Optimal Collateralization with Bilateral Default Risk

Daniel Bauer
Georgia State University

Enrico Biffis
Georgia State University

Luz Rocio Sotomayor
University of Washington at Vancouver

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OUTLINE

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MOTIVATION

Regulation of OTC derivative markets (Dodd-Frank/EMIR)

- Move to central clearing for standardized, liquid OTC derivatives
  - Do CCP increase systemic/counterparty risk? (Acharya/Bisin, 2011; Biais/al., 2012; Pirrong, 2011; Duffie/Zhu, 2011; Cont/Kokholm, 2013; etc.)
- How about **non-standardized, illiquid** OTC instruments that will **not** be centrally cleared?

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Table 3: Non-centrally cleared derivative activity before and after central clearing takes effect

	Total gross notional outstanding amount (EUR million)						
	Foreign exchange	**Interest rate**	**Credit**	**Equity**	**Commodity**	**Other**	**Total**
Before	54,958,056	230,135,986	24,264,950	6,596,400	2,026,853	514,734	318,496,980
After	47,863,156	107,208,907	12,132,371	2,908,279	1,211,562	408,843	171,733,118
% Reduction	13%	53%	50%	56%	40%	21%	46%

Note: The data above reflect the notional amount of non-centrally cleared derivative activity that will remain after central clearing mandates take effect (future portfolio). Each cell represents the simple sum of non-centrally cleared derivative notional amounts for each QIS respondent within each asset class and jurisdiction.

Source: BIS (2013)

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Counterparty risk mitigation

- Multi-curve valuation (OIS, EUREPO, etc.)
- Credit Support Annex (CSA), ISDA rules

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Counterparty risk is **bilateral**

- Bilateral CSA design/pricing
- xVA (CVA/DVA/FVA etc.) reporting/trading/hedging

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- Risk-averse agents A, B, trade in financial market, face illiquid exposure
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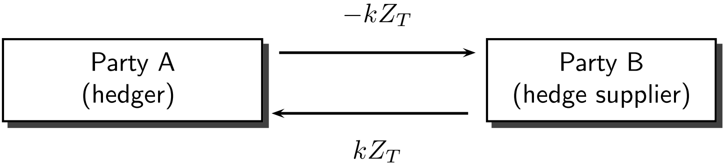

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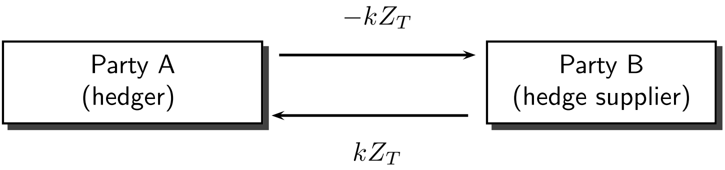

- Symmetric, exogenous default rate $\lambda > 0$. Collateral account: $C_t^A = -C_t^B$.

QUESTIONS AND FINDINGS

CSA pricing/design

- OTCD pricing with bilateral default risk (Duffie/Huang, 1996, Brigo/al., 2007, Crepey, 2011, Hull/White, 2010, etc.): exogenous pricing kernel, no collateral
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- Here **endogenous collateral** explaining observed CSAs

→ **What is the optimal collateral design in a marginal trade?**

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- Intuition:

	A defaults	B defaults
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- Borrowing costs, collateral segregation, contagion, etc. make result stronger

QUESTIONS AND FINDINGS

Impact of Dodd-Frank/EMIR provisions

- Standardized CSAs: **Initial Margin (IM)** and **Variation Margin (VM)**
- Aggregate cost of collateralization: Singh/Aitken (2009) [IMF], Heller/Vause (2012) [IMF], Sidanius/Zikes (2012) [BoE], BIS (2012,2013), ISDA (2013), etc.; **focus on IM**

→ **What is the impact of imposing collateral rules, particularly "full" collateralization?**

- Intuition:
  - Only way to shift resourced between default states is by extent of risk sharing

→ **Decreases volume of risk sharing arrangements** (liquidity)

OUTLINE

SETUP

- Agent $i \in \{A, B\}$, endowed with wealth w_0^i, CARA utility, default intensity $\lambda > 0$.
- Tradeable assets (only accessible in non-default states; Alvarez/Jermann, 2000)
  - Money market account yielding $r > 0$
  - Risky asset $\mathrm{d}S_t = S_t \left(\mu \mathrm{d}t + \sigma_S \mathrm{d}B_t^{(1)} \right)$

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$$\mathrm{d}W_t^i = W_t^i r \mathrm{d}t + \pi_t^i \left((\mu - r)\mathrm{d}t + \sigma_S \mathrm{d}B_t^{(1)} \right)$$

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$$\mathrm{d}Z_t = \sigma_Z \mathrm{d}B_t^{(2)}, \quad Z_0 = 0$$

  - Z illiquid: agent A has terminal wealth $W_T^A - Z_T$
  - Z unspanned: $B^{(1)} \perp B^{(2)}$

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- Agents can enter a **forward** agreement on k units of Z_T, but are exposed to **counterparty risk**

PROBLEMS

- Focus on A for convenience
- Symmetry between A's and B's views

- **Problem 1** (no counterparty risk)

$$\begin{cases} \sup_{(k,\pi^A)\in\mathbb{R}\times\mathcal{A}_\pi} & U\left(W_T^A - (1-k)Z_T\right) \\ s.t. & \mathrm{d}W_t^A = W_t^A r\mathrm{d}t + \pi_t^A\left((\mu - r)\mathrm{d}t + \sigma_S \mathrm{d}B_t^{(1)}\right) \end{cases}$$

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- Symmetry until $\tau := \tau^A \wedge \tau^B$ $(N_t := 1_{\tau \leq t})$

Problem 2 (counterparty risk, **Zero CSA**)

$$\begin{cases} \sup_{(k,\pi^A)\in\mathbb{R}\times\mathcal{A}_\pi} & U\left(W_T^A - (1 - k1_{\tau > T})Z_T\right) \\ s.t. & \mathrm{d}W_t^A = N_t^A W_t^A r \mathrm{d}t \\ & \quad + \text{ trading gains if no default} \\ & \quad + (1 - N_{t-})\left((R_{t-}^A)^+ \mathrm{d}N_t^A - (R_{t-}^A)^- \mathrm{d}N_t^B\right) \end{cases}$$

- τ^i default time of agent i and $N_t^i := 1_{\tau^i \leq t}$
- R^i (replacement cost) depends on close-out convention...

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Problem 3 (counterparty risk, **General CSA**)

$$\begin{cases} \sup_{(k, C^A, \pi^A)} & U\left(W_T^A - (1 - k1_{\tau > T})Z_T - 1_{\tau > T} C_T^A\right) \\ s.t. & \mathrm{d}W_t^A = N_t^A W_t^A r \mathrm{d}t \\ & \qquad + \text{ trading gains if no default} \\ & \qquad + (1 - N_{t-})\Big[\mathrm{d}C_t^A - rC_t^A \mathrm{d}t \\ & \qquad + \left((R_{t-}^A)^+ - (C_{t-}^A)^+\right) \mathrm{d}N_t^A \\ & \qquad + \left((R_{t-}^A)^- - (C_{t-}^A)^-\right) \mathrm{d}N_t^B\Big] \end{cases}$$

- Collateral fully fungible; interest rebated on cash collateral

OUTLINE

OPTIMAL CSAs

Replacement cost

- Default-risk-free, risk-neutral close-out convention
- $R_t^A = E_t\left[e^{-r(T-t)} Z_T\right] = k\, e^{-r(T-t)} Z_t$

Admissible CSAs

- Fractional collateral, $C_t^A = c(t) R_t^A$

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- Standardized margins (Dodd-Frank/EMIR), $C_t^A = \widetilde{c} + R_t^A$
  - ⋆ Two-way Initial Margin, $\widetilde{c}$ (VaR-based, segregated)
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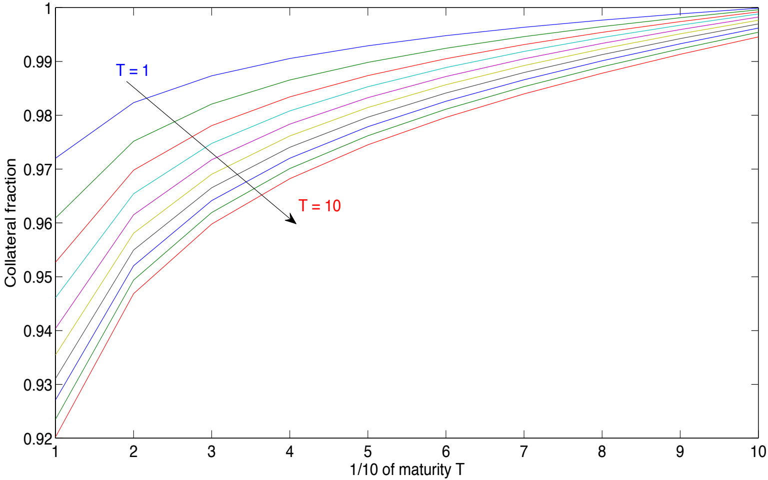
What is optimal for A, B?

- 'Less-than-full' collateralization (e.g., $c^*(t) < 1$)

OPTIMAL FRACTIONAL CSA



COMPARATIVE STATICS: FRACTIONAL COLLATERAL

Optimal collateral fraction

$$c^*(t) = 1 - \frac{1}{\gamma\,\sigma_Z\,\sqrt{t}}\,g^{-1}\left(1 + \frac{\lambda + \frac{1}{2}\,s^2\,\exp\left(-\left(\frac{1}{2}s^2 + \lambda\right)(T-t)\right)}{\frac{1}{2}\,s^2\,\left(1 - \exp\left(-\left(\frac{1}{2}s^2 + \lambda\right)(T-t)\right)\right)}\right),$$

with $s := \frac{\mu - r}{\sigma_S}$ Sharpe ratio, $g(x) := \Phi(x) + \frac{\phi(x)}{x}$, with Φ and ϕ the cdf and pdf of the standard Normal, respectively.

For fixed $t \in (0, \tau \wedge T]$, the optimal collateral fraction c^* is

- decreasing in the Sharpe ratio s
- increasing in σ_Z, λ, and the risk aversion coefficient γ

CONTINGENT COLLATERAL RULES

Same problem as before, same close-out convention, but larger CSA space

$$C_t^A = e^{-r(T-t)} \int_0^t \widehat{c}^A(s, W_s^A, -Z_s, C_s^A)\mathrm{d}Z_s$$

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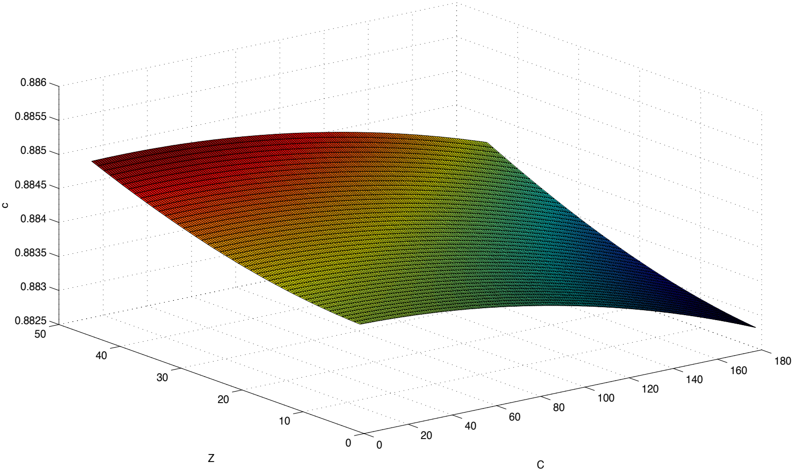
Results

- $\widehat{c}^{i,*}$ independent of W^i
- Optimal collateral fraction $\widehat{c}^{i,*}$ varies with (Z, C^i)
  - Consistent with collateral triggers/thresholds observed in practice
  - CSA can take into account collateral performance (relevant for type/quality other than cash)
- Same intuition as before, but larger utility gains

OPTIMAL CONTINGENT CSA



WHAT DRIVES THE OPTIMAL CSA?

Wedge between default states and no-default states

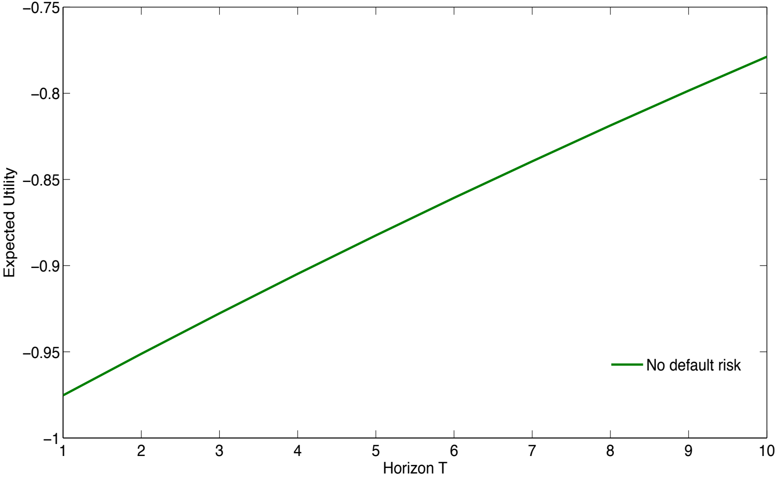
- Default penalties, exclusion from the financial market
- Collateral allows to move resources between states [I default & pay (OTM)] and [Other defaults & pays (ITM)]
- **Optimal trading volume** additional lever to transfer
- Optimal choice features “overhedging” $(k > 1)$ and partial collateral $(C < 1)$

Extensions

- Segregation will not affect results, fee by custodian strengthens results
- Borrowing cost will strengthen results
- Contagion in the sense that default rate increases (Jarrow/Yu, 2001) strengthens results

OUTLINE

THE COST OF COUNTERPARTY RISK



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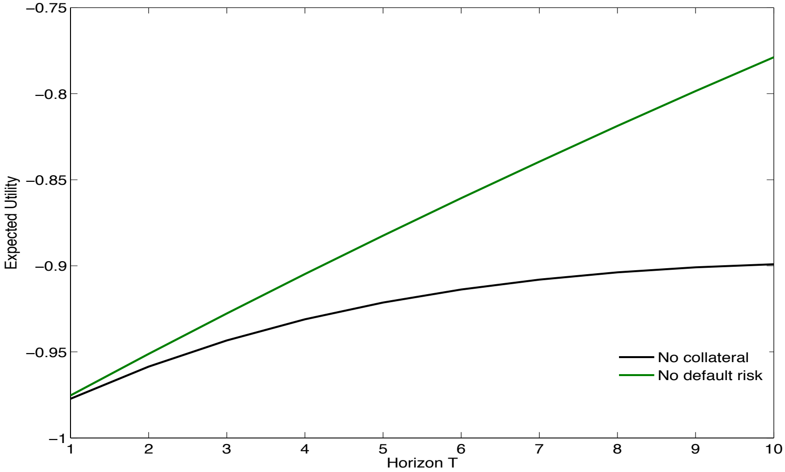

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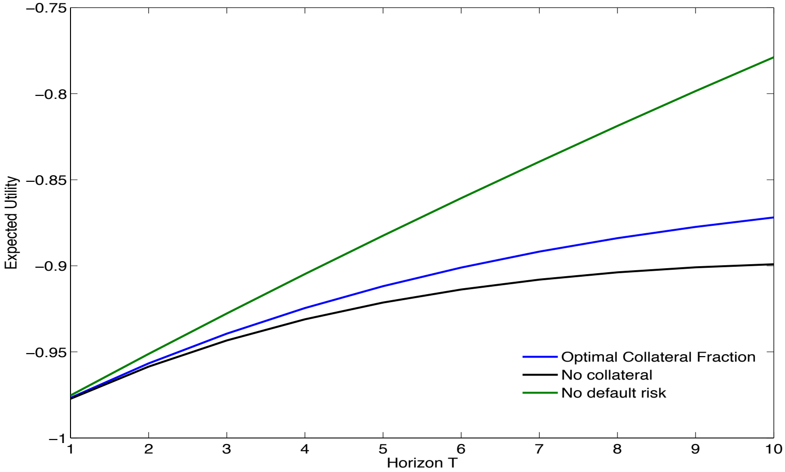

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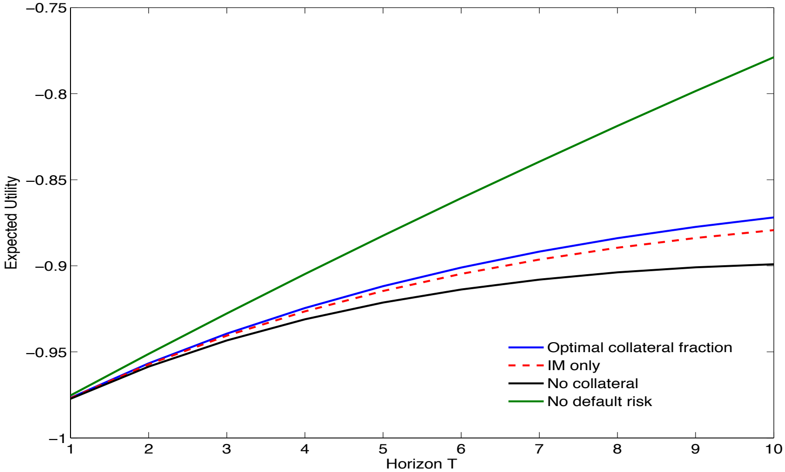

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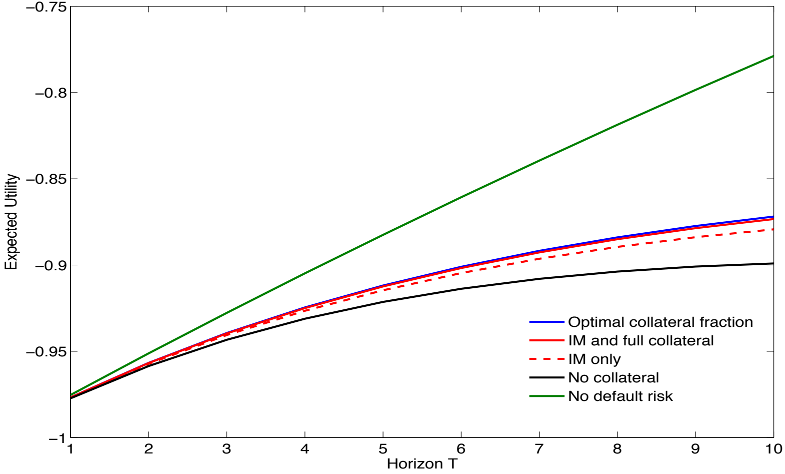

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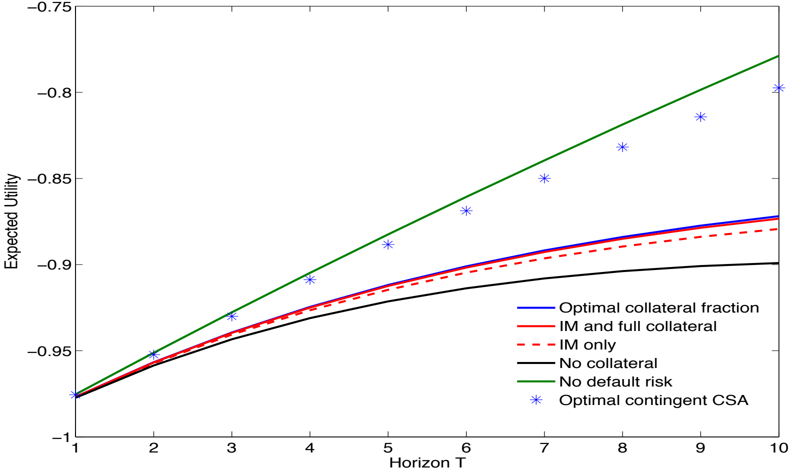
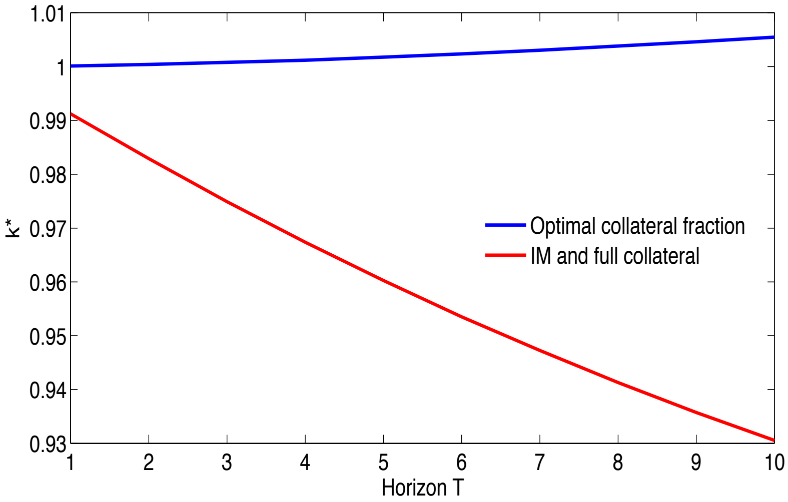


TRADING VOLUME



A POLICY EXPERIMENT

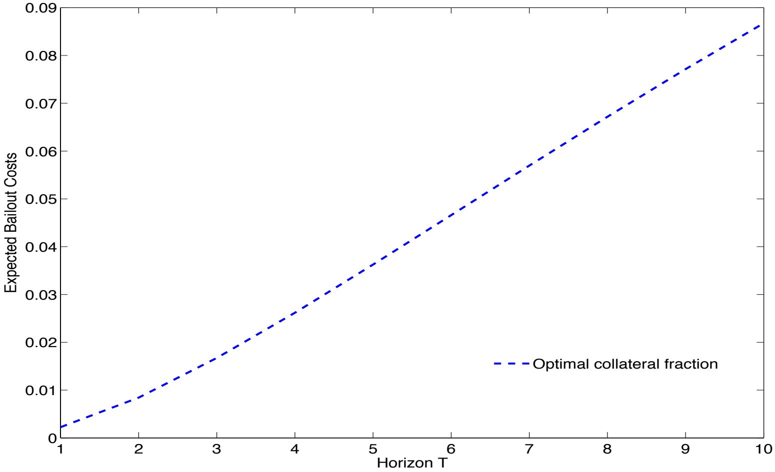
Benevolent social planner

- Maximizes the agents' expected utilities, while minimizing the expected shortfalls from defaults

$$\sum_{i\in\{A,B\}} E\left[1_{\tau^i\leq T}(W_T^i + Z_T^i)^-\right], \quad \text{with} \quad Z_T^i := (1_{i=B} - 1_{i=A})Z_T$$

- Standardized margins can be more costly than bilateral CSAs due to detrimental effect on risk sharing (lower trading volume)
  - Collateral (VM in particular) is overall **bad** in **single default states**
  - Collateral is **good** in **joint default states**, but IM is what matters

EXPECTED SHORTFALLS



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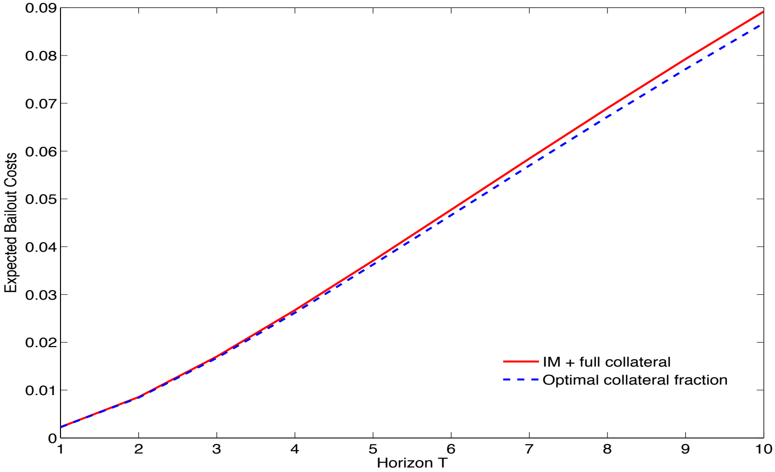

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- MTM proxies, valuation models
- Close out conventions

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Bilateral CSAs vs. **standardized margins** (Dodd-Frank/EMIR)

- Detrimental impact on risk sharing should be taken into account when assessing the costs/benefits of standardization
- Tradeoff liquidity vs. systematic risk?
- Both IM and VM matter, and in different ways

THANK YOU