

The Risky Capital of Emerging Markets

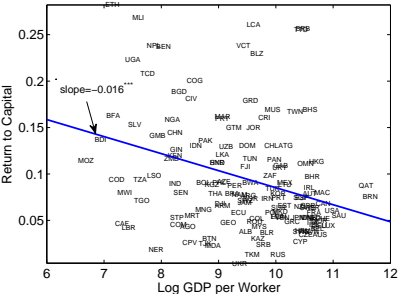
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December 12, 2014

Lucas Paradox Revisited



Lucas Paradox: Why don't returns to capital equalize?

Our answer: Capital in poorer countries is riskier.

⇒ Investor demands higher average returns there.

We show: differences in risk **quantitatively** account for differences in returns.

What we do...

First pass: workhorse CCAPM... works qualitatively; not quantitatively (paper)

- Given $cov(r_j, \Delta c_{US})$ from data, need $\gamma \approx 900$ to fit observed return diffs.

⇒ Lucas Paradox: just another asset pricing puzzle! [▶ Details](#)

Towards a resolution:

Key Building Blocks

- ① Aguiar and Gopinath (2007): shocks to trend growth key in poor/emerging market BC's +
- ② Bansal and Yaron (2004): implications of these shocks for asset prices/returns, i.e., compensation for “long-run risks”

What we find...

Investments in poor countries are more exposed to global long-run risk

⇒ US investor demands higher average returns.

- ① Risk accounts for 60-70% of return diff. btwn US & set of poorest cntrs
- ② Results robust to different levels of disaggregation; grouping of countries
- ③ LRR key; SRR implies (tiny) *negative* risk premia in poor countries

Measuring Returns to Capital for US Investor

Environment: J regions;

2 sectors: K is freely traded— P_I , C is not (builds on Hsieh and Klenow, 2007);

Representative US agent consumes, considers buying K in US, investing it in j .

Motivation: Poor countries import capital goods from rich.

(see Eaton and Kortum, 2001, Burstein et al., 2011, Mutreja et al., 2012)

Return from potential investment in j for US investor:

$$R_t^j = \underbrace{\alpha \frac{P_{Y,j,t} Y_{j,t}}{K_{j,t}} \frac{1}{P_{I,t}}}_{D_{j,t} \text{ in } C_{US,t}} + \underbrace{(1 - \delta_{j,t+1}) \frac{P_{I,t+1}}{P_{I,t}}}_{\text{capital gains, } \Delta P}$$

dividend yield, D/P

$P_I, P_{Y,j}$ are prices of K, Y relative to US price of C (Gomme et al., 2011).

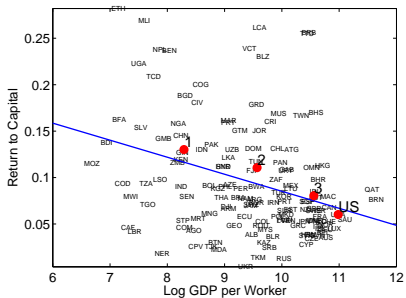
Returns Revisited: “Bundles” of countries

Assume common $\alpha = 0.3$.

▶ Robustness

▶ Open

144 countries bundled according to mean output per worker over period.



Data:

- PWT 8.0, 1950-2009: $K_{j,t}$, $\delta_{j,t}$, $P_{Y,US,2005} Y_{j,t}$ —adjust by $\frac{P_{Y,US,t}}{P_{Y,US,2005}}$
- BEA 1950-2009:
 - $P_{I,US}$ —price index of equipment + structures
 - $P_{Y,US}$ —price index of output
 - $P_{C,US}$ —price index of non-durables + services

Endowment economy in spirit of Bansal and Yaron (2004):

- EZ US investor consumes, gets dividends from inv'ts in US + abroad
 - ① Two sources of risk:
 - Small but persistent global component in $\Delta c^{(*)}$ and $\Delta d^{(*)}$ (*=foreign)
 - Small but persistent *-specific component; orthogonal to global component
 - ② US investor prices exposure to global component
 - ③ Key challenge in calibration: disentangle global from *-specific shocks

Model With Global and Local Shocks: US Processes

Representative US-based investor endowed with c , d , d^*

$$\Delta c_{t+1} = \mu_c + x_t + \eta_{t+1}$$

$$x_{t+1} = \rho x_t + e_{t+1}$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi \eta_{t+1} + \mu_{t+1}$$

$$\eta_{t+1} \sim N(0, \sigma_\eta), e_{t+1} \sim N(0, \sigma_e), \mu_{t+1} \sim N(0, \sigma_\mu)$$

US variables:

- μ_c (μ_d): unconditional mean growth rate of consumption (dividend)
- η : transitory shock to consumption growth
- ϕ : exposure of dividend growth to trend shock in consumption growth
- π : exposure of dividend growth to transitory shock in consumption growth
- μ : transitory shock to dividend growth

Model With Global and Local Shocks: Foreign Processes

$$\Delta c_{t+1}^* = \mu_c^* + \xi^* x_t + x_t^* + \pi_c^* \eta_{t+1} + \eta_{t+1}^*$$

$$x_{t+1}^* = \rho^* x_t^* + e_{t+1}^*$$

$$\Delta d_{t+1}^* = \mu_d^* + \tilde{\phi}^* (\xi^* x_t + x_t^*) + \pi^* \eta_{t+1} + \pi_d^* \mu_{t+1} + \pi_{cd}^* \eta_{t+1}^* + \mu_{t+1}^*$$

$$\eta_{t+1}^* \sim N(0, \sigma_{\eta^*}), e_{t+1}^* \sim N(0, \sigma_{e^*}), \mu_{t+1}^* \sim N(0, \tilde{\sigma}_{\mu^*})$$

- μ_c^* (μ_d^*): unconditional mean growth rate of consumption (dividend)
- ξ^* : exposure of cons. growth to trend shock in US cons. growth
- π_c^* : exposure of cons. growth to transitory shock in US cons. growth
- η^* : transitory shock to consumption growth
- $\tilde{\phi}^*$: exposure of dividend growth to local trend shock
- π^* : exposure of dividend growth to transitory shock in US cons. growth
- π_d^* : exposure of dividend growth to transitory shock in US dividend growth
- π_{cd}^* : exposure of dividend growth to transitory shock in cons. growth
- μ^* : transitory shock to dividend growth

Model: US Investor's Preferences

Representative US-based investor's preferences

$$V_t = \left[(1 - \beta) C_t^{\frac{\psi-1}{\psi}} + \beta \nu_t (V_{t+1})^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}$$
$$\nu_t (V_{t+1}) = \left[E_t \left(V_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}$$

- $\nu_t (V_{t+1})$ is certainty equivalent function
- ψ is IES; $\gamma \geq 0$ is risk aversion coefficient; CRRA special case iff $\gamma = 1/\psi$

Risk premia for US and foreign assets:

$$\mathbb{E} [R_t^e] = \underbrace{\left(\phi - \frac{1}{\psi} \right) \left(\gamma - \frac{1}{\psi} \right) \frac{\kappa_{m,1}}{1 - \kappa_{m,1}\rho} \frac{\kappa_1}{1 - \kappa_1\rho} \sigma_e^2}_{\text{long-run risk}} + \underbrace{\gamma \pi \sigma_\eta^2}_{\text{short-run risk}}$$
$$\mathbb{E} [R_t^{e*}] = \underbrace{\left(\phi^* - \frac{1}{\psi} \right) \left(\gamma - \frac{1}{\psi} \right) \frac{\kappa_{m,1}^*}{1 - \kappa_{m,1}^*\rho} \frac{\kappa_1}{1 - \kappa_1\rho} \sigma_e^2}_{\text{long-run risk}} + \underbrace{\gamma \pi^* \sigma_\eta^2}_{\text{short-run risk}}$$

where $\phi^* \equiv \tilde{\phi}^* \xi^*$ and κ 's are endogenous objects. ▶ SDF

Model: Calibration of US Parameters

$$\Delta c_{t+1} = \mu_c + x_t + \eta_{t+1}$$

$$x_{t+1} = \rho x_t + e_{t+1}$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi \eta_{t+1} + \mu_{t+1}$$

$\gamma = 10, \psi = 1.5, \beta = 0.99; \rho = 0.93$ (Ferson, 2013; Bansal, Kiku, Yaron, 2012)

$$\mathbb{E}[\Delta c_t] = \mu_c$$

$$\text{cov}(\Delta c_t, \Delta c_{t+1}) = \rho \frac{\sigma_e^2}{1 - \rho^2}$$

$$\text{var}(\Delta c_t) = \frac{\sigma_e^2}{1 - \rho^2} + \sigma_\eta^2$$

$$\mathbb{E}[\Delta d_t] = \mu_d$$

$$\sqrt{\frac{\text{cov}(\Delta d_{t+1}, \Delta d_t)}{\text{cov}(\Delta c_{t+1}, \Delta c_t)}} = \phi$$

$$\text{cov}(\Delta d_t, \Delta c_t) = \phi \frac{\sigma_e^2}{1 - \rho^2} + \pi \sigma_\eta^2$$

$$\text{var}(\Delta d_t) = \phi^2 \frac{\sigma_e^2}{1 - \rho^2} + \pi^2 \sigma_\eta^2 + \sigma_\mu^2$$

Model: Calibration of Foreign Parameters

Rewrite the foreign dividend process:

$$\Delta d_{t+1}^* = \mu_d^* + \phi^* x_t + \tilde{\phi}^* x_t^* + \pi^* \eta_{t+1} + \pi_d^* \mu_{t+1} + \pi_{cd}^* \eta_{t+1}^* + \mu_{t+1}^*$$

Iterative procedure to identify ϕ^* from:

$$\begin{aligned} \text{cov}(r_{m,t}^*, r_{m,t}) &= \frac{1}{\psi^2} \frac{\sigma_e^2}{1 - \rho^2} + \pi^* \pi \sigma_\eta^2 + \pi_d^* \sigma_\mu^2 + \\ &\quad \frac{\kappa_{m,1}}{1 - \kappa_{m,1}\rho} \frac{\kappa_{m,1}^*}{1 - \kappa_{m,1}^*\rho} \left(1 - \frac{1}{\psi}\right) \left(\phi^* - \frac{1}{\psi}\right) \sigma_e^2 \end{aligned}$$

+ moment conditions for remaining parameters.

For intuition, consider following moment:

$$\text{cov}(\Delta d_t^*, \Delta d_t) = \phi \phi^* \frac{\sigma_e^2}{1 - \rho^2} + \pi^* \pi \sigma_\eta^2 + \pi_d^* \sigma_\mu^2$$

- Both dividend growth and returns comove due to common transitory and persistent shocks, but return comovement is more sensitive to latter.
- Persistent shock $\Rightarrow \Delta$ asset prices due to change in future growth prospect $\Rightarrow \text{cov}(r_{m,t}^*, r_{m,t})$ relative to $\text{cov}(\Delta d_t^*, \Delta d_t)$ is larger.

Data: Moments

<i>US Moments</i>						
Consumption	$\mathbb{E} [\Delta c_t]$	$\text{cov} (\Delta c_{t+1}, \Delta c_t)$		$\text{std} (\Delta c_t)$		
	0.019	0.00024		0.022		
Dividends	$\mathbb{E} [\Delta d_t]$	$\sqrt{\frac{\text{cov} (\Delta d_{t+1}, \Delta d_t)}{\text{cov} (\Delta c_{t+1}, \Delta c_t)}}$	$\text{cov} (\Delta d_t, \Delta c_t)$		$\text{std} (\Delta d_t)$	
	-0.006	2.19	0.00018		0.026	
<i>Foreign Moments</i>						
Portfolio	$\mathbb{E} [\Delta d_t^*]$	$\text{cov} (\Delta d_{t+1}^*, \Delta d_t^*)$	$\text{cov} (\Delta d_t^*, \Delta c_t)$	$\text{std} (\Delta d_t^*)$	$\text{cov} (\Delta d_t^*, \Delta d_t)$	$\text{cov} (r_t^*, r_t)$
1	-0.017	0.00122	0.00011	0.083	0.00032	0.00109
2	-0.015	0.00156	0.00011	0.074	0.00033	0.00100
3	-0.011	0.00075	0.00015	0.063	0.00040	0.00085

- US Consumption TS: 1929-2009, Portfolio TS: 1950-2009
- Portfolio moment computed as mean of country-specific moments
- $\uparrow \text{cov} (\Delta d_t^*, \Delta d_t) + \downarrow \text{cov} (r^*, r)$ wrt income $\Rightarrow \uparrow \pi_d^* + \downarrow \phi^*$

Question: What are the implications for returns to capital?

▶ Parameters

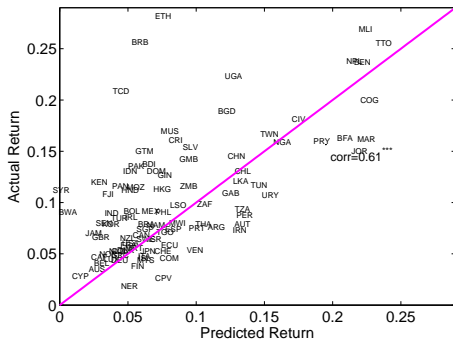
Predicted VS Actual Returns: 3, 5, 10 Bundles

	3 Portfolios		5 Portfolios			10 Portfolios		
	\hat{r}	r	\hat{r}	r		\hat{r}	r	
1	10.55	13.02	1	11.02	14.39	1	12.38	16.40
2	9.30	11.07	2	10.80	10.89	2	9.65	12.38
3	7.18	8.05	3	8.75	11.40	3	9.95	10.28
US	5.94	6.02	4	8.30	10.07	4	11.31	11.38
			5	6.28	6.75	5	7.47	10.65
			US	5.94	6.02	6	10.02	12.00
						7	8.06	9.76
						8	8.50	10.40
						9	6.60	7.63
						10	6.04	6.00
						US	5.94	6.02
Average:	8.24	9.54		8.52	9.92		8.72	10.26
Spread: 1-US	4.61	7.00		5.08	8.36		6.44	10.38
Percent of actual	66			61			62	
$\text{corr}(\hat{r}, r)$	1.00			0.92			0.91	

Model delivers 61-66% of spread.

► Rebalanced Portfolios

Predicted VS Actual Returns by Country



- 96 countries in PWT in or prior to 1961: $\text{corr}(\hat{r}, r)$ is 0.61
- predicted semi-elasticity of returns wrt income is 55% of actual: -0.023 (model) vs. -0.013 (data)

Decomposition: Long VS Short Run Risk

Portfolio	Actual	Predicted						
	r	\hat{r}	\hat{r}^f	\hat{r}_{sr}^e	\hat{r}_{lr}^e			
1	13.02	10.55	=	1.29	+	-0.28	+	9.54
2	11.07	9.30	=	1.29	+	-0.18	+	8.20
3	8.05	7.18	=	1.29	+	0.08	+	5.82
US	6.02	5.94	=	1.29	+	0.22	+	4.44

- High returns mainly driven by long-run risk
- Short-run risk compensation in low/middle-income countries

Conclusion

Can differences in risk **quantitatively** account for differences in returns?

Yes!

Risk (measured properly) accounts for 60-70% of difference in returns between poorest countries and US.

Key implication: Despite large return differentials, observed capital allocation is not so distant from that predicted by theory.

Future work should investigate sources of differences in long-run risk.

Model: Calibration of Foreign Parameters

Remaining moments on dividends similar to those for US.

Let $\rho^* = \rho$, $\sigma_{\mu^*}^2 \equiv \pi_{cd}^{*2} \sigma_{\eta^*}^2 + \tilde{\sigma}_{\mu^*}^2$.

Guess ϕ^* , then use $\text{cov}(\Delta d_t^*, \Delta d_t)$ and following moment conditions:

$$\begin{aligned}\mathbb{E}[\Delta d_t^*] &= \mu_d^* \\ \text{cov}(\Delta d_t^*, \Delta c_t) &= \phi^* \frac{\sigma_e^2}{1 - \rho^2} + \pi^* \sigma_{\eta}^2 \\ \text{cov}(\Delta d_{t+1}^*, \Delta d_t^*) &= (\tilde{\phi}^* \sigma_{e^*})^2 \frac{\rho^*}{1 - \rho^{*2}} + (\phi^* \sigma_e)^2 \frac{\rho}{1 - \rho^2} \\ \text{var}(\Delta d_t^*) &= (\phi^*)^2 \frac{\sigma_e^2}{1 - \rho^2} + \frac{(\tilde{\phi}^* \sigma_{e^*})^2}{1 - \rho^2} + \pi^{*2} \sigma_{\eta}^2 + \pi_d^{*2} \sigma_{\mu}^2 + \sigma_{\mu^*}^2\end{aligned}$$

Verify ϕ^* using $\text{cov}(r_{m,t}^*, r_{m,t})$ above.

Notable difference:

$\text{cov}(\Delta c_{t+1}^*, \Delta c_t^*)$ not needed b/c $\tilde{\phi}^*$ and σ_{e^*} need not be separately identified.

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Alternative Calibration Strategies and Biases

Alternative moment for ϕ^*

$$\phi_{\text{biased}}^* = \sqrt{\frac{\text{cov}(\Delta d_{t+1}^*, \Delta d_t^*)}{\text{cov}(\Delta c_{t+1}, \Delta c_t)}} = \phi^* \sqrt{1 + \frac{1}{\xi^{*2}} \frac{\sigma_{e^*}^2}{\sigma_e^2}}$$

- larger country-specific trend shock $\sigma_{e^*}^2 \Rightarrow$ larger bias
- $\sigma_{e^*}^2 = 0$ yields nearly identical results in our calibration
- $\pi_d^* = 0$ reverses returns; it ignores comovement due to transitory shocks

Portfolio	Actual	Benchmark	$\sigma_{e^*} = 0$		$\pi_d^* = 0$
			Autocovariance	Baseline	
1	13.02	10.55	11.06	10.56	8.76
2	11.07	9.30	12.05	9.33	9.28
3	8.05	7.18	9.59	7.19	10.47

Calibrated Model: Parameters

Preferences: $\gamma = 10$ $\psi = 1.5$ $\beta = 0.99$

Consumption: $\rho = 0.93$ $\mu_c = 0.019$ $\sigma_e = 0.006$ $\sigma_\eta = 0.02$

Portfolio	μ_d	ϕ	π	π_d	σ_μ	$\tilde{\phi}^* \sigma_{e^*}$
1	-0.017	5.14	-1.24	-0.16	0.074	0.005
2	-0.015	4.23	-0.81	-0.00	0.061	0.011
3	-0.011	2.87	0.34	0.27	0.056	0.008
US	-0.006	2.19	0.98	-	0.020	-

▶ Moments

Lucas Paradox: Just Another Asset Pricing Puzzle

With CRRA (γ) preferences:

$$\mathbb{E}[r_{jt}^e] \approx \gamma \text{cov}(r_{jt}, \Delta c_t)$$

- $r_{jt}^e \equiv r_{jt} - r_{ft}$ is excess (net) return on portfolio j over 3-month t-bill at t
- Δc_t is US (ND+S) consumption growth during 1950-2009

Portfolio	r^e	$\text{cov}(r, \Delta c)$	$\gamma =$	\hat{r}^e	
				10	889
1	11.80	0.00016	0.16	14.06	
2	9.85	0.00009	0.09	7.93	
3	6.83	0.00004	0.04	3.92	
US	4.80	0.00004	0.04	3.23	
Spread: 1-US	7.00	0.00012	0.12	10.83	

γ falls as granularity increases; at country level it is very high, ≈ 500 .

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Predicted VS Actual Returns—Annually Rebalanced Portfolios

	3 Portfolios		5 Portfolios			10 Portfolios		
	\hat{r}	r	\hat{r}	r		\hat{r}	r	
1	10.56	12.10	1	11.03	13.00	1	11.44	14.91
2	8.74	10.23	2	10.06	10.83	2	10.59	10.92
3	7.04	7.57	3	8.14	10.18	3	9.77	10.49
US	5.94	6.02	4	7.47	8.49	4	10.36	11.17
			5	7.18	7.26	5	8.25	10.51
			US	5.94	6.02	6	8.05	9.88
						7	8.33	8.96
						8	6.62	8.01
						9	7.28	6.87
						10	7.09	7.64
						US	5.94	6.02
Average:	8.07	8.98		8.31	9.30		8.52	9.58
Spread: 1-US	4.61	6.08		5.09	6.98		5.50	8.89
Percent of actual	76			73			62	
$\text{corr}(\hat{r}, r)$	1.00			0.96			0.91	

Details: SDF

Key asset-pricing equation for any asset:

$$\mathbb{E}_t[M_{t+1}R_{t+1}] = 1,$$

M_{t+1} is US investor's SDF.

In logs,

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{ct+1}$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$

and r_{ct+1} is return on asset that pays consumption as dividend.

Solving the Model

Standard approach; approximations to make exercise computationally feasible

Note that $z_t = \log\left(\frac{P_t}{D_t}\right)$ is enough to characterize returns

- For each asset $k = \text{home}, *$, and asset that pays off aggregate consumption

To solve:

- Conjecture: $z_t^k = A_0^k + A_1^k x_t$
- Approximate log returns: $r_{m,t+1}^k = \kappa_0^k + \kappa_1^k z_{t+1}^k + \Delta d_{t+1}^k - z_t^k$
where κ 's depend on \bar{z}^k
- Restrictions from Euler equation gives excess return (as in text) and rf rate
- Solve numerically for $\bar{z}^k = A_0^k(\bar{z}^k)$ and for $A_1^k(\bar{z}^k)$

► Model

Alternative Measurement Approaches

- 1 Caselli and Feyrer (2007): country-specific P_I, P_C, P_Y from PWT.
- 2 Country-time specific α 's + non-reproducible capital adjustment from WDI

Portfolio	All Years				1996			
	Baseline	Country prices	Country α 's	Country prices & α 's	Baseline	Country prices	Country α 's	Country prices & α 's
1	13.02*** (0.76)	12.00*** (0.66)	13.22*** (0.76)	13.63*** (1.05)	5.38** (.78)	3.32 (2.81)	8.24** (1.25)	7.27 (2.00)
2	11.07*** (0.61)	10.53*** (0.62)	13.15*** (0.75)	13.23*** (0.68)	5.21 (0.99)	5.89 (1.73)	8.12* (1.29)	7.51 (1.42)
3	8.05*** (0.45)	9.36** (0.33)	9.17*** (0.49)	11.39*** (0.44)	3.91 (0.85)	10.03* (1.48)	6.47 (1.29)	14.09*** (1.22)
US	6.02 (0.35)	8.22 (0.31)	6.20 (0.39)	9.39 (0.50)	3.64 -	7.23 -	5.52 -	9.55 -

Notes: Table reports the returns to capital across portfolios under a number of measurement approaches. Baseline uses US prices from BEA. Country prices uses country-specific P_Y, P_I, P_C from PWT. Country α 's uses country-year α from PWT and subtracts from α the share of payments to non-reproducible capital from WDI, dropping the countries that have negative α for at least one year. Country prices and α 's uses country prices and country-year α as described above. Baseline and Country prices cover years from 1950 to 2008. Country α 's and Country prices and α 's cover years from 1970 to 2008. The portfolios include only countries for which data are available. Standard errors are reported in parentheses. Asterisks denote significance of difference from US values: ***: difference significant at 99%, **: 95%, and *: 90%.

Returns for Countries With Open Capital Accounts

Countries with open capital accounts obey link btwn GDP and returns.

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Portfolio	Measure of Openness		
	Chinn, Ito	Quinn	Grilli, Milesi-Ferretti
1	10.38*** (0.66)	12.39*** (0.67)	11.48*** (0.59)
2	8.74*** (0.61)	11.27*** (0.62)	10.28*** (0.86)
3	5.61 (0.50)	6.66 (0.51)	7.57** (0.63)
US	5.22 (0.38)	6.06 (0.38)	5.85 (0.52)

Notes: Table reports the returns to capital across portfolios for economies that are characterized as open according to three indices: Chinn/Ito, Quinn, and Grilli/Milesi-Ferretti, respectively. Chinn/Ito and Quinn openness cutoff is median value in sample. Grilli/Milesi-Ferretti openness indicator is unity. Standard errors are reported in parentheses. Asterisks denote significance of difference from US values: ***: difference significant at 99%, **: 95%, and *: 90%.

Supplementary Results

Note:

- A 's depend on US parameters only
- σ_{μ}^{2*} , $\tilde{\phi}^* \sigma_e^2$ and calibrated parameters are sufficient to compute $\kappa_{m,1}^*$

$$A_{m,0}^* = \frac{\theta \log \beta - \gamma \mu + (\theta - 1) (\kappa_0 + A_0 (\kappa_1 - 1)) + \mu_d^* + \kappa_{m,0}^* + \frac{1}{2} \left(\frac{\kappa_{m,1}^*}{1 - \kappa_{m,1}^* \rho^*} \right)^2 (\tilde{\phi}^* \sigma_e^*)^2}{1 - \kappa_{m,1}^*}$$
$$+ \frac{\frac{1}{2} (\pi^* - \gamma)^2 \sigma_{\eta}^2 + \frac{1}{2} \left((\theta - 1) \kappa_1 A_1 + \frac{\kappa_{m,1}^*}{1 - \kappa_{m,1}^* \rho^*} \left(\phi^* - \frac{1}{\psi} \right) \right)^2 \sigma_e^2 + \frac{1}{2} \pi_d^{*2} \sigma_{\mu}^2 + \frac{1}{2} \sigma_{\mu}^{2*}}{1 - \kappa_{m,1}^*}$$

▶ Solution