Structural Stress Tests*

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Abstract

We develop a structural banking model for microprudential stress testing. We study the problem of a single bank that chooses its balance sheet structure, dividend policy and exit. It faces idiosyncratic and aggregate uncertainty, and it is subject to regulatory constraints. In this environment, the bank has an incentive to hold a buffer stock of capital above the minimum required to protect its charter value. In our main experiment, we explore the bank response to a stress situation. In contrast to state-of-the-art reduced-form stress tests, our structural approach offers a framework to evaluate stress scenarios that does not require making behavioral assumptions. Moreover, from the exit decision it is possible to derive an endogenous hurdle rate to the stress test. We calibrate the model using U.S. data. We find that, during the initial period of the stress scenario, the reduced form model underpredicts the decline in capital ratios and overestimates its reduction in the final periods. The main determinant of the difference is the balance sheet composition of the bank that is derived endogenously in our structural model but assumed to be constant in the reduced form model.

JEL classifications: C63, G11, G17, G21, G28

Key words: bank, stress testing, structural model, microprudential

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1 Introduction

State-of-the-art models for micro- and macroprudential stress tests derive bank capital shortfalls during counterfactual scenarios relying on a combination of exogenous, behavioral rules and reduced-form relationships that are extrapolated from historical data. A well known model that uses this approach is the Capital and Loss Assessment under Stress Scenarios (CLASS) model. This approach is susceptible to breakdowns in these relationships due to financial innovations, regulatory changes and large shocks; it is prone to the Lucas critique. This paper makes a first step towards a micro-founded stress testing framework.\footnote{Both models can be classified within the “top-down” approach. This approach is intended to complement the more detailed supervisory models of components of bank revenues and expenses, such as those used in the DFAST, CCAR and European Stress Test that are classified as using a “bottom-up” approach. A benefit of the “top-down” approach is the use of publicly available data. CCAR evaluates the capital planning processes and capital adequacy of bank holding companies with $50 billion or more in total consolidated assets.}

To this end we propose a quantitative banking model for microprudential stress testing based on Corbae and D’Erasmo (2014). Our model can be summarized by four features. First, we consider a single bank’s optimization problem in a partial equilibrium environment à la De Nicolo, Gamba, and Lucchetta (2014). To permit quantitative results, the model is closed by a bank-specific loan demand equation that is derived from an estimated discrete choice model. Second, the bank rationally anticipates the likelihood of stress, which influences optimal normal times behavior. Third, the bank can choose to exit the market by liquidating assets at the cost of losing its charter value. Fourth, the bank conducts maturity transformation between demandable external funding and term loans. We calibrate the model using balance sheet and income statement data for the top 1% of U.S. commercial banks and track its behavior, including the endogenous exit decision, during different stress scenarios.

Our main results are twofold: First, we show that the bank has an incentive to hold a buffer stock of capital above the regulatory requirement to reduce the likelihood of exit. Second, we contrast structural stress test results with those of a stylized non-structural stress test. Following the CLASS approach Hirtle, Kovner, Vickery, and Bhanot (2014), we show that stress tests that are based on the extrapolation of historical correlations can underestimate equity losses on impact but overestimate their reduction during a stress period. The main determinant of the difference is the balance sheet composition of the bank that is derived endogenously in the structural model but assumed to be constant in the CLASS model. Furthermore, “exogenous” exit rules based on historical data or existing capital regulation can bias stress results since these threshold might be bad predictors of the evolution of the market value of the bank (i.e., its charter value), the main determinant of the exit decision of the bank.

Related Literature. We contribute to two strands of literature: the literature on structural banking models and on microprudential stress testing. Our model is related to partial equilibrium models of banking such as Allen and Gale (2004); Boyd and Nicolo (2005); De Nicolo, Gamba, and Lucchetta (2014); Bianchi and Bigio (2014). We extend these models with a calibrated bank-specific loan demand equation to allow for quantitative results. In industrial organization there is a long tradition of estimating firm-specific demand using discrete choice models (see for example Berry, Levinsohn, and Pakes, 1995). In banking, Dick
Egan, Hortacsu, and Matvos (2015) apply this approach to the deposit market. Here we apply the approach to the loan market. Our approach is also related to the work of Elizalde and Repullo (2007) by quantifying the wedge between regulatory and economic bank capital.

Our major contribution is to the microprudential stress testing literature. To the best of our knowledge we are the first to employ a structural model for quantitative stress testing. State-of-the-art stress testing frameworks use a combination of reduced-form dependencies (Acharya, Engle, and Pierret, 2014; Covas, Rump, and Zakrajcek, 2014) and exogenous behavioral rules (Burrows, Learmonth, and McKeown, 2012; Board of Governors of the Federal Reserve System, 2013; Hirtle, Kovner, Vickery, and Bhanot, 2014; European Banking Authority, 2011, 2014) to map aggregate economic conditions to bank-specific variables. These frameworks do not identify structural parameters of the bank, which makes them prone to the Lucas critique. Therefore, these frameworks cannot conduct stress tests under counterfactual capital requirements or risk weights, as the estimated parameters are only implicit functions of these parameters. Our model replaces backward looking and exogenous rules by optimizing forward looking behavior. Thus, the policy functions that describe bank behavior become explicit functions of exogenous states and structural parameters. This offers a flexible laboratory for stress testing as a battery of counterfactual scenarios can be considered without having to extrapolate from observed conditions.

The remainder of the paper is structured as follows: Section 2 lays out the model, Section 3 presents the dynamic program of the bank, Section 4 shows the calibration to U.S. data and Section 5 provides intuition about bank behavior. Section 6 conducts stress testing exercises and compares structural with reduced-form stress test outcomes. Finally, Section 7 concludes.

2 Model

Here we present the decision problem of a competitive bank given estimated loan demand.

2.1 Loan Demand

To derive bank $i$- and sector $s$-specific loan demand we employ a discrete choice model a la Berry, Levinsohn, and Pakes (1995). Following Egan, Hortacsu, and Matvos (2015), a loan with discounted price $q_{ist}^L$ from bank $i$ in sector $s$ ($s \in \{\text{real estate, C&I, consumer}\}$) in period $t$, generates utility $\alpha_s q_{ist}^L$ for a potential borrower $\omega_j$. In addition, $\omega_j$ also receives non-interest utility $\gamma_{is} + \varepsilon_{jist}$ when borrowing from bank $i$, where $\gamma_{is}$ captures time-invariant but bank-specific factors and $\varepsilon_{jist}$ captures any borrower-specific bank preferences. We assume that $\varepsilon_{jist}$ are i.i.d. Type 1 Extreme Value.

Loans are long term contracts that mature probabilistically (e.g. Chatterjee and Eyigungor (2012)). More specifically, if an individual takes out a loan in period $t$, the loan matures next period with probability $m$; if the loan does not matures, it pays out coupon $c$.

\footnote{For a survey on state-of-the-art stress testing models see for example Foglia (2009); Borio, Drehmann, and Tsatsaronis (2012).}
make the agreed upon payments as long as the project they invest in does not fail. In case of
failure, the borrower returns only \((1 - \lambda)\) units. The failure probability is denoted by \((1 - p_t)\).

Potential borrower \(\omega_j\)'s total utility conditional on receiving a loan from bank \(i\) in sector \(s\) and period \(t\) is given by

\[
u(\varepsilon_{jist}) = \alpha_s q_ist + \gamma_{is} + \varepsilon_{jist}\]

Let \(U_{st}\) denote the expected utility of \(\omega_j\) when choosing optimally to take a loan from bank \(i\)

\[
U_{st} = \int_{-\infty}^{+\infty} \max_t \{u(\varepsilon_{jist})\} \, dG(\varepsilon)
\]

It can be shown that by properties of the Extreme Value distribution, this can be rearranged to

\[
U_{st}(q_{lst}) = \iota + \log \left( \sum_{i=0}^l \exp \left( \alpha_s q_{ist} + \gamma_{is} \right) \right),
\]

where \(\iota\) is the Euler constant and \(q_{lst}\) is the vector of loan prices. When not investing in

a risky project, potential borrower \(\omega_j\)'s utility is given by the stochastic realization of the

outside option \(\omega_{jt}\) that is distributed according to a cdf \(\Omega(\omega, z_t)\) (a function of the aggregate

shock \(z_t\)). Therefore, \(\omega_j\)'s first-stage problem is given by

\[
\max_{x \in \{0, 1\}} x U_{st}(q_{lst}) + (1 - x) \omega_{jt}
\]

where \(x\) is the choice of taking a loan \((x = 1)\) or not taking a loan \((x = 0)\). Integrating over

the mass of potential borrowers, we obtain a measure of borrowers in sector \(s\) and period \(t\)

(i.e. bank-\(i\)-specific loan demand):

\[
L_{ist}(q_{lst}, z_t) = \int_{-\infty}^{\infty} \Pi \left[ U_{st}(q_{lst}) > \omega \right] \, d\Omega(\omega, z_t). \tag{1}
\]

With the assumption of the extreme value distribution for \(\varepsilon_{jist}\), \(\sigma_{ist}(q_{lst})\) (bank \(i\)'s share) is
given by

\[
\sigma_{ist}(q_{lst}) = \frac{\exp(\alpha_s q_{ist} + \gamma_{is})}{\sum_{k=0}^l \exp(\alpha_s q_{kst} + \gamma_{ks})}. \tag{2}
\]

As a result, bank-\(i\)-specific loan demand can be written as

\[
L_{ist}(z_t, q_{lst}) = \sigma_{ist}(q_{lst}) \times M(z_t), \tag{3}
\]

where \(M(z_t)\) captures changes in the total demand for loans due to aggregate conditions.

### 2.2 Bank Environment

Banks operate in a competitive environment. The bank maximizes expected discounted dividends:

\[
\mathbb{E}_t \sum_{t=0}^{+\infty} \beta^t D_t, \tag{4}
\]
where $\beta$ is shareholders’ discount factor. At the beginning of each period, banks are matched with a random number of depositors $\delta_t$. These shocks capture liquidity variation derived from changes in the inflow of deposits and other short term funding. We assume that $\delta_t$ follows an AR(1) process. We denote its transition matrix by $G(\delta_t, \delta_{t+1})$. Deposits are assumed to be covered by deposit insurance and pay interest equal to $r^d$. Banks can invest in long term loans $\ell_t$ and risk-free securities $a_t$. We assume securities have a return equal to $r^a$. The discounted price of new long-term loans is determined endogenously and denoted by $q_{n,t}^L$. For reasons that will become clear later, the value of loans that were issued in the past, that have not matured and are in good standing is denoted by $q^L_{o,t}$. Loans can fail with probability $p_t$ that is a function of the aggregate state $z_t$. We assume that $z_t$ follows an AR(1) process. Performing bank loans generate cash flow of $[c(1 - m) + m]$. Non-performing loans pay no interest and a fraction $\lambda$ has to be written down.

Given a stock of loans, securities, deposits at hand, and the aggregate shock (that determines the loan price schedule), the bank chooses how many loans to extend, how many securities to hold, whether to pay dividends, issue equity and/or retain earnings. After the realizations of $z_t$, profits are

$$
\pi_t = [p(z_t)(1 - m)c - (1 - p(z_t))\lambda]\ell_t - r^d\delta_t + r^a a_t - I_{[i^L_t \geq q]}\phi(i^L_t) - \kappa,
$$

where $I_{[i^L_t \geq q]}$ is the indicator function and $\phi(i^L_t)$ denotes the initial cost of extending new loans. Once, profits are determined we can define bank cash-at-hand $n_t$

$$
n_t = \pi_t + [p(z_t)m + (1 - p(z_t))\lambda]\ell_t + a_t - \delta_t.
$$

After choosing the amount of new loans to extend $i^L_t$ (we allow $i^L_t$ to be negative in which case the bank is liquidating part of its portfolio), securities $a_{t+1}$ and a set of match deposits $\delta_{t+1}$, the cash flow for the bank is

$$
\mathcal{F}_t = n_t + \delta_{t+1} - a_{t+1} - [I_{[i^L_t \geq q]}q_{n,t}^L + I_{[i^L_t < q]}q_{o,t}^L]i^L_t.
$$

The law of motion for the stock of loans is

$$
\ell_{t+1} = p(z_t)(1 - m)\ell_t + i^L_t.
$$

The value of cash-at-hand $n_t$ and the choice of loans and securities determine whether the bank distributes dividends, retains earnings or issues new equity. The net-payoff to shareholders is

$$
\mathcal{D}_t = \begin{cases}
\mathcal{F}_t & \text{if } \mathcal{F}_t \geq 0 \\
\mathcal{F}_t - \nu(\mathcal{F}_t, z_t) & \text{if } \mathcal{F}_t < 0
\end{cases}
$$

where $\nu(\mathcal{F}_t, z_t)$ denote flotation costs per-unit of new funds. After loan and securities decisions have been made, we can define the present value of bank book equity capital $e_t$ as

$$
e_{t+1} \equiv p(z_t)(1 - m)\ell_t + [I_{[i^L_t \geq q]}q_{n,t}^L + I_{[i^L_t < q]}q_{o,t}^L]i^L_t + a_{t+1} - \delta_{t+1}.
$$
The bank’s portfolio choice is subject to a regulatory minimum capital constraint

\[ e_{t+1} \geq \varphi \left( w_t p(z_t)(1 - m) \ell_t + [I_{[i^L_t \geq 0]} q_{n,t}^L + I_{[i^L_t < 0]} q_{A,t}^L] + w_A a_{t+1} \right) \]  

(11)

where \( \varphi \) is the minimum regulatory common equity Tier 1 capital ratio requirement and \( w_k, k \in \{ \ell, A \} \), are regulatory risk-weights.

**Figure 1:** Timing

\[
\begin{array}{c}
\{a_t, \ell_t, z_{t-1}, \delta_t\} \quad z_t \quad \delta_{t+1} \\
\{\pi_t, n_t\} \quad \text{bank chooses} \quad \{a_{t+1}, \ell_{t+1}, z_t, \delta_{t+1}\} \quad \text{stay} \quad \text{exit} \\
\end{array}
\]

### 2.3 Loan Market

Perfect competition leads financial intermediaries to charge a price that in expectation earns zero profits.\(^3\) This implies that the price of \( i^L \) loans for \( i^L > 0 \) satisfy the following price equation:

\[
q_{n}^L(z, i^L) = \beta E_{z'}|z \left\{ p(z')[m + (1 - m)(c + q_{n}^L(z'))] + (1 - p(z'))(1 - \lambda) \right\} - \frac{\phi(i^L)}{i^L},
\]

(12)

where \( q_{n}^L(z) \) is

\[
q_{n}^L(z) = \beta E_{z'}|z \left\{ p(z')[m + (1 - m)(c + q_{n}^L(z'))] + (1 - p(z'))(1 - \lambda) \right\}
\]

(13)

The difference between \( q_{n}^L(z, i^L) \) and \( q_{n}^L(z) \) arises from the cost of extending a new loan \( \phi(i^L) \). The price function is independent of the exit probability of the bank because we assume that there are no liquidation costs (i.e. if the bank fails, other banks will bid the price of the existing loan portfolio down to the one that satisfies the expected zero profit condition).\(^4\)

Having specified the environment, we can now explain fundamental differences from the existing literature on structural banking models. While Egan, Hortacsu, and Matvos (2015) also use a logit approach to estimating demand, they focus on the deposit market while we focus on the loan market. By modeling the loan supply decision conditional on the estimated demand for a given bank’s loans we take a deep approach to determining cash flows as opposed to the reduced form approach in De Nicolo, Gamba, and Lucchetta (2014).

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\(^3\)To make this price consistent with the problem of the bank it is important to assume no bankruptcy costs. Bankruptcy costs which do not affect the value of loans, can easily be included. If bankruptcy costs which affect the value of the loan are included, then one needs to include the probability of bank failure into the pricing equations (12) and (13).

\(^4\)For a given \( p \) function and parameters \( m \) and \( c \), solving \( q_{n}^L(z) \) implies only solving a system of \( n_z \) equations with \( n_z \) unknowns (where \( n_z \) is the number of grid points for \( z \)). Once we obtain \( q_{n}^L(z) \), it is straightforward to compute \( q_{n}^L(z, i^L) \).
3 Recursive Formulation of Bank Problem

Due to the recursive nature of the bank’s problem, we can drop time subscripts. Let \( x_t = x \) and \( x_{t+1} = x' \). The value of the bank at the beginning of the period is given by

\[
V(a, \ell, \delta, z) = \max_{x \in \{0, 1\}} \{ V^{x=0}(a, \ell, \delta, z), V^{x=1}(a, \ell, \delta, z) \} \tag{14}
\]

where \( x \in \{0, 1\} \) denotes the exit decision of the bank, \( V^{x=0}(a, \ell, \delta, z) \) the value of the bank if it chooses to continue and \( V^{x=1}(a, \ell, \delta, z) \) the value in case of exit. The problem of the bank when it chooses to continue is

\[
V^{x=0}(a, \ell, \delta, z) = E_{\delta' | \delta} \left\{ \max_{\{i^L, a'\}} \mathcal{D} + \beta E_{z' | z} V(a', \ell', \delta', z') \right\}
\]

s.t.

\[
\pi = [p(z)(1-m)c - (1-p(z))\lambda] \ell - r^d\delta + r^A a - I_{[\delta \geq \phi(i^L)]} - \kappa, \tag{15}
\]

\[
n = \pi + [p(z)m + (1-p(z))] \ell + a - \delta, \tag{16}
\]

\[
\mathcal{F} = n + \delta' - a' - [I_{[\delta \geq \phi]} q_n^L(z, i^L) + I_{[\delta < \phi]} q_o^L(z)] i^L, \tag{17}
\]

\[
\mathcal{D} = \begin{cases} 
\mathcal{F} & \text{if } \mathcal{F} \geq 0 \\
\mathcal{F} - \nu(\mathcal{F}, z) & \text{if } \mathcal{F} < 0,
\end{cases} \tag{18}
\]

\[
e = p(z)(1-m)\ell + [I_{[\delta \geq \phi]} q_n^L + I_{[\delta < \phi]} q_o^L] i^L + a' - \delta', \tag{19}
\]

\[
e \geq \varphi \left( w_e (p(z)(1-m)\ell + [I_{[\delta \geq \phi]} q_n^L + I_{[\delta < \phi]} q_o^L] i^L + w_o a') \right), \tag{20}
\]

\[
\ell' = p(z)(1-m)\ell + i^L. \tag{21}
\]

The value in case of bank exit is given by

\[
V^{x=1}(a, \ell, \delta, z) = \max \{ n + p(z)q_o^L(1-m)\ell, 0 \}
\]

From the solution to this problem, we obtain the exit decision rule \( x(a, \ell, \delta, z) \), a loan decision rule \( i^L(a, \ell, \delta, z, \delta') \), a security decision rule \( a'(a, \ell, \delta, z, \delta') \), and a dividend policy \( \mathcal{D}(a, \ell, \delta, z, \delta') \).

4 Calibration

One period corresponds to a quarter. The bank in the model corresponds to an average bank in the top 1% of the asset distribution in the U.S. banking industry.\(^5\) At this stage we calibrate the model to allow for only one type of loan.\(^6\) We choose the real estate sector since it represents the larger share of the loan portfolio of banks in the top 1%. The data is taken from the Call Reports, which provides detailed information about individual U.S.

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\(^5\)The top 1% banks in the U.S. account for 74.52% of the loan market and 77.71% of the deposit market in the U.S. in 2015. There are 5,646 commercial banks in the U.S., so the top 1% represents approximately 56 banks.

\(^6\)We are currently working on a calibration with additional sectors.
commercial banks’ balance sheets and income statements.\textsuperscript{7} Our sample period is from 1984 to 2007.\textsuperscript{8} All parameters are in real terms. We deflate using total CPI index.

First, we describe the calibration of the loan demand and non-performing loans. In both cases, the relevant elasticities are pinned down directly from the data. Second, we present the calibration of the parameters that require solving the bank problem to then match a set of moments generated by the model with those from the data.

### 4.1 Loan Demand Estimation

To estimate bank $i$ loan demand curve $L^d_{ist}(z_t, q^L_{it})$, defined in Equation (3), we proceed as follows: first, we estimate market shares for the top 1% banks in the U.S. as predicted by the discrete choice model (Equation (2)). Second, we estimate the evolution of aggregate loan demand (Equation (1)) by aggregating the bank level data.

#### 4.1.1 Market Share Estimation

We estimate Equation (2) using interest income (from which we derive the implicit interest rate and the discounted price of the loan) and loan volume data for the top 1% U.S. banks. We compute each bank’s market share, $\sigma_{it}$, as loans by bank $i$ relative to total credit where total credit is the sum over all loans of all incumbent banks in the sample (i.e. not just the top 1%). Following Egan, Hortacsu, and Matvos (2015), we allow the quality of the bank to vary over time. Let $\zeta_{it}$ denote the time-varying quality component. Then total bank quality is given by $\delta_i + \zeta_{it}$. We treat credit from those banks outside the top 1% as an unobservable outside good, which we index by 0. We normalize non-interest utility of the outside good to zero, $\delta_0 + \zeta_{0t} = 0$. Dividing $\sigma_{it}$ in Equation (2) by $\sigma_{0t}$, taking logs and plugging in empirical counterparts, we get

$$
\log \sigma_{it} = \alpha q^L_{it} + \delta_i + \varpi_t + \zeta_{it},
$$

where $q^L_{it}$ denotes the inverse of the loan credit rate, $\delta_i$ is a firm-fixed effect, $\varpi_t \equiv \log(\sigma_{0t})-\alpha q^L_{0t}$ is a time-fixed effect. The time fixed-effects absorb any aggregate variation (including the outside good/credit by other banks) in market shares and ensures we capture the price elasticity correctly. This equation is identical to the equation estimated in Egan, Hortacsu, and Matvos (2015). To identify the demand curve and circumvent simultaneity bias, we use data on the cost of federal funds at the bank level as a supply shifter (i.e. we follow a standard instrumental variables approach). Table 1(a) shows the estimation results. The estimates parameters are used to calibrate Equations (1) and (2). With the estimate of $\alpha$ at hand, we set the average year and bank fixed (that we denote by $\mu_\sigma$) to match the average loan market share of the top 1% bank.

\textsuperscript{7}See Corbae and D’Erasmo (2014) for a detailed description of the data.

\textsuperscript{8}Data limitations prevents us from using data prior to 1984 and we decide to exclude the period since last financial crisis from the calibration exercise.
Table 1: Estimation Results: share and aggregate loan regression

<table>
<thead>
<tr>
<th>(a) Loan Market Share $\log \sigma_{ist}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{it}$</td>
<td>9.8767</td>
</tr>
<tr>
<td>$p-$value</td>
<td>0.00</td>
</tr>
<tr>
<td>obs.</td>
<td>2183</td>
</tr>
<tr>
<td>Period</td>
<td>1984 - 2007</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.3699</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) Aggregate Credit $M(z_t)$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(z_t)$</td>
<td>3.1576</td>
</tr>
<tr>
<td>$p-$value</td>
<td>0.000</td>
</tr>
<tr>
<td>obs.</td>
<td>31</td>
</tr>
<tr>
<td>Period</td>
<td>1984 - 2007</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.1169</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(c) Default Prob. $(1 - p_t)$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(z_t)$</td>
<td>-0.0754837</td>
</tr>
<tr>
<td>$p-$value</td>
<td>0.014</td>
</tr>
<tr>
<td>obs.</td>
<td>1,914</td>
</tr>
<tr>
<td>Period</td>
<td>1984 - 2007</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0082</td>
</tr>
</tbody>
</table>

**Aggregate Level Estimation.** Unlike Egan, Hortacsu, and Matvos (2015), we do not take the mass of borrowers to be constant, but let aggregate loan demand respond to changes in the aggregate conditions. We estimate (1) by

$$\log(M(z_t)) = (\eta_0 + \eta_1 \log(z_t)),$$

where $\log(M(z_t))$ represents aggregate HP-filtered log-loan demand and $\log(z_t)$ denotes log, HP-filtered log-real GDP. Since we will work with a normalization in our model (average $z = 1$), the estimated constant $\eta_0$ will be calibrated match average credit over GDP. Table 1(b) shows estimation results.

4.2 Non-Performing Loans Estimation

We estimate the elasticity of non-performing loans share, $[1 - p(z_t)]$, to changes in aggregate conditions by running the following panel regression for the top 1% banks in the U.S.

$$(1 - p_{it}) = \gamma_1 \log z_t + \gamma_i + \epsilon_{it},$$

(23)

where $(1 - p_{it})$ is measured as non-performing loans as a fraction of total loans of bank $i$ and quarter $t$ and $\log(z_t)$ is HP-filtered log real GDP. We account for time-invariant heterogeneity between banking groups by adding bank fixed effects, $\gamma_i$. Table 1(c) shows the estimation results.
4.3 Aggregate Shock Calibration

We relate the aggregate shock with the evolution of real GDP in the U.S. We detrend real log-GDP using the H-P filter and estimate the following equation:

$$\log(z_t) = \rho z \log(z_{t-1}) + u^z_t,$$

with $u_t \sim N(0, \sigma_{u^z})$. Once parameters $\rho$ and $\sigma_{u^z}$ are estimated, we discretized the process using the Tauchen (1986) method. We set the number of grid points to five, that is $z_t \in \mathbb{Z} = \{z_1, z_2, z_3, z_4, z_5\}$. We choose the grid in order to capture the infrequent crisis states we observe in the data and the stress scenario we aim to capture in our main experiment. In particular, we choose $z_4$ to match the mean of the process (i.e. $z_4 = 1$), select $z_3$ and $z_5$ so they are at 1.5 standard deviations from $z_5$, set the value of $z_2$ to be at 2.89 standard deviations from the mean to be consistent with the GDP levels observed during the 1982 crisis and the last financial crisis (years 2008/2009) and set $z_1$ to be at 5 standard deviations from the mean to be consistent with the severe stress scenario proposed by U.S. regulators. This large negative event has a very low probability of occurrence and the probability of transitioning into $z_1$ from $z_4$ or $z_5$ is zero (as determined by the Tauchen procedure).

4.4 Deposit Process Calibration

The idiosyncratic external funding shock process $\delta_{it}$ is calibrated using our panel of commercial banks in the U.S. In particular, after controlling for firm and year fixed effects as well as a time trend, we estimate the following autoregressive process for log-short term funds (deposits plus short term liabilities) for bank $i$ in period $t$:

$$\log(\delta_{it}) = (1 - \rho_d)k_0 + \rho_d \log(\delta_{it-1}) + k_1 t + k_2 t^2 + k_3 t + \gamma_i + u_{it},$$

where $t$ denotes a time trend, $k_{3,t}$ are year fixed effects, $\gamma_i$ are bank fixed effects, and $u_{it}$ is iid and distributed $N(0, \sigma^2_{u^d})$. Since this is a dynamic model we use the method proposed by Arellano and Bond (2004). To keep the state space workable, we apply the method proposed by Tauchen (1986) to obtain a finite state Markov representation $G^f(\delta', \delta)$ to the autoregressive process in (24). We work with a normalization in the model, the mean $k_0$ in (24) is not directly relevant. Instead, we leave the mean of the finite state Markov process, denoted $\mu_d$, as one of the parameters to be calibrated to match a target from the data (the most informative moment for this parameter is the average loan to deposit ratio).

4.5 Remaining Parameter Calibration

The parameters $\{\lambda, r^a, r^d, m\}$ are chosen to match the average charge off rate for top 1% banks, the return on securities (net of costs), the cost of deposits, and the average maturity of loans (see Black and Rosenb (2016)), respectively. We can pin down these parameters without the need to solve the model. We let the equity issuance cost function be $\nu(F, z) = (\nu_1 F) (z/z)^{\nu_3}$ (a linear function increasing in $z$) and calibrate the remaining parameters by minimizing the distance between a set of model simulated moments and their data counterpart. We are left with 11 parameters to calibrate $\{c, \zeta_0, \mu_\sigma, \eta_0, \beta, \kappa, \sigma_0, c_1, \nu_1, \nu_3, \mu_d\}$. 
Table 2 presents the parameters and the targets.

**Table 2: Model Parameters and Targets**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>z-process $\rho_z$</td>
<td>0.868</td>
<td>Real GDP</td>
</tr>
<tr>
<td>z-process $\sigma_{e,z}$</td>
<td>0.006</td>
<td>Real GDP</td>
</tr>
<tr>
<td>deposit process $\rho_d$</td>
<td>0.964</td>
<td>Evolution Short Term Liabilities</td>
</tr>
<tr>
<td>deposit process $\sigma_{e,d}$</td>
<td>0.084</td>
<td>Evolution Short Term Liabilities</td>
</tr>
<tr>
<td>Non-performing loans $\zeta_1$</td>
<td>-0.075</td>
<td>elasticity non-performing loans to GDP</td>
</tr>
<tr>
<td>Market Share elasticity $\alpha$</td>
<td>9.877</td>
<td>elasticity market share to loan price</td>
</tr>
<tr>
<td>Aggregate Loan Demand $\eta_1$</td>
<td>3.158</td>
<td>Elasticity Aggregate Loan Demand to GDP</td>
</tr>
<tr>
<td>loss given default $\lambda$</td>
<td>0.359</td>
<td>Avg. Charge off Rate</td>
</tr>
<tr>
<td>return securities $r^a$</td>
<td>0.030</td>
<td>Return on securities</td>
</tr>
<tr>
<td>deposit interest rate $r^d$</td>
<td>0.007</td>
<td>Cost of Funds</td>
</tr>
<tr>
<td>average maturity $m$</td>
<td>1/6.34</td>
<td>Avg. Maturity Loans</td>
</tr>
<tr>
<td>Capital Requirement $\varphi$</td>
<td>0.04</td>
<td>Regulation</td>
</tr>
<tr>
<td>Risk-weights $w_f$</td>
<td>1.00</td>
<td>Regulation</td>
</tr>
<tr>
<td>Risk-weights $w_a$</td>
<td>0.00</td>
<td>Regulation</td>
</tr>
<tr>
<td>coupon $c$</td>
<td>0.018</td>
<td>Avg. Interest Margin</td>
</tr>
<tr>
<td>Non-performing loans $\zeta_0$</td>
<td>0.020</td>
<td>Avg. non-performing loans</td>
</tr>
<tr>
<td>Market Share constant $\mu_s$</td>
<td>93750</td>
<td>Avg. Loan Market Share Top 1%</td>
</tr>
<tr>
<td>Aggregate Loan Deman $\eta_0$</td>
<td>-8.830</td>
<td>Loan to GDP Ratio</td>
</tr>
<tr>
<td>discount factor $\beta$</td>
<td>0.990</td>
<td>capital ratio (risk-weighted)</td>
</tr>
<tr>
<td>Fixed cost $\kappa$</td>
<td>0.0002</td>
<td>fixed cost to loans ratio</td>
</tr>
<tr>
<td>Cost new loans $c_0$</td>
<td>0.003</td>
<td>Avg. net cost</td>
</tr>
<tr>
<td>Cost new loans $c_1$</td>
<td>0.000</td>
<td>Loans to Asset Ratio</td>
</tr>
<tr>
<td>Equity issuance cost $\nu_1$</td>
<td>0.050</td>
<td>Equity Issuance over assets</td>
</tr>
<tr>
<td>Equity issuance cost $\nu_3$</td>
<td>100</td>
<td>Frequency of Equity Issuance</td>
</tr>
<tr>
<td>Deposit Process $\mu_d$</td>
<td>0.023</td>
<td>Loan to Deposit ratio</td>
</tr>
</tbody>
</table>

Note: Parameters above the line are set “off-line” (i.e., without the need to solve the model). Parameters below the line are chosen by minimizing the distance between the simulated model moments and the corresponding data moments.

Table 3 presents the data and model moments.
Table 3: Targets and Model Moments

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Non-Performing Loans</td>
<td>2.17</td>
<td>2.17</td>
</tr>
<tr>
<td>Loss given default</td>
<td>35.90</td>
<td>35.90</td>
</tr>
<tr>
<td>Return on securities (net of costs)</td>
<td>3.01</td>
<td>3.01</td>
</tr>
<tr>
<td>Cost of funds</td>
<td>0.73</td>
<td>0.73</td>
</tr>
<tr>
<td>Avg Maturity (quarters)</td>
<td>6.34</td>
<td>6.34</td>
</tr>
<tr>
<td>Avg Interest Margin</td>
<td>4.41</td>
<td>4.33</td>
</tr>
<tr>
<td>Avg. Market share top 1% banks</td>
<td>1.72</td>
<td>2.59</td>
</tr>
<tr>
<td>Loans to GDP Ratio</td>
<td>8.03</td>
<td>8.01</td>
</tr>
<tr>
<td>Capital Ratio (risk-weighted)</td>
<td>8.76</td>
<td>10.65</td>
</tr>
<tr>
<td>Fixed cost to loans ratio</td>
<td>0.83</td>
<td>3.51</td>
</tr>
<tr>
<td>Avg net cost top 1% banks</td>
<td>0.67</td>
<td>0.24</td>
</tr>
<tr>
<td>Loans to Asset Ratio</td>
<td>73.93</td>
<td>73.61</td>
</tr>
<tr>
<td>Loan to Deposit ratio</td>
<td>80.97</td>
<td>80.26</td>
</tr>
<tr>
<td>Frequency of Equity Issuance</td>
<td>3.65</td>
<td>13.65</td>
</tr>
<tr>
<td>Equity Issuance over assets</td>
<td>0.18</td>
<td>1.59</td>
</tr>
<tr>
<td>Dividends over assets</td>
<td>0.63</td>
<td>1.69</td>
</tr>
<tr>
<td>Frequency of Dividends Payments</td>
<td>95.41</td>
<td>55.16</td>
</tr>
<tr>
<td>Exit Probability</td>
<td>0.32</td>
<td>0.12</td>
</tr>
</tbody>
</table>

5 Bank Behavior Analysis

Before we move to stress testing, it is worthwhile to take a close look at the optimal choices of the bank. First, we describe the exit decision rule since it is one of the main determinants of the balance sheet composition. Second, we present the loan, securities and dividend policies together with the implied capital ratios. Finally, we present the analysis of a failure event by using our panel of simulated data.

In order to understand the exit decision rule, it is instructive to look at the continuation and exit values for the bank (i.e., $V^{x=0}$ and $V^{x=1}$, respectively). Figure 2 presents these value functions as a function of securities (top panels, evaluated at average loans) and as a function of loans (bottom panels, evaluated at average securities) for different values of $\delta \in \{\delta_L, \delta_M, \delta_H\}$. Average securities and loans correspond to the average values observed during the simulation of the model.
Figure 2: Value Functions $V^{x=0}(a, \ell, \delta, z)$ and $V^{x=1}(a, \ell, \delta, z)$

Notes: $V^{x=0}(a, \ell, \delta, z)$ and $V^{x=1}(a, \ell, \delta, z)$ as a function of securities (top panels, evaluated at average loans) and as a function of loans (bottom panels, evaluated at average securities) for different values of $\delta \in \{\delta_L, \delta_M, \delta_H\}$

Figure 2 shows that both, the continuation and exit value of the bank are increasing functions of securities and loans. Interestingly, value functions are a decreasing function of $\delta$. There are two effects at play. On one hand, the value of $\delta$ determines the level of low cost funds that the bank has access to, so the investment possibilities of a bank expand with $\delta$, so one would expect the continuation value of the bank to be an increasing function of $\delta$. On the other hand, since the process for $\delta$ is mean reverting, high levels of $\delta$ carry a downside risk. If a bank that suffers a reduction in its level of short term liabilities (similar to a roll-over crisis episode) will be forced to liquidate some of its assets or to inject new equity in order to cover the outflow. We observe in Figure 2 that the second effect dominates and also that if $\delta$ is sufficiently low ($\delta = \delta_L$), the continuation value of the bank is always below the exit value generating a non-linear response of exit to $\delta$.

Figure 2 provides relevant information regarding the exit decision. We note that for $\delta \geq \delta_M$ whenever the exit value of the bank is strictly positive, it is always below the continuation value. That is, exit is the dominating strategy whenever limited liability does not bind and $\delta$ is high enough. This is not a theorem but a quantitative result that arises at this parameterization of the model. It is evident from Panel (iii) (that shows the value
function for $z = \mu_z$ as a function of $a$) that the continuation value is concave for sufficiently low values of assets. Recall that banks in this environment are risk-neutral. However, being close to the minimum capital required induces this curvature and potentially can generate bank exit even when limited liability does not bind.

Figure 3 presents the exit decision rule $x(a, \ell, \delta, z) \in \{0, 1\}$. The left panels ((i), (iii) and (v)) present the exit decision rule as a function of loans and securities for different values of $z$, evaluated at average $\delta = \delta_M$. The right panels ((ii), (iv) and (vi)) present the exit decision rule as a function of loans and securities for different values of $\delta$, evaluated at average $z = z_M = \mu_z$. The dark color (green) represents the region where the bank chooses to continue $x(a, \ell, \delta, z) = 0$ and the light color (yellow) represents the region where the bank chooses to exit $x(a, \ell, \delta, z) = 1$.

**Figure 3:** Exit Decision Rule $x(a, \ell, \delta, z) \in \{0, 1\}$

Notes: $x(a, \ell, \delta, z)$ as a function of securities and loans for different values of $z$ (left panels, evaluated at average $\delta$) and for different values of $\delta$ (right panels, evaluated at average $z$).

In our model, the exit choice plays two important roles: first, the possibility of exit and the corresponding loss of the charter value induces the bank to hold a precautionary equity cushion. This affects the leverage ratio and therefore the stress performance of the bank. Second, the optimal exit choice of the bank induces an endogenous hurdle rate to stress testing. Panels (i), (iii) and (v) make clear that the region where the bank chooses to exit
shrinks as $z$ increases. A lower default frequency and higher continuation values are behind this decision. Of course, the choices of securities and loans are endogenous and, while we present this figure for a relevant range of $a$ and $\ell$, the figures do not say anything about the likelihood of exit. For that reason, at the end of this section we present an exit event analysis and show that the bank chooses to exit if its charter value is sufficiently low, which - for our calibrated bank - occurs on average during bad times.

Figure 4 presents the optimal value of securities $a'(\delta', a, \ell, \delta, z)$.

Figure 4: Securities Decision Rule $a'(\delta', a, \ell, \delta, z)$

Notes: $a'(\delta', a, \ell, \delta, z)$ as a function of securities (Panels (i)–(iii)) and as a function of loans (Panels (iv)–(vi)) for different values of $z$ and $\delta'$ (evaluated at average $\delta = \delta_M$).

This figure shows that the optimal level of future securities $a'$ is decreasing in loans (bottom panels), increasing in $\delta'$, and, for most part, increasing in initial securities $a$. Since issuing equity is costly, when surprised by a reduction in $\delta$ (i.e., moves from $\delta = \delta_M$ to $\delta' = \delta_L$) the bank exits, so it liquidates all of its current securities. On the other hand, when facing a sudden inflow of deposits, (i.e., moves from $\delta = \delta_M$ to $\delta' = \delta_H$) the bank chooses to accumulate most of these new funds for the future. This is consistent with the precautionary motive behavior that is evident from the observed balance sheet composition and capital ratios in the data.

Figure 5 presents the optimal value of net cash flow to shareholders $F(\delta', a, \ell, \delta, z)$. 

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Figure 5: Net cash flow to shareholders $F(\delta', a, \ell, \delta, z)$

Notes: $F(\delta', a, \ell, \delta, z)$ as a function of securities (Panels (i)−(iii)) and as a function of loans (Panels (iv)−(vi)) for different values of $z$ and $\delta'$ (evaluated at average $\delta = \delta_M$).

The net cash flow policy rule $F$ shows that if loans, securities and the inflow of short term liabilities are large enough the bank chooses to pay dividends $F > 0$. For intermediate values, the bank retains earnings in full and sets $F = 0$. When assets are sufficiently low, the bank is willing to issue equity $F < 0$ in order to increase its holdings of securities and its stock of loans. Moreover, when the bank faces an outflow of deposits ($\delta_L$) equity issuance or a reduction in dividend payments is more likely relative to cases where $\delta$ does not decrease ($\delta_M$ or $\delta_H$).

Figure 6 presents the optimal level of capital $e(\delta', a, \ell, \delta, z)$ (Panels (ii), (iv), (vi)) and the risk-weighted capital ratios $e/\left(p(z)(1 - m)\ell + [I_{[\ell \geq 0]}q^n_L + I_{[\ell < 0]}q^n_0]i^L\right)$ (Panels (i), (iii), (iv)).
Figure 6: Equity $e$ and Risk-weighted Capital Ratios $e/rwa$

Notes: Risk-weighted assets ($rwa$) is equal to $(p(z)(1-m)\ell + [I_{[L^L \geq 0]}q^L_M + I_{[L^L < 0]}q^L_0]q^L_L])$. Both risk-weighted capital ratios and equity are presented as functions of securities for different values of $z$ and $\delta'$ (evaluated at average $\delta = \delta_M$ and $\ell = \ell_M$).

Figure 4 made evident that future securities are increasing in current securities $a$. Figure 6 shows that this positive relation derives in equity levels that are increasing in $a$. The pattern observe in equity levels is maintained when looking at risk-weighted capital ratios as a function of securities $a$. For values of $\delta \geq \delta_M$, equity levels and risk-weighted capital ratios are decreasing in $\delta$. The higher the short-term borrowings of the bank the lower its capital since, as the funds increase, the bank chooses to distribute some of it as dividends.

Panels Panels $(i), (iii), (iv)$ show that capital ratios are increasing in $a$. However, risk-weighted capital ratios are decreasing in $\delta'$. The intuition is simple. The increase in equity is more than compensated by an increase in the stock of loans. When facing with a liquidity shortage (a reduction in $\delta$) banks are able to adjust rapidly its level of securities but the stock of loans remains at elevated levels. On the the other hand, when a sudden inflow of short term liabilities realizes, banks allocate funds to both type of assets but the increase is larger relative to that of securities.
5.1 Exit Event Analysis

The paper aims to provide a tool to analyze stress scenarios. But before we present our main experiment, it is instructive to analyze how a typical bank failure looks like in our model. For that reason, we study the model’s dynamics using a simulated panel of banks, large enough to capture the long-run properties of the model. This sample contains failures in 4% of the simulated banks (consistent with low failure probability of large banks in the U.S.). Thus, the model produces endogenous exit with a low failure probability in equilibrium.

Figure 7 shows an event analysis based on the simulated panel. In particular, from all the observed failures, we select the bank with median value of risk-weighted capital ratio at the start of the event. The plots show 3-year event windows (12 quarters) where the last period in each panel corresponds to the year of failure. Panel (i) shows the evolution of the aggregate state ($z$). Panel (ii) shows $\ell$, $a$, and $\delta$. Panel (iii) shows the evolution of the capital ratio as a fraction of risk-weighted assets ($e/rwa$) and as a fraction of total assets ($e/T.A$). Panel (iv) shows the evolution of dividends and profits as a fraction of total assets ($D/T.A$ and $\pi/T.A$, respectively). Panel (v) compares the loan return with the security return. Finally, Panel (vi) presents the equity value of the bank when it continues $v^x=0(\ell, a, \delta, z)$ and when it chooses to exit $v^x=1(\ell, a, \delta, z)$. 

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Panel (i): $z$

Panel (ii): $\ell$, $a$, and $\delta$

Panel (iii): Capital Ratios

Panel (iv): Profits and Dividends

Panel (v): Asset returns

Panel (vi): Equity Value

Notes: Exit event in last period. The figure presents the failure event for the bank with median risk-weighted capital at the start of the event window.

Panel (i) and (ii) show that bank failure happens when a sequence of low aggregate shocks is combined with a reduction in the flow of deposits. The bad state of the economy induces banks to cut on loans (the stock of loans decreases by 29.3 percent) and shift to safe securities (the ratio of securities to total assets increases from 27.36 percent to 45.01 percent during the event window) in order to prevent losses. Panel (iv) shows that profits over assets $\pi/TA$, while negative from start to end, do not decline sharply due the change in the composition of assets as well as the increase in loan interest rates (Panel (v)). The expected return on an additional unit of loans is higher than the expected return on assets but loans become sufficiently risky that the bank chooses to build a buffer stock against future losses. Absent profitable investment in loans, the bank also chooses to increase the distribution of dividends as the failure period approaches. Even though security holdings increase, the increase in dividend distribution combined with negative operating profits result in a notable decline in capital ratios. Panel (iii) shows that the risk-weighted capital ratio $e/rwa$ declines 34.9 percent and the leverage ratio (i.e., equity to total assets $e/TA$) declines by more than 40 percent to end at 4.34 percent. The decline in capital ratios is accompanied by a decrease in the continuation value of the bank $V^{x=0}$ that slowly approaches the exit value $V^{x=1}$ until it crosses it and the bank exits. Total assets decline 2.51 percent from the start of the exit
6 Structural Stress Tests

6.1 Benchmark Model vs CLASS Model

In this section, we perform a stress test using our quantitative model and contrast the results with a similar experiment when performed using the CLASS model methodology presented in Hirtle, Kovner, Vickery, and Bhanot (2014).

To set the stress scenario, we follow the guidelines presented in the Supervisory Stress Test Methodology and Results by the Federal Reserve Board in June 2016. We focus on the “Severely Adverse” scenario. According to the guidelines, in this stress scenario, the level of U.S. real GDP begins to decline in the first quarter of 2016 (the start of the stress window) and reaches a trough that is 6.25 percent below the pre-recession peak (similar to $z = z_1$). The crisis continues for about two years until output slowly goes back to trend. We feed our panel of simulated banks with a path for $z$ presented in Panel (i) of Figure 8. We let the value of $\delta$ evolve according to its stochastic process. We present the results from the average behavior (i.e., we take the average across banks of each variable where choices across banks in the structural model only differ due to the idiosyncratic realizations of $\delta$).

We compare the evolution of variables as predicted by the structural model with those derived from the CLASS model. In short, the CLASS model estimates a set of equations that determine the evolution of the net interest margin ($nim$), the net charge-off rate ($nco$) and net operating costs ($cost$) in order to predict the evolution of profits $\pi$. With the estimates of these equations at hand (and thus profits), it imposes a set of assumptions on dividend payments and the asset composition to then pin down the evolution of equity and each of the components of the balance sheet. More specifically, recall that profits are equal to

$$\pi = [p(z)(1-m)c - (1-p(z))\lambda]\ell - r^d \delta + r^A a - \kappa - \phi(i^L).$$

We can now define the key income ratios to be estimated as in the CLASS model approach:

$$nim = p(z)(1-m)c\ell - r^d \delta + r^A a$$
$$cost = \phi(i^L) + \kappa$$
$$nco = (1-p(z))\lambda\ell$$

It is straightforward to see that $\pi = nim - cost - nco$. The CLASS model assumes that these income ratios follow an AR(1) process and estimate their evolution controlling for bank-fixed effects and aggregate conditions using the following specification

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 z_t + \epsilon_t, \ y_t \in \{nim_t, nco_t, cost_t\}. \quad (25)$$

We use our simulated panel of banks and estimate equation (25) for $y_t \in \{nim_t, nco_t, cost_t\}$.

---

9 All documentation can be found in https://www.federalreserve.gov/bankinforeg/stress-tests/2016-Preface.htm

10 Note that fixing a path for $z$ does not imply that the bank has perfect foresight about this path.
We then use the estimated coefficients \( \{ \hat{\beta}_y \}_{i=0}^2 \) to generate the CLASS model stress projections \( \hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1 y_{t-1} + \hat{\beta}_2 z_t \) for \( y_t \in \{ \text{nim}_t, \text{nco}_t, \text{cost}_t \} \) as in Hirtle, Kovner, Vickery, and Bhanot (2014). From these estimates we can also derive profits for the CLASS model, that is \( \pi_t^C = \hat{\text{nim}}_t - \hat{\text{co}}_t - \hat{\text{nco}}_t \). Table 4 presents the estimated coefficients.

**Table 4: CLASS Model Income Ratios Estimation**

<table>
<thead>
<tr>
<th>Dep. Variable ( y_{it} )</th>
<th>( \text{nim} )</th>
<th>( \text{cost} )</th>
<th>( \text{nco} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.0016</td>
<td>0.0007</td>
<td>0.0342</td>
</tr>
<tr>
<td>( y_{it-1} )</td>
<td>0.9806</td>
<td>0.9833</td>
<td>0.0007</td>
</tr>
<tr>
<td>( z_t )</td>
<td>0.0002</td>
<td>-0.0001</td>
<td>-0.0027</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.968</td>
<td>0.966</td>
<td>0.999</td>
</tr>
<tr>
<td>obs.</td>
<td>318,097</td>
<td>318,097</td>
<td>318,097</td>
</tr>
</tbody>
</table>

In addition, the CLASS model assumes that dividends follow:

\[
D_t^C = \max\{0, 0.9D_t^C + (1 - 0.9)(D_t^* - D_{t-1}^C)\},
\]

where \( D_t^* = 0.45 \pi_t \) is the target level of dividends. Note that dividends are restricted to be non-negative. As in a version of our structural model where dividends are restricted to be non-negative, and with some abuse of notation, equity evolves according to the following equation

\[
e_{t+1} = e_t + \pi_t - D_t.
\]

A key issue in the CLASS model is how to determine the balance sheet composition since, given the evolution of liabilities \( \delta_t \) and equity \( e_t \), the level of total assets is well defined but not how much it corresponds to loans (or risky assets) and securities. As in Hirtle, Kovner, Vickery, and Bhanot (2014), we impose that in the CLASS model the composition of assets stays fixed at its historical average during the stress scenario (i.e., total assets evolves endogenously but the fraction allocated to loans and securities remains the same). This is not consistent with the data (and our structural model) that shows that balance sheet composition varies significantly with economic conditions. Furthermore, note that the CLASS model is silent about the charter value of the bank, the main determinant of bank failure. We follow the guidelines of the CLASS model and impose a closure rule. The closure rule assumes that, consistent with regulation, a bank is closed if their Tier 1 capital ratio falls below 4 percent of risk-weighted average.

We apply the stress test scenario to both models and obtain the results presented in Figure 8. Panel (i) presents the stress scenario (i.e., the evolution of \( z \) that we impose to represent the stress scenario). All other panels present the evolution of key variables for our Structural model and the Class model. More specifically, Panel (ii) to (vi) present the evolution of dividends to assets, the loan supply, the ratio of profits to assets, equity to assets,
Figure 8: Stress Test: Class vs Structural Model

Notes: The figure presents the average across all banks from the panel of simulated banks. Structural refers to our benchmark model. Class refers to the CLASS model.

We observe that on impact, profits decline in both models (Panel (iv)). However, the decline is much more persistent in the CLASS model than in our model. The reason is that while banks in the structural model adjust their portfolio composition (Panels (iii) and (vi)), banks according to the CLASS model keep the same asset composition inducing the bank to face larger losses. Risk-weighted capital ratios decline sharply in both models (Panel (v)) mostly driven by the decline in profits, even though banks in both models decide to reduce dividend payments. The decline in equity ratios in the structural model are mitigated by the increase in security holdings that also reduces the impact of bad economic times on loan losses.

The averages presented in Figure 8 hide the rich heterogeneity that arises due to idiosyncratic shocks to the banks in our simulated panel. Figure 9 presents the distribution of changes in capital ratios (e/TA) (relative to the initial period) for period (quarter) iv, viii, xii and xvi.
Figure 9: Stress Test: Distribution of Capital Ratios Changes over time

Notes: The figure presents the distribution of capital changes across all banks from the panel of simulated banks. Structural refers to our benchmark model. Class refers to the CLASS model.

Panel (i) of Figure 9 shows that the CLASS model distribution of capital changes is centered at a much lower value than that for the structural model. A small fraction of banks in both models observe capital ratio declines of more than 60 percent. The difference across models increases as we move to quarters viii, xii and xvi. At the end of the stress event, more than 80 percent of the banks in the CLASS model suffer a decline in capital to assets of 78 percent while the majority of banks in the structural model stay with changes in capital ratios below to 70 percent.

A key factor shaping the behavior of the banks during the stress scenario is failure. Recall that in the structural model exit is endogenous while in the CLASS model there is closure rule in which banks are closed if their capital ratio goes below 4 percent of risk weighted assets. This derives in an exit rate equal to 10 percent for the structural model and 9 percent for the CLASS model. However, capital ratios (equity over total assets and equity over risk-weighted assets) of banks that exit differs considerably across models. Figure 10 presents the comparison of the distribution of capital ratios of banks that exit (at the moment of exit) across models.
Figure 10: Stress Test: Distribution of Capital Ratios of Failing Banks

Notes: The figure presents the distribution of capital ratios across banks that exit (at the moment of exit) from the panel of simulated banks. Structural refers to our benchmark model. Class refers to the CLASS model.

Figure 10 shows that capital ratios for banks that exit are much higher in the Structural model than in the CLASS model. This makes evident one of the main shortcomings of the CLASS model that is its inability to capture changes in the charter value of the bank and asset composition that derive in bank exit. While the average risk-weighted capital ratio of a bank that exits in the Structural model is 18.24 percent, the average risk-weighted capital ratio for a bank that exits in the CLASS model is 4.58 percent. The timing of exit is very different. While it takes on average 12 quarters for a bank to fail during the stress scenario for a bank run according to the CLASS model, it takes only 7 quarters, on average, for a bank to exit according to the structural model.

6.2 Stress Test and Capital Requirements

In this section, we evaluate the performance of the banks when minimum capital requirements are higher. We explore a minimum risk-weighted capital ratio equal to 8.5 percent (i.e., $\varphi = 0.085$), consistent with Basel III and the Dodd-Frank Act that require an increase in the minimum capital required to all institutions to 6 percent and a 2.5 percent additional for large banks.

Figure 11 presents a comparison between our benchmark model and the one with higher...
capital requirements.

**Figure 11: Stress Test: Higher Capital Requirements**

Notes: The figure presents the average across all banks from the panel of simulated banks.

Panel (iv) shows that profits decrease more during the stress test window in the benchmark model than in the model with higher capital requirements. A higher minimum capital requirement induces the bank to substitute securities for loans (see Panels (iii) and (vi)) affecting profitability and importantly capital ratios (Panel (v)). Panel (iii) also shows that both models generate a similar response in terms of total credit.

Panels (i) and (ii) of Figure 12 present a comparison of the continuation value of the bank $V_{x=0}$ across models and the difference between the continuation value and the exit value $V_{x=0} - V_{x=1}$, respectively.
Notes: The figure presents the average across all banks from the panel of simulated banks.

Higher capital requirements result in a higher continuation value during stress times since it forces the bank to shift the composition of its assets towards riskless securities (Panel (i)). However, since there are no liquidation costs associated with safe securities, this shift also results in a reduction in the difference between continuing and exiting from start to end (Panel (ii)).

6.3 Stress Test and Countercyclical Capital Requirements

Introduced by the last Basel Accord and implemented by the Federal Reserve Board very recently, countercyclical capital buffers are a new policy tool aimed at reducing the risk of financial distress.\footnote{A countercyclical capital buffer is one of the requirements of the Dodd-Frank Act. On September 8, 2016 the Federal Reserve Board released a policy statement detailing the framework it will follow in setting the Countercyclical Capital Buffer (CCyB).} We evaluate such a policy under stress conditions and compare the results against our benchmark model. To implement this experiment, we assume that the capital requirement constraint takes the following form:

\[
e \geq \varphi(z) \left( w \ell (1 - m) \ell + [I_{[i^L \geq 0]} q_{\ell}^L + I_{[i^L < 0]} q_{\ell}^L] i^L + w a' \right),
\]

Figure 12: Stress Test: Higher Capital Requirements and Charter Value
with \( \varphi(z) \) defined as

\[
\varphi(z) = \frac{z_5 - z}{z_5 - z_1}\varphi + (1 - \frac{z_5 - z}{z_5 - z_1})(\varphi + 0.045)
\]  
(27)

that is a linear function of \( z \) with minimum value at the current level of capital requirements \( \varphi \) when \( z = z_1 \) (i.e., during a crisis) and maximum value at \( (\varphi + 0.045) \) when \( z = z_5 \) (i.e., during a boom). The 0.045 comes from the additional 2 percent risk-weighted capital that is required to large institutions and 2.5 percent that is the countercyclical buffer.

**Figure 13:** Stress Test: Countercyclical Capital Requirements

**Notes:** The figure presents the average across all banks from the panel of simulated banks.

### 6.4 Stress Test and Liquidity Requirements

We evaluate how liquidity requirements affect the prediction of the model during the stress scenario. In order to compute this experiment, we assume that the bank faces a liquidity constraint of the following form

\[
a' \geq \max\{0, -(n(i^L, a', \delta', z_L' + \delta_L))\},
\]  
(28)
that is the bank has to hold sufficient safe assets to cover a fraction $\vartheta$ of the outflows in the worst case scenario (i.e., given choices $i^L$ and $a'$ when $z' = z_L$ and $\delta'' = \delta_L$).

7 Conclusion

We propose a structural banking model for microprudential stress testing. We derive bank behavior during stress as the endogenous outcome of a bank’s dynamic optimization problem, including an exit decision. In contrast to reduced-form frameworks, the structural model identifies the effect of regulatory parameters on bank behavior. This allows us to gauge bank’s capital adequacy during stress scenarios that do not only feature counterfactual macro dynamics but also counterfactual regulatory parameters, like risk weights and capital requirements.
References


