Reverse Speculative Attacks*

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Abstract

We define a reverse speculative attack as a wave of purchases of assets denominated in a domestic currency, by foreign agents betting on the *appreciation* of that currency. The objective of this paper is to investigate what leads a central bank facing such an attack to abandon its currency peg and revalue its currency. We consider a model where the central bank maintains a peg, and responds to increases in demand for domestic currency by expanding its balance sheet. In contrast to the classic speculative attack a la Krugman, that are triggered by the depletion of foreign assets, we show that it is the fear of future losses that lead the central bank to an early abandonment of the peg and a currency revaluation. A decline in the world interest rate, and the possibility of hitting the lower bound on interest make the economy more vulnerable to reverse speculative attacks.

Keywords: Currency crises, Exchange rates, Zero Lower Bound

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1 Introduction

The objective of this paper is to understand what leads a central bank which is currently pegging its currency, to abandon the the peg and let its currency appreciate. Following Cochrane (2015) we name such an event 'Reverse Speculative Attack". A recent example of such an event is the January 2015 decision of the Swiss National Bank (henceforth SNB) to abandon the peg of the Swiss Franc against the Euro, decision which has lead to a 20% appreciation of the Swiss Franc against the Euro. Another example is the May 1971 decision of the Bundesbank to abandon the peg against the U.S. dollar, which also lead to a appreciation of the German currency (see Brunnermeier and James, 2015).

There exists a very large literature on standard speculative attacks, i.e. the case in which a central bank abandons a peg, and lets its currency depreciate.[1] However, to our knowledge, there is much less analysis on reverse speculative attacks, which are quite different in nature.[2] The key difference is that in the case of standard speculative attacks the central bank is forced to abandon the peg, as when its foreign currency reserves are depleted, it has no longer the ability of preventing its currency from depreciating. In the case of reverse speculative attack the central bank can, in principle, maintain the peg indefinitely, as to do so it just needs to print domestic currency to acquire foreign currency assets. So in the case of reverse speculative attack the decision to abandon the peg depends on so called "balance sheet concerns", i.e. on the willingness of the central bank to hold in its balance sheet large quantities of risky assets; although this an issue which is currently quite debated in monetary economics (see, among others, Del Negro and Sims, 2015), or Hall and Reis (2015) there is no clear consensus on when the assets of the central bank are too large and/or too risky.

Summarizing, it's not very clear why and under what condition a central bank under a reverse speculative attack would let go a peg; nevertheless this decision seems to matter a great deal for financial markets.[3] For this reason in this paper we develop a simple theory of reverse speculative attack, that allow us to better understand the timing of these attacks and how changes in fundamentals such as international interest rates can affect their likelihood.

After reviewing some basic data about the Swiss experience we start by developing a simple theory of a central bank's objective. We assume that the central bank would like, for

[1] See, among others, the seminal papers by Krugman (1979) and Flood and Garber (1984), or the very recent survey by Lorenzoni (2014).

[2] A notable exception is the work of Grilli (1986) who analyzes reverse speculative attacks assuming the Central Bank exogenous upper bound on reserves. Our work in a way attempts to better understand the reason for such a bound

[3] An article on the January 2015 edition of the Economist Magazine suggests that "The doffing of the cap surprised and upset the foreign exchange markets, hobbling several currency brokers," , while Brunnermeier and James (2015) state that "The risks created by the SNB's decision – as transmitted through the financial system – have a fat tail."

reason we do not model, to maintain a peg with a foreign currency. Were the central bank not to intervene, the currency would appreciate, as we assume, consistently with the Swiss situation after the Great Recession, that the central bank faces an increasing demand for domestic currency; thus maintaining the peg involves expanding its reserve holdings and its liabilities. We then make two key assumptions for our results. First we assume that reserves are risky, in the sense that they are subject to future loss in value (relative to the monetary liabilities issued) that the central bank cannot control. The second is that the central bank wants to keep its losses below a threshold value. Our first result is that the fear of future losses leads the central bank to an early abandonment of the peg and a currency revaluation. The idea is that be letting the currency appreciate the central bank realizes some losses today, but in doing so it reduces future appreciations and thus larger future losses.

Our second result is that the likelihood of attacks if higher when the economy operates close to or at the lower bound on interest.[4]. To understand this result recall that a reverse attack is a situation in which, because of future expected appreciation, the domestic currency is attractive relative to the foreign. If the domestic rate is far from the bound the central bank can make its currency less attractive by lowering its rate. But when the domestic rate is close to its bound, this is no longer a possibility for the central bank, and attacks can no longer be defended against, i.e. the only possibility of making the currency less attractive is appreciate instantly so future appreciation is reduced.

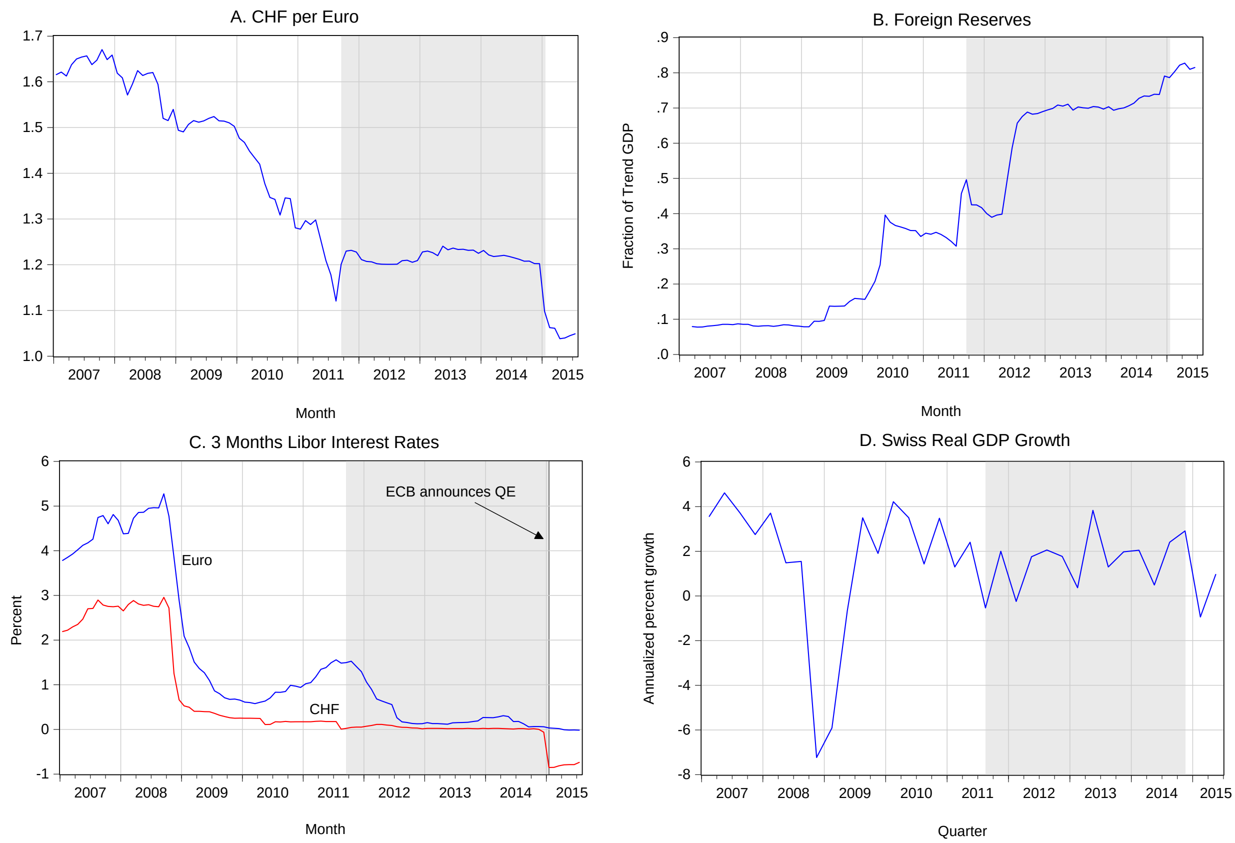
The paper is organized as follows. In section 2 we present some data that characterize the Swiss experience with the peg to the Euro, section 3 presents the model, section 4 contains our main results, and section 5 concludes.

2 Evidence on the Swiss experience

In this section we briefly provide some evidence on the experience of the Swiss National Bank with its peg and subsequent abandonment, as these events are the main motivation of our work. In September 2011 the SNB, mentioning overvaluation of the Swiss franc and its negative effect on the Swiss economy announced a peg with the Euro, stating that:

> "With immediate effect, it will no longer tolerate a EUR/CHF exchange rate below the minimum rate of CHF 1.20.The SNB will enforce this minimum rate with the utmost determination and is prepared to buy foreign currency in unlimited quantities."

[4]See the contributions of McCallum (2000) and Svensson (2003) for an analysis of the zero lower bound in open economies



Note The shaded areas represent the period in which the CHF was pegged to the Euro

Figure 1: The Swiss experience: before, during and after the peg

In January 2015 the SNB abandoned the peg, which resulted in a substantial devaluation of the Euro with respect to the CHF. Panel A of Figure 1 shows the path of the CHF/Euro exchange rate in the years preceding the peg, during the peg (the shaded area) and after the abandonment of the peg. Panel B shows instead the amount of foreign currency reserves held by the SNB (expressed as a fraction of trend GDP).[5]

Notice how in the first part of the sample (pre-peg) the CHF has appreciated quite substantially relative to the Euro, and the SNB has at the same time accumulated foreign reserves. During the peg the CHF has remained stable, while the SNB has continued accumulating reserves at a rapid pace. Panel C plots 3 months LIBOR rates on Swiss Franc and on Euros. Notice how, throughout the whole period, the CHF interest rate has been below the Euro rate, suggesting that, even during the peg, there were expectations that the CHF

[5]We normalized reserves by trend GDP (as opposed to actual GDP) in order to isolate the fluctuations in reserve holdings. We computed a linear trend using GDP data from 2007Q1 to 2015Q2.

was going to appreciate gainst the Euro, i.e. that the peg was not going to last.[6] Notice also that the abandonment of the Swiss peg coincides with the announcement by the ECB of the quantitative easing program. Later we will argue that these changes in Euro interest rate policy might be very important to understand the abandonment of the peg.

Finally panel D provides some evidence on the background macroeconomic conditions in which the SNB has been operating. Notice that the peg has been introduced at a time when real GDP growth was slowing down markedly, while the peg has been abandoned at a time in which Swiss growth was mildly accelerating.

3 The model

Let us consider a world composed of a small open economy, which uses a local currency (Swiss Francs) and a large trading partner, which has a different currency (Euros). We will denote with s_t the state of the economy at time t, and with s^t the history of states up to time t, i.e. $s^t = \{s_0, s_1, ..., s_t\}$. We let state to be such that $s \in S$ in a finite set S, and assume that it follows Markov chain, with a transition probability given by the function $\pi(s'|s)$.

3.1 The Central Bank

The key agent of our economy is the domestic central bank, which is a monopolist supplier of domestic currency, which can hold foreign currency reserves and make transfers to the central government. We denote by $M(s^t)$ the supply of domestic currency issued by the central bank in state s^t and by $F(s^t), T(s^t)$ foreign currency (Euros) reserves held and transfers to the central government made in state s^t.

The budget constraint (denominated in local currency) of the central bank is then given by

$$E(s^t)\left(F(s^t) - F(s^{t-1})\right) = F(s^{t-1})i^\star(s^{t-1})E(s^t) + M(s^t) - M(s^{t-1}) - T(s^t) \qquad (1)$$

where $E(s^t)$ denotes the nominal exchange rate of the economy i.e. the amount of local currency necessary to acquire 1 Euro, and $i^\star(s^{t-1})$ represents the foreign interest rate earned on reserves accumulated by the central bank in the previous period. This equation just states that the accumulation of foreign reserves, $E(s_t)[F(s_t) - F(s_{t-1})]$, is given by the income on accumulated foreign reserves $F(s^{t-1})i^\star(s^{t-1})E(s^t)$, plus the increase in money liabilities $M(s^t) - M(s^{t-1})$ minus the transfer to the central government $T(s^t)$.

[6] Jermann (2015) which uses option prices to back out probabilities of abandonment and found that over the duration of the peg probability of abandonment averaged 20%

We define the profits (or losses if negative) of the central bank $\Pi(s^t)$, as the sum of the earned interest income on foreign reserves $F(s^{t-1})E(s^t)i^\star(s^{t-1})$ plus the changes in valuation in foreign reserves $F(s^{t-1})(E(s^t) - E(s^{t-1}))$

$$\Pi(s^t) \equiv \left((1 + i^\star(s^{t-1}))E(s^t) - E(s^{t-1})\right) F(s^{t-1}). \tag{2}$$

Note that when the local currency appreciates, i.e. $E(s^t)$ falls below $E(s^{t-1})$, the central bank suffers a reduction in profits due to the fact that its existing reserves lose value. Equations (1) and (2) are an accounting relation and a definition, which lead us to our key restriction on central bank actions: we impose the following restrictions on profits and on the transfer policy

$$T(s^t) = \Pi(s^t) \tag{3}$$
$$\Pi(s^t) \geq -\bar{\Pi}, \tag{4}$$

for some $\bar{\Pi} \geq 0$.

In words, we assume that when the central bank makes positive profits, all of its profits are rebated to the treasury. If the bank makes losses smaller than a fixed threshold $-\bar{\Pi}$ then, the treasury recapitalizes the central bank. However, the central bank is not allowed to have losses that exceed a fixed limit $\bar{\Pi}$. The justification for this assumption is as follows. Since central banks are not for profit institutions, it seems reasonable to assume that when they make positive profits those are rebated to the treasury. When profits become losses, then they can significantly impact the net worth of the central bank, and that might make it impossible for the central bank to buy back (part of) its cash liabilities, and thus to conduct monetary policy. In this case the treasury would need to recapitalize the central bank; we assume that a recapitalization that is too large would not be politically feasible, hence we impose constraint 4), which limits the transfer the central bank can receive from the Treasury.[7] Note that substituting equation (3) into (1) and defining net worth of the central bank $NW(s^t)$ as the difference between its assets $E(s^t)F(s^t)$ and its liabilities $M(s^t)$ yields

$$NW(s^t) \equiv E(s^t)F(s^t) - M(s^t) = E(s^{t-1})F(s^{t-1}) - M(s^{t-1}) \equiv NW(s^{t-1}) \tag{5}$$

showing that under the assumed transfer policy the net worth of the central bank is constant.

[7] See Benigno and Nisticò (2015) for similar restrictions on the central bank transfer policy.

3.2 Money demand, exchange rates and interest rates

Another key ingredient of the model is the real (i.e. denominated in Euro) demand for the domestic currency, which we indicate as

$$L(i(s^t), s^t)$$

where $i(s^t)$ is the small open economy's nominal interest rate between period t and period $t+1$.

Equilibrium in the money market requires that

$$M(s^t) = E(s^t)L(i(s^t), s^t)$$

together with the restriction that local nominal interest rates cannot be below a fixed lower bound:

$$i(s^t) \geq i_{LB}$$

We assume that the trading partner is sufficiently large, that the following uncovered interest parity condition connects exchange rates and interest rates across the two countries:

$$1 + i(s^t) = \mathbb{E}\left[(1 + i^*(s^t))\frac{E(s_{t+1}, s^t)}{E(s^t)}\middle| s^t\right]$$

3.3 Exogenous Appreciation Risk

At this point, we have imposed very little structure on the state of the economy and its evolution. Consider now a subset $S^u \subset S$. We will assume that whenever $s_t \in S^u$, it will remain in this set forever (i.e., the set is absorbing). More importantly, we will assume that if the state ever transits into this set S^u, the Central Bank will exogenously follow a policy that will keep the currency pegged at a fixed level $\bar{E} < 1$.

This exogenous restriction captures the possibility of an appreciation risk that the central bank faces when pegging to parity. Although this is modeled in an exogenous fashion, it captures the point that even during the CHF peg, it was understood by the public and by policy makers that the peg was not going to last for ever, and that in the long run the CHF was eventually going to let the currency appreciate.

For the rest of the states, $S^e \equiv S - S^u$, the exchange rate will be determined in equilibrium, which we proceed to characterized now.

3.4 Recursive Competitive Equilibria

We focus attention to allocations that are recursive competitive equilibria given an initial state s_0, that is equilibrium mappings that are only function of the current state:

Definition 1 (Recursive Equilibrium)**.** *A* ***recursive competitive equilibrium*** *is a set of mappings for the exchange rate, money, reserves and interest rates as functions of the current state* $\{E(\cdot), M(\cdot), F(\cdot), i(\cdot)\}$ *such that for all possible* $s \in S^e$*,*

$$E(s)F(s) - M(s) = NW(s_0) \tag{6}$$

$$M(s) = E(s)L(i(s), s) \tag{7}$$

$$1 + i(s) = (1 + i^*(s)) \sum_{s' \in S} \pi(s'|s) \frac{E(s')}{E(s)} \tag{8}$$

$$\big((1 + i^\star(s))E(s') - E(s)\big)F(s) \geq -\bar{\Pi}, \text{ for all } s' \in S \text{ s.t. } \pi(s'|s) > 0 \tag{9}$$

$$i(s) \geq 0 \tag{10}$$

and $E(s) = \bar{E}$ *if* $s \in S^u$*.*

Equation (6) follows directly from equation (5) and imposes that along any equibrium path, because of the assumed transfer policy, the balance sheet of the central bank remains constant. Equation (7) is the money-market clearing condition while (8) the interest parity condition. Inequality (9) states that the central bank losses cannot exceed $\bar{\Pi}$, while the last inequality is the zero-lower-bound constraint on nominal rates. Combining equations (6), (7) and (8) we obtain

$$E(s)F(s) = NW(s_0) + E(s)L\left((1 + i^\star(s)) \sum_{s' \in S} \pi(s'|s) \frac{E(s')}{E(s)}, s\right) \tag{11}$$

which shows how the nominal value of the central bank's reserves $E(s)F(s)$ is a function of the exchange rate, exogenous processes and the constant net worth of the bank. Substituting (11) into (9) and taking (9) and (10) together yields the following useful result that characterizes equilibrium exchange rates

Proposition 1. *A function* $E(\cdot)$ *is part of a recursive equilibrium if and only if the following conditions are satified for all* $s \in S^e$*:*

$$\min_{\{s' \in S | \pi(s'|s) > 0\}} G(E(s), E(s'), i(s), s) \geq 0, \tag{12}$$

$$i(s) = (1 + i^\star(s)) \sum_{s' \in S} \pi(s'|s) \frac{E(s')}{E(s)} - 1 \geq i_{LB}; \tag{13}$$

and $E(s) = \bar{E}$ *for all* $s \in S^u$*; where* $G(e, e', i, s) \equiv (1 + i^{\star}(s))e' - e + \frac{\underline{\Pi}}{NW_0 + eL(i,s)}e$.

The first condition, (12), says that all equilibrium exchange rates, i.e. $E(s)$ have to be such that, under all possible realizations of the next period exchange rate, $E(s')$, the losses of the central bank are below the bound $\underline{\Pi}$. Condition (13), which is the traditional zero lower bound condition, imposes a restriction on expected appreciation: the more the currency is expected to appreciate, the lower the domestic interest rate has to be; so if expected depreciation is too large the lower bound on nominal interest rates will be violated, hence exchange rate functions which involves large expected depreciation cannot be equilibria. Note that this constraint is more likely to bind the lower $i^{\star}(s)$ is.

3.5 Markov Equilibrium

As it should be rather clear there are many potential recursive equilibrium functions $\{E(s)\}$: for example it is immediate to verify that $E(s) = \bar{E}$ with $i(s) = i^{\star}(s)$ for all s satify both (12) and (13). To select among this potentially large set of equilibria, we assume that central bank strictly prefers to maintain an exchange rate equal to one, as long as the economy remains in state $s \in S^e$. Specifically, we assume that the Central Bank evaluates the sequence of exchange rates in states $s \in S^e$ with the following recursive objective function:

$$V(s^t) = u(E, s) + \beta \sum_{s^{t+1}|s_{t+1} \in S^e} \pi(s^{t+1}|s^t) V(s^{t+1}) \tag{14}$$

where E denotes the exchange rate in state s^t. We impose that $u(E)$ is single peaked at 1. That is, the Central Bank strictly prefers to maintain the currency at parity. Note that we do not need impose an objective criterion for the CB once the economy switches to states in S^u, as we have assumed that in that case the CB has no remaining choices.

With the preferences at hand, we will focus attention on Markov equilibria:

Definition 2. *A* ***Markov equilibrium*** *is a function* $E^M(\cdot)$ *such that* $E^M(s) = \bar{E}$ *for* $s \in S^u$*, and there exists a value function* $V^M(\cdot)$ *which solves*

$$V^M(s) = \max_E \left\{ u(E, s) + \beta \sum_{s'|s' \in S^e} \pi(s'|s) V^M(s') \right\} \tag{15}$$

subject to:

$$\min_{s' \in S|\pi(s'|s) > 0} \{G(E, E^M(s'), i^M(s), s)\} \geq 0$$

$$i^M(s) \equiv (1 + i^{\star}(s)) \sum_{s'} \pi(s'|s) \frac{E^M(s')}{E} - 1 \geq i_{LB} \tag{16}$$

and $E^M(s)$ belongs to the argmax of (15) *for all $s \in S^e$.*

To show the next result, we will impose the following assumptions:

Assumption 1. *We impose the following restrictions:*

- *the Central Bank networth is non-negative, $NW_0 \geq 0$;*
- *for all $s \in S^e$, $i^\star(s) \geq 0$;*
- *the probability of reaching S^u from any state is always strictly positive, $\pi(s' \in S^u|s) > 0$ for all $s \in S^e$.*

In general, a Markov allocation will display periods of pegging (to 1) and periods of an abandonment of the parity, which we label reverse speculative attacks. The period of abandonment is one where either the loss-constraint of the CB is binding, or the zero lower bound constraint binds.

Proposition 2. *Under Assumption 1, the endogenous part of the exchange rate $E^M(\cdot)$ in any Markov Equilibrium is a fixed point of the following operator T:*

$$T(E^M(\cdot))(s) = \max\left\{E \in [\bar{E}, 1] \middle| G(E, \bar{E}, r(E, s, E^M(\cdot))) \geq 0 \text{ and } r(E, E^M(\cdot), s) \geq 0\right\}$$

for all $s \in S^e$; with $r(E, f(\cdot), s) \equiv (1 + i^\star(s)) \sum_{s' \in S} \pi(s') f(s')/E - 1$.

Proof. In the appendix. □

Note however, that $G(e, e', i)$ is increasing in i, and $r(e, s, E^M(\cdot))$ is increasing in $E^M(s')$ for all s'. It follows then that the operator T is monotone. We also know from the Proposition 2 that in any Markov equilibrium $E^M(s) \leq 1$ for all s. This means that iterating the operator T starting from $E(s) = 1$ for all s converges to a Markov equilibrium. We can also argue that the Markov equilibrium is unique:

Proposition 3 (Uniqueness)**.** *The operator T defined in Proposition 2 admits at most one fixed point.*

Proof. In the appendix. □

4 The Markov equilibrium: A Numerical Analysis

In order to characterize the dynamics surrounding the reverse speculative attack, we first impose more structure on the states of the economy and its evolution, as well as specify numerical values for the parameters of the model and characterize the patterns of key variables along the Markov equilibria described above. We would like to stress that, given the highly stylized model we are using, the goal of this exercise is just to provide the reader with some simple qualitative and quantitative insights on reverse speculative attacks; we will surely not provide a comprehensive quantitative evaluation on the issue.

4.1 States of the economy

Our economy is going to be subject to three exogenous disturbances, $A(s^t), b(s^t), i^*(s^t)$. The first regards the already discussed exogenous exchange rate policy state, the second captures shocks to money demand, and the third captures shocks to the foreign interest rate. We now describe the three sources in detail.

The A state. We let $A(s^t) = 1$ if $s \in S^u$ and zero otherwise. As discussed above, the set S^u is absorbing and $E(s^t) = \bar{E}$ for any $s_t \in S^u$. In addition, we will assume that the economy starts in a state $s_0 \in S^e$, and as long that $s_t \in S^e$, there is a constant probability λ that the state moves to S^u next period.

The b state. We assume that the money demand (as long as $a = 0$) has the following functional form:

$$L(i, s^t) = e^{g*b(s^t)} l(i)$$

where $b(s^t) \in \{0, 1, ..., N\}$ and represents possible shocks to money demand, $g > 0$ is a fixed parameter which determines how much money demand increases when a shock hits, and $l(i)$ captures the dependence of money demand on the nominal interest rate. Conventionally we assume that and $l' \leq 0$, i.e. as the nominal interest rate increases the real demand for cash balances falls. We assume that the state $b(s^t) = N$ is absorbing, i.e. that money demand shocks are bounded, and that once money demand shock reaches it maximum it will stay there. For all $b(s^t) < N$, $b(s^t)$ will stay constant with probability $1 - \gamma > 0$ or increase by 1 with probability $\gamma > 0$.In words, γ represent the probability that economy is hit by a shock that increases money demand by $g\%$.This probability is assumed to be independent from other events in the economy.

The i^* state. The third and final source of uncertainty in the economy regards the foreign interest rates. Our modeling of the foreign interest rates is loosely motivated by panel C in figure 1 where we observe that during the period of the Swiss peg first Euro rates fell (in late 2011). After that episode Euro rates did not move much, but toward the end of 2014 the European Central Bank announced the introduction of quantitative easing, which we are going to model as an announcement of a prolonged period of low interest rates. To capture these facts we assume that the foreign interest rate can be in three possible states: temporarily high, temporarily low (before QE),or permanently low (after QE). In the high state $i^\star(s) = i_h$ and in both low states $i^\star(s) = i_l$,with $i_h > i_l$. When the economy is in the high state it can move to the temporarily low state with probability θ_{hl} or to the permanently low with probability θ_{hp}, or stay in the high state with the residual probability. When the economy is in the temporarily low state it can revert to the high state with probability θ_{lh}, it can move to the permanently low state with probability θ_{ll}, or it can stay in the same state with the residual probability $1 - \theta_{lh} - \theta_{ll}$. Finally for simplicity, we assume that the permanently low state is absorbing, so when the foreign rates reaches that state they will just stay there. As with the previous shocks, we assume that these transition probabilities are independent on the realization of the other shocks. Note that when foreign rates transit from the temporarily to the permanently low, current foreign rates do not fall but expectation of future rates do, so this equivalent to a news shock to foreign interest rate (see, for example Siena (2014))

To sum-up, Figure 2 shows possible paths for the three sources of uncertainty. The value $\hat{T}(s^t)$ on the x axis represents the time in which the economy switches from $A(s^t) = 0$ to $A(s^t) = 1$. After $\hat{T}$ our model economy is not interesting, as by assumption exchange rate will be constant at $\bar{E} < 1$. Before $\hat{T}$ our economy faces a period of stochastically increasing demand for its own currency (due for example to global increased risk aversion, or fears of inflation in the trading partner) and/or stochastically decreasing international rates. In the plot the time $\tilde{t}$ represent the period in which foreign interest rates switch from temporarily low to permanently low. Our goal in the reminder of the paper is to analyze the central bank behavior between during this period, and to analyze its decision whether to keep a peg (i.e. keep $E(s^t) = 1$) or not. In order to do in the next sub-section we first characterize all possible equilibrium paths for the exchange rate and then we specify the preferences of the central bank, which will restrict the set of equilibrium allocations.

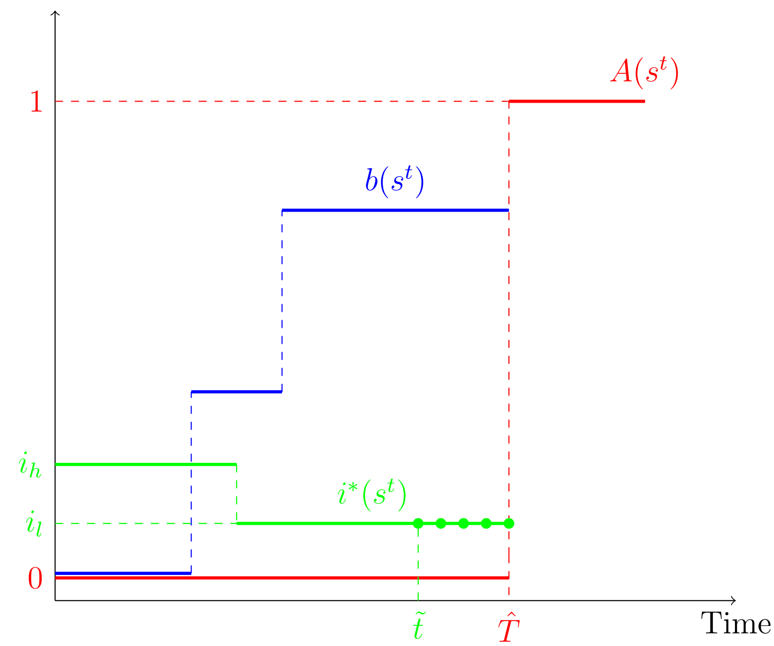


Figure 2: A possible path for the exogenous stochastic variables

4.2 Functional forms and parameter values

Our baseline parameters values are reported in table 1 below. We now briefly describe how we pick those values. Starting with money demand specify the interest elastic portion of money demand as a simple log linear function

$$l(i) = exp(-\psi i)$$

where the constant $\psi > 0$ captures the elasticity of money demand to the interest rate.[8]. In order to estimate the values for the parameters ψ we first construct a measure of money demand. The measure that is more consistent with our stylized model is monetary base, which is a measure of the monetary liabilities of the central bank. We construct this by adding currency in circulation plus deposits of domestic and foreign banks at the central bank (as reported in the balance sheet of the SNB),for the period 2007.1-2015.7, all converted in Euros.[9] We then use the series for the Swiss interest rate (CHF 3 months Libor, as shown in figure 1.C) to estimate equation 4.1 using non linear least squares. Figure 3 reports actual v/s fitted data, along with the estimated values of the parameters.

[8]See Lucas Jr (2000) for different specifications of the money demand

[9]This measure is narrower than more traditional measures of money demand such as M1 or M2, but is highly correlated with those

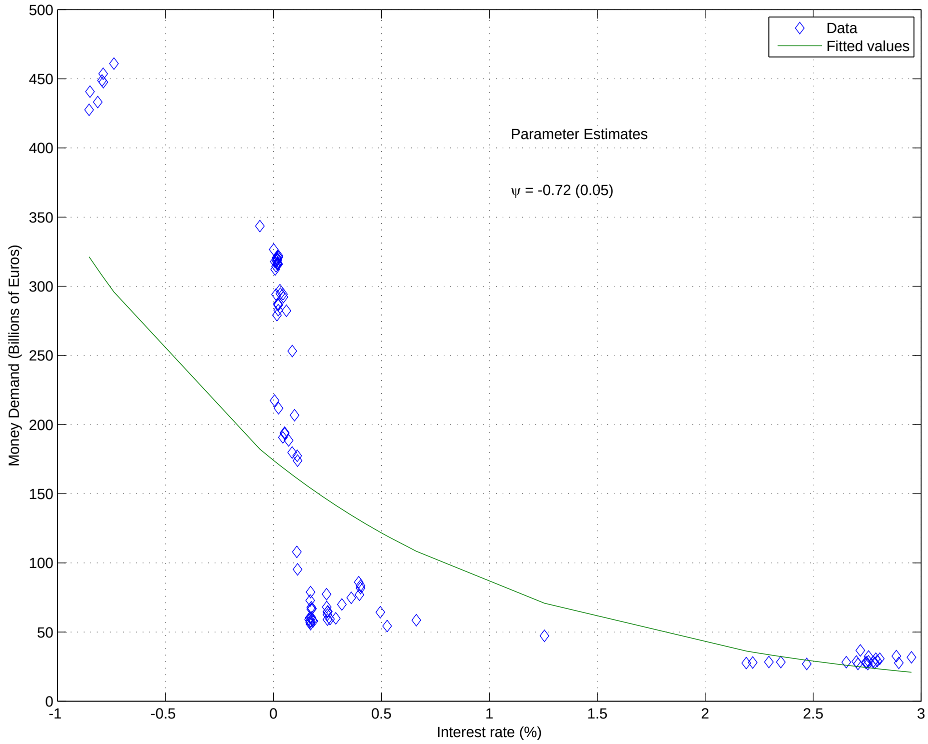


Figure 3: Money Demand in Switzerland: 2007.1-2015.7

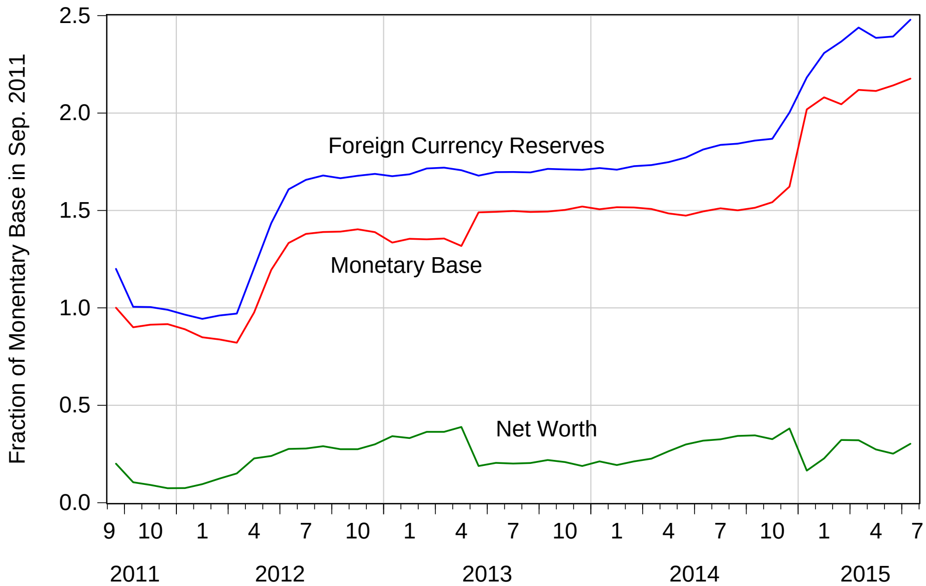


Figure 4: The Balance Sheet of the SNB during the Peg

The remaining part of the money demand equation is the shock process as specified in 4.1. Figure 4 plots monetary base during the peg.[10] Notice that in the second half of 2012 monetary base has increased with a stable interest rate and a fixed exchange rate; in the context of our model this increase cleanly identifies a shock to money demand. Since monetary bases in that period increases by roughly 50% we set g (the size of the increase in money demand) to 0.5. The probability of such an increase is harder to identify, but it seems reasonable to assume that these increase in money demand are associated to fairly rare events , such as risk of the collapse of the European Union: for this reason we assume a monthly probability of such a jump equal to 0.4%, which roughly means one such event in 20 years.

We move next to the values for the foreign interest rates. Figure 1 shows how in the early phase of the peg, Euro rates moved from 1.5% to about 0%. For this reason we set $i_h = 1.5\%$ and $i_l = 0\%$. Also we set the lower bound on interest rate to -1% which is the lowest value we observe for the Swiss interest rate. Transition probabilities for interest rates

[10]This is the same measure used in the estimation above

are harder to pin down; we think of transition from temporarily high to temporarily low as a standard transitions associated to changes of monetary policy over the business cycles so we set the monthly transition probability $\theta_{hl} = 1\%$ which translates into an expected duration of a high interest rate period (expansion phase) of 8 years, the probability $\theta_{lh} = 5\%$ which roughly translates in to a duration of the low interest period (recession period) of twenty months. We also view the transition from high and temporarily low to permanently low as a rare events (like the introduction of QE) and thus we set $\theta_{lp} = 0.4\%$ and $\theta_{hp} = 0.4\%$ which is is the same probability we assign to the jump in money demand.

The next parameters are related to the appreciation risk (i.e. the A shock), which are the value of the currency in case of appreciation $\bar{E}$ and the probability of such an appreciation λ. Note that these two parameters jointly determine the minimum expected appreciation of the domestic currency during the peg. We set the probability of appreciation to 0.4%, as we did for the other rare events, and we set the value of the currency in case of depreciation to 0.72, which implies an expected annual appreciation during the peg of about 1%. The final set of parameters concern the balance sheet of the central bank. Figure 4 shows that in September 2011 (the month in which the peg was introduced) the difference between foreign reserves and monetary base (which in our model corresponds to net worth) was about 20% of monetary base. So we set the value of the initial net worth, $NW(s_0)$ so that the model matches that ratio in the first period of the simulation, i.e. when the peg starts. Note from the figure that net worth of the central SNB stays fairly constant, despite large fluctuation in monetary base and in reserves, which is consistent with our assumption about the evolution of net worth (eq 5). Finally we set the maximum value of losses that can sustained by the central bank (Π) equal to twice the value of its initial net worth. This value is somehow arbitrary, but the parameter does not play a crucial role in our analysis [11]

[11] Numerically we find that setting a higher Π lengthens the expected duration of the peg, but it does not affect how the expected duration depends on fundamentals

Table 1. Parameter Values

	Symbol	Name	Value
Money Demand	ψ	Interest Elasticity of Money Demand	0.72
	g	Size of jump in money demand	0.5
	γ	Probability of a jump (monthly)	0.04%
Interest rate	i_{LB}	Lower bound on interest rate	-1%
	i_h	High foreign interest rate	1.5%
	i_l	Low foreign interest rate	0%
	θ_{hl}	Prob. from high to temp low	1%
	θ_{hp}	Prob from high to perm low	0.04%
	θ_{lh}	Prob. from temp low to high	5%
	θ_{lp}	Prob. from temp low to perm low	0.04%
Appreciation Risk	$\bar{E}$	Appreciated exchange rate	0.72
	λ	Probability of Appreciation	0.04%
Balance sheet[a]	NW_0	Net Worth of Central Bank	0.2
	Π	Maximum Loss	0.4

[a] NW_0 and Π are expressed as ratio to monetary base at the start of the peg

4.3 Results

Given these parameter values we can numerically characterize Markov equilibria and can characterize equilibria that feature reverse speculative attacks, i.e. abandonment of the peg. In particular we focus on two types of speculative attacks: attack driven by shocks to money demand, and attacks driven by reduction of (current or future) foreign interest rates. One result we would like to stress is the importance of the lower bound on interest rates for the likelihood of an attack: in order to understand this better, we start our analysis with a case in which we do not impose the lower bound on interest rates. Figure 5 displays key variables of the economy in all possible states. In each panel the x axis represents all possible shocks to money demand, while the different lines represent different states for the foreign interest rates. So, for example, the right most diamond on the top line in panel (a) represents the equilibrium exchange rate that will prevail when the money demand shock is in its sixth largest value and the foreign interest rate is high. To understand the first type of speculative attack consider an economy that moves along the lines with the diamond markers, i.e. an economy that facing a high foreign interest rate and experiencing a sequences of increase in money demand. Panel (a) shows how in the first four states the exchange rate is at 1. In those states the loss constraint is not binding, and thus the central bank can maintain the exchange rate pegged at 1, its preferred outcome. Panel (c) shows that maintaining the peg in face of an increasing money demand involves accumulation of reserves. In a sense, the jump between money demand shocks 1 and 2 (or 2 and 3) captures the experience of the SNB during the second half of 2012, where the peg was maintained through a large accumulation of foreign reserves. As reserves grow, so does the size of the losses of the central bank in case of appreciation (the A shock), and that makes the loss constraint more likely to bind. Indeed, state 4 is the last reserve state in which the bank can maintain the peg. Panel (a) shows that when the next money demand shock (state 5) hits, the central bank abandons the peg and the exchange rate appreciates by about 6.5%. When the appreciation happens the central experiences losses and sets an exchange rate far from its preferred, but it does so because a current appreciation prevents a larger appreciation in the future that would lead to larger losses. Panel (b) plots domestic interest rates. The top line shows that appreciation is anticipated by investors, and that it induces a decline in domestic interest rate in state 4 (this simply follows from the UIP condition). The fall in interest rates causes a further increase in money demand (over and above the shock) that forces an even larger increase in reserves just before the peg is abandoned. We found interesting that before the attack, the model displays patterns that resembles a pattern a defense against the attack: as abandonment of the parity becomes more likely (state 3), demand for reserves increase, the peg is maintained and domestic interest rates fall.

The figure also suggests another possible driver of an attack. Consider, for example, state 4 in panel (a) and consider a change in the foreign rate from high to permanently low; in this case the central bank will abandon the parity and the exchange rate would depreciate by about 8%. The logic for this attack is similar to the one described above: a fall in foreign interest causes (should the central bank not abandon the peg) a fall in the domestic rate, and the fall in domestic rate triggers a large increase in demand for local currency reserves, which causes the loss constraint of the central bank to be binding. Panel (d) shows the increase in reserve that result from such an attack. So for example the line with diamonds shows the increase in reserves that result as the economy moves from the high to the permanently low interest rate.

To sum up we have highlighted two possible causes of attacks, and under both drivers the central bank abandons the peg because keeping the peg would involve too much appreciation risk, which coupled with large reserves might lead to losses that are too large. By letting the currency appreciate, the central bank realizes some losses when reserves are still low, and by doing so it reduces the size of future losses.

So far we have ignored the zero lower bound. Figure 6 displays the same equilibrium values as in figure 5, using the same exact parameters with the only difference that now the lower bound on interest rate is imposed. The key result here is that a binding lower bound on interest rate can lead the central bank to abandon the parity, well before the loss constraint binds. Focusing on panel (a), consider, for example, the money demand shock 2 and the high interest rate state. In this state the central bank can maintain the parity, but should the economy switch to a lower interest rate (either temporary or permanent) the parity will be abandoned. By contrast figure 5 shows that without the lower bound in state 2 the peg is maintained for all possible states of the international rates. Notice also from the panel (c) in figure 5 that the parity will be abandoned when reserves are low, and thus in a state when the loss constraint 16 is not binding. What drives the attack in this case?

To understand this keep in mind that the lower bound on nominal interests effectively imposes a lower bound on the expected appreciation of the currency (again this is simply follows from the uncovered interest parity). In particular, if in some future period, the exchange rate is expected to move to a lower level (because of say, a future binding of the central bank's loss constraint), then it could be that only a sufficiently low exchange rate today is consistent with the lower bound on interest rates. A higher value of the exchange rate would not be consistent, because moving to the low would involve a depreciation that is too large to respect the interest rate lower bound. That is, a future abandonment of the peg can lead to an abandonment today, even if today's loss constraint is not binding. In this way, the zero lower bound propagates low exchange rates in the future into the present.

Let us try to understand, by arguing away from equilibrium, what would happen if the central bank were to decide to try to maintain the parity, even though this would violate the zero bound. So let's consider a situation where the Central Bank decides to exchange foreign currency for domestic currency at a rate of 1. Given that the interest parity condition is violated, the foreigners would like to invest in mass in domestic bonds. As a result, they will borrow in foreign currency, use the foreign currency and exchange it to domestic currency with the Central Bank, and use the domestic currency to buy domestic bonds. The Central Bank, on the other hand, will use the collected foreign currency to accumulate foreign assets. This cycle however never ends: the Central Bank ends up expanding its balance sheet to infinity. But at some point, the Central Bank loss constraint will bind; and these strategy of maintaining a peg in the presence of a failure of the UIP cannot be sustained.

Quantitatively we note that this simple framework is able to replicate patterns roughly

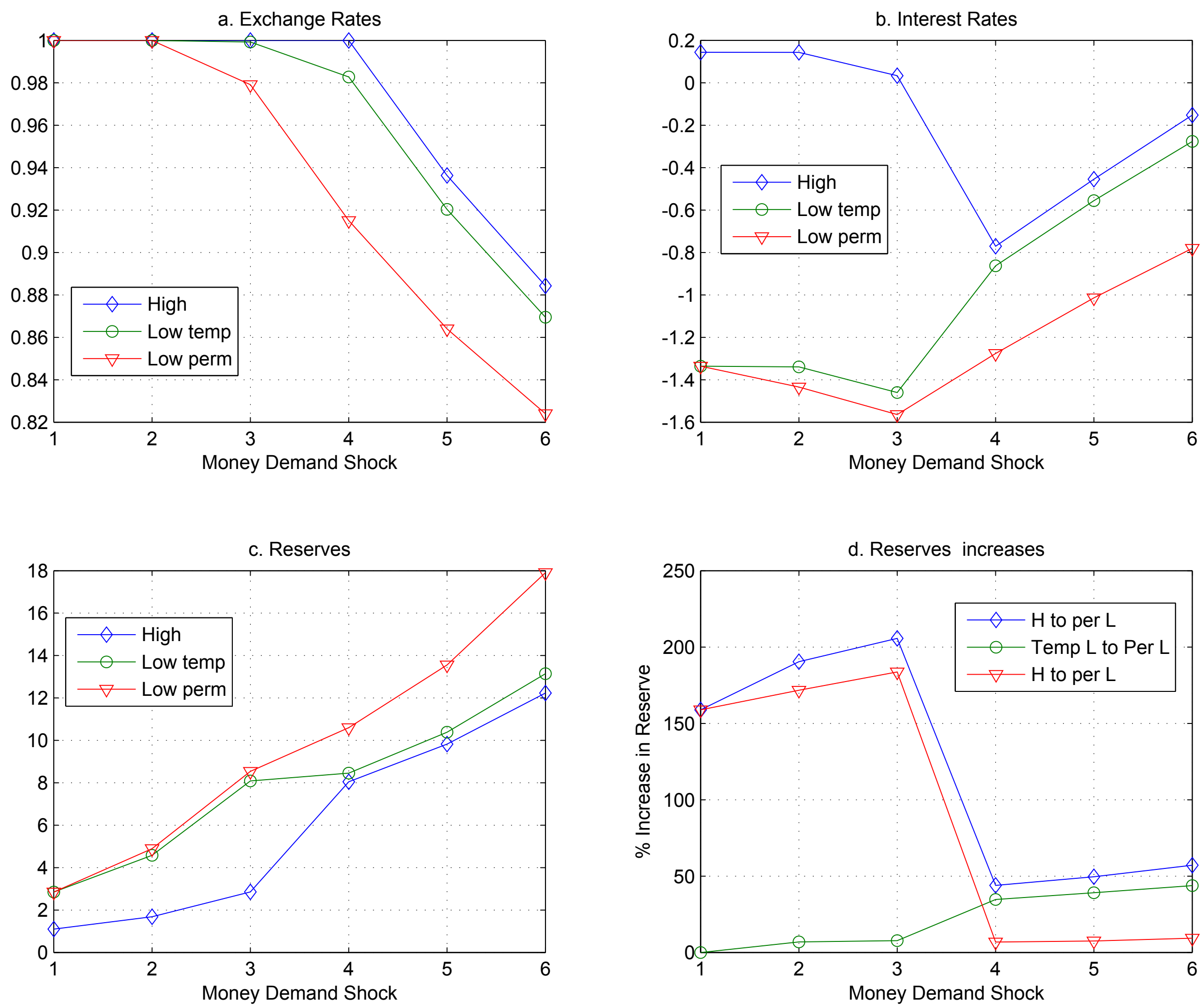


Figure 5: Markov Equilibria without the lower bound on interest rate

consistent with the abandonment of the Swiss peg in January. Consider for example state 3 and assume the announcement of QE is a move in foreign interest rate from temporarily low to permanently low.[12]. Our model suggests that such a move induce an appreciation that exceeds 10% and a jump in reserves of around 50%, which is not too far from the observed patterns in figure 1. One aspect that is at odds with the data, is the model predicts stable interest rate (which are anchored at the lower bound) while in the data we observed that Swiss rates fell further after the abandonment.

To sum up, our last, and perhaps more novel, point is that when the domestic interest is close to its lower bound it is hard for a central bank to maintain a peg, i.e. to stave off a reverse speculative attack. At a basic level the intuition for this result is simple: a

[12]Although short term rates did not move much with announcement of QE, rates on DM long term bonds fell significantly around the announcement of QE

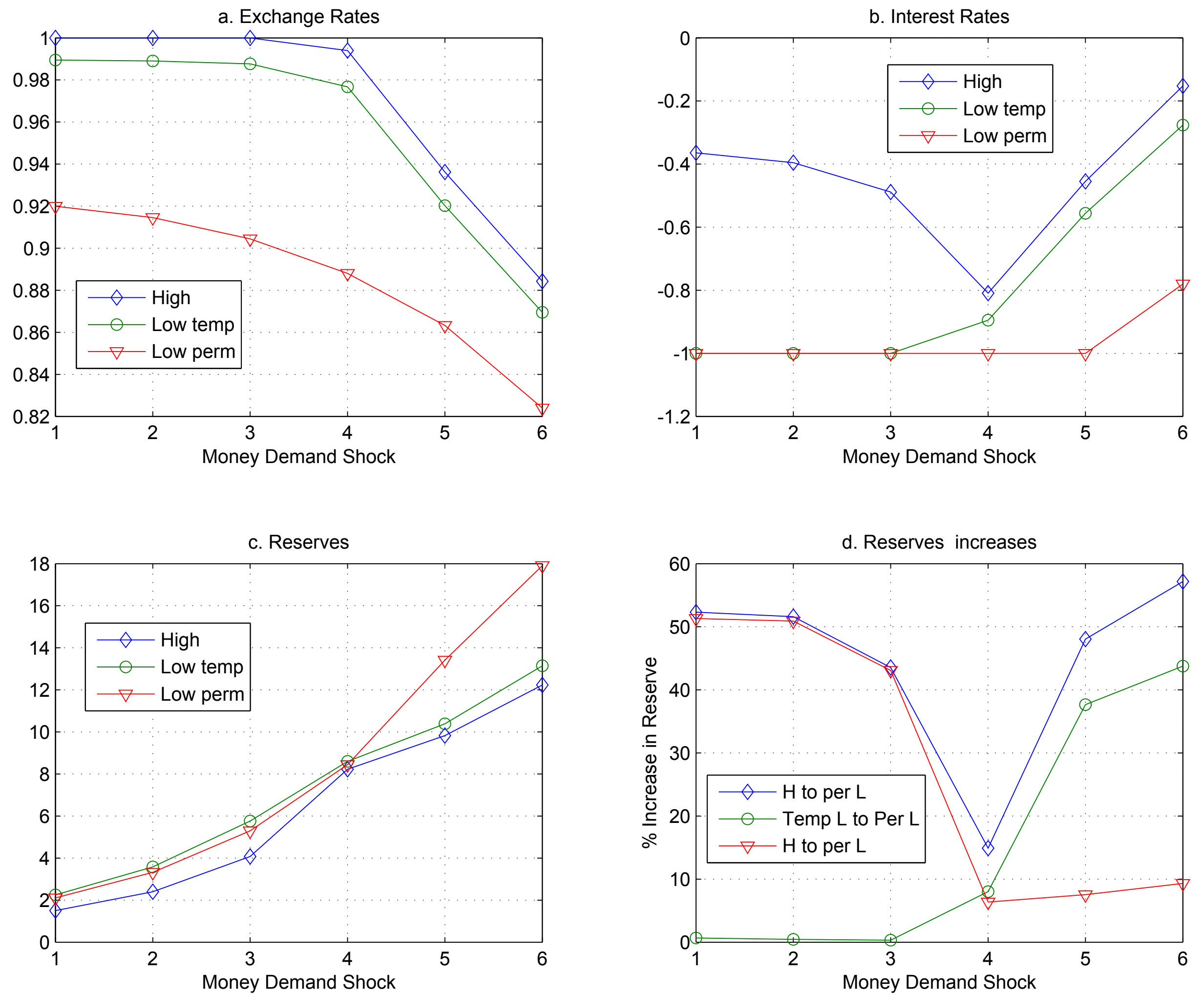


Figure 6: Markov Equilibria with the lower bound on interest rate

reverse attack is a situation in which, because of future expected appreciation, the domestic currency is very attractive relative to the foreign. If the domestic rate is far from the bound the central bank can make its currency less attractive by lowering its rate. But when the domestic rate is close to its bound, this is no longer a possibility for the central bank, and attacks can no longer be defended against.

5 Conclusions

This paper has presented a stylized framework to understand reverse speculative attacks. We have shown that reverse speculative attacks can be triggered by increases in the demand for local currency, or by reduction in foreign interest rates. Maintaining a peg in this context involves accumulation of risky foreign reserves, and the central bank might abandon the peg to limit its exposure to this risk. We have also shown that if/when the central bank operates in a regime close to the lower bound on interest, attacks are more likely and/or pegs are more vulnerable. Reverse speculative attack arise when there is strong demand for domestic currency. If the economy is away from the lower bound on interest rates, this demand can be curbed by lowering the rate. When the domestic interest rates is close to its lower bound, this is no longer possible, i.e. there is no way for the central bank to make holding its currency less attractive. Our framework is highly stylized in several dimensions. In particular we have assumed that foreign reserves are risky, and that the central bank faces a hard constraint on the size of this risk. In reality risk of foreign reserves can itself be affected by actions of the central bank, and the limit on reserve accumulation is not likely to be a hard one, but to be one that also depends on current and future conditions and on monetary policy objectives. Future research could enrich our analysis with these elements, to connect the likelihood of attacks to more structural features of the economy.

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Proof of Proposition 2

The first step is to argue that $E(s) \geq \bar{E}$ for all $s \in S^e$. Suppose not, then for some s_i we have that $E(s_i) < \bar{E}$, and without loss of generality, let s_i be such that $E(s_i) \leq E(s_j)$ for all s_j. This means that:

$$G(E, E(s_i), i^M(s_i), s_i) \leq G(E, E^M(s_j), i^M(s_i), s_i),$$

Now note that $G(E(s_i), E(s_i), i^M(s_i), s_i) = i^\star(s_i)E(s_i) + \underline{\Pi}/(NW_0 + eL(i^M(s_i), s_i)) > 0$, and thus

$$G(E(s_i), E^M(s_j), i^M(s_i), s_i) > 0 \text{ for all } s_j$$

Also note that:

$$i^M(s_i) = (1 + i^\star(s_i)) \sum_{s' \in S} \pi(s'|s_i)E(s')/E(s_i) - 1 > i^\star(s_i) \geq i_{LB}$$

where the inequality follows from $E(s_i) \leq E(s')$. As result, all of the constraints in (15) are not binding. But this means that $E(s^i) = 1$, a contradiction that $E(s^i) < \bar{E}$.

Having shown that $E(s) \geq \bar{E}$, the result is now straightforward. The central bank in any state will choose the highest possible $E \leq 1$, unless one of the constraints of problem (15) bind. Note that $G(E, \bar{E}, i^M(s), s) \leq G(E, E^M(s'), i^M(s), s)$ for all s', as G is increasing in its second argument. And thus only checking the G constraint at $\bar{E}$ is sufficient. As a result, an equilibrium exchange rate must be the one that maximzes the left hand side of the operator T, which is what we wanted to prove.

Proof of Proposition 3

Before attempting the proof, let us first argue the following. Let $\gamma(s) > 0$ for all $s \in S^e$, then $G(E, \overline{E}, r(E, s, E \times \gamma(\cdot)), s)$ and $r(E, s, E \times \gamma(\cdot))$ are strictly decreasing in E for all $s \in S^e$. The proof of this claim is as follows. First, note that r is:

$$\hat{r}(E) \equiv r(E, s, E \times \gamma(\cdot)) = (1 + i^\star(s)) \left(\sum_{s' \notin S^e} \pi(s'|s)\overline{E}/E + \sum_{s' \in S^e} \pi(s'|s)\gamma(s') \right) - 1 \qquad (17)$$

Written this way, $\hat{r}(E)$ is clearly decreasing in E. Going back to G, it follows then that $\hat{G}(E) \equiv G(E, \overline{E}, \hat{r}(E), s)$ is decreasing in E as G is decreasing in its first argument and increasing in its third.

We can now prove Proposition 3 by contradiction. Suppose there are two fixed points:

E_A and E_B. Without loss of generality, let state s_1 be such that $E_A(s_1) < E_B(s_1)$ and

$$\frac{E_A(s_1)}{E_B(s_1)} \leq \frac{E_A(s)}{E_B(s)} \text{ for all } s \in S^e$$

Note that this implies that $E_A(s_1) < E_B(s_1) \leq 1$, and as a result, it must be that:

$$G(E_A(s_1), \overline{E}, r(E_A(s_1), s_1, E_A(\cdot)), s_1) = 0 \text{ or } r(E_A(s_1), s_1, E_A(\cdot)) = 0, \tag{18}$$

as otherwise, $E_A(s_1)$ could have been raised towards 1.

Using our definition of s_1, we note that

$$\gamma_B(s) \equiv \frac{E_B(s)}{E_B(s_1)} = \frac{E_B(s)}{E_A(s)} \frac{E_A(s)}{E_A(s_1)} \frac{E_A(s_1)}{E_B(s_1)} \leq \frac{E_A(s)}{E_A(s_1)} \equiv \gamma_A(s)$$

where we have also defined γ_B and γ_A. Then we have that

$$r(E, s, E \times \gamma_B(\cdot)) \leq r(E, s, E \times \gamma_A(\cdot)) \text{ for all } E \text{ and } s \in S^e$$

Using that G is increasing in its third argument, it follows also that

$$G(E, \bar{E}, r(E, s, E \times \gamma_B(\cdot)), s) \leq G(E, \bar{E}, r(E, s, E \times \gamma_A(\cdot)), s) \text{ for all } E \text{ and } s \in S^e$$

But note that $\hat{G}$ and $\hat{r}$ are decreasing in E. So we have that:

$$\begin{aligned} G(E_B(s_1), \bar{E}, r(E_B(s_1), s_1, E_B(\cdot)), s_1) &= G(E_B(s_1), \bar{E}, r(E_B(s_1), s_1, E_B(s_1) \times \gamma_B(\cdot)), s_1) \\ &< G(E_A(s_1), \bar{E}, r(E_A(s_1), s, E_A(s_1) \times \gamma_B(\cdot)), s_1) \\ &\leq G(E_A(s_1), \bar{E}, r(E_A(s_1), s_1, E_A(s_1) \times \gamma_A(\cdot)), s_1) \\ &= G(E_A(s_1), \bar{E}, r(E_A(s_1), s_1, E_A(\cdot)).s_1) \end{aligned}$$

Using a similar argument as above, we obtain that

$$r(E_B(s_1), s_1, E_B(\cdot)) < r(E_A(s_1), s_1, E_A(\cdot))$$

But these means that $r(E_A(s_1), s_1, E_A(\cdot)) > 0$ and $G(E_A(s_1), \bar{E}, r(E_A(s_1), s_1, E_A(\cdot)), s_1) > 0$, a contradiction of (18)