Optimal Collateralization with Bilateral Default Risk

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CenFIS-CEAR workshop – The Role of Liquidity in the Financial System
November 20, 2015
1. Overview
2. A model
3. Optimal CSAs
4. Policy Implications
5. Conclusion
MOTIVATION

Regulation of OTC derivative markets (Dodd-Frank/EMIR)

- Move to central clearing for standardized, liquid OTC derivatives
  - Do CCP increase systemic/counterparty risk? (Acharya/Bisin, 2011; Biais/al., 2012; Pirrong, 2011; Duffie/Zhu, 2011; Cont/Kokholm, 2013; etc.)

- How about non-standardized, illiquid OTC instruments that will not be centrally cleared?
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Table 3: Non-centrally cleared derivative activity before and after central clearing takes effect

<table>
<thead>
<tr>
<th></th>
<th>Total gross notional outstanding amount (EUR million)</th>
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<tbody>
<tr>
<td></td>
<td>Foreign exchange</td>
</tr>
<tr>
<td>Before</td>
<td>54,958,056</td>
</tr>
<tr>
<td>After</td>
<td>47,863,156</td>
</tr>
<tr>
<td>% Reduction</td>
<td>13%</td>
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Note: The data above reflect the notional amount of non-centrally cleared derivative activity that will remain after central clearing mandates take effect (future portfolio). Each cell represents the simple sum of non-centrally cleared derivative notional amounts for each QIS respondent within each asset class and jurisdiction.

Source: BIS (2013)
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- How about **non-standardized, illiquid** OTC instruments that will **not** be centrally cleared?

**Counterparty risk** mitigation
- Multi-curve valuation (OIS, EUREPO, etc.)
- Credit Support Annex (CSA), ISDA rules
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Counterparty risk is bilateral
- Bilateral CSA design/pricing
- xVA (CVA/DVA/FVA etc.) reporting/trading/hedging
STYLIZED TRANSACTION

- Risk-averse agents $A, B$, trade in financial market, face illiquid exposure
- Party $A$ exposed to random outflow $-Z_T$, party $B$ to random inflow $Z_T$
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\[ -kZ_T \]

\[ kZ_T \]

Party A (hedger) \hspace{2cm} Party B (hedge supplier)
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- Party A (hedger)
- Party B (hedge supplier)

• Symmetric, exogenous default rate $\lambda > 0$. Collateral account: $C_t^A = -C_t^B$. 
QUESTIONS AND FINDINGS

CSA pricing/design

- OTCD pricing with bilateral default risk (Duffie/Huang, 1996, Brigo/al., 2007, Crepey, 2011, Hull/White, 2010, etc.): exogenous pricing kernel, no collateral
- Biffis/al. (2011), Brigo/al. (2012): CSA pricing, but exogenous collateral rules
  - Here endogenous collateral explaining observed CSAs
  → What is the optimal collateral design in a marginal trade?
**QUESTIONS AND FINDINGS**

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→ **What is the optimal collateral design in a marginal trade?**

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**Intuition:**

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→ Partial collateralization optimal
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- Intuition:
  - A in the money
  - B in the money

- Collateral augments risk sharing opportunities
- **BUT**: Different valuation in default vs. non-default states

→ **Partial** collateralization optimal
- Borrowing costs, collateral segregation, contagion, etc. make result stronger
QUESTIONS AND FINDINGS

Impact of Dodd-Frank/EMIR provisions

- Standardized CSAs: **Initial Margin (IM)** and **Variation Margin (VM)**

→ **What is the impact of imposing collateral rules, particularly “full” collateralization?**

- Intuition:
  - Only way to shift resourced between default states is by extent of risk sharing

→ **Decreases volume of risk sharing arrangements** (liquidity)
SETUP

- Agent $i \in \{A, B\}$, endowed with wealth $w_0^i$, CARA utility, default intensity $\lambda > 0$.
- Tradeable assets (only accessible in non-default states; Alvarez/Jermann, 2000)
  - Money market account yielding $r > 0$
  - Risky asset $dS_t = S_t \left( \mu dt + \sigma S \, dB_t^{(1)} \right)$
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- Trading account dynamics

\[
dW_t^i = W_t^i r dt + \pi_t^i \left( (\mu - r) dt + \sigma_S dB_{t}^{(1)} \right)
\]
Agent $i \in \{A, B\}$, endowed with wealth $w_i^0$, CARA utility, default intensity $\lambda > 0$.

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- Trading account dynamics
  \[
dW_t^i = W_t^i rd t + \pi_t^i \left( (\mu - r)dt + \sigma_S dB_t^{(1)} \right)
  \]

- A has exposure $-Z_T$ at time $T > 0$, B has exposure $+Z_T$, with
  \[
dZ_t = \sigma_Z dB_t^{(2)}, \quad Z_0 = 0
  \]
  - $Z$ illiquid: agent A has terminal wealth $W_T^A - Z_T$
  - $Z$ unspanned: $B^{(1)} \perp B^{(2)}$
Agent \( i \in \{A, B\} \), endowed with wealth \( w^i_0 \), CARA utility, default intensity \( \lambda > 0 \).

Tradeable assets (only accessible in non-default states; Alvarez/Jermann, 2000)
- Money market account yielding \( r > 0 \)
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Trading account dynamics

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- \( Z \) illiquid: agent A has terminal wealth \( W^A_T - Z_T \)
- \( Z \) unspanned: \( B^{(1)} \perp B^{(2)} \)

Agents can enter a **forward** agreement on \( k \) units of \( Z_T \), but are exposed to **counterparty risk**
PROBLEMS

- Focus on A for convenience
- Symmetry between A’s and B’s views

**Problem 1** (no counterparty risk)

\[
\begin{cases}
\sup_{(k, \pi^A) \in \mathbb{R} \times A} U \left( W^A_T - (1 - k) Z_T \right) \\
\text{s.t.} \\
\quad dW^A_t = W^A_t r dt + \pi^A_t \left( (\mu - r) dt + \sigma dB^{(1)}_t \right)
\end{cases}
\]
Focus on A for convenience

Symmetry until \( \tau := \tau^A \land \tau^B \) \((N_t := 1_{\tau \leq t})\)

**Problem 2** (counterparty risk, **Zero CSA**)

\[
\sup_{(k, \pi^A) \in \mathbb{R} \times A_\pi} U \left( W_T^A - (1 - k1_{\tau > T})Z_T \right)
\]

\[
s.t. \quad dW_t^A = N_t^A W_t^A r dt
\]

\[
+ \text{ trading gains if no default}
\]

\[
+ (1 - N_{t-}) \left((R_{t-}^A)^+ dN_t^A - (R_{t-}^A)^- dN_t^B \right)
\]

\( \tau^i \) default time of agent \( i \) and \( N_t^i := 1_{\tau^i \leq t} \)

\( R^i \) (replacement cost) depends on close-out convention...
PROBLEMS

- Focus on A for convenience
- Symmetry until $\tau := \tau^A \land \tau^B$ ($N_t := 1_{\tau \leq t}$)

**Problem 3** (counterparty risk, **General CSA**)

\[\begin{align*}
\sup_{(k, C^A, \pi^A)} & \quad U \left( W_T^A - (1 - k1_{\tau > T})Z_T - 1_{\tau > T}C_T^A \right) \\
\text{s.t.} & \quad dW_t^A = N_t^A W_t^A rdtdN_t^A + \text{trading gains if no default} \\
& \quad + (1 - N_{t-}) \left[ dC_t^A - rC_t^A dt \\
& \quad + ((R_t^A)^+ - (C_t^A)^+) \right] dN_t^A \\
& \quad + ((R_t^A)^- - (C_t^A)^-) \right] dN_t^B
\end{align*}\]

- Collateral fully fungible; interest rebated on cash collateral
OPTIMAL CSAs

Replacement cost

- Default-risk-free, risk-neutral close-out convention
- \[ R_t^A = E_t \left[ e^{-r(T-t)} Z_T \right] = k e^{-r(T-t)} Z_t \]

Admissible CSAs

- Fractional collateral, \( C_t^A = c(t) R_t^A \)
OPTIMAL CSAs

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OPTIMAL CSAs

Replacement cost

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- Contingent collateral, $$C_t^A = f(t, W^A, Z, C^A, R^A)$$
- Standardized margins (Dodd-Frank/EMIR), $$C_t^A = \tilde{c} + R_t^A$$
  - Two-way Initial Margin, $$\tilde{c}$$ (VaR-based, segregated)
  - Variation Margin, $$c(t) = 100\%$$ (‘full’ collateralization)
OPTIMAL CSAs

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What is optimal for \( A, B \)?

- ‘Less-than-full’ collateralization (e.g., \( c^*(t) < 1 \))
OPTIMAL FRACTIONAL CSA

![Graph showing collateral fraction vs. 1/10 of maturity T]

- **T = 1**
- **T = 10**

The graph illustrates the relationship between collateral fraction and 1/10 of maturity T for different values of T, highlighting the optimal fractional CSA.
COMPARATIVE STATICS: FRACTIONAL COLLATERAL

Optimal collateral fraction

\[ c^*(t) = 1 - \frac{1}{\gamma \sigma_Z \sqrt{t}} g^{-1} \left( 1 + \frac{\lambda + \frac{1}{2} s^2 \exp \left( - \left( \frac{1}{2} s^2 + \lambda \right) (T-t) \right)}{\frac{1}{2} s^2 \left( 1 - \exp \left( - \left( \frac{1}{2} s^2 + \lambda \right) (T-t) \right) \right)} \right), \]

with \( s := \frac{\mu - r}{\sigma_S} \) Sharpe ratio, \( g(x) := \Phi(x) + \frac{\phi(x)}{x} \), with \( \Phi \) and \( \phi \) the cdf and pdf of the standard Normal, respectively.

For fixed \( t \in (0, \tau \wedge T] \), the optimal collateral fraction \( c^* \) is

- decreasing in the Sharpe ratio \( s \)
- increasing in \( \sigma_Z, \lambda \), and the risk aversion coefficient \( \gamma \)
CONTINGENT COLLATERAL RULES

Same problem as before, same close-out convention, but larger CSA space

\[ C_t^A = e^{-r(T-t)} \int_0^t \hat{c}^A(s, W_s^A, -Z_s, C_s^A) dZ_s \]
CONTINGENT COLLATERAL RULES

Same problem as before, same close-out convention, but larger CSA space

\[ C^A_t = e^{-r(T-t)} \int_0^t \widehat{c}^A(s, W^A_s, -Z_s, C^A_s) dZ_s \]

Results

- \( \widehat{c}^i,^* \) independent of \( W^i \)
- Optimal collateral fraction \( \widehat{c}^i,^* \) varies with \( (Z, C^i) \)
  - Consistent with collateral triggers/thresholds observed in practice
  - CSA can take into account collateral performance (relevant for type/quality other than cash)
- Same intuition as before, but larger utility gains
WHAT DRIVES THE OPTIMAL CSA?

Wedge between default states and no-default states
- Default penalties, exclusion from the financial market
- Collateral allows to move resources between states [I default & pay (OTM)] and [Other defaults & pays (ITM)]
- Optimal trading volume additional lever to transfer
- Optimal choice features “overhedging” ($k > 1$) and partial collateral ($C' < 1$)

Extensions
- Segregation will not affect results, fee by custodian strengthens results
- Borrowing cost will strengthen results
- Contagion in the sense that default rate increases (Jarrow/Yu, 2001) strengthens results
OUTLINE

1. Overview
2. A model
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THE COST OF COUNTERPARTY RISK

-0.75
-0.8
-0.85
-0.9
-0.95
-1

Horizon T

Expected Utility

No default risk
THE COST OF COUNTERPARTY RISK

![Graph showing the expected utility for different horizons with and without collateral or default risk.](image-url)
(SUB)OPTIMAL COLLATERALIZATION

Expected Utility

Horizon T

-0.75
-0.8
-0.85
-0.9
-0.95

Optimal Collateral Fraction
No collateral
No default risk
(SUB)OPTIMAL COLLATERALIZATION

![Graph showing Expected Utility against Horizon T for different scenarios: Optimal collateral fraction, IM only, No collateral, No default risk. The y-axis represents Expected Utility ranging from -1 to 0, and the x-axis represents Horizon T ranging from 1 to 10. The graph illustrates the impact of collateralization on expected utility over different horizons.]
(SUB)OPTIMAL COLLATERALIZATION

Expected Utility vs. Horizon T

- Optimal collateral fraction
- IM and full collateral
- IM only
- No collateral
- No default risk

Overview
A model
Optimal CSAs
Policy Implications
Conclusion
(SUB)OPTIMAL COLLATERALIZATION

![Graph showing Expected Utility vs. Horizon T]

- **Expected Utility** vs. **Horizon T**
- Lines represent different scenarios:
  - Optimal collateral fraction
  - IM and full collateral
  - IM only
  - No collateral
  - No default risk
  - Optimal contingent CSA

The graph illustrates the expected utility for different collateralization strategies over time.
TRADING VOLUME

![Graph showing the relationship between horizon T and the optimal collateral fraction \( \kappa^* \). The graph includes two lines: one for the optimal collateral fraction and another for IM and full collateral.](image-url)
A POLICY EXPERIMENT

Benevolent social planner

- Maximizes the agents’ expected utilities, while minimizing the expected shortfalls from defaults

\[
\sum_{i \in \{A,B\}} E \left[ 1_{\tau^i \leq T} (W^i_T + Z^i_T)^- \right], \quad \text{with} \quad Z^i_T := (1_{i=B} - 1_{i=A}) Z_T
\]

- Standardized margins can be more costly than bilateral CSAs due to detrimental effect on risk sharing (lower trading volume)
  - Collateral (VM in particular) is overall **bad** in **single default states**
  - Collateral is **good** in **joint default states**, but IM is what matters
EXPECTED SHORTFALLS

![Graph showing Expected Bailout Costs vs. Horizon T]

- **Expected Bailout Costs**
- **Optimal collateral fraction**

The graph illustrates the relationship between Expected Bailout Costs and Horizon T, emphasizing the optimal collateral fraction as a function of Horizon T.
EXPECTED SHORTFALLS

![Graph showing expected bailout costs vs horizon T]

- **Expected Bailout Costs**
- **Horizon T**

Lines:
- Red: IM + full collateral
- Blue dashed: Optimal collateral fraction
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CONCLUSION

Hedging demand and **CSA design** for bilateral OTC trades

- Optimal CSA results in **undercollateralization**
- Overcollateralization lowers **risk sharing** (hedging volume)
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CSAs have **several** important dimensions

- Collateral rules (what is ‘full’ collateralization?)
- MTM proxies, **valuation** models
- **Close out** conventions
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Bilateral CSAs vs. **standardized margins** (Dodd-Frank/EMIR)

- Detrimental impact on **risk sharing** should be taken into account when assessing the costs/benefits of standardization
- Tradeoff liquidity vs. systematic risk?
- Both IM and VM matter, and in different ways
THANK YOU