Quantifying Contagion Risk in Funding Markets: A Model-Based Stress-Testing Approach

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The subprime crisis was put in motion on Aug 9th, 2007
- BNP Paribas announced it had suspended withdrawals from three investment funds exposed to U.S. subprime mortgages

News triggered general market anxiety about the extent of other banks’ exposures to sub-prime mortgages and solvency
- Exacerbated by the opacity of banks’ balance sheets

Funding conditions deteriorated for all banks
“Good news”

- Flip side – good news can have a positive market impact

- The Supervisory Capital Assessment Program (SCAP)
  - Stress-tests conducted by the Federal Reserve on U.S. banks
  - First conducted in 2009 – midst of the crisis
  - Yielded credible results for prospective losses for banks
  - Helped restore confidence in the banking system
Information contagion and stress testing

- **Information contagion** – key driver in financial crises
- Modeling / quantifying contagion is crucial for **stress testing**
  - Identify vulnerabilities within financial systems
  - Support crisis management and resolution
Our contribution

- **We present a model-based stress-testing framework**
  - Banks’ solvency risks, funding liquidity risks and market risks are intertwined due to information contagion

- **Frictions**
  - Coordination failure
  - Asymmetric information

- **Used by the BoC in regular stress-testing of banks (MFRAF)**
Outline of Presentation

Overview

Model

Equilibrium

Stress testing

Conclusion
Overview
Our model

- Solvency risks
  - Exogenous
  - Stress-test scenario

- Funding liquidity risks
  - Endogenous
  - Coordination failures between a bank's creditors
  - Global games (Morris and Shin, 2009)
Our model

- **Market risks**
  - Collateral haircuts – influences banks' recourse to liquidity
  
  \[
  \text{Macro-economy} = \begin{cases} 
  \text{“Good”} & \rightarrow \text{low haircuts} \\
  \text{“Bad”} & \rightarrow \text{large haircuts} 
  \end{cases}
  \]

- Investors entertain prior beliefs on the macro-economy

- Bank failure → Beliefs updated → “Bad” state more probable
Our results

- **Vicious illiquidity:** Investors’ pessimism over the macro-economy hampers the bank’s recourse to liquidity
  - Influences the incidence of bank runs
  - Investors turn more pessimistic
  - Driving down other banks’ recourse to liquidity

- **Virtuous liquidity:** Investors’ are optimistic to start with
  - Banks are more likely to survive solvency shocks
  - Investors turn more optimistic over asset quality
  - Other banks’ recourse to liquidity improves
Our results

- **Haircut spread**: An increase in the haircut-spread heightens the illiquidity channel
  - Larger spread $\rightarrow$ greater uncertainty over macro-economy
  - Investors are more inclined to believe that banks fail because the macro-economy is in the “bad” state

- **Convergence**: For a system of $N \geq 2$ banks, a unique equilibrium is always reached after, at most, $N$ iterations
  - Simple induction argument
MODEL
Agents and environment

- Three dates $t = 0, 1, 2$, and no time discounting
  - Map to an annual time-horizon

- $N = 2$ banks, $b \in \{1, 2\}$

- Two groups of risk-neutral agents
  - Banks’ creditors; can consume in $t = 1$ or $t = 2$
  - Outside deep-pocketed investors; consume at $t = 2$

- Interim date $t = 1$ is divided into two rounds
## Balance sheet in period 2

<table>
<thead>
<tr>
<th></th>
<th>Risky Investments</th>
<th>“Short-term” Debt</th>
<th>“Long-term” Debt</th>
<th>Liquid Assets</th>
<th>Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y^b - S_1^b - S_2^b$</td>
<td>$ST^b$</td>
<td>$LT^b$</td>
<td>$M^b$</td>
<td>$E^b = CET1 + In - Div - S_1^b - S_2^b$</td>
</tr>
</tbody>
</table>
Insolvency

- Bank $b$ is insolvent in period 2 whenever $E^b - S^b_1 - S^b_2 < 0$

- However, illiquidity in period 1 can also trigger insolvency
Recourse to liquidity in period 1 (round 1)

- Banks repo risky assets with investors for liquidity
  - Reversed in period 2

- **Pro-cyclical haircuts**: depend on the macro-economy
  - “Good” ($m = 1$) – small haircut; $\psi_H < 1$ of liquidity
  - “Bad” ($m = 0$) – large haircut; only $\psi_L < \psi_H$ of liquidity
Recourse to liquidity in period 1 (round 1)

- State $m$ realized in period 2 – **no one knows the state**
  - Investors do not observe banks’ shocks
  - Prior belief: $w_1 = \text{Prob}(m = 1)$

- Bank $b$’s recourse to liquidity is

$$M^b + \left\{ w_1 \psi_H + (1 - w_1) \psi_L \right\} (Y - S^b_1) = \psi^1$$
Rollover risk in period 1 (round 1)

- The rollover decisions of bank $b$'s “short-term” creditors at round 1 modeled as a binary-action simultaneous move game

<table>
<thead>
<tr>
<th></th>
<th>Solvent</th>
<th>Insolvent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not to withdraw</td>
<td>$1 + r^b$</td>
<td>0</td>
</tr>
<tr>
<td>Withdraw</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Rollover risk in period 1 (round 1)

- If a fraction $\ell^b_1 \in [0, 1]$ creditors withdraw, bank $b$ is illiquid if

$$\ell^b_1 > \lambda^b \left( S^b_1; \overline{\psi}^1 \right) \equiv \frac{M^b + \overline{\psi}^1 \left[ Y^b - S^b_1 \right]}{ST^b}$$

- We refer to $\lambda^b$ as the **balance sheet liquidity** for bank $b$.
Rollover risk in period 1 (round 2)

- Indicator $\eta_1^b \in \{0, 1\}$ for the outcome of bank $b$ after round 1

- End of round 1, bank $b$ is
  \[
  \begin{cases} 
  \text{liquid} & \rightarrow \eta_1^b = 0 \\
  \text{illiquid} & \rightarrow \eta_1^b = 1 
  \end{cases}
  \]

- Investors update their belief $w_2 = \text{Prob (} m = 1 | \eta_1^1, \eta_1^2 \text{)}$
Rollover risk in period 1 (round 2)

- Change to liquid bank(s) recourse to liquidity ("margin call")

\[ \bar{\psi}^2 = w_2 \psi_H + (1 - w_2) \psi_L \]

- Creditors of liquid bank(s) decide to withdraw in round 2
  - Payoffs same as in round 1

- If a fraction \( \ell^b_2 \in [0, 1] \) of "short-term" creditors from (liquid) bank \( b \) withdraw, then bank \( b \) is illiquid if

\[ \ell^b_2 > \lambda^b \left( S^b_1; \bar{\psi}^2 \right) \]
## Model timeline

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>$t = 1$ (round 1)</th>
<th>$t = 1$ (round 2)</th>
<th>$t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Initial balance sheet</td>
<td>1. Interim shock</td>
<td>1. Belief updated</td>
<td>1. Final shock</td>
</tr>
<tr>
<td>2. Private signals</td>
<td>2. “Margin calls”</td>
<td>2. Incomes accrued</td>
<td></td>
</tr>
<tr>
<td>3. $ST$ debt withdrawals</td>
<td>3. New private signals</td>
<td>3. Dividends paid</td>
<td></td>
</tr>
<tr>
<td>4. $ST$ debt withdrawals</td>
<td></td>
<td>4. State $m$ realized</td>
<td></td>
</tr>
</tbody>
</table>
EQUILIBRIUM
Tripartite classification of shock

- With common knowledge about the shock, in each round
  
  - Solve for the Bayes-Nash equilibrium in each round
    - Creditors of bank $b$ receive a noisy signal on $S^b$
    - The noise is i.i.d across creditors and rounds
Critical illiquidity threshold

In the limit of vanishing private noise, there exists a unique equilibrium in threshold strategies, \( S_{d}^{b*} \), where bank \( b \) is illiquid if and only if \( S_{1}^{b} > S_{d}^{b*} \).

The threshold is implicitly defined by the indifference condition for the expected payoff to a creditor between rolling over and withdrawing:

\[
F_{2}^{b}(E^{b} - S_{d}^{b*}) \lambda^{b}(S_{d}^{b*} ; \overline{\psi}^{d}) = \frac{1}{1 + r^{b}}.
\]

21 / 31
Virtuous liquidity

If both banks are liquid at the end of round 1, then \( w_2 > w_1 \). Consequently, both banks remain liquid at the end of round 2.
Suppose bank $i$ is liquid and bank $j$ is illiquid after round 1. The investors become more pessimistic, $w_2 < w_1$, whenever:

\[
\frac{\text{Prob}(\eta_1^i = 0 \mid m = 1)}{\text{Prob}(\eta_1^i = 0 \mid m = 0)} < \frac{\text{Prob}(\eta_1^j = 1 \mid m = 0)}{\text{Prob}(\eta_1^j = 1 \mid m = 1)}.
\]

If the downward revision of the belief is large enough, then bank $i$ will also become illiquid at the end of round 2.
Price and spread effects

For a given initial belief, $w^1$, and “bad” state haircut, $\psi_L$, an increase in the “good” state haircut, $\psi_H$, increases the spread, $\Delta = \psi_H - \psi_L$. This, in turn, strengthens the pessimism condition and increases the range of parameters where the investor’s belief is revised downwards.

On the other hand, for a given “good” state haircut, $\psi_H$, an increase in the “bad”, $\psi_L$, leads to a decrease in the spread. This weakens the pessimism condition and reduces the range of parameters where the investor’s belief is revised downwards.
In a game involving $N \geq 2$ banks, the cycles of Bayesian updating by investors and withdrawal by creditors terminates after, at most, $N$ rounds.
STRESS TESTING
Macro Stress Tests in Canada

- Annual exercise involving Canadian D-SIBS

- **Objective**: Assess the resilience of the financial system to extreme but plausible shocks

- MST scenario development

- Bottom-up exercise
  - Banks apply MST scenario to their balance sheets
  - Focus on solvency risk only

- Top-down exercise
  - The Macro Financial Risk Assessment Framework (MFRAF)
The MFRAF: Structure

Solvency risk module
- Macroeconomic and financial shocks materialize.
- Banks suffer losses due to credit risk and market risk.

Liquidity risk module
- Creditors have concerns over banks’ funding strategies and solvency.
- Creditors withdraw their claims on banks.

Systemic risk module
- Contagion between investors' beliefs and creditors' withdrawals and interbank spillovers.
- System-wide losses distribution.
The MFRAF: Calibration

- Macroeconomic scenario draws on Canada’s 2013 FSAP
- Canadian D-SIBs’ balance sheet – 2013Q1
  - Average CET1 ratio – 8.9%
  - Liabilities maturity within 6 months – 35% of all liabilities
- Front-load income onto bank’s capital
- “Insolvency” if capital falls below 7% CET1 capital
- Baseline
  - Identical asset portfolios and losses
  - Banks differ in their liability structures
  - Market liquidity parameters: $\psi_H = 0.3$ and $\psi_L = 0.2$
The MFRAF: Results

- Average balance sheet liquidity $= 1.08$

<table>
<thead>
<tr>
<th>Bank</th>
<th>Solvency</th>
<th>Liquidity</th>
<th>Contagion</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47.0</td>
<td>22.9</td>
<td>0.0</td>
<td>69.9</td>
</tr>
<tr>
<td>2</td>
<td>47.0</td>
<td>0.0</td>
<td>0.0</td>
<td>47.0</td>
</tr>
<tr>
<td>3</td>
<td>47.0</td>
<td>23.0</td>
<td>0.6</td>
<td>70.6</td>
</tr>
<tr>
<td>4</td>
<td>47.0</td>
<td>0.0</td>
<td>19.2</td>
<td>66.2</td>
</tr>
<tr>
<td>5</td>
<td>47.0</td>
<td>0.0</td>
<td>0.0</td>
<td>47.0</td>
</tr>
<tr>
<td>6</td>
<td>47.0</td>
<td>22.2</td>
<td>0.8</td>
<td>70.0</td>
</tr>
</tbody>
</table>
The MFRAF: System-wide loss distribution
Conclusion

- We offer a model-based stress-testing framework
  - Information contagion amplifies banks’ funding liquidity risks
  - Use Global games to solve for unique equilibrium

- Uses in policy
  - Consistency check for bottom-up results
  - Considers impact of second-round effects over and above the (solvency only) bottom-up stress-test
  - Quantifies liquidity assistance required to avoid runs

Thank you!
Related literature

- Chen (1999) – Heterogeneous information amongst depositors are responsible for runs

- Acharya and Yorulmazer (2008) – Ex-post information contagion leads to ex-ante herding, with banks undertaking correlated investments

- Li and Ma (2013) – Most similar to our paper; coordination failure and adverse selection mutually reinforce each other, leading to bank runs and fire-sales

- Many models of stress-testing, e.g., Elsinger et al. (2006), Alessandri et al. (2009), and Gauthier et al. (2012)
References


