Bank Regulation under Fire Sale Externalities

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November 19, 2015

Disclaimer: The analysis and the conclusions set forth are those of the authors and do not indicate concurrence by other members of the research staff or the Board of Governors.
Motivation: Background

- The recent crisis was characterized by liquidity problems.

- The regulation before the crisis was predominantly micro-prudential and focused on capital requirements.

- Basel III supplements capital regulations with liquidity requirements (such as LCR and NSFR) and focuses on macro-prudential measures.
Research Questions

This paper investigates the optimal design of capital and liquidity regulations in a model characterized by systemic externalities generated by asset fire sales. Our research questions are:

- Can we trust the institutions to properly manage their liquidity, once excessive risk taking has been controlled by the capital requirement?

- What are -if any- the advantages and disadvantages of liquidity requirements that supplement the capital regulations?
Agents

A continuum of banks with a unit mass.
A continuum of consumers with a unit mass.
A continuum of outside investors with a unit mass.
A financial regulator (e.g. a central bank).
Related Literature

**Financial Regulation**

**Asset Fire Sales**

**Incomplete Markets**
The Model: Basic Setup

Three dates: $t = 0, 1, 2$.

Two goods:
- A consumption good (liquid/safe asset)
- An investment good (illiquid/risky asset)

Banks can convert consumption goods into investment goods one-to-one at $t = 0$.

Banks choose risky asset level, $n_i$, at $t = 0$. 
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Two states of the world at $t = 1$:
- Good state with probability $1 - q$
- Bad state with probability $q$

The risky assets pay a return of $R$ at $t = 2$. 
Safe assets: Banks are endowed with a storage technology with unit returns.

A bank chooses how much safe assets to hold per unity of risky assets, \( b_i \in [0, 1] \).

A bank hoards total safe assets of \( n_i b_i \) at \( t = 0 \).

The total assets of a bank is \( n_i + n_i b_i = (1 + b_i) n_i \).
Banks are endowed with $E$ units of fixed equity capital. Banks raise $L_i = (1 + b_i) n_i - E$ units of consumption goods from depositors. Risk weighted capital ratio of bank is $E/n_i$. Capital regulation limits risky investment $n_i$ since the equity is fixed.

**Table:**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risky assets ($n_i$)</td>
<td>Deposits ($L_i$)</td>
</tr>
<tr>
<td>Cash ($n_i b_i$)</td>
<td>Equity ($E$)</td>
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Banks are endowed with $E$ units of fixed equity capital.

Banks raise $L_i = (1 + b_i)n_i - E$ units of consumption goods from depositors.

Risk weighted capital ratio of bank is $E/n_i$.

Capital regulation limits risky investment $n_i$ since the equity is fixed.
Cost of funding and operating a bank

Banks’ initial equity is sufficiently large to avoid default in equilibrium.

As a result, deposits are safe, and the net interest rate on deposits is zero.

The operational cost of a bank is $\Phi((1 + b_i)n_i)$, where $\Phi'(\cdot) > 0$ and $\Phi''(\cdot) > 0$.

$\Phi(\cdot)$ is convex, that is, $\Phi'(\cdot) > 0$ and $\Phi''(\cdot) > 0$. Van den Heuvel (2008) and Acharya (2003, 2009).

The total cost of a bank is $D((1 + b_i)n_i) = \Phi((1 + b_i)n_i) + (1 + b_i)n_i$. 
Liquidity Shock at $t = 1$

Three dates: $t = 0, 1, 2$.

Two states of the world at $t = 1$:
- Good state with probability $1 - q$
- Bad state with probability $q$

Good state:
- No liquidity shock.
- Bank’s assets yield $Rn_i + n_ib_i$ units of consumption goods at $t = 2$. 
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Good state:
- No liquidity shock.
- Bank’s assets yield $Rn_i + n_ib_i$ units of consumption goods at $t = 2$.

Bad state:
- Investment distressed, has to be restructured to remain productive.
- Restructuring costs are $c$ units per risky asset.
- Banks can use safe assets $n_ib_i$ to carry out the restructuring.
- Banks fire sale assets if safe assets are not sufficient.

The net expected return on the risky asset is positive: $R > 1 + qc$. 
Outside Investors’ Problem

Outside investors are endowed with large liquid resources at $t = 0$ and 1. They can purchase assets from banks and employ them in a technology $F$. $F$ is concave ($F' > 0$ and $F'' < 0$), and satisfies $F'(0) \leq R$.

They choose how much investment goods $y$ to buy from banks at $t = 1$

$$\max_{y \geq 0} \quad F(y) - Py$$
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First order conditions: $F'(y) = P$.

Outside investors’ demand function $y = Q^d(P) \equiv F'(P)^{-1}$ is downward sloping!

Assume outside investors’ demand is elastic to rule out multiple equilibria:

$$\epsilon_{P,y} = -\frac{\partial y}{\partial P} \frac{P}{y} = -\frac{F'(y)}{yF''(y)} > 1$$
A bank decides what fraction of investment to sell \((1 - \gamma_i)\)

\[
\max_{0 \leq \gamma_i \leq 1} \pi_i = R\gamma_i n_i + P(1 - \gamma_i) n_i + b_i n_i - c n_i
\]

subject to the budget constraint

\[
P(1 - \gamma_i) n_i + b_i n_i - c n_i \geq 0.
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$$P(1 - \gamma_i) n_i + b_i n_i - cn_i \geq 0.$$ 

In equilibrium $c < P \leq R$. Hence, the BC binds, and we obtain

$$1 - \gamma_i = \frac{c - b_i}{P}$$

and the total supply of assets is

$$(1 - \gamma)n = \frac{c - b}{P}n \quad \iff \text{Downward Sloping Supply}$$
Asset Market Equilibrium at \( t=1 \)

Equilibrium price, \( P \), and the fraction of assets sold in equilibrium, \( 1 - \gamma = (c - b)/P \), are functions of \( n \) and \( b \).
Lemma: A higher initial risky investment \((n)\) or a lower a liquidity ratio \((b)\) increases the severity (lower asset prices) and the cost (more asset fire-sales) of financial crises.
Three Cases

We will compare and contrast three cases:

- Competitive Equilibrium: No regulation $(n, b)$.

- Partial Regulation: Only the risky investment level $(n_i)$ is regulated, i.e. pre-Basel III regulation $(n^*, b^*)$.

- Complete Regulation: Both risky investment level $(n_i)$ and liquidity ratio $(b_i)$ are regulated $(n^{**}, b^{**})$. 
Competitive Equilibrium

Banks’ problem at $t = 0$:

$$
\max_{n_i, b_i} \Pi_i(n_i, b_i) = (1 - q)(R + b_i)n_i + qR \gamma_i n_i - D(n_i(1 + b_i))
$$

where $\gamma_i = 1 - \frac{c - b_i}{P}$ as obtained from banks’ problem at $t = 1$. 

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where $\gamma_i = 1 - \frac{c - b_i}{P}$ as obtained from banks’ problem at $t = 1$.

First order conditions with respect to $n_i$ and $b_i$ are respectively:

$$(1 - q)(R + b_i) + qR\gamma_i = D'(n_i(1 + b_i))(1 + b_i)$$

$$(1 - q)n_i + qR\frac{1}{P}n_i = D'(n_i(1 + b_i))n_i$$
Partial Regulation: Regulating only capital

Regulator moves first and sets \( n \). Given \( n_i = n \), banks choose the liquidity ratio \( (b_i) \) to maximize their expected profits. FOCs of banks’ problem wrt \( b_i \) yields:
Partial Regulation: Regulating only capital

Regulator moves first and sets $n$. Given $n_i = n$, banks choose the liquidity ratio $(b_i)$ to maximize their expected profits. FOCs of banks’ problem wrt $b_i$ yields:

$$(1 - q) + qR \frac{1}{P} = D'(n(1 + b_i)) \implies b_i = \frac{D'^{-1}(1 - q + q \frac{R}{P})}{n} - 1$$
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The regulator maximizes:

\[
\max_n W(n) = (1 - q)(R + b(n))n + qR\gamma n - D((1 + b(n))n)
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The regulator maximizes:

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Using $\gamma = 1 - \frac{c - b(n)}{P}$ the first order conditions with respect to $n$ is:

$$\frac{1 - q}{P} \left[ R + b(n) + nb'(n) \right] + qR \left[ \gamma + n \left( \frac{c - b}{P^2} \frac{dP}{dn} + \frac{1}{P} b'(n) \right) \right] = D'(n(1 + b(n))) \left[ 1 + b(n) + nb'(n) \right]$$
Complete Regulation: Regulating both capital and liquidity

Regulator’s problem at \( t = 0 \):

\[
\max_{n,b} W(n, b) = (1 - q)(R + b)n + qR \gamma n - D(n(1 + b))
\]

First order conditions with respect to \( n, b \) are respectively:

\[
(1 - q)(R + b) + qR\left\{ \gamma + n\frac{c - b}{P^2} \frac{\partial P}{\partial n} \right\} = D'(n(1 + b))(1 + b)
\]

\[
(1 - q)n + qR\left\{ \frac{1}{P} + \frac{c - b}{P^2} \frac{\partial P}{\partial b} \right\} n = D'(n(1 + b))n
\]
Demand side: $F(y) = R \ln(1 + y)$.

For this return function we obtain the (inverse) demand function as

$$P = F'(y) = \frac{R}{1 + y} \quad \text{and hence} \quad y = F'^{-1}(P) = \frac{R - P}{P} \equiv Q^d(P)$$
Functional Assumptions

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The operational cost of a bank: \( \Phi(x) = dx^2 \), and hence

\( \Phi'(\cdot) \) is increasing, that is, \( \Phi'(x) = 2dx \).
Proposition 3

Banks decrease their liquidity ratio as the regulator tightens the limit on risky investment, i.e. \( b_i(n) > 0 \).

- Stricter limits on risky investment \( \rightarrow \) lower liquidity ratios.

- Banks are restricted to take risk on the investment side, they switch to the liquidity channel.
Lemma 2

\[ n > n^* \]
\[ b > b^* \]

- There is over investment in the risky asset under competitive equilibrium.
- Banks are less liquid under partial regulation: They undermine the purpose of regulation.
Comparing Risky Holdings ($n$)

**Proposition 4 (a)**

$$n > n^{**} > n^*$$

Diagram: Risky Holdings

- **Red** line: Competitive Equilibrium
- **Blue** line: Partial Regulation
- **Green** line: Complete Regulation

$c$ : size of liquidity shock

Kara and Ozsoy (Fed/OzU)
Comparing Liquidity Hoarding ($b$)

**Proposition 4 (b)**

$b^{**} > b > b^*$

**Liquidity Holdings**

$c : $ size of liquidity shock

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Bank Regulation  
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Fire-sale price of risky asset

Proposition 4 (c)

\[ P^{**} > P^* > P \]

Prices under Fire Sale

\( c: \) size of liquidity shock

- Red: Competitive Equilibrium
- Blue: Partial Regulation
- Green: Complete Regulation

Kara and Ozsoy (Fed/OzU)
Severity of the crisis: fraction of risky assets sold

Proposition 4 (c)

\[ 1 - \gamma > 1 - \gamma^* > 1 - \gamma^{**} \]

Fire Sale: fraction of risky assets sold

c : size of liquidity shock

Kara and Ozsoy (Fed/OzU)  Bank Regulation

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Severity of the crisis: total amount of risky assets sold

Proposition 4 (c)

\[(1 - \gamma)n > (1 - \gamma^*)n^* > (1 - \gamma^{**})n^{**}\]

Fire Sale: amount of risky assets sold

- Competitive Equilibrium
- Partial Regulation
- Complete Regulation

\[c : \text{size of liquidity shock}\]
Proposition 4 (d)

\[(1 + b)n = (1 + b^{**})n^{**} > (1 + b^{*})n^{*}\]
Partial vs Complete Regulation

- Looking at $n^{**} > n^*$, one may think that entering the interim period with $n^*$ rather than $n^{**}$ should be safer.

- However, fire-sales are bigger under partial regulation:
  - Ratio: $1 - \gamma^* > 1 - \gamma^{**}$
  - Level: $(1 - \gamma^*)n^* > (1 - \gamma^{**})n^{**}$

- Level of risky investment is not as informative for fire-sales.

- The important thing is not the level of risky investment; it is how the risky investment is backed by liquid assets.
Advantages of Regulating Liquidity

- More funds for high return projects: $n^{**} > n^*$
- More liquidity: $b^{**} > b^*$
- Less fire-sales:
  - Ratio: $1 - \gamma^* > 1 - \gamma^{**}$
  - Level: $(1 - \gamma^*)n^* > (1 - \gamma^{**})n^{**}$
- Higher fire sale prices: $P^{**} > P^*$
Conclusion

- If we regulate capital but not liquidity, banks will undermine the regulation by taking more risk through the liquidity channel.

Basel III liquidity regulations are a step in the right direction.

Kara and Ozsoy (Fed/OzU)
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- Regulation of liquidity is essential to address fire sales related financial instability.
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Basel III liquidity regulations are a step in the right direction.
Appendix I: Endogeneizing the deposit rate

Let $L_i = (1 + b_i)n_i - E$ be the initial deposits at bank $i$.

Each bank is a local monopsony and chooses $n_i, b_i, r_i$ to maximize:

$$(1 - q)[(R + b_i)n_i - r_iL_i] + q \max\{R\gamma_in_i - r_iL_i, 0\} - E - \Phi(n_i(1 + b_i))$$

subject to the Individual Rationality (IR) condition of its depositors:

$$(1 - q)r_iL_i + q \min\{R\gamma_in_i, r_iL_i\} \geq L_i$$

IR will bind. We have two cases, depending on parameters:

Case 1: No bank failure in equilibrium and hence banks will set $r_i = 1$.

Case 2: Bank failure in equilibrium. The IR condition will imply:

$$(1 - q)r_iL_i + qR\gamma_in_i = L_i \Rightarrow r_i = \frac{L_i - qR\gamma_in_i}{(1 - q)L_i}$$

(1)

In both cases, substituting optimal $r_i$ back into bank’s problem yields the same problem as before:

$$(1 - q)(R + b_i)n_i + qR\gamma_in_i - (1 + b_i)n_i - \Phi(n_i(1 + b_i))$$
Appendix II: Deposit insurance

Fairly priced deposit insurance: Banks pay deposit insurance fees in good times, and in exchange the deposit insurance agency covers any deficits in bad times.

Banks can offer zero net interest to depositors.

$$(1 - q)[(R + b_i)n_i - L_i - \tau_i L_i] + q \max\{R\gamma_i n_i - L_i, 0\} - E - \Phi(n_i(1 + b_i))$$

The fair pricing of deposit insurance requires

$$(1 - q)\tau_i L_i = q \max\{L_i - R\gamma_i n_i, 0\}$$

Substitute this back into the bank’s problem above:

$$(1 - q)(R + b_i)n_i + qR\gamma_i n_i - E - L_i - \Phi(n_i(1 + b_i))$$

Using $L_i = (1 + b_i)n_i - E$ this can be written as:

$$(1 - q)(R + b_i)n_i + qR\gamma_i n_i - n_i(1 + b_i) - \Phi(n_i(1 + b_i))$$