

Monetary Policy and the Uncovered Interest Rate Parity Puzzle*

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March 2013

(First Draft: May 2010)

Abstract

High interest rate currencies tend to appreciate. This is the *uncovered interest rate parity (UIP) puzzle*. It is primarily a statement about *short-term* interest rates and how they are related to exchange rates. Short-term interest rates are strongly affected by monetary policy. The UIP puzzle, therefore, can be restated in terms of monetary policy. When one country has a high interest rate *policy* relative to another, why does its currency tend to appreciate? We represent monetary policy as foreign and domestic Taylor rules. Foreign and domestic pricing kernels determine the relationship between these Taylor rules and exchange rates. We examine different specifications for the Taylor rule and ask which can resolve the UIP puzzle. We find evidence in favor of *asymmetries*. A foreign Taylor rule that is *more* procyclical than its domestic counterpart, generates a positive excess expected return on foreign currency. If foreign policy reacts relatively passively to inflation, the same is true. *The combination of a weak inflation policy and a strong employment policy makes for a risky currency*. This is broadly consistent with empirical evidence on ‘carry trade’ funding and recipient currencies. When we calibrate our model to a particular currency pair — the United States and Australia — we find that our model is consistent with many empirical characteristics of real and nominal exchange rates, including the negative correlation between interest rate differentials and currency depreciation rates.

*Thanks to Bernard Dumas, Michael Evers, Jordi Galí, Isaac Kleshchelski, Robert Kollman, Kristoffer Nimark, Klaus Schmidt-Hebbel, Jaume Ventura, Adrien Verdelhan and, in particular, Burton Hollifield for many helpful comments and discussions. Telmer thanks CREI, Universitat Pompeu Fabra, for generous hospitality while some of this work was completed.

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1 Introduction

Uncovered interest rate parity (UIP) predicts that high interest rate currencies will depreciate relative to low interest rate currencies. Yet for many currency pairs and time periods we seem to see the opposite (Bilson (1981), Fama (1984), Tryon (1979)). The inability of asset-pricing models to reproduce this fact is what we refer to as the *UIP puzzle*.

The UIP evidence is primarily about *short-term* interest rates and currency depreciation rates. Monetary policy exerts substantial influence over short-term interest rates. Therefore, the UIP puzzle can be restated in terms of monetary policy: Why do countries with high interest rate *policies* have currencies that tend to appreciate relative to those with low interest rate *policies*?

The risk-premium interpretation of the UIP puzzle asserts that high interest rate currencies pay positive risk premiums. The question, therefore, can also be phrased in terms of currency risk: When a country pursues a relatively high-interest rate monetary policy, why does this make its currency risky? For example, when the Fed sharply lowered rates in 2001 and the ECB did not, why did the euro become relatively risky? When the Fed sharply reversed course in 2005, why did the dollar become the relatively risky currency? This paper formulates a model of interest rate policy and exchange rates that can potentially answer these questions.

To understand what we do it's useful to understand previous work on monetary policy and the UIP puzzle.¹ Most models are built upon the basic Lucas (1982) model of international asset pricing. The key equation in Lucas' model is

$$\frac{m_{t+1}^*}{m_{t+1}} \equiv \frac{n_{t+1}^* e^{-\pi_{t+1}^*}}{n_{t+1} e^{-\pi_{t+1}}} = \frac{S_{t+1}}{S_t} \quad (1)$$

where S_t denotes the nominal exchange rate (price of foreign currency in units of domestic), n_t denotes the intertemporal marginal rate of substitution of the domestic representative agent, π_t is the domestic inflation rate, $m_t \equiv n_t \exp(-\pi_t)$ is the *nominal* marginal rate of substitution, and asterisks denote foreign-country variables. Equation (1) holds by virtue of complete financial markets. It characterizes the basic relationship between interest rates, nominal exchange rates, real exchange rates, preferences and consumption.

Previous work has typically incorporated monetary policy into Equation (1) via an explicit model of *money*. Lucas (1982), for example, uses cash-in-advance constraints to map Markov processes for foreign and domestic money supplies into the inflation term, $\exp(\pi_t - \pi_t^*)$, and thus into exchange rates. His model, and many that have followed it, performs poorly in accounting for data. This is primarily a reflection of the weak empirical link between measures of money and exchange rates.

¹Examples are Alvarez, Atkeson, and Kehoe (2009), Backus, Gregory, and Telmer (1993), Bekaert (1994), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Canova and Marrinan (1993), Dutton (1993), Grilli and Roubini (1992), Macklem (1991), Marshall (1992), McCallum (1994) and Schlagenhauf and Wrase (1995).

Our approach is also built upon Equation (1). But — like much of the modern theory and practice of monetary policy — we abandon explicit models of money in favor of interest rate rules. Following the New Keynesian macroeconomics literature (*e.g.*, Clarida, Galí, and Gertler (1999)), the policies of the domestic and foreign monetary authorities are represented by Taylor (1993) rules. The simplest forms we consider are

$$\dot{i}_t = \tau + \tau_\pi \pi_t + \tau_x x_t \tag{2}$$

$$\dot{i}_t^* = \tau^* + \tau_\pi^* \pi_t^* + \tau_x^* x_t^* \tag{3}$$

where i_t is the domestic short-term (nominal) interest rate, x_t is consumption growth (analogous to the output gap in a model with nominal frictions), τ , τ_π and τ_x are policy parameters, and, as above, asterisks represent foreign variables and parameters. Basically, where Lucas (1982) uses money to describe how monetary policy affects the variables in Equation (1), we use Taylor rules. An intuitive sketch of how we do so is provided in Section 2, followed by a formalization in Sections 4 and 5.

We can now state our question more precisely. The variables m_t^* and m_t in Equation (1) are called foreign and domestic *pricing kernels*. A number of papers (*e.g.*, Backus, Foresi, and Telmer (2001), Lustig, Roussanov, and Verdelhan (2011)) have demonstrated the importance of *asymmetries* between foreign and domestic pricing kernels for explaining the UIP puzzle. Inspection of Equation (1) shows why. Exchange rates are all about *differences* in pricing kernels. If there are no differences, then the exchange rate is a constant. Many previous papers have come up with *statistical* models of such differences, but far fewer have come up with *economic* models. Lustig, Roussanov, and Verdelhan (2011), for example, argue in favor of asymmetric pricing-kernel parameters (“prices of risk”) loading on global sources of risk that are common across countries. They present empirical evidence supporting such a statistical representation. But what is the interpretation of these parameter asymmetries? Why do certain country’s pricing kernels load on global factors in such a way so as to make their currencies risky?

Herein lies our basic question. Perhaps the differences in pricing-kernel parameters reflect differences in monetary policies? That is, if the Fed’s policy is described by $[\tau \ \tau_\pi \ \tau_x]$, and the ECB’s by $[\tau^* \ \tau_\pi^* \ \tau_x^*]$, so that the pricing kernels can be written $m_t(\tau, \tau_\pi, \tau_x)$ and $m_t^*(\tau^*, \tau_\pi^*, \tau_x^*)$, then does the exchange rate implied by Equation (1) help us understand the UIP puzzle? If so, does the model offer economic insights into why high-interest-rate policy countries have risky currencies? We find in the affirmative on both counts.

Our findings are summarized as follows. We refer to $\tau_x < \tau_x^*$ as foreign monetary policy being *relatively procyclical*. We refer to $\tau_\pi > \tau_\pi^*$ as foreign policy being *relatively accommodative* (to inflation). We find that the combination of the two generates a *foreign* currency that is risky in that the unconditional expected return to “borrow domestic, lend foreign” is positive. This also affects the conditional distribution. An appropriately-defined “carry trade” — “borrow domestic, lend foreign, but only when the foreign interest rate is *high enough*” — has a higher Sharpe ratio, the larger are these differences in policy parameters. We demonstrate these results both analytically and quantitatively. We argue that they are broadly

consistent with the salient features of carry-trade recipient and funding countries, and use Australia-U.S. as a case study in our calibration analysis.

The technical economic intuition goes as follows. First, a procyclical monetary policy generates a negative correlation between inflation and consumption growth: *monetary policy generates inflation risk*. This is a fairly general feature of Taylor-rule-implied inflation. Second, since $\log m_{t+1} = \log n_{t+1} - \pi_{t+1}$, this negative correlation generates $\text{Var}_t(\log m_{t+1}) < \text{Var}_t(\log n_{t+1})$: *monetary policy reduces nominal risk relative to real risk*. Third, an accommodative policy generates high inflation volatility and *amplifies* this effect. Fourth, if $\text{Var}_t(\log m_{t+1}^*) < \text{Var}_t(\log m_{t+1})$ then the *foreign* currency pays a positive risk premium. Finally, if the first three items are considered in *relative* terms (foreign compared to domestic), then they imply the fourth and the story is concluded.

The plainer-language intuition — necessarily a bit loose — goes as follows. Variation in exchange rates arises from differences in domestic shocks relative to foreign shocks. The larger are the former, the larger will be the extent to which domestic shocks and exchange rate shocks are the *same thing*. Hence, if U.S. (domestic) shocks are relatively large, then the U.S. investor views currency as being risky, relative to the Australian (foreign) investor. The result is that the *foreign* currency — the Australian dollar — pays a risk premium. This is a fairly general characteristic of models of currency risk. Now consider monetary policy. A procyclical policy *stabilizes* nominal state prices relative to real state prices (*i.e.*, it generates a negative correlation between inflation and consumption). An accommodative policy exacerbates this effect by generating larger inflation volatility. *Ceteris paribus*, if Australian policy is both procyclical and accommodative, relative to U.S. policy, then Australian nominal state prices will be stable relative to their U.S. counterparts, the USD/AUD nominal exchange rate will be relatively reflective of U.S. “shocks,” and the AUD will be the risky currency. End of the story. Note that it is *not* a story about the relative size of Australian and U.S. macroeconomic shocks. It is, instead, a story about how differences in monetary policies can take a *given* joint distribution of shocks and impart them into nominal exchange rates in a way that moves us in the direction of some standard carry-trade facts.

We conduct a calibration analysis that shows that this comparative-static story is empirically substantive. The data are from the U.S. and Australia, a typical “carry trade” country-pair. The Taylor-rule parameters are calibrated to U.S. and Australian inflation and consumption data, leaving open the implications for nominal interest rates and exchange rates. Specifically, the six parameters in Equations (2) and (3) are chosen to match the mean and standard deviation of the two country’s inflation rates along with the correlations between their consumption growth rates and their inflation rates. The above discussion highlights why *these* correlations are pivotal. We find that the solution to this identification scheme is unique and that it yields parameter estimates that are both economically sensible and similar to those typically found in the literature. The implications for interest rates and exchange rates are *qualitatively* consistent with our paper’s main point. This means that the calibrated Australian Taylor-rule parameters are both relatively procyclical and relatively accommodative, and that the model features (i) a realistic mean, volatility and autocorrela-

tion in the nominal exchange rate, (ii) a negative UIP regression-coefficient, (iii) a positive *unconditional* risk premium on the Australian dollar, and (iv) a carry-trade Sharpe ratio that is indicative of time-variation in *conditional* risk premiums.

Quantitatively, our model generates risk premia that are far too small. We go on to ask why by exploring some alternative identification schemes. The first shows that the Taylor parameters can be identified with exchange rate moments — volatility and the UIP regression coefficient — instead of the consumption-inflation correlations. This seems interesting in-and-of-itself. It is supportive of our main point, that monetary policy asymmetries are important for currency risk. It also improves the model’s quantitative performance by an order of magnitude, and at very little cost in terms of the consumption-inflation behavior. We also calibrate the Taylor parameters to match the unconditional currency risk premium itself. The implied U.S./Australian inflation dynamics are strongly counterfactual, with, for instance, U.S. inflation volatility being 0.30% compared to 1.9% for Australia (sample moments are about 0.9% and 1.0%, respectively). Our paper is about how policy differences are manifest in currency risk. The point here is that U.S. and Australian policies are not different *enough* to fit the facts.

Our model features a number of other counterfactual behaviors. Chief among them is that consumption growth is far too highly correlated across countries (Brandt, Cochrane, and Santa-Clara (2006)). In the Appendix we develop and calibrate an enhanced model that uses the two-country long-run risk setup, borrowing from Bansal and Shaliastovich (2013) and Colacito and Croce (2011). We show that the counterfactual behaviors are diminished substantially while at the same time leaving our main points in tact and robust. Additional shortcomings of our model — both quantitative and conceptual — are discussed in Section 2.

The distinction between unconditional and conditional currency risk plays an important role in our paper. We place much emphasis on the former. The reason is apparent from looking at Equations (2) and (3). We interpret the Taylor rules as reflecting deep, immutable *policies* (more on this below). Cross-country differences in them are primarily reflected in the model’s unconditional distributions, including the distribution of the risk premium. While we do show that the model’s conditional distributions *are* affected — via the effect on the variability of the pricing kernel — this effect turns out to be relatively small. A better model of conditional risk might feature some sort of time-variation in the cross-country policy differences (*e.g.*, regime-switching in the τ parameters). This is beyond the scope of our study. As to what is more empirically relevant, it is important to acknowledge that the unconditional risk premium appears to be small or zero for many, but not all, currency pairs. As we’ve accumulated more time series evidence, the size of the latter set seems to have grown (*c.f.*, Engel (2011), Hassan (2010) and Lustig, Roussanov, and Verdelhan (2011)) Prominent among these, no surprise, are the currencies that play an active role in the carry-trade phenomenon, such as Australia, Japan and New Zealand. Our paper is more empirically relevant for pairs involving these currencies than it is for, say, dollar/pound.

The remainder of our paper is organized as follows. In Section 2 we provide a relatively

non-technical overview of our methodology, its strengths and its weaknesses. Section 3 reviews some previous results that are used extensively in Section 4, where we develop and solve our model. Sections 5 and Section 6 analyze the model analytically and then quantitatively. Section 7 concludes. Proofs and detailed calculations are relegated to a series of appendices. In addition, the appendices contain an analysis of some alternative Taylor rule formulations — incorporating the nominal exchange and lagged interest rates as in McCallum (1994) and Woodford (2003)) — and a discussion of the sense in which our setting, with time-varying volatility and exchange rates, overcomes some of the identification issues that Cochrane (2011) shows are associated with Taylor rules.

2 Overview

Our approach is essentially a two-country version of Gallmeyer, Hollifield, Palomino, and Zin (2007). It can also be viewed as a version of Bansal and Shaliastovich (2013) with endogenous inflation. We assume that markets are complete, so that Equation (1) can be viewed in terms of allocations, not just a change-of-units. The simplest Taylor rule we consider is

$$i_t = \tau + \tau_\pi \pi_t + \tau_x x_t \quad , \quad (4)$$

where i_t is the nominal short-term interest rate, π_t is the inflation rate, x_t is consumption growth (analogous to the output-gap in a model with nominal frictions), and τ , τ_π and τ_x are policy parameters. We assume that the private sector can trade bonds. Therefore the nominal interest rate must also satisfy the standard (nominal) Euler equation,

$$i_t = -\log E_t n_{t+1} e^{-\pi_{t+1}} \quad , \quad (5)$$

where n_{t+1} is the real marginal rate of substitution. We specify n_{t+1} using Epstein and Zin (1989) preferences and use the Hansen, Heaton, and Li (2008) linearization to express it in terms of consumption growth, x_t , its volatility, u_t , and their innovations. Note that, as we make very clear below, time-varying volatility is not an option for the question of currency risk. It is a necessity (Backus, Foresi, and Telmer (2001)). The only issue is where it comes from. In our paper it is an exogenously-specified characteristic of the consumption-growth process.

An equilibrium inflation rate process must satisfy both of these equations at each point in time, which requires inflation to solve the nonlinear stochastic difference equation:

$$\pi_t = -\frac{1}{\tau_\pi} (\tau + \tau_x x_t + \log E_t n_{t+1} e^{-\pi_{t+1}}) \quad . \quad (6)$$

A solution to Equation (6) is an endogenous inflation process, π_t , that is a stationary function of the model's state variables, x_t and u_t : $\pi(x_t, u_t)$. By substituting it back into the Euler equation (5), we arrive at what Gallmeyer, Hollifield, Palomino, and Zin (2007) refer to as

a ‘monetary policy consistent pricing kernel:’ a (nominal) pricing kernel that depends on the Taylor-rule parameters τ , τ_π and τ_x . Doing the same for the foreign country, and then using Equation (1), we arrive at a nominal exchange rate process that also depends on the Taylor-rule parameters. Equations (1)–(6) (along with specifications for the shocks) fully characterize the joint distribution of interest rates and exchange rates and, therefore, any departures from UIP.

We now outline the model’s strengths and limitations. Its main strength is that it delivers processes for inflation and exchange rates that are jointly determined by the response of the monetary authority and the private sector to the *same* underlying shocks. This is a defining characteristic of New Keynesian macroeconomic models. Its main limitation is that this dependence only goes one way. Ours is an endowment economy in which consumption is specified exogenously. The monetary rule responds to consumption shocks, but consumption outcomes are unaffected by the existence of the monetary rule. We could add “nominal shocks” to the model — shocks to the Taylor rule are quite common in the literature — but this property would remain. ‘Monetary policy’ affects only nominal outcomes. There can be no discussion of whether a given policy is good or bad. We simply rely on empirical support for the connection between Taylor rules and monetary policy, and the fact that better-articulated monetary economies often feature interest rate behavior that looks like Taylor rules. Moreover, since real exchange rates are, in our model, ratios of (real) marginal rates of substitution, they are also unaffected by monetary policy. Our is a model with realistic real and nominal exchange rate behavior, but only the latter is a result of the mechanism we study. The former derives from properties of consumption and Epstein-Zin preferences and has previously been pointed out by Bansal and Shaliastovich (2013) and Colacito and Croce (2011).

A second limitation is that we specify consumption to be exogenous for *both* the foreign and the domestic country. That is, we specify the consumption processes that underly the ratio n_{t+1}^*/n_{t+1} in Equation (1) to match observed consumption data from two countries, but we are silent on the model of international trade that gives rise to such consumption allocations. While it is true that, in any equilibrium (with our preferences) that fits the observed consumption facts, Equation (1) must hold, it is also true that we have nothing to say about *why* there is cross-country variation in marginal utility (and thus in real exchange rates). Bansal and Shaliastovich (2013), Colacito and Croce (2011), Gavazzoni (2008), Verdelhan (2010) and others follow a similar approach. Hollifield and Uppal (1997), Sercu, Uppal, and Hulle (1995) and the appendix in Verdelhan (2010) — all building upon Dumas (1992) — are examples of more fully-articulated complete markets models in which imperfectly-correlated cross-country consumption is generated by transport costs. Basically, our approach is to these models what Hansen and Singleton’s (1983) first-order-condition-based approach was to Mehra and Prescott’s (1985) general equilibrium model.

3 Pricing Kernels and Currency Risk Premiums

We begin with a terse treatment of existing results in order to fix notation. The level of the spot and one-period forward exchange rates, in units of U.S. dollars (USD) per unit of foreign currency (say, British pounds, GBP), are denoted S_t and F_t . Logarithms are s_t and f_t . USD and GBP one-period interest rates (continuously compounded) are denoted i_t and i_t^* . Covered interest parity implies that $f_t - s_t = i_t - i_t^*$. Fama’s (1984) decomposition of the interest rate differential (forward premium) is

$$\begin{aligned} i_t - i_t^* = f_t - s_t &= (f_t - E_t s_{t+1}) + (E_t s_{t+1} - s_t) \\ &\equiv p_t + q_t \end{aligned}$$

This decomposition expresses the forward premium as the sum of q_t , the expected USD depreciation rate, and p_t , the expected payoff on a forward contract to receive USD and deliver GBP. We define the latter as the *currency risk premium*. We define *uncovered interest parity* (UIP) as $p_t = 0$. The well-known rejections of UIP are manifest in negative estimates of the parameter b from the regression (Bilson (1981), Fama (1984), Tryon (1979))

$$s_{t+1} - s_t = c + b(i_t - i_t^*) + \text{residuals} \quad . \quad (7)$$

The population regression coefficient — we’ll call it the “UIP coefficient” — can be written

$$b = \frac{\text{Cov}(q_t, p_t + q_t)}{\text{Var}(p_t + q_t)} \quad . \quad (8)$$

Fama (1984) noted that necessary conditions for $b < 0$ are

$$\text{Cov}(p_t, q_t) < 0 \quad (9)$$

$$\text{Var}(p_t) > \text{Var}(q_t) \quad (10)$$

Our approach revolves around the standard (nominal) pricing-kernel equation,

$$b_t^{n+1} = E_t m_{t+1} b_{t+1}^n \quad , \quad (11)$$

where b_t^n is the USD price of a nominal n -period zero-coupon bond at date t and m_t is the pricing kernel for USD-denominated assets. The one-period interest rate is $i_t \equiv -\log b_t^1$. An equation analogous to (11) defines the GBP-denominated pricing kernel, m_t^* , in terms of GBP-denominated bond prices, b_t^* .

Backus, Foresi, and Telmer (2001) translate Fama’s (1984) decomposition into pricing kernel language. First, assume complete markets so that the currency depreciation rate is

$$s_{t+1} - s_t = \log(m_{t+1}^*/m_{t+1})$$

Fama's (1984) decomposition becomes

$$i_t - i_t^* = \log E_t m_{t+1}^* - \log E_t m_{t+1} \quad (12)$$

$$q_t = E_t \log m_{t+1}^* - E_t \log m_{t+1} \quad (13)$$

$$p_t = (\log E_t m_{t+1}^* - E_t \log m_{t+1}^*) - (\log E_t m_{t+1} - E_t \log m_{t+1}) \quad (14)$$

$$= \text{Var}_t(\log m_{t+1}^*)/2 - \text{Var}_t(\log m_{t+1})/2 \quad , \quad (15)$$

where Equation (15) is only valid for the case of conditional lognormality. Basically, Fama's (1984) conditions (9) and (10) translate into the restriction that the means and the variances must move in opposite directions and that the variation in the variances must exceed that of the means.

Our objective is to write down a model of m_{t+1} and m_{t+1}^* in which $b < 0$ and the associated currency risk is realistic. Inspection of Equations (9) and (15) indicate that a necessary condition is that p_t vary over time and that, for the lognormal case, the log kernels *must* exhibit stochastic volatility.

4 Model

Consider two countries, home and foreign. The home-country representative agent's consumption is denoted c_t and preferences are of the Epstein and Zin (1989) (EZ) class:

$$U_t = [(1 - \beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}$$

where β and ρ characterize patience and intertemporal substitution, respectively, and the certainty equivalent of random future utility is

$$\mu_t(U_{t+1}) \equiv E_t[U_{t+1}^\alpha]^{1/\alpha} \quad ,$$

so that α characterizes (static) relative risk aversion (RRA). The relative magnitude of α and ρ determines whether agents prefer early or late resolution of uncertainty ($\alpha < \rho$, and $\alpha > \rho$, respectively). Standard CRRA preferences correspond to $\alpha = \rho$. The marginal rate of intertemporal substitution, defined as n_{t+1} , is

$$n_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{\rho-1} \left(\frac{U_{t+1}}{\mu_t(U_{t+1})} \right)^{\alpha-\rho} \quad . \quad (16)$$

We also refer to n_{t+1} as the *real* pricing kernel. The *nominal* marginal rate of substitution — the pricing kernel for claims denominated in USD units — is then

$$m_{t+1} = n_{t+1}e^{-\pi_{t+1}} \quad , \quad (17)$$

where π_{t+1} is the (continuously-compounded) rate of inflation between dates t and $t+1$. The foreign-country representative agent's consumption, c_t^* , and preferences are defined analogously. Asterisks are used to denote foreign variables. Foreign inflation is π_{t+1}^* .

The domestic pricing kernel satisfies $E_t(m_{t+1}R_{t+1}) = 1$ for all USD-denominated asset returns, R_{t+1} . Similarly, $E_t(m_{t+1}^*R_{t+1}^*) = 1$ for all GBP-denominated returns. The domestic pricing kernel must also price USD-denominated returns on GBP-denominated assets:

$$E_t\left(m_{t+1}\frac{S_{t+1}}{S_t}R_{t+1}^*\right) = 1 . \quad (18)$$

We assume that international financial markets are complete for securities denominated in goods units, USD units and GBP units. This implies the uniqueness of the nominal and real pricing kernels and therefore, according to Equation (18),

$$\frac{S_{t+1}}{S_t} = \frac{m_{t+1}^*}{m_{t+1}} = \frac{n_{t+1}^*e^{-\pi_{t+1}^*}}{n_{t+1}e^{-\pi_{t+1}}} . \quad (19)$$

Equation (19) must hold in any equilibrium with complete financial markets. This is true irrespective of the particular goods-market equilibrium that gives rise to the consumption allocations c_t and c_t^* that are inherent in n_t and n_t^* . Our approach is to specify c_t and c_t^* *exogenously* and calibrate them to match the joint behavior of data on domestic and foreign consumption. We are silent on the model of international trade that gives rise to such consumption allocations. Bansal and Shaliastovich (2013), Colacito and Croce (2011), Gavazzoni (2008), Verdelhan (2010) and others follow a similar approach. Hollifield and Uppal (1997), Sercu, Uppal, and Hulle (1995) and the appendix in Verdelhan (2010) — all building upon Dumas (1992) — are examples of more fully-articulated complete markets models in which imperfectly-correlated cross-country consumption is generated by transport costs. Basically, our approach is to these models what Hansen and Singleton’s (1983) first-order-condition-based approach was to Mehra and Prescott’s (1985) general equilibrium model.

Domestic consumption growth, $x_{t+1} \equiv \log(c_{t+1}/c_t)$, follows an AR(1) process with stochastic volatility u_t .

$$\begin{aligned} x_{t+1} &= (1 - \varphi_x)\theta_x + \varphi_x x_t + \sqrt{u_t}\epsilon_{t+1}^x \\ u_{t+1} &= (1 - \varphi_u)\theta_u + \varphi_u u_t + \sigma_u\epsilon_{t+1}^u \end{aligned}$$

The innovations ϵ_t^x and ϵ_t^u are standard normal and independent of each other. The analogous foreign consumption process is denoted with asterisks: $x_{t+1}^* \equiv \log(c_{t+1}^*/c_t^*)$, with volatility u_t^* . Foreign parameter values are assumed to be identical (‘symmetric’) to their domestic counterparts and the innovation-pairs, $(\epsilon_t^x, \epsilon_t^{x*})$ and $(\epsilon_t^u, \epsilon_t^{u*})$, have correlations η_{x,x^*} and η_{u,u^*} . The assumption of symmetry in the consumption processes serves to isolate the effect of asymmetry in monetary policy.

The final ingredients are domestic and foreign Taylor rules:

$$i_t = \bar{r} + \tau_\pi \pi_t + \tau_x x_t \quad (20)$$

$$i_t^* = \bar{r}^* + \tau_\pi^* \pi_t^* + \tau_x x_t^* \quad (21)$$

For future reference we denote $\Upsilon \equiv [\tau \ \tau_\pi \ \tau_x]$ and $\Upsilon^* \equiv [\tau^* \ \tau_\pi^* \ \tau_x^*]$. Here, we make the foreign specification explicit so as to emphasize the asymmetry that is the focal point of our paper, $\tau_\pi \neq \tau_\pi^*$ and $\tau_x \neq \tau_x^*$.

The Taylor rules (20) and (21) are fairly typical in the literature, the main exception being that we use consumption growth instead of the ‘output gap.’ In our model, which abstracts from any frictions that can give rise to a ‘gap,’ the distinction is meaningless. In Appendix A we extend the basic specification (20, 21) to include exchange rates, lagged interest rates and ‘policy shocks.’

4.1 Solution

What *is* a ‘solution?’ Since we take foreign and domestic consumption to be exogenous, it is just a stochastic process for domestic inflation and one for foreign inflation such that the nominal interest rates implied by the nominal pricing kernels, (17), are the same as those implied by the Taylor rules, (20) and (21). A process for the nominal exchange rate follows immediately by virtue of Equation (19).

We proceed as follows. Starting with the domestic country, we derive an expression for the *real* pricing kernel in terms of the model’s state variables, x_t and u_t . Next, we solve for domestic inflation and, therefore, obtain an endogenous expression for the domestic *nominal* pricing kernel. We can do this independently of the foreign country because (i) consumptions are exogenous, and (ii) there is no cross-country interaction in the Taylor rules (condition (ii) is relaxed in Appendix A). Next, we do the same things for the foreign country. Finally, we compute the nominal exchange rate as a ratio of the foreign and domestic nominal pricing kernels.

Following Hansen, Heaton, and Li (2008), we linearize the logarithm of the real pricing kernel, Equation (16):

$$-\log n_{t+1} = \delta^r + \gamma_x^r x_t + \gamma_u^r u_t + \lambda_x^r \sqrt{u_t} \epsilon_{t+1}^x + \lambda_u^r \sigma_u \epsilon_{t+1}^u \quad , \quad (22)$$

where

$$\begin{aligned} \gamma_x^r &= (1 - \rho)\varphi_x \quad , \quad \gamma_u^r = \frac{\alpha}{2}(\alpha - \rho)(\omega_x + 1)^2 \quad , \\ \lambda_x^r &= (1 - \alpha) - (\alpha - \rho)\omega_x \quad , \quad \lambda_u^r = -(\alpha - \rho)\omega_u \quad , \end{aligned} \quad (23)$$

where ω_x and ω_u are linearization coefficients, expressions for which are given (along with δ^r) in Appendix B. Following the affine term structure literature, we refer to $\gamma^r = [\gamma_x^r \ \gamma_u^r]^\top$ as ‘real factor loadings’ and to $\lambda^r = [\lambda_x^r \ \lambda_u^r]^\top$ as ‘real prices of risk.’ The one-period real interest rate is $r_t = -\log E_t n_{t+1} = \bar{r} + \gamma_x^r x_t + (\gamma_u^r - \frac{1}{2}(\lambda_x^r)^2)u_t$, where $\bar{r} = \delta^r - \frac{1}{2}(\lambda_u^r \sigma_u)^2$.

The Euler equation for the *nominal* one-period interest rate is

$$i_t = -\log E_t n_{t+1} e^{-\pi_{t+1}} \quad . \quad (24)$$

This, combined with the domestic Taylor rule (20), implies that a solution for endogenous inflation, $\pi(x_t, u_t)$, must solve the difference equation.

$$\pi_t = -\frac{1}{\tau_\pi} (\bar{\tau} + \tau_x x_t + \log E_t n_{t+1} e^{-\pi_{t+1}}) . \quad (25)$$

We guess that the solution is of the form

$$\pi_t = a + a_x x_t + a_u u_t \quad (26)$$

for coefficients a , a_x and a_u to be determined. The unique “minimum state variable” solution (McCallum (1981)) is

$$\begin{aligned} a_x &= \frac{(1 - \rho)\varphi_x - \tau_x}{\tau_\pi - \varphi_x} \\ a_u &= \frac{\frac{\alpha}{2}(\alpha - \rho)(\omega_x + 1)^2 - \frac{1}{2}\left((1 - \alpha) - (\alpha - \rho)\omega_x + a_x\right)^2}{\tau_\pi - \varphi_u} , \end{aligned}$$

with the (relatively inconsequential) solution for a relegated to Appendix B.2. Putting together Equations (22), (24) and (26), we arrive at what Gallmeyer, Hollifield, Palomino, and Zin (2007) call the “monetary-policy-consistent nominal pricing kernel:”

$$-\log m_{t+1} = \delta + \gamma_x x_t + \gamma_u u_t + \lambda_x \sqrt{u_t} \epsilon_{t+1}^x + \lambda_u \sigma_u \epsilon_{t+1}^u , \quad (27)$$

with (nominal) factor loadings and prices of risk

$$\delta = \delta^r + a + a_x(1 - \varphi_x)\theta_x + a_u(1 - \varphi_u) ;$$

$$\gamma_x = \gamma_x^r + a_x \varphi_x ; \quad \gamma_u = \gamma_u^r + a_u \varphi_u ;$$

$$\lambda_x = \lambda_x^r + a_x ; \quad \lambda_u = \lambda_u^r + a_u .$$

The nominal one-period interest rate is

$$\begin{aligned} i_t &\equiv -\log E_t(m_{t+1}) \\ &= \delta - \frac{1}{2}(\lambda_u \sigma_u)^2 + \gamma_x x_t + \left(\gamma_u - \frac{1}{2}\lambda_x^2\right)u_t . \end{aligned} \quad (28)$$

Analogous calculations for the foreign country yield solutions for n_{t+1}^* , π_{t+1}^* , m_{t+1}^* , r_t^* and i_t^* . We omit these calculations since they are almost identical, the only differences being that (i) the state variables, x_t^* and u_t^* are imperfectly correlated with x_t and u_t , and (ii) the foreign Taylor-rule coefficients, Υ^* , are distinct from their domestic counterparts, Υ . See Appendix B for explicit details.

4.2 Exchange Rates

Using Equation (19), the nominal depreciation rate is $d_{t+1} \equiv \log(S_{t+1}/S_t) = \log(m_{t+1}^*/m_{t+1})$, the difference between equation (27) and its analogous foreign counterpart. Using this, and following Section 3, the forward premium, expected depreciation rate and risk premium can be written as linear functions of the state variables:

$$\begin{aligned} f_t - s_t &= i_t - i_t^* = (\iota - \iota^*) + (\gamma_x x_t - \gamma_x^* x_t^*) + (\gamma_u - \frac{1}{2}\lambda_x^2)u_t - (\gamma_u^* - \frac{1}{2}(\lambda_x^*)^2)u_t^* \\ q_t &= (\delta - \delta^*) + (\gamma_x x_t - \gamma_x^* x_t^*) + (\gamma_u u_t - \gamma_u^* u_t^*) \\ p_t &= (\iota - \iota^*) - (\delta - \delta^*) - \frac{1}{2}(\lambda_x^2 u_t - (\lambda_x^*)^2 u_t^*) \quad , \end{aligned} \tag{29}$$

where $\iota \equiv \delta - \frac{1}{2}(\lambda_u \sigma_u)^2$ and $\iota^* \equiv \delta^* - \frac{1}{2}(\lambda_u^* \sigma_u^*)^2$.

The UIP coefficient (from Equation (7)) is $b = Cov(f_t - s_t, q_t) / Var(f_t - s_t)$. A useful reference point is the special case of $\varphi_x = 0$. If the Taylor rule parameters are symmetric (*i.e.*, $\Upsilon = \Upsilon^*$), then

$$b = \frac{\gamma_u}{\gamma_u - \frac{1}{2}\lambda_x^2} \quad .$$

Examining the above expressions for λ_u and λ_x , we see that, if volatility is positively auto-correlated ($\varphi_u > 0$), then, for given Taylor coefficients, sufficient conditions for $b < 0$ are (i) $\alpha < 0$, and (ii) $\rho - \alpha > 0$ and large enough, so that the representative agent has a *strong* preference for the early resolution of uncertainty. Our quantitative analysis demonstrates what ‘strong’ means in this context (it is in line with the norm in the asset pricing literature).

This last point highlights an important feature of our setup thus far. The extent to which the UIP coefficient is negative hinges on *preferences*. The sign of the UIP coefficient is driven by real exchange rate behavior, not by the properties of endogenous inflation. Indeed, in Appendix B.3 we show that (i) the *real* UIP coefficient — the slope coefficient of a regression of the real depreciation rate on the real interest rate differential — is unambiguously negative if $\rho > \alpha$, and (ii) the nominal UIP coefficient is typically greater than its real counterpart. Endogenous inflation, in other words, pushes us *toward* UIP. Exogenous inflation, in contrast, imposes no such restrictions. Papers such as Bansal and Shaliastovich (2013) and Lustig, Roussanov, and Verdelhan (2011) take the latter route and show that an exogenously calibrated inflation process is consistent with the nominal UIP deviations observed in the data. However, these papers assume — rather than endogenously derive — that inflation is driven by the same factors driving the real pricing kernel, and freely specify the sensitivity of inflation to those factors. We don’t have this freedom. The Euler equation and Taylor rule, together, tell us what this sensitivity must be.

Two related points are as follows. First, this is not a general feature of Taylor-rule-implied inflation. In Appendix A.2 we show that more elaborate Taylor rules — *e.g.*, incorporating exchange rates and/or lagged nominal interest rates — *can* generate a nominal UIP coefficient that is less than its real counterpart. Here, we use the simpler setting because it articulates

our main point most clearly. Second, suppose that our model *did* generate nominal deviations from UIP that were substantially different from real deviations. While this would support our cause — our view that monetary policy is important for exchange rate behavior — it would also give rise to an important empirical tension. We know that real and nominal exchange rates behave quite similarly (Mussa (1986)), and that real and nominal UIP regressions also look quite similar (Engel (2011) is a recent example). Were our model to generate big differences in nominal and real exchange rate behavior, it would thus contradict some important features of the data. The most obvious way around this — if one wants to continue down the road in which monetary policy plays an important role — is to consider environments in which there is some feedback between nominal variables/frictions and real exchange rates. This is beyond the scope of this paper.

5 Asymmetric Monetary Policy

What is the effect of asymmetries in the Taylor rule parameters, $[\tau \ \tau_\pi \ \tau_x]$, and $[\tau^* \ \tau_\pi^* \ \tau_x^*]$?

To answer this, we work with the log excess expected return on a forward contract that is long GBP and short USD. The definition of p_t above is the opposite. Therefore we work with $-p_t$.² From Equation (29), after slight rearrangement,

$$\begin{aligned} -p_t &= \frac{\lambda_u^2 - (\lambda_u^*)^2}{2} \sigma_u^2 + \frac{\lambda_x^2 u_t - (\lambda_x^* u_t^*)^2}{2} \\ &\approx \frac{\lambda_u^2 - (\lambda_u^*)^2}{2} \sigma_u^2 + \frac{\lambda_x^2 - (\lambda_x^*)^2}{2} u_t \end{aligned} \quad (30)$$

where the approximation is that the innovations in volatility are highly correlated across countries: $\eta_{u,u^*} \approx 1$. This captures the main point of Lustig, Roussanov, and Verdelhan (2011), the possibility that currency risk is best characterized as ‘asymmetric loadings on a global risk factor.’ Our calibration will reflect this. Note that, according to Equation (30), these asymmetric loadings (on the random factor) involve the prices of *consumption* risk, λ_x and λ_x^* , not the prices of volatility risk, λ_u and λ_u^* . This will play an important interpretive role.

Result 1: *Relatively procyclical monetary policy generates currency risk*

The risk premium on foreign currency is increasing in $(\tau_x^* - \tau_x)$, provided that $\tau_x^* > \tau_x$. In words, foreign currency risk is associated with a foreign policy rule that is relatively procyclical. The larger the difference, the greater the risk.

²We do so because it is more intuitive to say “this is the risk premium on holding *foreign* currency,” as opposed to the premium on holding domestic currency. Our notational convention for p_t , from Section 3, follows that which is common in the literature.

Result 2: *Relatively accommodative monetary policy generates currency risk*

The risk premium on foreign currency is increasing in $(\tau_\pi - \tau_\pi^*)$, provided that $\tau_\pi > \tau_\pi^*$ and that τ_x and τ_x^* are large enough. In words, provided that domestic and foreign policies are sufficiently procyclical, foreign currency risk is associated with a foreign policy rule that is relatively accommodative toward inflation. The larger the difference, the greater the risk.

Proofs are provided in Appendix C. Note that these two results are broadly consistent with historical evidence. German monetary policy, for example, has been widely perceived as having placed a relatively small weight (small τ_x) on the ‘dual mandate,’ and a relatively large weight (big τ_π) on *not* accommodating inflation. The German currency has typically been a ‘safe haven,’ paying a *negative* risk premium vis-a-vis countries with more procyclical, accommodative monetary policies. We provide empirical substantiation in Section 6.

5.1 Parametric Interpretation

We now explore the forces at work behind Results 1 and 2. Equation (30) indicates that the monetary policy parameters affect the risk premium through their effect on the prices of consumption risk, λ_x and λ_x^* and the prices of volatility risk, λ_u and λ_u^* (functional forms given in Section 4.1). We now hold the foreign parameter values $[\tau_\pi^* \tau_x^*]$ fixed and outline the relevant comparative static calculations involving the domestic parameter values $[\tau_\pi \tau_x]$. This will yield insights into how our model works and where the important empirical restrictions lie.

Recall that the nominal prices of risk are related to the real prices of risk as $\lambda_x = \lambda_x^r + a_x$ and $\lambda_u = \lambda_u^r + a_u$, and that the real prices of risk, λ_x^r and λ_u^r , are unaffected by the policy parameters. Inspection of the coefficients

$$\begin{aligned} a_x &= \frac{(1 - \rho)\varphi_x - \tau_x}{\tau_\pi - \varphi_x} \\ a_u &= \frac{\gamma_v^r - \lambda_x^2/2}{\tau_\pi - \varphi_u} \end{aligned}$$

indicates that any effect on the price of consumption risk, λ_x , affects also the price of volatility risk, λ_u , but not the reverse. Inspection of the risk premium (30) indicates that this is important for understanding its mean and variance. The premium has two pieces, a constant term involving λ_u and a random term involving λ_x . The price of consumption risk, therefore, affects both the mean and the variance of the premium, whereas the price of volatility risk affects only its mean. This will play an important interpretive role in what follows.

5.1.1 Effect of the Cyclical Parameter: τ_x

The risk premium (30) depends on the *squared* values of the prices of risk. Therefore the *signs* of λ_x and λ_u are important for understanding how the comparative static calculations $\partial\lambda_x/\partial\tau_x$ and $\partial\lambda_u/\partial\tau_x$ affect the premium (*i.e.*, the sign of $\partial x(y)^2/\partial y$ depends on the sign of $x(y)$). These signs are determined as follows.

First, the real prices of risk satisfy $\lambda_x^r > 0$ and $\lambda_u^r < 0$. This holds by virtue of a preference for the early resolution of uncertainty ($\alpha < \rho$) (see Equation (23)), a standard specification in the literature. Second, $a_x < 0$ if the autocorrelation in consumption growth is small and τ_x is ‘large enough.’ The former is an empirical fact (at the monthly frequency) and the latter is a driver of our calibration because it generates the observed negative correlation between inflation and consumption growth. Third, $a_u < 0$ if λ_x is large enough as is determined by $\gamma_v^r - (\lambda_x)^2/2 < 0$. A necessary condition for this to be true is that $\gamma_v^r - (\lambda_x^r)^2/2 < 0$. The latter condition is familiar in any affine model — sometimes called the “precautionary savings condition” — and, with symmetric coefficients, it is necessary in our model for a negative *real* UIP coefficient. Fourth, it is easily shown that $\partial\lambda_x/\partial\tau_x < 0$ and $\partial\lambda_u/\partial\tau_x > 0$. Putting these four conditions together (details provided in Appendix C.2) we can unambiguously conclude that $-p_t$, from Equation (30), satisfies $\partial(-p_t)/\partial\tau_x < 0$: a more procyclical domestic policy reduces the foreign currency risk premium.

To summarize, the point of these calculations is to better identify the mapping between sample moments of the data, the parameter values of our model, and Results 1 and 2. Result 1 states that currency risk is associated with a relatively procyclical monetary policy. A preference for the early resolution of uncertainty plays an important role. The pivotal sample moments turn out to be $Cov(x_{t+1}, x_t) \approx 0$, $Cov(x_t, \pi_t) < 0$ and $b < 0$. Each will play a prominent role in our calibration and our identification scheme for the policy parameters.

5.1.2 Effect of the Accommodative Parameter: τ_π

Given the above sign restrictions, most of the work here is done. All that remains is to compute the partial derivatives, $\partial\lambda_x/\partial\tau_\pi$ and $\partial\lambda_u/\partial\tau_\pi$. In Appendix C.2 we show that the former is positive but that the sign of the latter is ambiguous. This ambiguity is what underlies the dependence of Result 2 on τ_x being ‘large enough.’ In our calibration we find (we do not impose) that it is. In this case both derivatives are positive and we have the result that a relatively accommodative *foreign* monetary policy is associated with a risky *foreign* currency.

The main insights here are inherent in the above sign and preference parameter restrictions. For example, the covariance between inflation and consumption growth is once again pivotal for how τ_π affects the premium. One additional insight involves how τ_π affects the constant and the variable terms in the risk premium (30). A tighter domestic policy (large τ_π) increases the price of consumption risk, λ_x , thus amplifying the *variable* part of the premium. Where the ambiguity comes from is that, for relatively small values of τ_x , a tighter

policy decreases the price of volatility risk, thus muting the *constant* part of the premium. The constant part is only relevant for the mean risk premium. The variable part matters for both the mean (since $u_t > 0$) and the variance and, therefore, for the UIP coefficient. It will also play an important interpretive role when we explain the economic intuition in the next section. This intuition will involve how the *variance* of the foreign and domestic pricing kernels are affected by the policy parameters and will use the insights developed by Hansen and Jagannathan (1991).

5.2 Economic Interpretation

At the heart of the economic intuition lies the general expression for the risk premium, Equation (15), which applies to *any* lognormal model. We reproduce it here for clarity:

$$-p_t = \text{Var}_t(\log m_{t+1})/2 - \text{Var}_t(\log m_{t+1}^*)/2 . \quad (31)$$

Recalling that *minus* p_t is the risk premium on *foreign* currency, this expression says something that, at first blush, might seem counterintuitive. It says that the country with the *low* pricing-kernel variability is the country with the *risky* currency. Low variance ... high risk! At second blush, however, it makes perfect sense. It is a general characteristic of the “change-of-units risk” that distinguishes currency risk from other forms of risk. Change-of-units risk is a relative thing. It measures how I perceive the risk in unit-changing *relative* to how you perceive it.

To understand this, recall that the (log) depreciation rate is the difference between the two (log) pricing kernels. If they are driven by the same shocks (and if loadings are symmetric), then the conditional variances in Equation (31) are the same and currency risk is zero for both foreign and domestic investors. If, instead, domestic shocks are much more volatile than foreign shocks, then variation in the exchange rate and variation in the *domestic* pricing kernel are more-or-less the same thing. The domestic investor then views foreign currency as risky because its value is being dominated by the same shocks as is his marginal utility. The foreign investor, in contrast, feels *relatively* sanguine about exchange rate variation. It is *relatively* unrelated to whatever it is that is affecting his marginal utility. Hence, the risk premium on the foreign currency must be positive and the premium on the domestic currency (which, of course is the “foreign” currency for the foreign investor) must be negative. While the latter might seem counterintuitive — the foreign investor views currency as a *hedge* — it is inescapable. If GBP pays a positive premium then USD *must* pay a negative premium.³

³One should take care not to confuse this with “Siegel’s Paradox,” the statement that — because of the ubiquitous Jensen’s inequality term — the forward rate cannot equal the conditional mean of the future spot rate, irrespective of the choice of the currency numeraire. This *is not* what is going on here. The Jensen’s term is (one half of) the variance in the difference of the (log) kernels, not the difference in the variances from Equation (31). To make this crystal-clear, consider the case in which this entire discussion *would be* a futile exercise in Siegel’s Paradox. Suppose that the log kernels are independent of one-another with constant and identical conditional variances. Then the log risk premium, p_t is zero, and the level risk

Remember it like this; “high variability in marginal utility means low tolerance for exchange-rate risk and, therefore, a high risk premium on foreign currency.”

Now that we’ve established intuition for Equation (31) we can turn to monetary policy. Recall the basic definition of the nominal pricing kernel:

$$\begin{aligned} \log m_{t+1} &= \log n_{t+1} - \pi_{t+1} \\ \implies \text{Var}_t(\log m_{t+1}) &= \text{Var}_t(\log n_{t+1}) + \text{Var}_t(\pi_{t+1}) - 2\text{Cov}_t(\log n_{t+1}, \pi_{t+1}) . \end{aligned}$$

In Appendix B.2 we show something quite intuitive. If τ_x is large enough, then the covariance term is positive enough so that $\text{Var}_t(\log m_{t+1}) < \text{Var}_t(\log n_{t+1})$; the nominal pricing kernel is less variable than the marginal rate of substitution. This is not the case for any set of parameter values, but it is for those that arise in any of our calibrations. It is a fairly typical characteristic of most New Keynesian models and was first pointed out in our specific class of models by Gallmeyer, Hollifield, Palomino, and Zin (2007). It is also empirically-plausible in the sense that a large value for τ_x implies an endogenous inflation process that is negatively correlated with consumption growth — thus implying a positive correlation between inflation and marginal utility in Equation (32) — something we typically see in the data.

We find this implication of monetary policy to be interesting, irrespective of how things work out for exchange rates. What’s intriguing is that a policy which seeks to fulfill the “dual mandate” by reacting to real economic activity will typically generate *inflation risk*; a negative (positive) correlation between consumption growth (marginal utility) and inflation. This means that securities denominated in nominal units will have expected returns that incorporate an inflation risk premium. What it also means, however, is that *nominal risk is less than real risk*; the nominal pricing kernel is less variable than its real counterpart. Sharpe ratios on nominal risky assets will therefore tend to be *less* than those on real risky assets (Hansen and Jagannathan (1991)). Of course, we don’t observe data on the latter, but the implication seem interesting nevertheless.

Now for our main point. Given a sufficiently large value for τ_x , why does a relatively large value for τ_π translate into a larger *foreign* currency premium? Because it undoes the effect of τ_x . That is, as τ_π gets large, the variance of endogenous inflation decreases (*i.e.*, the as the central bank cares more about inflation it drives the variability of inflation to zero). Thus, the extent to which $\text{Var}_t(\log m_{t+1}) < \text{Var}_t(\log n_{t+1})$ is mitigated which, because $\text{Var}_t(\log n_{t+1})$ is exogenous in our setting, must mean that $\text{Var}_t(\log m_{t+1})$ increases. Thus the foreign-currency risk premium from Equation (31) increases.

Summarizing, then, the economic intuition goes as follows. A sufficiently procyclical interest rate rule makes the nominal economy less risky than the real economy in the sense that the nominal pricing kernel is less variable than the real pricing kernel. A stronger interest rate reaction to inflation *undoes* this. It makes domestic state prices more variable so that

premium is $-\text{Var}_t(d_{t+1})/2$, the familiar Siegel-Jensen term. The above discussion, and our model, presume no such independence nor homoscedasticity. The name-of-the-game is the covariance term, $\text{Cov}_t(d_{t+1}, m_{t+1})$ (or its foreign-kernel equivalent).

domestic residents view currency as being more risky *relative to* foreign residents. ‘Weak’ interest rate rules make for riskier currencies. This seems to accord with the data. The countries with (supposedly) strong anti-inflation stances — *e.g.*, Germany, Japan, Switzerland, the United States — have had, on average, low interest rates, low inflation and negative risk premiums.

6 Quantitative Results: U.S. and Australia

We now turn to a quantitative examination of our model, the main idea being to confirm the main mechanism outlined above, articulate it better, and to see how it behaves in a non-local manner.

Before beginning, it is important to note that, in its current form, there are certain dimensions of the joint distribution of consumption, exchange rates and interest rates that our model will necessarily perform poorly on. This is well known in the literature. The model’s primary limitation is that pointed out by Brandt, Cochrane, and Santa-Clara (2006) . It cannot account for the high observed volatility in pricing kernels (Hansen and Jagannathan (1991)) at the same time as the low observed cross-country correlation in consumption, without having ridiculously volatile exchange rates. The approach that we take throughout our paper is to ignore the cross-country-consumption evidence. We will maintain a large cross-country correlation. This allows us to ask our primary question — “How do Taylor rule asymmetries map into exchange rate risk premia?” — in an environment with realistic behavior in asset returns and exchange rates. In Appendix D we present an enhanced model that follows Bansal and Shaliastovich (2013) and Colacito and Croce (2011) by using long-run risk to overcome the Brandt, Cochrane, and Santa-Clara (2006) puzzle. Qualitatively almost all of what we emphasize in this section is unaffected. The enhanced model offers the same insights, but in a much more complex environment. Quantitatively, the enhanced model delivers what we’ll demonstrate here (and more), while at the same time having a realistic cross-country consumption correlation.

Our basic approach is as follows. First, we calibrate the real side of our model — preferences and the processes for foreign and domestic consumption — to match U.S. data on consumption, real interest rates and real exchange rates. We treat the foreign and domestic country symmetrically, so that, although the shocks are not the same across countries, the parameter values and real moments are. Second, given the real model, we calibrate the six Taylor rule parameters — three domestic parameters, $[\tau \tau_\pi \tau_x]$, and three foreign parameters, $[\tau^* \tau_\pi^* \tau_x^*]$ — to match nominal data from the U.S. and Australia. This country-pair accords well with our basic story of a low and high inflation/interest-rate pair of countries that can be associated with asymmetries in the Taylor parameters. Finally, we examine the moments of the model that were not equated to their sample counterparts via the calibration exercise. These moments are primarily related to exchange rates.

Figure 1 summarizes the nominal evidence that we focus upon. The top panel reports

the U.S.-Australia one-month interest rate differential and the nominal exchange rate, 1986-2012. Several things are readily apparent. First, Australia has typically had relatively high interest rates. The average Australian rate (U.S. rate) over the period was 7.25% (4.48%), for an average spread of 2.77%. No surprise, this has partly been a reflection of high Australian inflation, which averaged 3.67% versus 2.80% for the U.S., for a spread of 0.87%. More of a surprise has been the tendency for AUD to appreciate. Over the entire sample, the average monthly (log) appreciation rate has been just over 2% (annualized), while since 2000 it has been 3.75%.

For our question, this is all nicely summarized by the average excess return on a monthly carry trade that is (typically) long AUD, funded by borrowing in USD. The month-by-month payoffs, per unit AUD notional principal, are shown in the lower panel of Figure 1. We report both unconditional and conditional payoffs, the former being ‘always long AUD and short USD,’ whereas the latter conditions the long/short position on the predicted value from the UIP regression (details are described in Figure 1’s caption). The sample means — displayed in the figure as constant horizontal lines — are 0.0038 and 0.0069, with t-statistics of 2.0835 and 3.6793 respectively. These means translate into annualized excess returns and Shape ratios of 4.46% and 0.39 for the unconditional strategy and 7.93% and 0.71 for the conditional strategy. The latter — the conditional moments — are in-line with previous studies (perhaps a bit large), whereas the former are less so. Previous work (*e.g.*, Backus, Foresi, and Telmer (2001)) has estimated that the unconditional currency risk premium, $E(p_t)$, is zero, thus implying that all of the action is in the conditional distribution. More recently, however, papers such as Hassan (2010) and Lustig, Roussanov, and Verdelhan (2011) have documented some exceptions. The exceptions tend to be countries that are (i) relatively small in terms of global share of GDP, and (ii) typically the recipients of the carry-trade capital flows. Our focus on Australia, therefore, should not be viewed narrowly. The evidence is more comprehensive and we use Australia as an illustrative example.

6.1 Calibration of Real Variables

Our consumption calibration is fairly standard. We treat the two countries symmetrically — meaning that the parameter values are the same but that the realizations of the shocks are not — and calibrate using U.S. data. There are seven parameters: the unconditional mean and autocorrelation of the growth rate, (θ_x, φ_x) , the AR(1) parameters for volatility, $(\theta_u, \varphi_u, \sigma_u)$, and the cross-country correlation of the shocks, $(\eta_{x,x^*}, \eta_{u,u^*})$.

We proceed sequentially, starting with parameters that have isolated effects on the model’s moments, and continuing on to those that have increasingly inter-related effects. Table 1 summarizes. The unconditional mean of consumption growth, θ_x , is set to its monthly sample counterpart. The cross-country correlation in consumption growth is set to be very close to unity: $\eta_{x,x^*} = 0.99$. As discussed above, this is the major counterfactual aspect of the calibration of our simple model. We rectify this in the enhanced model of Appendix D and show that the insights afforded by the simple model are unaffected. The autocorrelation

in consumption growth is set to $\varphi_x = 0$, motivated in part by the empirical evidence and in part by a desire to remain consistent with the analytical results from above. The unconditional mean of volatility, θ_u , is set to match the sample standard deviation of consumption growth. The autocorrelation of volatility is taken from Bansal and Yaron (2004) (and others who use very similar values): $\varphi_u = 0.987$. This generates reasonable autocorrelation in our model’s real interest rate. Given these values for θ_u and φ_u , the conditional variance of volatility, σ_u^2 , is set to be as large as possible subject to the constraint that the probability of observing a negative realization of volatility does not exceed 5%. This constraint turns out to be quite binding in the sense that our economy requires high risk aversion in order to generate realistic variability in interest rates and asset returns.

Turning to preference parameters, we choose a standard value for $\rho = 1/3$ (an elasticity of intertemporal substitution of 1.5) and then choose risk aversion to match the volatility of the real interest rate. This results in $\alpha = -89.4$, admittedly very high but not without many recent precedents of papers seeking to examine issues conditional on having realistic asset returns (*e.g.*, Campbell (1996), Ludvigson, Lettau, and Wachter (2008), Piazzesi and Schneider (2007), Tallarini (2000)). Moreover, the enhanced model of Appendix D features low risk aversion via the inclusion of long-run-risk. Finally, we choose the time preference parameter, β and the cross-country correlation in volatility, η_{u,u^*} , to (simultaneously) match the sample average of the U.S. real interest rate and the volatility of the U.S.-Australian *real* exchange rate.

6.2 Calibration of Nominal Variables

We now arrive at the crux of our exercise, asking how the Taylor rule coefficients affect endogenous inflation, interest rates and exchange rates. We begin in what seems the most natural and disciplined way, calibrating to inflation only and then seeing what happens to interest rates and exchange rates. We calibrate the six Taylor-rule parameters, $[\tau \tau_\pi \tau_x]$ and $[\tau^* \tau_\pi^* \tau_x^*]$, to match the mean and variance of U.S. and Australian inflation, respectively, as well as the two correlations, $Corr(x_t, \pi_t)$ and $Corr(x_t^*, \pi_t^*)$. The motivation for the using these correlations derives from Section 5.1.1, where they played a key role in the comparative statics exercise. Here, our quantitative findings confirm the sense in which these correlations, in addition to the means and variances, are useful for identification. Based on numerical experimentation, we find that, for the economically-admissible region of the parameter space, the mapping between the six Taylor parameters and the six moments that we use for identification is unique.

Panel B of Table 2 reports results in the column labeled ‘Model I.’ The domestic and foreign inflation processes capture, by construction, what we want them to capture. Foreign inflation is higher on average and more volatile. The difference in the latter is small, but that is a reflection of the data (we’ll see shortly that the model performs substantially better if the inflation volatility spread is larger). The dynamics, on the other hand, are unrealistic. Theoretical inflation is much more highly autocorrelated, at roughly 0.90, than

its sample counterpart at 0.43 (monthly data).⁴ This reflects the following sequence of tensions. First, the data compel us to have near-iid consumption growth. However our real calibration emphasizes highly autocorrelated real interest rates (which then play a critical role in the highly-autocorrelated nature of nominal interest rates). This then compels us to have highly autocorrelated consumption volatility (something which, reassuringly, previous work has argued in favor of based on consumption data alone). Second, the autocorrelation of inflation is a convex combination of that of consumption growth and consumption volatility. The coefficients in this combination are basically the a_x and a_u coefficients from above. They are, in turn, governed by the Taylor-rule parameters. In order to have moderate autocorrelation in inflation, the weight on the consumption-growth term must be large, which must be manifest in a large value for τ_x , the cyclicity parameter. However, such a large value generates a correlation between consumption and inflation that is far too negative. We’ve chosen to match this correlation so, consequently, our model must have highly autocorrelated inflation. To summarize, the class of Taylor rules we consider, given the behavior of consumption growth, cannot simultaneously account for a slightly negative correlation between consumption and inflation and moderate autocorrelation observed in (recent) inflation data.

Turning to what we are really interested in, exchange rates, we see numbers that are consistent with Results 1 and 2 above, and therefore the overall message of our paper. What we find is that, given a foreign country with higher and more volatile inflation, the Taylor-rule coefficients that are (uniquely) associated with this generate (i) a realistic mean, volatility and autocorrelation in the nominal exchange rate, (ii) a negative UIP coefficient, and (iii) a positive unconditional (and therefore conditional) risk premium on the *foreign* currency. Our model’s weaknesses, obviously, are the magnitudes of the risk premia which are minuscule relative to the data. We now turn to two alternative calibrations that will yield insights into why this is the case.

6.2.1 Two Alternative Nominal Calibrations

Model II. FX Volatility and UIP Slope. The third column of Table 2 (labeled ‘Model II’) corresponds to the following modification of the above (‘Model I’) calibration. Instead of targeting the domestic and foreign inflation-consumption correlations, we target the volatility of the nominal currency depreciation rate and the UIP slope coefficient. This, again, leaves us with six parameters and six moments and, again, we find that the exercise is uniquely identified in the admissible region of the parameter space.

⁴Note that measuring the autocorrelation in U.S. inflation is somewhat problematic. Among other things, it seems to have dropped substantially in the post-Volcker years. Estimates based on quarterly data are in the neighborhood of 0.8-0.9 for the 70s and 80s, and drop to 0.0-0.4 in the 90s and 2000s. See, for example, Fuhrer (2009). Australian inflation is even more problematic, being unavailable at the monthly frequency. Our estimates in Table 1 are based on using quarterly data and then scaling things down by factors that match the ratio of U.S. quarterly-to-monthly inflation persistence.

What we find is that the model’s implications for the inflation-consumption correlation are not drastically altered — the foreign correlation goes from -0.3 down to -0.45 — and that this is primarily a reflection of a more procyclical foreign monetary policy, with τ_x^* increasing from 0.21 to 0.30 (see Table 1) and the other parameters largely unaffected. Encouragingly, the unconditional currency risk premium increases by a factor of 60, from 0.007 to 0.421. This is still an order of magnitude shy of the data, but it’s a lot of progress at a fairly small cost. The lack of monthly Australian consumption (and inflation) data makes it difficult to say exactly how small. What the model is telling us is that a high-inflation, high-currency-risk country is predicted to have a consumption-inflation correlation that is *relatively* more negative than its counterpart.

Model III. Unconditional Risk Premium and Sharpe Ratio. Models I and II tell us that asymmetries in the Taylor-rule coefficients move everything in the right direction, but not far enough. Here we ask how large the asymmetries must be to account for the risk premium, and at what cost? We abandon the target of the nominal exchange rate volatility and, instead target the unconditional risk premium of 4.60%. The other 5 moments remain as the mean and variance of domestic and foreign inflation and the UIP slope coefficient. In addition, we fix $\tau_x = 0.2$ because (i) we know that this will generate a realistic *domestic* inflation-consumption correlation and, (ii) unconstrained, this parameter wants to be negative, which is not economically interesting. The exercise, then, is not exactly identified. Nevertheless, we find that there is a unique global minimum in the admissible region of the parameter space.

The column of Table 2 labeled ‘Model III’ summarizes what we learn here. A realistically-large risk premium requires (i) a large ‘cyclicality differential’ of $(\tau_x^* - \tau_x) = (0.87 - 0.20)$, and (ii) a large ‘accommodation differential’ of $(\tau_\pi - \tau_\pi^*) = (4.43 - 1.25)$. Both are exact reflections of what the analytical results of Section 5 told us; a risky currency is one from a country with a relatively procyclical and relatively accommodative monetary policy. The counterfactual aspects of our model are simply how large these differentials need to be. Relative to the data, the model generates too large of a differential in average inflation and the volatility of inflation. U.S. and Australian inflation just don’t look different enough in order for Taylor-rule asymmetries to account for a large fraction of the currency risk premium. In addition, the model generates foreign inflation that is now close to *i.i.d.*, and has a correlation with consumption that is far too negative, at -0.94. We could ‘fix’ these things by allowing for asymmetries in preferences and/or the foreign and domestic consumption processes. However this would only serve to cloud our point: realistic asymmetries in Taylor rule parameters contribute to our understanding about what drives currency risk.

6.3 Implications

In spite of the quantitative limitations of our baseline model, what we take from Table 1 is support for our main mechanism and point. What this means is basically three things.

First, we *have not* imposed that $\tau_x < \tau_x^*$ and $\tau_\pi > \tau_\pi^*$: “the foreign policy is both more procyclical and more accomodative.” They are the outcome of the identification procedure which is driven by observed differences in inflation, the UIP coefficient, the covariance of inflation and consumption, and several measures of currency risk. Second, the calibrated values of the Taylor coefficients imply an *unconditional* risk premium on AUD that has the right sign and, depending on the calibration, a non-trivial magnitude. This lends credence to the comparative static exercise of Section 5; a country with a *tighter* monetary policy is predicted to have a currency with a *negative* risk premium. Finally, because the mapping between the Taylor coefficients and the unconditional premium is basically their effect on the variability of the nominal pricing kernel, there is also an effect on the *conditional* risk premium. This is apparent in Table 2 and in our final set of results, to which we now turn.

Figure 2 shows the effect of varying τ_x and τ_π , relative to their calibrated values from Table 1, holding fixed the foreign Taylor rule parameters. The objects of interest are the Sharpe ratios on unconditional and conditional ‘carry trade’ portfolios. By the former we simply mean ‘always long foreign currency, funded by borrowing in domestic currency.’ By the latter we mean ‘go long foreign currency, but only when the expected excess return is positive ... otherwise go short.’⁵ Formally, defining $rx_{t+1} \equiv s_{t+1} - f_t$ as the log excess return on foreign currency over domestic currency, the unconditional Sharpe ratio is $E(rx_{t+1})/Stdev(rx_{t+1})$. Its conditional counterpart is $E(rx_{t+1}I_t)/Stdev(rx_{t+1}I_t)$, where $I_t = 1$ if $E_t(s_{t+1} - f_t) > 0$ and $I_t = -1$ otherwise.

Figure 2 makes several interesting points:

1. At our calibrated value of $\tau_\pi \approx 4$, the Sharpe ratio on foreign currency is increasing in the parameter τ_π . This is true both conditionally and unconditionally. It captures and confirms our main result: a weak-policy country (low τ_π) is predicted to have a risky currency.
2. Although the immediate effect of the asymmetry $\tau_\pi > \tau_\pi^*$ is on the *unconditional* distribution — this *is* Result 2 — it also affects the conditional distribution, the focal point of almost all previous research on the UIP coefficient. The reason is coincident with the basic intuition offered in Section 5.2. *Ceteris paribus*, a larger τ_π *increases* unconditional domestic pricing-kernel variability because it reduces inflation variability. Since it has a very small effect on the variability in the conditional mean of the kernel, it must also increase the average conditional variance and, therefore, the *conditional* currency risk premium.
3. In Figure 2 the nominal Sharpe ratios exceed their real counterparts. Nominal currency risk is *larger* than real currency risk. This is true, in spite of the fact that nominal

⁵Strictly speaking, the language ‘carry trade,’ as used here, is inconsistent with its typical usage. Typically, it means ‘go long the high interest rate currency.’ However this will only coincide with the rule that says ‘go long the currency with the highest expected return’ under special circumstances, such as (i) the *unconditional* risk premium is zero, *and* the average interest rate spread is zero. Neither apply in our setting. We define ‘carry trade’ in terms of expected returns, not interest rate spreads.

pricing kernel variability is less than real pricing kernel variability (‘nominal-unit risk is less than real-unit risk’). Symbolically, the average *nominal* foreign currency risk premium, $E(\text{Var}_t \log m_{t+1} - \text{Var}_t \log m_{t+1}^*)$ exceeds its real counterpart, $E(\text{Var}_t \log n_{t+1} - \text{Var}_t \log n_{t+1}^*)$, in spite of the fact that $E(\text{Var}_t \log m_{t+1}) > E(\text{Var}_t \log n_{t+1})$ and $E(\text{Var}_t \log m_{t+1}^*) > E(\text{Var}_t \log n_{t+1}^*)$. This result — like Results 1 and 2 — seems intrinsic to having an endogenous model of inflation. It is saying something about the link between the real and nominal economy that would not be possible were one to exogenously specify inflation processes and append them to a model of the real pricing kernel.

4. The lower graph in Figure 2 shows that, holding τ_π fixed, an increase in τ_x *reduces* the Sharpe ratio. A stronger procyclicality in monetary policy makes currency risk *smaller*. Or, put in the language used in Section 5.2, the extent to which a tight inflation policy (a large value for τ_π) will undo the risk-stabilizing effect of a procyclical policy becomes mitigated. This suggests another tension in the so-called ‘dual mandate.’ Procyclical policies reduce nominal risk relative to real risk, but the inflation-stabilizing role of policy serves as an antagonist.

7 Conclusions

It is obvious that monetary policy affects exchange rates. Purchasing Power Parity is a good low-frequency model. What is less obvious is that monetary policy affects exchange rate *risk*. For this to be so, at least in the case of lognormal models, policy must interact with *volatility* in some way. Ours is a model of such an interaction. It is a model in which the monetary authority reacts to the same volatility shocks as does the private sector, so that such shocks become manifest in nominal interest rates and exchange rates. We’ve shown that a particular parameterization of this interaction can help explain data on currency risk, as well as offer some economic interpretation. The main limitation of our setup is that the interaction goes just one way. Policy reacts to volatility, but there is no sense in which volatility reacts, or is affected by, policy. Hence, volatility in *real* exchange rates is independent of monetary policy. This is clearly an important issue, but one we leave for future work.

We close with some broader observations and how they relate to our study. How is monetary policy related to the UIP puzzle? Ever since we’ve known about the apparent profitability of the currency carry trade people have speculated about a lurking role played by monetary policy. The story is that, for some reason, central banks find themselves on the short side of the trade, borrowing high yielding currencies to fund investments in low yielding currencies. In certain cases this has seemed almost obvious. It’s well known, for instance, that in recent years the Reserve Bank of India has been accumulating USD reserves and, at the same time, sterilizing the impact on the domestic money supply through contractionary open-market operations. Since Indian interest rates have been relatively high, this policy basically *defines* what it means to be on the short side of the carry trade. This leads one to ask if carry trade losses are in some sense a *cost* of implementing Indian monetary policy?

If so, is this a good policy? Is there some sense in which it is *causing* the exchange rate behavior associated with the carry trade?

Our paper’s questions, while related, are admittedly less ambitious than these speculations about India and related situations. What we’ve shown goes as follows. It is almost a tautology that we can represent exchange rates as ratios of nominal pricing kernels in different currency *units*:

$$\frac{S_{t+1}}{S_t} = \frac{n_{t+1}^* \exp(-\pi_{t+1}^*)}{n_{t+1} \exp(-\pi_{t+1})}.$$

It is less a tautology that we can write down sensible stochastic processes for these four variables that are consistent with the carry trade evidence.⁶ Previous work has shown that such processes have many parameters that are difficult to identify with sample moments of data. Our paper shows two things. First, that by incorporating a Taylor rule for interest rate behavior we reduce the number of parameters. Doing so is sure to deteriorate the model’s fit. But the benefit is lower dimensionality and parameters that are economically interpretable. Second, we’ve shown that some specifications of Taylor rules work and others don’t. Specifically, cross-country differences in Taylor-rule coefficients give rise to cross-country differences in how pricing kernels load on global sources of risk, and this works in certain dimensions. It also provides an economic interpretation of parameter asymmetries emphasized by previous work, work that is mostly statistical and econometric in nature. It also fits some basic empirical facts about cross-country differences in inflation and interest rates (and, presumably, policies), and is driven by estimates of Taylor-rule coefficients that are common in the literature. Moreover, such asymmetries seem intuitively plausible. It is clear that the U.S., for example, plays an ‘asymmetric’ role in the foreign exchange market. It seems natural to associate this special role with (i) U.S. monetary policy, and, therefore (ii) a differential response of the U.S. pricing kernel to a global shock.

Finally, it’s worth noting that India is much more the exception than the rule. Most central banks — especially if we limit ourselves to those from OECD countries — don’t have such explicit, foreign-currency related policies. However, many countries *do* use nominal interest rate targeting to implement domestic policy and, therefore, we can think about central banks and the carry trade in a *consolidated* sense. For example, in early 2004 the UK less U.S. interest rate differential was around 3%. Supposing that this was, to some extent, a policy *choice*, consider the open-market operations required to implement such policies. The Bank of England would be contracting its balance sheet — selling UK government bonds — while (at least in a relative sense) the Fed would be expanding its balance sheet by buying U.S. government bonds. If the infamous carry-trader is in between, going long GBP and short USD, then we can think of the Fed funding the USD side of the carry trade and the Bank of England providing the funds for the GBP side. In other words, the consolidated

⁶See, for example, Backus, Foresi, and Telmer (2001), Bakshi and Chen (1997), Bansal (1997), Brenna and Xia (2006), Frachot (1996), Lustig, Roussanov, and Verdelhan (2011), and Saá-Requejo (1994).

balance sheets of the Fed and Bank of England are short the carry trade and the carry-trader is, of course, long. In this sense, central banks and their interest-rate policies may be playing a more important role than is apparent by just looking at their foreign exchange reserves.

References

- Alvarez, Fernando, Andrew Atkeson, and Patrick J. Kehoe, 2008, If exchange rates are random walks, then almost everything we say about monetary policy is wrong, *Federal Reserve Bank of Minneapolis Quarterly Review* 32, 2–9.
- , 2009, Time-varying risk, interest rates, and exchange rates in general equilibrium, *Review of Economic Studies* 76, 851–878.
- Ang, Andrew, Sen Dong, and Monika Piazzesi, 2007, No-arbitrage taylor rules, Unpublished manuscript, Columbia University.
- Backus, David K., Silverio Foresi, and Christopher I. Telmer, 2001, Affine term structure models and the forward premium anomaly, *Journal of Finance* 56, 279–304.
- Backus, David K., Allan W. Gregory, and Christopher I. Telmer, 1993, Accounting for forward rates in markets for foreign currency, *Journal of Finance* 48, 1887–1908.
- Backus, David K., and Stanley E. Zin, 2008, Notes on identification of taylor rules, Unpublished notes.
- Bakshi, Gurdip, and Zhiwu Chen, 1997, Equilibrium valuation of foreign exchange claims, *Journal of Finance* 52, 799–826.
- Bansal, Ravi, 1997, An exploration of the forward premium puzzle in currency markets, *Review of Financial Studies* 10, 369–403.
- , and Ivan Shaliastovich, 2013, A long-run risks explanation of predictability puzzles in bond and currency markets, *Review of Financial Studies* 26, 1–33.
- Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run, a potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.
- Bekaert, Geert, 1994, Exchange rate volatility and deviations from unbiasedness in a cash-in-advance model, *Journal of International Economics* 36, 29–52.
- Bilson, John, 1981, The speculative efficiency hypothesis, *Journal of Business* 54, 435–452.
- Brandt, Michael W., John H. Cochrane, and Pedro Santa-Clara, 2006, International risk sharing is better than you think, or exchange rates are too smooth, *Journal of Monetary Economics* 53, 671–698.
- Brenna, Michael J., and Yihong Xia, 2006, International capital markets and foreign exchange risk, *Review of Financial Studies* 19, 753–795.
- Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski, and Sergio Rebelo, 2011, Do peso problems explain the returns to the carry trade?, *Review of Financial Studies* 24, 853–91.

- Campbell, John Y., 1996, Understanding risk and return, *Journal of Political Economy* 104, 298–345.
- Canova, Fabio, and Jane Marrinan, 1993, Profits, risk and uncertainty in foreign exchange markets, *Journal of Monetary Economics* 32, 259–286.
- Clarida, Richard, Jordi Galí, and Mark Gertler, 1999, The science of monetary policy: A new keynesian perspective, *Journal of Economic Literature* XXXVII, 1661–1707.
- Cochrane, John H., 2011, Determinacy and identification with taylor rules: A critical review, *Journal of Political Economy* 119, 565–615.
- Colacito, Ricardo, and Mariano Massimiliano Croce, 2011, Long run risks and the real exchange rate, *Journal of Political Economy* 119, 153–181.
- Dumas, Bernard, 1992, Dynamic equilibrium and the real exchange rate in a spatially separated world, *Review of Financial Studies* 5, 153–180.
- Dutton, John, 1993, Real and monetary shocks and risk premia in forward markets for foreign exchange, *Journal of Money, Credit and Banking* 25, 731–754.
- Eichenbaum, Martin S., and Charles L. Evans, 1995, Some empirical evidence on the effects of monetary policy shocks on exchange rates, *Quarterly Journal of Economics* 110, 975–1009.
- Engel, Charles, 2011, The real exchange rate, real interest rates, and the risk premium, Unpublished manuscript, University of Wisconsin.
- , and Kenneth D. West, 2006, Taylor rules and the deutschmark-dollar real exchange rate, *Journal of Money, Credit and Banking* 38, 1175–1194.
- Epstein, L. G., and Stanley Zin, 1989, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.
- Fama, Eugene F, 1984, Forward and spot exchange rates, *Journal of Monetary Economics* 14, 319–38.
- Frachot, Antoine, 1996, A reexamination of the uncovered interest rate parity hypothesis, *Journal of International Money and Finance* 15, 419–437.
- Fuhrer, Jeffrey C., 2009, Inflation persistence, Working paper No. 09-14, Federal Reserve Bank of Boston.
- Gallmeyer, Michael F., Burton Hollifield, Francisco Palomino, and Stanley E. Zin, 2007, Arbitrage-free bond pricing with dynamic macroeconomic models, Working paper, Carnegie Mellon University.

- Gavazzoni, Federico, 2008, Uncovered interest rate parity puzzle: An explanation based on recursive utility and stochastic volatility, Working Paper.
- Grilli, Vittorio, and Nouriel Roubini, 1992, Liquidity and exchange rates, *Journal of International Economics* 32, 339–352.
- Hansen, Lars Peter, John C. Heaton, and Nan Li, 2008, Consumption strikes back? measuring long-run risk, *Journal of Political Economy* 116, 260–302.
- Hansen, Lars P., and Ravi Jagannathan, 1991, Implications of security market data for models of dynamic economies, *Journal of Political Economy* 99, 225–262.
- Hansen, Lars Peter, and Kenneth Singleton, 1983, Stochastic consumption, risk aversion and the temporal behavior of stock returns, *Journal of Political Economy* 91, 249–265.
- Hassan, Tarek, 2010, Country size, currency unions, and international asset returns, Unpublished manuscript, University of Chicago.
- Hollifield, Burton, and Raman Uppal, 1997, An examination of uncovered interest rate parity in segmented international commodity markets, *Journal of Finance* 52, 2145–2170.
- Lochstoer, Lars, and Georg Kaltenbrunner, 2010, Long-run risk through consumption smoothing, *Review of Financial Studies* 23, 3141–3189.
- Lucas, Robert E, 1982, Interest rates and currency prices in a two-country world, *Journal of Monetary Economics* 10, 335–60.
- Ludvigson, Sydney C., Martin Lettau, and Jessica A. Wachter, 2008, The declining equity premium: What role does macroeconomic risk play?, *Review of Financial Studies* 21, 1653–1687.
- Lustig, Hanno N., Nikolai L. Roussanov, and Adrien Verdelhan, 2011, Common risk factors in currency markets, *Review of Financial Studies* 24, 3731–3777.
- Macklem, Tiff, 1991, Forward exchange rates and risk premiums in artificial economies, *Journal of International Money and Finance* 10, 365–391.
- Marshall, David, 1992, Phd thesis,, Carnegie Mellon University.
- McCallum, Bennett T., 1981, Price level determinacy with an interest rate policy rule and rational expectations, *Journal of Monetary Economics* 8, 319–329.
- , 1994, A reconsideration of the uncovered interest rate parity relationship, *Journal of Monetary Economics* 33, 105–132.
- Mehra, R, and E Prescott, 1985, The equity premium puzzle, *Journal of Monetary Economics* 15, 145–61.

- Mussa, Michael, 1986, Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications, *Carnegie-Rochester Series on Public Policy* 25 (Autumn): 117-214.
- Piazzesi, Monika, and Martin Schneider, 2007, *Equilibrium Yield Curves* (MIT Press: Cambridge MA) In *NBER Macroeconomics Annual*, Daron Acemoglu, Kenneth Rogoff, and Michael Woodford eds.
- Saá-Requejo, Jesús, 1994, The dynamics and the term structure of risk premium in foreign exchange markets, Unpublished manuscript, INSEAD.
- Schlagenhauf, Don E., and Jeffrey M. Wrase, 1995, Liquidity and real activity in a simple open economy model, *Journal of Monetary Economics* 35, 431–461.
- Sercu, Piet, Raman Uppal, and C. Van Hulle, 1995, The exchange rate in the presence of transactions costs: implications for tests of purchasing power parity, *Journal of Finance* 50, 1309–1319.
- Tallarini, Thomas D., 2000, Risk sensitive business cycles, *Journal of Monetary Economics* 45.
- Taylor, John B., 1993, Discretion versus policy rules in practice, *Carnegie Rochester Conference Series on Public Policy* 39, 195–214.
- Tryon, Ralph, 1979, Testing for rational expectations in foreign exchange markets, International Finance Discussion Paper #139, Board of Governors of the Federal Reserve System.
- Verdelhan, Adrien, 2010, A habit-based explanation of the exchange rate risk premium, *Journal of Finance* 65, 123–145.
- Woodford, Michael, 2003, *Interest and Prices* (Princeton University Press: Princeton NJ 08540 USA).

Table 1
Calibrated Parameter Values

Description	Parameter	Value		
Panel A: The Real Economy				
Discount factor	β	0.993		
Relative risk aversion	$1 - \alpha$	90.408		
Elasticity of intertemporal substitution	$(1 - \rho)^{-1}$	1.5		
Mean of consumption growth	θ_x	0.0015		
Autocorrelation of consumption growth	φ_x	0		
Cross-Country correlation in consumption innovations	η_{x,x^*}	0.999		
Mean volatility level	θ_u	$6.165e^{-5}$		
Autocorrelation of volatility	φ_u	0.987		
Volatility of volatility	σ_u	$6.000e^{-6}$		
Cross-Country correlation in volatility innovations	η_{u,u^*}	0.999		
Panel B: The Nominal Economy				
		<i>Model I</i>	<i>Model II</i>	<i>Model III</i>
Constant in the domestic interest rate rule	$\bar{\tau}$	-0.002	-0.002	-0.008
Constant in the foreign interest rate rule	$\bar{\tau}^*$	-0.002	-0.002	0.002
Domestic response to consumption growth	τ_x	0.198	0.194	0.200
Foreign response to consumption growth	τ_x^*	0.205	0.304	0.866
Domestic response to inflation	τ_π	1.968	1.965	4.423
Foreign response to inflation	τ_π^*	1.884	1.874	1.264

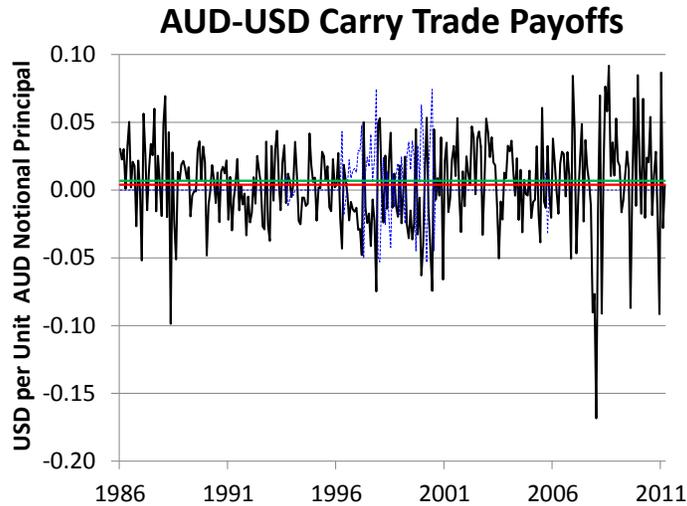
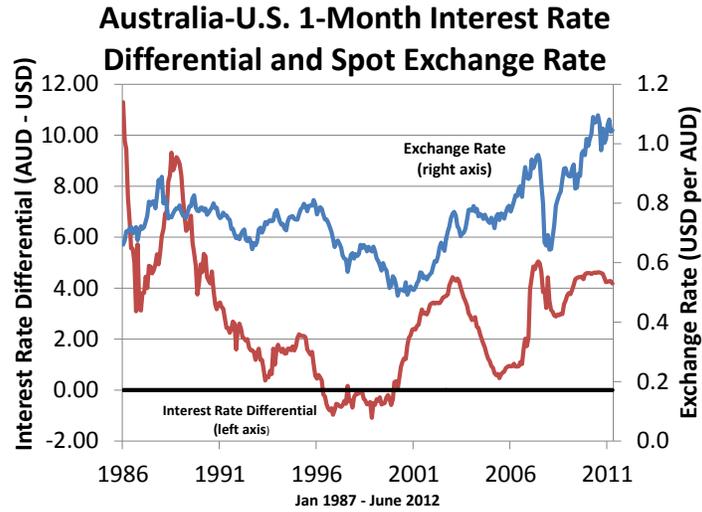
Table 1 reports the parameter values associated with the calibration exercise described in Sections 6.1 and 6.2. These parameter values underly the various population moments reported in Table 2. Table 2 reports sample moments in the second column and population moments from our model in the remaining columns. Sample moments derive from a variety of sources. The data frequency is monthly and, where appropriate the moments are reported as annualized percentages. The notation ‘-’ indicates a moment for which the data are either absent or unreliable. For example, we are not aware of a study that estimates the *real* Bilson-Fama coefficient using real interest rates (which are different than realized real returns on nominal bonds). Similarly, the unreliability of monthly U.S. consumption growth for ascertaining persistence is well known. Consumption moments that are reported are based on the standard monthly U.S. series. The cross-country consumption correlation is representative of data reported by Brandt, Cochrane, and Santa-Clara (2006). Real interest rate moments are taken from Lochstoer and Kaltenbrunner (2010). Data on foreign and domestic inflation are based on authors own calculations using monthly data from *Datastream*, 1987-2012. The foreign country is Australia whereas the domestic country is the U.S.. Note that the Australian inflation data is problematic relative to its U.S. counterpart. Among other things, it is only available at the quarterly frequency. The above estimates are based on using quarterly data and then scaling things down by factors that match the ratio of U.S. quarterly-to-monthly inflation moments. Calculations and data are available upon request.

Table 2
Sample and Population Moments

Moment	Data	Model		
<i>Panel A: The Real Economy</i>				
Consumption Growth (x_t, x_t^*)				
Mean	1.800	1.800		
Standard Deviation	2.720	2.720		
Autocorrelation	–	0		
Cross-Country Correlation	0.350	0.999		
Real Interest Rate (r_t, r_t^*)				
Mean	0.860	0.860		
Standard Deviation	0.970	0.970		
Autocorrelation	0.840	0.987		
Real Depreciation Rate ($\log(n_t^*/n_t)$)				
Standard Deviation	11.410	11.410		
Real UIP Coefficient	–	-53.486		
<i>Panel B: The Nominal Economy</i>				
Inflation (π_t, π_t^*)				
<i>Domestic, U.S.</i>				
Mean	2.833	<i>Model I</i>	<i>Model II</i>	<i>Model III</i>
Standard Deviation	0.911	2.833	2.834	2.833
Autocorrelation	0.428	0.911	0.914	0.294
Correlation(x_t, π_t)	-0.300	0.898	0.902	0.814
<i>Foreign, Australia</i>				
Mean	3.199	-0.300	-0.294	-0.418
Standard Deviation	0.985	3.199	3.199	3.199
Autocorrelation	0.429	0.985	0.985	1.964
Correlation(x_t^*, π_t^*)	-0.300	0.898	0.788	0.098
Nominal Interest Rate (i_t, i_t^*)				
<i>Domestic, U.S.</i>				
Mean	4.304	-0.300	-0.449	-0.949
Standard Deviation	2.584	4.304	3.786	3.773
Autocorrelation	0.992	3.786	3.773	3.820
<i>Foreign, Australia</i>				
Mean	7.076	4.159	4.559	8.213
Standard Deviation	3.558	1.771	1.648	0.784
Autocorrelation	0.994	0.987	0.987	0.987
Nominal Depreciation Rate ($\log(m_t^*/m_t)$)				
Mean	1.675	0.342	0.357	0.274
Standard Deviation	11.398	11.398	11.396	11.505
Autocorrelation	0.052	0.000	0.001	0.000
Nominal Currency Risk Variables				
Nominal UIP Coefficient	-1.019	-0.127	-1.019	-0.894
Uncond. Risk Premium on AUD, $-E(p_t)$	4.459	0.007	0.421	4.028
Unconditional Sharpe Ratio	0.389	0.001	0.039	0.361
Conditional Risk Premium on AUD	7.933	0.982	1.080	4.326
Conditional Sharpe Ratio	0.709	0.084	0.091	0.365

See caption for Table 1.

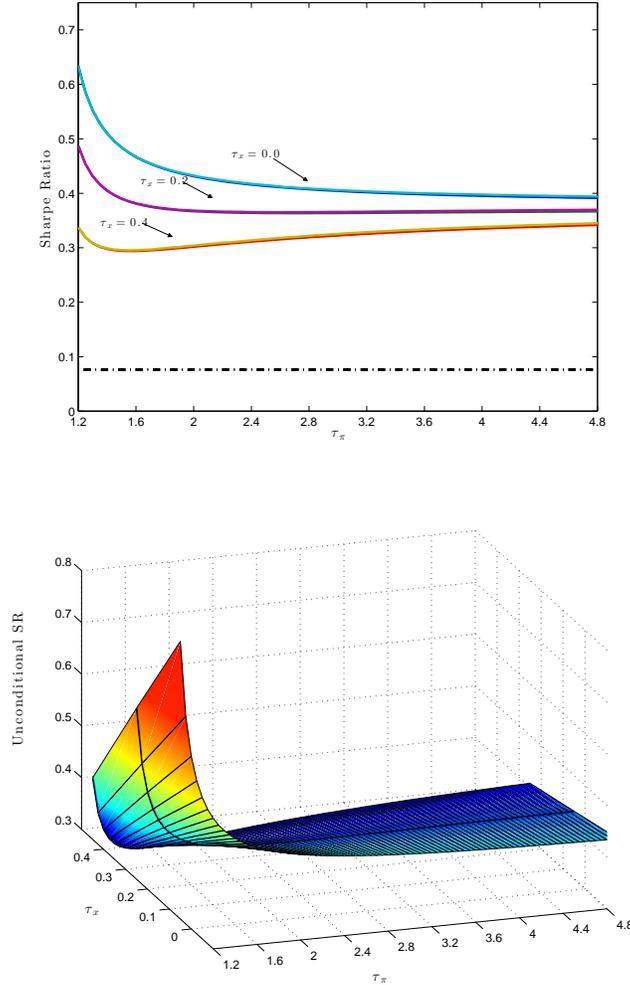
Figure 1
Australia, U.S. Interest Rates and Exchange Rates



The top graph shows the Australian less U.S. dollar 1-month eurocurrency interest rate differential and the USD/AUD spot exchange rate (price of Australian dollar in units of U.S. dollar). The bottom graph plots the month-by-month, continuously-compounded excess return associated with two “carry-trade” strategies. The first, plotted as a solid black line, is the ‘Unconditional Strategy:’ always long AUD and short USD. The second, plotted as a dashed blue line, is the ‘Conditional Strategy:’ long AUD whenever the estimated UIP regression function predicts that the excess return on AUD is positive, and short AUD otherwise (whenever the dashed-blue line is not visible, it is equal to the solid black line).

More specifically, the unconditional line is $s_{t+1} - s_t - (i_t - i_t^*)$ whereas the conditional line is $I_t[s_{t+1} - s_t - (i_t - i_t^*)]$ where $I_t = 1$ if $\hat{a} + (\hat{b} - 1)(i_t - i_t^*) > 0$ and $I_t = -1$ otherwise, and the parameter estimates are $\hat{a} = -0.00116$ and $\hat{b} = 1.17749$, based on monthly data, 1987-2012. The two solid, constant lines are the sample means of each strategy, with the conditional sample mean lying above its unconditional counterpart. The means (t-statistics) are, respectively, 0.0038 (2.0835) and 0.0069 (3.6793). The percentage-annualized means and Sharpe ratios are (4.46, 0.39) for the unconditional strategy and (7.93, 0.71) for the conditional strategy. If one omits the years 2007-2012 these values are (4.26, 0.44) and (8.44, 0.88), respectively. *Data Source:* Datastream.

Figure 2
Comparative Statics: Sharpe Ratios and Taylor Coefficients



Both graphs show how population Sharpe ratios on carry trade portfolios vary with the Taylor rule coefficients τ_π and τ_x , holding the corresponding foreign parameter values fixed at their values described by ‘Model III’ from Table 1. The top graph plots both unconditional and conditional *nominal* Sharpe ratios, as well as the conditional *real* Sharpe ratio (the unconditional real Sharpe ratio is zero by construction because the real side of the model does not feature any asymmetries, implying that the unconditional real risk premium is zero). The unconditional Sharpe ratio is defined as that on a monthly portfolio which is always long foreign currency funded by borrowing in domestic currency. It is $E(rx_{t+1})/Stdev(rx_{t+1})$, where $rx_{t+1} \equiv s_{t+1} - f_t$, the log excess return on foreign currency. The conditional Sharpe ratio is $E(rx_{t+1}I_t)/Stdev(rx_{t+1}I_t)$, where $I_t = 1$ if $E_t(s_{t+1} - f_t) > 0$ and $I_t = -1$ otherwise.

Supplemental Appendix

Monetary Policy and the Uncovered Interest Rate Parity Puzzle

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A Abstracting from Real Exchange Rates

A.1 A Simple Taylor Rule

The crux of our question asks “how does Taylor-rule-implied inflation affect exchange rates?” In this appendix we try to clarify things further by abstracting from real exchange rate variation. We set $n_t = n_t^*$, implying that $\log(S_t/S_{t-1}) = \pi_t - \pi_t^*$, so that relative PPP holds exactly. We don’t take this specification seriously for empirical analysis. We use it to try to understand exactly how the Taylor rule restricts inflation dynamics and, therefore, nominal exchange rate dynamics. As we’ll see in Appendix D, the lessons we learn carry over to more empirically-relevant models with both nominal and real variability.

We use simplest possible variant of the Taylor rule that allows us to generate time-varying risk premia:

$$i_t = \tau + \tau_\pi \pi_t + z_t , \tag{A1}$$

where z_t is a ‘policy shock’ that follows the process

$$z_{t+1} = (1 - \varphi_z)\theta_z + \varphi_z z_t + \sqrt{v_z}\epsilon_{t+1}^z \tag{A2}$$

$$v_{t+1} = (1 - \varphi_v)\theta_v + \varphi_v v_t + \sigma_v \epsilon_{t+1}^v , \tag{A3}$$

where ϵ^z and ϵ^v are *i.i.d.* standard normal. There are, of course, many alternative specifications. A good discussion related to asset pricing is Ang, Dong, and Piazzesi (2007). Cochrane (2011) uses a similar specification to address issues related to price-level determinacy and the identification of the parameters in Equation (A1). We begin with it for reasons of tractability and clarity. In Appendix A.2, we then go on to include the nominal depreciation rate and the lagged interest rate.

In addition to $n_t = n_t^*$, we abstract from real interest rate variation by setting $n_t = n_t^* = 1$. For exchange rates, conditional on having $n_t = n_t^*$, this is without loss of generality. The (nominal) short interest rate, $i_t = -\log E_t m_{t+1}$, is therefore

$$\begin{aligned} i_t &= -\log E_t e^{-\pi_{t+1}} \\ &= E_t \pi_{t+1} - \frac{1}{2} \text{Var}_t(\pi_{t+1}) . \end{aligned} \tag{A4}$$

The Taylor rule (A1) and the Euler equation (A4) imply that inflation must satisfy the following difference equation:

$$\pi_t = -\frac{1}{\tau_\pi} \left(\tau + z_t + E_t \pi_{t+1} - \frac{1}{2} \text{Var}_t(\pi_{t+1}) \right) . \tag{A5}$$

Given the log-linear structure of the model, guess that the solution has the form,

$$\pi_t = a + a_1 z_t + a_2 v_t . \quad (\text{A6})$$

Instead of solving Equation (A5) forward, just substitute Equation (A6) into the Euler equation (A4), compute the moments, and then solve for the a_i coefficients by matching up the result with the Taylor rule (A1). This gives,

$$\begin{aligned} a &= \frac{C - \tau}{\tau_\pi} \\ a_1 &= \frac{1}{\varphi_z - \tau_\pi} \\ a_2 &= \frac{1}{2(\varphi_z - \tau_\pi)^2(\varphi_v - \tau_\pi)} , \end{aligned}$$

where

$$C \equiv a + a_1 \theta_z (1 - \varphi_z) + a_2 \theta_v (1 - \varphi_v) - (a_2 \sigma_v)^2 / 2 .$$

It's useful to note that

$$a_2 = \frac{a_1^2}{2(\varphi_v - \tau_\pi)} .$$

Note that this is the same as saying that

$$\frac{\partial i_t}{\partial v_t} = \tau_\pi \frac{\partial \pi_t}{\partial v_t} = \frac{\partial E_t \pi_{t+1}}{\partial v_t} - \frac{1}{2} \frac{\partial \text{Var}_t \pi_{t+1}}{\partial v_t} .$$

Similarly, $a_1 = 1/(\varphi_z - \tau_\pi)$ is the same as saying that

$$\frac{\partial i_t}{\partial z_t} = \tau_\pi \frac{\partial \pi_t}{\partial z_t} + 1 = \frac{\partial E_t \pi_{t+1}}{\partial z_t} - \frac{1}{2} \frac{\partial \text{Var}_t \pi_{t+1}}{\partial z_t} .$$

Both of these things are kind of trivial. They just say that the effect of a shock on the Taylor rule equation must be consistent with the effect on the Euler equation.

Inflation and the short rate can now be written as:

$$\begin{aligned} \pi_t &= \frac{C - \tau}{\tau_\pi} + \frac{1}{\varphi_z - \tau_\pi} z_t + \frac{1}{2(\varphi_z - \tau_\pi)^2(\varphi_v - \tau_\pi)} v_t \\ i_t &= C + \frac{\varphi_z}{\varphi_z - \tau_\pi} z_t + \frac{\tau_\pi}{2(\varphi_z - \tau_\pi)^2(\varphi_v - \tau_\pi)} v_t \\ &= C + \varphi_z a_1 z_t + \tau_\pi a_2 v_t , \end{aligned}$$

and the pricing kernel as

$$\begin{aligned}
-\log m_{t+1} &= C + (\sigma_v a_2)^2/2 + a_1 \varphi_z z_t + a_2 \varphi_v v_t + a_1 v_t^{1/2} \epsilon_{t+1}^z + \sigma_v a_2 \epsilon_{t+1}^v \\
&= D + \frac{1}{\varphi_z - \tau_\pi} \varphi_z z_t + \frac{\varphi_v}{2(\varphi_z - \tau_\pi)^2 (\varphi_v - \tau_\pi)} v_t \\
&\quad + \frac{1}{\varphi_z - \tau_\pi} v_t^{1/2} \epsilon_{t+1}^z + \frac{\sigma_v}{2(\varphi_z - \tau_\pi)^2 (\varphi_v - \tau_\pi)} \epsilon_{t+1}^v \quad ,
\end{aligned} \tag{A7}$$

where

$$D \equiv C + (\sigma_v a_2)^2/2 \quad .$$

Now consider a foreign country. Denote all foreign variables with an asterisk. The foreign Taylor rule is

$$i_t^* = \tau^* + \tau_\pi^* \pi_t^* + z_t^* \quad .$$

with z_t^* and its volatility following processes analogous to Equations (A2–A3). For now, z_t and z_t^* can have any correlation structure. Repeating the above calculations for the foreign country and then substituting the results into Equations (12–15) in the main text we get

$$\begin{aligned}
i_t - i_t^* &= \varphi_z a_1 z_t - \varphi_z^* a_1^* z_t^* + \tau_\pi a_2 v_t - \tau_\pi^* a_2^* v_t^* \\
q_t &= D - D^* + a_1 \varphi_z z_t - a_1^* \varphi_z^* z_t^* + a_2 \varphi_v v_t - a_2^* \varphi_v^* v_t^* \\
p_t &= -\frac{1}{2} (a_1^2 v_t - a_1^{*2} v_t^* + \sigma_v^2 a_2^2 - \sigma_v^{*2} a_2^{*2}) \quad .
\end{aligned}$$

It is easily verified that $p_t + q_t = i_t - i_t^*$.

Result A.1 *Symmetry and $\varphi_z = 0$*

If all foreign and domestic parameter values are the same and $\varphi_z = \varphi_z^* = 0$, then the UIP regression parameter (8) is:

$$b = \frac{Cov(i_t - i_t^*, q_t)}{Var(i_t - i_t^*)} = \frac{Cov(p_t + q_t, q_t)}{Var(p_t + q_t)} \tag{A8}$$

$$= \frac{\varphi_v}{\tau_\pi} \tag{A9}$$

The sign of $Cov(p_t, q_t)$ does not depend on φ_z . That is, $Cov(p_t, q_t)$ is essentially the covariance between the kernel's mean and its variance and, while v_t appears in both, z_t appears only in the mean. The assumption $\varphi_z = 0$ is therefore relatively innocuous in the sense that it has no effect on one of the two necessary conditions (9) and (10).

We require $\tau_\pi > 1$ for the solution to make sense. Therefore, according to Equation (A9), $0 < b < 1$ unless $\varphi_v < 0$. The latter is implausible. Nevertheless, the UIP regression coefficient can be significantly less than unity and the joint distribution of exchange rates and interest rates will admit positive expected excess returns on a suitably-defined trading strategy.

We cannot, at this point, account for $b < 0$. But the model does deliver some insights into our basic question of how Taylor rules restrict inflation dynamics and, consequently, exchange rate dynamics. We summarize with several remarks.

Remark A.1.1: *This is not just a relabeled affine model*

Inspection of the pricing kernel, Equation (A7), indicates that it is basically a log-linear function of two unobservable factors. Is what we are doing just a relabeling of the class of latent-factor affine models described in Backus, Foresi, and Telmer (2001)? The answer is no and the reason is that the Taylor rule imposes economically-meaningful restrictions on the model's coefficients.

To see this consider a pricing kernel of the form

$$-\log m_{t+1} = \alpha + \chi v_t + \gamma v_t^{1/2} \varepsilon_{t+1} \quad (\text{A10})$$

where v_t is an arbitrary, positive stochastic process, and an analogous expression describes m_{t+1}^* . Backus, Foresi, and Telmer (2001) show that such a structure generates a UIP coefficient $b < 0$ if $\chi > 0$ and $\chi < \gamma^2/2$. The former condition implies that the mean and variance of negative the log kernel move in the same direction — this gives $\text{Cov}(p_t, q_t) < 0$ — and the latter implies that the variance is more volatile so that $\text{Var}(p_t) > \text{Var}(q_t)$.

Now compare Equations (A10) and (A7). The Taylor rule imposes the restrictions that χ can only be positive if φ_v is negative (because $a_2 < 0$ since $\tau_\pi > 1$) and that $\chi/\gamma = \varphi_v/(2\tau_\pi(\tau_\pi - \varphi_v))$. Both χ and γ are restricted by value of the policy parameter τ_π , and the dynamics of the volatility shocks. In words, the UIP evidence requires the mean and the variance of the pricing kernel to move in particular ways relative to each other. The Taylor rule and its implied inflation dynamics place binding restrictions on how this can happen. The unrestricted pricing kernel in Equation (A10) can account for $b < 0$ irrespective of the dynamics of v_t . Imposing the Taylor rule says that v_t must be negatively autocorrelated.

Remark A.1.2: *Reason that negatively-correlated volatility is necessary for $b < 0$?*

First, note that $a_2 < 0$, so that an increase in volatility v_t decreases inflation π_t . Why? Suppose not. Suppose that v_t increases. Then, since $\tau_\pi > 1$, the Taylor rule implies that the interest rate i_t must increase *by more* than inflation π_t . However this contradicts the stationarity of inflation which implies that the conditional mean must increase *by less* than the contemporaneous value. Hence $a_2 < 0$. A similar argument implies that $a_1 < 0$ from

Equation (A6). The point is that the dynamics of Taylor-rule implied inflation, at least as long as the real exchange rate is constant, are driven by the *muted response of the interest rate* to a shock, relative to that of the inflation rate.

Next, to understand why $\varphi_v < 0$ is necessary for $b < 0$, consider again an increase in volatility v_t . Since $a_2 < 0$, the U.S. interest rate i_t and the contemporaneous inflation rate π_t must decline. But for $b < 0$ USD must be expected to *depreciate*. This means that, although π_t decreases, $E_t\pi_{t+1}$ must increase. This means that volatility must be negatively autocorrelated.

Finally, consider the more plausible case of positively autocorrelated volatility, $0 < \varphi_v < 1$. Then $b < 1$ which is, at least, going in the right direction (*e.g.*, Backus, Foresi, and Telmer (2001) show that the vanilla Cox-Ingersoll-Ross model generates $b > 1$). The reasoning, again, derives from the ‘muted response of the interest rate’ behavior required by the Taylor rule. This implies that $Cov(p_t, q_t) > 0$ — thus violating Fama’s condition (9) — which says that if inflation and expected inflation move in the same direction as the interest rate (because $\varphi_v > 0$), then so must the USD currency risk premium. The Tryon-Bilson-Fama regression (7) can be written

$$q_t = c + b(p_t + q_t) - \text{forecast error} \ ,$$

where ‘forecast error’ is defined as $s_{t+1} - s_t - q_t$. Since $Cov(p_t, q_t) > 0$, then $Var(p_t + q_t) > Var(q_t)$ and, therefore, $0 < b < 1$.

Even more starkly, consider the case of $\varphi_v = 0$ so that $b = 0$. Then the exchange rate is a random walk — *i.e.*, $q_t = 0$ so that $s_t = E_t s_{t+1}$ — and all variation in the interest rate differential is variation in the risk premium, p_t . Taylor rule inflation dynamics, therefore, say that for UIP to be a good approximation, changes in volatility must show up strongly in the conditional mean of inflation and that this can only happen if volatility is highly autocorrelated.

Remark A.1.3: *Identification of policy parameters*

Cochrane (2011) provides examples where policy parameters like τ_π are impossible to distinguish from the parameters of the unobservable shocks. Result A.1 bears similarity to Cochrane’s simplest example. We can estimate b from data but, if we can’t estimate φ_v directly then there are many combinations of φ_v and τ_π that are consistent with any estimate of b .

Identification in our special case, however, is possible because of the conditional variance term in the interest rate equation: $i_t = E_t\pi_{t+1} - Var_t\pi_{t+1}$. To see this note that, with $\varphi_z = 0$, the autocorrelation of the interest rate is φ_v and, therefore, φ_v is identified by observables. Moreover,

$$\frac{i_t}{E_t\pi_{t+1}} = \frac{\tau_\pi}{\varphi_v} \ ,$$

which identifies τ_π because the variables on the left side are observable.

The more general case of $\varphi_z \neq 0$ doesn't work out as cleanly, but it appears that the autocorrelation of inflation and the interest rate jointly identify φ_z and φ_v and the above ratio again identifies the policy parameter τ_π . These results are all special cases of those described in Backus and Zin (2008).

A.2 Exotic Taylor Rules

Asymmetric Taylor Rules

Suppose that foreign and domestic Taylor rules depend on the exchange rate in addition to domestic inflation and a policy shock:

$$i_t = \tau + \tau_\pi \pi_t + z_t + \tau_3 \log(S_t/S_{t-1}) \quad (\text{A11})$$

$$i_t^* = \tau^* + \tau_\pi^* \pi_t^* + z_t^* + \tau_3^* \log(S_t/S_{t-1}) \quad (\text{A12})$$

The asymmetry that we'll impose is that $\tau_3 = 0$ so that the Fed does not react to the depreciation rate whereas, say, the Bank of England does. Foreign central banks reacting more to USD exchange rates seems plausible. It's also consistent with some empirical evidence in, for example, Clarida, Galí, and Gertler (1999), Engel and West (2006), and Eichenbaum and Evans (1995).

Assuming the same processes for the state variables as Equations (A2) and (A3) (and their foreign counterparts), guess that the inflation solutions look like:

$$\begin{aligned} \pi_t &= a + a_1 z_t + a_2 z_t^* + a_3 v_t + a_4 v_t^* \equiv a + A^\top X_t \\ \pi_t^* &= a^* + a_1^* z_t + a_2^* z_t^* + a_3^* v_t + a_4^* v_t^* \equiv a^* + A^{*\top} X_t \end{aligned}$$

where we collected the state variables into the vector

$$X_t^\top \equiv [z_t \ z_t^* \ v_t \ v_t^*]^\top .$$

Interest rates, from Euler equations with real interest rates equal to zero, must satisfy

$$\begin{aligned} i_t &= C + B^\top X_t \\ i_t^* &= C^* + B^{*\top} X_t \end{aligned}$$

where

$$\begin{aligned}
B^\top &\equiv \left[a_1\varphi_z \quad a_2\varphi_z^* \quad \left(a_3\varphi_v - \frac{a_1^2}{2}\right) \quad \left(a_4\varphi_v^* - \frac{a_2^2}{2}\right) \right] \\
C &\equiv a + a_1\theta_z(1 - \varphi_z) + a_2\theta_z^*(1 - \varphi_z^*) + a_3\theta_v(1 - \varphi_v) + a_4\theta_v^*(1 - \varphi_v^*) - \frac{1}{2}(a_3^2\sigma_v^2 + a_4^2\sigma_v^{*2}) \\
B^{*\top} &\equiv \left[a_1^*\varphi_z \quad a_2^*\varphi_z^* \quad \left(a_3^*\varphi_v - \frac{a_1^{*2}}{2}\right) \quad \left(a_4^*\varphi_v^* - \frac{a_2^{*2}}{2}\right) \right] \\
C^* &\equiv a^* + a_1^*\theta_z(1 - \varphi_z) + a_2^*\theta_z^*(1 - \varphi_z^*) + a_3^*\theta_v(1 - \varphi_v) + a_4^*\theta_v^*(1 - \varphi_v^*) - \frac{1}{2}(a_3^{*2}\sigma_v^2 + a_4^{*2}\sigma_v^{*2}) .
\end{aligned}$$

The Taylor rules become:

$$\begin{aligned}
i_t &= \tau + \tau_\pi(a + A^\top X_t) + z_t + \tau_3(a + A^\top X_t - a^* - A^{*\top} X_t) \\
&= \tau + \tau_\pi a + \tau_3(a - a^*) + (\tau_\pi A^\top + \iota_z^\top + \tau_3[A^\top - A^{*\top}])X_t \\
i_t^* &= \tau^* + \tau_\pi^*(a^* + A^{*\top} X_t) + z_t^* + \tau_3^*(a + A^\top X_t - a^* - A^{*\top} X_t) \\
&= \tau^* + \tau_\pi^* a^* + \tau_3^*(a - a^*) + (\tau_\pi^* A^{*\top} + \iota_z^{*\top} + \tau_3^*[A^\top - A^{*\top}])X_t ,
\end{aligned}$$

where $\iota_z^\top \equiv [1 \ 0 \ 0 \ 0]$ and $\iota_z^{*\top} \equiv [0 \ 1 \ 0 \ 0]$. Matching-up the coefficients means

$$\begin{aligned}
C &= \tau + \tau_\pi a + \tau_3(a - a^*) \\
C^* &= \tau^* + \tau_\pi^* a^* + \tau_3^*(a - a^*) \\
B &= \tau_\pi A^\top + \iota_z^\top + \tau_3(A^\top - A^{*\top}) \\
B^* &= \tau_\pi^* A^{*\top} + \iota_z^{*\top} + \tau_3^*(A^\top - A^{*\top})
\end{aligned}$$

To solve for the constants (the first two equations):

$$\begin{bmatrix} 1 - \tau_\pi - \tau_3 & \tau_3 \\ -\tau_3^* & 1 - \tau_\pi^* + \tau_3^* \end{bmatrix} \begin{bmatrix} a \\ a^* \end{bmatrix} = \begin{bmatrix} \tau - stuff \\ \tau^* - stuff^* \end{bmatrix}$$

where *stuff* and *stuff*^{*} are everything on the LHS of the solutions for *C* and *C*^{*}, except the first terms, *a* and *a*^{*}.

The *B* equations are eight equations in eight unknowns, *A* and *A*^{*}. Conditional on these, the *C* equations are two-in-two, *a* and *a*^{*}. The *B* equations can be broken into 4 blocks of

2. It's useful to write them out because you can see where the singularity lies:

$$\begin{aligned} \begin{bmatrix} (\tau_\pi + \tau_3 - \varphi_z) & -\tau_3 \\ \tau_3^* & (\tau_\pi^* - \tau_3^* - \varphi_z) \end{bmatrix} \begin{bmatrix} a_1 \\ a_1^* \end{bmatrix} &= \begin{bmatrix} -1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} (\tau_\pi + \tau_3 - \varphi_z^*) & -\tau_3 \\ \tau_3^* & (\tau_\pi^* - \tau_3^* - \varphi_z^*) \end{bmatrix} \begin{bmatrix} a_2 \\ a_2^* \end{bmatrix} &= \begin{bmatrix} 0 \\ -1 \end{bmatrix} \\ \begin{bmatrix} (\tau_\pi + \tau_3 - \varphi_v) & -\tau_3 \\ \tau_3^* & (\tau_\pi^* - \tau_3^* - \varphi_v) \end{bmatrix} \begin{bmatrix} a_3 \\ a_3^* \end{bmatrix} &= \begin{bmatrix} -a_1^2/2 \\ -a_1^{*2}/2 \end{bmatrix} \\ \begin{bmatrix} (\tau_\pi + \tau_3 - \varphi_v^*) & -\tau_3 \\ \tau_3^* & (\tau_\pi^* - \tau_3^* - \varphi_v^*) \end{bmatrix} \begin{bmatrix} a_4 \\ a_4^* \end{bmatrix} &= \begin{bmatrix} -a_2^2/2 \\ -a_2^{*2}/2 \end{bmatrix} . \end{aligned}$$

Two singularities exist:

- *UIP holds exactly.* If $\tau_3 = 0$ (so that the Fed ignores the foreign exchange rate), $\varphi_v = \varphi_v^*$ and $\tau_\pi = \tau_\pi^*$ (complete symmetry in parameters, save τ_3 and τ_3^*) then a singularity is $\tau_3^* = \tau_\pi - \varphi_v$. As τ_3^* approaches this from below or above, the UIP coefficient goes to 1.
- *Anomaly resolved.* Similarly, if $\tau_3 = 0$, $\varphi_v = \varphi_v^*$ and $\tau_\pi = \tau_\pi^*$ then a singularity is $\tau_3^* = \tau_\pi$. As τ_3^* approaches from *below*, the UIP coefficient goes to infinity. As τ_3^* approaches from *above*, it goes to *negative* infinity.

The latter condition is where the UIP regression coefficient changes sign. This says that we need $\tau_3^* > \tau_3$. This may seem pathological. It says that — if we interpret these coefficients as policy responses (which we shouldn't) — the Bank of England responds to an appreciation in GBP by increasing interest rates *more* than 1:1 (and more than the ‘Taylor principle’ magnitude of $\tau_\pi > 1$). We can now write the following result.

Result A.2: *Asymmetric reaction to exchange rates*

If foreign and domestic Taylor rules are Equations (A11) and (A12), with $\tau_3 = 0$ and all remaining foreign and domestic parameter values the same, then $b < 0$ if $\tau_3^* > \tau_\pi$.

Remark A.2.1: *Pathological policy behavior?*

Interpreted literally, $\tau_3^* > 0$ means that the Bank of England reacts to an *appreciation* in GBP by increasing the British interest rate. However, at the same time, there exist sensible calibrations of the model in which $Cov(i_t^*, \log(S_t/S_{t-1})) > 0$. This makes the obvious point that the Taylor rule coefficients must be interpreted with caution since all the endogenous variables in the rule are responding to the same shocks.

McCallum's Model

McCallum (1994), Equation (17), posits a policy rule of the form

$$i_t - i_t^* = \lambda(s_t - s_{t-1}) + \sigma(i_{t-1} - i_{t-1}^*) + \zeta_t ,$$

where ζ_t is a policy shock. He also defines UIP to include an exogenous shock, ξ_t , so that

$$i_t - i_t^* = E_t(s_{t+1} - s_t) + \xi_t .$$

McCallum solves the implicit difference equation for $s_t - s_{t-1}$ and finds that it takes the form

$$s_t - s_{t-1} = -\sigma/\lambda(i_t - i_{t-1}) - \lambda^{-1}\zeta_t + (\lambda + \sigma)^{-1}\xi_t$$

He specifies values $\sigma = 0.8$ and $\lambda = 0.2$ — justified by the policy-makers desire to smooth interest rates and ‘lean-into-the-wind’ regarding exchange rates — which resolve the UIP puzzle by implying a regression coefficient from our Equation (7) of $b = -4$. McCallum’s insight was, recognizing the empirical evidence of a risk premium in the interest rate differential, to understand that the policy rule and the equilibrium exchange rate must respond to the same shock that drives the risk premium.

In this section we show that McCallum’s result can be recast in terms of our pricing kernel model and a policy rule that targets the interest rate itself, not the interest rate differential. The key ingredient is a lagged interest rate in the policy rule:

$$i_t = \tau + \tau_\pi \pi_t + \tau_4 i_{t-1} + z_t , \tag{A13}$$

where the processes for z_t and its volatility v_t are the same as above. Guess that the solution for endogenous inflation is:

$$\pi_t = a + a_1 z_t + a_2 v_t + a_3 i_{t-1} , \tag{A14}$$

Substitute Equation (A14) into the pricing kernel and compute the expectation:

$$i_t = \frac{1}{1 - a_3} \left(C + a_1 \varphi_z z_t + (a_2 \varphi_v - a_1^2/2) v_t \right) ,$$

where

$$C \equiv a + a_1 \theta_z (1 - \varphi_z) + a_2 \theta_v (1 - \varphi_v) - (a_2 \sigma_v)^2 / 2 .$$

Match-up the coefficients with the Taylor rule and solve for the a_j parameters:

$$\begin{aligned}
a &= \frac{C}{\tau_\pi + \tau_4} - \frac{\tau}{\tau_\pi} \\
a_1 &= \frac{\tau_\pi + \tau_4}{\tau_\pi(\varphi_z - \tau_\pi - \tau_4)} \\
a_2 &= \frac{(\tau_\pi + \tau_4)^2}{2\tau_\pi^2(\varphi_z - \tau_\pi - \tau_4)^2(\varphi_v - \tau_\pi - \tau_4)} \\
a_3 &= -\frac{\tau_4}{\tau_\pi} .
\end{aligned}$$

It's useful to note that

$$a_2 = \frac{a_1^2}{2(\varphi_v - \tau_\pi - \tau_4)}$$

and that matching coefficients imply

$$\frac{a_1\varphi_z}{1 - a_3} = 1 + \tau_\pi a_1 \quad ; \quad \frac{a_2\varphi_v - a_1^2/2}{1 - a_3} = \tau_\pi a_2 .$$

Inflation and the short rate are:

$$\begin{aligned}
\pi_t &= \frac{C}{\tau_\pi + \tau_4} - \frac{\tau}{\tau_\pi} + \frac{\tau_\pi + \tau_4}{\tau_\pi(\varphi_z - \tau_\pi - \tau_4)} z_t + \\
&\quad + \frac{(\tau_\pi + \tau_4)^2}{2\tau_\pi^2(\varphi_z - \tau_\pi - \tau_4)^2(\varphi_v - \tau_\pi - \tau_4)} v_t - \frac{\tau_4}{\tau_\pi} i_{t-1} \\
i_t &= \frac{\tau_\pi}{\tau_\pi + \tau_4} C + \frac{\varphi_z}{\varphi_z - \tau_\pi - \tau_4} z_t + \frac{(\tau_\pi + \tau_4)^2}{2\tau_\pi(\varphi_z - \tau_\pi - \tau_4)^2(\varphi_v - \tau_\pi - \tau_4)} v_t \\
&= \frac{1}{1 - a_3} \left(C + \varphi_z a_1 z_t + (\tau_\pi + \tau_4) a_2 v_t \right) .
\end{aligned}$$

The pricing kernel is

$$-\log m_{t+1} = D + \frac{a_1\varphi_z}{1 - a_3} z_t + \frac{a_2\varphi_v - a_3 a_1^2/2}{1 - a_3} v_t + a_1 v_t^{1/2} \epsilon_{t+1}^z + \sigma_v a_2 \epsilon_{t+1}^v$$

where

$$D \equiv \frac{C}{1 - a_3} + (\sigma_v a_2)^2/2 .$$

The foreign currency denominated kernel and variables are denoted with asterisks. If we assume that all foreign and domestic parameter values are the same, the interest-rate differ-

ential, the expected depreciation rate, q_t , and the risk premium, p_t , are:

$$\begin{aligned} i_t - i_t^* &= \frac{a_1 \varphi_z}{1 - a_3} (z_t - z_t^*) + \frac{a_2 \varphi_v - a_1^2/2}{1 - a_3} (v_t - v_t^*) \\ q_t &= \frac{a_1 \varphi_z}{1 - a_3} (z_t - z_t^*) + \frac{a_2 \varphi_v - a_3 a_1^2/2}{1 - a_3} (v_t - v_t^*) \\ p_t &= -\frac{1}{2} a_1^2 (v_t - v_t^*) . \end{aligned}$$

It is easily verified that $p_t + q_t = i_t - i_t^*$.

The nominal interest rate and the interest rate differential have the same autocorrelation:

$$\begin{aligned} \text{Corr}(i_{t+1}, i_t) &= \text{Corr}(i_{t+1} - i_{t+1}^*, i_t - i_t^*) \\ &= 1 - (1 - \varphi_z)(1 + \tau_\pi a_1)^2 \frac{\text{Var}(z_t)}{\text{Var}(i_t)} - (1 - \varphi_v)(\tau_\pi a_2)^2 \frac{\text{Var}(v_t)}{\text{Var}(i_t)} . \end{aligned}$$

If we set $\varphi_z = 0$, then the regression parameter is:

$$\begin{aligned} b &= \frac{\text{Cov}(i_t - i_t^*, q_t)}{\text{Var}(i_t - i_t^*)} \\ &= \frac{\varphi_v - \tau_4}{\tau_\pi} \end{aligned}$$

To see the similarity to McCallum's model define $\zeta \equiv z_t - z_t^*$, and subtract the foreign Taylor rule from its domestic counterpart in (A13). Assuming symmetry, we get

$$\begin{aligned} i_t - i_t^* &= \tau_\pi (\pi_t - \pi_t^*) + \tau_4 (i_t - i_t^*) + \zeta_t \\ &= \tau_\pi (s_t - s_{t-1}) + \tau_4 (i_t - i_t^*) + \zeta_t , \end{aligned}$$

where the second equality follows from market completeness and our simple pricing kernel model. This is the same as McCallum's policy rule with $\tau_\pi = \lambda$ and $\tau_4 = \sigma$. His UIP "shock" is the same as our $p_t = -a_1^2(v_t - v_t^*)/2$, with $\varphi_z = \varphi_v = 0$. With $\varphi_v = 0$ we get the same UIP regression coefficient, $-\tau_4/\tau_\pi$. McCallum's model is basically a two-country Taylor rule model with a lagged interest rate in the policy rule and no dynamics in the shocks. Allowing for autocorrelated volatility diminishes the model's ability to account for a substantially negative UIP coefficient, a feature that McCallum's approach does not recognize. A value of $b < 0$ can only be achieved if volatility is less autocorrelated than the value of the interest rate smoothing policy parameter.

A.3 Summary

The goal of this section has been to ascertain how the imposition of a Taylor rule restricts inflation dynamics and how these restrictions are manifest in the exchange rate. What have we learned?

A good context for understanding the answer is the Alvarez, Atkeson, and Kehoe (2008) (AAK) paper. The nuts and bolts of their argument goes as follows. With lognormality, the nominal interest is

$$i_t = -E_t(\log m_{t+1}) - \text{Var}_t(\log m_{t+1})/2 .$$

AAK argue that if exchange rates follow a random walk then variation in the conditional mean term must be small.¹ Therefore (according to them), “almost everything we say about monetary policy is wrong.” The idea is that, in many existing models, the monetary policy transmission mechanism works through its affect on the conditional mean of the nominal marginal rate of substitution, m_t . But if exchange rates imply that the conditional mean is essentially a constant — so that ‘everything we say is wrong’ — then the mechanism must instead be working through the conditional variance.

If one takes the UIP evidence seriously, this isn’t quite right. The UIP puzzle *requires* variation in the conditional means (*i.e.*, it says that exchange rates *are not* a random walk).² Moreover, it also requires that this variation be negatively correlated with variation in the conditional variances, and that the latter be larger than the former. In terms of monetary policy the message is that the standard story — that a shock that increases the mean (of the marginal rate of substitution) *decreases* the interest rate — is wrong. The UIP evidence says that we need to get used to thinking about a shock that increases the mean as *increasing* the interest rate, the reason being that the same shock must *decrease* the variance, and by more than it increases the mean.

Now, to what we’ve learned. We’ve learned that symmetric monetary policies as represented by Taylor rules of the form (A1) can’t deliver inflation dynamics that, by themselves, satisfy these requirements. The reason is basically what we label the ‘muted response of the short rate’. The evidence requires that the conditional mean of inflation move by more than its contemporaneous value. But the one clear restriction imposed by the Taylor rule — that the interest rate must move *less* than contemporaneous inflation because the interest rate

¹*i.e.*, random walk exchange rates mean that $E_t \log(S_{t+1}/S_t) = 0$, and, from Equation (13), $E_t \log(S_{t+1}/S_t) = -E_t(\log m_{t+1} - \log m_{t+1}^*)$. Random walk exchange rates, therefore, imply that the *difference* between the mean of the log kernels does not vary, not the mean of the log kernels themselves. More on this below.

²Of course, the variation in the forecast error for exchange rates dwarfs the variation in the conditional mean (*i.e.*, the R^2 from the Tryon-Bilson-Fama-regressions is very small). Monthly changes in exchange rates certainly exhibit ‘near random walk’ behavior, and for policy questions the distinction may be a second-order effect. This argument, however, does not affect our main point regarding the AAK paper: that exchange rates are all about *differences* between pricing kernels and its hard to draw definitive conclusions about their *levels*.

must also be equal to the conditional mean future inflation — says that this can’t happen (unless volatility is negatively autocorrelated).

This all depends heavily on the real interest rate being a constant. What’s going on is as follows. In general, the Euler equation and the simplest Taylor rule can be written as

$$i_t = r_t + E_t \pi_{t+1} - \frac{\text{Var}_t(\pi_{t+1})}{2} + \text{Cov}_t(n_{t+1}, \pi_{t+1}) \quad (\text{A15})$$

$$\dot{i}_t = \tau + \tau_\pi \pi_t + z_t \quad (\text{A16})$$

The Euler equation (A15) imposes restrictions between the current short rate and *moments* of future inflation. The Taylor rule (A16) imposes an additional *contemporaneous* restriction between the current interest rate and current inflation. To see what this does, first ignore the real parts of Equation (A15), r_t and the covariance term. Recalling that endogenous inflation will be a function $\pi(z_t, v_t)$, consider a shock to volatility that increases inflation by 1%.³ The Taylor rule says that i_t must increase by *more than* 1%, say 1.2%. But, if inflation is a positively autocorrelated stationary process, then its conditional mean, $E_t \pi_{t+1}$, must increase by *less than* 1%, say 0.9%. Equation (A15) says that the only way this can happen is if the conditional variance *decreases* by 0.2%; a volatility shock that increases π_t must decrease $\text{Var}_t \pi_{t+1}$. Therefore the mean and variance of the pricing kernel must move in the *same* direction, thus contradicting what Fama (1984) taught us is necessary for $b < 0$.

Phrased in terms of the exchange rate, the logic is equally intuitive. The increase in the conditional mean of inflation implies an expected devaluation in USD — recall that relative PPP holds if we ignore real rates — which, given the increasing interest rate implied by the Taylor rule, moves us in the UIP direction: high interest rates associated with a devaluing currency. Note that, if volatility were negatively autocorrelated, $E_t \pi_{t+1}$ would *fall* and the reverse would be true; we’d have $b < 0$.⁴

So, the contemporaneous restriction implied by the Taylor rule is very much a binding one for our question. This points us in two directions. First, it suggests that an interaction with the real interest rate is likely to be important. None of the above logic follows if r_t and $\text{Cov}_t(n_{t+1}, \pi_{t+1})$ also respond to a volatility shock. We follow this path in the main text. Second it points to something else that the AAK story doesn’t get quite right. Exchange rate behavior tells us something about the *difference* between the domestic and foreign pricing kernels, not necessarily something about their *levels*. The above logic, and AAK’s logic, is about levels, not differences. Symmetry makes the distinction irrelevant, but with asymmetry it’s important. What our asymmetric example delivers is (i) inflation dynamics that, in each currency, satisfies ‘muted response of the short rate’ behavior, and (ii) a *difference* in inflation

³A shock to z_t isn’t particularly interesting in this context because it doesn’t affect both the mean and variance of the pricing kernel.

⁴This intuition is also useful for understanding why we get $0 < b < 1$ with positively autocorrelated volatility. The RHS of the regression, the interest rate spread, contains *both* the mean and the variance of inflation. The LHS contains only the mean. If (negative) the mean and the variance move in the same direction, then the RHS is moving more than the LHS and the population value of b is less than unity.

dynamics that gets the *difference* in the mean and the variance of the kernels moving in the right direction.

To see this, recall that $X_t^\top \equiv [z_t \ z_t^* \ v_t \ v_t^*]^\top$ and consider the foreign and domestic pricing kernels in the asymmetric model:

$$\begin{aligned} -\log m_{t+1} &= \text{constants} + a_1\varphi_z z_t + a_3\varphi_v v_t + a_1 v_t^{1/2} \epsilon_{t+1}^z + a_3 \sigma_v \epsilon_{t+1}^v \\ -\log m_{t+1}^* &= \text{constants} + A^\top \Lambda X_t + V(X_t)^{1/2} [\epsilon_{t+1}^z \ \epsilon_{t+1}^{z^*} \ \epsilon_{t+1}^v \ w_{t+1}^*]^\top, \end{aligned}$$

where Λ is a diagonal matrix of autoregressive coefficients, and $V(X_t)$ is a diagonal matrix of conditional standard deviations. The asymmetric restriction that $\tau_3 = 0$ and $\tau_3^* \neq 0$ effectively makes this a ‘common factor model’ with asymmetric loadings on the common factors. A number of recent papers, Lustig, Roussanov, and Verdelhan (2011) for example, have argued persuasively for such a specification. What we’ve developed is one economic interpretation of their statistical exercise.⁵

More explicitly, consider the *difference* in the mean and variance of the log kernels from the symmetric and asymmetric examples of Appendix A.1–A.2. For the symmetric case we have

$$\begin{aligned} p_t &= -\frac{1}{2}a_1^2(v_t - v_t^*) \\ q_t &= a_2\varphi_v(v_t - v_t^*) \end{aligned}$$

whereas for the asymmetric case we have

$$\begin{aligned} p_t &= -\frac{1}{2}(a_1^2 - a_1^{*2})v_t + \frac{1}{2}a_4^*v_t^* \\ q_t &= \varphi_v(a_3 - a_3^*)v_t - a_4^*v_t^*, \end{aligned}$$

where the a coefficients are functions of the model’s parameters. What’s going on in the symmetric case is transparent. p_t and q_t can only be negatively correlated if $\varphi_v < 0$ (since $a_2 < 0$). The asymmetric case is more complex, but it turns out that what’s critical is that $(a_3 - a_3^*) < 0$. This in turn depends on the difference $(\tau_\pi - \tau_3^*)$ being negative. Overall, what the asymmetric Taylor rule does is that it introduces an asymmetry in how a common factor between m and m^* affect their conditional means. This asymmetry causes the common factor to show up in exchange rates, and it can also flip the sign and deliver $b < 0$ with the right combination of parameter values.

⁵Note that if the conditional mean coefficients on z_t and v_t were the same across m and m^* then, contrary to AAK’s assertion, monetary policy *could* affect the mean of the pricing kernel while still allowing for a random walk exchange rate. This is simply because z_t and v_t *would not appear* in the difference between the means of the two log kernels.

B The Model

B.1 Linearization of the Pricing Kernel

The log of the equilibrium domestic marginal rate of substitution in Equation (16) is given by

$$\log(n_{t+1}) = \log \beta + (\rho - 1)x_{t+1} + (\alpha - \rho)[\log W_{t+1} - \log \mu_t(W_{t+1})] , \quad (\text{B1})$$

where $x_{t+1} \equiv \log(c_{t+1}/c_t)$ is the log of the ratio of domestic observed consumption in $t + 1$ relative to t and W_t is the value function. The first two terms are standard expected utility terms: the pure time preference parameter β and a consumption growth term times the inverse of the negative of the intertemporal elasticity of substitution. The third term in the pricing kernel is a new term coming from recursive preferences.

We work on a linearized version of the real pricing kernel, following the findings of Hansen, Heaton, and Li (2008). In particular, we focus on the the value function of each representative agent, scaled by the observed equilibrium consumption level

$$\begin{aligned} W_t/c_t &= [(1 - \beta) + \beta(\mu_t(W_{t+1})/c_t)^\rho]^{1/\rho} \\ &= \left[(1 - \beta) + \beta \mu_t \left(\frac{W_{t+1}}{c_{t+1}} \times \frac{c_{t+1}}{c_t} \right)^\rho \right]^{1/\rho} , \end{aligned}$$

where we use the linear homogeneity of μ_t . In logs,

$$wc_t = \rho^{-1} \log[(1 - \beta) + \beta \exp(\rho g_t)] ,$$

where $wc_t = \log(W_t/c_t)$ and $g_t \equiv \log(\mu_t(\exp(wc_{t+1} + x_{t+1})))$. Taking a linear approximation of the right-hand side as a function of g_t around the point \bar{m} , we get

$$\begin{aligned} wc_t &\approx \rho^{-1} \log[(1 - \beta) + \beta \exp(\rho \bar{m})] + \left[\frac{\beta \exp(\rho \bar{m})}{1 - \beta + \beta \exp(\rho \bar{m})} \right] (g_t - \bar{m}) \\ &\equiv \bar{\kappa} + \kappa g_t , \end{aligned}$$

where $\kappa < 1$. Approximating around $\bar{m} = 0$, results in $\bar{\kappa} = 0$ and $\kappa = \beta$, and for the general case of $\rho = 0$, the “log aggregator”, the linear approximation is exact with $\bar{\kappa} = 1 - \beta$ and $\kappa = \beta$.

Given the state variables of the economy, x_t and u_t , and the log-linear structure of the model, we conjecture a solution for the value function of the form,

$$wc_t = \bar{\omega} + \omega_x x_t + \omega_u u_t ,$$

where $\bar{\omega}$, ω_x , and ω_u are constants to be determined. Therefore,

$$wc_{t+1} + x_{t+1} = \bar{\omega} + (\omega_x + 1)x_{t+1} + \omega_u u_{t+1}$$

and, using the properties of lognormal random variables, g_t can be expressed as

$$\begin{aligned}
g_t &\equiv \log(\mu_t(\exp(wc_{t+1} + x_{t+1}))) \\
&= \log(E_t[\exp(wc_{t+1} + x_{t+1})^\alpha]^\frac{1}{\alpha}) \\
&= E_t[wc_{t+1} + x_{t+1}] + \frac{\alpha}{2}\text{Var}_t[wc_{t+1} + x_{t+1}] \\
&= \bar{\omega} + (\omega_x + 1)(1 - \varphi_x)\theta_x + \omega_u(1 - \varphi_u)\theta_u + (\omega_x + 1)\varphi_x x_t + \omega_u\varphi_u u_t \\
&\quad + \frac{\alpha}{2}(\omega_x + 1)^2 u_t + \frac{\alpha}{2}\omega_u^2 \sigma_u^2 .
\end{aligned}$$

Using the above expression, we solve for the value-function parameters by matching coefficients:

$$\begin{aligned}
\omega_x &= \kappa(\omega_x + 1)\varphi_x \\
\Rightarrow \omega_x &= \left(\frac{\kappa}{1 - \kappa\varphi_x}\right)\varphi_x \\
\omega_u &= \kappa[\omega_u\varphi_u + \frac{\alpha}{2}(\omega_x + 1)^2] \\
\Rightarrow \omega_u &= \left(\frac{\kappa}{1 - \kappa\varphi_u}\right)\left[\frac{\alpha}{2}\left(\frac{1}{1 - \kappa\varphi_x}\right)^2\right] \\
\bar{\omega} &= \frac{\bar{\kappa}}{1 - \kappa} + \frac{1}{1 - \kappa}\left[(\omega_x + 1)(1 - \varphi_x)\theta_x + \omega_u(1 - \varphi_u)\theta_u + \frac{\alpha}{2}\omega_u^2 \sigma_u^2\right] .
\end{aligned}$$

The solution allows us to simplify the term $[\log W_{t+1} - \log \mu_t(W_{t+1})]$ in the pricing kernel in Equation (B1):

$$\begin{aligned}
\log W_{t+1} - \log \mu_t(W_{t+1}) &= wc_{t+1} + x_{t+1} - \log \mu_t(\exp(wc_{t+1} + x_{t+1})) \\
&= (\omega_x + 1)[x_{t+1} - E_t x_{t+1}] + \omega_u[u_{t+1} - E_t u_{t+1}] \\
&\quad - \frac{\alpha}{2}(\omega_x + 1)^2 \text{Var}_t[x_{t+1}] - \frac{\alpha}{2}\omega_u^2 \text{Var}_t[u_{t+1}] \\
&= (\omega_x + 1)u_t^{1/2}\varepsilon_{t+1}^x + \omega_u\sigma_u\varepsilon_{t+1}^u - \frac{\alpha}{2}(\omega_x + 1)^2 u_t - \frac{\alpha}{2}\omega_u^2 \sigma_u^2 .
\end{aligned}$$

Equation (22) in the main text follows by collecting terms. In particular,

$$-\log n_{t+1} = \delta^r + \gamma_x^r x_t + \gamma_u^r u_t + \lambda_x^r \sqrt{u_t} \varepsilon_{t+1}^x + \lambda_u^r \sigma_u \varepsilon_{t+1}^u , \quad (\text{B2})$$

where

$$\delta^r = -\log \beta + (1 - \rho)(1 - \varphi_x)\theta_x + \frac{\alpha}{2}(\alpha - \rho)\omega_u^2 \sigma_u^2$$

$$\gamma_x^r = (1 - \rho)\varphi_x \quad ; \quad \gamma_u^r = \frac{\alpha}{2}(\alpha - \rho)(\omega_x + 1)^2$$

$$\lambda_x^r = (1 - \alpha) - (\alpha - \rho)\omega_x \quad ; \quad \lambda_u^r = - \left(\frac{\alpha}{2} \right) \left(\frac{\kappa(\alpha - \rho)}{1 - \kappa\varphi_u} \right) \left(\frac{1}{1 - \kappa\varphi_x} \right)^2$$

$$\omega_x = \left(\frac{\kappa}{1 - \kappa\varphi_x} \right) \varphi_x \quad ; \quad \omega_u = \left(\frac{\kappa}{1 - \kappa\varphi_u} \right) \left[\frac{\alpha}{2} \left(\frac{1}{1 - \kappa\varphi_x} \right)^2 \right] .$$

B.2 Endogenous Inflation and Monetary Policy Consistent Pricing Kernel

We find the unique minimum state variable solution by guessing a solution for the endogenous inflation process and then applying the method of undetermined coefficients as in McCallum (1981). Given the lognormal structure of the economy we guess that

$$\pi_t = a + a_x x_t + a_u u_t . \quad (\text{B3})$$

Substitute the guess in (B3) into the Euler condition and compute the expectation:

$$\begin{aligned} i_t &= -\log E_t n_{t+1} e^{-\pi_{t+1}} \\ &= C + (\gamma_x^r + a_x \varphi_x) x_t + \left(\gamma_u^r + a_u \varphi_u - \frac{1}{2} (\lambda_x^r + a_x)^2 \right) u_t , \end{aligned} \quad (\text{B4})$$

where $C = \delta^r + a + a_x(1 - \varphi_x)\theta_x + a_u(1 - \varphi_u) - \frac{1}{2}(\lambda_u^r + a_u)^2\sigma_u^2$. Then, plug in the guess in (B3) into the Taylor rule:

$$\begin{aligned} i_t &= \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t \\ &= (\bar{\tau} + \tau_\pi a) + (\tau_\pi a_x + \tau_x) x_t + \tau_\pi a_u u_t . \end{aligned} \quad (\text{B5})$$

Matching coefficients in (B4) and (B5) gives the solution for the a_i parameters. In particular,

$$\begin{aligned} a_x &= \frac{\gamma_x^r}{\tau_\pi - \varphi_x} \\ a_u &= \frac{\gamma_u^r - \frac{1}{2}(\lambda_x^r + a_x)^2}{\tau_\pi - \varphi_u} \\ a &= \frac{1}{\tau_\pi - 1} \left(\delta - \bar{\tau} + a_x(1 - \varphi_x)\theta_x + a_u(1 - \varphi_u)\theta_u - \frac{(\lambda_u^r + a_u)^2\sigma_u^2}{2} \right) . \end{aligned}$$

Putting together (B2), (B3) and (B4), we arrive at what Gallmeyer, Hollifield, Palomino, and Zin (2007) call the ‘‘monetary-policy-consistent nominal pricing kernel.’’

$$-\log m_{t+1} = \delta + \gamma_x x_t + \gamma_u u_t + \lambda_x \sqrt{u_t} \epsilon_{t+1}^x + \lambda_u \sigma_u \epsilon_{t+1}^u , \quad (\text{B6})$$

where

$$\delta = \delta^r + a + a_x(1 - \varphi_x)\theta_x + a_u(1 - \varphi_u) \ ;$$

$$\gamma_x = \gamma_x^r + a_x\varphi_x; \quad \gamma_u = \gamma_u^r + a_u\varphi_u;$$

$$\lambda_x = \lambda_x^r + a_x; \quad \lambda_u = \lambda_u^r + a_u \ .$$

Finally, recall that the nominal pricing kernel is related to the real pricing kernel and inflation according to $\log m_{t+1} = \log n_{t+1} - \pi_{t+1}$. Therefore, we can write

$$Var_t(\log m_{t+1}) = Var_t(\log n_{t+1}) + Var_t(\pi_{t+1}) - 2Cov_t(\log n_{t+1}, \pi_{t+1}) \ .$$

The negative correlation between consumption growth and inflation we observe in the data implies $Cov_t(\log n_{t+1}, \pi_{t+1}) > 0$. Together with a modest volatility of inflation — relative to the volatility of the real marginal rate of substitution — we have that, for all our quantitative exercises, $Var(\log m_{t+1}) < Var(\log n_{t+1})$. The nominal pricing kernel is less volatile than the real marginal rate of substitution. *Nominal risk is less than real risk.*

B.3 Properties of the UIP coefficient

The *real* UIP coefficient is $b^r = Cov(f_t^r - s_t^r, q_t^r)/Var(f_t^r - s_t^r)$, where $f_t^r - s_t^r$ is the real forward discount and q_t^r is the real depreciation rate. Assume symmetric coefficients across countries. A useful reference point is the special case of $\varphi_x = 0$. When this is the case, we have

$$b^r = \frac{\gamma_u^r}{\gamma_u^r - \frac{1}{2}(\lambda_x^r)^2} = \frac{\frac{\alpha}{2}(\alpha - \rho)}{\frac{\alpha}{2}(\alpha - \rho) - \frac{(1-\alpha)^2}{2}} \ .$$

Examining the above expression, we see that the *real* UIP coefficient is negative if and only if $\gamma_u^r = \frac{\alpha}{2}(\alpha - \rho) > 0$ and $\gamma_u^r - \frac{1}{2}(\lambda_x^r)^2 = \frac{\alpha}{2}(\alpha - \rho) - \frac{(1-\alpha)^2}{2} < 0$. One can verify that (i) $\alpha < 0$, and (ii) a preference for the early resolution of risk (that is, $\rho > \alpha$) are sufficient conditions to satisfy both the above inequalities.

Likewise, the *nominal* UIP coefficient is $b = Cov(f_t - s_t, q_t)/Var(f_t - s_t)$. When all parameters are symmetric across countries (including the Taylor parameters), and when $\varphi_x = 0$, we have

$$b = \frac{\gamma_u}{\gamma_u - \frac{1}{2}(\lambda_x)^2} = \frac{\gamma_u^r + a_u\varphi_u}{\gamma_u^r + a_u\varphi_u - \frac{1}{2}(\lambda_x + a_x)^2} \ .$$

Examining the above expression, we see that *nominal* UIP coefficient is negative if and only if $\gamma_u = \gamma_u^r + a_u\varphi_u > 0$ and $\gamma_u - \frac{1}{2}(\lambda_x)^2 = \gamma_u^r + a_u\varphi_u - \frac{1}{2}(\lambda_x + a_x)^2 < 0$. For the relevant parameter space in our quantitative exercise, the sensitivity of inflation to volatility, a_u , is

negative. Therefore, when volatility is positively autocorrelated ($\varphi_u > 0$), we have that $\gamma_u = \gamma_u^r + a_u \varphi_u < \gamma_u^r$, so that, in general, it is harder to obtain $\gamma_u > 0$. For γ_u to be positive we must have that (i) $\alpha < 0$ (as was the case for the real UIP coefficient) and (ii) $\rho > \alpha$ and *large enough*. The same conditions guarantee that $\gamma_u - \frac{1}{2}(\lambda_x)^2 < 0$. A numerical analysis of the relative magnitude of the real and nominal UIP coefficients shows that the nominal UIP coefficient is typically greater than its real counterpart. In other words, endogenous inflation pushes us *toward* UIP. Results are available upon request.

C Proofs

C.1 Proofs of Results 1 and 2

In this Appendix, we show analytical proofs of Results 1 and 2 in the main text. We focus on the specific case in which the *difference* in the Taylor rule parameters across countries changes, while keeping their *sum* constant. This allows us to obtain clean analytical expressions. We verified numerically that our results hold more generally for the set of parameters that is relevant to our quantitative exercise. Additional comparative static results are provided in Appendix C.2.

Result 1: *Relatively procyclical monetary policy generates currency risk*

The risk premium on foreign currency is increasing in $(\tau_x^* - \tau_x)$, provided that $\tau_x^* > \tau_x$. In words, foreign currency risk is associated with a foreign policy rule that is relatively procyclical. The larger the difference, the greater the risk.

Proof: Let $\Delta_x = \tau_x^* - \tau_x$ and $\Sigma_x = \tau_x^* + \tau_x$. Express the USD denominated risk premium on holding foreign currency as a function of Δ_x and Σ_x , that is

$$\begin{aligned}
 -p_t(\Delta_x, \Sigma_x) &= \frac{\lambda_u^2(\Delta_x, \Sigma_x) - (\lambda_u^*)^2(\Delta_x, \Sigma_x)}{2} \sigma_u^2 + \frac{\lambda_x^2(\Delta_x, \Sigma_x) u_t - (\lambda_x^*)^2(\Delta_x, \Sigma_x) u_t^*}{2} \\
 &\approx \frac{\sigma_u^2}{2} (\lambda_u^2(\Delta_x, \Sigma_x) - (\lambda_u^*)^2(\Delta_x, \Sigma_x)) + \frac{u_t}{2} (\lambda_x^2(\Delta_x, \Sigma_x) - (\lambda_x^*)^2(\Delta_x, \Sigma_x)) \\
 &= \frac{\sigma_u^2}{2} \left(\left(\lambda_u^r + \frac{\gamma_u^r - \frac{1}{2} \left(\lambda_x^r + \frac{\gamma_x^r + \frac{1}{2} (\Delta_x - \Sigma_x)}{\tau_\pi - \varphi_x} \right)^2}{\tau_\pi - \varphi_u} \right)^2 \right) - \left(\lambda_u^r + \frac{\gamma_u^r - \frac{1}{2} \left(\lambda_x^r + \frac{\gamma_x^r + \frac{1}{2} (-\Delta_x - \Sigma_x)}{\tau_\pi^* - \varphi_x} \right)^2}{\tau_\pi^* - \varphi_u} \right)^2 \right) \\
 &\quad + \frac{u_t}{2} \left(\left(\lambda_x^r + \frac{\gamma_x^r + \frac{1}{2} (\Delta_x - \Sigma_x)}{\tau_\pi - \varphi_x} \right)^2 - \left(\lambda_x^r + \frac{\gamma_x^r + \frac{1}{2} (-\Delta_x - \Sigma_x)}{\tau_\pi^* - \varphi_x} \right)^2 \right),
 \end{aligned}$$

where the approximation holds for the case of very high correlation between stochastic volatilities across countries ($\eta_{u,u^*} \approx 1$). Therefore,

$$\begin{aligned}
 \frac{\partial(-p_t(\Delta_x, \Sigma_x))}{\partial \Delta_x} &= \frac{\sigma_u^2}{2} \left(-\frac{\lambda_u \lambda_x}{(\tau_\pi - \varphi_u)(\tau_\pi - \varphi_x)} - \frac{\lambda_u^* \lambda_x^*}{(\tau_\pi^* - \varphi_u)(\tau_\pi^* - \varphi_x)} \right) \\
 &\quad + \frac{u_t}{2} \left(\frac{\lambda_x}{\tau_\pi - \varphi_x} + \frac{\lambda_x^*}{\tau_\pi^* - \varphi_x} \right) > 0,
 \end{aligned}$$

since $\lambda_x > 0$, $\lambda_x^* > 0$, $\lambda_u < 0$, $\lambda_u^* < 0$, $\tau_\pi > 1$, $\tau_\pi^* > 1$, $|\varphi_x| < 1$, and $|\varphi_u| < 1$. \square

Result 2: *Relatively accommodative monetary policy generates currency risk*

The risk premium on foreign currency is increasing in $(\tau_\pi - \tau_\pi^*)$, provided that $\tau_\pi > \tau_\pi^*$ and that τ_x and τ_x^* are large enough. In words, provided that domestic and foreign policies are sufficiently procyclical, foreign currency risk is associated with a foreign policy rule that is relatively accommodative toward inflation. The larger the difference, the greater the risk.

Proof: Let $\Delta_\pi = \tau_\pi - \tau_\pi^*$ and $\Sigma_\pi = \tau_\pi + \tau_\pi^*$. Express the USD denominated risk premium on holding foreign currency as a function of Δ_π and Σ_π . When $\eta_{u,u^*} \approx 1$, we can write

$$\begin{aligned} -p_t(\Delta_\pi, \Sigma_\pi) &\approx \frac{\sigma_u^2}{2} (\lambda_u^2(\Delta_\pi, \Sigma_\pi) - (\lambda_u^*)^2(\Delta_\pi, \Sigma_\pi)) + \frac{u_t}{2} (\lambda_x^2(\Delta_\pi, \Sigma_\pi) - (\lambda_x^*)^2(\Delta_\pi, \Sigma_\pi)) \\ &= \frac{\sigma_u^2}{2} \left(\left(\lambda_u^r + \frac{\gamma_u^r - \frac{1}{2} \left(\lambda_x^r + \frac{\gamma_x^r - \tau_x}{\frac{1}{2}(\Delta_\pi + \Sigma_\pi) - \varphi_x} \right)^2}{\frac{1}{2}(\Delta_\pi + \Sigma_\pi) - \varphi_u} \right)^2 - \left(\lambda_u^r + \frac{\gamma_u^r - \frac{1}{2} \left(\lambda_x^r + \frac{\gamma_x^r - \tau_x^*}{\frac{1}{2}(-\Delta_\pi + \Sigma_\pi) - \varphi_x} \right)^2}{\frac{1}{2}(-\Delta_\pi + \Sigma_\pi) - \varphi_u} \right)^2 \right) \\ &\quad + \frac{u_t}{2} \left(\left(\lambda_x^r + \frac{\gamma_x^r - \tau_x}{\frac{1}{2}(\Delta_\pi + \Sigma_\pi) - \varphi_x} \right)^2 - \left(\lambda_x^r + \frac{\gamma_x^r - \tau_x^*}{\frac{1}{2}(-\Delta_\pi + \Sigma_\pi) - \varphi_x} \right)^2 \right). \end{aligned}$$

Therefore,

$$\frac{\partial(-p_t(\Delta_\pi, \Sigma_\pi))}{\partial\Delta_\pi} = \frac{\sigma_u^2}{2} \left(2\lambda_u \frac{\partial\lambda_u}{\partial\Delta_\pi} - 2\lambda_u^* \frac{\partial\lambda_u^*}{\partial\Delta_\pi} \right) + \frac{u_t}{2} \left(2\lambda_x \frac{\partial\lambda_x}{\partial\Delta_\pi} - 2\lambda_x^* \frac{\partial\lambda_x^*}{\partial\Delta_\pi} \right),$$

where we have suppressed the dependence of the prices of risk on Δ_π and Σ_π . We have $\lambda_x > 0$, $\lambda_x^* > 0$, $\lambda_u < 0$, $\lambda_u^* < 0$, and

$$\frac{\partial(\lambda_x(\Delta_\pi, \Sigma_\pi))}{\partial\Delta_\pi} = -\frac{\gamma_x^r - \tau_x}{2(\tau_\pi - \phi_x)^2} > 0,$$

since $(\gamma_x^r - \tau_x)$ is typically negative (see Section 5.1.1 in the main text). Similarly,

$$\frac{\partial(\lambda_x^*(\Delta_\pi, \Sigma_\pi))}{\partial\Delta_\pi} = \frac{\gamma_x^r - \tau_x^*}{2(\tau_\pi^* - \phi_x)^2} < 0.$$

The sign of the partial derivatives $\frac{\partial\lambda_u}{\partial\Delta_\pi}$ and $\frac{\partial\lambda_u^*}{\partial\Delta_\pi}$ is ambiguous. A sufficient condition for $\frac{\partial(-p_t(\Delta_\pi, \Sigma_\pi))}{\partial\Delta_\pi} > 0$ is that $\frac{\partial\lambda_u}{\partial\Delta_\pi} < 0$ and $\frac{\partial\lambda_u^*}{\partial\Delta_\pi} > 0$. We have

$$\frac{\partial(\lambda_u(\Delta_\pi, \Sigma_\pi))}{\partial\Delta_\pi} = -\frac{\gamma_u^r - \frac{1}{2}\lambda_x^2}{2(\tau_\pi - \phi_u)^2} + \frac{\lambda_x(\gamma_x^r - \tau_x)}{2(\tau_\pi - \phi_u)(\tau_\pi - \phi_x)^2}.$$

The first term, $-\frac{\gamma_u^r - \frac{1}{2}\lambda_x^2}{2(\tau_\pi - \phi_u)^2}$ is positive, and the second term, $\frac{\lambda_x(\gamma_x^r - \tau_x)}{2(\tau_\pi - \phi_u)(\tau_\pi - \phi_x)^2}$ is negative. However, the first term is decreasing in τ_x , while the second term is increasing in τ_x .⁶ Therefore, there exists a *large enough* τ_x such that $\frac{\partial \lambda_u}{\partial \Delta \pi} < 0$. Similar calculations show that there exists a *large enough* τ_x^* such that $\frac{\partial \lambda_u^*}{\partial \Delta \pi} > 0$. \square

C.2 The effect of cyclical and accommodative policy

Write the USD denominated risk premium on holding foreign currency as

$$\begin{aligned} -p_t &= (\bar{i} - \bar{l}) - (\delta^* - \delta) - \frac{1}{2}((\lambda_x^*)^2 u_t^* - \lambda_x^2 u_t) \\ &= \frac{\lambda_u^2 - (\lambda_u^*)^2}{2} \sigma_u^2 + \frac{\lambda_x^2 u_t - (\lambda_x^* u_t^*)^2}{2} \\ &\approx \frac{\lambda_u^2 - (\lambda_u^*)^2}{2} \sigma_u^2 + \frac{\lambda_x^2 - (\lambda_x^*)^2}{2} u_t \\ &= \kappa(p) + v(p) \quad . \end{aligned}$$

We refer to $\kappa(p)$ as the constant part of the risk premium, and to $v(p)$ as to its variable part.

C.2.1 The Effect of τ_x on the Price of Consumption Risk

Recall that the nominal price of consumption risk λ_x is

$$\lambda_x = \lambda_x^r + a_x \quad . \quad (\text{C1})$$

where

$$a_x = \frac{(1 - \rho)\varphi_x - \tau_x}{\tau_\pi - \varphi_x} \quad . \quad (\text{C2})$$

Result C.1 *An increase (decrease) in $\tau_x > 0$ reduces (increases) the nominal price of consumption risk, λ_x .*

Proof: From equations (C1) and (C2),

$$\frac{\partial \lambda_x}{\partial \tau_x} = \underbrace{\frac{\partial \lambda_x}{\partial a_x}}_{+} \underbrace{\frac{\partial a_x}{\partial \tau_x}}_{-} < 0 \quad .$$

⁶More precisely, $\frac{\partial}{\partial \tau_x} \left(\frac{\lambda_x(\gamma_x^r - \tau_x)}{2(\tau_\pi - \phi_u)(\tau_\pi - \phi_x)^2} \right) > 0$ for a large enough price of consumption risk λ_x , which is the case for all the set of parameters we consider.

□

Note that: i) the real price of consumption risk λ_x^r is positive and independent of monetary policy, so that $\frac{\partial \lambda_x^r}{\partial a_x} > 0$; ii) a_x is typically negative, so that $\frac{\partial a_x}{\partial \tau_x} < 0$ (see discussion in Sections 5.1.1 and 5.2); and iii) λ_x^r is typically large enough to ensure $\lambda_x > 0$.

In words, an increase in $\tau_x > 0$ makes a_x more negative, reduces the nominal price of risk $\lambda_x > 0$, and thus reduces the variable part of the foreign currency risk premium, $v(p)$. *The effect of a more procyclical domestic monetary policy on the price of consumption risk is to reduce the foreign currency risk premium.*

C.2.2 The Effect of τ_x on the Price of Volatility Risk

Recall that the nominal price of volatility risk λ_u is

$$\lambda_u = \lambda_u^r + a_u \quad , \quad (\text{C3})$$

where

$$a_u = \frac{\gamma_v^r - \lambda_x^2/2}{\tau_\pi - \varphi_u} \quad . \quad (\text{C4})$$

Result C.2 *An increase (decrease) in $\tau_x > 0$ increases (reduces) the nominal price of volatility risk, λ_u .*

Proof: From equations (C3) and (C4), and using the result in Appendix C.2.1, we have

$$\frac{\partial \lambda_u}{\partial \tau_x} = \underbrace{\frac{\partial \lambda_u}{\partial a_u}}_+ \underbrace{\frac{\partial a_u}{\partial \lambda_x}}_- \underbrace{\frac{\partial \lambda_x}{\partial \tau_x}}_- > 0 \quad .$$

□

Note that: i) the real price of volatility risk λ_u^r is negative with early resolution of risk ($\alpha < \rho$) and independent of monetary policy; ii) for the set of parameters we consider, the coefficient a_u is negative. This is the case whenever the nominal price of consumption risk, λ_x , is large enough relative real factor loading of volatility, γ_v^r (see Equation (C4)). A necessary condition is that $\gamma_v^r - (\lambda_x^r)^2/2 < 0$ which, in a model with symmetric parameters across countries, is a necessary condition for the real UIP slope to be negative (see Appendix B.3).

In words, an increase in $\tau_x > 0$ increases a_u (that is, it makes it *less* negative), increases the nominal price of risk λ_u (that is, it makes it *less* negative), and thus reduces the constant part of the foreign currency risk premium, $\kappa(p)$. *The effect of a more procyclical domestic monetary policy on the price of volatility risk is to reduce the foreign currency risk premium.*

C.2.3 The Effect of τ_x on the Prices of Risk: Summary

The results in Appendix C.2.1 and C.2.2 above tell us that the overall effect of a change in τ_x on the foreign currency risk premium is unambiguous. *A more procyclical domestic monetary policy reduces the foreign currency risk premium both through its effect on the price of consumption risk — and therefore on the variable component of the foreign currency risk premium, $v(p)$ — and through its effect on the price of volatility risk — and therefore on the constant component of the foreign currency risk premium, $\kappa(p)$.*

C.2.4 The Effect of τ_π on the Price of Consumption Risk

Result C.3 *An increase (decrease) in $\tau_\pi > 1$ increases (reduces) the nominal price of consumption risk, λ_x .*

Proof: From Equations (C1) and (C2), and the fact that $a_x < 0$,

$$\frac{\partial \lambda_x}{\partial \tau_\pi} = \underbrace{\frac{\partial \lambda_\pi}{\partial a_x}}_+ \underbrace{\frac{\partial a_x}{\partial \tau_\pi}}_+ > 0 .$$

□

An increase in $\tau_\pi > 1$ increases a_x (it makes it *less* negative), increases the nominal price of risk λ_x , and thus increases the variable part of the foreign currency risk premium, $v(p)$. In words, *the effect of a less accommodative domestic monetary policy on the price of consumption risk is to increase the foreign currency risk premium.*

C.2.5 The Effect of τ_π on the Price of Volatility Risk

Result C.4 *The effect of a change in τ_π on the nominal price of volatility risk is ambiguous.*

Proof: Let $num(a_x)$ and $den(a_x)$ denote the numerator and denominator of a_x , respectively. From Equations (C3) and (C4), and using the result in Appendix C.2.4, we have

$$\begin{aligned} \frac{\partial \lambda_u}{\partial \tau_\pi} &= \frac{\partial \lambda_u}{\partial a_u} \frac{\partial a_u}{\partial \tau_\pi} \\ &= \underbrace{\frac{\partial \lambda_u}{\partial a_u}}_+ \frac{\overbrace{\frac{den(a_x)}{\partial \lambda_x}}^+ \overbrace{\frac{\partial num(a_x)}{\partial \lambda_x}}^- \overbrace{\frac{\partial \lambda_x}{\partial \tau_\pi}}^+ - \overbrace{num(a_x)}^- \overbrace{\frac{\partial den(a_x)}{\partial \tau_\pi}}^+}{\underbrace{den(a_x)^2}_+} , \end{aligned}$$

so that the sign of $\frac{\partial \lambda_u}{\partial \tau_\pi}$ is ambiguous. \square

The ambiguity arises from the fact that, for relatively small values of τ_x , a less accommodative domestic policy increases the price of volatility risk, λ_u (it makes it *less* negative). For values of τ_x that are large enough, the sensitivity is reversed and a less accommodative domestic policy decreases the price of volatility risk, λ_u , thus contributing *positively* to the foreign currency risk premium.

C.2.6 The Effect of τ_π on the Price of Risk: Summary

The results in Appendix C.2.4 and C.2.5 tell us that the overall effect of a change in τ_π on the foreign currency risk premium is ambiguous. A less accommodative domestic monetary policy contributes positively to the foreign currency risk premium through its effect on the price of consumption risk (and therefore on the variable component of the foreign currency risk premium, $v(p)$). However, a less accommodative domestic monetary policy has an ambiguous effect on the price of volatility risk (and therefore on the constant component of the foreign currency risk premium, $\kappa(p)$). Which of the two components prevails depends on the parameters of the model. *For values of τ_x that are large enough, a less accommodative domestic monetary policy increases the foreign currency risk premium.*

D Enhanced Model

D.1 A Model with Long Run Risk

We now attempt to improve the overall quantitative performance of our model by following Bansal and Yaron (2004), and the application to exchange rates of Bansal and Shaliastovich (2013), in modeling consumption growth, x_{t+1} and x_{t+1}^* , as containing a small and persistent component (its ‘long-run risk’) with stochastic volatility:

$$\log(c_{t+1}/c_t) \equiv x_{t+1} = \mu + l_t + \sqrt{u_t} \epsilon_{t+1}^x \quad (\text{D1})$$

$$l_{t+1} = \varphi_l l_t + \sqrt{w_t} \epsilon_{t+1}^l, \quad (\text{D2})$$

where

$$u_{t+1} = (1 - \varphi_u) \theta_u + \varphi_u u_t + \sigma_u \epsilon_{t+1}^u \quad (\text{D3})$$

$$w_{t+1} = (1 - \varphi_w) \theta_w + \varphi_w w_t + \sigma_w \epsilon_{t+1}^w. \quad (\text{D4})$$

Foreign consumption growth, x_{t+1}^* is defined analogously. The innovations are assumed to be multivariate normal and independent *within*-country: $(\epsilon^x, \epsilon^l, \epsilon^u, \epsilon^w)' \sim \text{NID}(0, I)$, but we allow for correlation *across* countries: $\eta_j \equiv \text{Corr}(\epsilon^j, \epsilon^{j*})$, for $j = (x, l, u, w)$. Note that there is no direct relation between the symbols used in the model of this Appendix and the symbols previously used in the main text.

The process (D1)–(D4) looks complicated, but each of the ingredients are necessary. Stochastic volatility is necessary because without it the currency risk premium would be constant and the UIP regression parameter, b , would be 1. Long-run risk — by which we mean time variation in the conditional mean of consumption growth, l_t — decouples the conditional mean of consumption growth from other moments of consumption growth, thereby permitting persistent and volatile interest rates to co-exist with relatively smooth and close-to-*i.i.d.* consumption growth. Finally, cross-country correlation in the innovations is critical for achieving realistic cross-country consumption correlations. The latter imposes substantial discipline on our calibration (*c.f.*, Brandt, Cochrane, and Santa-Clara (2006)).

As before, we use the Hansen, Heaton, and Li (2008) linearization of the real pricing kernel. The log wealth consumption ratio $wc_t = \log(W_t/c_t)$ is, up to a first order approximation, related to $g_t \equiv \log(\mu_t(\exp(wc_{t+1} + x_{t+1})))$ as follows:

$$\begin{aligned} wc_t &\approx \rho^{-1} \log[(1 - \beta) + \beta \exp(\rho \bar{m})] + \left[\frac{\beta \exp(\rho \bar{m})}{1 - \beta + \beta \exp(\rho \bar{m})} \right] (g_t - \bar{m}) \\ &\equiv \bar{\kappa} + \kappa g_t, \end{aligned}$$

where \bar{m} is the point around which the approximation is taken and $\kappa < 1$.

Given the state variables of the economy, l_t , u_t and w_t , and the loglinear structure of the

model, we conjecture a solution for the value function of the form,

$$wc_t = \bar{\omega} + \omega_l l_t + \omega_u u_t + \omega_w w_t \quad ,$$

where $\bar{\omega}$, ω_l , ω_u and ω_w are constants to be determined. Therefore

$$wc_{t+1} + x_{t+1} = \bar{\omega} + \omega_l l_{t+1} + \omega_u u_{t+1} + \omega_w w_{t+1} + x_{t+1}$$

and, using the properties of lognormal random variables, g_t can be expressed as

$$\begin{aligned} g_t &\equiv \log(\mu_t(\exp(wc_{t+1} + x_{t+1}))) \\ &= \log(E_t[\exp(wc_{t+1} + x_{t+1})^\alpha]^{\frac{1}{\alpha}}) \\ &= E_t[wc_{t+1} + x_{t+1}] + \frac{\alpha}{2} \text{Var}_t[wc_{t+1} + x_{t+1}] \quad . \end{aligned}$$

Using the above expression, we solve for the value-function parameters by matching coefficients

$$\begin{aligned} \omega_l &= \kappa(\omega_l \varphi_l + 1) \\ \Rightarrow \omega_l &= \left(\frac{\kappa}{1 - \kappa \varphi_l} \right) \\ \omega_u &= \kappa(\omega_u \varphi_u + \frac{\alpha}{2}) \\ \Rightarrow \omega_u &= \frac{\alpha}{2} \frac{\kappa}{1 - \kappa \varphi_u} \\ \omega_w &= \kappa(\omega_w \varphi_w + \frac{\alpha}{2} \omega_l^2) \\ \Rightarrow \omega_w &= \frac{\alpha}{2} \omega_l^2 \frac{\kappa}{1 - \kappa \varphi_u} \quad . \end{aligned}$$

The solution allows us to simplify the term $[\log W_{t+1} - \log \mu_t(W_{t+1})]$ in the pricing kernel in Equation (B1):

$$\begin{aligned} \log W_{t+1} - \log \mu_t(W_{t+1}) &= wc_{t+1} + x_{t+1} - \log \mu_t(\exp(wc_{t+1} + x_{t+1})) \\ &= \omega_l \sqrt{w_t} \epsilon_{t+1}^l + \omega_u \sigma_u \epsilon_{t+1}^u + \omega_w \sigma_w \epsilon_{t+1}^w + \sqrt{u_t} \epsilon_{t+1}^x \\ &\quad - \frac{\alpha}{2} (\omega_l^2 w_t + \omega_u^2 \sigma_u^2 + \omega_w^2 \sigma_w^2 + u_t) \quad . \end{aligned}$$

Collecting terms, the real pricing kernel can be expressed as

$$\begin{aligned} -\log(n_{t+1}) &= \delta^r + \gamma_l^r l_t + \gamma_u^r u_t + \gamma_w^r w_t \\ &\quad + \lambda_x^r \sqrt{u_t} \epsilon_{t+1}^x + \lambda_l^r \sqrt{w_t} \epsilon_{t+1}^l + \lambda_u^r \sigma_u \epsilon_{t+1}^u + \lambda_w^r \sigma_w \epsilon_{t+1}^w \quad , \end{aligned}$$

where

$$\gamma_l^r = (1 - \rho); \quad \gamma_u^r = \frac{\alpha}{2}(\alpha - \rho); \quad \gamma_w^r = \frac{\alpha}{2}(\alpha - \rho)\omega_l^2$$

$$\lambda_x^r = (1 - \alpha); \quad \lambda_l^r = -(\alpha - \rho)\omega_l; \quad \lambda_v^r = -(\alpha - \rho)\omega_u; \quad \lambda_w^r = -(\alpha - \rho)\omega_w$$

$$\delta^r = -\log \beta + (1 - \rho)\mu + \frac{\alpha}{2}(\alpha - \rho)[(\omega_u\sigma_u)^2 + (\omega_w\sigma_w)^2] .$$

The conditional mean of the real pricing kernel is equal to

$$E_t \log n_{t+1} = -(\delta^r + \gamma_l^r l_t + \gamma_u^r u_t + \gamma_w^r w_t)$$

and its conditional variance is

$$Var_t \log n_{t+1} = (\lambda_x^r)^2 u_t + (\lambda_l^r)^2 w_t + (\lambda_u^r \sigma_u)^2 + (\lambda_w^r \sigma_w)^2 .$$

The conditional mean depends both on expected consumption growth and stochastic volatility, whereas the conditional variance is a linear function of current stochastic volatility processes only.

Next, the real short rate is

$$\begin{aligned} r_t &\equiv -\log E_t(n_{t+1}) \\ &= \bar{r} + \gamma_l^r l_t + r_u^r u_t + r_w^r w_t , \end{aligned}$$

where

$$\bar{r} = \delta^r - \frac{1}{2}[(\lambda_u^r \sigma_u)^2 + (\lambda_w^r \sigma_w)^2]$$

and

$$r_u^r = \gamma_u^r - \frac{1}{2}(\lambda_x^r)^2; \quad r_w^r = \gamma_w^r - \frac{1}{2}(\lambda_l^r)^2 .$$

Assuming symmetry, the expression for the expected real depreciation, q_t^r , the real forward premium, $f_t^r - s_t^r$, and the real risk premium, p_t^r , are:

$$\begin{aligned} q_t^r &= \gamma_l^r (l_t - l_t^*) + \gamma_u^r (u_t - u_t^*) + \gamma_w^r (w_t - w_t^*) , \\ f_t^r - s_t^r &= \gamma_l^r (l_t - l_t^*) + r_u^r (u_t - u_t^*) + r_w^r (w_t - w_t^*) , \\ p_t^r &= -\frac{1}{2} ((\lambda_x^r)^2 (u_t - u_t^*) + (\lambda_l^r)^2 (w_t - w_t^*)) . \end{aligned}$$

Result D.1: *The real UIP slope coefficient*

If all foreign and domestic parameter values are the same, then the real UIP regression parameter, obtained by the regressing the real interest rate differential on the real depreciation rate is:

$$\begin{aligned} b^r &= \frac{\text{Cov}(f_t^r - s_t^r, q_t^r)}{\text{Var}(f_t^r - s_t^r)} \\ &= \frac{(\gamma_l^r)^2 \text{Var}(l_t - l_t^*) + \gamma_u^r r_u^r \text{Var}(u_t - u_t^*) + \gamma_w^r r_w^r \text{Var}(w_t - w_t^*)}{(\gamma_l^r)^2 \text{Var}(l_t - l_t^*) + (r_u^r)^2 \text{Var}(u_t - u_t^*) + (r_w^r)^2 \text{Var}(w_t - w_t^*)} . \end{aligned}$$

Without the presence of both stochastic volatility and EZ preferences, b^r is equal to one and, in real terms, UIP holds identically. Also, when the long-run state variables, l_t and w_t , are perfectly correlated across countries, the slope coefficient reduces to $b^r = \gamma_u^r / r_u^r$. This is the case considered by Bansal and Shaliastovich (2013).

For b^r to be negative, we require $\text{Cov}(f_t^r - s_t^r, q_t^r) < 0$. The expression above makes it evident that only stochastic volatility terms can contribute negatively to this covariance. In particular, a necessary condition for a negative real slope coefficient is that the $\gamma^r = (\gamma_u^r, \gamma_w^r)$ and $r^r = (r_u^r, r_w^r)'$ coefficients have opposite sign, for at least one of the stochastic volatility processes. A preference for the early resolution of risk ($\alpha < \rho$) and an EIS larger than one ($\rho < 0$) deliver the required covariations.

D.2 Taylor Rule, Inflation and the Nominal Pricing Kernel

We consider the following domestic Taylor rule:⁷

$$i_t = \tau + \tau_\pi \pi_t + \tau_l l_t . \quad (\text{D5})$$

An analogous equation, denoted with asterisks, characterizes the foreign-country Taylor rule. Following the technique developed above, we guess that the solution for endogenous inflation has the form

$$\pi_t = a + a_1 l_t + a_2 u_t + a_3 w_t , \quad (\text{D6})$$

substitute it into the Euler equation, compute the moments, and then solve for the a_j coefficients by matching up the result with the Taylor rule (D5). This gives,

$$a_1 = \frac{\gamma_l - \tau_l}{\tau_\pi - \varphi_l}; \quad a_2 = \frac{\gamma_u - \frac{1}{2}\lambda_x^2}{\tau_\pi - \varphi_u}; \quad a_3 = \frac{\gamma_w - \frac{1}{2}\lambda_l^2}{\tau_\pi - \varphi_w};$$

⁷For parsimony, we use expected consumption growth, l_t , and not its current level, x_t , as is instead standard in the literature. Doing so reduces our state space by one variable. The model can readily be extended to allow for a specification that includes x_t instead of l_t .

$$a = \frac{1}{\tau_\pi - 1} [\delta - \tau + a_2(1 - \varphi_u)\theta_u + a_3(1 - \varphi_w)\theta_w - \frac{1}{2}[(\lambda_u\sigma_u)^2 + (\lambda_w\sigma_w)^2]$$

where the constant term, the factor loadings and the pricing of risk of the nominal pricing kernel are

$$\delta = \delta^r + a + a_2(1 - \varphi_u)\theta_u + a_3(1 - \varphi_w)\theta_w$$

$$\gamma_l = \gamma_l^r + a_1\varphi_l; \quad \gamma_u = \gamma_u^r + a_2\varphi_u; \quad \gamma_w = \gamma_w^r + a_3\varphi_w;$$

$$\lambda_x = \lambda_x^r; \quad \lambda_l = \lambda_l^r + a_1; \quad \lambda_u = \lambda_u^r + a_2; \quad \lambda_w = \lambda_w^r + a_3 .$$

The linearized nominal pricing kernel is

$$\begin{aligned} -\log m_{t+1} &= -\log n_{t+1} + \pi_{t+1} \\ &= \delta + \gamma_l l_t + \gamma_u u_t + \gamma_w w_t \\ &\quad + \lambda_x \sqrt{u_t} \epsilon_{t+1}^x + \lambda_l \sqrt{w_t} \epsilon_{t+1}^l + \lambda_u \sigma_u \epsilon_{t+1}^u + \lambda_w \sigma_w \epsilon_{t+1}^w , \end{aligned}$$

and the nominal short rate is

$$\begin{aligned} i_t &\equiv -\log E_t(m_{t+1}) \\ &= \bar{i} + \gamma_l l_t + r_u u_t + r_w w_t , \end{aligned}$$

where

$$\bar{i} = \delta - \frac{1}{2}[(\lambda_u\sigma_u)^2 + (\lambda_w\sigma_w)^2] ;$$

$$r_u = \gamma_u - \frac{1}{2}\lambda_x^2; \quad r_w = \gamma_w - \frac{1}{2}\lambda_l^2 .$$

The nominal interest rate differential, the expected depreciation rate and the risk premium can be derived from Equations (12–15). Assuming symmetry across countries, we have

$$\begin{aligned} q_t &= \gamma_l(l_t - l_t^*) + \gamma_u(u_t - u_t^*) + \gamma_w(w_t - w_t^*) , \\ f_t - s_t &= \gamma_l(l_t - l_t^*) + r_u(u_t - u_t^*) + r_w(w_t - w_t^*) , \\ p_t &= -\frac{1}{2}(\lambda_x^2(u_t - u_t^*) + \lambda_l^2(w_t - w_t^*)) . \end{aligned}$$

Result D.2: *The nominal UIP slope coefficient*

If all foreign and domestic parameter values are the same, the nominal UIP slope coefficient is

$$\begin{aligned} b &= \frac{\text{Cov}(f_t - s_t, q_t)}{\text{Var}(f_t - s_t)} \\ &= \frac{\gamma_l^2 \text{Var}(l_t - l_t^*) + \gamma_u r_u \text{Var}(u_t - u_t^*) + \gamma_w r_w \text{Var}(w_t - w_t^*)}{\gamma_l^2 \text{Var}(l_t - l_t^*) + r_u^2 \text{Var}(u_t - u_t^*) + r_w^2 \text{Var}(w_t - w_t^*)}. \end{aligned}$$

As was the case for the real UIP slope coefficient, without EZ preferences and stochastic volatility in consumption growth, $b = 1$.

D.3 Quantitative Results

Ideally, we'd like our model to be able to account for a broad set of sample moments of exchange rates and interest rates. Foremost, of course, are the negative nominal UIP slope coefficient and the unconditional currency risk premium that have been discussed at length in the main text. Others are (i) high correlation between real and nominal exchange rates (Mussa (1986)), (ii) high exchange rate volatility relative to volatility in inflation differentials, (iii) near random-walk behavior in exchange rates but obvious stationarity in interest rate differentials, (iv) highly autocorrelated domestic real and nominal interest rates with means and volatilities that match data, and (v) highly correlated foreign and domestic pricing kernels but low correlation in cross-country consumption growth (Brandt, Cochrane, and Santa-Clara (2006)). As we will see below, while the model with long run risk delivers most of the required sample moments, including the low correlation in cross country consumption growth that the main model could not account for, it struggles to replicate large unconditional foreign risk premia.

We calibrate our model using a monthly frequency such that foreign and domestic consumption processes are the same (but with different shocks). Table (3) reports the calibrated parameters. The unconditional mean of consumption growth, μ , is set to 0.0015. The autocorrelation coefficients, φ_l , φ_v , and φ_w , are taken from Bansal and Shaliastovich (2013). Given φ_l , we choose θ_v and θ_w to match an annualized sample standard deviation of consumption growth of 2.72%, while keeping the ratio of short-run versus long-run volatility as in Bansal and Shaliastovich (2013). As was the case in the main text, the conditional variances of short-run and long-run volatilities, σ_w^2 and σ_v^2 , are set to be as large as possible subject to the constraint that the probability of observing a negative realization of volatility does not exceed 5%. We set the cross-country correlations, η_l and η_w , equal to unity (as do Bansal and Shaliastovich (2013) and Colacito and Croce (2011)), the idea being that long-run consumption growth shocks are global. In contrast, we set $\eta_u = 0$, so that shocks

to ‘short-run’ consumption volatility are country-specific. Given this, we choose η_x to match the observed cross-country correlation in consumption growth of 0.35. Finally, we choose the preference parameters as follows. As is standard in the long run risk literature, we fix the intertemporal elasticity of substitution at 1.5 ($\rho = 1/3$). Then, we choose the risk aversion coefficient, α , to match the volatility of the real depreciation rate and the subjective discount factor, β , to pin down the unconditional mean of the real interest rate.

Panel A of Table 4 shows that the real model with long run risk *partially* delivers what we set out to achieve. In particular, the cross country correlation in consumption growth, at 0.35, is now as low as in the data, but the need for a low risk aversion coefficient in achieving sensible values for the volatility of the real depreciation rate drives down the standard deviation of the real interest rate to 0.53%, roughly half of our sample estimate of 0.97%.

Turning to the nominal side of the model, we follow an approach that is similar to that of Section 6.2, asking how the Taylor rule coefficients affect endogenous inflation, interest rates and exchange rates. We propose two alternative calibrations. We begin in what seems the most natural and disciplined way, calibrating to inflation only and then seeing what happens to interest rates and exchange rates. We calibrate the six Taylor-rule parameters, $[\tau \tau_\pi \tau_l]$ and $[\tau^* \tau_\pi^* \tau_l^*]$, to match the mean and variance of U.S. and Australian inflation, respectively, as well as the two correlations, $Corr(x_t, \pi_t)$ and $Corr(x_t^*, \pi_t^*)$.

Panel B of Table 4 reports results in the column labeled ‘Model LRR I.’ The domestic and foreign inflation processes capture, by construction, what we want them to capture. Foreign inflation is higher on average and more volatile. As was the case in the model of the main text, theoretical inflation is much more autocorrelated than its sample counterpart. The reason for this is discussed at length in Section 6.2. Also, unlike the simpler model of Section 5, we run into some constraints. The correlation between consumption growth and inflation hits a lower bound at -0.283 , a touch short of its empirical counterpart of -0.300 . This is due to the following series of tensions. First, the correlation between consumption growth and inflation is affected by the Taylor parameters only through their effect on the volatility of inflation, $Stdev(\pi_t)$, and the coefficient a_l , which governs the sensitivity of inflation to long run risk. However, our calibration of the real part of the model, which we borrow from the long run risk literature, implies that $Stdev(\pi_t) \approx a_1 Stdev(l_t)$. Taken together, the ratio $Stdev(\pi_t)/a_1$ is roughly independent of monetary policy.

Turning to what we are really interested in, exchange rates, we see that the model with long run risk struggles to replicate, at the same time, a negative Bilson-Fama-Tryon coefficient and a sizeable risk premium. The foreign risk premium is minuscule, albeit with the correct sign, and the regression coefficient is positive at $+0.248$, versus a sample counterpart of -1.000 . On the positive side, the mean, volatility and autocorrelation of the nominal depreciation rate is consistent with the data.

Importantly, while the foreign risk premium is minuscule relative to the data in our sample, its sensitivity to the Taylor parameters is consistent with Results 1 and 2, and therefore the overall message of our paper. Quantitative exercises confirm that relatively more

procyclical monetary policy and relatively more accommodative monetary policy generate currency risk (that is, $-E(p)$ is increasing in τ_π and τ_l^* , and decreasing in τ_l and τ_{π^*}).

Panel B of Table 4 also reports results for an alternative calibration labeled ‘Model LRR II.’ This alternative calibration corresponds to the following modification of the above ‘Model LRR I.’ Instead of targeting the domestic and foreign inflation-consumption correlations, we target the volatility of the depreciation rate and the UIP coefficient.

What we find is that the model’s implications for the inflation-consumption correlation are unaltered — a consequence of the calibration of the real part of the model, as explained above — but the unconditional risk premium is (roughly) zero, confirming the difficulties of the model to simultaneously account for a negative UIP coefficient and a sizeable unconditional foreign currency risk premium. The requirement in the calibration exercise to have a UIP coefficient around -1.000 makes it impossible for the model to generate the spread in the volatilities of inflation that we observe in our sample. With a perfectly symmetric real world, and very similar inflation processes at home and abroad, the currency risk premium *must* be close to zero, as it turns out to be. This constraint can be relaxed by allowing for a richer specification of the correlation structure of the volatility shocks. While comparative statics exercises for the currency risk premium still deliver results consistent with the main message of our paper, we leave to future work the task of finding a better specification for real pricing kernels and Taylor rules that can deliver our story both qualitatively and quantitatively.

Table 3
Calibrated Parameter Values

Description	Parameter	Value	
Panel A: The Real Economy			
Discount factor	β	0.999	
Relative risk aversion	$1 - \alpha$	3.678	
Elasticity of intertemporal substitution	$(1 - \rho)^{-1}$	1.5	
Mean of consumption growth	μ	0.0015	
Autocorrelation of long-run risk	φ_l	0.991	
Mean of short-run volatility level	θ_u	$5.661e^{-5}$	
Autocorrelation of short-run volatility	φ_u	0.800	
Volatility of short-run volatility	σ_u	$2.070e^{-5}$	
Mean of long-run volatility level	θ_w	$9.057e^{-8}$	
Autocorrelation of long-run volatility	φ_w	0.980	
Volatility of long-run volatility	σ_w	$1.100e^{-8}$	
Cross-Country correlation in short-run consumption innovations	η_{x,x^*}	0.292	
Cross-Country correlation in long-run risk innovations	η_{l,l^*}	1.000	
Cross-Country correlation in short-run volatility innovations	η_{u,u^*}	0	
Cross-Country correlation in long-run volatility innovations	η_{w,w^*}	1.000	
Panel B: The Nominal Economy			
Constant in the domestic interest rate rule	$\bar{\tau}$	<i>Model LRR I</i>	<i>Model LRR II</i>
Constant in the foreign interest rate rule	$\bar{\tau}^*$	-0.008	-0.008
Domestic response to long run risk	τ_l	5.146	5.202
Foreign response to long run risk	τ_l^*	5.003	5.225
Domestic response to inflation	τ_π	4.817	4.732
Foreign response to inflation	τ_π^*	4.417	4.713

Table 3 reports the parameter values associated with the calibration exercise described in Appendix D.3. These parameter values underly the various population moments reported in Table 4. Table 4 reports sample moments in the second column and population moments from our model in the remaining columns. Sample moments derive from a variety of sources. The data frequency is monthly and, where appropriate the moments are reported as annualized percentages. The notation ‘-’ indicates a moment for which the data are either absent or unreliable. For example, we are not aware of a study that estimates the *real* Bilson-Fama coefficient using real interest rates (which are different than realized real returns on nominal bonds). Similarly, the unreliability of monthly U.S. consumption growth for ascertaining persistence is well known. Consumption moments that are reported are based on the standard monthly U.S. series and taken from Bansal and Shaliastovich (2013). The cross-country consumption correlation is representative of data reported by Brandt, Cochrane, and Santa-Clara (2006). Real interest rate moments are taken from Lochstoer and Kaltenbrunner (2010). Data on foreign and domestic inflation are based on authors own calculations using monthly data from *Datastream*, 1987-2012. The foreign country is Australia whereas the domestic country is the U.S.. Note that the Australian inflation data is problematic relative to its U.S. counterpart. Among other things, it is only available at

the quarterly frequency. The above estimates are based on using quarterly data and then scaling things down by factors that match the ratio of U.S. quarterly-to-monthly inflation moments. Calculations and data are available upon request.

Table 4
Sample and Population Moments

Moment	Data	Model	
<i>Panel A: The Real Economy</i>			
Consumption Growth (x_t, x_t^*)			
Mean	1.800	1.800	
Standard Deviation	2.720	2.720	
Autocorrelation	–	0.081	
Cross-Country Correlation	0.350	0.350	
Real Interest Rate (r_t, r_t^*)			
Mean	0.860	0.860	
Standard Deviation	0.970	0.531	
Autocorrelation	0.840	0.990	
Real Depreciation Rate ($\log(n_t^*/n_t)$)			
Standard Deviation	11.410	11.410	
Real UIP Coefficient	–	-1.476	
<i>Panel B: The Nominal Economy</i>			
Inflation (π_t, π_t^*)			
<i>Domestic, U.S.</i>		<i>Model LRR I</i>	<i>Model LRR II</i>
Mean	2.833	2.833	2.833
Standard Deviation	0.911	0.911	0.943
Autocorrelation	0.428	0.991	0.991
Correlation(x_t, π_t)	-0.300	-0.283	-0.283
<i>Foreign, Australia</i>			
Mean	3.199	3.199	3.199
Standard Deviation	0.985	0.985	0.953
Autocorrelation	0.429	0.991	0.991
Correlation(x_t^*, π_t^*)	-0.300	-0.283	-0.283
Nominal Interest Rate (i_t, i_t^*)			
<i>Domestic, U.S.</i>			
Mean	4.304	3.733	3.734
Standard Deviation	2.584	0.406	0.437
Autocorrelation	0.992	0.988	0.989
<i>Foreign, Australia</i>			
Mean	7.076	4.102	4.101
Standard Deviation	3.558	0.477	0.446
Autocorrelation	0.994	0.989	0.989
Nominal Depreciation Rate ($\log(m_t^*/m_t)$)			
Mean	1.675		
Standard Deviation	11.398	11.410	11.410
Autocorrelation	0.052		
Nominal Currency Risk Variables			
Nominal UIP Coefficient	-1.019	0.248	-0.997
Uncond. Risk Premium on AUD, $-E(p_t)$	4.459	0.003	0.000

See caption for Table 3.