in interest rate equilibria. We believe this can be a reading of a sovereign debt crisis. If so, then policies of large purchases of sovereign debt, at penalty rates, such as the ones announced by the ECB back in 2012, can have the effect that they seem to have had, of bringing down sovereign debt spreads.

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